Cal Poly Nano Hydro

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2013
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Executive Summary

In many parts of the developing world, access to electricity can be extremely limited. The goal of this project is to create and demonstrate a possible method for solving this problem. The proposed solution is a portable, inexpensive hydro-kinetic turbine and generator capable of producing a usable amount of power from streams or rivers. With little power generally available from most river systems, a high efficiency turbine and modular approach to portability was needed. The power output goal was in the range of 50 to 100 watts at a water flow speed of 3 knots, limiting the system’s practical applications to high efficiency systems (like LEDs) and charging batteries.

The field of generating electricity purely from the kinetic energy of water is a recently emerging one as the alternative energy industry grows. Universities and researchers in Washington have performed significant work in this area as their extensive canal system is looked to as an untapped resource of energy. Similar research is being conducted into using tidal currents with similar applications. One of the resultant technologies of this research is the Gorlov helical turbine. This modified wind turbine (derived from the Darrieus turbine) can perform at a relatively high efficiency compared to other reaction-type designs. Using airfoil shaped blades wrapped around its helical shape, the Gorlov can theoretically rotate faster than flowing fluid velocity. Because of this and other positive attributes (such as its ability to self-start), the Gorlov was the turbine design chosen for this device.

The next design consideration was to determine how to apply the turbine to the fluid in a consistent and stable fashion. Although a number of designs were considered (including a sunken box frame and cantilever systems), a raft concept was chosen for the final device. The raft concept allowed for a modular design approach and ability to use lightweight materials. In order to accommodate a developing world market, materials were chosen with widespread availability in mind as to remove the need to ship large or bulky components while instead providing instructions to create them locally. The raft consists of two polyurethane foam pontoons, a PVC frame, and a plywood platform. Although the
dimensions of the raft are 1.6m. x 1.0m. x 0.4m., the PVC tubing can be broken down for easy transportation and the overall weight is under 16 kg.

Finally, the last major component for design was the generator. While the main considerations when designing this project were portability and cost (aside from performance), reparability and maintenance were also a focus. A component determined to be ill-suited for consistent upkeep and ease of repairs was a gearbox. As such, choosing a generator that does not require a high input angular velocity became the goal (ruling out most standard DC generators). What was finally selected was a two phase stepper motor run in reverse to produce an alternating current from its normal input lines. This device is capable of producing high voltages at even sub 100 rpm inputs.

The entire assembly can be easily constructed and installed by one or two people. The turbine is fitted to a shaft protruding from the bottom side of the raft. The shaft passes through a hole in the plywood platform and is secured to two bearings supported by a steel A-shaped mount. Attached to the top of the mount is the generator which is directly coupled to the drive shaft. On board are power electronics to convert the varying voltage output of the generator to a regulated output voltage for use.

In order to initially construct the turbine, a half-scale model was produced using rapid prototyping techniques. Although a usable half-scale could be produced within two days on available equipment, a full scale prototype would be too large to build. As such, the half model served as the basis for testing turbine performance while full scale turbine manufacturing continued to be explored. The rest of the assembly was much more easily constructed and were produced in full scale.

Testing of the turbine began with basic proof of concept experiments. The turbine was fitted to a shorter test shaft and mounted to a portable board using the A-shaped bearing mount. Velocities were determined using a light based tachometer and torques were calculated by timing spool up to steady state velocities and measuring moments of inertia. The initial experiments consisted of finding a steadily flowing river or stream (due to the lack of available facilities like water tunnels on campus), manually
applying the turbine assembly to the flow, and observing self-start characteristics as well as rotational outputs at the shaft. Due to the low amount of rain during the time of testing, finding a suitable river or stream proved difficult. As such, much of the water testing was done at Morro Bay using the current produced by the protected coastline. This testing proved inconsistent at best due to poor flow characteristics, bearing sizing, and controllability. Facing these problems, the testing approach was changed and the more controlled environment of wind tunnel testing was selected.

Using Buckingham Pi groups and other dimensionless parameters, the wind tunnel half-scale testing model could be directly correlated to a full-scale water model. New bearings were constructed that increased the model’s performance significantly and were fixtured to the wind tunnel in the mechanical engineering fluid dynamics lab. By varying the wind speed, spool up tests were used to derive average torque values and compute the resultant power output. This output, though measurable, proved to be lower than the specified 50 watts at 3 knots.

Testing the remaining components of the system yielded more positive results. The raft was very stable both under load and free floating, and aligns itself along the flow direction. The power electronics developed by the two electrical engineers assigned to this project produced a stable output voltage under varying load conditions.

Producing a set of full scale turbine blades proved to be more difficult than expected. Given the helical shape and airfoil cross section of the blade, machining it would be both costly and time consuming. Cost estimates from external machine shops for a set of four blades were around $400 per blade and with a working half scale model available, it was deemed unreasonable given the constraints of a budget. Casting was attempted but with a number of failed attempts, the team ran out of time before producing a workable prototype.

Full-scale production plans for this system should it be distributed worldwide are fairly straightforward. The device will be reduced to the essential, hard to produce components and packaged as a kit.
The kit would contain four turbine blades, two turbine end caps, a drive shaft, a generator, a power conversion box, fasteners, and instructions for both assembly and construction of remaining components. The turbine blades would be produced via die-casting, as this would allow for efficient mass production with a good surface finish. The end caps would be stamped from aluminum with slots to easily fit and install the blades in the correct orientation. The remaining components would be sourced and purchased from outside vendors and suppliers.

The constraints of cost and portability combined with the performance demanded by this task proved a significant hurdle to overcome in a year. Poor early decisions (such as not considering the poor manufacturability of a Gorlov turbine) led to wasted time better served for prototyping and testing. With such low amounts of power available from the source itself combined with low device efficiencies, component redesigns would be considered (such as the bearing mounts) to improve the performance of the system and create opportunities for more significant testing. Even with redesigns and better manufacturing methods, the concept of a portable power source from flowing water sources appears to be impractical. Larger or more permanent designs such as low head Pelton wheels can more efficiently and effectively produce energy from similar water sources with minimal additional labor.
Chapter 1: Introduction

This senior project provides a possible solution to small scale hydrokinetic power generation for use in low power situations. The term “nanopower” generally refers to energy generation on the order of 100W. Chapter 1 provides an overview of the needs and objectives of this project. Chapter 2 discusses the background of existing nano-scale hydrokinetic energy generation. Chapter 3 focuses on the design development for the turbine and implementation methods of the power generator. Chapter 4 gives a detailed outline of the finalized design of the system. Chapter 5 delves into the processes and planning that went into bringing the project from the design stage to building the device. Chapter 6 outlines the testing procedures used by the team in verifying the final design. Finally, Chapter 7 provides the conclusions and recommendations drawn from the finished product and the team’s experiences along the way.

1.1 Background and Needs

In many developing countries, isolated communities struggle to secure a consistent and reliable source of electricity. While solar panel and wind turbine technology continue to progress towards more effective methods of energy generation, difficult maintenance and unreliable production makes them unsuitable for solving this problem. Water turbines for power generation are often associated with large dams such as the Hoover Dam, but kinetic energy can be harnessed from flowing water as well. As there is no energy gathered from the vertical displacement of the water (like with dams), the design considerations for the turbine differ little from that of a wind turbine. A small, portable system that can generate between 10 and 200 watts from a nearby stream or river would be sufficient to sustain the low power demands of an isolated developing community. This small amount of power (an average incandescent light bulb can use about 50 watts) can be utilized to charge batteries for wireless devices or power high efficiency LEDs. Should additional power be needed, multiple systems could be chained together.
1.2 Problem Definition

The main objective of this project was to design and develop a portable, low power, hydrokinetic generator that can optimally perform in low flow streams or rivers. The goal was to provide power in a remote location (away from the main power grid) by installing this generator in a nearby stream, resulting in access to consistent power generation regardless of wind or daylight.

1.3 Objective/Specification Development

There are several design specifications that were considered integral to the success of this system. The first parameter was weight; our proposed system is designed to be portable. As such, the maximum design weight was set at 250 Newton. This was set with the idea that this is about the maximum weight a person can maneuver without assistance. Another important parameter is the overall power output extracted from the flowing water.

In order to consider this project a success, a minimum power output of 50W, at a water flow speed of 3 knots, was made a design requirement. This amount of power would limit the generator’s practicality to that of a trickle charger (e.g. a cell phone overnight while camping). Unfortunately, the field of hydrokinetics is fairly unexplored at scales below a kilowatt and little information was available in order to base these power expectations. Because of this, many of the turbine related specifications, including efficiencies and blade sizing, were determined using the results of unconfirmed previous research work. Cost was considered a less critical parameter (due to the exploratory nature of an under-defined project) and will be bounded by the maximum available budget.

1.4 Project Management

Due to the interdisciplinary nature of this project, the tasks necessary to successfully design and build a portable hydrokinetic generator were split between two teams, one focused on the mechanical design project and one the electrical. The mechanical team was comprised of three mechanical engineering undergraduate students: Andrew Del Prete, Brandon Fujio, and Alex Sobel. This team was
tasked with creating a suitable method of power generation using the flow of a river. Andrew Aw and James Biggs were the two electrical engineering students responsible for creating a system to transform a varying input voltage from the generator to a constant output voltage able to charge a battery. There was some interaction between the two teams, mostly based around idea and data sharing, but since the tasks of the two teams were seen as separate projects that integrated at the generator, the roles and responsibilities for the teams were determined separately.

Andrew Del Prete served as the lead contact for the mechanical team. As part of this role, he was responsible for documenting the team’s progress and ensuring part orders were received. In addition, Andrew was held responsible for the manufacturing of both the half-scale and full-scale turbines. Alex Sobel was in charge of ensuring the validity of the team’s calculations and solid modeling. Additionally, Alex led the mechanical team in the construction of the implementation apparatus for the selected turbine. Brandon Fujio was tasked with testing methods and evaluation of the prototypes. His main responsibility was to find suitable ways to test different aspects of the selected design and to ensure the validity of these tests. Although these roles served as guidelines for which team member was accountable for different aspects of the project, no one member of the team worked exclusively on any one area of the project.

Chapter 2: Background

Before any design activity could occur, it was important to understand what already exists in the field of research as well as commercially. These pre-existing solutions for similar problems yielded valuable insight into possible obstacles and options. This section focuses on existing projects with a similar objective.
2.1 Existing Products

In 2011, a group of students and faculty at St. Martin’s University in Washington tackled a problem similar to ours. They wanted to produce 1kW of energy using a run-of-the-river style hydroelectric plant. A modular design was used, resulting in a system weighing about 200 lbs. The team had an approximate site location for installation as opposed to the general approach of this project, allowing them to specifically characterize the size and speed of the river. Though the scale of their project was an order of magnitude larger than ours, our team benchmarked our design upon their results. Some of this information includes: blade profile specification functions, turbine performance data, relative pricing and sizing of parts, as well as river and stream characterizations for power output estimates.

In addition to the report out of St. Martin’s University, a student named Adam Niblick from the University of Washington wrote his master’s thesis on generating hydroelectric power at small scales. His goal was to charge oceanographic instruments using the hydrokinetic energy from tidal currents. These instruments would require 20 watts of continuous power using fluid flow that oscillates, thus requiring a turbine that can react to multidirectional flow while still rotating unidirectionally. One of the most important correlations between this project and ours is the similarity of scope. His estimation of tidal current speed (~1.5 m/s) is similar to our own estimation of the average river flow speed. This allows us to gather an immense amount of turbine data and characterization from his helical turbine tests. Some of the information we used included: advanced blade profile characterization, dimensionless parameters, testing processes, as well as test data relating to part sizing.

2.2 Current State of the Art

Although many ideas exist for how to best extract energy from the flow of moving water, very few of these ideas have actually been realized as prototypes and no portable hydrokinetic generator currently exists on the market. Bourne Energy, an alternative energy company based in Los Angeles,
developed what appears to be the best solution for a portable hydrokinetic generator with its RiverStar Backpack Power Plant. The Backpack Power Plant is advertised as a renewable energy generator measuring 3 feet in length and weighing less than 30 pounds. Figure 1 below shows the expanded design for the Backpack Power Plant. Each unit is self-contained with its own integrated power, control, cooling and sensor systems and collapses into a backpack size module with the generator, hub and folded turbine blades stored inside. With the ability to generate up to 500 Watts of continuous power in a flow of four knots, Bourne’s product would meet the goal of a portable hydrokinetic generator, but the expected $3000 price tag makes it an unsuitable solution for developing countries. In addition, having not received a research grant critical to their research and development, it appears Bourne Energy no longer exists.

![Figure 1: BackPack Power Plant by Bourne Energy](image)

2.3 Applicable Standards

Very little legislation exists regarding hydrokinetic projects on this scale. According to the Washington Department of Fish and Wildlife, “any device you use for pumping water from fish-bearing waters must be equipped with a fish guard to prevent passage of fish into the pump intake. You must screen the pump intake with material that has openings no larger than 5/64 inch for square openings, measured side to side, or 3/32 inch diameter for round openings, and the screen must have at least one square inch of functional screen area for every gallon per minute (gpm) of water drawn through it.” [2]
While this does not specifically mention turbines, the emphasis on fish safety prompted an emphasis on environmentally conscientious decisions in regards to how to extract energy from water and how this generator would be implemented.

Chapter 3: Design Development

Several designs were considered for both the turbine design and the implementation methods for the system. These decisions, as well as the rationale for the generator, are outlined in this section. In addition, the basic calculations used to help pick the conceptual are detailed.

3.1 Discussion of Conceptual Design and Selection

This section contains the rationale used to select the main components of our final design.

3.1.1 Turbine Selection

As shown in the table in Appendix A, many different types of turbines were considered and evaluated for practical application. While there were many options for turbine selection, most types require significant amounts of head and therefore did not make sense to use on a small-scale low-power basis. Due to this constraint, only five types of turbine were seriously considered for use in the design: Pelton, Kaplan, Gorlov, Darrieus, and Savonius.

The Pelton wheel is an impulse turbine that extracts energy from the impulse of moving fluid. It works by having specially shaped buckets mounted on the perimeter of a wheel hit by the water, causing the wheel to turn. Typically, a nozzle is used with a Pelton wheel in order to increase the velocity of the flow into the specially designed paddles that leave the water with very little speed, extracting most of its energy. The Pelton wheel is highly efficient at low flow rates, but works better with large head.
The Kaplan turbine is a type of reaction turbine generally used in applications with low head and large discharge. This turbine utilizes axial flow, meaning that fluid enters and leaves the turbine axially, producing rotation in the propeller as it flows through as it is deflected through the guide vanes. A key aspect of the Kaplan turbine is that it has automatically adjusted propeller blades, which allows the turbine to achieve efficiency over a wide range of flow and water level.

The Darrieus turbine is a lift-type vertical axis turbine that can function effectively regardless of which direction the fluid is flowing. The Darrieus is well suited for energy generation as the design on the turbine allows for the blades to reach speeds that are higher than the speed of the moving fluid. However, due to this high speed and low torque generation, the Darrieus has difficulties with self-starting.
The Gorlov helical turbine, which was specifically designed for hydroelectric applications in water with little to no head, is based on the design of the Darrieus H-shape turbine. However, unlike the Darrieus turbine, the Gorlov has helical blades. These helical blades help to increase the efficiency of the turbine as well as alleviate the self-starting issues of the Darrieus. Both the Darrieus and Gorlov turbines create lift due to the airfoil shape of the blades. The blades of these turbines cut through the fluid with an angle of an attack that causes a pressure differential. The resulting pressure differential causes a lifting force, which propels the blade forward.

The Savonius turbine is a drag type vertical axis turbine, and it operates by using two or three scoops to cup and drag the moving fluid, causing the rotor to turn. Unlike the Darrieus or Gorlov turbines, the Savonius Turbine cannot rotate faster than the speed of the moving fluid, but this type of turbine yields a large amount of torque from rotation. The Savonius turbine is very simple and economical, leading it to be used whenever cost and reliability are more important than efficiency.
To select the top turbine design for our application, a comparative matrix was established using many different parameters we deemed as relevant to creating a turbine that meets the established design goal. The parameters were weighted by how important each was to completing this goal and each turbine was evaluated in each of these parameters. A score of one was given to turbines that perform poorly in the listed parameter, two for average for performance, and three for exceptional performance.

Portability was determined to be the most important factor for comparison. Since this is one of our design requirements, it is necessary that we select a turbine where portability is achievable. Both the Kaplan and Pelton types of turbine did not score well in this category as they would require large and bulky apparatuses in order to be implemented, whereas the Darrieus, Gorlov, and Savonius are much more compact. Next, self-start ability and efficiency with no head were determined to be equally important. Self-starting is important as we want for the turbine to be able to begin power generation on its own with the only outside force being the movement of the fluid. Without the ability to self-start, it is necessary to either install a small motor that can push start the turbine or to manually push the turbine blades. The efficiency of the turbine with no head was deemed to be of high important because our design must be very efficient in order to produce the desired amount of power with the expected flow conditions. For this same reason, efficiency at low speed was deemed to be the next most valuable criterion for comparison along with versatility.

Versatility, a parameter we defined as the ability of the turbine to work in varying flows and flows from different is valuable as we do not expect to see consistent conditions with the flow of the
river water. The Gorlov is specifically designed for varying conditions, and thus scored better than all the other turbines in this category. After this, safety was deemed next most important. While safety of the use is a key aspect of design, all of the turbines considered are relatively safe and thus this factor seemed less meaningful. Manufacturability and durability were deemed to follow in importance. Durability is a key factor as we do not want our design to fail or break during operation. Additionally, due to the portability of the design, we do not want the turbine to break if the design is accidentally dropped while being moved, making durability a necessary factor in the turbine selection.

Manufacturability is important as the turbine will eventually be built, and having a simple design will allow for easier construction. However, since the final goal for this design is as a consumer product, the mass production of this turbine makes this factor less important. Finally, cost and environmental were the final two parameters we elected to consider. While we felt factors were necessary to compare, they do not have a direct effect on the overall performance of the turbine and thus were considered the least important.

Using these factors, a decision matrix (Appendix A) was generated to help select the best turbine for this application. The decision matrix revealed that the Gorlov and Darrieus turbines would be the best decisions for our turbine. We ultimately selected the Gorlov due to its increased self-start capabilities.

3.1.2 Turbine Implementation

After completing preliminary designing and prototyping a turbine design, our next goal was selecting from our three structural housing concepts. The concepts considered for the structural base were a submerged frame, a stabilized tripod with adjustable legs, and a floating raft tied to shore. The submerged frame had the benefits of being out of the way of floating debris as well as being the easiest to install; however, sealing power electronics and the fact that the bottom of a stream has the slowest fluid flow lead us away from this concept. Our second idea of the tripod also shared the benefit of ease of installation while allowing for simple adjustments for different types of river beds. Where this
concept fell short was its major susceptibility to floating debris, the inability to handle streams with significant depth, and its lack of stability when considering variable flow conditions from storms or upstream rain. This left us with the raft design. The risk of floating debris is limited to only major obstacles such as logs and the buoyancy of the raft ensures the turbine is delivered perpendicularly to the highest rate of flow at the top of the stream. Weight and buoyancy concerns can be solved by moving the battery and power electronics offshore with cables wrapped around the support lines and the implementation of floatation devices such as pontoons or inflatables.

![Figure 7: Implementation methods. Tripod (left), Raft (middle), Submerged Box (right)](image)

### 3.1.3 Generator Selection

When exploring generator options, we first started with a synchronous DC generator. Using a multi pole design, we hoped to be able to produce a usable amount of power from the relatively low angular velocity outputted by the turbine (≈100-200rpm). However, our research showed that this velocity was much too low for this type of generator and that a gearbox would need to be constructed to ramp up the velocity. Constructing a gearbox is something elected avoid due to the increased complexity of the system, the loss of mechanical efficiency, and increased weight on a raft already depending on buoyancy for its performance. An alternative to the DC generator we explored was a stepper motor run backwards to produce an AC signal. Preliminary analysis showed that even at <100rpm, the stepper motor was able to produce a usable amount of rectified voltage. Our goal was to keep our overall system as simple as possible and the ability to omit a gear box with the use of the stepper motor made it the most desirable choice for power generation.
### 3.2 Preliminary Analysis

Preliminary calculations were used to determine the necessary inlet area to generate 50 Watts of power for varying turbine sizes at various water speeds using the equation:

\[ P = \frac{1}{2} \times \rho_w \times (A \times \varepsilon_t \times V_w^3) \]  

where \( P \) is the power of the turbine in Watts, \( \varepsilon_t \) is the turbine efficiency, \( \rho_w \) is the density of water (1000 kg/m\(^3\)), \( A \) is the inlet area of the turbine and \( V_w \) is the velocity of the water. Based on these calculations, an inlet area of 0.1 m\(^2\) was determined to be the best for our application. This calculation is found in Appendix E.

![Figure 8: Expected Power Output for Turbine of 0.1m\(^2\) Inlet](image)

### Chapter 4: Final Design

This section will provide an in depth discussion of the final design. This includes an overall layout description, component design and analysis, and a cost analysis breakdown. The overall layout will describe how the components come together. The component design provides a discussion of the
materials selected for different components and the dimensions proposed. The cost analysis breaks down the expected costs of the raft, generator, and turbine. The maintenance and repair section discuss how certain components were designed for easy upkeep.

4.1 Overall Description

The final design consists of two main components, the selected turbine and a raft to support the turbine. The raft is a simple square frame supported by pontoons on either side for buoyancy. Attached to the top of the raft frame is a platform for hosting the generator as well as any additional electronic equipment. A hole in the center of the platform allows a shaft to run from the generator to the Gorlov turbine.

4.2 Detailed Design Description

This section discusses the design of each component in detail.

4.2.1 Frame

The frame of the raft, designed to be both study and modular, is constructed of 25.4 mm diameter PVC pipe. PVC pipe was selected as it allows for a very durable frame, but is still easy to pull apart and piece back together. Furthermore, it is a relatively inexpensive material and very easy to find, making repairing or replacing the frame simple. The frame of the raft is 1 meter wide by 1 meter long by 0.171 meters in height. It consists of roughly 8 meters of piping connected with 8 T-joins and 4 elbow-joins to form a square raft with 2 legs where the pontoons attach.

4.2.2 Platform

A simple square platform is anchored to the raft frame in order to provide a support for the generator as well as any other electronic devices. The 12.7-millimeter thick platform will attach to the raft frame using eyebolts that will allow the platform to be easily removed. Cedar plywood was selected for this application as it is a cheap and readily available material while still being strong enough to support the expected weight of the generator. Other types of wood were also suitable for the platform,
but cedar was selected as it was considered best for decay resistance. In addition, the plywood was treated with a waterproof coating to further protect it from the outdoor elements. Mounted to the top of the platform is an A-shaped steel frame to house the bearings and to install the generator.

4.2.3 Pontoons

Pontoons are used to provide buoyancy for the raft. The pontoons are made of foam with a density of 128 kg/m$^3$ as this provides a low density, but still durable solution for floating the raft frame, platform, generator, and turbine. The foam selected, poured urethane foam, allows us to form the pontoons around the two legs of the raft. The pontoons are 1.5 meters long, 150 millimeters thick, and 150 millimeters wide, while being shaped on the end as to remain forward facing in the water. The foam is closed-cell; meaning that the pontoons will not absorb water, and thus the buoyancy of the raft will not be compromised.

4.2.4 Generator

The generator converts mechanical power input into electrical power output. In order to achieve reasonable generator efficiency at low speeds, we have elected to run a stepper motor backwards to generate AC power. The stepper motor’s multi pole design allows it to generate relatively high voltages at low angular velocities. Testing conducted by the electrical engineering members of the project determined that this generator will produce over 24V at rotational speeds under 60 rpm. This is sufficient to power the power electronics and ensures trickle charging capability at even the slowest angular velocities.

4.2.5 Shaft

The shaft, responsible for the mechanical transfer of rotation from the turbine to the stepper motor, measures 0.6 meters long as this is the length from the bottom of the turbine to the top of the wooden platform and mounting plate where the generator is placed. The material for the shaft is stainless steel; this will provide a stable shaft that will not be corroded by water. The shaft has an outer diameter of 12.5 millimeters and is a solid cylinder.
4.2.6 Electronics Housing

The electronics housing of the raft went undeveloped due to a lack of decision by the electrical engineering team whether to house the battery on the raft or on the shore. However, a tarp could be used as a simple solution since there is very minimal heat generation by the generator.

4.2.7 Gorlov Helical Turbine

An entry area of 0.1 m\(^2\) was selected for the turbine in order to keep the size reasonable while still generating the desired amount of power. We determined that a height to diameter ratio, also called the aspect ratio (AR), of approximately 1.5 is appropriate as this aspect ratio would allow for the appropriate inlet area while still being compact and providing necessary support to the turbine blades. To achieve the desired inlet area, we elected to use a turbine with a diameter of 0.25 meters and a height of 0.4 meters.

Once we determined the physical size of the turbine, we next considered the sizing of the turbine blades. Based on existing test results published by Dr. Mitsuhiro Shiono, a professor from Nihon University in Japan, the optimum solidity ratio ($\sigma$) for maximum efficiency is between 20 and 40 percent. Solidity ratio is a measure of how much of the surface area is taken up by the blades. The solidity of the turbine will affect the turbines ability to capture the energy from the flowing water. This is especially important when considering the startup capabilities. A lower solidity ratio will allow the turbine to spin faster through the water, but it will not generate as much torque. Thus, there is a limit to how solid a helical turbine can be while maintaining a reasonable rotational speed. As the solidity ratio increases above 40 percent, the efficiency of the turbine begins to drop off. Since we are trying to generate as much power as possible, we selected a solidity ratio of 27 percent to optimize starting torque with rotational speed.

Testing performed by Niblick indicated that a Gorlov turbine with four blades performs better than one with three blades, so we chose a four blade design. Based on these parameters, we were able to calculate a chord, or nose to tip, length for the blades using the equation:
\[ \sigma = \frac{n \cdot c}{\pi \cdot D} \]  \hspace{1cm} (2)

where \( n \) is the number of blades, \( c \) is the chord length, and \( D \) is the turbine diameter. This comes out to a chord length of 7 cm for our full-scale turbine. The blades are designed using a NACA 0018 cross section. The NACA designation indicates that the widest portion blade is 18 percent of the chord length; in this case, 1.26 cm. This profile was selected as it will yield reasonably durable midsized blades.

Blade wrap is another aspect of the design under consideration. The term “blade wrap” refers to the percentage of the perimeter that the blades span. This is the major difference between the Gorlov helical turbine and the Darrieus straight blade turbine. The blade wrap allows each blade to generate torque for a longer portion of its revolution. This creates a more continuous torque supplied to a generator. For smooth torque transmission, a blade wrap that is a multiple of 100 percent is preferred. Ratios greater than 100 percent generate double the torque at certain angles, because two blades are being pushed at the same time, while ratios of less than 100 percent have locations with zero torque because there is no blade being pushed. For simplicity, we have elected to use a 100 percent blade wrap, instead of 200 or 300 percent.

In order to determine the blade wrap, we needed the helical pitch angle (\( \delta \)). The helical pitch angle represents the angle that the blades make with the bottom plane, as seen in Figure 10 below.
Shiono’s tests indicate that the optimum pitch angle is between 43.70 and 60 degrees. However, since we already knew approximately what the other parameters would be, we used simple trigonometry to determine a pitch angle of 63.855 degrees.

Detail drawings of all the components as well as exploded views of the raft and the entire apparatus can be found in Appendix B.
4.3 Analysis Results

Multiple calculations were performed to ensure that the material selection was adequate and used to help size the components of the raft and to select materials. For the raft, buoyancy calculations were determined in order to ensure that the apparatus is able to float despite the weight of the generator and additional electronic devices. Originally, our plan was to use the buoyancy of the PVC pipe to float the raft and turbine setup, but found that this would not be able to support the entire weight of the system. Because of this analysis, pontoons were added to the final design. Furthermore, the originally selected material for the raft platform was Plexiglas. However, looking at the deflection of the plate allowed us to conclude that using wood would be more than suitable for the application at a lighter weight and cheaper cost. Next, the diameter of the shaft was determined by considering the deflection and stresses of the shaft. Our ideal shaft size was determined to be 16 millimeters, but after researching bearing availability, we concluded that using a shaft with an outside diameter of .5 inches would make more sense. Finally, multiple calculations were used to find the proper sizing of the turbine to ensure maximum efficiency. The calculations used to determine or verify sizing and material selection can be found in Appendix E.

4.4 Cost Analysis

We worked to design a first generation hydrokinetic generator system. As such, some elements of this design were not optimized for cost. Instead, we focused on finding a reasonable solution for the problem we were asked to solve. Further iterations of this project may be able to reduce costs by using different materials. In addition, large scale production of the blades would dramatically reduce the cost of each turbine. The individual costs of each component as well as the total cost for the prototype can be found in Appendix C.
4.5 Maintenance and Repair

One of our goals as the mechanical engineers on this project was to keep the overall design as simple as possible to enable the end user to easily repair and maintain the product. Where possible, components incorporate modularity such as removable turbine blades, PVC piping in the frame, and removable shaft couplers. This allows for damaged components to be repaired with minimal machining and required parts. Ease of repair was also the main driving force behind using a stepper motor as a generator. If a standard DC generator had been used, a gear box would have been required to step up the angular velocity to achieve reasonable efficiency. Being able to omit the gear box with the stepper motor’s direct coupling to the shaft greatly reduces the mechanical complexity of the system and reduces the skill requirement for repair of the overall system while also reducing the number of parts that can break.

Chapter 5: Product Realization

This section discusses the steps taken by the team to take the concept designed and realize it as an actual product. Manufacturing steps in creating the initial prototype are detailed as well as future considerations for larger scale production.

5.1 Pontoon

The pontoon is a relatively simple component consisting of poured urethane foam with a PVC chassis attachment set in the center. In order to create the desired shape for the pontoon, a mold was constructed out of wood and plastic tarp. The wood was fashioned into a box with an open top. Tarp was then laid at the bottom and covering the side walls to create a semi-smooth surface while also preventing leakage from the container while the foam sets. The shape of the mold was a long rectangular prism with a square cross section. In order to allow for post-processing, shaping, and finishing after molding, the dimensions of the square were about 2 inches larger with the long dimension being about 6 inches longer.
Before pouring the foam, the attachment site for the chassis as well as the embedded PVC needed to be fixtured to the mold as the expanding plastic is more apt to push the piping out than to form around it. This was done through wooden extensions to the mold and manually securing extension piping to prevent lifting.
Future production of the pontoon could be accomplished a number of ways. For local large scale production, precision made silicone or resin based molds could be constructed negating the need to shape and finish the plastic after the molding process. Another option for large scale production would be to purchase large lots of foam from a foam supplier and shape it using either power tools or a CNC mill. In the case of delivering a low cost kit to a customer in a developing country, it would not make sense to produce a pair of bulky pontoons and ship them around the world. As such, the kit could come with the foam plastic packaged as its separated liquid constituents along with some plastic sheeting. The sheeting would have instructions for the mold frame construction as well as serving as the internal lining.

5.2 Turbine

The turbine is a very difficult component when it comes to producing a single prototype at a reasonable cost. Early in the project, the Gorlov helical turbine was chosen for its high efficiency and desirable characteristics (such as the ability to self-start). However, the four helical blades that make that performance possible also create problems from a manufacturing standpoint. The first method of production explored was rapid prototyping. Using an additive method of manufacturing, a half scale
plastic model could be created of the turbine as a whole with a reasonable surface finish for baseline testing. Although the initial attempt of this method resulted a pile of detached blades (due to insufficient reinforcement at fragile locations), after making the necessary corrections, the subsequent runs were successful.

![Failed (L) and Final (R) Half-Scale Turbine Rapid Prototypes](image)

**Figure 15: Failed (L) and Final (R) Half-Scale Turbine Rapid Prototypes**

The next step came later in the project timeline, as a full-scale prototype was now desired. The rapid prototyping machine could not handle the size of model required for a full-scale design, including that of a single blade. Because of this, outside prototyping and machine shops were contacted for quotes regarding the production of the four helical blades individually. It would be much easier and cheaper to create the turbine in parts than produce it as a single component. The cheapest quote available was around $2000, barely within the remaining budget but still too expensive to justify spending all of the available funds. This high cost lead to looking within the university for resources that could possibly accomplish this at a more reasonable cost. With machining the blades not viable due to difficulty and cost of execution, casting became the top candidate. Working with Martin Koch, the team learned about the various methods of casting and worked to determine which technique would be most
viable for the production of a turbine blade. The first required component would be a model of the blade from which to create the mold. As the rapid prototyping machine could not handle to the full-scale blade model, the model was split in half and produced in two batches. These two batches were then glued together and sanded down to produce a full sized blade prototype.

![Figure 16: Attempt to Green Sand Cast Full-Scale Turbine Blade Half](image)

Future full-scale production of the turbine would likely employ the use of a precision casting die. Although the initial cost of producing the die would be very high, it would facilitate the production of cheap and consistent turbine blades en mass. The end caps of the turbine cylinder would be stamped from a sheet of aluminum with cutout slots for fitting the blades into. The disassembled turbine would be included in the kit with a diagram showing the correct orientation and installation of the blades into the ends.

### 5.3 Chassis

With the chassis being the least technical component of the overall system, much of the design focus was centered on using widely available materials in simple, but effective ways. The main support of the raft comes from a rectangular PVC pipe frame. The frame attaches to the pontoons with an
extended T joint sticking out from an embedded pipe in the foam. PVC was chosen due to its relative lightweight yet sturdy characteristics, as well as its resistivity to water corrosion and its wide spread availability. The PVC frame is attached to a meter-by-meter plywood sheet using hose clamps. The clamps are threaded through drilled holes in the plywood and fastened to the underside of the raft. To prevent water damage, the plywood was treated with a water resistant coating and painted. To create the A-frame desired, three Simpson Strong-Tie Half Bases were pieced together and fastened with machine screws. The bearing housings were made of wood and attached to the A-frame and the entire apparatus was attached to the plywood platform using L-brackets.

Figure 17: A-Frame Bracket

Figure 18: Chassis Design
With worldwide distribution of a kit-based product in mind, shipping the chassis as a whole or even in components would be frivolously expensive and impractical. To accommodate this, the materials chosen could be purchased locally and constructed on site. There is very limited labor involved in the construction and hand tools are adequate to complete the task. The only thing required to be placed in the kit would be a bill of materials and dimensions for assembly.

Figure 19: Final Build

Chapter 6: Design Verification

The major components in the final design required extensive testing at both a component level and an overall system level. The most extensive testing revolved around the characterization of the helical turbine, as this was the least understood component on a conceptual level. Testing for the turbine included tip speed scaling as it relates to flow speeds, self-start capabilities, and acceleration and torque testing to determine the overall power. The raft was tested for buoyancy and stability in real world conditions. The final component tested was the stepper motor. Stepper motors are not well characterized for their power generating capabilities so developing our understanding of that would be the major goal. Testing and its results are discussed below.
6.1 Turbine Testing

All of the turbine testing used a half scale turbine due to the size constraints of the manufacturing and testing facilities.

6.1.1 Weir Flow Test

Initial testing involved used a pump-weir-sump system located in the Cal Poly Mechanical Engineering fluid dynamics laboratory to simulate the flow of a slow moving stream or river. The turbine was placed both in front of and behind the weir in order to test the turbine in different flow conditions and at different flow speeds. While the turbine was able to turn if properly oriented behind the weir, a consistent result was not able to be achieved, thus leaving this initial test inconclusive. This failure was not completely unexpected due to the random and turbulent flow field produced by the weir and the slow speeds of the moving water.

6.1.2 Morro Bay Testing

The turbine was tested again using the current in Morro Bay and a hand-held turbine set up. Kayaks were used to reach the center of the channel, where flow was fastest, but as with the weir system, testing proved to be inconclusive. Due to the low flow rate in the bay as well as excessive bearing friction, the turbine was unable to turn.

Figure 20: Attempted Testing in Morro Bay
6.1.3 Wind Tunnel Testing

Testing on the half scale turbine occurred in the Cal Poly Mechanical Engineering fluid dynamics laboratory using a wind tunnel. Hydrokinetic turbines and wind turbines are very similar since they both extract energy from the flow of a low-pressure fluid. The main differences are the vastly different densities of the fluid as well as the compressibility of air. Using Buckingham Pi groups and dimensionless parameters such as torque coefficients and tip speed ratios, wind tunnel testing can generate significant results in lieu of full scale water tunnel testing.

The wind tunnel has a one-foot square cross section testing area and can produce wind speeds of up to 110 miles per hour. The top plate of the tunnel was removed and a replace was fashioned with bearing mounts to support a shaft with the half scale rapid prototype of the turbine suspended in the middle of the air stream. One problem immediately apparent was the ratio of drag forces to torque produced by the turbine. With a cantilever style mount consisting of a thrust and ball bearing to support the shaft, the precise measurements and a rigid frame is required to prevent the drag forces from torquing the shaft into the bearings. Due to a small clearance between the ball bearing and the shaft, the friction became much too high to generate significant results, especially at high wind speeds. To fix this, a new bearing system was designed that was much simpler and yielded extremely low friction even at high loads. Instead of the cantilever design, a new base plate was designed and mounted to the bottom of the wind tunnel. To hold the axial load, sharpened pieces of steel were used in a “V” shape to restrict the shaft from sliding while maintaining minimal friction. The thrust load was supported by turning the shaft to a fine point and grounding it into a tungsten carbide bearing plate. Figure 21 shows the testing apparatus for wind tunnel testing.
6.1.4 Testing Results

The conditions under which testing occurred were not ideal. Since air is a compressible fluid, the available energy in the fluid flow is dramatically less than what should be available in an incompressible flow. The theoretical model used to predict the available in a fluid showed 50 watts at a flow speed of 1.5 m/s to be possible. However, the actual tests, using Pi groups to switch to water conditions, shows that only around 6 watts is available. Figure 22 shows the power output from the system.
The low power output likely occurred primarily because of the incompressible nature of the flow. In addition, friction in the bearing setup increased as the wind speed increased dramatically. Occasionally the bearings hummed because of the vibration and friction between the knives and the shaft. The increased friction would have reduced the top speed of the turbine, as well as slowed it down.

The torque was calculated using the equation:

$$ T = I \alpha $$  \hspace{1cm} (3)

where I is the moment of inertia of the system and alpha is the angular acceleration. Torque is directly proportional to angular acceleration and the mass moment of inertia so any decrease in the magnitude of acceleration will decrease the torque.

Power was then calculated using the torque and angular velocity using:

$$ P = T \omega $$  \hspace{1cm} (4)

where omega is the angular velocity. Inconsistencies in the physical model caused the angular velocities to vary dramatically. The friction in the system and weaknesses of the plastic reduced the rotational speed. The high speed of the wind and fast rotational speed in the air needed to generate any useable information caused the plastic blades of the turbine to bow out at higher velocities. This created a
limiting factor on the possible range under which data could be collected. This bowing was only noticed at much higher air speeds, but it likely occurred at every velocity tested. This action reduced the effectiveness of the model, thereby generating less power. Future tests could remedy this problem by using metal blades instead of plastic, or by putting struts from the blade to the shaft to support the expanse of material. The bowing and friction in the system created an upper limit for the rotational speed. This is easiest to see in the no-load conditions on Figure 23.

![Figure 23: Rotational speed at various water velocities](image.jpg)

The bowing at high rotational speeds causes this critical velocity so these measurements were neglected to generate the power curves in water. This was valid since the rotational speed and fluid velocity will not be nearly as large in water, and metal blades would be used which will not warp as easily.

Other inconsistencies in the system made data collection impossible beyond a certain wind speed. The loads used to create different conditions consisted of blocks of wood with a hole drilled in the middle. Unfortunately, since the system rotates, any unevenness in the weight distribution caused wobbles during testing. The first load, which was also the smallest, spun easily, but with each subsequent load, the imperfections became worse. The wobbling became worse as the wind speed, and therefore the rotational speed, increased. The third load caused so much wavering that only three air
velocities provided viable results. There was an upper limit because the blades arched, but there was also a lower limit to how slow the air could flow. Below about 20 m/s, the shaft would start to move away from its supports. This may be a function of imperfections in the shaft point, a slope in the tungsten carbide plate it was spinning against, or even the turbine because of the lift generated by the blades. When the airspeed increased, the force of the air against the system held the turbine in place.

Further testing using a metal turbine should occur, but the testing performed using the rapid prototyped model pointed out flaws in the existing system. Additionally, testing in water will provide a better understanding of the available power. Based on the air testing results, some power is available from streams, but it seems unlikely that the desired 50 watts is possible with this size system.

6.2 Raft Testing

The raft testing focused on two main properties: how much weight could it hold, and how stable was the raft. The raft must support at least the weight of the turbine, shaft, bearing mounts, and generator. It was deemed necessary to have enough extra buoyancy to support a large battery, if necessary. The raft was taken to Morro Bay to do this testing. The system was placed in the water and loads were added until the pontoons sank below the water’s surface. The maximum load that the raft will support is 45 kg. The rest of the system weighs only 10 kg, so there is a factor of safety of 4.5. After the raft’s buoyancy was tested, point loads were added to test how the raft would respond to an uneven weight distribution. As long as the load did not exceed the 40 kg, the raft remained level. Additional tests were performed to see how the raft did with moving water. The raft was pushed against the current in the Bay, and the reactions were observed. The raft seemed to move freely in the direction it was pushed, regardless of the relative direction of the current. This indicated that, as long as the raft is pointed mostly up stream, the raft would remain stable.
6.3 Generator Testing

Running a stepper motor as a generator is not a common solution to the need for low angular velocity power generation, but it has been shown to be effective in particular low power applications. A large stepper motor was selected with the intention of obtaining high voltage outputs. To determine the output of the motor, the leads were hooked into an oscilloscope and the shaft was spun at a constant speed. At 20 rpm, the motor delivered on the order of 25 volts. This was with no added load to the system, so the current was very low. However, the most important part was that the motor output more than 2.5 volts at a minimum. At this point, with the generator successfully supplying enough power, testing was passed on to electrical engineers to determine the best configuration for the circuit board and motor attachments.

Chapter 7: Conclusion and Recommendations

7.1 Component Redesign

Although aspects of the final design were successful, further analysis of components such as the turbine and the generator could easily be the subject of a yearlong project. Extensive research into these apparatuses would result in greater efficiencies. Since the scale of the power for this design is
so small, any improvement of efficiency would cause a great impact on the final amount of power generated.

Additionally, one of the key flaws of the final design was the bearing friction created due to fluid drag over the turbine as a result of the cantilever implementation design. This issue could be resolved by attaching a metal box to the bottom of the raft that supported a third bearing to hold the bottom of the shaft. Since the raft was designed to hold more weight than necessary, this addition would not be difficult to add to the existing design.

**7.2 Reduce Constraints**

Based on the testing and analysis conducted, it is has become increasingly evident that the goals set for this project were not fully practical. Efficiently extracting a usable amount of energy from a low energy density source while maintaining portability and affordability is a daunting task, especially when creating a single prototype. Since higher efficiencies mean greater costs, either the performance of the turbine or the overall cost of the system had to be sacrificed when creating and building this project. Ultimately, while the design as is could work to generate small amounts of power from a flowing stream, the overall price of the system does not make it an applicable solution for developing countries.
References


<http://www.nrdc.org/onearth/05spr/gorlov1.asp>.


Appendix A: Decision Matrix

Table 1: Decision matrix for turbine design selection

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Weight</th>
<th>Pelton</th>
<th>Weighted</th>
<th>Kaplan</th>
<th>Weighted</th>
<th>Darrieus</th>
<th>Weighted</th>
<th>Gorlov</th>
<th>Weighted</th>
<th>Savonius</th>
<th>Weighted</th>
</tr>
</thead>
<tbody>
<tr>
<td>Efficient at low speed</td>
<td>10</td>
<td>1</td>
<td>10</td>
<td>2</td>
<td>20</td>
<td>3</td>
<td>30</td>
<td>3</td>
<td>30</td>
<td>2</td>
<td>20</td>
</tr>
<tr>
<td>Efficient with no head</td>
<td>15</td>
<td>1</td>
<td>15</td>
<td>1</td>
<td>15</td>
<td>3</td>
<td>45</td>
<td>3</td>
<td>45</td>
<td>2</td>
<td>30</td>
</tr>
<tr>
<td>Manufacturability</td>
<td>7</td>
<td>3</td>
<td>21</td>
<td>2</td>
<td>14</td>
<td>3</td>
<td>21</td>
<td>1</td>
<td>21</td>
<td>3</td>
<td>21</td>
</tr>
<tr>
<td>Durability</td>
<td>7</td>
<td>3</td>
<td>21</td>
<td>3</td>
<td>21</td>
<td>2</td>
<td>14</td>
<td>2</td>
<td>14</td>
<td>3</td>
<td>21</td>
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<td>Environmental Impact</td>
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<td>3</td>
<td>1</td>
<td>3</td>
<td>3</td>
<td>9</td>
<td>3</td>
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<td>Portability</td>
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<td>Safety</td>
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<td>24</td>
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<td>16</td>
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<td>Cost</td>
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<td>10</td>
<td>1</td>
<td>10</td>
<td>3</td>
<td>15</td>
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<tr>
<td>Versatility</td>
<td>10</td>
<td>1</td>
<td>10</td>
<td>2</td>
<td>20</td>
<td>2</td>
<td>20</td>
<td>3</td>
<td>20</td>
<td>2</td>
<td>20</td>
</tr>
<tr>
<td>Self-start Ability</td>
<td>15</td>
<td>3</td>
<td>45</td>
<td>1</td>
<td>15</td>
<td>2</td>
<td>30</td>
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<tr>
<td>Total</td>
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Table 2: Turbine design characteristic table

<table>
<thead>
<tr>
<th>Type</th>
<th>Description</th>
<th>Head Ranges</th>
<th>Efficiency</th>
<th>RPM Range</th>
<th>Size</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Francis</td>
<td>Encased reaction turbine</td>
<td>10 - 650 meters</td>
<td>Large Scale (≈ 85%)</td>
<td>83 - 1000 rpm</td>
<td>1 - 10 meters diameter</td>
<td>Most widely used in the world, mostly large scale operations</td>
</tr>
<tr>
<td>Kaplan</td>
<td>Propeller type, inward flow turbine</td>
<td>10 - 70 meters (as little as 2 ft. in some applications)</td>
<td>Large Scale (≈ 85%)</td>
<td>79 - 429 rpm</td>
<td>2 - 8 meters diameter</td>
<td>Micro hydro applications, used in high flow low head situations</td>
</tr>
<tr>
<td>Tyson</td>
<td>Propeller type, reaction turbine mounted on a raft</td>
<td>No head</td>
<td>≈ 74%</td>
<td>?</td>
<td>?</td>
<td>Low research available, appears to be fairly inefficient but is a no head solution</td>
</tr>
<tr>
<td>Darrieus</td>
<td>Vertical, reaction hydrofoil</td>
<td>No head</td>
<td>≈ 40%</td>
<td>Spins slightly faster than moving fluid</td>
<td>Depends on application</td>
<td>Wind turbine design, generates maximum torque in two locations</td>
</tr>
<tr>
<td>Savonius</td>
<td>Vertical, scoop</td>
<td>No head</td>
<td>≈ 15%</td>
<td>Spins at speed of moving fluid</td>
<td>Depends on application</td>
<td>Wind turbine design, good when cost and reliability important</td>
</tr>
<tr>
<td>Gorlov</td>
<td>Vertical, reaction hydrofoil with curved blades</td>
<td>No head</td>
<td>≈ 35%</td>
<td>Spins slightly faster than moving fluid</td>
<td>&gt; 1 meter</td>
<td>No head solution, requires deeper water, based on Darrieus</td>
</tr>
<tr>
<td>Waterwheel</td>
<td>Traditional water wheel</td>
<td>&gt; 1 meter</td>
<td>&lt; 60%</td>
<td>Low</td>
<td>1 - 22 meters diameter</td>
<td>Inefficient compared to turbines, low head applications but large size?</td>
</tr>
<tr>
<td>Pelton</td>
<td>Impulse turbine</td>
<td>15 - 1800 meters</td>
<td>Per turbine conditions</td>
<td>Depends on head</td>
<td>Depends on head</td>
<td>High head, low flow application. Edge spins at half the speed of water jet</td>
</tr>
<tr>
<td>Turgo</td>
<td>Impulse turbine</td>
<td>15 - 300 meters</td>
<td>≈ 87%</td>
<td>Higher than Pelton</td>
<td>Smaller than Pelton</td>
<td>Runs at double the specific speed of the Pelton, for middle head range applications</td>
</tr>
<tr>
<td>Crossflow</td>
<td>Crossflow impulse turbine</td>
<td>Low head (&gt;10 meters)</td>
<td>≈ 75%</td>
<td>?</td>
<td>?</td>
<td>Has a flat efficiency curve from 1/6th to max loads, useful for seasonal flows</td>
</tr>
<tr>
<td>Archimedes' Screw</td>
<td>Cylindrically housed screw</td>
<td>Low head (&gt;10 meters)</td>
<td>High (?)</td>
<td>Low (?)</td>
<td>1 - 10 meters diameter</td>
<td>Common in English rivers, large initial energy require to start rotation, suitable for varying flow</td>
</tr>
</tbody>
</table>
### Table 3: Decision matrix for system design selection

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Weight</th>
<th>Sunken Box</th>
<th>Tripod</th>
<th>Raft</th>
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</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Score</td>
<td>Weighted</td>
<td>Score</td>
</tr>
<tr>
<td>Varying River Depths</td>
<td>12</td>
<td>1</td>
<td>12</td>
<td>2</td>
</tr>
<tr>
<td>Durability</td>
<td>15</td>
<td>3</td>
<td>45</td>
<td>1</td>
</tr>
<tr>
<td>Compactness</td>
<td>5</td>
<td>3</td>
<td>15</td>
<td>3</td>
</tr>
<tr>
<td>Maintainability</td>
<td>10</td>
<td>2</td>
<td>20</td>
<td>2</td>
</tr>
<tr>
<td>Portability</td>
<td>10</td>
<td>1</td>
<td>10</td>
<td>3</td>
</tr>
<tr>
<td>Manufacturability</td>
<td>5</td>
<td>3</td>
<td>15</td>
<td>1</td>
</tr>
<tr>
<td>Environmental Impact</td>
<td>9</td>
<td>3</td>
<td>27</td>
<td>3</td>
</tr>
<tr>
<td>Safety</td>
<td>7</td>
<td>2</td>
<td>14</td>
<td>3</td>
</tr>
<tr>
<td>Cost</td>
<td>12</td>
<td>3</td>
<td>36</td>
<td>1</td>
</tr>
<tr>
<td>Stability</td>
<td>10</td>
<td>3</td>
<td>30</td>
<td>1</td>
</tr>
<tr>
<td>Ease of Implementation</td>
<td>5</td>
<td>1</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>100</td>
<td>25</td>
<td>229</td>
<td>22</td>
</tr>
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</table>
Figure 25: Full schematic diagram of the raft and turbine system

<table>
<thead>
<tr>
<th>ITEM NO.</th>
<th>PART NUMBER</th>
<th>DESCRIPTION</th>
<th>Material</th>
<th>QTY.</th>
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<tbody>
<tr>
<td>1</td>
<td>CPNH1</td>
<td>PVC frame</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>CPNH2</td>
<td>Pontoon</td>
<td>Polyurethane foam rigid</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>CPNH3</td>
<td>Platform</td>
<td>Cedar</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>CPNH4</td>
<td>Mounting bracket</td>
<td>1060 Alloy</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>CPNH5</td>
<td>Turbine shaft</td>
<td>Stainless Steel (fermic)</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>CPNH6</td>
<td>Turbine Assembly</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>7</td>
<td>CPNH7</td>
<td>Aluminum Plate</td>
<td>1060 Alloy</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>S7BYGH420</td>
<td>Generator</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>9</td>
<td>60715K11</td>
<td>Thrust Bearing</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>10</td>
<td>7208K52</td>
<td>Ball Bearing</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>11</td>
<td>9404135</td>
<td>Eye Bolt</td>
<td></td>
<td>2</td>
</tr>
<tr>
<td>12</td>
<td>P1590A1022</td>
<td>Snap Ring</td>
<td>1020 Carbon Steel Sheet</td>
<td>1</td>
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<tr>
<td>13</td>
<td>98611932</td>
<td>Shaft Coupler</td>
<td></td>
<td>1</td>
</tr>
</tbody>
</table>
Figure 26: Exploded view of overall assembly
Figure 27: Frame is built using 1 inch, Schedule 40 PVC piping and standard Elbow Joints and T joints
Figure 28: 1 in, schedule 40 PVC pipe for the raft frame
Figure 29: 1 inch, Schedule 40 PVC pipe for raft frame
Figure 30: 1 inch, Schedule 40 PVC pipe for raft frame
Figure 31: 1 inch, Schedule 40 PVC pipe for raft frame
Figure 32: Pontoon made from poured polyurethane foam
Figure 33: Cedar plywood for the raft platform
Figure 34: Mounting bracket for bearings and shaft, made of 1060 Aluminum Alloy
Figure 35: Steel shaft to transfer rotation of the turbine to the generator
Figure 36: Design for Gorlov Helical turbine assembly
Figure 37: End plates for turbine blade support
Figure 38: Turbine blade. The blades have a NACA 0018 airfoil profile.
Figure 39: Aluminum mounting plate for added stability
Figure 40: Stepper Motor from Spark Fun
Figure 41: Thrust Bearing from McMaster-Carr
Figure 42: Ball bearing from McMaster-Carr
Figure 43: Eyebolt from McMaster-Carr. This piece is available at any local hardware store.
Figure 44: Snap ring from McMaster-Carr
Figure 45: Flexible shaft coupler from McMaster-Carr
Appendix C: Vendors, Contact information, Pricing

Table 4: Cost estimates and pricing for bill of materials

<table>
<thead>
<tr>
<th>Component</th>
<th>Quantity</th>
<th>Cost Per</th>
<th>Total Cost</th>
<th>Dimensions</th>
<th>Manufacturer/Distributor</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Poured Polyurethane Foam</td>
<td>3</td>
<td>$67.00</td>
<td>$201.00</td>
<td>8 lb. per cu. ft.</td>
<td>US Composites</td>
<td>16 lb. kit size</td>
</tr>
<tr>
<td>Shaft</td>
<td>1</td>
<td>$76.52</td>
<td>$76.52</td>
<td>12.7 mm Dia.</td>
<td>McMaster-Carr</td>
<td>Part #:6253K41; Hardened Stainless Steel</td>
</tr>
<tr>
<td>Bearing (Thrust)</td>
<td>1</td>
<td>$16.84</td>
<td>$16.84</td>
<td>12.7mm Shaft Dia. (31 mm OD)</td>
<td>McMaster-Carr</td>
<td>Part #: 6071SK11; Steel</td>
</tr>
<tr>
<td>Bearing</td>
<td>1</td>
<td>$11.15</td>
<td>$11.15</td>
<td>12.7mm Shaft Dia. (35mm. OD)</td>
<td>McMaster-Carr</td>
<td>Part #: 6384K363; Steel Flanged Dbl. Shielded</td>
</tr>
<tr>
<td>PVC T-Joints</td>
<td>8</td>
<td>$2.25</td>
<td>$18.00</td>
<td>25.4mm ID</td>
<td>Home Depot</td>
<td></td>
</tr>
<tr>
<td>PVC Elbow Joints</td>
<td>4</td>
<td>$1.80</td>
<td>$7.20</td>
<td>24.4 mm ID</td>
<td>Home Depot</td>
<td></td>
</tr>
<tr>
<td>PVC Piping</td>
<td>3</td>
<td>$3.38</td>
<td>$10.14</td>
<td>25.4 mm OD, 3m long</td>
<td>Home Depot</td>
<td></td>
</tr>
<tr>
<td>Platform</td>
<td>1</td>
<td>$27.97</td>
<td>$27.97</td>
<td>1m x 1m (12.7m thickness)</td>
<td>Home Depot</td>
<td>Cedar Plywood</td>
</tr>
<tr>
<td>Stepper Motor</td>
<td>1</td>
<td>$23.95</td>
<td>$23.95</td>
<td>Input rated: 2A/3V</td>
<td>Spark Fun</td>
<td>Part #: ROB-10847</td>
</tr>
<tr>
<td>Shielded Hose Clamps</td>
<td>9</td>
<td>$1.85</td>
<td>$16.65</td>
<td>5 Inch</td>
<td>Home Depot</td>
<td>Stainless Steel</td>
</tr>
<tr>
<td>Turbine</td>
<td>1</td>
<td>$1,500.00</td>
<td>$1,500.00</td>
<td>25m D, .40m H</td>
<td>ProtoLabs</td>
<td>Cost Estimation</td>
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<td>Retaining Ring</td>
<td>1</td>
<td>$9.37</td>
<td>$9.37</td>
<td>12.7mm Shaft Dia.</td>
<td>McMaster-Carr</td>
<td>Part #:91590A122; 10 Pack</td>
</tr>
<tr>
<td>Shaft Coupling</td>
<td>1</td>
<td>$47.59</td>
<td>$47.59</td>
<td>12.7mm Shaft Dia. x 9.5mm bore</td>
<td>McMaster-Carr</td>
<td>Part #:9861T81; Aluminum Helical Beam</td>
</tr>
<tr>
<td>Half Base</td>
<td>3</td>
<td>$4.27</td>
<td>$12.81</td>
<td>4in x 4in</td>
<td>Home Depot</td>
<td></td>
</tr>
<tr>
<td>Angle Bracket</td>
<td>2</td>
<td>$0.88</td>
<td>$1.76</td>
<td>1/2in x 2in x 2in</td>
<td>Home Depot</td>
<td></td>
</tr>
<tr>
<td>Machine Screws</td>
<td>3</td>
<td>$4.41</td>
<td>$13.23</td>
<td>Various Lengths/Diameters</td>
<td>Home Depot</td>
<td>84-Piece Combo Pack</td>
</tr>
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</table>

Total: $1,994.18
Appendix D: Gantt Chart

<table>
<thead>
<tr>
<th>Task Name</th>
<th>Duration</th>
<th>Start</th>
<th>Finish</th>
</tr>
</thead>
<tbody>
<tr>
<td>ME328</td>
<td>54 days</td>
<td>Tue 1/6/13</td>
<td>Fri 3/22/13</td>
</tr>
<tr>
<td>Develop Turbine Design</td>
<td>41 days</td>
<td>Tue 1/6/13</td>
<td>Tue 3/5/13</td>
</tr>
<tr>
<td>RP Turbine</td>
<td>7 days</td>
<td>Tue 3/5/13</td>
<td>Wed 3/6/13</td>
</tr>
<tr>
<td>Write Conceptual Design Report</td>
<td>8 days</td>
<td>Tue 3/5/13</td>
<td>Thu 3/14/13</td>
</tr>
<tr>
<td>Conceptual Design Review</td>
<td>8 days</td>
<td>Fri 3/15/13</td>
<td>Fri 3/15/13</td>
</tr>
<tr>
<td>Develop Mount Concept</td>
<td>7 days</td>
<td>Thu 3/14/13</td>
<td>Fri 3/22/13</td>
</tr>
<tr>
<td>ME329</td>
<td>50 days</td>
<td>Tue 4/2/13</td>
<td>Mon 6/10/13</td>
</tr>
<tr>
<td>Mount Design Finalization</td>
<td>5 days</td>
<td>Tue 4/2/13</td>
<td>Mon 4/6/13</td>
</tr>
<tr>
<td>Testing Development</td>
<td>6 days</td>
<td>Mon 4/6/13</td>
<td>Mon 4/19/13</td>
</tr>
<tr>
<td>Determine/Order Long Lead Time Parts</td>
<td>6 days</td>
<td>Mon 4/19/13</td>
<td>Mon 4/22/13</td>
</tr>
<tr>
<td>Critical Design Report</td>
<td>14 days</td>
<td>Mon 4/15/13</td>
<td>Thu 5/2/13</td>
</tr>
<tr>
<td>Critical Design Review</td>
<td>0 days</td>
<td>Fri 5/3/13</td>
<td>Fri 5/3/13</td>
</tr>
<tr>
<td>Turbine Testing</td>
<td>23 days</td>
<td>Tue 4/16/13</td>
<td>Thu 5/16/13</td>
</tr>
<tr>
<td>Manufacturing and Test Review</td>
<td>0 days</td>
<td>Fri 5/17/13</td>
<td>Fri 5/17/13</td>
</tr>
<tr>
<td>Assembly Design and Part Ordering</td>
<td>17 days</td>
<td>Fri 5/17/13</td>
<td>Mon 6/10/13</td>
</tr>
<tr>
<td>Summer Break</td>
<td>76 days</td>
<td>Mon 6/10/13</td>
<td>Mon 9/23/13</td>
</tr>
<tr>
<td>Review Design and Finalize BOM</td>
<td>76 days</td>
<td>Mon 6/10/13</td>
<td>Mon 9/23/13</td>
</tr>
<tr>
<td>ME330</td>
<td>60 days</td>
<td>Mon 9/23/13</td>
<td>Fri 12/13/13</td>
</tr>
<tr>
<td>Manufacture Assembly with Generator</td>
<td>15 days</td>
<td>Mon 9/23/13</td>
<td>Thu 10/1/13</td>
</tr>
<tr>
<td>Assembly Level Testing</td>
<td>5 days</td>
<td>Thu 10/1/13</td>
<td>Thu 10/24/13</td>
</tr>
<tr>
<td>Project Assembly Demo</td>
<td>0 days</td>
<td>Fri 10/25/13</td>
<td>Fri 10/25/13</td>
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<tr>
<td>Continued Assembly Lab Testing</td>
<td>12 days</td>
<td>Sat 10/26/13</td>
<td>Sat 11/9/13</td>
</tr>
<tr>
<td>River Testing</td>
<td>7 days</td>
<td>Sun 11/10/13</td>
<td>Sun 11/17/13</td>
</tr>
<tr>
<td>Senior Project Expo</td>
<td>8 days</td>
<td>Mon 11/18/13</td>
<td>Fri 11/22/13</td>
</tr>
<tr>
<td>Final Report</td>
<td>18 days</td>
<td>Sat 12/1/13</td>
<td>Thu 12/5/13</td>
</tr>
<tr>
<td>Final Report Due</td>
<td>8 days</td>
<td>Fri 12/6/13</td>
<td>Fri 12/6/13</td>
</tr>
<tr>
<td>Upload Report to Library</td>
<td>8 days</td>
<td>Sat 12/7/13</td>
<td>Fri 12/23/13</td>
</tr>
</tbody>
</table>

Figure 46: Gantt chart for yearlong project cycle
Appendix E: Detailed Analysis

Testing Data and Results

Table 5: Measured wind speeds and rotational speeds

<table>
<thead>
<tr>
<th>Load</th>
<th>Fan Frequency (hz)</th>
<th>Self-Start Frequency (hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>25</td>
<td>27.5</td>
</tr>
<tr>
<td>0</td>
<td>540</td>
<td>572</td>
</tr>
<tr>
<td>1</td>
<td>547</td>
<td>557</td>
</tr>
<tr>
<td>2</td>
<td>418</td>
<td>517</td>
</tr>
</tbody>
</table>

Where Loads refer to added inertia

Table 6: Added inertial loads

<table>
<thead>
<tr>
<th>Load</th>
<th>Added Inertia (kg-m^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0.000193</td>
</tr>
<tr>
<td>2</td>
<td>0.000532</td>
</tr>
<tr>
<td>3</td>
<td>0.001319</td>
</tr>
</tbody>
</table>

Table 7: Derived results for water with load 0

<table>
<thead>
<tr>
<th>Fan Frequency (hz)</th>
<th>Velocity (m/s)</th>
<th>Rotational Speed (rpm)</th>
<th>Torque (N-m)</th>
<th>Power Output (w)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>air</td>
<td>water</td>
<td>air</td>
<td>water</td>
</tr>
<tr>
<td>25</td>
<td>20.49</td>
<td>0.61</td>
<td>540</td>
<td>7.97</td>
</tr>
<tr>
<td>27.5</td>
<td>22.54</td>
<td>0.67</td>
<td>572</td>
<td>8.45</td>
</tr>
<tr>
<td>30</td>
<td>24.59</td>
<td>0.73</td>
<td>650</td>
<td>9.60</td>
</tr>
<tr>
<td>32.5</td>
<td>26.64</td>
<td>0.79</td>
<td>725</td>
<td>10.71</td>
</tr>
<tr>
<td>35</td>
<td>28.69</td>
<td>0.85</td>
<td>820</td>
<td>12.11</td>
</tr>
<tr>
<td>37.5</td>
<td>30.73</td>
<td>0.91</td>
<td>840</td>
<td>12.40</td>
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<tr>
<td>40</td>
<td>32.78</td>
<td>0.97</td>
<td>800</td>
<td>11.81</td>
</tr>
<tr>
<td>42.5</td>
<td>34.83</td>
<td>1.03</td>
<td>800</td>
<td>11.81</td>
</tr>
<tr>
<td>45</td>
<td>36.88</td>
<td>1.09</td>
<td>785</td>
<td>11.59</td>
</tr>
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</table>
Table 8: Derived results for water with load 1

<table>
<thead>
<tr>
<th>Fan Frequency (hz)</th>
<th>Velocity (m/s)</th>
<th>Rotational Speed (rpm)</th>
<th>Torque (N-m)</th>
<th>Power Output (w)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>air</td>
<td>water</td>
<td></td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>20.49</td>
<td>0.61</td>
<td>547</td>
<td>8.08</td>
</tr>
<tr>
<td>27.5</td>
<td>22.54</td>
<td>0.67</td>
<td>557</td>
<td>8.23</td>
</tr>
<tr>
<td>30</td>
<td>24.59</td>
<td>0.73</td>
<td>562</td>
<td>8.30</td>
</tr>
<tr>
<td>32.5</td>
<td>26.64</td>
<td>0.79</td>
<td>576</td>
<td>8.51</td>
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<tr>
<td>35</td>
<td>28.69</td>
<td>0.85</td>
<td>593</td>
<td>8.76</td>
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<tr>
<td>37.5</td>
<td>30.73</td>
<td>0.91</td>
<td>624</td>
<td>9.21</td>
</tr>
<tr>
<td>40</td>
<td>32.78</td>
<td>0.97</td>
<td>685</td>
<td>10.11</td>
</tr>
<tr>
<td>42.5</td>
<td>34.83</td>
<td>1.03</td>
<td>671</td>
<td>9.91</td>
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<tr>
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<td>36.88</td>
<td>1.09</td>
<td>695</td>
<td>10.26</td>
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</tbody>
</table>

Table 9: Derived results for water with load 2

<table>
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<tr>
<th>Fan Frequency (hz)</th>
<th>Velocity (m/s)</th>
<th>Rotational Speed (rpm)</th>
<th>Torque (N-m)</th>
<th>Power Output (w)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>air</td>
<td>water</td>
<td></td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>20.49</td>
<td>0.61</td>
<td>418</td>
<td>6.17</td>
</tr>
<tr>
<td>27.5</td>
<td>22.54</td>
<td>0.67</td>
<td>517</td>
<td>7.63</td>
</tr>
<tr>
<td>30</td>
<td>24.59</td>
<td>0.73</td>
<td>530</td>
<td>7.83</td>
</tr>
<tr>
<td>32.5</td>
<td>26.64</td>
<td>0.79</td>
<td>557</td>
<td>8.23</td>
</tr>
<tr>
<td>35</td>
<td>28.69</td>
<td>0.85</td>
<td>597</td>
<td>8.37</td>
</tr>
<tr>
<td>37.5</td>
<td>30.73</td>
<td>0.91</td>
<td>599</td>
<td>8.845</td>
</tr>
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</table>

Table 10: Derived results for water with load 3

<table>
<thead>
<tr>
<th>Fan Frequency (hz)</th>
<th>Velocity (m/s)</th>
<th>Rotational Speed (rpm)</th>
<th>Torque (N-m)</th>
<th>Power Output (w)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>air</td>
<td>water</td>
<td></td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>20.49</td>
<td>0.61</td>
<td>301</td>
<td>4.44</td>
</tr>
<tr>
<td>27.5</td>
<td>22.54</td>
<td>0.67</td>
<td>339</td>
<td>5.01</td>
</tr>
<tr>
<td>30</td>
<td>24.59</td>
<td>0.73</td>
<td>383</td>
<td>5.66</td>
</tr>
</tbody>
</table>
Theoretical Power Calculations

Figure 47: Power calculation for varying speeds of a turbine with 0.2 m$^2$ inlet area

Figure 48: Power calculation for varying speeds of a turbine with 0.3 m$^2$ inlet area
Figure 49: Power calculation for varying speeds of a turbine with 0.4 m$^2$ inlet area

Figure 50: Power calculation for varying speeds of a turbine with 0.5 m$^2$ inlet area

Equation Used:

$$P = \frac{1}{2} \rho_w \cdot (A \cdot \varepsilon_t \cdot V_w^3)$$

- $P \equiv$ Power (Watts)
- $A \equiv$ Area (m$^2$)
- $V_w \equiv$ Stream Velocity (m/s)
- $\varepsilon_t \equiv$ Efficiency Coefficient
- $\rho_w \equiv$ Density of water (kg/m$^3$)
## Gorlov Helical Turbine Sizing

<table>
<thead>
<tr>
<th>Dimensionless Parameter</th>
<th>Equation</th>
<th>Value</th>
<th>Ideal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solidity Ratio</td>
<td>$\sigma = \frac{Bc}{\pi D}$</td>
<td>0.36</td>
<td>0.30</td>
</tr>
<tr>
<td>Aspect Ratio</td>
<td>$AR = \frac{H}{D}$</td>
<td>1.60</td>
<td>?</td>
</tr>
<tr>
<td>Blade Wrap Ratio</td>
<td>$\omega_B = \frac{BH}{\pi D \tan \delta}$</td>
<td>100.00%</td>
<td>100.00%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>B</th>
<th>c</th>
<th>D</th>
<th>H</th>
<th>$\delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Blades</td>
<td>Chord Length</td>
<td>Turbine Diameter</td>
<td>Turbine Height</td>
<td>Helical Blade Pitch</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Area (m)</th>
<th>0.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>B (#)</td>
<td>4</td>
</tr>
<tr>
<td>c (m)</td>
<td>0.07</td>
</tr>
<tr>
<td>D (m)</td>
<td>0.25</td>
</tr>
<tr>
<td>H (m)</td>
<td>0.4</td>
</tr>
<tr>
<td>$\delta$ (°)</td>
<td>63.855</td>
</tr>
</tbody>
</table>

| Solidarity Ratio | 0.3 |
| Aspect Ratio     | 1.5 |
| Blade Wrap       | 100 |

<table>
<thead>
<tr>
<th>Thickness (mm)</th>
<th>Chord location (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-10</td>
<td>0.00</td>
</tr>
<tr>
<td>-8.00</td>
<td>0.00</td>
</tr>
<tr>
<td>-6.00</td>
<td>1.75</td>
</tr>
<tr>
<td>-4.00</td>
<td>3.50</td>
</tr>
<tr>
<td>-2.00</td>
<td>5.25</td>
</tr>
<tr>
<td>0.00</td>
<td>7.00</td>
</tr>
<tr>
<td>2.00</td>
<td>9.75</td>
</tr>
<tr>
<td>4.00</td>
<td>12.50</td>
</tr>
<tr>
<td>6.00</td>
<td>15.25</td>
</tr>
<tr>
<td>8.00</td>
<td>18.00</td>
</tr>
</tbody>
</table>

### NACA 0018 Blade

<table>
<thead>
<tr>
<th>Length</th>
<th>Width</th>
</tr>
</thead>
<tbody>
<tr>
<td>70.00</td>
<td>0.1323</td>
</tr>
<tr>
<td>66.50</td>
<td>0.847</td>
</tr>
<tr>
<td>63.00</td>
<td>1.5204</td>
</tr>
<tr>
<td>56.00</td>
<td>2.7545</td>
</tr>
<tr>
<td>49.00</td>
<td>3.8472</td>
</tr>
<tr>
<td>42.00</td>
<td>4.7915</td>
</tr>
<tr>
<td>35.00</td>
<td>5.5587</td>
</tr>
<tr>
<td>28.00</td>
<td>6.0935</td>
</tr>
<tr>
<td>17.50</td>
<td>6.2384</td>
</tr>
<tr>
<td>14.00</td>
<td>6.0242</td>
</tr>
<tr>
<td>10.50</td>
<td>5.6126</td>
</tr>
<tr>
<td>7.00</td>
<td>4.9168</td>
</tr>
<tr>
<td>5.25</td>
<td>4.41</td>
</tr>
<tr>
<td>3.50</td>
<td>3.7324</td>
</tr>
<tr>
<td>1.75</td>
<td>2.7944</td>
</tr>
<tr>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>
This program finds the necessary diameter for a raft made of PVC

**Inputs**

\[ L_{\text{Longpipe}} = 1 \text{ [m]} \quad \text{Length of Long Pipe} \]

\[ L_{\text{Shortpipe}} = 1 \text{ [m]} \quad \text{Length of Short Pipe} \]

\[ D_{\text{OP}} = 0.0762 \text{ [m]} \]

3.00 inch

\[ D_{\text{IP}} = 0.0254 \text{ [m]} \]

1.00 inch

\[ \text{SF} = 1 \quad \text{Submersive Factor} \]

**Converts and Constants**

\[ L_{\text{Pipes}} = 5 \cdot L_{\text{Longpipe}} + 2 \cdot L_{\text{Shortpipe}} \quad \text{Length of Pipes} \]

\[ \rho_{\text{Water}} = 1000 \quad \text{[kg/m}^3\text{]} \quad \text{Density of Water} \]

\[ \rho_{\text{PVC}} = 1400 \quad \text{[kg/m}^3\text{]} \quad \text{Density of PVC} \]

\[ g = 9.81 \quad \text{[m/s}^2\text{]} \quad \text{Gravity} \]

\[ M_{\text{LumpSys}} = 27.2 \quad \text{[kg]} \quad \text{Mass of system} \]

**Calculations**

\[ V_{\text{WaterDisp}} = \text{SF} \cdot 3.14 \cdot L_{\text{Pipes}} \cdot \left( \frac{D_{\text{OP}}}{2} \right)^2 \]

\[ V_{\text{Pipes}} = \text{SF} \cdot 3.14 \cdot L_{\text{Pipes}} \cdot \left[ \left( \frac{D_{\text{OP}}}{2} \right)^2 - \left( \frac{D_{\text{IP}}}{2} \right)^2 \right] \]

\[ F_{\text{Bouyant}} = \rho_{\text{Water}} \cdot g \cdot V_{\text{WaterDisp}} \quad \text{Buoyant Force} \]

\[ F_{\text{Weight}} = \rho_{\text{PVC}} \cdot g \cdot V_{\text{Pipes}} \quad \text{Weight of Pipes} \]

\[ F_{\text{SysWeight}} = M_{\text{LumpSys}} \cdot g \quad \text{Weight of System} \]

\[ F_{\text{Diff}} = F_{\text{Bouyant}} - F_{\text{Weight}} - F_{\text{SysWeight}} \quad \text{Difference between buoyancy and weights} \]

**Parametric Table: Table 1**

<table>
<thead>
<tr>
<th>( D_{\text{OP}} )</th>
<th>( D_{\text{IP}} )</th>
<th>( F_{\text{Diff}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>[m]</td>
<td>[m]</td>
<td>[N]</td>
</tr>
<tr>
<td>Run 1</td>
<td>0.02134</td>
<td>0.0158</td>
</tr>
</tbody>
</table>
### Parametric Table: Table 1

<table>
<thead>
<tr>
<th>Run</th>
<th>D$_{OP}$</th>
<th>D$_{IP}$</th>
<th>F$_{Diff}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Run 2</td>
<td>0.02692</td>
<td>0.02093</td>
<td>-249.4</td>
</tr>
<tr>
<td>Run 3</td>
<td>0.0334</td>
<td>0.02664</td>
<td>-237.3</td>
</tr>
<tr>
<td>Run 4</td>
<td>0.04216</td>
<td>0.03505</td>
<td>-212.4</td>
</tr>
<tr>
<td>Run 5</td>
<td>0.04826</td>
<td>0.04089</td>
<td>-190.8</td>
</tr>
<tr>
<td>Run 6</td>
<td>0.06033</td>
<td>0.0525</td>
<td>-137.3</td>
</tr>
<tr>
<td>Run 7</td>
<td>0.07303</td>
<td>0.06271</td>
<td>-85.01</td>
</tr>
<tr>
<td>Run 8</td>
<td>0.0889</td>
<td>0.07793</td>
<td>21.05</td>
</tr>
<tr>
<td>Run 9</td>
<td>0.1143</td>
<td>0.1023</td>
<td>240.7</td>
</tr>
<tr>
<td>Run 10</td>
<td>0.1413</td>
<td>0.1282</td>
<td>542.9</td>
</tr>
<tr>
<td>Run 11</td>
<td>0.1657</td>
<td>0.1541</td>
<td>931.9</td>
</tr>
<tr>
<td>Run 12</td>
<td>0.2191</td>
<td>0.2027</td>
<td>1800</td>
</tr>
<tr>
<td>Run 13</td>
<td>0.2731</td>
<td>0.2545</td>
<td>3014</td>
</tr>
<tr>
<td>Run 14</td>
<td>0.3239</td>
<td>0.3033</td>
<td>4413</td>
</tr>
<tr>
<td>Run 15</td>
<td>0.3556</td>
<td>0.3332</td>
<td>5388</td>
</tr>
</tbody>
</table>
Power = 50 [W]  
\[ P = \frac{1}{2} \cdot \omega \cdot R \cdot v \]  
\( R = 0.25 \cdot 1 \) [m]  
\( x = 1 \) tip speed ratio  
\( v \cdot x = R \cdot \omega \) calculate omega, hz  
\( \omega = \frac{v \cdot x}{R} \)  
\( T = F \cdot R \) calculate Force  
\( E = 206.8 \text{ [GPa]} \cdot 1 \times 10^9 \text{ [N/m}^2] \) youngs modulus for steel  
\( L = 0.6 \) [m] shaft length  
\( m_{\text{shaft}} = 0.75 \) [kg]  
\( \delta_{\text{shaft}} = m_{\text{shaft}} \cdot g \cdot \frac{L^3}{8 \cdot E \cdot I} \) deflection of shaft from its own weight  
\( \delta_{\text{turbine}} = F \cdot \frac{L^3}{3 \cdot E \cdot I} \) deflection caused by the turbine  
\( \delta = \delta_{\text{shaft}} + \delta_{\text{turbine}} \) total deflection  
\( I = \pi \cdot \left[ \frac{D_o^4 - D_i^4}{64} \right] \) Moment of Inertia of Shaft  
\( D_o = 0.02 \) [m] shaft outer diameter  
\( D_i = 0.014 \) [m] shaft inner diameter  
\( N_c = \frac{30}{\pi} \cdot \sqrt{\frac{g}{\delta}} \cdot 9.549 \cdot \frac{\text{rev/min}}{\text{hz}} \) Rayleigh Ritz Critical Speed  
\( g = 9.81 \) [m/s\(^2\)] gravity  

SOLUTION

Unit Settings: SI C kPa kJ mass deg
\( \delta = 0.002105 \) [m]  
\( \delta_{\text{shaft}} = 0.000161 \) [m]  
\( \delta_{\text{turbine}} = 0.001945 \) [m]  
\( D_o = 0.02 \) [m]  
\( D_i = 0.014 \) [m]  
\( E = 2.068E+11 \) [N/m\(^2\)]  
\( F = 33.33 \) [N]  
\( \omega = 12 \) [1/s]  
\( m_{\text{shaft}} = 0.75 \) [kg]  
\( L = 0.6 \) [m]  
\( N_c = 6224 \) [rev/min]  
\( \text{Power} = 50 \) [W]
R = 0.125 [m] \hspace{2cm} T = 4.167 [N-m]

v = 1.5 [m/s] \hspace{2cm} x = 1

No unit problems were detected.
This program is used to find the proper bearing

\[ a = 3 \text{ for ball-bearings, } 10/3 \text{ for other} \]

\[ F_D = 33.33 \text{ Radial Load} \]

\[ L_D = 1 \times 10^9 \text{ Desired Life} \]

\[ a_f = 1.3 \text{ Load Factor - Table 11-5} \]

\[ R_D = 0.995 \text{ Reliability} \]

\[ L_R = 1000000 \text{ Rated Life (provided by manufacturer)} \]

\[ x_0 = 0.02 \text{ Minimum value of the variate (provided by manufacturer)} \]

\[ \phi - x_0 = 4.439 \text{ Percentile Value of the variate (provided by manufacturer)} \]

\[ b = 1.483 \text{ Weibull Parameter (provided by manufacturer)} \]

\[ k_r = x_0 + \left[ \phi - x_0 \right] \cdot \ln \left[ \frac{\phi}{R_D} \right] \text{ reliability factor} \]

\[ C_{10} = a_f \cdot F_D \cdot \left[ \frac{L_D}{L_R \cdot k_r} \right] \text{ } 1/a } \text{ C}_{10} \text{ rating} \]

**SOLUTION**

**Unit Settings: SI C kPa kJ mass deg**

\[ a = 3 \]

\[ b = 1.483 \]

\[ F_D = 33.33 \]

\[ L_D = 1.000E+09 \]

\[ R_D = 0.995 \]

\[ x_0 = 0.02 \]

\[ a_f = 1.3 \]

\[ C_{10} = 825 \]

\[ k_r = 0.1449 \]

\[ L_R = 1000000 \]

\[ \phi = 4.459 \]

No unit problems were detected.
This program calculates the deflection of the plate used on the platform.

**Inputs**

\[ a = 1 \text{ [m]} \quad \text{Minor length of plate in meters} \]

\[ b = 1 \text{ [m]} \quad \text{Major length of plate in meters} \]

\[ E = 9.23 \times 10^9 \text{ [Pa]} \quad \text{Young's modulus of plate material} \]

\[ v = 0.081 \quad \text{Poisson's ratio of plate material} \]

\[ t = 0.0127 \text{ [m]} \quad \text{Plate thickness} \]

\[ P = 200 \text{ [N]} \quad \text{Concentrated load} \]

\[ e' = 0.1 \text{ [m]} \quad \text{Radius of small area load acts over in meters} \]

\[ k_1 = \text{Interpolate1} \left[ \text{LOOKUP}, 'Ratio', 'k_1', 'Ratio' = \frac{b}{a} \right] \quad \text{Table Lookup} \]

\[ k_2 = \text{Interpolate1} \left[ \text{LOOKUP}, 'Ratio', 'k_2', 'Ratio' = \frac{b}{a} \right] \quad \text{Table Lookup} \]

\[ \sigma = \frac{1.5 \cdot P}{\pi \cdot t^2} \cdot \left[ (1 + v) \cdot \ln \left( \frac{2 \cdot a}{\pi \cdot e'} \right) + 1 - k_2 \right] \quad \text{Max Pressure in N/m}^2 \]

\[ y = k_1 \cdot \frac{P \cdot a^2}{E \cdot t^3} \quad \text{Max Deflection in meters} \]

**SOLUTION**

**Unit Settings:** SI \( C \) kPa \( kJ \) mass \( \text{deg} \)

\[ a = 1 \text{ [m]} \]

\[ b = 1 \text{ [m]} \]

\[ E = 9.230E+09 \text{ [Pa]} \]

\[ k_1 = 0.127 \]

\[ k_2 = 0.564 \]

\[ P = 200 \text{ [N]} \]

\[ t = 0.0127 \text{ [m]} \]

\[ y = 0.001343 \text{ [m]} \]

\[ \sigma = 1.443E+06 \text{ [N/m}^2\] \]

\[ v = 0.081 \]

No unit problems were detected.
Torque coefficient, variables (rho, v, D, omega, mu)

variables

no added load

\[ p_{i01,\text{air}} = \frac{\text{torque}_{0,\text{air}}}{\frac{1}{2} \cdot \rho_{\text{air}} \cdot v_{\text{air}}^2 \cdot R \cdot A_c} \]

\[ p_{i02,\text{air}} = \frac{\mu_{\text{air}}}{\rho_{\text{air}} \cdot d \cdot v_{\text{air}}} \]

\[ p_{i03,\text{air}} = \omega_{\text{air},0} \cdot \left[ \frac{v_{\text{air}}}{d} \right]^{-1} \]

\[ p_{i01,\text{water,full}} = \frac{\text{torque}_{0,\text{water,full}}}{\frac{1}{2} \cdot v_{\text{water,full},0}^2 \cdot R_f \cdot A_{c,\text{full}} \cdot \rho_{\text{water}}} \]

\[ p_{i02,\text{water,full}} = \frac{\mu_{\text{water}}}{\rho_{\text{water}} \cdot d_f \cdot v_{\text{water,full}}} \]

\[ p_{i03,\text{water,full}} = \omega_{\text{water,full},0} \cdot \left[ \frac{v_{\text{water,full},0}}{d_f} \right]^{-1} \]

\[ p_{i01,\text{air}} = p_{i01,\text{water,full}} \]

\[ p_{i02,\text{air}} = p_{i02,\text{water,full}} \]

\[ p_{i03,\text{air}} = p_{i03,\text{water,full}} \]

load 1

\[ p_{i11,\text{air}} = \frac{\text{torque}_{1,\text{air}}}{\frac{1}{2} \cdot \rho_{\text{air}} \cdot v_{\text{air}}^2 \cdot R \cdot A_c} \]

\[ p_{i12,\text{air}} = \frac{\mu_{\text{air}}}{\rho_{\text{air}} \cdot d \cdot v_{\text{air}}} \]

\[ p_{i13,\text{air}} = \omega_{\text{air},1} \cdot \left[ \frac{v_{\text{air}}}{d} \right]^{-1} \]

\[ p_{i11,\text{water,full}} = \frac{\text{torque}_{1,\text{water,full}}}{\frac{1}{2} \cdot v_{\text{water,full},1}^2 \cdot R_f \cdot A_{c,\text{full}} \cdot \rho_{\text{water}}} \]

\[ p_{i12,\text{water,full}} = \frac{\mu_{\text{water}}}{\rho_{\text{water}} \cdot d_f \cdot v_{\text{water,full}}} \]

\[ p_{i13,\text{water,full}} = \omega_{\text{water,full},1} \cdot \left[ \frac{v_{\text{water,full},1}}{d_f} \right]^{-1} \]
\pi_{11,\text{air}} = \pi_{11,\text{water,full}} \\
\pi_{12,\text{air}} = \pi_{12,\text{water,full}} \\
\pi_{13,\text{air}} = \pi_{13,\text{water,full}} \\
\text{load 2} \\
\pi_{21,\text{air}} = \frac{\text{torque}_{2,\text{air}}}{\frac{1}{2} \cdot \rho_{\text{air}} \cdot v_{\text{air}}^2 \cdot R \cdot A_c} \\
\pi_{22,\text{air}} = \frac{\mu_{\text{air}}}{\rho_{\text{air}} \cdot d \cdot v_{\text{air}}} \\
\pi_{23,\text{air}} = \omega_{\text{air},2} \cdot \left( \frac{v_{\text{air}}}{d} \right)^{-1} \\
\pi_{21,\text{water,full}} = \frac{\text{torque}_{2,\text{water,full}}}{\frac{1}{2} \cdot v_{\text{water,full},2}^2 \cdot R_f \cdot A_{c,\text{full}} \cdot \rho_{\text{water}}} \\
\pi_{22,\text{water,full}} = \frac{\mu_{\text{water}}}{\rho_{\text{water}} \cdot d_f \cdot v_{\text{water,full},2}} \\
\pi_{23,\text{water,full}} = \omega_{\text{water,full},2} \cdot \left( \frac{v_{\text{water,full},2}}{d_f} \right)^{-1} \\
\pi_{21,\text{air}} = \pi_{21,\text{water,full}} \\
\pi_{22,\text{air}} = \pi_{22,\text{water,full}} \\
\pi_{23,\text{air}} = \pi_{23,\text{water,full}} \\
v_{\text{water,full},2} = v_{\text{water,full}} \\
\text{load 3} \\
\pi_{31,\text{air}} = \frac{\text{torque}_{3,\text{air}}}{\frac{1}{2} \cdot \rho_{\text{air}} \cdot v_{\text{air}}^2 \cdot R \cdot A_c} \\
\pi_{32,\text{air}} = \frac{\mu_{\text{air}}}{\rho_{\text{air}} \cdot d \cdot v_{\text{air}}} \\
\pi_{33,\text{air}} = \omega_{\text{air},3} \cdot \left( \frac{v_{\text{air}}}{d} \right)^{-1} \\
\pi_{31,\text{water,full}} = \frac{\text{torque}_{3,\text{water,full}}}{\frac{1}{2} \cdot v_{\text{water,full},3}^2 \cdot R_f \cdot A_{c,\text{full}} \cdot \rho_{\text{water}}} \\
\pi_{32,\text{water,full}} = \frac{\mu_{\text{water}}}{\rho_{\text{water}} \cdot d_f \cdot v_{\text{water,full},3}}
\[ p_{33, \text{water, full}} = \frac{\omega_{\text{water, full,} 3}}{d_f} \left( \frac{v_{\text{water, full,} 3}}{d_f} \right)^{-1} \]

\[ p_{31, \text{air}} = p_{31, \text{water, full}} \]

\[ p_{32, \text{air}} = p_{32, \text{water, full}} \]

\[ p_{33, \text{air}} = p_{33, \text{water, full}} \]

\[ v_{\text{water, full,} 3} = v_{\text{water, full}} \]

\textit{tip speed ratio}

\[ \lambda_{\text{air, 0}} = R \cdot \frac{\omega_{\text{air, 0}}}{v_{\text{air}}} \]

\[ \lambda_{\text{water, full, 0}} = R_f \cdot \frac{\omega_{\text{water, full, 0}}}{v_{\text{water, full}}} \]

\[ \lambda_{\text{air, 1}} = R \cdot \frac{\omega_{\text{air, 1}}}{v_{\text{air}}} \]

\[ \lambda_{\text{water, full, 1}} = R_f \cdot \frac{\omega_{\text{water, full, 1}}}{v_{\text{water, full}}} \]

\[ \lambda_{\text{air, 2}} = R \cdot \frac{\omega_{\text{air, 2}}}{v_{\text{air}}} \]

\[ \lambda_{\text{water, full, 2}} = R_f \cdot \frac{\omega_{\text{water, full, 2}}}{v_{\text{water, full}}} \]

\[ \lambda_{\text{air, 3}} = R \cdot \frac{\omega_{\text{air, 3}}}{v_{\text{air}}} \]

\[ \lambda_{\text{water, full, 3}} = R_f \cdot \frac{\omega_{\text{water, full, 3}}}{v_{\text{water, full}}} \]

\textit{power coefficient}

\[ c_{p, \text{air, 0}} = \frac{\text{torque}_{0, \text{air}}}{\frac{1}{2} \cdot \rho_{\text{air}} \cdot v_{\text{air}}^3 \cdot A_c} \cdot \frac{\omega_{\text{air, 0}}}{v_{\text{air}}} \]

\[ c_{p, \text{air, 0}} = c_{p, \text{water, full, 0}} \]

\[ c_{p, \text{water, full, 0}} = \frac{\text{torque}_{0, \text{water, full}}}{\frac{1}{2} \cdot \rho_{\text{water}} \cdot v_{\text{water, full}}^3 \cdot A_{c, \text{full}}} \cdot \frac{\omega_{\text{water, full, 0}}}{v_{\text{water, full}}} \]

\[ c_{p, \text{air, 1}} = \frac{\text{torque}_{1, \text{air}}}{\frac{1}{2} \cdot \rho_{\text{air}} \cdot v_{\text{air}}^3 \cdot A_c} \cdot \frac{\omega_{\text{air, 1}}}{v_{\text{air}}} \]

\[ c_{p, \text{air, 1}} = c_{p, \text{water, full, 1}} \]
\[ cp,\text{water,full,1} = \frac{\text{torque}_{1,\text{water,full}}}{\frac{1}{2} \cdot \rho_{\text{water}} \cdot v_{\text{water,full}}^3 \cdot A_{c,\text{full}}} \cdot \omega_{\text{water,full,1}} \]

\[ cp,\text{air,2} = \frac{\text{torque}_{2,\text{air}}}{\frac{1}{2} \cdot \rho_{\text{air}} \cdot v_{\text{air}}^3 \cdot A_{c}} \cdot \omega_{\text{air,2}} \]

\[ cp,\text{water,full,2} = \frac{\text{torque}_{2,\text{water,full}}}{\frac{1}{2} \cdot \rho_{\text{water}} \cdot v_{\text{water,full}}^3 \cdot A_{c,\text{full}}} \cdot \omega_{\text{water,full,2}} \]

\[ cp,\text{air,3} = \frac{\text{torque}_{3,\text{air}}}{\frac{1}{2} \cdot \rho_{\text{air}} \cdot v_{\text{air}}^3 \cdot A_{c}} \cdot \omega_{\text{air,3}} \]

\[ cp,\text{water,full,3} = \frac{\text{torque}_{3,\text{water,full}}}{\frac{1}{2} \cdot \rho_{\text{water}} \cdot v_{\text{water,full}}^3 \cdot A_{c,\text{full}}} \cdot \omega_{\text{water,full,3}} \]

**parameters**

\[ A_{c} = d \cdot H \quad \text{area of half scale turbine} \]

\[ A_{c,\text{full}} = d_{f} \cdot H_{f} \quad \text{area of full scale turbine} \]

\[ d_{f} = 2 \cdot d \quad \text{diameter of full scale turbine} \]

\[ H_{f} = 2 \cdot H \quad \text{height of full scale turbine} \]

\[ R_{f} = R \cdot 2 \quad \text{radius of full scale turbine} \]

\[ d = 0.125 \quad \text{[m] diameter of half scale turbine} \]

\[ R = \frac{d}{2} \quad \text{radius of half scale turbine} \]

\[ H = 0.2 \quad \text{[m] height of half scale turbine} \]

\[ \rho_{\text{water}} = \rho_{\text{water}, T = \text{T}, P = \text{P}} \quad \text{density of water} \]

\[ \mu_{\text{water}} = \text{Visc}_{\text{water}, T = \text{T}, P = \text{P}} \quad \text{viscosity of water} \]

\[ \rho_{\text{air}} = \rho_{\text{Air}, T = \text{T}, P = \text{P}} \quad \text{density of air} \]

\[ \mu_{\text{air}} = \text{Visc}_{\text{Air}, T = \text{T}} \quad \text{viscosity of air} \]

\[ P = 14.7 \cdot \frac{6895}{\text{psia}} \quad \text{atmospheric pressure} \]

\[ T = \left[75 - 32\right] \cdot \frac{5}{9} \cdot 1 \quad \text{[C] average temperature} \]

**dimensions for loads**

\[ a \text{ is side } 1 \]

\[ b \text{ is side } 2 \]

\[ m \text{ is the mass of the load} \]
\begin{align*}
\mathbf{i}_i &= \frac{1}{12} \cdot m_i \cdot \left[ a_i^2 + b_i^2 \right] + I_r + I_t \quad \text{for } i = 1 \text{ to } 3 \quad \text{moment of inertia for the whole system} \\
\mathbf{I}_0 &= I_r + I_t \quad \text{moment of inertia of the shaft and turbine} \\
\mathbf{I}_r &= 0.00000996 \quad \text{[kg} \cdot \text{m}^2] \quad \text{moment of inertia of the shaft} \\
\mathbf{I}_t &= 0.02561 \quad \text{[kg} \cdot \text{m}^2] \quad \text{moment of inertia of the turbine} \\
\mathbf{a}_1 &= 3.25 \cdot 0.0254 \cdot \frac{\text{m}}{\text{in}} \\
\mathbf{b}_1 &= 4 \cdot 0.0254 \cdot \frac{\text{m}}{\text{in}} \\
\mathbf{m}_1 &= 0.135 \quad \text{[kg]} \\
\mathbf{a}_2 &= 2.5 \cdot 0.0254 \cdot \frac{\text{m}}{\text{in}} \\
\mathbf{b}_2 &= 6 \cdot 0.0254 \cdot \frac{\text{m}}{\text{in}} \\
\mathbf{m}_2 &= 0.234 \quad \text{[kg]} \\
\mathbf{a}_3 &= 3.25 \cdot 0.0254 \cdot \frac{\text{m}}{\text{in}} \\
\mathbf{b}_3 &= 9 \cdot 0.0254 \cdot \frac{\text{m}}{\text{in}} \\
\mathbf{m}_3 &= 0.268 \quad \text{[kg]} \\

\text{Torque Calculations} \\
\mathbf{\text{torque}}_{0,\text{air}} &= \mathbf{\alpha}_{\text{air},0} \cdot I_0 \\
\mathbf{\text{torque}}_{1,\text{air}} &= \mathbf{\alpha}_{\text{air},1} \cdot I_1 \\
\mathbf{\text{torque}}_{2,\text{air}} &= \mathbf{\alpha}_{\text{air},2} \cdot I_2 \\
\mathbf{\text{torque}}_{3,\text{air}} &= \mathbf{\alpha}_{\text{air},3} \cdot I_3 \\
\mathbf{v}_{\text{air}} &= \frac{x}{60} \cdot \mathbf{v}_{\text{max}} \cdot 0.44704 \cdot \frac{\text{m/s}}{\text{mph}} \\
\mathbf{v}_{\text{max}} &= 110 \quad \text{[mph]} \\
\mathbf{\omega}_{\text{air},0} &= \mathbf{\omega}_{\text{rpm},0} \cdot 0.1047 \cdot \frac{\text{rad/s}}{\text{rev/min}} \quad \text{convert rotational speed (rpm) to angular velocity (rad/s)} \\
\mathbf{\alpha}_{\text{air},0} &= \frac{\mathbf{\omega}_{\text{air},0}}{\text{time}}
\end{align*}
\( \omega_{\text{rpm}, w, 0} = \omega_{\text{water, full}, 0} \cdot 9.549 \cdot \frac{\text{rev/min}}{\text{rad/s}} \)

*convert angular velocity (rad/s) to rotational speed (rpm)*

\( \omega_{\text{air}, 1} = \omega_{\text{rpm}, 1} \cdot 0.1047 \cdot \frac{\text{rad/s}}{\text{rev/min}} \)

*convert rotational speed (rpm) to angular velocity (rad/s)*

\( \alpha_{\text{air}, 1} = \frac{\omega_{\text{air}, 1}}{\text{time}} \)

\( \omega_{\text{rpm}, w, 1} = \omega_{\text{water, full}, 1} \cdot 9.549 \cdot \frac{\text{rev/min}}{\text{rad/s}} \)

*convert angular velocity (rad/s) to rotational speed (rpm)*

\( \omega_{\text{air}, 2} = \omega_{\text{rpm}, 2} \cdot 0.1047 \cdot \frac{\text{rad/s}}{\text{rev/min}} \)

*convert rotational speed (rpm) to angular velocity (rad/s)*

\( \alpha_{\text{air}, 2} = \frac{\omega_{\text{air}, 2}}{\text{time}} \)

\( \omega_{\text{rpm}, w, 2} = \omega_{\text{water, full}, 2} \cdot 9.549 \cdot \frac{\text{rev/min}}{\text{rad/s}} \)

*convert angular velocity (rad/s) to rotational speed (rpm)*

\( \omega_{\text{air}, 3} = \omega_{\text{rpm}, 3} \cdot 0.1047 \cdot \frac{\text{rad/s}}{\text{rev/min}} \)

*convert rotational speed (rpm) to angular velocity (rad/s)*

\( \alpha_{\text{air}, 3} = \frac{\omega_{\text{air}, 3}}{\text{time}} \)

\( \omega_{\text{rpm}, w, 3} = \omega_{\text{water, full}, 3} \cdot 9.549 \cdot \frac{\text{rev/min}}{\text{rad/s}} \)

*convert angular velocity (rad/s) to rotational speed (rpm)*

**power calculations**

\[
\begin{align*}
\text{Power}_1 &= \text{torque}_{1, \text{water, full}} \cdot \omega_{\text{water, full}, 1} \\
\text{Power}_2 &= \text{torque}_{2, \text{water, full}} \cdot \omega_{\text{water, full}, 2} \\
\text{Power}_3 &= \text{torque}_{3, \text{water, full}} \cdot \omega_{\text{water, full}, 3} \\
\text{Power}_0 &= \text{torque}_{0, \text{water, full}} \cdot \omega_{\text{water, full}, 0} \\
\text{time} &= 10 \ [\text{s}] 
\end{align*}
\]