Minimum expected cost-oriented optimal maintenance planning for deteriorating structures: application to concrete bridge decks

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Abstract

Civil engineering structures are designed to serve the public and often must perform safely for decades. No matter how well they are designed, all civil engineering structures will deteriorate over time and lifetime maintenance expenses represent a substantial portion of the total lifetime cost of most structures. It is difficult to make a reliable prediction of this cost when the future is unknown and structural deterioration and behavior are assumed from a mathematical model or previous experience. An optimal maintenance program is the key to making appropriate decisions at the right time to minimize cost and maintain an appropriate level of safety. This study proposes a probabilistic framework for optimizing the timing and the type of maintenance over the expected useful life of a deteriorating structure. A decision tree analysis is used to develop an optimum lifetime maintenance plan which is updated as inspections occur and more data is available. An estimate which predicts cost and behavior over many years must be refined and reoptimized as new information becomes available. This methodology is illustrated using a half-cell potential test to evaluate a deteriorating concrete bridge deck. The study includes the expected life of the structure, the expected damage level of the structure, costs of inspection and specific repairs, interest rates, the capability of the test equipment to detect a flaw, and the management approach of the owner towards making repairs.

Keywords: Deteriorating structures; Cost-based optimal maintenance; Inspection/repair planning; Probability; Updating

1. Introduction

The lifetime maintenance of a deteriorating structure can comprise a far greater portion of the total lifetime cost than the original cost of construction. The cost of construction, while high, is a one-time cost while the cost of maintenance can span decades. The infrastructure of the United States consists of thousands of deteriorating structures and the national cost of maintaining them is a substantial portion of the budget. Of the almost 600,000 bridges and culverts in the National Bridge Inventory, over 35\% of the bridges are either structurally deficient, functionally obsolete, or both. The estimated cost to eliminate the backlog of bridge deficiencies and maintenance repair levels is about $80 billion [1]. With such huge expenditures, any realized efficiency or optimization can result in significant savings.

It is difficult to make a reliable estimate of these lifetime costs when the future is unknown and structural deterioration and behavior are assumed from a mathematical model or previous experience. This paper is based on a methodology developed by the writers [7,9] for optimizing the lifetime inspection and repair of any deteriorating structure based on the information available at the time using a decision tree analysis. The optimized strategy is revised and updated as new inspection information becomes available and repair/no repair decisions are made on the structure. The methodology is illustrated using a concrete bridge deck whose steel reinforcement is corroding. The results are a series of optimum strategies throughout the life of the structure which specify the inspection technique, the number and timing of the inspections, and the expected lifetime maintenance cost of the structure.

The general methodology for optimizing the lifetime maintenance of a deteriorating structure is as follows [9]:

- Define the structure and the criteria which constitute failure of the structure.
- Develop a deterioration model which predicts how the structure will change over time.
- Specify the inspection methods available to detect this deterioration. Quantify the inspection costs and capability
of these methods to detect the relevant flaws or changes in the structure.
- Define the available repair options, their effect on the structure, and their costs.
- Quantify the probability of making a repair if a defect is detected.
- Formulate the optimization problem based on the optimization criterion, failure constraints, expected life of the structure, and any other imposed constraints.
- Use an event tree to account for all the repair/no repair decision possibilities that occur after every inspection. The event tree shows the probability of taking a particular path and provides the expected value of the optimization criterion.
- For a discrete number of lifetime inspections, optimize the timing of these inspections for a specific inspection technique.
- Repeat the problem for other inspection techniques and numbers of lifetime inspections. The optimum strategy is the one which provides the best expected value of the optimization criterion.
- As inspections are conducted and the repair/no repair decisions are actually made, use the new information to update the optimum strategy.

2. Concrete bridge deck

The structure whose lifetime inspection and repair strategy is optimized is a 42.1 m × 12.2 m concrete bridge deck which deteriorates over time as spalls and delaminations appear in the concrete. The deterioration is caused by corroding reinforcing steel in the bridge deck. Consistent with the Colorado Department of Transportation (CDOT) repair policy, deck failure occurs when active corrosion is underway in at least 50% of the deck [4].

The concrete deteriorates as chlorides from deicing salts penetrate the concrete and reach the steel reinforcing. At a critical chloride concentration, the reinforcing corrodes which causes the concrete deck to spall. The corrosion initiation time which is the amount of time between the application of surface chloride and the onset of corrosion is expressed as [11]

\[
T_i = \left(\frac{d_i - D_t/2}{4D_e}\right)^2 \left(\text{erf}^{-1}\left(\frac{C_{cr} - C_0}{C_i - C_0}\right)\right)^{-2}
\]

where \(d_i\) is the concrete cover, \(D_t\) the initial diameter of the reinforcing bar, \(C_0\) the equilibrium chloride concentration on the concrete surface, \(C_i\) the initial chloride concentration, \(D_e\) the chloride diffusion coefficient, and \(C_{cr}\) is the critical chloride concentration that will initiate corrosion. Using the distributions and parameters listed in Ref. [6] for all of these random variables, \(T_i\) was calculated to be normally distributed with a mean value \(\mu_{T_i} = 19.6\) years and standard deviation \(\sigma_{T_i} = 7.51\) years. The deterioration model can predict the percentage of corrosion in the deck at any time.

There are a variety of tests which may be performed individually or used in combination to detect concrete deterioration. These include measured crack widths, chain drag to detect delaminations, percentage of spalls, observed efflorescence, and chloride content. This study uses the half-cell potential test because it remains the most useful source of information regarding active corrosion in the deck [3]. It is inexpensive, simple, and non-destructive. The half-cell potential survey measures the electrical potential difference between a standard portable half-cell placed on the surface of the concrete and the embedded reinforcing steel. The voltage readings are compared to empirically derived values which indicate relative probabilities of active corrosion [8].

The correlation between the half-cell readings and the presence of active corrosion has been the subject of considerable research. The ASTM guideline prescribes that half-cell readings more positive than \(-0.20\) V indicate at least 90% probability of no corrosion activity. Similarly, values more negative than \(-0.35\) V indicate at least 90% probability of corrosion activity. Marshall [10] studied the data from 89 bridges to determine the probability density functions of the half-cell potentials for both sound and damaged deck areas. The half-cell potentials in areas where the deck was known to be undamaged was a normal distribution with a mean of \(\mu = -0.207\) V and a standard deviation \(\sigma = 0.0804\) V and the half-cell potentials in areas where the deck was known to be damaged was a normal distribution with a mean of \(\mu = -0.354\) V and a standard deviation \(\sigma = 0.0697\) V.

The uncertainty associated with assessing the condition of the entire deck from a finite number of half-cell readings was considered. Three different inspection techniques were used where the number of readings varied from one every 5 ft (1.52 m) to one every 20 ft (6.10 m). Table 1 shows the techniques and their associated costs. The inspection costs, developed in consult with CDOT [5] included fixed costs such as travel time to site, traffic control, equipment set-up, and writing the final report and variable costs such as marking the grid pattern, prewetting the test locations,

<table>
<thead>
<tr>
<th>Inspection techniques</th>
<th>Spacing of readings</th>
<th>Total readings</th>
<th>Inspection cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>5 ft (1.52 m)</td>
<td>8 × 27 = 216</td>
<td>1027</td>
</tr>
<tr>
<td>B</td>
<td>10 ft (3.05 m)</td>
<td>4 × 13 = 52</td>
<td>604</td>
</tr>
<tr>
<td>C</td>
<td>20 ft (6.10 m)</td>
<td>2 × 6 = 12</td>
<td>408</td>
</tr>
</tbody>
</table>
taking readings, and traffic control while the test was being conducted.

Although several repair options such as a concrete overlay, waterproofing membrane, and cathodic protection were considered, the only repair option used in this study was replacement of the deck at a cost of $225,600 [5]. It is assumed that the cost is in year 2000 dollars. The effect of the repair is to return the deck to its original condition. Local damage will be patched and repaired as necessary to keep the bridge deck serviceable.

The probability of making a repair once a defect has been detected is a function of the bridge manager’s willingness to make a repair, which could be based on past performance. Availability of funds, competing priorities, and political considerations become relevant variables. Four repair approaches are shown in Fig. 1. The delayed approach waits the longest to make a repair (only 30% chance of making the repair when the deck is 50% damaged) and the proactive approach employs a preventive strategy (80% chance of repair when the deck is 50% damaged).

A discrete optimization of the bridge deck was conducted for two, three, and four lifetime inspections and the objective to be minimized was the expected total cost $E(C_{tot})$ which equaled the actual inspection cost $C_{insp}$ plus the expected cost of repair $E(C_{rep})$. The optimization problem for three lifetime inspections is formulated as follows:

Minimize: $E(C_{tot})$ such that

\begin{align*}
E(Damage)_{t_1} & \leq 0.50 \quad (2a) \\
E(Damage)_{t_2} & \leq 0.50 \quad (2b) \\
E(Damage)_{t_3} & \leq 0.50 \quad (2c) \\
E(Damage)_{t_{life}} & \leq 0.50 \quad (2d) \\
2.0 \leq t_1 & \leq 20.0 \quad (2e) \\
2.0 \leq t_2 - t_1 & \leq 20.0 \quad (2f) \\
2.0 \leq t_3 - t_2 & \leq 20.0 \quad (2g) \\
t_3 & \leq t_{life} \quad (2h)
\end{align*}

where $t_1$, $t_2$, and $t_3$ are the inspection times (in years) for the

---

Fig. 1. Four approaches to the probability of replacing the deck based on inspection results: delayed, linear, proactive, and idealized.
three inspections. The expected damage $E(Damage)_t$ never exceeds the 50% damage limit established by the replacement policy. The additional constraints ensure that inspection times are at least two years apart but not more than 20 years. Fig. 2 shows the optimized inspection strategy and expected level of damage for three lifetime inspections on a bridge deck where the planned service life $t_{life}$ is 45 years. The optimal inspection times are 10.05, 19.76, and 35.45 years. Inspection Technique A (5 ft spacing of readings) was used with a proactive approach to repair and a 2% discount rate on money. There appears to be little likelihood of replacing the deck after the first inspection but a substantial probability of replacement after the second or third inspection.

After every inspection, a decision is made to repair or not repair the deck. The decision tree which illustrates the possible paths in the three inspection example is shown in Fig. 3. The number in parentheses at every branch is the expected deterioration at the time of inspection based on the deterioration model. The total expected cost and the expected amount of damage are based on the probability of taking a particular path. The expected amount of damage at any point in time is

$$E(Damage)(t) = \sum_{i=1}^{\infty} E(Damage(t)|Branch_i)(P_{b_i})$$  \hspace{1cm} (3)$$

where the sum is taken over all branches of the tree, $E(Damage(t)|Branch_i)$ the expected damage given that

![Figure 2. Optimum inspection strategy and expected damage for a bridge deck with 45 year service life, three lifetime inspections, proactive approach, inspection technique A, and 2% discount rate.](image)

![Figure 3. Event tree for the optimum inspection strategy for a 45 year bridge deck using a proactive repair approach and three lifetime inspections.](image)
branch $i$ is taken, $P_{b_i}$ the probability of taking branch $i$; and $m$ is the number of inspections.

Similarly, the expected cost of replacement of the bridge deck $E(C_{rep})$ is the sum over all the branches of the discounted replacement costs associated with an individual branch multiplied by the probability of taking that branch

$$E(C_{rep}) = \sum_{i=1}^{m} \sum_{r=1}^{nr} C_{rep_i} (1 + r)^r (P_{b_i})$$

where $nr$ is the total number of repairs, and $r$ is the discount rate of money.

The expected cost of repair for the three inspection example from Fig. 3 is $E(C_{rep}) = $172,130 as computed in Table 2. Similarly, the cost of the inspection $C_{insp}$ is

$$C_{insp} = \sum_{i=1}^{m} C_{insp_i} (1 + r)^r$$

where $C_{insp}$ is the cost of a single inspection as determined by the cost of Technique A, B or C from Table 1, and $t_i$ is the time of the inspections. The cost of inspection for the three inspection example using Technique A and a discount rate of 2% is $C_{insp} = $2045. The expected total cost $E(C_{tot})$ is the sum of the expected repair cost and the inspection costs

$$E(C_{tot}) = C_{insp} + E(C_{rep})$$

which for the three inspection example equals $174,175$.

Fig. 4 shows the lifetime expected damage for each of the eight possible outcomes formed by the branches of the event tree in Fig. 3 along with their likelihood of occurrence. The optimum solution can be seen as a weighted average of the eight branches shown in Figs. 3 and 4. The timing of the inspections is optimized to meet all the constraints. The decrease in the expected damage to the deck after each inspection is based on the probability of taking a branch in which the deck is replaced after that inspection. For example, the probability of making a repair after the first inspection is only 6.7% (i.e. $(0.1 + 0.3 + 5.8 + 0.5)%$). The expected effect of this repair is very small. The probability of making a repair after the second inspection is 75.3% and the expected effect of the repair is therefore very large. The probability of repair after the third inspection is 40.2%.

Branches 1, 2, 4, and 8 in Figs. 3 and 4 have almost no chance of occurring. The most likely path is Branch 6 which would involve one repair after 19.76 years. This branch taken alone would not meet the constraints of the problem. It is the combined effect of all eight paths and their relative
<table>
<thead>
<tr>
<th>Event tree</th>
<th>Discounted cost of repair ($)</th>
<th>Probability of taking branch</th>
<th>Expected cost of repair ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Repair 1 $t_1 = 10.05$</td>
<td>Repair 2 $t_2 = 19.76$</td>
<td>Repair 3 $t_3 = 35.45$</td>
</tr>
<tr>
<td>Branch 1</td>
<td>184,887</td>
<td>152,455</td>
<td>111,805</td>
</tr>
<tr>
<td>Branch 2</td>
<td>184,887</td>
<td>152,455</td>
<td>0</td>
</tr>
<tr>
<td>Branch 3</td>
<td>184,887</td>
<td>0</td>
<td>111,805</td>
</tr>
<tr>
<td>Branch 4</td>
<td>184,887</td>
<td>0</td>
<td>184,887</td>
</tr>
<tr>
<td>Branch 5</td>
<td>0</td>
<td>152,455</td>
<td>111,805</td>
</tr>
<tr>
<td>Branch 6</td>
<td>0</td>
<td>152,455</td>
<td>0</td>
</tr>
<tr>
<td>Branch 7</td>
<td>0</td>
<td>0</td>
<td>111,805</td>
</tr>
<tr>
<td>Branch 8</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Total expected cost of repair</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. Optimizing and updating

After the first inspection, the deterioration model can be updated. The first decision to replace or not replace the deck is made. After that, inspection is made, and half of the eight paths are eliminated. With that information, the probability of occurrence that determines the optimum (i.e., least-cost) inspection strategy (Table 1) is updated. None of these eight paths will be taken. The optimized strategy at this time is the optimum strategy, and is the strategy that is updated after the first inspection. To account for the new information that the inspection provides.

Table 2
Expected cost of repair for concrete bridge deck with 45 year service life, three lifetime inspections, proactive approach, inspection technique A, and 2% discount rate

<table>
<thead>
<tr>
<th>Event tree</th>
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<th>Probability of taking branch</th>
<th>Expected cost of repair ($)</th>
</tr>
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<td>111,805</td>
</tr>
<tr>
<td>Branch 2</td>
<td>184,887</td>
<td>152,455</td>
<td>0</td>
</tr>
<tr>
<td>Branch 3</td>
<td>184,887</td>
<td>0</td>
<td>111,805</td>
</tr>
<tr>
<td>Branch 4</td>
<td>184,887</td>
<td>0</td>
<td>184,887</td>
</tr>
<tr>
<td>Branch 5</td>
<td>0</td>
<td>152,455</td>
<td>111,805</td>
</tr>
<tr>
<td>Branch 6</td>
<td>0</td>
<td>152,455</td>
<td>0</td>
</tr>
<tr>
<td>Branch 7</td>
<td>0</td>
<td>0</td>
<td>111,805</td>
</tr>
<tr>
<td>Branch 8</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Total expected cost of repair</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
and each of the five successive optimizations (steps 2–6) that were updated as new information was obtained through inspections. The updated optimization inspection strategy after the inspection at year 18 (step 3) calls for one remaining lifetime inspection and is based on a new undamaged deck with a remaining service life of 27 years. The one-inspection event tree and the expected lifetime damage for both paths are shown in Fig. 7. The optimum inspection time is 17.38 years (35.38 years of service) with an expected cost of $72,976 which in year 2000 dollars is $51,095. Because the replacement decision on the deck is now known, the expected cost of the project is $210,612 which is the sum of the money already spent ($159,517) plus the expected future cost ($51,095).

The probabilities of taking either branch in Fig. 7 are quite close (45.2 versus 54.8%). When the deck gets inspected at 17.38 years (16 years if it coincides with the biennial visual inspection), the deck will have only 11 years of service remaining and will be damaged over 38.4%. It will probably be a close decision whether to replace the deck at that time or allow the deck to deteriorate to the point where there is damage over 83.8% of the deck when its service life ends. The effect on the final lifetime cost of the deck will be substantial.

Continuing with the assumption that the most probable path will be chosen, branch 2 (54.8%) is chosen and the deck is not replaced after 16 years (year 34 in the service life of the deck) and the optimization process is repeated. The event tree associated with the new optimum strategy (step 4, Table 3) is shown in Fig. 8. The plan requires a single lifetime inspection to take place at 2 years which is the minimum interval allowed in the optimization problem. At this point the expected damage covers 41.5% of the deck and the probability of replacing the deck (branch 1) has increased to 54.4% making it the most probable failure path. If the deck is neglected, the algorithm would continue to require an inspection every two years and the probability of making the repair would increase until the constraints of the optimization were violated.

Following the most probable path, the deck is replaced at year 36 of service life (step 5, Table 3) for a cost in year 2000 dollars of $110,594 which brings the actual cumulative lifetime cost of the deck to $271,137. The final optimization strategy requires one lifetime inspection 7 years after the deck replacement at year 43 of service life as shown on the event tree in Fig. 9. The deck is expected to experience damage over only 5.3% of the deck at this time and the probability of replacement (branch 1) is minimal (i.e. 4.5%).

![event tree](image)

**Fig. 5.** Two-inspection event tree for the updated optimum inspection strategy for the 45 year bridge deck after 10 years of service.

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**Table 3**

Updated optimization results for inspection of a deck with a 45 year service life using a proactive approach, inspection technique A, and discount rate \( r = 2\% \)

<table>
<thead>
<tr>
<th>Step</th>
<th>Time (years)</th>
<th>Number inspections</th>
<th>Inspection times (years)</th>
<th>Percent damage (%)</th>
<th>Time since repair (years)</th>
<th>Remaining service life (years)</th>
<th>Projected remaining cost (year 2000 $)</th>
<th>Actual cumulative cost (year 2000 $)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>3</td>
<td>10.05</td>
<td>0</td>
<td>0</td>
<td>45</td>
<td>174,175</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>19.76</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>35.45</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>2</td>
<td>8.96</td>
<td>10.2</td>
<td>10</td>
<td>35</td>
<td>159,558</td>
<td>842</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>24.51</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>18</td>
<td>1</td>
<td>17.38</td>
<td>41.5/0</td>
<td>18/0</td>
<td>27</td>
<td>51,095</td>
<td>159,517</td>
</tr>
<tr>
<td>4</td>
<td>34</td>
<td>1</td>
<td>2.0</td>
<td>38.4</td>
<td>16</td>
<td>11</td>
<td>60,831</td>
<td>160,041</td>
</tr>
<tr>
<td>5</td>
<td>36</td>
<td>1</td>
<td>7.4</td>
<td>41.5/0</td>
<td>10/0</td>
<td>9</td>
<td>4,765</td>
<td>271,137</td>
</tr>
<tr>
<td>6</td>
<td>43</td>
<td>0</td>
<td>–</td>
<td>5.3</td>
<td>7</td>
<td>2</td>
<td>0</td>
<td>271,575</td>
</tr>
<tr>
<td>7</td>
<td>45</td>
<td>0</td>
<td>–</td>
<td>7.9</td>
<td>9</td>
<td>0</td>
<td>0</td>
<td>271,575</td>
</tr>
</tbody>
</table>
In fact, the inspection is required by the optimization and justified economically only because the cost of the deck replacement is so much greater than the cost of the inspection. The expected remaining lifetime cost of the project in year 2000 dollars is the expected cost of replacement $4332 plus the cost of the remaining inspection $433 for a total of $4765. The expected total lifetime cost is $275,902 which is the sum of the expected remaining cost and the money already spent as shown in Table 3 (step 5).

At year 43, the deck is not replaced after the inspection (step 6, Table 3). The optimization strategy calls for no new inspections. All information is known, expected damage of the deck at the end of its service life is 7.9%, and the total lifetime cost (in year 2000 dollars) was $271,575. Considering that the projected cost at year 0 of service life was $174,175, one may argue that this method is not very accurate. If the deck had not been replaced with only nine years of service remaining, the total lifetime cost would have been less than $165,000 and the original projected cost would have been too high. It is impossible to accurately predict 45 years into the future, but some sort of estimate and strategy is needed. The most reasonable approach is to
create an optimized plan and base an estimate on the probabilities as they are known at the time and periodically update the information at the appropriate times. As time passes and decisions are made, the predictions become more accurate and the total lifetime cost becomes more certain.

4. Updating the deterioration model

Projected structural deterioration is initially based on a model developed for similar material under similar conditions with regard to load, exposure, and usage. This model may or may not accurately predict what will actually happen to a structure over time. One of the purposes of the inspection is to update the deterioration model.

Using the deterioration model described earlier based on chloride penetration through the concrete [11], the corrosion initiation time, $T_i$ was a normally distributed variable with a mean value $\mu_{T_i} = 19.6$ years and standard deviation $\sigma_{T_i} = 7.51$ years. The initial optimization was based on this model and produced the initial strategy shown in Figs. 3 and 4. At the first inspection which occurred at 10.05 years (10 years to coincide with visual inspection), there were real data points to evaluate. Inspection Technique A (Table 1), which involved measurements every 5 ft (1.53 m) resulted in 216 half-cell potential readings on the deck. Based on the actual half-cell readings and the prior or posterior distribution of the deterioration model, a Bayesian approach [2] can be taken to revise the corrosion initiation time parameters.

The deterioration model predicted damage over 10.2% of the deck at the time of the first inspection. Let us say that after analysis of the inspection results actual detected damage is only over 5.5% of the deck and the revised deterioration model provides corrosion initiation time parameters: $\mu_{T_i} = 44.8$ years and $\sigma_{T_i} = 15.0$ years. The
optimization strategy is updated for a 10 year old bridge with damage over 5.5% of the deck and a remaining service life of 35 years. The solution is one lifetime inspection at 10.80 years (20.80 years of service) as shown on the event tree in Fig. 10. The most probable path is branch 2 (95.4%) where the deck is not replaced after the inspection. The expected damage at year 45 if no replacement is made is 50.5%.

The next inspection occurs at year 20 of service life where half-cell potential readings are again made at 216 locations on the deck. The inspection results showed damage over 16.9% of the deck, rather than the 6.4% expected damage. The posteriori (updated) probability density function for the corrosion initiation time has values \( \mu_{t_1} = 49.0 \) years and \( \sigma_{t_1} = 28.2 \) years, which reflects a small change in the mean value but a large change in the standard deviation. With the revised damage assessment, the deck is not replaced at 20 years of service and the new optimization strategy calls for a single inspection in 19.6 years (year 39.6 of service) as shown in the event tree in Fig. 11.

Assuming that future inspections verify and are consistent with the revised deterioration model, the optimization process proceeds in a similar manner. As the deck sustains greater damage and the decision is made to not replace the deck, more inspections are required. The optimization at year 38 of bridge life requires another inspection at year 40. If the most probable path is strictly followed, the deck would be replaced at year 40 as shown in steps 1–6 of Table 4 and the total lifetime cost of the deck would be $104,654 in year 2000 dollars. In reality, the deck would probably not be replaced at year 40 to allow just five extra years of service and the total lifetime cost would have been $2,928 resulting from the inspections alone. This is a great deal less than the $174,175 predicted in the original strategy.

5. Conclusions

While nobody can predict the future with certainty, engineers are often required to plan for the future and prepare economic assessments to support that plan. The probabilistic methodology outlined herein offers a rational and logical approach for optimizing the inspection/repair strategy for a deteriorating structure. Adequate safety and minimum cost are the objectives. The approach focuses on the likelihood of events occurring, defines the decision points, and uses the best information available at the time. The updating process is critical since inspection results and repair/no repair decisions provide critical new information. The projected cost eventually converges to the actual lifetime.

Fig. 8. The one-instruction event tree for the updated optimum inspection strategy for a 45 year bridge deck after 34 years of service.

Fig. 9. The one-instruction event tree for the updated optimum inspection strategy for a 45 year bridge deck after 36 years of service.

Fig. 10. Single-instruction event tree for the updated optimization for a 45 year deck at 10 years of service using a revised deterioration model.

Fig. 11. Single-instruction event tree for the updated optimization for a 45 year deck at 20 years of service using a revised deterioration model.
cost of the structure and owners are not unexpectedly surprised when this is done in a progressive, rational manner.

The initial expected cost is not necessarily accurate, but it presents the best information available at the time. This study evaluated a deteriorating concrete bridge deck with a 45 year service life to illustrate the methodology. The original expected lifetime cost of the deck was $174,175. During the updating process, the actual lifetime cost ranged from $271,575 down to $2,928 based on inspection results and decisions made. The difference in costs however was not unexpected. The successive strategy iterations show how and why this was occurred and gives planners time to prepare.

The analysis was also performed for different expected service lives of the structure, other management repair approaches, alternative repair policies, and different discount rates. The method requires a great deal of input data that is not readily available and demands investment of time and research. Additional research is needed in the areas of quantifying the probabilistic capability of NDE inspection techniques, probability of making repairs, and the modeling of deterioration. An optimization strategy that considers the results of several different inspection techniques also merits further study.

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