Closure by Rakesh K. Goel, Anil K. Chopra

The writers wish to thank the discussers for their interest in the paper.

The discussion by Makarios and Anastasiadis focused on the question of how the static eccentricities for multistory buildings should be defined. As the authors pointed out in the original paper, several alternative definitions exist for these eccentricities; the various definitions were evaluated elsewhere (Hejaj and Chopra 1987). However, the purpose of the paper was to simply present an approach for the convenient implementation of the code procedure, with the eccentricity defined as the distance between the center of mass (CM) and center of rigidity (CR)—the definition used in the engineering field of several countries.

As pointed out by the discussers, the locations of CRs in most multistory buildings may be highly irregular over the height. The locations of the shear centers (SCs), on the other hand, are generally much more regular. However, the floor forces applied at the CRs and the story shears applied at the SCs lead to identical story torsional moments and, hence, the same design forces in resisting elements (Tso 1990). Therefore, the highly irregular locations of the CRs should not be of concern during the implementation of code procedures.

The discussion by Toro pointed out a discrepancy in an intermediate step of the procedure presented in the original paper. In particular, he illustrated that forces associated with either (1a) or (1b), for some of the members obtained by the suggested procedure, may not always be equivalent to those obtained from the procedure using CRs. However, the final design forces obtained by the suggested procedure are always correct (column 8 of Table 8).

The reason for the discrepancy in intermediate calculations is the unknown direction of the accidental eccentricity in the absence of information regarding locations of CRs. The accidental eccentricity in (1a) is additive to the first term involving the static eccentricity, indicating that the accidental eccentricity is in the same direction as the static eccentricity. On the other hand, the accidental eccentricity in (1b) is subtractive, implying that it is in the direction opposite to the static eccentricity. This requires that the directions of the static eccentricities and, hence, the locations of the CRs with respect to the CMs be known. However, these locations are not known in the procedure presented in the paper. For this reason, it is possible to start with the incorrect algebraic sign of the floor torques resulting from the accidental eccentricity in step 3 of the procedure. The alleviate this problem, the procedure in the original paper specified that the sign of $r^{(3)}$—the result of step 3—should always be the one that increases the magnitude obtained from the sum of the first two terms in (23) and (24). In other words, the response $r^{(3)}$, resulting from the accidental eccentricity, is always additive in the suggested procedure. Since it should be additive in one equation and subtractive in the other, the sign of $r^{(3)}$ in one of the two equations would be incorrect, leading to the discrepancy pointed out by the discusser.

Although it is difficult to imagine when the intermediate results might be useful, their correct values, if desired, can be obtained by modifying the procedure presented in the original paper.

---


2Johnson Prof. of Civ. Engrg., Univ. of California, Berkeley, CA 94720.
as follows. If \( r^{(1)} \) is less than \( r^{(2)} \) in magnitude, the sign of \( r^{(3)} \) should be such that it increases the magnitude obtained from the sum of the first two terms in (23); conversely, it reduces the magnitude obtained from the sum of the first two terms in (24). On the other hand, if \( r^{(1)} \) is greater than \( r^{(2)} \) in magnitude, the sign of \( r^{(3)} \) should be taken such that it increases the magnitude obtained by the sum of the first two terms in (24) and reduces the magnitude in (23).

To illustrate the modified procedure, let us calculate values of forces \( V^{(a)}_1 \) and \( V^{(a)}_2 \) in frame \( B \), and \( V^{(a)}_3 \) and \( V^{(a)}_4 \) in frame \( C \), using the results presented in Table 8 by the discusser. Since \( V^{(1)}_j \) is greater than \( V^{(2)}_j \) for all four members, according to the modified procedure the algebraic sign \( V^{(3)}_j \) should be selected such that it reduces the magnitude of the sum of the first two terms in (23). This leads to values of

\[
\begin{align*}
V^{(a)}_4 &= -0.5V^{(1)}_4 + 1.5V^{(2)}_4 - V^{(3)}_4 = -0.5 \times 1.542 + 1.5 \times 1.424 - 0.186 = 1.179 \\
V^{(a)}_3 &= -0.5V^{(1)}_3 + 1.5V^{(2)}_3 - V^{(3)}_3 = -0.5 \times 2.845 + 1.5 \times 2.540 - 0.342 = 2.045
\end{align*}
\]

for frame \( B \) and

\[
\begin{align*}
V^{(a)}_2 &= -0.5V^{(1)}_2 + 1.5V^{(2)}_2 - V^{(3)}_2 = -0.5 \times 2.528 + 1.5 \times 1.305 - 0.483 = 0.211 \\
V^{(a)}_1 &= -0.5V^{(1)}_1 + 1.5V^{(2)}_1 - V^{(3)}_1 = -0.5 \times 2.448 + 1.5 \times 1.398 - 0.561 = 0.312
\end{align*}
\]

for frame \( A \). These are the same values as those in column 6 of Table 8 obtained by the discusser using the procedure involving the CRs. The values of \( V^{(b)}_j \) and the final design forces remain unchanged.

**Errata.** The following corrections should be made to the original paper: Page 3039, (1b): the positive sign in front of \( \beta b_j \) should be negative.