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Ultrasonic flowmeters in determination of correlation functions of velocity and ultrasound wave fluctuations in grid-generated turbulence.

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The paper is devoted to the experimental investigation of the statistical characteristics of the grid-generated turbulence produced in a wind tunnel. Ultrasonic time-of-flight method using dual transducers is utilized to develop a methodology for determination of the correlation functions of turbulent velocity and sound speed fluctuations. The ultrasonic flowmeter equation is considered in the form that includes effects of turbulent velocity and sound speed fluctuations. The influence of temperature inhomogeneities on ultrasonic wave propagation is investigated using a set of experiments with a heated grid. Utilization of high-speed digital data acquisition cards and LabView software for the experiments allows collecting a significant amount of statistical data.

I. Introduction

The ultrasonic technique for measuring flows offers good prospects for turbulent flow diagnostic[1]. There has been intensive research work focusing on ultrasonic flow meters and their capabilities for measuring non-ideal flows[2,3]. However, despite its advantages over traditional methods, current ultrasonic applications fail to achieve theoretical accuracies[4]. We believe the explanation lies in understanding the effect of turbulence on the ultrasonic wave propagation.

The research into the problem of wave propagation in turbulent media is a complex problem. Firstly, it aims to understand the interaction mechanism of acoustic waves with turbulent media. Secondly, the fact that an acoustic wave carries some structural information regarding the turbulent medium after interaction makes it possible to use the statistical characteristics of the acoustic wave as a diagnostic tool to obtain some statistical information about the medium (Chernov, 1960; Ostashev (references therein), 1994; Daigle, 1986; Rasmussen, 1986; Bass, 1991; Ph. Blanc-Benon, 1991; Karweit, 1991; Iooss, 2000; Ostashev, 2001; Andreeva and Durgin, 2002). Early theory and results on sound wave propagation in inhomogeneous moving medium are summarised in the books by Rytov[16], Tatarskii[15]. The modern theory of sound propagation in a moving random medium has been developing intensively since mid-1980s. The results are systematically described by Ostashev [17].

We consider a locally isotropic, passive temperature field coupled with a locally isotropic velocity field, which is realized by introducing a heated grid in a uniform flow (Yeh and Van Atta (and references therein), 1973). Temperature fluctuations are sufficiently small, density is effectively constant and buoyancy forces are negligible. Statistic turbulence data from hot and cold wire anemometry that describe the temperature and velocity field downstream of a heated grid in a low speed wind tunnel given in the following form: downstream decay of turbulence intensities, energy spectra, autocorrelations, spatially separated cross correlations, phase, coherence [Comte-Bellot and Corrsin, 1971; Yeh and Van Atta, 1973; Sepri, 1976; Warhaft and Lumley, 1978; Sreenivasan et al, 1980; Van Atta, 1991; Nelkin, 1994; Sreenivasan and

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Antonia, 1997; Warhaft (and references therein), 2000]. The original goal of the present investigation was firstly, to develop a methodology of determination of correlation functions of turbulent velocity and sound speed fluctuations. Secondly, to demonstrate quantitatively that effect of thermal fluctuations is as important as the effect of velocity fluctuations on acoustic wave propagation. The ultrasonic flowmeter equation is reconsidered, where the effects of turbulent velocity and sound speed fluctuations are included. The result is integral equation in terms of correlation functions for travel time, turbulent velocity and sound speed fluctuations. Experimentally measured travel time statistic data with and without grid heating are approximated by Gaussian function and used to solve integral equation analytically. Turbulence spectral models for sound propagation in turbulent media were addressed by different authors and a summary of recent works presented in Wilson (2000).

Some of the main results presented and compared here are space cross correlation functions of turbulent velocity and sound speed fluctuations.

II. Methodology

In the experimental part of the study we utilize ultrasonic pulses traveling in straight paths as shown in Figure1. The sound propagates across a grid-generated turbulence from a transmitter to a receiver separated by a distance *s*.

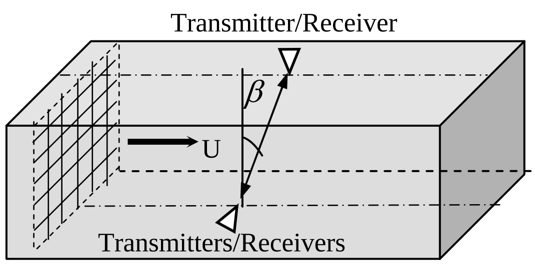

Figure 1. Sketch of wind-tunnel test section with ultrasonic flowmeter.

The flowmeter equation may be used to derive an expression for a travel time of a wave traveling from the speaker to microphone t.

$$t = \int_0^L \frac{dy}{c-u} \approx t_0 + \frac{1}{c^2}\int_0^L u' dy \qquad (1)$$

$$u = U\sin\beta + u'$$

where t_0 is a travel time in the undisturbed media, U is a mean velocity, c is a sound speed, u' are fluctuations of the mean flow velocity. In Eq. (1) we neglected the terms of order $U/c,\ U^2/c^2$.

In the experiment the only parameter that is measured is a travel time of ultrasound pulses $t(s)$. The data from experiments performed for a finite number of different lengths s and s' are collected. For each of those lengths a value $t(s)t(s')$ is calculated. Averaging the latter through the entire ensemble results in a space correlation function of travel time t as follows

$$K_t(s,s') = \frac{\sum_N t(s)t(s')}{N} \qquad (2)$$

Our objective here, then, is to construct spatial correlation functions for turbulent velocity u' and sound speed fluctuations c'. Perturbation analysis applied to (1) leads to the following expression:

$$t(s) = \int_s \frac{dx}{U+u'+\bar{c}+c'} = \frac{1}{\bar{c}}\int_s \frac{dx}{1+\frac{U}{\bar{c}}+\frac{u'}{\bar{c}}+\frac{c'}{\bar{c}}} \cong \frac{1}{\bar{c}}\int_s\left(1-\frac{u'}{\bar{c}}-\frac{c'}{\bar{c}}\right)dx = \frac{s}{\bar{c}} - \frac{1}{\bar{c}^2}\int_s (u'+c')dx \qquad (3)$$

Here we assumed that $U \ll \bar{c}$ and terms $u'/\bar{c}$ and $c'/\bar{c}$ are the same order infinitesimal compare to unity. In order to construct the correlation function we introduce new variable:

$$\Delta^0 t(s) = \Delta t(s) - \langle \Delta t(s) \rangle \qquad (4).$$

where angular brackets mean an operation of averaging. Consequently,

$$t^0(s) = -\frac{1}{\bar{c}^2}\int_s (u'+c')dx$$
$$t^0(s') = -\frac{1}{\bar{c}^2}\int_{s'} (u'+c')dx' \qquad (5).$$

Then,

$$t^0(s)t^0(s') = \frac{1}{\bar{c}^4}\left[\int_s\int_{s'}\left\{u'(x)u'(x') + c'(x)c'(x')\right\}dxdx'\right] + \frac{1}{\bar{c}^4}\left[\int_s\int_{s'}\left\{c'(x)u'(x') + u'(x)c'(x')\right\}dxdx'\right] \qquad (6).$$

In the classical theory of isotropic, homogeneous turbulence cross correlation between velocity and temperature, and, consequently between velocity and sound speed must necessarily vanish everywhere. For the velocity measurement used here, the data were collected in the isotropic region of flow, so that u' and c' are not correlated. Consequently, the space correlation function of time can be defined from the equation, known as Volterra equation:

$$\left\langle \Delta^0 t(s)\Delta^0 t(s')\right\rangle = K_{\Delta t}\left(s, s'_m\right) = \frac{1}{\bar{c}^4}\left[\int_s\int_{s'}\left(K_{u'}\left(x,x'\right)+K_{c'}\left(x,x'\right)\right)dxdx'\right] \qquad (7).$$

It was observed experimentally (Sepri) that the velocity statistics were identical within experimental limits with and without grid heating. It is important to emphasize that the space correlation function $K_{u'}(x,x')$ alone can be defined based on data from room temperature experiments. Then, data from heated air experiments can be used to identify $K_{c'}(x,x')$, knowing $K_{u'}(x,x')$ that is taken unchanged from the room temperature experiment and $K_{\Delta t}(s,s')$.

III. Experimental arrangement

The experiments were carried out in the $11.75''\times 11.62''\times 45.25''$ test section of low turbulence, low speed open circuit type wind tunnel. The velocity and temperature fluctuations were generated simultaneously using a heated grid. The square-mesh biplane heated grid was composed of 16 round chromalox heating rods, model TSSM 14XX[24]. To insure uniformity of the grid, the heating rods were inserted in hollow aluminum rods with diameter of $0.25''$ positioned $1''$ between centers. The mesh, M, was therefore $1''$ and the grid solidity was 0.64. Nine cases of different distances L for two different temperatures $\mathrm{T}=59^\circ F$ and $\mathrm{T}=159^\circ F$, are studied. The defined temperatures correspond to the temperature of aluminum rods of the grid. The angle β is changed from 0 to 40 degrees with 5-degree step.

It was experimentally demonstrated that for an open loop tunnel thermal stratification is not significant even for low-speed flows, and also that the mean air temperature is independent of downstream location within the range considered[18,19]. In the earlier works[18,19] it was found that near the grid the wall temperature is higher than that of the fluid because of the radiation from the hot grid. Therefore, the measurements were collected at x/M=35, where radiation effects are negligible.

The mean flow velocity U was 3.5 m/s. During the recording of turbulent data the low velocity was chosen in favor of higher velocity in order to maximize the temperature fluctuations. The Reynolds number R_M based on M and U was about 6000 and the corresponding Péclet number $Pe_M = \mathrm{Pr}\,\mathrm{Re}_M \sim 4350$; $\mathrm{Pr}=0.725$ for the working fluid air. The techniques employed were used in our previous experimental work and no fundamentally novel procedures have been developed for the experiment. A more detailed description of the experimental particulars may be found in[14]. The main discussion here shall be in the presentation of the experimental data and in the developing of new methodology based on experimental data.

IV. Experimental results and discussion

Following the strategy described in the Section II we have to convey to the flowmeter integral equation for the case of temperature of $59^\circ F$,

$$K_t^{F59}\left(s,s'\right) = \frac{1}{c^4}\int_0^s\int_0^{s'} K_{u'}\left(x,x'\right)dxdx' \qquad (8)$$

In many practical problems, the form of the correlation function is not known. However, its general shape is often approximated by a Gaussian function. It is very convenient for analytical studies of wave propagation in random media, and, besides, it allows taking into consideration the effect of the largest inhomogeneities in a medium on the statistical moments of a sound field[17]. In addition, the combination of numerical simulations and analytical approximations confirmed that a sound pulse tend to effectively obey Gaussian statistics[25]. We represent the correlation function in Eq. (9) by

$$K_t^{59F}\left(s,s'\right) = \sigma_t^2\Big|_{F59}\exp\left(-\left(s-s'\right)^2/l^2\right) = \sigma_t^2\Big|_{F59}\exp\left(-\tau^2/l^2\right) \qquad (9)$$

Here σ_t^2 is a variance of travel time fluctuations. Selection of l is problematic. In some applications l is chosen to be equivalent to a Taylor microscale. A better procedure is to choose l on the basis of the integral length scale of the turbulence[17]. Since we are considering travel time fluctuations in rigorously speaking diffractive media in the approximation of ray acoustic, we should realize, that there is some uncertainty up to numerical coefficient[16]. Figure 2 demonstrate correlation function of travel time obtained using experimental data as a function of separation distance x compared with Gaussian curve providing the best fit.

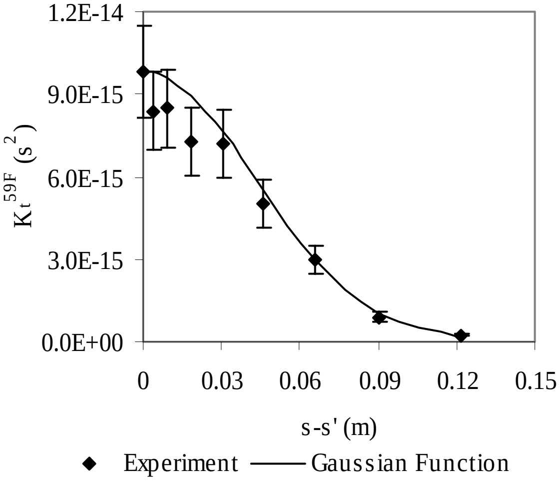

Figure 2 Correlation function of travel time along with Gaussian function providing the best fit.

Experimental data allow us to determine unknown coefficients, $\sigma_t^2 = 9.85e-15$ and $l^2 = 0.0036$. Integration of Eq. (9) with known leads to the following form of correlation function of turbulent velocity

$$K_{u'}^{F59^\circ}(\tau) = c^4 \left[2\frac{\sigma_t^2\big|_{F59^\circ}}{l^2} \exp\left(-\tau^2/l^2\right) \right] - c^4 \left[4\frac{\sigma_t^2\big|_{F59^\circ}}{l^4} \tau^2 \exp\left(-\tau^2/l^2\right) \right] \qquad (10)$$

It is apparent that correlation function of turbulent velocity is no longer Gaussian, although a Gaussian part is present in the first term in Eq. (11) and the second term vanishes rapidly with distance. Figure 3 shows the correlation function of turbulent velocity for our particular experimental data. The variance of velocity fluctuations is $\sigma_{u'}^2 = 2c^4 \frac{\sigma_t^2}{l^2} = 0.0801$. At the same time we know, that $\sigma_{u'} = \left\langle u'^2 \right\rangle^{0.5}$, meaning that for our experimental conditions we have very small values of $u'^2/c^2 \approx 6.9 \cdot 10^{-7}$, which is in a very good correspondence with data[26] The ratio of a turbulent velocity to the mean velocity is $\alpha = u'/U \cdot 100\% \approx 6\%$, which is typical for experiments performed in grid turbulence.

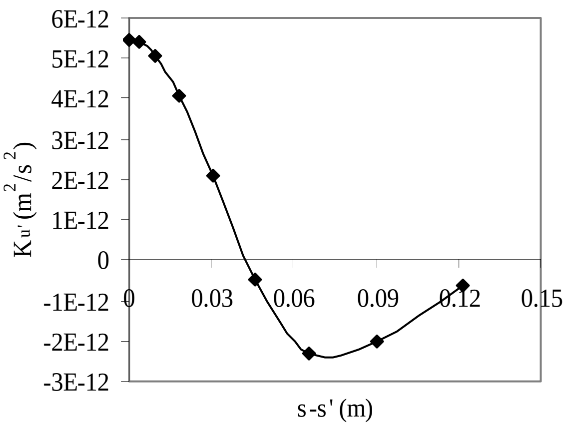

Figure 3 Experimentally obtained correlation function of turbulent velocity.

Figure 4 shows the cross correlation function of travel time at temperature 159F, again along with Gaussian function providing the best fit

$$K_t^{F159}(s, s') = \sigma_t^2\big|_{F159} \exp\left(-\tau^2/l^2\right) \qquad (11)$$

The unknown coefficient is determined to be $\sigma_t^2\big|_{F159} = 2.05\text{E-}13$

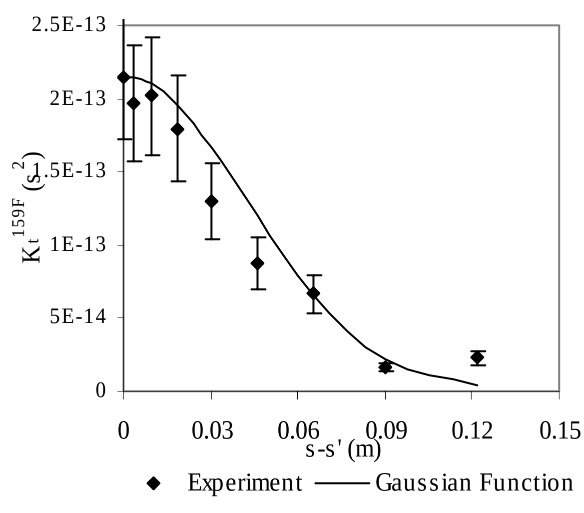

Figure 4. Correlation function of travel time obtained from experimental data collected at temperature of $159^\circ F$

In accordance with the methodology, the next step is to find the correlation function of sound speed fluctuations. Correlation function of sound speed fluctuations can be found from the following equation

$$K_{\Delta t}^{F159^\circ}\left(s,s'\right)-K_{\Delta t}^{F59^\circ}\left(s,s'\right)= = \frac{1}{c^4}\int_0^s\int_0^{s'}K_{c'}\left(x,x'\right)dxdx' \quad (12),$$

where

$$K_t^{F159^\circ}\left(s,s'\right)-K_t^{F59^\circ}\left(s,s'\right)= =\left(\sigma_t^2\Big|_{F159^\circ}-\sigma_t^2\Big|_{F59^\circ}\right)\exp\left(-\left(s-s'\right)^2/l^2\right)= =\Delta\sigma_t^2\exp\left(-\tau^2/l^2\right) \quad (13)$$

Substitution of Eq.(14) into Eq. (13) yelds

$$K_{c'}\left(s,s'\right)= =c^4\left[2\frac{\Delta\sigma_t^2}{l^2}\exp\left(-\tau^2/l^2\right)\right]- -c^4\left[4\frac{\Delta\sigma_t^2}{l^4}\tau^2\exp\left(-\tau^2/l^2\right)\right] \quad (14)$$

Figure 7 shows the correlation function of sound speed fluctuations.

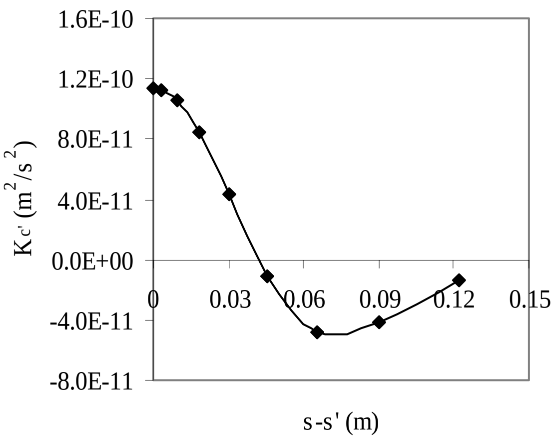

Figure 5 Corelation function of sound speed fluctuations.

The variance of the sound speed fluctuations is $\sigma_{c'}^2=c^4 2\Delta\sigma_t^2/l^2\sim 1m^2/s^2$. Neglecting humidity fluctuations, a speed of sound fluctuation is given to the first order by $\left\langle c'^2\right\rangle^{0.5}=\left(c_0/2T_0\right)\left\langle T'^2\right\rangle^{0.5}$ [23], where T_0 is a representative value of the temperature. Turbulence-level measurements made with heating the grid are consistent with results of Yeh and Van Atta (1973), namely

$$\beta\left\langle u'^2\right\rangle^{0.5}/C_pR \sim 10^{-9};\ \beta\left\langle T'^2\right\rangle^{0.5}/R\,\mathrm{Pr}\sim 10^{-5} \quad (15),$$

where β is a coefficient of thermal expansion for air, C_p is a specific heat and R is a universal gas constant. Measurements of this type were made by Sepri (1976), Yeh and Van Atta (1973) and others; however, theirs were for velocity and temperature turbulence mostly.

V. Conclusions

The significance of the present work is that we attempted to show the influence of turbulence on ultrasound wave propagation by solving the integral flowmeter equation that included the sound speed fluctuation term.

The methodology was developed for determination of statistic characteristics of isotropic, homogeneous turbulence. The methodology involves experimental and theoretical parts. The experimental part utilizes ultrasonic time-of-flight method using dual transducers in a wind tunnel. Utilization of high-speed digital data acquisition cards and LabView software for the experiments allows collecting a significant amount of statistical data.

The theoretical part of the methodology deals with ultrasonic flowmeter equation, which is reconsidered in order to include the term corresponding to sound speed fluctuations, which previously were neglected. The result is an integral equation for the corresponding correlation functions. The influence of temperature inhomogeneous on ultrasonic wave propagation is investigated using a set of experiments with a heated grid. Experimentally measured travel time statistical data with and without grid heating were approximated by Gaussian function and used to solve integral equation analytically in terms of correlation functions of turbulent velocity and sound speed fluctuations, and demonstrated qualitatively and quantitatively the effect of turbulence on ultrasound wave propagation. The assumption that the grid thermal input did not actively influence the turbulent velocity statistics as well as assumption of isotropy and homogeneity that allows neglecting by cross correlation between velocity and temperature were used to solve the integral flowmeter equation and demonstrate the effect (importance) of sound speed fluctuations and its statistics. During the experiment we observed that thermal fluctuations have significant influence on the sound speed propagation and thus it may not be neglected as has often been supposed previously both in experiment and theory.

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