Electronic Supplementary Material (ESI) for Lab on a Chip.

Supplementary Information for:

Two-ply Channels for Faster Wicking in Paper-based Microfluidic Devices

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Figure S1

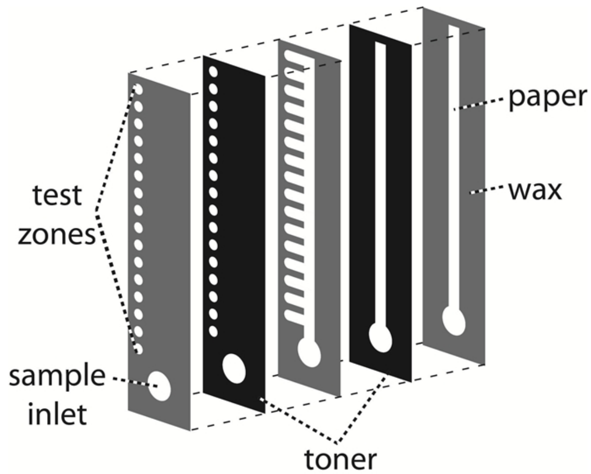


Figure S1. Schematic representation of the layers of patterned paper and toner used to fabricate the titration device.

Figure S2

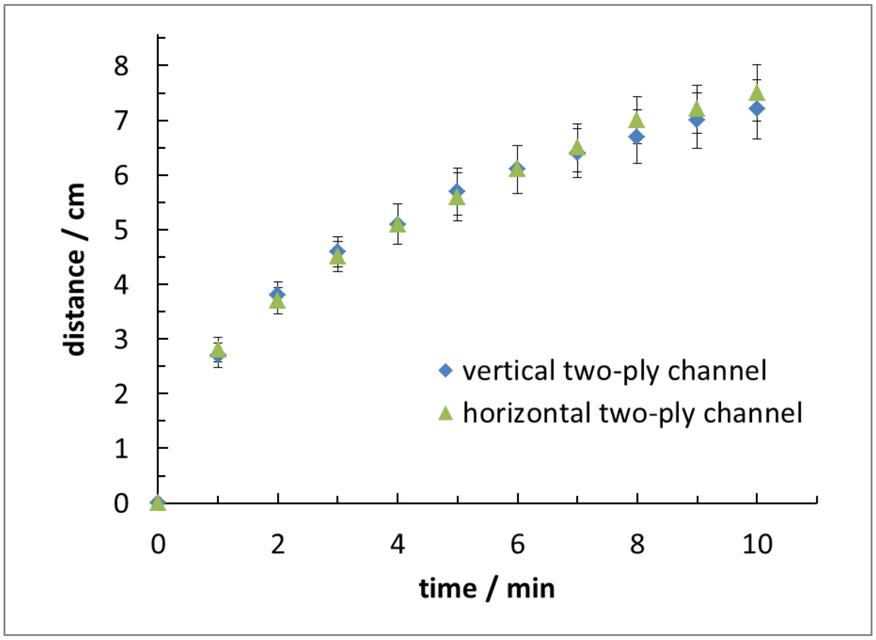


Figure S2. Plot of distance wicked by the fluid versus time for two-ply channels that were held either vertically or horizontally as they wicked liquid from a reservoir. Data points represent the mean of nine measurements and the error bars represent one standard deviation from the mean.

Figure S3

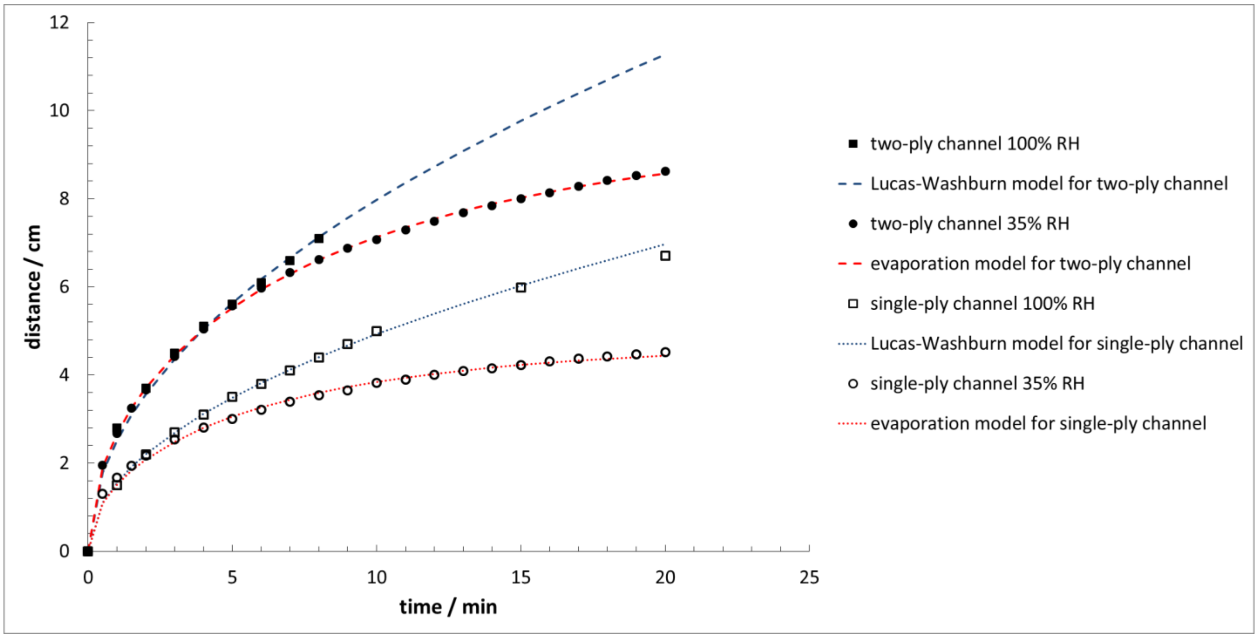
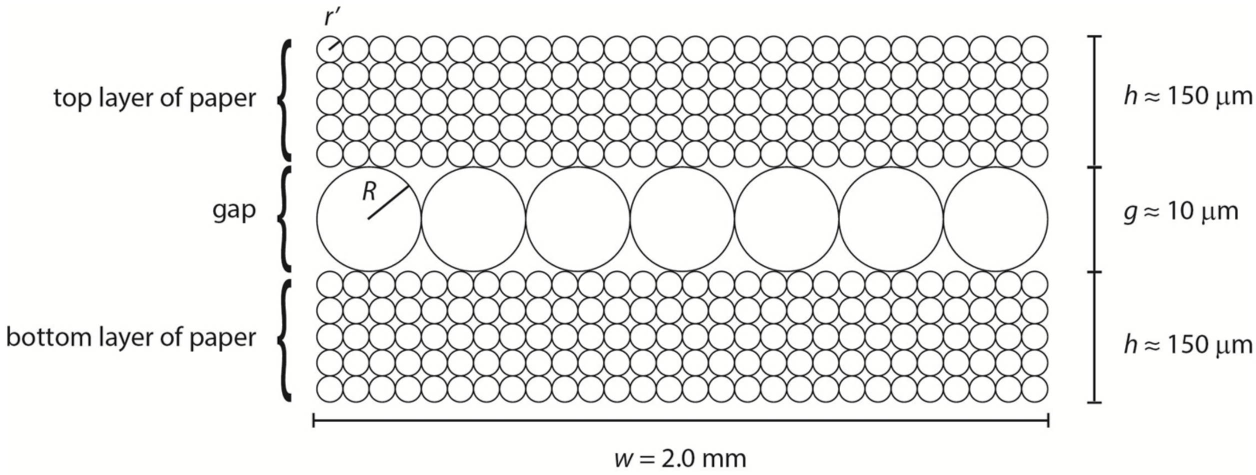


Figure S3. Plot of distance wicked by the fluid versus time for single-ply and two-ply channels under 100% and 35% RH. Data points represent the mean of 9 measurements and error bars were omitted for clarity, but can be seen in Figure 2. The dashed lines represent the modeled results using the magnitudes for r and q_0 in Table 1.

Figure S4



For a two-ply channel: $r = \dfrac{2r'(hw) + R(gw)}{2hw + gw}$, and $R = g/2$

Figure S4. Schematic diagram of the cross-section of a two-ply channel modelling the two layers of paper as bundles of uniform capillaries of radius *r'* and the gap between the layers of paper as a layer of capillaries of radius *R*. The effective pore radius for the two-ply channel *r* can be estimated by calculating the cross-sectional-surface-area-weighted average of the capillary radii in each layer of paper and in the gap.

Figure S5

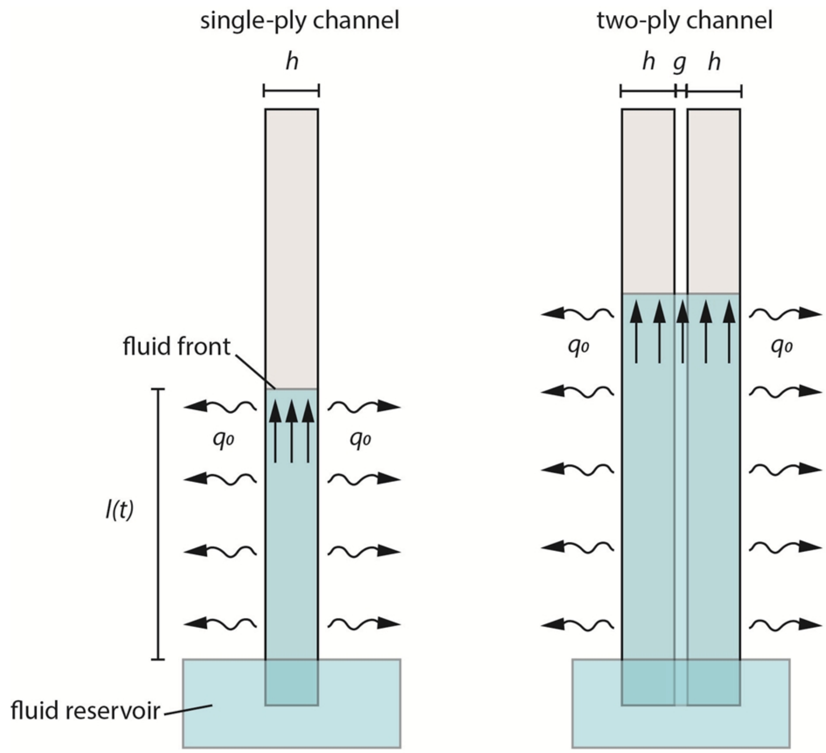


Figure S5. Schematic of a longitudinal section of single-ply and two-ply channels wicking liquid from a reservoir.

Derivation of an equation to describe wicking in a paper-based porous channel including a term for loss of fluid due to evaporation.

Starting with Darcy's law:

$$\frac{\partial P}{\partial z} = -\frac{\mu v}{k} = -\frac{\mu}{k} \times \frac{dl}{dt} \quad (1)$$

where *P* is pressure (N/m^2), *z* is distance (m), μ is dynamic viscosity (Ns/m^2), *v* is velocity (m/s) and *k* is interstitial permeability (m^2), *l* is the position of the fluid front (m) and *t* is time (s).

We write conservation of mass as:

$$\frac{\partial v}{\partial z} = -\frac{2q_0}{\phi h} \quad (2)$$

where q_0 (m/s) is loss of fluid due to evaporation in terms of volume per second per unit area from each face of the channel, the factor of 2 accounting for the fact that evaporation is occurring from the two faces of the channel, *h* (m) is the cross-sectional thickness of the channel and ϕ is the porosity. For a two-ply channel, the term ϕh must account for the two layers of paper, with porosity ϕ, and the gap, with a porosity of 1 (Figure S5).

We take the partial derivative of (1) to obtain:

$$\frac{\partial^2 P}{\partial z^2} = -\frac{\mu}{k} \times \frac{\partial v}{\partial z} \quad (3)$$

then substitute (2) into (3) to obtain:

$$\frac{\partial^2 P}{\partial z^2} = \frac{2\mu q_0}{k\phi h} \quad (4)$$

and integrate (4) twice with respect to *z* to give:

$$P = \frac{\mu q_0 z^2}{k\phi h} + C_1 z + C_2 \quad (5)$$

where C_1 and C_2 are constants.

We also know that at $z=0$, $P=0$, and at $z=l(t)$, $P=-2\gamma\cos\theta/r$ (from Young-Laplace where γ is surface tension (N/m), θ is contact angle and *r* is effective pore radius of the capillaries in paper), so we can find C_1 and C_2 in (5):

$$C_1 = -\frac{2\gamma cos\theta}{rl} - \frac{\mu q_0 l}{k\phi h}; C_2 = 0 \ (6)$$

We can substitute (6) into (5) to obtain:

$$P(z,t) = \frac{\mu q_0 z^2}{k\phi h} - \left(\frac{2\gamma cos\theta}{rl} + \frac{\mu q_0 l}{k\phi h}\right)z \quad (7)$$

Next we take the partial derivative of *P* with respect to *z* in (7) to find speed of the interface *dl/dt*:

$$\frac{\partial P}{\partial z} = \frac{\mu q_0 l}{k\phi h} - \frac{2\gamma cos\theta}{rl} \quad (8)$$

Substituting (1) into (8) gives:

$$-\frac{\mu}{k} \times \frac{dl}{dt} = \frac{\mu q_0 l}{k\phi h} - \frac{2\gamma cos\theta}{rl} \quad (9)$$

and we rearranged (9) to find:

$$\frac{d}{dt}l^2 + \frac{2q_0}{\phi h}l^2 - \frac{4\gamma k cos\theta}{r\mu} = 0 \quad (10)$$

Solving (10) we find:

$$l(t) = \sqrt{\frac{2\gamma k\phi h cos\theta}{\mu r q_0}\left(1 - e^{-\frac{2q_0}{\phi h}t}\right)} \quad (11)$$

where we have used *l(0)=0.*

We also know that for a simple channel, such as a circular pipe, interstitial permeability can be approximated as:[1]

$$k = \frac{r^2}{8} \quad (12)$$

where *r* (m) in this case is the effective pore radius of the channels in a piece of paper.

Substituting (12) into equation (11) gives our final equation:

$$l(t) = \sqrt{\frac{\gamma r\phi h cos\theta}{4\mu q_0}\left(1 - e^{-\frac{2q_0}{\phi h}t}\right)} \quad (13)$$

For fitting the experimental results in Kaleidagraph, we set $\gamma = 0.0728\ N/m$, $\mu = 0.001\ Ns/m^2$, and $\theta = 0$. For a single-ply channel, we estimated $\phi = 0.34$ and $h = 150\ \mu m$. For a two-ply channel, the term $\phi h = 2(0.34)(150\ \mu m) + 10\ \mu m = 112\ \mu m$.

In the limit where q_0 goes to zero, we can use a Taylor series to make the following approximation:

$$\lim_{q_0 \to 0} e^{-\frac{2q_0}{\phi h}t} = 1 - \frac{2q_0}{\phi h}t \quad (14)$$

Substituting (14) into (13) gives:

$$l(t) = \sqrt{\frac{\gamma cos\theta rt}{2\mu}} \quad (15)$$

which is the Lucas-Washburn equation.

References.

1. H. A. Stone, in *CMOS Biotechnology*, 2007, pp. 5–30.