A Compensated Acoustic Actuator for Systems with Strong Dynamic Pressure Coupling

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This study improves the performance of a previously developed acoustic actuator in the presence of an acoustic duct system with strong pressure coupling. The speaker dynamics and the acoustic duct dynamics are first modeled separately. The two systems are then coupled, and the resulting system is modeled. A velocity sensor is developed and used in feedback compensation. The resulting speaker system has minimal magnitude and phase variation over a 20–200 Hz bandwidth. These conclusions are verified through experimental results.

Introduction

Active noise control is an expanding field in the automotive and aircraft industries. Commercial products are currently available (Bradley, 1995; Warner, 1995) which use passive and active controls to treat unwanted noise. Passive control treats high frequency noise with dampening material which must be of the same physical dimension as the wavelength of the sound wave to be effective (Radcliffe et al. 1994). Below 200 Hz the wave length in air is approximately 0.26 m or longer, requiring approximately 0.25 m of damping material. For such low frequency noise, active control methods which rely on a combination of sensors, a controller, and actuators can be used.

One system that has received much attention in this field is the acoustic duct, which consists of a long, hard walled enclosure. Hull showed that the resonances excited by a noise source in an acoustic duct can be attenuated using feedback active noise control (Hull, 1993). Attempts at wide band noise control were hindered by actuator dynamics that caused the measured control input to deviate from the desired control. Gogate proposed a strategy for eliminating the effects of speaker dynamics through feedback compensation (Radcliffe et al., 1996). The original design did not include the effects of the coupled dynamics through the interaction of the plant pressure and the actuator. Figure 1 shows the speaker face velocity to primary coil voltage frequency response of the original compensator with two cases: the dashed line represents the response of the speaker face exposed to a large room, and the solid line represents the response when the speaker is coupled with an acoustic duct. In a large room there is little magnitude and phase variation from 20–200 Hz, but the measured response of the speaker face matches the input signal and the relative magnitude and phase between the 2 signals is small. When the speaker is coupled with the acoustic duct, the magnitude and phase differ dramatically from the input signal at the resonance frequencies of the duct.

In many noise control applications it is desirable to use an actuator which does not exhibit dynamic response in the frequency range that is of interest in the process. Such an "ideal actuator" could be said to have a unity gain and zero phase in the controller bandwidth. In this study, the controller bandwidth is defined by the 20–200 Hz bandwidth achieved by the experimental system. This bandwidth is acceptable since noise above this frequency can be easily treated with passive noise control. Although actuator dynamics could be compensated for in the controller design, it would add significant complexity. The strategy used here is to develop an actuator that is self compensated to decouple the actuator dynamics from the noise control design.

This study presents an acoustic actuator that compensates for both actuator and plant dynamics. The acoustic duct is presented to demonstrate the robustness of the actuator to a plant with strong pressure coupling. A model of a dual voice coil speaker is first presented, and a velocity sensor is developed. It is shown that the speaker dynamics can be minimized through feedback compensation. A model of an acoustic duct is presented next. Finally, the two systems are coupled, and it is shown that the speaker compensation minimizes both the speaker and the acoustic duct dynamics. The results are verified through experimental testing, Figure 2 shows the experimental setup.

Acoustic Actuator Speaker Model

Audio speakers are commonly used as acoustic actuators in noise control systems. They are relatively inexpensive, widely available in commercial sizes and models, and a small applied voltage can generate a strong control effort. However, they have the disadvantage that the response can be strongly affected by both the dynamics associated with the free-air resonance of the speaker and the dynamics of the coupled acoustic system. If a speaker is to be used as an acoustic actuator, these dynamic effects must be minimized. Ideally, an actuator will have a pure gain over some bandwidth, that is, the measured output will exactly match an arbitrary desired input signal. When used in closed-loop, the acoustic actuator input voltage, or "desired velocity" is the output from the noise reduction controller. In experimental verification the speaker face velocity can be measured by a laser velocity transducer and improved performance of the system is indicated by reductions in the relative magnitude and phase between the measured and the input signals.

One method of minimizing magnitude and phase variations is to apply feedback compensation to the speaker. If the speaker velocity can be measured, then the signal can be applied to a proportional feedback controller; the response can be driven to the desired output; and the magnitude and phase variation can be reduced.

One variety of speaker named the "dual voice coil" speaker has certain characteristics that make it ideal for use as an acoustic actuator (Radcliffe et al., 1996). The dual voice coil speaker has 2 independent wire coils intertwined and wrapped around...
A bobbin, which is allowed to slide over a permanent magnet. This configuration is shown in Fig. 3.

A transfer function model of the system can be developed which relates the inputs: primary coil voltage, secondary coil current and speaker face pressure to the outputs: secondary coil voltage, primary coil current, and speaker face velocity (Birdsong, 1996).

An infinite impedance is applied to the secondary coil forcing the current to zero, eliminating the secondary current as an input. A current measuring resistor, \( R_{\text{ip}} \), is placed in series with the secondary coil so that the current, \( i_{\text{p}} \), can be computed from the measured voltage drop across \( R_{\text{ip}} \).

The speaker parameters necessary to define the model are the mechanical inertia of speaker, \( I_{\text{spkr}} \); mechanical compliance of speaker, \( C_{\text{spkr}} \); viscous friction of speaker, \( f_{\text{spkr}} \); electromagnetic coupling factor, \( b_{\text{l}} \); speaker coil resistance, \( R_{\text{coil}} \); and the equivalent speaker area. So with the exception of mutual inductance, \( M_{\text{coil}} \), these electrical and mechanical parameters are defined in IEEE standard 219-1975 (IEEE Standard 219-1975) for loudspeaker measurements.

The transfer function model of the system is given by

\[
\begin{bmatrix}
    e_{\text{fb}}(s) \\
    i_{\text{fb}}(s) \\
    v_{\text{fb}}(s)
\end{bmatrix} =
\begin{bmatrix}
    G_{\text{e}_{\text{fb}}/e_{\text{fb}}} & G_{\text{e}_{\text{fb}}/i_{\text{fb}}} & G_{\text{e}_{\text{fb}}/v_{\text{fb}}}
\end{bmatrix}
\begin{bmatrix}
    e_{\text{p}}(s) \\
    i_{\text{p}}(s) \\
    P(s)
\end{bmatrix}
\]

Each element in the transfer function matrix \( G(s) \) is given by

\[
G_{\text{e}_{\text{fb}}/e_{\text{fb}}} = \frac{C_{\text{spkr}}M_{\text{coil}}}{G_{\text{den}}} \left[ I_{\text{spkr}}s^3 + R_{\text{spkr}}s^2 + \left( \frac{b_{\text{l}}}{M_{\text{coil}}} + \frac{1}{C_{\text{spkr}}} \right)s \right]
\]

\[
G_{\text{i}_{\text{fb}}/i_{\text{fb}}} = \frac{C_{\text{spkr}}I_{\text{spkr}}}{G_{\text{den}}} \left[ (C_{\text{spkr}}I_{\text{spkr}})s^2 + (C_{\text{spkr}}R_{\text{spkr}})s + 1 \right]G_{\text{den}}^{-1}
\]

\[
G_{\text{e}_{\text{fb}}/v_{\text{fb}}} = \frac{blC_{\text{spkr}}s^2}{G_{\text{den}}} \left[ (I_{\text{coil}} - M_{\text{coil}})s^3 + R_{\text{coil}}s \right]
\]

and the denominator of the \( G(s) \) matrix is given as

\[
G_{\text{den}} = (C_{\text{spkr}}I_{\text{spkr}}I_{\text{coil}})s^3 + (C_{\text{spkr}}I_{\text{spkr}}R_{\text{coil}} + C_{\text{spkr}}R_{\text{spkr}})s^2 + (b_{\text{l}}^2C_{\text{spkr}} + I_{\text{coil}} + C_{\text{spkr}}R_{\text{coil}}R_{\text{spkr}})s + R_{\text{coil}}
\]

Equations (4) and (7) are new and important results not used in previous work. These transfer function equations will be useful when designing a velocity sensor for the speaker and when modeling the speaker coupled with the acoustic duct.

Velocity Feedback Compensation of Speaker. The velocity of the speaker, \( V_{\text{spkr}} \), is strongly affected by the dynamics of the speaker and the pressure input, \( P \). These effects will combine to create magnitude and phase variations in the primary coil voltage to speaker velocity response. One method of eliminating these unwanted effects is to apply a proportional feedback controller as shown in Fig. 4. The transfer function for this system is given by (9), where \( K_p \) is the proportional gain and \( H(s) \) is a velocity sensor. If the sensor transfer function is a real constant, \( k \), over some bandwidth; then as \( K_p \) is increased, the closed-loop transfer function, \( T(s) \), will approach a constant, with zero phase. This compensation forces the speaker cone velocity to accurately follow the desired velocity input. The result is independent of the speaker dynamics and the input pressure provided that the sensor has a constant transfer function over some bandwidth.

\[
T_{\text{spkr}}(s) = \frac{V_{\text{spkr}}(s)}{V_{\text{ip}}(s)} = \frac{K_p G_{\text{spkr}}}{1 + K_p G_{\text{spkr}}H(s)}
\]

This approach assumes that the velocity of the speaker face can be measured. A speaker velocity sensor is therefore needed to accurately predict the speaker velocity in the presence of speaker and plant dynamics. The relation between the speaker velocity and the two other measurable outputs (secondary coil voltage, \( e_{\text{fb}} \) and primary coil current, \( i_{\text{p}} \)) is given in (1). The
speaker velocity, \( V_{\text{spkr}} \), can be solved for in terms of \( E_{\text{in}} \) and \( I_p \) yielding
\[
V_{\text{spkr}}(s) = H_0 E_{\text{in}}(s) - H_f(s) I_p(s) \tag{10}
\]
where \( H_0 = 1/ bl \) and \( H_f(s) = sM_{\text{coil}}/bl \).

The secondary coil voltage, \( E_{\text{in}} \), can be measured directly from the speaker coil; and the primary coil current, \( I_p \), can be determined from the voltage across the resistor, \( R_a \). The term \( H_f(s) \) is an improper transfer function that represents a time derivative and cannot be realized exactly. However, an approximation \( \hat{H}_f(s) \) can be used where
\[
\hat{H}_f(s) = \frac{M_{\text{coil}}}{bl} \left( \frac{s}{s + p_1} \right) \tag{11}
\]
where \( p_1 \) is a pole location selected such that (11) approximates \( H_f(s) \) over some bandwidth. An electronic dynamic filter can be created using analog devices to realize (10) and (11), to generate a voltage that is proportional to the actual speaker face velocity. This filter creates the electronic velocity sensor used by the feedback control.

Feedback compensation can now be implemented using the signal from the velocity sensor to compute the error between the desired velocity and the sensor velocity and a proportional controller to drive the speaker velocity to the desired velocity. It should be noted that the development of the velocity sensor did not assume that the pressure at the speaker face was constant, as in previous work. This new velocity sensor includes the effects of pressure as an input to the system. As a result, the closed-loop system minimizes magnitude and phase variations from not only the speaker dynamics (as in previous work); but in addition, the dynamics associated with the acoustic system, coupled through the pressure interaction with the speaker are minimized as well. This improvement over the previous velocity sensor is essential for the speaker to perform as an effective actuator in a coupled system such as the acoustic duct.

**Acoustic Duct System Model**

The acoustic duct is a system that exhibits strong dynamics when coupled with the speaker system will cause large magnitude and phase variations in the speaker response. These effects can then be minimized through feedback compensation. A mathematical model is needed for the acoustic duct before these effects can be demonstrated. In this section, a model that accurately represents the duct pressure response is developed. System equations are first presented, then they are transformed into state space and transfer function representations.

**System Model.** An accurate system model of the acoustic duct is needed for modeling and analysis. The linear second order wave equation modeling particle displacement in a hard-walled, one-dimensional duct is (Seto, 1971; Doak, 1973)
\[
\frac{\partial^2 u(x, t)}{\partial x^2} - c^2 \frac{\partial^2 u(x, t)}{\partial t^2} = -\frac{\partial}{\partial x} \left[ \delta(x) P(t) \right] - \sum_{i=1}^{4} \left[ \delta(x - x_i) \right] \frac{\partial}{\partial t} \left[ \frac{M_i(t)}{\rho S} \right] \tag{12}
\]
where \( u(x, t) \) is the particle displacement, \( c \) is sound speed (m/s), \( x \) is spatial location (m), \( t \) is time (s), \( \rho \) is density of the medium (kg/m³), \( M_i(t) \) is mass flow input in the domain (kg/s), \( x_i \) is location of mass flow input (m), \( S \) is the speaker area driving the mass flow input (m²), \( P(t) \) is pressure excitation at \( x = 0 \) (N/m²), and \( \delta(x) \) is the Dirac delta function. The partially reflective boundary condition at location \( x = L \) is the relationship between the spatial gradient and the time gradient of the particle displacement and is expressed as (Seto, 1971; Spiekerman, 1986)
\[
\frac{\partial u}{\partial x} (L, t) = -K \left( \frac{1}{c} \right) \frac{\partial u}{\partial t} (L, t); \quad K \neq 0 + 0i, 1 + 0i, \infty \tag{13}
\]
where \( K \) is complex end impedance of the termination end (dimensionless). The duct end at \( x = 0 \) is modeled as a totally reflective end. This boundary condition is
\[
\frac{\partial u}{\partial x} (0, t) = 0 \tag{14}
\]
which corresponds to an open duct end. The acoustic pressure of the system is related to the spatial gradient of the particle displacement by (Seto, 1971)
\[
P(x, t) = -pc^2 \frac{\partial u}{\partial x} (x, t) \tag{15}
\]
The above four equations represent a mathematical model of the duct. It should be noted that the acoustic end impedance \( K \) is related to the acoustic impedance, \( Z \) (Ns/m²) used in some texts by
\[
Z = \rho c K \tag{16}
\]

**State Space Representation.** To derive the state equations used throughout the analysis, separation of variables is applied to the unforced version of (12), (13), and (14). Solving for the separation constant and the eigenfunctions yields (Spiekerman, 1990)
\[
\lambda_n = \frac{1}{2L} \log_e \left( \frac{1 - K}{1 + K} \right) - \frac{n \pi i}{L}, \quad n = 0, \pm 1, \pm 2, \ldots \tag{17}
\]
\[
\phi_n(x) = e^{\lambda_n x} + e^{-\lambda_n x} \tag{18}
\]
where \( \lambda_n \) are the natural frequencies and \( \phi_n(x) \) are the eigenfunctions of the duct. For a duct with one mass flow rate as the input, the above equations can be manipulated such that the following state space representation is produced (Hull, 1990)
\[
a(t) = A_{\text{duct}} a(t) + B_{\text{duct}} m(t) \tag{19}
\]
where \( a(t) \) is the vector of modal wave amplitudes
\[
A_{\text{duct}} = \text{the diagonal matrix } [c \lambda_n]
\]
\[
B_{\text{duct}} = \text{the matrix } \left\{ \frac{1}{4c^2 \lambda_n^2 L_0 S} \frac{d \phi_n(x)}{dx} \right\}
\]
and \( m(t) \) is the mass flow input \( \partial M/\partial t \). The system output is the pressure at any position in the duct
\[
P(x_m, t) = C^T_{\text{duct}} a(t) \tag{20}
\]
where \( P(x_m, t) \) is the pressure in the duct at \( x = x_m \), and \( C_{\text{duct}} = \text{the column vector } [-pc^2(d \phi_n(x_m)/dx)] \).

Equations (19) and (20) represent the state space formulation of the acoustic duct with complex end impedance, \( K \), on the termination end.

**Duct Transfer Function.** A velocity to duct pressure transfer function can be computed from the state space representation of the acoustic duct model for the case with one mass flow input. The transfer function representation will be used for the coupled speaker-duct system model.

The duct transfer function can be computed numerically from
\[
G_{\text{duct}}(s) = \frac{P_{\text{duct}}}{m} = C_{\text{duct}}(sI - A_{\text{duct}})^{-1} B_{\text{duct}} \tag{21}
\]
where $s$ is the Laplace variable times an identity matrix and $G_{\text{duct}}(s)$ is the speaker velocity to duct pressure transfer function. The mass flow rate, $m(t)$, can be replaced by the speaker face velocity, $v_{\text{spkr}}$, by the relation, $m(t) = S_P v_{\text{spkr}}(t)$, where $S_P$ is the speaker area. The transfer function, $G_{\text{duct}}(s)$, will have a numerator which consists of a polynomial of order $2n$ and a denominator of order $2n + 1$, where $n$, the number of modes can be chosen to represent as many modes as required.

**Coupled Speaker-Duct System**

In the previous discussions both the dynamics of a speaker and a duct were modeled separately. The model of the speaker assumed that the speaker face was exposed to atmospheric pressure. This implied that the speaker velocity was the only input to the system. The model of the duct gave the pressure at a point in the duct given a velocity input.

These two systems can be coupled by allowing the velocity output of the speaker to be the input to the duct and the pressure output of the duct to be the input to the speaker. The velocity of the speaker face is then no longer affected only by the primary speaker voltage but also by the pressure generated in the duct, which must be determined from the coupled dynamics of the two systems. This coupling is illustrated by Figure 5.

The coupled system can be modeled by combining the transfer functions of the speaker and duct models. The resulting transfer function can be used to model the open-loop response of the coupled speaker-duct system. The speaker velocity, $v_{\text{spkr}}$, is given by (1) as

$$V_{\text{spkr}}(s) = G_{\text{spkr}}(s)E_{\text{p}} + G_{\text{spkr}}(s)P(s)$$

The duct pressure to speaker velocity transfer function (21) is given by

$$P_{\text{duct}}/V_{\text{spkr}} = G_{\text{duct}}$$

The pressure can be eliminated from (22) by substituting (23), which gives

$$V_{\text{spkr}}(s) = G_{\text{spkr}}(s)E_{\text{p}} + G_{\text{spkr}}(s)G_{\text{duct}}(s)V_{\text{spkr}}(s)$$

which can be solved for the transfer function of speaker velocity to primary speaker voltage as

$$V_{\text{spkr}}/E_{\text{p}} = \frac{G_{\text{spkr}}(s)}{1 - G_{\text{spkr}}(s)G_{\text{duct}}(s)}$$

**Velocity Sensor.** The coupled system transfer function (25) can be used to model the response of the velocity sensor presented in the previous section. The velocity sensor model will include the effect of estimating the derivative of the primary current (Birdsong, 1996).

The secondary speaker voltage was given by (1). The pressure can be eliminated by replacing $P$ with (23), giving

$$E_{\text{st}}(s) = G_{\text{spkr}}E_{\text{p}} + G_{\text{spkr}}G_{\text{duct}}V_{\text{spkr}}$$

The velocity can be eliminated by replacing $V_{\text{spkr}}$ with (25) giving the secondary speaker voltage to primary speaker voltage transfer function as

$$E_{\text{st}}/E_{\text{p}} = \frac{G_{\text{spkr}}}{1 - G_{\text{spkr}}G_{\text{duct}}}$$

**Equation (29) can be used to simulate the sensor velocity response of the coupled system. It should be noted that the model (29) must include the dynamics associated with the acoustic duct in order to accurately predict the response of the sensor-speaker-duct system. However, the design and implementation of the velocity sensor (10) does not require any information about the acoustic system connected to the speaker. This means that changes in the acoustic system do not require changes in the velocity sensor design or calibration.**

The feedback compensation strategy can be applied to the coupled system. The sensor velocity accounts for the pressure input as well as the primary voltage input, and the closed-loop system compensates for the dynamics associated with both the speaker and the duct.

**Coupled Speaker-Duct Model Verification**

The coupled speaker-duct system model was verified through experimental testing. The speaker velocity model was first compared to experimental results, then the velocity sensor was shown to accurately predict the measured velocity. Finally, the velocity sensor was used in closed-loop feedback compensation.

**Speaker Velocity Model.** The speaker-duct system was set up as shown in Fig. 6. A 3.96-meter long, 3-inch diameter, schedule-40 PVC pipe was used as the duct. A 6-inch dual voice coil speaker was used for the actuator. The end of the duct was left open with no dampening material added for this experiment. This was done to emphasize the resonant effects of the duct. In successive experiments some foam damping material was added to the end. This foam passively damps the duct modes and is consistent with the method of using passive noise control where possible and treating the remaining noise with active controls. The speaker velocity to primary coil volt-
The open-loop response of the actuator is not acceptable but can be improved by closed-loop feedback. Above approximately 200 Hz, the deviation between the response of the actuator and the velocity sensor circuit was then applied to the coupled speaker-duct system as shown in Figure 8. A 2-inch foam plug was placed in the termination end to add some passive damping to the system. An end impedance was measured and the value of 0.6 + 0.1j was used in the model.

The sensor velocity to desired velocity transfer functions were then measured from 0-200 Hz using the signal analyzer. Figure 9 shows good agreement between the measured velocity and the velocity sensor signal was fed back in a proportional controller. The proportional gain, \( K_p \), was varied from 0 to 100; and the measured velocity to desired velocity transfer function was measured from 0–200 Hz using the signal analyzer. Figure 10 shows that the measured speaker velocity response approached the desired velocity as the gain was increased.

The error between the 2 signals is attributed to the physical limitations of the electronic velocity sensor circuit. Above approximately 200 Hz, the deviation between the response of the sensor and the speaker grows due to the limitation in the circuit's ability to estimate the derivative of \( i_v \). The derivative estimate diverges from the actual derivative above 200 Hz as describe by (11). This property of the circuit imposes the bandwidth limitation on the sensor and consequently on the closed-loop compensated system.

Velocity Feedback Compensation of Speaker. The velocity feedback compensation strategy was then applied to the coupled speaker-duct system. The agreement in Fig. 7 verifies that the model accurately represents the response of the open-loop system within the frequency range of 20–400 Hz. Having shown that the model accurately predicts the response of the system and that the model based velocity sensor circuit accurately estimates the speaker velocity; (9) shows that the closed-loop response of the actuator will approach unity gain and zero phase as \( K_p \) is increased.

The velocity sensor signal was fed back in a proportional controller. The proportional gain, \( K_p \), was varied from 0 to 100; and the measured velocity to desired velocity transfer function was measured from 0–200 Hz using the signal analyzer. Figure 10 shows that the measured speaker velocity response approached the desired velocity as the gain was increased.

The magnitude and phase variations exhibited in open-loop have been reduced. Specifically, the effect of the duct resonances and the free-air resonance of the speaker are significantly reduced. With a value of \( K_p = 100 \) there is less than 5 dB and 45 degrees magnitude and phase variation compared with 30 dB and 180 degrees in the uncompensated system. Although the phase variation is not zero in Figure 10 (as desired in an ideal actuator) it has been reduced from 180 to 45 degrees. Higher values of \( K_p \) would further reduce the phase; however, the maximum value of \( K_p \) is limited by system stability. For
the system used here, a theoretical maximum value of $K_p = 170$ was calculated from the gain margin of the open-loop system model. The effect of phase on application of the actuator in a closed-loop system is to reduce the gain margin, which reduces the relative stability of the closed-loop system. The phase variation limits the level of stable feedback gain.

**Conclusions**

This paper addresses the use of a compensated audio speaker as an actuator for systems with strong dynamic pressure coupling. It was shown that the response of an actuator is degraded by both the internal dynamics of the actuator and the interaction with the plant. Previous solutions are not effective for such applications because they only compensate for internal dynamics and not the pressure interactions from the plant. A new velocity sensor that uses a combination of speaker cone motion induced secondary coil voltage and primary coil current is developed and applied in a proportional feedback controller. An acoustic duct is used as an example of a system with strong dynamic pressure interactions. It is demonstrated through modeling and experimentation that the compensated speaker response minimizes the effect of both internal actuator dynamics and coupling through the pressure with the acoustic plant. The effectiveness, i.e., bandwidth of the system is limited by the physical abilities of the velocity sensor circuit and the maximum value of $K_p$. The experimental results could be further improved by using additional techniques to compute the derivative and by further increasing the value of $K_p$.

The work presented here represents an actuator whose design is independent of the acoustic plant. The compensation yields a feasible actuator for acoustic systems with strong pressure coupling.

**References**


