MODEL-BASED DESIGN OF AN OPTIMAL LQG REGULATOR FOR A PIEZOELECTRIC ACTUATED SMART STRUCTURE USING A HIGH-PRECISION LASER INTERFEROMETRY MEASUREMENT SYSTEM

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TITLE: Model-Based Design of an Optimal LQG Regulator for a Piezoelectric Actuated Smart Structure Using a High-Precision Laser Interferometry Measurement System

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ABSTRACT

Model-Based Design of an Optimal LQG Regulator for a Piezoelectric Actuated Smart Structure Using a High-Precision Laser Interferometry Measurement System

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Smart structure control systems commonly use piezoceramic sensors or accelerometers as vibration measurement devices. These measurement devices often produce noisy and/or low-precision signals, which makes it difficult to measure small-amplitude vibrations. Laser interferometry devices pose as an alternative high-precision position measurement method, capable of nanometer-scale resolution. The aim of this research is to utilize a model-based design approach to develop and implement a real-time Linear Quadratic Gaussian (LQG) regulator for a piezoelectric actuated smart structure using a high-precision laser interferometry measurement system to suppress the excitation of vibratory modes.

The analytical model of the smart structure is derived using the extended Hamilton Principle and Euler-Bernoulli beam theory, and the equations of motion for the system are constructed using the assumed-modes method. The analytical model is organized in state-space form, in which the effects of a low-pass filter and sampling of the digital control system are also accounted for. The analytical model is subsequently validated against a finite-element model in Abaqus, a lumped parameter model in Simscape Multibody, and experimental modal analysis using the physical system. A discrete-time proportional-derivative (PD) controller is designed in a heuristic fashion to serve as a baseline performance criterion for the LQG regulator. The Kalman Filter observer and Linear Quadratic Regulator (LQR) components of the LQG regulator are also derived from the state-space model.

It is found that the behavior of the analytical model closely matches that of the physical system, and the performance of the LQG regulator exceeds that of the PD controller. The LQG regulator demonstrated quality estimation of the state variables of the system and further constitutes an exceptional closed-loop control system for active vibration control and disturbance rejection of the smart structure.
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Chapter 1

INTRODUCTION

1.1 Project Motivation

The primary motivation for this project is to investigate the application of a high-precision laser measurement system and its use in vibration measurement and active vibration control. This paper discusses the design, fabrication, and testing of a smart structure closed-loop control system using a high-precision laser interferometry measurement system. The objectives of this paper are outlined as followed:

- Design and fabricate a smart structure that utilizes laser interferometry for measurement feedback.
- Develop an analytical model that encapsulates the dynamics of the smart structure.
- Design an LQG regulator using a model-based design approach.
- Design a PD controller to act as a reference for the performance of the LQG regulator.
- Validate the performance of the LQG controller through simulation and experimental implementation.

1.2 Smart Structures and Materials

A smart structure is defined as a structure which is capable of reacting to external perturbations by sensing, actuation, and control. Unlike passive control, which is designed to permanently alter the dynamic properties of a structure or system, active control is capable of changing the mechanical and dynamic properties of a structure or system as a function of time. Smart structures are capable of performing active vibration control to rapidly dampen vibrations caused by external stimuli much faster than passive control methods; although, active control methods are considerably more complex.
While smart structures are a relatively new area of research, smart materials have been well studied and documented since the late 1800's. The most common form of smart material is piezoelectric. Pierre and Jacques Curie [1] were among the first to document the natural piezoelectric effect in 1880 while researching quartz crystals for use as a transducer for ultrasonic pulse echo detection. The piezoelectric effect was also observed independently in three countries during World War II while documenting ferroelectric materials for use in radar equipment [2, 3, 4]. The most widely used piezoelectric material today is lead zirconate titanate (PZT), which was discovered in 1951 by Shirane, Suzuki, Takeda et al. [5]; however, in 1971 Jaffe et al. [6] popularized the suitability of PZT as a piezoelectric material in ceramics by demonstrating its sensitivity and higher operating temperature compared to other piezoelectric materials of the time. Cross [7] developed a more in-depth timeline of the history and advancement of piezoelectric materials.

While the properties and formulation of piezoceramic materials continue to be studied, their applications in engineering and smart structures remain a highly active field of research. Smart structures are currently being researched in a variety of industries. Because of their very small footprint, fast response times, and ability to simultaneously sense, actuate, and control, piezoelectric transducers are commonly used in aerospace structures. In 2001, Grewal et al. [8] researched the control of piezoceramic transducers for acoustic damping of aircraft cabins. In 2013, Bilgen et al. [9] proposed using piezoceramic actuators for shape control of a variable-camber wing design. Zhou et al. [10] and Wang et al. [11] further discussed how piezoelectric materials can be used for active shape control of wing structures in 2021 and 2022, respectively.

In addition to improving the performance of aircrafts, piezoceramic smart structures have also been researched for their applications in structural health monitoring. In 2018, Kscica et al. [12] developed a virtual model for the optimal placement of piezoceramic patches for the active structural health monitoring of aerospace structures. In the same year, Hoshyarmanesh and Abbasi [13] experimentally verified the efficacy of using piezoceramic transducers for damage detection in aerospace rotary structures. In 2020, Jiao et al. [14] constructed a comprehensive review of techniques for using piezoceramic transducers to employ self-monitoring and self-powering health monitoring systems.

In addition to aerospace structures, smart materials are also being incorporated into civil structures. In 2006, Song et al. [15] discussed in-depth techniques of incorporating piezoceramics in structural beams, trusses, and bridges. In 2013, Bitarafan et al. [16] proposed an optimal design for the allocation of smart materials in bridges to mitigate damage from earthquakes—in which the design,
research, cost, performance, and maintenance of smart structures are considered. Smart materials
are also being applied to more dynamic civil projects. In 2013, Supeni et al. [17] proposed the
application of using smart-structured wind turbine blades for improving the performance of wind
turbines, while also reducing maintenance and blade wear.

1.3 Laser Interferometry

A laser interferometer is a high-precision metrology device that is used to directly measure the
position or velocity of a target. The term "interferometry" comes from the principle of using the
superposition and interference of waves as a form of measurement. A simplified diagram of a het-
erodyne interferometer setup, used in this project, is illustrated in Figure 1.1.

![Laser interferometer diagram](image)

Figure 1.1: Optical path of a general heterodyne Michelson interferometer setup.

The helium-neon laser source outputs two continuous low-frequency light waves. The two incident
waves are then projected onto a half-mirrored surface called a beam splitter. The beam splitter
allows half of the light, carrying both frequencies, to pass through the beam splitter, while the other
half is reflected at a 45° angle. The light that is initially reflected is called the reference beam, and
the light that passes through the beam splitter is called the returned beam.

The reference beam is directed toward a retroreflector, which reflects the light back at a 180° angle,
where it is then reflected off of the beam splitter again. Finally, the reference beam is directed
back in the direction of the laser source where it is detected by an optical sensor. Since there are
two frequencies of light in the coincident beam, an interference pattern is created. The interference
pattern moves radially at the beating frequency specified by the offset frequency of the two known
beams of light.
The returned beam is directed to another retroreflector which is attached to the moving target. As the target and the retroreflector move together, the frequency of the light is shifted because of the Doppler effect. The returned beam is then reflected back at a 180° angle through the beam splitter and finally into the detector. The interference pattern created by the superposition of the reference beam and the returned beam in turn produces an interference pattern that moves at a velocity determined by the shift in frequency of the returned light. Thus, knowing the frequency of the reference beam, the frequency of the returned beam can be detected by the frequency of the interference pattern at the detector, and the movement of the stage can be calculated. The fine resolution of the wavelengths of light and their frequency shift are what allow interferometry to produce nanometer measurement resolution.
Chapter 2

MATHEMATICAL MODELING

The mathematical model of the smart structure is fundamentally developed by using the extended Hamilton’s principle. The equations of motion are derived using Euler-Lagrange equations, wherein the steel substructure of the system is assumed to be an Euler-Bernoulli beam. Subsequently, the equations of motion of the smart structure are constructed using the assumed-modes method. Finally, the analytical model is formulated into state-space and transfer function equations, in which the dynamics of a low-pass filter and power amplifier are also considered.

2.1 Euler-Bernoulli Beam Theory

The smart structure, represented in Figure 2.1, is primarily composed of a cantilevered homogeneous isotropic steel beam. Two pairs of piezoceramic patches, located at the base and tip of the structure, are used for actuation and control. Finally, a retroreflector, located at the tip of the structure, is used to measure the transverse deflection.

![Figure 2.1: Top view diagram of smart structure.](image)

The steel beam has a length $L$, and a transverse deflection profile $w$ which is perpendicular to $x$. Furthermore, the steel beam is assumed to behave as an ideal Euler-Bernoulli beam in the plane of motion. It is assumed that the beam has no torsional or out-of-plane displacements. The
Euler-Bernoulli assumption entails that the beam is relatively long, the planar sections remain perpendicular to the neutral axis during bending, and that the effects of shear force are negligible.

The piezoceramic patches have specified lengths $l_p$, and the two collocated pairs are located at distances $x_1$ and $x_2$ from the base of the structure, respectively. Each pair of piezoceramic actuators add to the flexural rigidity and mass of the structure. They are also capable of producing bending moments, $M_1$ and $M_2$, on the structure. Finally, the retroreflector is used to measure the transverse deflection $w$ of the structure at the axial location $x_r$. The retroreflector is also assumed to have a point mass $m_r$ which does not add to the flexural rigidity of the system.

2.2 Extended Hamilton’s Principle

Hamilton’s principle is an energy-based modeling technique that describes the dynamics of the system by the conservation of its kinetic and potential energy. The extended version of Hamilton’s principle incorporates the consequences of dissipative and nonconservative forces. For the smart structure, the total change in kinetic energy $\delta T$, the total change in potential energy $\delta U$, and the total work performed by non-conservative forces $\delta W_{nc}$ between any two points in time, $t_1$ and $t_2$, is described by the Euler-Lagrange equation [18]

$$\int_{t_1}^{t_2} (\delta T - \delta U + \delta W_{nc}) \, dt = 0. \quad (2.1)$$

where $T$ is the total kinetic energy of the system, $V$ is the total potential energy of the system, $W_{nc}$ is the work done on the system by non-conservative forces. Moreover, the symbol $\delta$ is used to represent the change or difference of the respective quantities. The virtual work of the internal and non-conservative forces acting upon the system can be expressed as a linear function of generalized coordinates

$$T = T(q_1, q_2, ..., q_n, \dot{q}_1, \dot{q}_2, ..., \dot{q}_n), \quad (2.2)$$

$$U = U(q_1, q_2, ..., q_n), \quad (2.3)$$

$$\delta W_{nc} = (f_1\delta q_1, f_2\delta q_2, ..., f_n\delta q_n), \quad (2.4)$$

where $[q_1, q_2, ..., q_n]$ are generalized coordinates and $[f_1, f_2, ..., f_n]$ are forces represented in the generalized coordinates. Substituting Eqs. (2.2-2.4) into Eq. (2.1) produces the following Euler-Lagrange
equation which is used to define the equations of motion of the system

\[
\frac{\partial}{\partial t} \left( \frac{\partial T}{\partial \dot{q}_j} \right) - \frac{\partial T}{\partial q_j} + \frac{\partial U}{\partial q_j} = F_j, \tag{2.5}
\]

where \( j \) is a single term of the \( n \) number of Lagrange equations. Furthermore, the strain forces generated by the piezoceramic patches and viscous damping of the steel substructure are represented by the generalized non-conservative forces \( F_j \). The potential strain energy of the Euler-Bernoulli beam can be written as

\[
U = \frac{1}{2} \int_0^L G(x) \left( \frac{\partial^2 w(x, t)}{\partial x^2} \right)^2 \, dx, \tag{2.6}
\]

where \( w(x, t) \) is the displacement profile of the smart structure as a function of time and position along the structure. Furthermore, the local flexural rigidity of the beam \( G(x) \) is assumed to be of the form

\[
G(x) = E_b I_b + E_p I_p \sum_{m=1}^2 \left[ u(x - x_m) - u(x - x_m - l_p) \right], \tag{2.7}
\]

where \( u \) is the unit step function, \( E \) is the local elastic modulus of the system, \( I \) is the local moment area of inertia of the system, and the subscripts \( b \) and \( p \) denote the mechanical properties of the beam and piezoceramic patches, respectively. Furthermore, the subscript \( m \) is used to identify the pair of collocated piezoceramic patches; e.g., \( m = 1 \) denotes the properties of the piezoceramic patches located near the base of the structure and \( m = 2 \) denotes the properties of the piezoceramic patches located near the tip of the structure.

The assumption that the local flexural rigidity \( G(x) \) can be written as a continuous function allows for the mass and stiffness matrices of the system to be expressed symmetrically—allowing the modal equations of motion of the system to be decoupled. With both patches offset from the neutral axis, their area moment of inertia is given by

\[
I_p = \frac{ht_p}{2} \left( t_b^2 + 2t_bt_p + \frac{4t_p^3}{3} \right), \tag{2.8}
\]

where \( h \) is the height of the structure, \( t_b \) is the thickness of the beam, and \( t_p \) is the thickness of the piezoceramic patch. The cross-sectional view of the smart structure at the location of the piezoceramic actuators is shown in Figure 2.2.
The geometry of the smart structure is simplified to equate the height of the piezos and the beam [19]. Furthermore, the kinetic energy of the Euler-Bernoulli beam is given by

\[
T = \frac{1}{2} \int_0^L P(x) \left( \frac{\partial w(x, t)}{\partial t} \right)^2 dx, \tag{2.9}
\]

in which the linear density of the beam \( P(x) \) is assumed to be in the form

\[
P(x) = \rho_b A_b + \rho_p A_p \sum_{m=1}^2 [u(x - x_m) - u(x - x_m - l_p)] + m_r \delta(x - x_r), \tag{2.10}
\]

where \( m_r \) is the mass of the retroreflector, \( \rho_b \) and \( \rho_p \) are the densities for the beam and piezoceramic patches, and \( A_b \) and \( A_p \) are the cross-sectional areas for the beam and piezoceramic patches, respectively.

### 2.3 Assumed-Modes

Using the assumed-modes method, the kinetic and potential energy of the system can be used to extract the deflection profile of the structure. The deflection profile of the structure \( w(x, t) \) can be expressed as the superposition of its generalized displacements and shape functions

\[
w(x, t) = \sum_{j=1}^n \Psi_j(x)q_j(t), \tag{2.11}
\]

where \( \Psi_j(x) \) is a shape function describing the assumed profile of the structure and \( q_j(t) \) is the generalized displacement of the structure along the shape function, where \( n \) is the total degrees of
freedom of the model. For each degree of freedom \( j \), the shape function is assumed to have the general hyperbolic form

\[
\Psi_j(x) = A_1 \cos(\beta_j x) + A_2 \sin(\beta_j x) + A_3 \cosh(\beta_j x) + A_4 \sinh(\beta_j x),
\]

(2.12)

where the coefficients \( A_1 - A_4 \) must satisfy the boundary conditions of the physical system, and the set of all values \( \beta_j \) forms a linearly independent set of shape functions \( \Psi_j \) for \( j = 1, 2, \ldots, n \). Thus, the boundary conditions for the Euler-Bernoulli beam with fixed-free boundary conditions are such that the fixed end of the beam does not experience any deflection

\[
w(x = 0, t) = 0,
\]

(2.13)

the fixed end of the beam does not experience any angular deflection

\[
\frac{\partial w}{\partial x}(x = 0, t) = 0,
\]

(2.14)

the free end of the beam does not experience any internal bending moment

\[
\frac{\partial^2 w}{\partial^2 x}(x = L, t) = 0,
\]

(2.15)

and the free end of the beam does not experience any internal shear force

\[
\frac{\partial^3 w}{\partial^3 x}(x = L, t) = 0.
\]

(2.16)

Applying the boundary conditions from Eqs. (2.13-2.16) to Eq. (2.12), the shape function can be simplified to

\[
\Psi_j(x) = \sin(\beta_j x) - \sinh(\beta_j x) + \sigma_j \cos(\beta_j x) - \cosh(\beta_j x),
\]

(2.17)

such that

\[
\sigma_j = \frac{\sinh(\beta_j L) + \sin(\beta_j L)}{\cosh(\beta_j L) + \cos(\beta_j L)}.
\]

(2.18)

The only nontrivial solutions to Eq. (2.17) are such that

\[
\cosh(\beta_j L)\cos(\beta_j L) = -1,
\]

(2.19)
where the values of $\beta_j$ satisfy the fixed-free boundary conditions and closely represent the deflection profiles of the $j^{th}$ vibratory mode shape. Subsequently, the modal deflection profiles represented by the shape function for $j = 1, 2, \ldots, 5$ are illustrated in Figure 2.3.

![Figure 2.3: Plot of the normalized shape functions used for the first five vibratory modes of the cantilevered Euler-Bernoulli beam model.](image)

The displacement contribution from each mode at a specified location can be determined by evaluating the shaping function at that location.

The total kinetic energy $T$ and total potential energy $U$ of the system can be now used to reexpress the equations of motion of the structure in terms of its generalized mass and stiffness in matrix form. Assuming that the matrices diagonalized, the equations of motion can then be represented as a system of second-order differential equations, which are easier to solve compared to partial differential equations.

First, the potential energy of the system can be reorganized as a function of the generalized stiffness matrix $K$ by inserting Eq. (2.11) into Eq. (2.6) to yield

$$U = \frac{1}{2} \sum_{i=1}^{n} \left( \sum_{j=1}^{n} [K_{ij}q_i(t)q_j(t)] \right),$$

(2.20)

where the elements of the generalized stiffness matrix $K_{ij}$ are defined as

$$K_{ij} = \int_0^L G(x)\Psi''_i(x)\Psi''_j(x)dx.$$  

(2.21)
Furthermore, the potential energy of the system can be reorganized as a function of the generalized mass matrix $M$ by inserting Eq. (2.11) into Eq. (2.9)

$$T = \frac{1}{2} \sum_{i=1}^{n} \left( \sum_{j=1}^{n} [M_{ij}q_i(t)q_j(t)] \right),$$

(2.22)

where the elements of the generalized mass matrix $M_{ij}$ are defined as

$$M_{ij} = \int_{L}^{l} P(x)\Psi_i(x)\Psi_j(x)dx. \quad (2.23)$$

### 2.4 Piezoelectric Actuator Equations

The beam is subjected to external forces by the piezoceramic patches which can be expressed in terms of the virtual work in generalized coordinates

$$\delta W_{nc} = \int_{0}^{L} f(x,t)\delta w(x,t)dx. \quad (2.24)$$

Substituting Eq. (2.5) into Eq. (2.24) leaves

$$F_j = \int_{0}^{L} f(x,t)\Psi_j(x)dx. \quad (2.25)$$

where $f(x,t)$ is the generalized, distributed transverse force acting on the beam of each pair of co-located piezoceramic patches. The actuation force of each pair of piezos is defined as

$$f_m(x,t) = \left( \frac{\partial^2 M_m(x,t)}{\partial x^2} \right) \Psi_j(x), \quad (2.26)$$

where $M_m(x,t)$ is the moment generated by the $m^{th}$ pair of piezoelectric patches indicated by Figure 2.1. The moment generated by a single pair of collocated electrically-excited piezoceramic patches is

$$M_m(x,t) = kV_m(t) [u(x - x_m) - u(x - x_m - l_p)], \quad (2.27)$$

where $V_m(t)$ is the high-voltage potential across each piezoceramic patch for $m = 1, 2$. In particular, the linear stress-strain relationship for each piezoceramic patch is defined as

$$k = E_b I_b \gamma, \quad (2.28)$$

where $\gamma$ considers the cross-sectional dimensions of the piezos and their electrocoupling constant

$$\gamma = \frac{d_{31}}{t_p} \left[ \frac{3E_p ((\frac{t_b}{2} + t_p)^2 - (\frac{t_b}{2})^2)}{2 (E_p((\frac{t_b}{2} + t_p)^3 - (\frac{t_b}{2})^3) + E_b(\frac{t_b}{2})^3)} \right]. \quad (2.29)$$

where $t_b$ and $t_p$ are the thickness of the beam and piezoceramic patches, respectively (see Figure 2.2).

Eq. (2.25) can be evaluated more easily by integration by parts

$$F_{m,j}(x, t) = kV_m(t) \left[ \Psi_j(L)(\delta(L - x_m) - \delta(L - x_m - l_p)) - \int_0^L \frac{\partial \psi_j(x)}{\partial x}(\delta(x - x_m) - \delta(x - x_m - l_p)) dx \right]. \quad (2.30)$$

Furthermore, it is desired to express the actuation force of the piezoceramic actuators as a function of the actuation voltage

$$F_{m,j}(t) = \tilde{F}_{m,j}V_m(t), \quad (2.31)$$

which can be further simplified as

$$\tilde{F}_{m,j} = k \left[ \frac{\partial \psi_j(x_m - l_p)}{\partial x} - \frac{\partial \psi_j(x_m)}{\partial x} \right]. \quad (2.32)$$

### 2.5 Rayleigh Damping

Rayleigh proportional damping is used to introduce viscous damping in the analytical model [20]. While various methods of modeling damping were considered, such as direct modal damping, Rayleigh damping was chosen because of its ease of implementation and tunability. In many analytical models, it is very difficult, if not impossible, to accurately replicate the damping characteristics of physical structures. Therefore, empirical methods are commonly used to determine the damping characteristics of physical structures, which are then employed in analytical models.

Rayleigh damping is a form of proportional damping which incorporates the mass and stiffness of the system, as shown in Figure 2.4.
While mass-proportional damping typically underdamps the higher frequencies of a model, stiffness-proportional damping typically overdamps the higher frequencies of the model. Rayleigh damping has the benefit of instantiating the damping coefficients over a span of specified frequencies.

The Rayleigh structural damping matrix $C_R$ is defined as a linear combination of the structural mass and stiffness matrices

$$
C_R = \eta_1 M + \eta_2 K,
$$

where $\eta_1$ is the mass-proportional Rayleigh damping coefficient and $\eta_2$ is the stiffness-proportional Rayleigh damping coefficient. In turn, these coefficients are what give the Rayleigh damping model better tunability over the mass-proportional and stiffness-proportional damping models. The coefficients $\eta_1$ and $\eta_2$ can be determined by applying modal damping ratios to the minimum and maximum observable frequencies in the system.

$$
\begin{bmatrix}
\eta_1 \\
\eta_2
\end{bmatrix} = \frac{1}{2}\begin{bmatrix}
\frac{1}{\omega_1} & \omega_1 \\
\frac{1}{\omega_n} & \omega_n
\end{bmatrix}
\begin{bmatrix}
\zeta_1 \\
\zeta_n
\end{bmatrix}
$$

where $\omega_1$ and $\zeta_1$ are the frequency and damping ratio of first mode of the system, respectively, and $\omega_n$ and $\zeta_n$ are the frequency and damping ratio of the $n^{th}$ mode of the $n$-degree of freedom model.
2.6 Equations of Motion

By inserting Eqs. (2.21, 2.23, 2.32, 2.33) into (2.1), the equations of motion for the smart structure are defined as

\[ M\ddot{q} + C_R\dot{q} + Kq = \tilde{F}_1 V_1 + \tilde{F}_2 V_2, \quad (2.35) \]

where \( q = [q_1, q_2, ..., q_n]^T \) are the generalized coordinates of the system.

To represent the dynamics of the vibratory modes of the system, the modal matrix \( \Phi \) is constructed by calculating the eigenvectors of the structural mass matrix \([21, 22]\).

\[ q = \Phi g \quad (2.36) \]

The modal matrix provides a linear transform between generalized coordinates \( q \) and modal coordinates \( g \) while also decoupling the equations of motion—so long as the shape functions are permissible \([23]\). Substituting Eq. (2.36) into Eq. (2.35)

\[ M\Phi\ddot{g} + C_R\Phi\dot{g} + K\Phi g = \tilde{F}_1 V_1 + \tilde{F}_2 V_2, \quad (2.37) \]

then multiplying both sides of Eq. (2.37) by the transpose of modal matrix

\[ \Phi^T M\Phi\ddot{g} + \Phi^T C_R\Phi\dot{g} + \Phi^T K\Phi g = \Phi^T \tilde{F}_1 V_1 + \Phi^T \tilde{F}_2 V_2, \quad (2.38) \]

The result can be further simplified to

\[ \bar{M}\ddot{g} + \bar{C}_R\dot{g} + \bar{K}g = \Phi^T \tilde{F}_1 V_1 + \Phi^T \tilde{F}_2 V_2 \quad (2.39) \]

where \( \bar{M} \), \( \bar{C} \), and \( \bar{K} \) represent the generalized mass, generalized damping, and generalized stiffness matrices, respectively.

2.7 Low-pass Filter and Amplifier Equations

The piezoceramic actuators require a high-voltage signal to produce a significant actuation force on the smart structure. A power amplifier is used to amplify the voltage used to drive the piezoceramic
However, the power amplifier has a relatively low current limit. To prevent damage to the power amplifier, the control signal must be kept at a low slew rate. As a result, the control signal must not contain any sharp edges or high frequencies. To prevent damage to the power amplifier, the slew rate is limited by placing a low-pass filter (LPF) inline with the power amplifier. The power amplifier and LPF are shown in Figure 2.5.

![Figure 2.5: Diagram of the electrical connections between the real-time target, LPF and amplifier, and piezoceramic patches.](image)

The circuit used to create the LPF is a simple active LPF which uses an op-amp, two resistors, and a capacitor. A circuit diagram of the active LPF is shown in Figure 2.6.

![Figure 2.6: Circuit schematic of an active LPF.](image)

The voltage gain associated with the LPF is defined as the ratio between the two resistors

$$K_{lpf} = \frac{R_2}{R_1},$$  \hspace{1cm} (2.40)

and the cutoff frequency of the LPF is

$$\omega_c = \frac{1}{R_2 C_{lpf}}.$$  \hspace{1cm} (2.41)
The LPF behaves as a first-order system by attenuating high-frequency signals. At the cutoff frequency $\omega_c$, the input-output ratio of the LPF is specified as -3 dB. Incorporating the amplifier gain $K_{amp}$ into the input of the LPF, the ordinary differential equation representing the LPF and the amplifier can be expressed as

$$\dot{V}_m = -K_{lpf}\omega_c V_m + K_{amp}\omega_c S_m$$

(2.42)

where $V_p = [V_1, V_2]$ is the filtered output voltage supplied to each pair of piezoceramic patches and $V_c = [V_1, V_2]$ is the supplied control voltage.

### 2.8 State-Space Formulation

The dynamics of the smart structure can be represented as a continuous linear time-invariant state-space system [24]

$$\dot{x}(t) = Ax(t) + Bu(t).$$

(2.43)

Before including the dynamics of the LPF and amplifier, the dynamics of the smart structure can be represented in state-space form by reorganizing Eq. (2.39) in the form of Eq. (2.43). Subsequently, the state matrix can be expressed as

$$A^* = \begin{bmatrix} 0_{n \times n} & I_{n \times n} \\ -\bar{M}^{-1} \bar{K} & -\bar{M}^{-1} \bar{C}_R \end{bmatrix},$$

(2.44)

the control matrix is

$$B^* = \begin{bmatrix} 0_{n \times 1} & 0_{n \times 1} \\ \bar{M}^{-1} \Phi^T \bar{F}_1 & \bar{M}^{-1} \Phi^T \bar{F}_2 \end{bmatrix},$$

(2.45)

the state vector is

$$x^* = \begin{bmatrix} g \\ \dot{g} \end{bmatrix}^T,$$

(2.46)

and the input vector is

$$u^* = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}^T,$$

(2.47)

in which $V_1$ and $V_2$ are the amplified voltages at the first and second pair of piezoceramic patches, respectively. The transverse displacement of the beam at the location of the retroreflector $w(x_r, t)$
is defined as

$$y^*(t) = Cx^*(t),$$  \hspace{1cm} (2.48)$$

where the output matrix is

$$C^* = \begin{bmatrix} \Psi(x_r) \Phi & 0_{1 \times n} \end{bmatrix}. \hspace{1cm} (2.49)$$

Note that Eqs. (2.44-2.49) are denoted by a star since they do not yet contain the expressions for the LPF and amplifier, which both have a significant impact on the system model. Consequently, the state-space system can be augmented to incorporate the states of the amplified voltage of the LPF. Since there are two piezoceramic actuators, Eq. (2.42) can be appended to Eqs. (2.44-2.49) to construct the augmented state-space matrices

$$A = \begin{bmatrix} 0_{n \times n} & I_{n \times n} & 0_{n \times 1} & 0_{n \times 1} \\
-M^{-1}\bar{K} & -M^{-1}\bar{C}_R & M^{-1}\Phi^TF_1 & M^{-1}\Phi^TF_2 \\
0_{1 \times n} & 0_{1 \times n} & K_{lpf}\omega_c & 0 \\
0_{1 \times n} & 0_{1 \times n} & 0 & K_{lpf}\omega_c \end{bmatrix}, \hspace{1cm} (2.50)$$

$$B = K_{amp}\omega_c \begin{bmatrix} 0_{2n \times 2} \\
I_{2 \times 2} \end{bmatrix}, \hspace{1cm} (2.51)$$

$$x = \begin{bmatrix} g & \dot{g} & V_1 & V_1 \end{bmatrix}^T, \hspace{1cm} (2.52)$$

$$u = \begin{bmatrix} S_1 & S_2 \end{bmatrix}^T, \hspace{1cm} (2.53)$$

$$y(t) = Cx(t), \hspace{1cm} (2.54)$$

$$C = \begin{bmatrix} \Psi(x_r) \Phi & 0_{1 \times (n+2)} \end{bmatrix}, \hspace{1cm} (2.55)$$

where $S_1$ and $S_2$ are the non-amplified controller outputs for the first and second pair of piezoceramic patches, respectively.

2.9 Transfer Function Formulation

Unlike modern control theory, where the use of state-space equations are employed for multi-input multi-output (MIMO) systems, classical control theory surrounds the analysis of single-input single-output (SISO) systems represented in the frequency domain by transfer functions; however, state-
space equations can also be used to model SISO systems. The PD controller for the smart structure system will be developed with respect to the input of the piezoceramic patches located at the base of the beam, and the piezoceramic patches located at the tip of the beam will serve as the disturbance perturbation. Thus, the transfer function can be converted from state-space form as such

\[ G_{\text{plant}}(s) = C^* [sI - A^*]^{-1} \tilde{B}^*, \]  

(2.56)

where \( G_{\text{plant}}(s) \) is the transfer function representing the input-output relationship between the control signal voltage and the transverse displacement of the smart structure at the location of the retroreflector and \( \tilde{B}^* \) is the first column of \( B^* \) from Eq. (2.45) representing the signal voltage for the base piezoceramic patches. Moreover, the input-output relationship can be more simply expressed as

\[ G_{\text{plant}}(s) = \frac{W(s)}{S_1(s)}, \]  

(2.57)

where \( W \) is the transverse displacement at the retroreflector and \( S_1 \) is the control signal voltage. Finally, \( s \) is a complex variable

\[ s = \sigma + i\Omega, \]  

(2.58)

where \( \sigma \) is the magnitude of the system and \( \Omega \) is the angular frequency of the system. Substituting Eqs. (2.43) and (2.48) into (2.55) yields

\[ G_{\text{plant}}(s) = \frac{b_{2n-1}s^{2n-1} + b_{2n-2}s^{2n-2} + \ldots + b_0}{a_{2n+2}s^{2n+2} + b_{2n+1}s^{2n+1} + \ldots + a_0}, \]  

(2.59)

where \( n \) is the number of degrees of freedom of the model defined by the number of shape functions used to construct the state space model. The highest power of \( s \) in the denominator of Eq. (2.58) represents the order of the transfer function. For example, a state-space model with \( n = 1 \) degrees of freedom can be expressed as a second-order transfer function, and a state-space model with \( n = 5 \) degrees of freedom can be expressed as a 12\textsuperscript{th} order transfer function.
For classical control of the smart structure system, a PD controller is proposed. The objective of designing a PD controller is to establish a performance baseline to compare with the performance of the LQG controller for vibration control. Moreover, classical control theory has been studied for many years, and there exist a variety of well-known controller tuning methods, making it a good choice for designing a baseline controller.

3.1 Closed-Loop Transfer Function

While most closed-loop systems can be modeled in continuous time, the controller for the smart structure system is derived in discrete time because its performance is significantly impacted by the sampling rate of the system. The system is more sensitive to the sampling rate because it directly influences the ability to observe higher-order modes in the system. The higher-order modes can produce higher frequencies in the measurement signal, which inhibits the performance of the PD controller. As a result, the sampling frequency of the controller significantly impacts the stability of the closed-loop system.

The continuous-time transfer function can be converted to a discrete-time transfer function by using a zero-order hold (ZOH). The ZOH samples the continuous-time signal and holds its value for the duration of a sampling period. The block diagram of the discrete-time closed-loop control system is shown in Figure 3.1

Figure 3.1: Closed-loop block diagram for the transfer function model of the smart structure with inclusion of sampling and ZOH.
where $G_{plant}(s)$ is the continuous-time transfer function for the smart structure from Eq. (2.58), $G_p(s)$ is sampled plant transfer function, and $D(z)$ is the general discrete controller transfer function. Located before the plant is the ZOH, which acts as a digital-to-analog converter (DAC) by transforming the output of the digital controller into a quantized continuous-time signal. Located between the summing junction and the controller is a sampler that converts the continuous-time measurements into discrete data points and acts as an analog-to-digital converter (ADC).

The first step in discretizing the plant transfer function involves taking the z-transform of the continuous components following the sampler. Consider the following transfer function of the feed-forward path

$$G_p(s) = G_{ZOH}(s)G_{plant}(s). \tag{3.1}$$

The z-transform of the transfer function using the inversion integral method is the sum of the residuals of the function. The exponential term from the ZOH is expressed as the coefficient of the sum, and the integrator is moved inside of the residuals

$$G_p(z) = (1 - z^{-1}) \sum_{\text{for } \lambda} \text{Res} \left[ \frac{1}{\lambda} G_{plant}(\lambda) \left( \frac{1}{1 - z^{-1}e^{T_s \lambda}} \right) \right], \tag{3.2}$$

where $G_p(z)$ is the discretized plant transfer function, $T_s$ is the sampling period of the controller, and the values of $\lambda$ are the open-loop poles of the plant transfer function $G_{plant}(s)$. Substituting the continuous-time transfer function from Eq. (2.58) into (3.2) yields the general discrete-time transfer function

$$G_p(z) = \frac{b_{2n+1}z^{2n+1} + b_{2n}z^{2n} + \ldots + b_0}{a_{2n+2}z^{2n+2} + b_{2n+1}z^{2n+1} + \ldots + a_0}, \tag{3.3}$$

Furthermore, the comparison between the step response of the continuous model and the discrete model using the inversion integral method is shown in Figure 3.2.
Figure 3.2: Step response comparison of continuous-time and discrete-time transfer function models of the smart structure utilizing the ZOH.

Both the continuous and discrete systems were modeled in Simulink and a unit step was applied. The sampled step response shows characteristics incredibly similar to the continuous step response, as expected. At each sample period, the ZOH maintains the current value of the measured displacement for 0.3 ms until the next point is sampled. It’s important to note that while the sampled system is held with a ZOH, it is not quantized. This means that there is still an infinite amount of resolution on the measurement system. In a real digital system, the measurement signal is digitized and the held value has a finite amount of resolution determined by the number of bits used by the DAC.

With the plant transfer function expressed in the z-domain, it is now possible to define the open-loop and closed-loop transfer functions for the system. From Figure 3.1, the open-loop transfer function for the system is expressed as the product of the feed-forward path on the top. The open-loop transfer function for the second-order approximated system is

\[ G_{open}(z) = D(z)G_p(z). \] (3.4)

where \( G_{open} \) is the open-loop system transfer function and \( D(z) \) is the generalized controller transfer function. Next, the closed-loop transfer function can be calculated by reducing the unity feedback loop in the system

\[ G_{closed}(z) = \frac{D(z)G_{open}(z)}{1 + D(z)G_{open}(z)} \] (3.5)

where \( G_{closed} \) is the transfer function representing the closed-loop system. Using the closed-loop transfer function, the parameters of \( D(z) \) can be determined to develop the appropriate controller
for a given application. In addition, the closed-loop transfer function can be used to observe the stability of the system and to simulate its performance and response.

3.2 PD Controller

From Eq. (3.5), the transfer function $D(z)$ can now be explicitly defined for the PD controller. First, the continuous transfer function $D(s)$ is expressed as

$$D(s) = K_p + K_d \frac{N s}{s + N}$$  (3.6)

where $K_p$ is the proportional gain, $K_d s$ is the derivative gain, and $N$ is the filter coefficient. The added term $s$ in the numerator of the controller transfer function allows the control system designer to place a zero by varying the gains $K_p$ and $K_d$. Since implementing an improper transfer function such as $K_d s$ is impossible, a filter coefficient $N$ is introduced; thus, as $N$ tends toward infinity, the limit of the derivative term approaches $K_d s$. Furthermore, since the smart structure control system is implemented digitally and the effects of sampling are considered significant to the impact of the underdamped closed-loop system, the PD controller must be discretized. The controller can be discretized using Euler’s backward-difference method

$$D(z) = D(s) \big|_{s=\frac{z-1}{z+1}}.$$  (3.7)

While there are other methods of mapping between the continuous and discrete domains, such as Euler’s forward-difference method and the bilinear transform (aka bilinear or Tustin’s method), the backwards difference method results in a much more stable closed-loop system. This is because it maps the negative, real plane in the s-domain to a region within the unit circle in the z-domain. A visual illustration of the mapping is shown below.
Since the negative real plane in the s-domain is stable and the region within the unit circle is stable, this ensures that any system which was stable before the mapping will remain stable. Substituting Eq. (3.7) into (3.6) and simplifying yields

\[ D(z) = K_p + K_d \frac{N(z - 1)}{(1 + NT_s)z - 1}, \]  

which is the closed-form transfer function of the PD controller.

3.3 Jury Stability Criterion

The Jury stability test [25], much like the Routh-Hurwitz stability test [26], is an analytical method to determine the stability of a closed-loop system through its transfer function. However, the Routh-Hurwitz test can only be utilized for continuous-time systems, whereas the Jury test is utilized for discrete-time functions. The Jury test can be used to identify the maximum values of the proportional and derivative gains that will produce a marginally stable system. Subsequently, it can be used heuristically to establish a baseline controller, much like the Ziegler-Nichols method [27].

It is first necessary to identify the characteristic polynomial equation for the discrete-time, closed-loop transfer function, from Eq. (3.3), which is considered to be the denominator

\[ f(z) = a_{2n+2}z^{2n+2} + b_{2n+1}z^{2n+1} + ... + a_0. \]  

\[ \]
For the characteristic polynomial equation, there exist two necessary conditions for stability that include

\[ f(1) > 0, \]  
(3.10)

and

\[ (-1)^n f(-1) > 0. \]  
(3.11)

If and only if Eqs. (3.10) and (3.11) are satisfying can the sufficient conditions for stability be checked. The sufficient conditions are obtained by constructing the Jury table as followed in Table 3.1.

Table 3.1: Generalized Jury table for the stability analysis of a \( n \)th order discrete-time system.

<table>
<thead>
<tr>
<th>Row</th>
<th>( z^0 )</th>
<th>( z^1 )</th>
<th>( z^3 )</th>
<th>( z^4 )</th>
<th>...</th>
<th>( z^{n-k} )</th>
<th>...</th>
<th>( z^{n-1} )</th>
<th>( z^n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( a_0 )</td>
<td>( a_1 )</td>
<td>( a_2 )</td>
<td>( a_3 )</td>
<td>...</td>
<td>( a_{n-k} )</td>
<td>...</td>
<td>( a_{n-1} )</td>
<td>( a_n )</td>
</tr>
<tr>
<td>2</td>
<td>( a_n )</td>
<td>( a_{n-1} )</td>
<td>( a_{n-2} )</td>
<td>( a_{n-3} )</td>
<td>...</td>
<td>( a_k )</td>
<td>...</td>
<td>( a_1 )</td>
<td>( a_0 )</td>
</tr>
<tr>
<td>3</td>
<td>( b_0 )</td>
<td>( b_1 )</td>
<td>( b_2 )</td>
<td>( b_3 )</td>
<td>...</td>
<td>( b_{n-k} )</td>
<td>...</td>
<td>( b_{n-1} )</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>( b_{n-1} )</td>
<td>( b_{n-2} )</td>
<td>( b_{n-3} )</td>
<td>( b_{n-3} )</td>
<td>...</td>
<td>( b_k )</td>
<td>...</td>
<td>( b_0 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
</tr>
<tr>
<td>2n-5</td>
<td>( p_0 )</td>
<td>( p_1 )</td>
<td>( p_2 )</td>
<td>( p_3 )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
</tr>
<tr>
<td>2n-4</td>
<td>( p_3 )</td>
<td>( p_2 )</td>
<td>( p_1 )</td>
<td>( p_0 )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
</tr>
<tr>
<td>2n-3</td>
<td>( q_0 )</td>
<td>( q_1 )</td>
<td>( q_2 )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
<td>( \vdots )</td>
</tr>
</tbody>
</table>

where the first row is constructed from the polynomial coefficients of the characteristic equation in ascending order and the second row is equal to the first row in reverse conjugated order. The subsequent odd-numbered rows can be calculated using the two rows above; first, creating a matrix using the first column of values and choosing the second column as given by

\[
b_k = \begin{vmatrix} a_0 & a_{n-k} \\ a_n & a_k \end{vmatrix}.
\]  
(3.12)

where \( k \) is the selected polynomial order. Finally, the sufficient conditions for stability of the closed-loop system are satisfied under the given circumstances

\[
\begin{align*}
|a_0| &< a_n \\
|b_0| &< |b_{n-1}| \\
&\vdots \\
|p_0| &< |p_1| \\
|q_0| &< |q_2|
\end{align*}
\]  
(3.13)
Thus, the values chosen for $K_p$ and $K_d$ to construct the closed-loop transfer function of Eq. (3.8) which satisfies Eqs. (3.12) and (3.13) can be described as stable. In the method of using the Jury stability test to design a PD controller, it is possible to determine both the values of $K_p$ and $K_d$ at the same time; however, the result will be a parametric inequality function that confines both values $K_p$ and $K_d$. An easier and more traditional approach to designing the PD controller is to define $D(z)$ from Eq. (3.8) as the proportional term only and determine the range of stable values. The proportional gain can then be reduced from its maximum value, and the range of stable values for the derivative gain can be determined [27].
Chapter 4

OPTIMAL CONTROL

For the optimal control of the smart structure system, an LQG regulator is proposed. The LQG regulator is developed by using a Kalman filter in conjunction with a linear quadratic regulator, as shown in Figure 4.1

The Kalman filter is an *optimal* state observer which is used to estimate the states of the system in the presence of noise using the inputs and output linear state-space model, process noise covariance, and measurement noise covariance. Then, the Kalman filter compares the weighted output of the linear model with the weighted measurement output to determine an estimation of the states of the system. The linear quadratic regulator is an optimal controller which minimizes a cost function defined by the states and inputs of the system, and minimizes its value according to the specified weights of the system states and controller. Together, the optimal observer and the optimal controller create the LQG regulator which retains optimally. The primary objective of the LQG regulator is disturbance rejection of the smart structure system.

Figure 4.1: Simplified block diagram of an LQG regulator subject to process noise and measurement noise.

The Kalman filter is an *optimal* state observer which is used to estimate the states of the system in the presence of noise using the inputs and output linear state-space model, process noise covariance, and measurement noise covariance. Then, the Kalman filter compares the weighted output of the linear model with the weighted measurement output to determine an estimation of the states of the system. The linear quadratic regulator is an optimal controller which minimizes a cost function defined by the states and inputs of the system, and minimizes its value according to the specified weights of the system states and controller. Together, the optimal observer and the optimal controller create the LQG regulator which retains optimally. The primary objective of the LQG regulator is disturbance rejection of the smart structure system.
4.1 Kalman Filter

The physical system is subject to different forms of process noise, which influence the dynamic behavior of the system

\[ \dot{x}(t) = Ax(t) + Bu(t) + z(t), \quad (4.1) \]

in addition to measurement noise

\[ y(t) = Cx(t) + v(t), \quad (4.2) \]

where \( z(t) \) and \( v(t) \) are the process noise and measurement noise, respectively; both having continuous, zero-mean Gaussian distributions expressed as

\[ z(t) \sim \left(0, Q_f \right), \quad (4.3) \]

\[ v(t) \sim \left(0, R_f \right). \quad (4.4) \]

Herein, the Kalman filter can be digitally modeled by discretizing the linear state-space model

\[ x[k + 1] = A_d x[k] + B_d u[k] + z[k] \quad (4.5) \]

\[ y[k] = C_d x[k] + v[k]. \quad (4.6) \]

The state-transition matrix \( A_d \) and the process noise covariance matrix \( Q_d \) can be estimated numerically using Van Loan’s method [28]. First, the intermediate matrix is calculated

\[ \Lambda = \begin{bmatrix} -A & BWB^T \\ 0 & A^T \end{bmatrix} \Delta t, \quad (4.7) \]

then the state-transition matrix and noise covariance matrices can be extracted by the exponent of the intermediate matrix

\[ e^{\Lambda} = \begin{bmatrix} \cdots & A_d^{-1}Q_f[k] \\ 0 & A_d^T \end{bmatrix}, \quad (4.8) \]

and finally, the state-transition matrix \( A_d \) and the discretized process noise covariance matrix \( Q_d \) are obtained in the upper right and lower right quadrants of Eq. (4.8). The control matrix can be
determined using the state-transition matrix as follows

$$B_d = A^{-1} (A_d - I) B.$$  \hfill (4.9)

Finally, the discretized input matrix can be defined as

$$C_d = C.$$  \hfill (4.10)

The Kalman filter loop begins by obtaining the initial values for the \textit{a priori} estimate of the states of the system $\hat{x}_0^-$ and the \textit{a priori} error covariance matrix $P_0^-$. With these initial estimates, the Kalman gain is computed

$$K_f[k] = P^- [k] C_d^T [k] \left( C_d[k] P^- [k] C_d^T [k] + R_f[k] \right)^{-1}. \hfill (4.11)$$

Next, the state estimates and the error covariance matrix of the system are updated with the measurement at the current sampling time

$$\hat{x}[k] = \hat{x}^- [k] + K_f[k] \left( z[k] - C_d[k] \hat{x}^- [k] \right), \hfill (4.12)$$

$$P[k] = (I - K_f[k] C_d[k]) P^- [k]. \hfill (4.13)$$

Finally, before the next sampling period, the \textit{a priori} state estimates and the \textit{a priori} error covariance matrix are projected ahead

$$\hat{x}^- [k + 1] = A_d[k] \hat{x}[k], \hfill (4.14)$$

$$P^- [k + 1] = A_d[k] P[k] A_d^T [k] + Q_f[k]. \hfill (4.15)$$

Finally, the updated estimates of the states of the system can be expressed as

$$\hat{x}[k] = (I - K_f[k] C_d[k]) \hat{x}^- [k] + K_f[k] \left( C_d[k] x[k] + v[k] \right). \hfill (4.16)$$
4.2 Linear Quadratic Regulator

The Linear Quadratic Regulator calculates the optimal control gains associated with the states of the system using the estimated states of the Kalman filter [29]. The cost function is defined as

$$J = \frac{1}{2} \int_{0}^{\infty} (\hat{x}^T Q_c \hat{x} + u^T R_c u) \, dt$$  \hspace{1cm} (4.17)

wherein $Q_c$ and $R_c$ are positive semidefinite and symmetric matrices that place weights on the states of the system and the actuator effort, respectively. The state weighting matrix is defined as

$$Q_c = \begin{bmatrix} q_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & q_c \end{bmatrix}$$  \hspace{1cm} (4.18)

where $q_c$ for $c = 1, 2, \ldots, 2n + m$ places weight on the $c^{th}$ state in Eq. (4.19). Subsequently, the control weighting matrix is

$$R_c = \begin{bmatrix} r_1 & 0 \\ 0 & r_2 \end{bmatrix}$$  \hspace{1cm} (4.19)

where $r_1$ and $r_2$ are the weights of the piezo actuators at the base and tip of the smart structure, respectively. Thus, the control law that minimizes the cost of the input can be defined as

$$u(t) = -K_c \hat{x}(t),$$  \hspace{1cm} (4.20)

where $K_c$ is the controller gain that is defined as

$$K_c = R_c^{-1} B^T P,$$  \hspace{1cm} (4.21)

and $P$ is the solution to the continuous algebraic Riccati equation

$$P A + A^T P - P B R_c^{-1} B^T P = 0.$$  \hspace{1cm} (4.22)
The closed-form solution to the continuous algebraic Ricatti equation can be found by first constructing the Hamiltonian matrix [30]

\[
H = \begin{bmatrix}
A & BR_c^{-1}B^T \\
-Q_c & -A^T
\end{bmatrix},
\tag{4.23}
\]

then constructing the matrix of eigenvectors

\[
T_H = \begin{bmatrix}
T_{11} & T_{12} \\
T_{21} & T_{22}
\end{bmatrix},
\tag{4.24}
\]

where the columns of \( T_{11} \) and \( T_{21} \) are the eigenvectors for the stable component of the system and the columns of \( T_{12} \) and \( T_{22} \) are the eigenvectors for the unstable component of the system. Finally, the solution to Eq. (4.23) can be expressed as

\[
P = T_{21}T_{11}^{-1}.
\tag{4.25}
\]
5.1 Model Validation

An accurate plant model is critical in the model-based design of a closed-loop controller because the controller is tuned accordingly to the dynamics of the system. If the experimental system exhibits dynamics which are not accounted for in the analytical model, then the performance of the controller on the experimental closed-loop system will differ drastically from the simulated closed-loop system. Moreover, the experimental system can become unstable or erratic—causing poor performance, damage to equipment, or even harm to nearby individuals. Thus, it is important to validate the accuracy of the analytical model in the model-based design approach.

The accuracy of the analytical model was verified by comparing it with two different forms of simulation models: a finite-element (FE) model in Abaqus and a lumped parameter model in Simscape Multibody. The finite-element method (FEM) is a powerful tool that makes modeling not only the substructures of the smart structure easy, but the joints and damping as well. Subsequently, the Simscape Multibody model is used as a second validation tool to provide further evidence of analytical model accuracy.

The steel body of the smart structure is composed of AISI-1005 and the properties of the piezoceramic patches correspond to the piezo ceramic material APC 855 [31]. From herein, the geometric and material properties used in all models are displayed in 5.1.
Table 5.1: Geometric and material properties of the physical smart structure.

<table>
<thead>
<tr>
<th>Property</th>
<th>Variable</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam Length</td>
<td>$L$</td>
<td>0.603 m</td>
<td></td>
</tr>
<tr>
<td>Beam Thickness</td>
<td>$t_b$</td>
<td>0.00635 m</td>
<td></td>
</tr>
<tr>
<td>Beam Height</td>
<td>$h_b$</td>
<td>0.0381 m</td>
<td></td>
</tr>
<tr>
<td>Beam Density</td>
<td>$\rho_b$</td>
<td>8000 kg/m$^3$</td>
<td></td>
</tr>
<tr>
<td>Beam Stiffness</td>
<td>$E_b$</td>
<td>190.0 GPa</td>
<td></td>
</tr>
<tr>
<td>Mass-proportional Damping Coeff.</td>
<td>$\eta_1$</td>
<td>7.89e−2</td>
<td>1/s</td>
</tr>
<tr>
<td>Stiffness-proportional Damping Coeff.</td>
<td>$\eta_2$</td>
<td>2.75e−6</td>
<td>s</td>
</tr>
<tr>
<td>Retroreflector Mass</td>
<td>$m_r$</td>
<td>0.130 kg</td>
<td></td>
</tr>
<tr>
<td>Retroreflector Location</td>
<td>$x_r$</td>
<td>0.584 m</td>
<td></td>
</tr>
<tr>
<td>Piezo Length</td>
<td>$l_p$</td>
<td>0.030 m</td>
<td></td>
</tr>
<tr>
<td>Piezo Thickness</td>
<td>$t_p$</td>
<td>0.0008 m</td>
<td></td>
</tr>
<tr>
<td>Piezo Height</td>
<td>$h_p$</td>
<td>0.030 m</td>
<td></td>
</tr>
<tr>
<td>Piezo Location 1</td>
<td>$x_1$</td>
<td>0.002 m</td>
<td></td>
</tr>
<tr>
<td>Piezo Location 2</td>
<td>$x_2$</td>
<td>0.534 m</td>
<td></td>
</tr>
<tr>
<td>Piezo Stiffness</td>
<td>$E_p$</td>
<td>51.0 GPa</td>
<td></td>
</tr>
<tr>
<td>Electrocoupling Constant</td>
<td>$d_{31}$</td>
<td>−2.76e−10 m/V</td>
<td></td>
</tr>
<tr>
<td>LPF Cutoff Frequency</td>
<td>$\omega_c$</td>
<td>324.8 Hz</td>
<td></td>
</tr>
<tr>
<td>LPF Gain</td>
<td>$K_{lpf}$</td>
<td>1 V/V</td>
<td></td>
</tr>
<tr>
<td>Amplifier Gain</td>
<td>$K_{amp}$</td>
<td>20 V/V</td>
<td></td>
</tr>
</tbody>
</table>

Although the material properties of these components have been well documented, they still serve only as an approximation. It is impossible to determine the true material properties of the smart structure are without experimental testing or system identification. The batch of AISI-1005 steel used to manufacture the steel beam could have more carbon in it than advertised due to process error, which could affect the density and elastic modulus of the steel substructure. The piezoceramic patches could have cracks or defects in them caused by shipping, which can influence their stiffness and electrical capacitance. Moreover, the dynamics of the smart structure can vary with time as the temperature changes or fatigue begins to influence the behavior of the smart structure. As a result, the metrics listed in Table 5.1 serve as only an approximation for the experimental model. Because the simulated models each use the same material properties inputted by the user, the purpose of the simulation is to verify that the analytical model is well-derived—not that the material properties are accurate.

5.1.1 Analytical Model

The state-space model was constructed entirely in Matlab by substituting the system parameters from Table 5.1 into the equations developed in Chapter 2. Herein, a 5$^{th}$ degree of freedom ($n = 5$) is used to formulate the state-space model. The model used for the entirety of Chapter 5 follows that of
the nonaugmented state-space matrix from Eqs. (2.44-2.49). The block diagram of the uncontrolled analytical model is shown in Figure 5.1.

![Simulink block diagram of the state-space model representing the uncontrolled smart structure system with inclusion of noise and sampling.](image)

Figure 5.1: Simulink block diagram of the state-space model representing the uncontrolled smart structure system with inclusion of noise and sampling.

For simulations comparing the analytical model, Abaqus model, and Simscape Multibody model, the state-space block utilizes the matrices from Eqs. (2.44-2.49) to model a unit line load input, whereas the state-space block used for comparison with the experimental model utilizes matrices from Eqs. (2.50-2.55) to model the piezoceramic actuators. Zero-order holds (ZOHs) are used to sample the input and output of the plant at a rate of 0.0003 s. Additionally, band-limited white noise blocks are used to simulate the zero-mean, Gaussian-distributed process and measurement noise. The input to the system is shown to be a zero and a step input. The constant zero is used to model the control of the piezoceramic patches located at the base of the smart structure, while the step input is used to model the line load applied along the edge of the tip of the smart structure.

It is important to note that the input matrix of the state-space model has been altered to simulate a unit step force being applied to the tip of the beam, rather than using the piezoceramic patches as transducers. This alteration was done to reduce the variability and complexity of the model for the sake of making the comparison with other simulated models more straightforward.

The models constructed in Abaqus and Simscape Multibody are used to verify the dynamics of the smart structure, not the actuation of the piezoceramics. Moreover, the piezoceramics of the physical system poses nonlinearities and hysteresis, which are difficult to model in a variety of software. Furthermore, the actuation of the piezoceramics is more appropriately compared to the physical model than to other simulated models.
5.1.2 Finite-Element Model

The finite-element method (FEM) is a powerful computer simulation tool used to model the dynamics of physical structures and fluid mechanics. For this study, the FE software used to model the smart structure was Abaqus CAE. Abaqus can handle the modeling, preprocessing, and postprocessing steps concerning finite-element analysis (FEA). Although it is not as easy to model the structure in Abaqus as it is compared to SolidWorks or AutoCAD Fusion, Abaqus provides the designer with an abundance of tools and freedom needed to produce accurate FE models—which includes more power over field output variables, boundary conditions, component mates, meshing, element types, integration techniques, processing methods, and more.

In Abaqus, the smart structure assembly was modeled using a combination of solid bodies, shells, and lumped-inertia components—the material properties and dimensions used for the FE model are found in Table 5.1. The steel beam made of AISI-1005 was constructed using solid elements, extruded along its weak axis. The piezoceramic patches were constructed using shell elements because they are better suited for thin features and are less susceptible to negative Jacobian errors. Furthermore, fewer elements are required to model a feature with shell elements, which reduces computational effort and saves time. The steel beam and piezoceramic patches were joined using a tie constraint as depicted in Figure 5.2. Surface friction and viscous damping were not considered, and the contact between the piezoceramic patches and the steel beam was considered explicitly adhered.

![Figure 5.2: Abaqus FE model of the smart structure comprised of solid elements for the AISI-1005 steel beam (grey), shell elements for the APC 855 piezoceramics (yellow), and a lumped inertia feature for the retroreflector (red).](image)

Finally, the retroreflector and magnetic mount were modeled as a singular point mass, as shown by the red dot on the right side of the structure in Figure 5.2 whose mass is listed in 5.1. The mass moment of inertia for the retroreflector is difficult to determine due to its nonhomogeneous make-up. The frame of the retroreflector is made of stainless steel, whereas the interior is composed of various
flat-plane mirrors at a number of angles. As a result, the mass moment of inertia about each of its axes is ignored because it is anticipated to be insignificant compared to its mass contribution. To accurately place the piezoceramic patches and retroreflector along the beam, the steel beam body was partitioned. Furthermore, this allows for more precise mesh control along these critical regions.

Various boundary conditions were applied to the model to accurately represent the fixed-free smart structure, as well as isolate the transverse deflection along the weak axis as depicted in Figure 5.3.

![Abaqus FE model of the smart structure with applied boundary conditions (orange).](image)

Figure 5.3: Abaqus FE model of the smart structure with applied boundary conditions (orange).

The fixed end of the structure was encastréed to prevent transversal and rotation movement along each direction. Subsequently, the short edges along the length of the beam were fixed along the y direction—only allowing for transverse movement along the x and z directions.

The last step in preprocessing was determining the seed size and element types for the model. The C3D20, which is a 20-node quadratic brick element, was chosen for its accuracy and shape. The quadratic shape function more accurately captures the dynamics of the beam under linear deformation. The C3D20 element also provides more accurate interpolation results compared to its reduced-integration variant, the C3D20R, which can lead to models that are less rigid than they otherwise should be. Finally, the smart structure contains many flat square edges, making the brick element an ideal fit. To determine the seed size of the mesh, a convergence study was performed with the results depicted in Figure 5.4.
Figure 5.4: Mesh convergence plot of FE model utilizing the 5th natural frequency as the field variable of interest.

The convergence study provides the designer with insight into how many elements are needed to obtain relatively accurate results. Although more elements are typically considered better, they also require significantly more time to obtain results and can even lead to significant error accumulation. Using too few elements will also lead to inaccurate results because the mass and stiffness matrices of the system do not reflect the desired system. For the convergence study, all parameters of the model were kept constant and only the seed size, which correlates with the number of elements, was varied. The fifth natural frequency was chosen as the field output of interest. As depicted in Figure 5.4, the use of more than 1000 elements produces slightly more accurate results at a rate of diminishing returns. While 1000 elements may not seem a lot of computational effort for a static analysis, dynamics analyses require hundreds or thousands of steps which can take a significant amount of time and computational power to perform. As a result, a total of 1120 elements were used to define the mesh for subsequent results, as shown in Figure 5.5.

Figure 5.5: Abaqus FE model mesh of the smart beam structure created from C3D20 elements of approximately 4mm in size.
In most FE models, it is a general rule of thumb to use at least three elements throughout the length of any single component. In this case, using only one element across the thickness of the beam is acceptable, since the geometry and boundary conditions are simple and the results need only be approximate.

5.1.3 Simscape Multibody Model

Simscape Multibody is a simulation software that utilizes a block-based design approach for mechanical systems. One advantage of using Simscape Multibody over other simulation software is that it runs natively on Simulink. Simscape Multibody systems can be constructed inside the Simulink design environment, which provides ease of interface with a number of useful analysis tools found in MATLAB and Simulink. Moreover, the simulation data does not have to be exported to a separate software to be analyzed.

Since the analytical model was developed inside the MATLAB environment, the system parameters used in the analytical model can be easily accessed by Simscape via the user workspace. The system parameters used can be found once again in Table 5.1. During the design process, a series of flexible rectangular beam elements were used to construct a finite-element model of the smart structure. The complete block diagram model of the smart structure is shown in Figure 5.7.

![Figure 5.7: Complete block diagram model of the smart structure.]

Figure 5.6: Block diagram of Simscape Multibody model with the retroreflector located at 23 inches.

The flexible elements contain all parameters necessary to accurately model each segment, including geometric dimensions, material properties, and damping coefficients for Rayleigh damping. To account for the added stiffness of the piezoceramic patches, the elements containing the collocated
pair of piezoceramic patches are modeled as a homogeneous section of width

\[ w_{pzt,seg} = 2t_p \frac{E_p}{E_b} + t_b, \]  

(5.1)

and elastic modulus

\[ E_{pzt,seg} = E_b. \]  

(5.2)

Thus, Eqs. (5.1) and (5.2) account for the reflected stiffness of the piezoceramic-sandwiched segments. The flexible beam elements are joined together via weld-joint blocks which fix the surface ends of each flexible element. All the blocks highlighted inside the gray area make up the smart structure. Highlighted in purple are three blocks that make up the fixed end boundary condition of the system. The lumped mass and measurement sensor for the retroreflector are highlighted in red. Finally, shown in the top right of Figure 5.7, is a unit step input that is applied to the tip of the beam to serve as a comparison for the analytical model. When the simulation is run, a 3D rendering of the system is created, which can be seen in Figure 5.7.

![Figure 5.7: 3D rendering of the Simscape Multibody model with the vise fixture (green), piezoceramic-sandwiched segments (yellow), homogeneous beam segments (green), and retroreflector (red) located at 23 inches.](image)

It is important to note that the vise components highlighted in green do not influence the dynamics of the smart structure and are solely meant as a visual aid. The yellow segments are the elements that contain the sandwiched piezoceramic elements with reflected stiffness. Finally, the red cylinder represents the lumped mass and the measurement location of the retroreflector.

Using the Bode command, the input and output of the system are linearized and the frequency response of the system is generated. For both the Simscape and analytical models, a 5\textsuperscript{th} order approximation was used (i.e. \( n = 5 \)).
Similarly to the FE model developed in Abaqus, Simscape Multibody is also capable of generating dynamic analyses. Like before, a unit force is applied to the tip of the smart structure along the weak axis, and the displacement at the location of the retroreflector is recorded.

To verify that the state-space model can accurately extract the displacement profile of the smart structure at various locations along the length of the structure, the location of the retroreflector was also moved to the region between the piezoceramic patches 12 inches from the location of the vise. The altered block diagram and 3D rendering is displayed in Figure 5.8.

Figure 5.8: Block diagram (a) of Simscape Multibody model (b) with the retroreflector located at 12 inches from the vise.

5.1.4 Results

The first type of analysis conducted to identify any discrepancies or similarities between the models was modal analysis. In the modal analysis, the frequency response of the system was analyzed by inspecting the magnitude and phase characteristics of the system as a function of the input frequency. Perhaps the most distinguishing characteristics of a structure are the locations of its natural frequencies in its magnitude response. The first four mode shapes of the smart structure, constructed in Abaqus, can be seen in 5.9.
The fixed-free boundary conditions highlighted in Figure 5.3 are easy to visualize in 5.9. The free end of the smart structure (right) is unconstrained and able to oscillate, whereas the fixed end of the smart structure (left) is held stationary. Furthermore, the nodes and antinodes of the structure are clearly colored in blue and red, respectively.

The modal analysis of each simulated model was conducted in a similar fashion. The natural frequencies of the models were determined by calculating the eigenvectors of their mass-normalized stiffness matrices. The natural frequencies of the analytical, Abaqus, and Simscape Multibody models are organized in Table 5.2.

Table 5.2: Simulated modal frequencies of the smart structure generated by the analytical, Abaqus, and Simscape Multibody models and their percent difference taken with respect to the analytical model.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Analytical</th>
<th>Abaqus</th>
<th>Simscape</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency</td>
<td>Frequency</td>
<td>Difference</td>
<td>Frequency</td>
</tr>
<tr>
<td>Hz</td>
<td>Hz</td>
<td>[%]</td>
<td>Hz</td>
</tr>
<tr>
<td>1</td>
<td>11.68</td>
<td>11.54</td>
<td>-1.21</td>
</tr>
<tr>
<td>2</td>
<td>77.41</td>
<td>80.20</td>
<td>3.54</td>
</tr>
<tr>
<td>3</td>
<td>233.95</td>
<td>234.65</td>
<td>0.30</td>
</tr>
<tr>
<td>4</td>
<td>466.05</td>
<td>471.08</td>
<td>1.07</td>
</tr>
<tr>
<td>5</td>
<td>786.32</td>
<td>798.97</td>
<td>1.60</td>
</tr>
</tbody>
</table>

From the table of natural frequencies, it is evident that the analytical model matches closely with that of other simulation methods. The frequencies of the first vibratory mode of each model are within 0.14 Hz of one another, which is only an approximate 1.2% difference. For the second mode, the analytical and Simscape models are very similar at 77.41 Hz and 78.26 Hz, respectively. Meanwhile, the second natural frequency of the Abaqus model is slightly higher at 80.2 Hz. The trend of Abaqus having slightly higher values of natural frequencies, compared to the other models, continues for the
higher-order modes. This result is expected, as finite-element models typically are typically stiffer than their physical or analytical counterparts. This effect is sometimes a contribution of the mesh quality, unrealistic boundary condition applications, and sometimes the integration method used to interpolate between nodes.

Overall, the analytical, Abaqus, and Simscape Multibody models demonstrated acceptable differences between modal frequencies. The largest percent difference between the Simscape and analytical models was -1.17%, at the third modal frequency. The first, fourth, and fifth modal frequencies were less than one percent different. Furthermore, the third mode of the Abaqus model had the greatest percent difference of 3.54%, and the additional modal frequencies had a percent difference less than two percent.

In addition to the numerical values of the natural frequencies, the magnitude responses of the analytical and Simscape Multibody models were also generated, as shown in Figure 5.10.

![Figure 5.10: Magnitude response of analytical and Simscape multibody models with the retroreflector located at 12 inches (a) and the retroreflector located at 23 inches (b).](image)

From the magnitude plot, the first five modal frequencies of the two models further agree with the values presented in Table 5.2; however, the magnitude response also depicts the system response at different input frequencies. The magnitude response of the two models appears to be in agreement over the full 1-1000 Hz band of input frequencies. Furthermore, Figure 5.10 also demonstrates the damping characteristics of the models by the "sharpness" of each peak. Figure 5.10 also depicts that the system is capable of producing antimodes, as depicted by the "valleys" in its magnitude response. When driven at these frequencies using the tip piezoceramic patches, the smart structure experiences very little displacement at the location of the retroreflector.

Furthermore, it is shown that moving the retroreflector to 12 inches from the base of the smart structure not only increases its first natural frequency, but it also reduces the level of measured vibrations.
about the third mode. This is not an indication that the system is not displacing less—rather, it is indication that the system is displacing less at the location of the retroreflector. By looking at Figure 5.10, it can be seen that the antimode of the third natural frequency occurs very close to the location at half the length of the structure. Although, the added stiffness and mass of the piezoceramic patches and the added mass of the retroreflector slightly influence the shape of the third mode. With this in mind, it is likely that the retroreflector is near the antimode at 12 inches, which makes it difficult to observe the response of the smart structure. Adding more retroreflectors or placing the retroreflector away from the antimodes of the structure can mitigate this problem.

The linear dynamic analysis for the analytical, Abaqus, and Simscape models were conducted by applying a unit step force to the tip of the smart structure. Analyzing the structure response to the retroreflector located 23 inches from the base produces the time response shown in Figure 5.11.

Figure 5.11: Simulated step response of analytical, Abaqus, and Simscape models measured with the retroreflector at 23 inches, perturbed by a one Newton line load at the tip of the structure.

With equal initial conditions, boundary conditions, and loading conditions, the three simulated models exhibit similar time response characteristics. While it appears that the three models have the same step response characteristics, the discrepancies in their first natural frequencies can be seen over a period of time. When one looks at the oscillations occurring after 350 ms, they begin to move out of phase with each other. The analytical response appears to have the smallest frequency, followed by the Simscape model, and lastly the Abaqus model. Comparing this information with the natural frequencies displayed in Table 5.2, the results appear to be in agreement. Moreover, the Abaqus model appears to have a slightly smaller amplitude of vibration, as noticeable between 350 and 500 milliseconds.

For further model verification, the lumped mass representing the retroreflector is moved from 23 inches to 12 inches from the fixed end of the structure. Moving the mass closer to the base of the
fixed-free structure alters the natural frequencies of the system in addition to providing more insight on the response profile of the beam. The dynamic responses of the simulated systems are shown in Figure 5.12.

![Figure 5.12: Simulated step response of analytical, Abaqus, and Simscape models measured with the retroreflector at 12 inches, perturbed by a one Newton line load at the tip of the structure (a) and a reduced timescale (b).]

The higher order modes are clearly more visible in the transient response of the system with the retroreflector located 12 inches from the base compared to Figure 5.23. This is a direct result of the measurement location being near the antinode of the second natural frequency, as shown in Figure 2.3. In mathematical terms, the shape function, Eq. (2.17), is evaluated at the location of the retroreflector, which scales the respective generalized mode shapes. This causes the first mode to be scaled down more compared to the second mode—this, of course, is only true if all modes are excited simultaneously, as is the case for a sudden impact or random noise.

The transient response of the Simscape model with the retroreflector located at 12 inches matches that of the analytical model more so than the FE model. Once again, the FE model appears to have a smaller amplitude compared to the analytical and Simscape models. The discrepancy in the
amplitudes of Figure 5.12 is more noticeable than that of 5.11. Although both the Abaqus and Simscape models match the analytical model appropriately, this is not evidence alone to suggest that the analytical model is an accurate representation of the physical system. The geometric and physical properties used to construct each model may not reflect the true properties of the physical system. Although the material properties of the piezoceramics and steel beam have been thoroughly studied and made available, it is impossible to determine what their true values are. The beam could be denser than advertised due to a manufacturing defect. The length of the beam could have been recorded incorrectly due to human error. The piezoceramic patches could have cracks or defects that are invisible to the naked eye, which impedes their force generation or reduces their stiffness. Thus, system identification or experimental testing are the only methods that can truly verify the accuracy of the analytical model of the physical system.

In total, the transient response of the analytical model is in agreement with the Simscape and Abaqus models. Moreover, the transient responses of the models match when relocating the measurement and lumped mass of the retroreflector along the length of the beam. While these results do not conclude that the analytical model perfectly represents the physical system, comparing the two models provides evidence that the mathematical derivation and implementation of the analytical model agrees with another method.

5.2 Closed-Loop Control

Following the model-based design approach, the closed-loop control systems of the PD controller and the LQG regulator are simulated and tuned before being deployed to the experimental hardware. The smart structure and both controllers are constructed in Matlab and Simulink to analyze the closed-loop responses. Simulating the controllers before physical implementation provides more information on the stability and performance characteristics of the controllers, as an unstable closed-loop system can cause serious damage to nearby equipment and personnel. The block diagram in fig. 5.1 was used to construct the uncontrolled model. The step input is used to actuate the piezoceramics at the tip of the smart structure.

While the same block diagram format was used to construct the uncontrolled model as in Figure 5.1, the augmented state-space model, including the LPF, was used as the state-space equation block. Moreover, the step input is used to model the control of piezoceramic patches located at the tip.
5.2.1 PD Implementation

Since the PD controller is designed heuristically using the Jury stability criterion, it is paramount that the controller is tuned using simulation methods before implementation on the physical system.

The continuous-time transfer function of the $n = 5$ degree-of-freedom model is calculated using Eqs. (2.50-2.55) and (2.56), and the values found in Table 5.1

$$G_{plant}(s) = \frac{(0.014) s^8 + (0.057) s^7 + (1.9e4) s^6 + (5.2e6) s^5 + (9.4e11) s^4}{s^{10} + (94.0) s^9 + (3.4e7) s^8 + (1.5e9) s^7 + (2.7e14) s^6 + (3.9e15) s^5} - \frac{(1.3e13) s^3 - (1.6e18) s^2 + (1.6e18) s + (1.5e23)}{s^{10} + (7.3) s^9 + (25.0) s^8 - (55.0) s^7 + (85.0) s^6 - (97.0) s^5}.$$  

(5.3)

Subsequently, the pole-zero plot of the open-loop, continuous-time plant transfer function $G_{plant}(s)$ is shown in

![Pole-Zero Plot](image)

Figure 5.13: Continuous-time pole-zero plot of the open-loop, uncontrolled smart structure model using the base piezoceramic actuators as the input.

Next, the continuous-time transfer function is discretized using the ZOH with a sampling period of $T_s = 0.0003$ seconds and substituting Eq. (5.3) into (3.2) to produce the following discrete-time plant transfer function

$$G_p(z) = \frac{(4.9e-10) z^9 - (3.3e-9) z^8 + (9.6e-9) z^7 - (1.4e-8) z^6 + (7.1e-9) z^5}{z^{10} - (7.3) z^9 + (25.0) z^8 - (55.0) z^7 + (85.0) z^6 - (97.0) z^5} + \frac{(6.6e-9) z^4 - (1.3e-8) z^3 + (9.4e-9) z^2 - (3.3e-9) z + (4.9e-10)}{z^{10} - (7.3) z^9 + (25.0) z^8 - (55.0) z^7 + (85.0) z^6 - (97.0) z^5}.$$  

(5.4)
As previously displayed in Figure 3.3, the ZOH maps the poles and zeros in the left half-plane of the s-domain into the unit circle of the z-domain. The poles and zeros for the \( n = 5 \) smart structure model mapped into the discrete z-domain are illustrated by the pole-zero plot shown in Figure 5.14.

![Pole-zero plot](image)

**Figure 5.14:** Discrete-time pole-zero plot of the open-loop, uncontrolled smart structure model, sampled at 0.3 milliseconds, using the base piezoceramic actuators as the input (a) and a close-up view of the dominant poles (b).

In the s-domain, all poles are located in the left half-plane—where the dominant poles are approximately \(-0.047 \pm 73.19i\omega\). In the z-domain, all of the poles are mapped within the unit circle—where the dominant poles are located at approximately \(0.999 \pm 0.0219i\omega\). All poles of the discrete plant model are located within the unit circle, indicating that the uncontrolled, open-loop system is stable after sampling—which it should be.

Subsequently, the proportional gain \( K_p \) is considered as the controller transfer function \( D(z) \) without the inclusion of the derivative term. Using the Jury stability criterion, the largest value of \( K_p \) was determined to be

\[
K_{p,max} = 4283.
\]  

(5.5)

Various simulations of step response inputs showed an acceptable system response using \(0.7K_{p,max}\). The derivative gain was then calculated in a similar fashion using values of \( N = 100 \) and \( K_p \approx 3000 \), producing the maximum discrete gain for a marginally stable system

\[
K_{d,max} = 461.
\]  

(5.6)
After a series of simulations, a derivative gain of $0.55K_{d,max}$, or $K_d = 254$, was determined to produce a controller with the desired balance of robustness and aggressiveness. Thus, the closed-loop discrete-time transfer function for the PD control is defined as

$$D(z) = 3000 + \frac{(24660)z - 24660}{z - 0.9708}. \tag{5.7}$$

Furthermore, the marginally stable gains can be used in conjunction with the root locus plot of the system with the integrated PD controller is shown in Figure 5.15

![Figure 5.15: Discrete-time root locus of the PD-controlled smart structure model, sampled at 0.3 milliseconds, using the base piezoceramic actuators as the input (a) and a close-up view of the dominant poles (b).](image)

The root locus is a powerful tool that shows the behavior of the closed-loop system, using the open-loop transfer function. Because the derivative term is not ideal (from the filtering coefficient), it adds a zero and a pole to the system. The additional pole and zero can be seen in Figure 5.15 on the real-axis to the left of the dominant poles. Furthermore, the additional pole and zero added to the system can be seen to shift the root locus near the dominant poles, dragging the dominant poles closer inside the unit circle and increasing the stability of the closed-loop system. Finally, the closed-loop system with PD control is constructed in Simulink, as shown by the block diagram model in Figure 5.16.
In addition to the state-space model, process noise, measurement noise, and ZOHs... the output measurement of the system is negatively fed back to the proportional and derivative terms. Since the objective of the controller is to reject disturbance noise, there is no reference input, and the error is equal to the negative value of the measurement.

5.2.2 LQG Implementation

Compared to the PD controller, the LQG regulator is much easier to tune; however, it also requires a more complicated model to approximate the states of the system. In addition to the state-space model found in Eqs. (2.50-2.55), the power spectral density (PSD) of the process noise and measurement noise are also required.

The PSD of the process noise was approximated to be $6 \times 10^{-7}$ V$^2$/Hz by taking a long-duration measurement of the system at steady state. Using the time response data, a fast Fourier transform (FFT) was applied to the data and the autocorrelation was calculated. Finally, the PSD was calculated by the Wiener-Khinchin theorem [32]. Due to the fact that the laser interferometry equipment is so accurate, it was difficult to determine the measurement noise since the measurement system was able to detect even the smallest movements of stationary objects. Thus, the measurement accuracy was approximated to be $5 \times 10^{-20}$ by using technical data sheets.

Due to the relatively fast sampling of the closed-loop control system, the initial A Priori estimates of the system were assumed to be zero. As the Kalman filter continues to sample the data, the A Priori error estimates converge to a small value in approximately less than 100 samples, or 30 milliseconds in this case.
The last tuning parameters used for the LQR are the state and control weighting matrices, $Q_c$ and $R_c$. The LQR method works to find the optimal actuation effort which minimizes the states of the system, where a large magnitude of $Q_c$ creates a more aggressive controller and a large magnitude of $Q_c$ creates a less aggressive controller. Since the two values are mutually dependent, the LQR was tuned by first placing weights on the state control matrix. With emphasis on controlling the modal positions of the smart structure, the state weighting matrix was defined as

$$Q_c = \begin{bmatrix} I_{5 \times 5} & 0_{5 \times 7} \\ 0_{7 \times 5} & 0_{7 \times 7} \end{bmatrix}.$$  

(5.8)

The first five rows/columns place weights on the modal positions of the smart structure, the subsequent five rows/columns place weights on the modal velocities of the smart structure, and the last two rows/columns place weights on the output voltage of the low-pass filters for the first and second piezoceramic patches, respectively. Control over individual modes can also be achieved by changing the values of the first five diagonals—where the first diagonal places weight on the first mode, the second diagonal places weight on the second mode, etc.

With the state weighting matrix defined, the control weighting matrix was varied until the desired simulation performance was achieved. Due to the fact that the piezoceramic patches at the tip of the smart structure were used as actuators for the sine sweep tests, their respective weights were set to a large value. Thus, the control weighting matrix was defined as

$$R_c = \begin{bmatrix} 1e-11 & 0 \\ 0 & 1e10 \end{bmatrix}.$$  

(5.9)

For good measure, a gain of zero was also placed in the feedback loop after the control gain, to block all control signals from the second pair of piezoceramic patches. Following the equations derived in Chapter 4, the feedback gain was determined to be

$$K_c = \begin{pmatrix} 1.59e5 & -1.33e4 & -1.57e5 & 2.08e5 & -9.67e4 & 3400.0 & -592.0 & 162.0 & -26.0 & -4.56 & 0.0107 & -8.7e-5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$  

(5.10)

Finally, the LQR gain $K_c$ is placed in the feedback path of the system as shown in Figure 4.1, while the new pole-zero plot of the closed-loop system with the LQG regulator is shown in Figure 5.17.
Compared to the open-loop uncontrolled system (Figure 5.13) and the PD-controlled system in Figure 5.15, the LQG regulator *dramatically* drags the dominant poles of the closed-loop system closer within the unit circle—further increasing the stability of the system. As seen by other pole placement (full-state feedback) methods, the LQG regulator has a much stronger ability to manipulate the position of the poles of the closed-loop system by being able to place multiple poles and zeros. Furthermore, the closed-loop system with LQG control is constructed in Simulink as shown in Figure 5.18.

In addition to the state-space model with noise and sampling, the Kalman filter block is placed after the measurement location. The Kalman filter block, shown in Figure 5.18, uses the same discrete-
time algorithm derived by Eqs. (4.12-4.17), using the input and output of the plant to develop a stochastic model that estimates the states and measurement of the system. The state estimates are then fed to the LQR gain, which produces the optimal control signal that is fed back through the ZOH into the plant.

5.2.3 Results

The open-loop and closed-loop models previously discussed were constructed and analyzed using Simulink, a visual programming environment that is an extension of Matlab. The two primary metrics used to analyze the open-loop and closed-loop performance of the smart structure system involve its frequency response and time response.

In Simulink, the Bode block was used to estimate the frequency response of the uncontrolled and closed-loop systems depicted in Figures 5.1, 5.16 and 5.18. The Bode block linearizes the block diagram model into a state-space system and analyzes the frequency response using classical methods. Because the objective of the closed-loop controllers is to reject input disturbances, the frequency response characteristics are measured with respect to the base piezoceramic patches. The magnitude responses of the uncontrolled, PD-controlled, and LQG-controlled systems are highlighted in Figure 5.19.
The PD controller was able to slightly attenuate the magnitude response near the first natural frequency. Additionally, the PD controller managed to shift the first natural frequency of the closed-loop system, slightly increasing it. In some cases, controllers can be used to shift the modes of a system higher or lower so that there is a larger bandwidth of operating frequencies; however, the PD controller was unable to shift the first mode by more than 5%. Finally, the PD controller had an insignificant impact on the higher order modes.

The closed-loop LQG system had a dramatic effect on the magnitude response of the system. It was able to almost fully dampen out the first mode, along with its antimode, and continued to dampen out the second and third modes. It is interesting to note that the LQG regulator was able to attenuate the magnitude response at the first and second natural frequencies to significantly lower levels than that of the uncontrolled and PD-controlled systems. As shown from the pole-zero plot in Figure 5.17, the LQG controller is capable of placing multiple poles to better shape the response of the system, compared to the PD controller which does not have as much tuneability.

Furthermore, the LQG regulator demonstrated the ability for direct modal suppression in simulation. This can be achieved by varying the coefficients of the control weighting matrix $Q_c$. To demonstrate

---

**Figure 5.19:** Simulated Bode plot of uncontrolled, PD-controlled, and LQG-controlled smart structure systems.
this phenomenon in a more visual manner, control was excluded from a specified mode and the rest of the modes were left to be controlled. This allowed the amplitudes of the uncontrolled mode to pass through and remain visible in the transient response. Figure 5.20 shows the system response when the first mode is excluded from the control.

![Figure 5.20: Simulated magnitude (a) and finite-impulse (b) response of LQG-controlled smart structure without control over the first mode.](image)

From the frequency response plot, the magnitude of the LQG system at the first mode appears relatively unchanged; however, the antinode has been flattened. Subsequently, the second and third modes are attenuated as in the full-mode control model. More interestingly, the difference between the uncontrolled and mode-isolated models is apparent. The second mode is clearly visible in the uncontrolled model, whereas the mode-isolated model responded almost as a perfect sine wave. Figure 5.21 illustrates isolation of the second mode.

![Figure 5.21: Simulated magnitude (a) and finite-impulse (b) response of LQG-controlled smart structure without control over the second mode.](image)

From the frequency response graph, the magnitude of the second mode was left unchanged, whereas the other modes were attenuated. Subsequently, the transient response of the second mode is clearly visible in the transient response. It is even possible to see the period between the peak amplitudes
and how the second mode constructively adds to the first mode to create the total response. Finally, the third mode is isolated in Figure 5.21.

Figure 5.22: Simulated magnitude (a) and finite-impulse (b) response of LQG-controlled smart structure without control over the third mode.

Herein, the same trend occurs for the third mode. For the transient response of the third mode-isolated system, the amplitude of vibration decreases and the frequency of oscillation increases. When looking at the transient responses of Figures 5.20 to 5.22, it can be visualized how the superposition of these modes constructs to form the total response of the uncontrolled system.
The uncontrolled system has a dramatically long settling time. Because the base of the smart structure is made up of steel, which has very little damping, it takes approximately 35 seconds for the uncontrolled system to settle within 2% of its steady-state value. The numerical performance metrics for the time response of the simulated, controlled and uncontrolled systems are organized in Table 5.3.

Table 5.3: Simulated time response metrics of uncontrolled, PD-controlled, and LQG-controlled smart structure systems subjected to a 10 volt finite-impulse of the base piezoceramics, lasting 30 milliseconds.

<table>
<thead>
<tr>
<th>Metric</th>
<th>Units</th>
<th>Uncontrolled</th>
<th>PD</th>
<th>LQG</th>
</tr>
</thead>
<tbody>
<tr>
<td>Settling Time</td>
<td>sec</td>
<td>49.99</td>
<td>1.93</td>
<td>0.11</td>
</tr>
<tr>
<td>Max Displacement</td>
<td>µm</td>
<td>121.53</td>
<td>106.52</td>
<td>21.89</td>
</tr>
<tr>
<td>Max Controller Effort</td>
<td>V</td>
<td>0</td>
<td>1.04</td>
<td>−9.30</td>
</tr>
</tbody>
</table>
The PD controller demonstrated a quicker response time than the uncontrolled system, reducing the settling time from almost 50 seconds to just about 1.93 seconds; however, the response of the PD system had a lot more "jitter", which is common with PD control systems that incorporate a lot of derivative action. The derivative term is very sensitive to noise and sudden events. As a result, the higher-order modes remain active for longer duration. The LQG regulator performed significantly better than the uncontrolled and PD controlled systems, with a settling time of approximately 11 ms. The first large amplitude of each of the system responses is due to the finite-time impulse, although the LQG controller demonstrated significantly less undershoot while returning to steady state.

While the LQG regulator dampened the system much quicker, it also utilized significantly more voltage at the piezos, as shown in Figure 5.24.

![Figure 5.24: Simulated control voltage of PD and LQG controllers responding to a 10 volt finite-impulse using the base piezoceramics, lasting 30 milliseconds.](image)

Once the controllers were turned on (after the impulse signal), the LQG regulator peaked out at -9.30 volts, or approximately 180 volts at the power amplifier. It is also possible to see how the frequencies present in the LQG control signal almost match those of the vibrations seen in Figure 5.23, cancelling them out; whereas the PD controller has an almost monotonous, sinusoidal control voltage which is seems to primarily match the first mode and slightly the second mode.

While the LQG regulator used significantly more voltage than the PD controller, initially, the PD controller had a larger total root-mean-square (RMS) voltage for a longer period of time. Figure 5.25 contains a plot of the integrated RMS voltage for both the PD and LQG controlled systems.
Figure 5.25: Integrated, simulated RMS control voltage of PD and LQG controllers responding to a 10 volt finite-impulse using the base piezoceramics, lasting 30 milliseconds.

The integrated RMS voltage is used to illustrate the total amount of power used by the controller. The power used by the piezoceramic patch can be assumed proportional to its voltage. As a result, the controller which requires more RMS voltage over a longer period of time can be considered to use more power. From Figure 5.25, the PD controller can be seen to have more RMS voltage over a period of time. In conjunction with the slower settling time of the PD controller, it can be shown that the PD controller is not only slower in damping out the response of the system, but less efficient as well.

Additionally, Figures 5.24 and 5.25 show that the LQG regulator utilizes significantly more control voltage initially. While this is acceptable in the case where there is no limit on the amount of actuation effort, it is not always achievable due to power limitations and heat dissipation of the electronics. However, the actuation effort of the LQG regulator can easily be reduced by more tuning. Lagrange multipliers can be applied to the cost function of the LQG to bound the limits of the output function. The control weighting matrix can also be altered to place more weight on the controller effort and reduce the actuation voltage. Additionally, the easiest method to reduce the actuation voltage is to incorporate a saturation limiter into the controller. In short, the LQG regulator behaves more optimally than the PD controller and has more tunability for the desired application.
Chapter 6

EXPERIMENTATION

6.1 Experimental Setup

The experimental design of the smart structure is shown in Figure 6.1.

![Figure 6.1: Photograph of the experimental smart structure with two pairs of piezoceramic patches located at the base and tip, and a retroreflector attached to the tip using a magnet.]

The primary component of the smart structure is a cantilevered steel beam composed of AISI1005 hot rolled steel. The beam is considered long and slender and has a length-to-width ratio greater than 10:1—which warrants the use of Euler-Bernoulli beam theory. Euler-Bernoulli beam theory makes the assumption that the structure is only capable of accommodating bending moments to induce stress, unlike Timoshenko beam theory, which also incorporates the effects of shear stress. Since the beam is long and slender, it can be assumed that the primary stress of the structure comes from the bending moments. In addition, the choice to use a relatively simple mechanical structure, such as a slender cantilevered beam, reduces the complexity of the model. Cantilevered Euler-Bernoulli beams are one of the most fundamental structural design components that have been studied and documented for many years. This in turn allows for less time to be spent on the analytical modeling and more time focused on the design, implementation, and testing of a closed-loop controller.
Located at the base and near the tip of the beam, two pairs of collocated piezoceramic patches, summing to four in total. The piezoceramic patches are attached in pairs to preserve the isotropic behavior of the smart structure. Additionally, a 24 gauge wire is soldered to the surfaces of the piezoceramic patches for actuation. A 20 gauge wire is attached at the base of the vise, where the steel structure behaves as the ground. The positive and negative terminals are connected to the Bayonet-Neil-Concelman (BNC) terminals, which are used to deliver the control signals.

Located at the tip of the beam is a retroreflector which is used to measure the transverse displacement of the smart structure. Figure 6.2 shows an overhead view of the experimental setup involving the optical path of the laser interferometry measurement system.

![Figure 6.2: Overhead photograph of the experimental smart structure and laser interferometry system showing laser interferometry optical path.](image)

The retroreflector is attached to a strong neodymium magnet using a high-strength adhesive that is magnetically attached to the beam. The use of a magnet allows the retroreflector to move to various positions along the length of the smart structure without permanently altering the structure. While the added mass of the retroreflector and magnet causes some torsional bending of the smart structure, the effects are considered negligible. Shown in the bottom right of Figure 6.2, is the laser head of the
interferometry measurement system. The laser head creates a ray of light which is directed through the interferometer, to the stationary and moving retroreflectors, back to the interferometer, and finally to one of the three optical receivers. A more detailed diagram showing the optical pathway of an interferometry setup is shown in Figure 1.1.

A block diagram of the closed-loop control system hardware interface is shown in Figure 6.3

![Block diagram of real-time closed-loop control hardware interface.](image)

First, the control algorithm is constructed in Matlab and Simulink Realtime on the development PC. When the control algorithm is to be implemented, Simulink Realtime generates a C++ script from the Simulink block diagram which is uploaded to the Speedgoat Realtime target. The control signal was outputted from the Speedgoat using the IO191 DAC module, which has 16 bits of resolution and a maximum output voltage of ±10 volts. From the IO191, the nonamplified control signal is fed through a low-pass filter (LPF) and to a power amplifier which has a gain of approximately 20 volts per volt. After the smart structure is actuated using the piezoceramic patches, the laser interferometry signal is measured by the Keysight E1708A optical sensor. The sensor signal is converted from an analog signal to a digital signal using the Keysight N1231B board. The Xilinx Genesys Zu FPGA is used to process and package the measurement signal into UDP packets, which is fed back to the Speedgoat Realtime target over a high-speed EtherCat line.

6.2 Model Validation

The final method used to verify the accuracy of the analytical mode is experimental testing. Although computer simulations provide good insight on the qualitative response of the system, they do not represent the true dynamics of the system. Subsequently, the boundary conditions, material properties, geometric properties, or other factors used to model the system in simulations could be
intrinsically different from the physical system. For example, piezoceramic patches can be thinner than advertised due to a manufacturing defect, the measured length of the smart structure can differ from the recorded value due to human measurement error, or the boundary conditions used do not accurately account for the stiffness of the vise and optical table. Thus, the only way to compare whether the analytical model accurately represents the physical system is to compare its response to the physical system through experimental procedures.

The natural frequencies and the magnitude response of the smart structure are determined by performing a sine sweep. For the sine sweep test, the collocated pair of piezoceramic patches actuated with a 5 volt chirp signal independently. When considering the gain of the power amplifiers, the voltage potential across the piezoceramic patches is approximately 100 volts in total. The start and end frequencies of the chirp signal were set to 1 Hz and 1000 Hz, respectively, to excite the first five modes of interest. A series of two sine sweeps were conducted for the magnitude response of the piezos located near the fixed end and the free end of the smart structure, respectively. Throughout the duration of the sine sweep, the actuation frequency is held periodically constant to allow the system to reach steady state for the given frequency. After a specified period, the signal frequency is incremented and the process repeats until reaching the end frequency.

The experimental setup of the smart structure system is discussed in Chapter 1. While Matlab computer simulation packages were used to extract the frequency response and time response of the system, two altered physical testing methods were employed to document the response characteristics of the experimental system.

6.2.1 Results

To extract the frequency response of the physical system, a custom sine sweep algorithm was developed. The piezoceramic patches at the tip of the smart structure were used as actuators for the sine sweep, while the piezoceramic patches at the base of the structure were used for control. The actuating piezoceramic patches were driven by a discrete chirp signal to drive the system at, or near, its natural frequencies. The chirp signal begins at a user-specified starting frequency, and increases discretely in a logarithmic fashion until reaching a user-specified ending frequency. At each interval, the chirp signal holds its frequency for a period of time to allow the system to reach a steady state. The input voltage, input frequency, frequency interval duration, and measured displacement of the
smart structure were then used to identify the magnitude and phase between the input and output of the system using a least means square (LMS) algorithm, developed by project partner Jordan Kochavi. The magnitude response of the experimental system was derived by performing a sine sweep over a band of 1-1000 Hz. The results are shown in Figure 6.5.

![Figure 6.4](image)

(a) Base actuation.  
(b) Tip actuation.

Figure 6.4: Experimental magnitude responses of uncontrolled smart structure using the base piezoceramics (a) and the tip piezoceramics (b).

It is critical to note that while the resonant frequencies of the experimental system are approximately the same as those of the Simscape model (see Figure 5.19), the profile of the magnitude response is very different—the piezoceramics at the tip of the smart structure are capable of inducing antimodes whereas the piezoceramics at the base of the smart structure can not. Moreover, the magnitude of the response is lesser because the piezoceramic patches were used to drive the system, not a force applied to the tip. Furthermore, the analytical model was altered to reflect this change. As seen from Figure 6.4, the analytical model approximates the experimental system exceptionally well. The numerical values of the natural frequencies, as well as their percent differences, are shown in Table 6.1.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Analytical [Hz]</th>
<th>Experimental [Hz]</th>
<th>Difference [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>11.68</td>
<td>11.65</td>
<td>-0.26</td>
</tr>
<tr>
<td>2</td>
<td>77.41</td>
<td>78.80</td>
<td>1.65</td>
</tr>
<tr>
<td>3</td>
<td>233.95</td>
<td>228.90</td>
<td>-2.18</td>
</tr>
<tr>
<td>4</td>
<td>466.05</td>
<td>458.33</td>
<td>-1.67</td>
</tr>
<tr>
<td>5</td>
<td>786.32</td>
<td>769.08</td>
<td>-2.21</td>
</tr>
</tbody>
</table>

Moreover, the model accurately accounts for the actuation of the piezoceramics in addition to the structural dynamics. The resonant frequencies appear to match well; however, there is a lot of noise in the experimental data in the higher frequency range.
The noisy data seen in Figure 6.4 is likely attributed to the intrusion of the LPF by reducing the actuation voltage sent to the piezoceramic actuators. As a result, the piezoceramics are unable to drive the system properly to generate a clean frequency response. The cut-off frequency of the LPF is specified as 324.8 Hz, which is lower than the frequency of the fourth mode, to limit the slew rate of the control signal and prevent damage to the power amplifier. Additionally, higher-order modes require exponentially more energy to excite than lower-order modes. As a result, only the first three modes (under 400 Hz) will be analyzed in subsequent experiments. Not only will this produce cleaner results, it will also reduce the amount of time to conduct sine sweep tests by decreasing the frequency range.

6.3 Closed-Loop Control

The performance of the LQG regulator was compared with the PD-controlled and uncontrolled systems. Two metrics were used to compare the performance of the LQG regulator against the PD-controlled and uncontrolled systems.

The first metric used to evaluate the performance of the closed-loop system was its frequency response. To develop the frequency response of the physical closed-loop and open-loop systems, a sine sweep was conducted using the piezoceramic patches at the tip of the smart structure as the actuators. Subsequently, the piezoceramic patches at the base of the smart structure were used as the actuators for the controllers.

The second method used to evaluate the performance of the closed-loop system was its impulse response. To develop the time-response characteristics of the physical system, a finite-impulse signal was used to excite the system, and the measured displacement of the retroreflector was recorded. The piezoceramic patches located at the tip of the smart structure were not capable of driving the system at large-enough amplitudes, so the piezoceramic patches at the base were used.

Because the piezoceramic patches at the base of the structure were used for both actuation and control, careful precaution was taken to ensure the uncontrolled and controlled systems were actuated in a repeatable manner. Throughout the duration of the finite-impulse, the controllers were deactivated by initializing the gain of the feedback path to zero. After the finite-impulse signal stopped, the falling edge was used as a trigger to reinitialize the gain of the feedback path to one. This allowed the controllers to reactivate and compensate for the input disturbance. Thus, each
During the duration of the impulse, the feedback path of the closed-loop control systems was initialized to zero, preventing all control signals from interfering with the perturbation signal. This is to ensure that the uncontrolled, PD-controlled, and LQG-controlled systems are all subjected to the same finite-impulse signal without interference from the controller. Subsequently, the feedback path is re-initialized to one by the trigger of the falling edge of the finite-impulse. In turn, this process reengages the closed-loop controllers immediately following the perturbation, and the displacement of the smart structure, measured at the location of the retroreflector, was recorded.

6.3.1 Results

As shown in Table 5.1 and discussed in Chapter 5, the LPF has a cutoff frequency of 324.8 Hz, which significantly reduces the voltage applied to the piezoceramic patches at frequencies larger than that. Additionally, the time required to perform a sine sweep across a large bandwidth of frequencies requires significantly more time than a sine sweep over a small band of frequencies. As a result, the sine sweep is used to produce the experimental magnitude responses in Figure 6.5 spans 5 to 400 Hz—with a total of 1200 sampling points.

![Experimental Bode plot of the uncontrolled, PD-controlled, and LQG-controlled smart structure systems.](image)

Figure 6.5: Experimental Bode plot of the uncontrolled, PD-controlled, and LQG-controlled smart structure systems.
The PD controller was capable of moderately attenuating the response of the first mode. The PD controller also slightly shifted the frequency of the first mode from 11.65 to 11.75 Hz. However, the PD controller showed little to no effect in reducing the higher-order modes of the system.

Alternatively, the LQG regulator demonstrated superior attenuation to a larger band of frequencies. The LQG controller was not only capable of significantly flattening out the first mode, but demonstrated the ability to attenuate higher-order modes as well. In particular, the antimodes of the smart structure system were also flattened, which resulted in a small magnification of the magnitude response. While this may indicate that the LQG regulator increases the magnitude of the response at the frequencies of these antimodes, an overall flatter system response is far more desirable. From Figure 6.5, it is also possible to see that the LQG regulator was able to suppress the first mode more than that of the higher-order modes.

While it is possible to control the higher-order modes, there are three factors which may have inhibited the ability to do so. The first reason is that the smart structure system incorporates a LPF which attenuates high-frequency signals. The cutoff frequency for the LPF is at 324.8 Hz, which means that the input-output ratio of the LPF is -3 dB. Input signals which are higher frequency than the cutoff frequency are even further attenuated. Although this frequency is lower than that of the fourth mode, it was necessary to use such an aggressive filter to limit the slew rate of the control signal and prevent damage to the power amplifier. Secondly, it requires more energy to excite or control higher-order modes than it does for lower-order modes. As the frequency of the input signal increases, so does the current in the piezoceramic patches. As a result, the amount of power required to excite the higher-order modes is significantly more than that of the lower-order modes. Finally, the piezoceramic patches contain nonlinearities and hysteresis effects. These nonlinearities are not currently modeled. Frequency, heat generation, and other time parameters may influence the performance of the piezoceramics.

The finite-impulse signal used to excite the smart structure had an amplitude of 5 volts and a duration of 500 samples, or 150 milliseconds. The recorded responses are shown in Figure 6.6.
Figure 6.6: Experimental disturbance response of uncontrolled, PD-controlled, and LQG-controlled smart structure systems (a), and a reduced timescale (b), subjected to a 5 volt finite-impulse using the base piezoceramics, lasting 150 milliseconds.

Compared to the frequency responses of the closed-loop systems depicted in Figure 6.5, it is much clearer to distinguish the performance of the closed-loop systems from the finite-time impulse response. The performance metrics for the time response of the controlled and uncontrolled systems are also organized in Table 6.2.

Table 6.2: Experimental time response performance metrics of the uncontrolled, PD-controlled, and LQG-controlled smart structure systems subjected to a 5 volt finite-impulse using the base piezoceramics, lasting 150 milliseconds.

<table>
<thead>
<tr>
<th>Performance Metric</th>
<th>Units</th>
<th>Uncontrolled</th>
<th>PD</th>
<th>LQG</th>
</tr>
</thead>
<tbody>
<tr>
<td>Settling Time</td>
<td>sec</td>
<td>53.07</td>
<td>1.86</td>
<td>0.10</td>
</tr>
<tr>
<td>Max Displacement</td>
<td>µm</td>
<td>131.75</td>
<td>233.51</td>
<td>85.63</td>
</tr>
<tr>
<td>Max Controller Effort</td>
<td>V</td>
<td>0</td>
<td>3.21</td>
<td>6.00</td>
</tr>
</tbody>
</table>
The uncontrolled system took much longer to decay compared to both the closed-loop systems. The PD controller was able to reduce the settling time of the system from approximately 53.07 seconds to approximately 1.86 seconds; however, the PD controller also caused the system to have a larger amplitude of vibration after the impulse signal, which is shown in the time before 400 milliseconds in Figure 6.6. From the finite-impulse response, it can also be seen that the PD controller increased the frequency of the first mode—as also seen in Figure 6.5.

In general, the LQG regulator demonstrated a superior transient response compared to the PD controller by producing a faster system settling time and reduced amplitude of oscillations. The LQG regulator was able to dramatically reduce the settling time of the uncontrolled system to 100 milliseconds—approximately 176 milliseconds or 18 times faster than the PD controller. It was also able to reduce the settling time to only one oscillation period. Moreover, the LQG regulator had a maximum amplitude of only 85.63 micrometers, which is about 65% of maximum amplitude of the uncontrolled system. However, as also observed in closed-loop simulations, the LQG regulator demanded more controller effort than the PD controller. The LQG regulator even hit the saturation limit of 6 volts, which was put in place to prevent damage to the power amplifier. Even with the saturation limit in place, the LQG regulator was able to outperform the PD controller.
Chapter 7

SUMMARY AND CONCLUSIONS

In this paper, an analytical model for a smart structure was derived by using Euler-Bernoulli beam theory and the extended Hamilton principle. The analytical model was validated using a combination of simulation and experimental tests. After the model was validated, it was used in the model-based design of a classical PD controller and optimal LQG regulator. Finally, the closed-loop performance of the smart structure system equipped with the LQG regulator was compared against the PD-controlled and uncontrolled systems using simulation and experimental analyses.

In simulation and experimentation, the LQG regulator demonstrated superior performance over the PD controller. For the smart structure, the LQG regulator was able to dramatically reduce the magnitude response of the first natural frequency, attenuate the magnitude response of the higher-order modes, and significantly reduce the settling time from input perturbations. Furthermore, the LQG regulator also demonstrated the ability to selectively dampen out the vibratory modes of the system, independently of others.

In addition to optimal closed-loop control, the smart structure and laser interferometry systems are currently being used to investigate the applications of system identification, hardware communication acceleration, and machine learning. In the future, the experimental system may serve as an ideal platform for further research of the effects of hysteresis and time variant systems, forced shape control, nonlinear control, machine learning control, and other advanced topics.
BIBLIOGRAPHY


