Evaluation of Code Period Formula for Concrete MRF Buildings

Rakesh K. Goel\textsuperscript{1}, M. ASCE and Anil K. Chopra\textsuperscript{11}, M. ASCE

Abstract

The empirical formula specified in US building codes for fundamental vibration period of concrete moment resisting frame buildings is evaluated using periods of buildings measured from their strong motion records. The presented results show that the current code formula may lead to fundamental periods that are much shorter than the measured periods. In order to obtain better correlation between measured and computed periods, regression analysis technique has been used. The results indicate that the value of $C_i$ in the current code formula should be increased from its present value of 0.030 to 0.035.

Introduction

The empirical formula specified in US building codes -- UBC-94 (Uniform Building Code, 1994), SEAOC-90 (Recommended Lateral Force Requirements, 1990), and NEHRP-94 (NEHRP, 1994) -- for fundamental vibration period is of the form:

$$T = C_i H^{3/4}$$

where $H$ is the height of the building in feet above the base and $C_i$ is a numerical coefficient related to the lateral-force-resisting system. The values of $C_i$ specified in these codes are: 0.035 for steel moment resisting frame (MRF) buildings, 0.03 for concrete MRF buildings and eccentrically braced steel frames, and 0.02 for other buildings which include buildings with shear walls.

The codes specify that the period calculated from any rational analysis should not be longer than that estimated from empirical formula (Eq. 1) by a certain factor. The factors specified in various US codes are: 1.3 for high seismic region (Zone 4) and 1.4 for other regions (Zones 3, 2, and 1) in UBC-94; and a range of values with

\textsuperscript{1}Assistant Professor, Department of Civil and Environmental Engineering, Syracuse University, Syracuse, NY 13244-1190.

\textsuperscript{11}Johnson Professor, Department of Civil Engineering, University of California, Berkeley, CA 94720.
1.2 for regions of high seismicity to 1.7 for regions of very low seismicity in NEHRP-94. SEAOC-90 specifies that the base shear calculated using the period from the rational analysis shall not be less than 80 percent of the value obtained by using the period from the empirical formula, which corresponds to a value of 1.4.

The formula for fundamental period (Eq. 1) in US building codes is based largely on motions of buildings recorded during the 1971 San Fernando earthquake. This formula should be re-evaluated in light of the wealth of data that has become available from recent earthquakes. This research investigation is aimed towards filling this need. For this purpose, a comprehensive database has been developed on periods of buildings identified (measured) from their strong motions recorded during the 1971 and subsequent earthquakes. The database included 37 records for 27 concrete MRF buildings; number of records exceeded the number of buildings because some buildings yielded data for more than one earthquake or because the relevant information was reported by more than one investigator for the same earthquake. This database is then used to evaluate the code formula for estimating the fundamental period of buildings with concrete moment resisting frames.

Evaluation of Code Period Formula

In order to evaluate the code period formulas, the measured building periods identified from their strong motion records are compared with those obtained from the empirical code formula (Eq. 1) in Figure 1 where they are plotted against the building height. The measured periods in two orthogonal directions are shown by solid circles connected by a vertical line, whereas code periods are shown by the curve denoted as $T$. Note that the code formula provides the same period in the two directions as long as the lateral resisting systems in these directions are identical. Also included are curves for $1.2T$ and $1.4T$ representing restrictions on the period from rational analysis imposed by various US codes for high seismic regions like California. These results for concrete MRF buildings show that:

- The code formula leads to periods that are generally shorter than measured periods.
- The code formula tends to provide a lower bound, with only a few exceptions, of measured periods for building up to 160 ft in height.
- The code formula leads to periods much shorter than the measured periods for buildings in the range of 160 ft to 225 ft. For such buildings, the lower bound tends to be about 1.2 times the code period.
- The data for concrete MRF buildings taller than 225 ft is limited. However, it appears that the code formula would lead to much shorter period compared to measured periods for such buildings as well.
- The measured periods of most concrete MRF buildings fall between the curves for $1.2T$ and $1.4T$. This indicates that the limits imposed by various seismic codes on the period from the rational analysis are reasonable for high seismic regions like California. This data, however, can not be used to verify the much higher limits,
for example $1.7T$ imposed by NEHRP-94, in regions of low seismicity. The higher limits should be verified using measured periods of buildings in regions of low seismicity.

![Figure 1. Periods of concrete MRF buildings.](image1)

![Figure 2. Periods of strongly shaken concrete MRF buildings.](image2)

Since it is most useful to examine the periods of buildings that have been shaken strongly but did not reach their yield limit, the results are also presented in Figure 2 for concrete MRF buildings that were subjected to base accelerations in excess of 0.15g. The cutoff value of base acceleration was selected based on judgment. This value does not necessarily imply that all such buildings were stressed close to their yield limits. An exact determination would require detailed structural analyses which were not implemented as part of this research investigation. These results show the following trends:

- The code formula leads to periods much shorter than the measured periods.
- The lower bound of measured periods of strongly shaken buildings appears to be 1.2 times the code period for the range of building heights considered.
- In general, fundamental periods of strongly shaken buildings tend to be longer compared to less strongly shaken buildings. This becomes apparent by comparing results of Figure 2 with those of Figure 1. The longer periods are due to reduced stiffness resulting from increased cracking of concrete during stronger shaking.
- The coefficient $C_r=0.030$ in current codes may be too conservative. A value close to 0.035 may be more appropriate. This trend is consistent with the results of regression analysis presented latter in this paper.

**Improved Code-Type Formula**

Although the results presented in the preceding section indicate that the code formula in general provides periods that are shorter compared to the measured periods, this formulas could be improved to provide better correlation with the measured periods. For this purpose, the following form (or structure) of the theoretical formula (Housner and Brady, 1963) was utilized:
\[ T = \alpha H^\beta \]  \hspace{1cm} (3)

in which \( \alpha \) and \( \beta \) are the constants which depend on the building behavior with \( 0.5 \leq \beta \leq 1 \). It is useful to note that the code formula (Eq. 1) is of the same form as Eq. (3) but with \( \alpha = C_i \) and \( \beta = 3/4 \).

The theoretical form of Eq. (3) was adopted in the present investigation and constants \( \alpha \) and \( \beta \) were determined by regression analysis of the measured period data. The regression analysis led to the formula that represented the best fit, in the least squared sense, to the measured period data. However, the formula that is of interest for code-type application is the one that provides a lower bound to the measured data. Such formula was obtained by lowering the best fit curve in such a way that it represented the 84.1 percentile, mean-minus-one-standard-deviation solution. This bound implies that only 15.9 percent of the measured (or actual) periods would fall below the lower bound curve.

The formulas were obtained from two regression analyses. In the first analysis, both constants \( \alpha \) and \( \beta \) in Eq. (3) were considered as variables. This unconstrained regression analysis led to the best possible fit and thus the minimum possible error between the measured and calculated periods. In the second analysis, only \( \alpha \) was considered as variable; \( \beta \) was fixed at 0.75. For cases where \( \beta \) from the first analysis was much different than 0.75, this analysis led to higher errors for obvious reasons. However, this constrained regression analysis was included in order to investigate by how much the value of \( \alpha (= C_i \) in current codes) should be modified in light of the new data. For brevity, the results are presented only for the second analysis.

The results of regression analysis are presented in Figures 3 and 4 where the measured period data are shown in solid circles with periods of a building in the two orthogonal directions connected by a vertical line. The results obtained from the regression analysis are plotted as two solid curves. The first one, labeled as \( T_R \), corresponds to the best fit where as the second, denoted as \( T_L \), corresponds to the lower bound to the measured period data. For each of the two curves, the associated formula is included in the figure.

These results of Figure 3 show that the lower bound formula obtained from regression analysis of concrete MRF buildings is identical to the one in current US building codes (Eq. 1). This indicates that the current code formula leads to an excellent estimate of the fundamental period for such buildings.

The preceding conclusion is based on data that include periods of buildings that were not necessarily shaken very strongly. Ideally it would be useful to consider only those buildings which are shaken close to the elastic limit but not damaged. Therefore, regression analysis was also implemented for strongly shaken buildings and the results are presented in Figure 4. These results show that the values of constant \( \alpha \), and hence periods, obtained from regression analysis of strongly shaken buildings...
concrete MRF buildings are higher compared to the less strongly shaken buildings. This confirms the generally accepted wisdom that periods of concrete buildings increase with level of shaking.

As mentioned previously, it is desirable to develop the lower bound formula for fundamental period that is based on data from strongly shaken buildings. Therefore, results of Figure 4 should be more appropriate for code-type applications. This indicates that value of $C$, in the code formula (Eq. 1) should be increased from its present value of 0.030 to 0.035.

![Figure 3. Regression analysis for concrete MRF buildings.](image)

![Figure 4. Regression analysis of strongly shaken concrete MRF buildings.](image)

**Acknowledgments**

This research investigation is supported by the National Science Foundation under Grant CMS-9416265. The authors are grateful for this support.

**References**


