INTRODUCTION - To develop a noninvasive method of detecting arterial occlusive disease, pulse waveforms were recorded at two locations on the lower extremities of young normal volunteers and patients with arteriographically confirmed arterial occlusive disease. Pulses were monitored using impedance plethysmography at the knee and the iliac regions. The frequency spectra of the abnormal iliac waveforms contained 4-5 harmonics as compared to the 2-3 harmonics present in normals'. It was hypothesized that the occurrence of high frequencies resulted from pulse wave interactions with diseased portions of the vessel. This paper will present the results of a theoretical and experimental model developed to test this hypothesis.

CLINICAL BACKGROUND - Pulses were monitored at two levels of the extremities in 4 normal volunteers and 13 patients. Information about distal flow in the leg was provided by waveforms obtained when pairs of strip electrodes were placed directly proximal and distal to the knee. Volume pulses from the iliac arteries were acquired using an original electrode array, consisting of two ECG electrodes placed just below the navel and two electrodes placed on the top of the thigh. Figure 1 shows sample pulse waveforms recorded at each location in: a normal volunteer, a patient with severe disease at one level of the limb, and a patient with severe disease at a combination of levels.

The knee pulses of the normal volunteer exhibited a large primary peak and a second, smaller peak. Patient tracings display a damped primary peak and a highly attenuated or absent second wave. This observation may indicate disease and correlates well with the findings of other investigators [1].
The morphology of the iliac pulses reveals some anomalies. The normal volunteer waveforms showed clear primary pulses with solitary secondary pulses. The pulses for the patient with single disease location had many smaller peaks present. This was an unexpected finding. It is hypothesized that these harmonics are the result of pulse interactions with arterial obstructive disease (AOD).

This observation can be quantified by performing spectral analysis on this signal. Figure 2 shows the power spectrum of a normal volunteer and a patient with AOD. The normal spectrum shows no significant peaks beyond the second harmonic. Five harmonics are present in the diseased case. The goal of the present study is to understand the fundamental basis of this difference in arterial pulse characteristics.

![Figure 2 - Power Spectrum Comparison](image)

**EXPERIMENTAL ANALYSIS** - An experimental model was developed to study wave propagation in a straight compliant tube. A silicone rubber tube was used to model the aorta. Experiments modeling a perfect reflecting site, were achieved with a 96 cm. tube having a completely closed end. An oscillating diaphragm provided a sinusoidal input of specified frequency. Flow was monitored at the proximal end of the tube with an electromagnetic flowmeter and transducers located a distance 15 and 92 cm. away from the diaphragm recorded pressure. The two pressure readings provided a means of comparing variations in pulse morphology and allowed calculation of wave speed and attenuation.

Experiments were performed using oscillations from 1 to 10 Hz. Flow, pressure and sinusoidal input were monitored with a 4-channel signal processor. The signal processor provided a means of obtaining averaged time and frequency domain data. A plotter, provided a permanent record of results.

Figure 3 depicts the results obtained from the model at a frequency of 8 Hz. All driving frequencies tested displayed similar results, an essentially sinusoidal signal for both pressure and flow. The power spectrum (average of 50 records) had one major peak at the fundamental driving frequency and only minor peaks at the harmonics.
THEORETICAL MODEL - The theoretical model developed to simulate wave propagation incorporated a frequency response analysis based upon steady oscillatory forced vibrations. The linearized equations for oscillatory flow were solved by a separation of variables technique to determine constitutive equations for flow and pressure head. The functions were represented as the product of spatial and time varying quantities [2]:

\[
\text{Flow: } \quad q' = \text{Re} \left[ Q(x)e^{i\omega t} \right]
\]

\[
\text{Press. Head: } \quad h' = \text{Re} \left[ H(x)e^{i\omega t} \right]
\]

Transfer equations for the complex valued spatial component of flow and pressure head were determined from the particular solutions. The equations were:

\[
H(x) = H_{D,u} \cosh \gamma x \pm Z_c Q_{D,u} \sinh \gamma x
\]

\[
Q(x) = -(H_{D,u}/Z_c) \sinh \gamma x \pm Q_{D,u} \cosh \gamma x
\]

where:

- \(H_{D,u}\) = downstream or upstream pressure head
- \(Q_{D,u}\) = downstream or upstream flow
- \(\gamma\) = propagation constant, a function of: resistance, capacitance, and inerance
- \(Z_c\) = characteristic impedance, a parameter based on constants of the system

Hydraulic impedance, the ratio of pressure head to flow, was the basis of the third constitutive equation. The transfer equation for impedance was:

\[
Z(x) = \frac{(Z_{D,u} \pm Z_c \tanh \gamma x)}{(1 \pm (Z_{D,u}/Z_c) \tanh \gamma x)}
\]

The impedance at any point along the tube may also be determined from the reflection coefficient. Reflection in the system is caused by a change in characteristic impedance and is quantified through the complex valued reflection coefficient. An open end tube has a coefficient of \((-1.0,0.0)\), a closed end tube \((1.0,0.0)\), and an infinitely long tube a coefficient of \((0.0,0.0)\). The hydraulic impedance and reflection coefficient are related by:

\[
Z(x) = Z_c \left( \frac{\Gamma(x) + 1}{\Gamma(x) - 1} \right)
\]

where: \(\Gamma = \text{reflection coefficient}\)
STRAIGHT ELASTIC TUBE RESULTS - To solve these equation, boundary conditions were stated at the proximal and distal points of the tube. The proximal end boundary condition specified flow magnitude from the experimental data. At the distal end, impedance was specified from the reflection coefficient for a closed end. Upstream impedance was calculated using equation (5) and upstream pressure was determined from the impedance and known flow. With the upstream parameters evaluated, the pressure and flow profile along the tube was calculated from eqns. (3) and (4). The pressures at 15 and 92 cm. were compared to the values obtained experimentally, Figure 4.

![Figure 4 - Comparison of Theory and Experiment](image)

Both models displayed a steady pressure with one resonante peak, at about 5 Hz. The theory and model agree quantitatively with both pressures being of the same order of magnitude. Broader resonance peaks in the experimental model may be due to the uncontrolled source impedance induced by the oscillator and electromagnetic flowmeter.

CONCLUSIONS - The behavior of sinusoidal pulses in a straight compliant tube has been demonstrated in both a theoretical and an experimental model study. While the amplitude of the pulses was seen to vary with location along the tube as well as with the source frequency; higher harmonics, similar to those seen in human data, were not demonstrated. It is concluded that these harmonics are not due to the effect of a complete arterial obstruction. Further experiments are underway to evaluate the contribution of nonlinear elements, including less than complete obstruction, branching and variable compliance.

REFERENCES