DESIGN, FABRICATION, AND TESTING OF MECHANICAL HINGES WITH SNAP-FIT LOCKING MECHANISMS IN RIGID ORIGAMI STRUCTURES

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ABSTRACT

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The ancient art of ‘origami’ has recently become the source of inspiration for engineers to create structures that can unfold from a compact state to a fully deployed one. For instance, researchers have currently adopted origami designs in various engineering disciplines, including aerospace engineering, robotics, biomedical engineering, and architecture. In particular, architects have been interested in designing origami-inspired rigid walled structures that can be deployed as disaster-relief shelters. This type of design has three main advantages: transportability, constructability, and rigidity. Although there has been increased interests in deployable structures, limited research has been conducted on evaluating their structural performance, specifically the mechanical performance of the hinges that allow for the rotation of the rigid panels. To address the limitation, this thesis proposes a novel design of hinge connections for rigid origami structures. The hinges utilize snap fit connections to allow for the structure to achieve and maintain a locked state once unfolded without the need for any additional connections. Prototypes of the hinge design were fabricated using a 3D printer and their flexural strength was experimentally and computationally studied. It was concluded that the design could resist typical flexural loads for residential structures, and future research should be performed to minimize deflection.

Keywords: Active disassembly, deployable structures, disaster-relief shelters, origami
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1.1. Background & Motivation

Throughout history, the world has witnessed devastating natural disasters and conflicts that have caused the displacement of massive amounts of people from their homes. For instance, in 2020, the U.S. experienced 22 separate natural disasters that each caused over $1 billion worth of damages and $95 billion cumulatively [1]. As of June 2020, 80 million people worldwide have been forcibly displaced from their homes because of conflict, disaster, or persecution [2]. This has created a need for innovative ways to design and build structures rapidly across the globe.

This is a difficult problem to solve because it requires the balance of many competing design constraints as shown in Figure 1.1. Certain constraints are qualitative, such as a structure's livability while other metrics, such as the packing efficiency, can be quantified.

![Figure 1.1. Design constraints for deployable structures.](image_url)
One of the most promising solutions to this problem is deployable rigid-origami structures. These are rigid-walled structures that can unfold from a packed state that is a fraction of the size of the fully deployed state. This design has significant advantages over soft-walled tent-like structures mainly because it is both stiffer and more dependable for long-term use while still maintaining a high packing efficiency [3].

While typical construction methods require skilled contractors to construct the structure entirely on-site which is both time and labor-intensive, deployable origami structures can be constructed and stored entirely offsite. Once they are needed, the structure can be shipped to site and unfolded as one continuous element allowing for rapid construction that is far less labor intensive [3].

Although research has been done considering different folding patterns for rigid walled deployable structures, a limited amount of research has been done regarding the hinges and load bearing connections in these structures [3]. Specifically, there are two significant design issues that this study aims to address:

1. There are currently no procedures for the structural design of rigid deployable structures and specifically their connections. This limits the ability of engineers to create cost-effective designs that ensure life-safety.

2. Limited testing has been done regarding the performance of mechanical hinges and snap fit connections as load transfer elements in civil and structural applications.

This thesis project proposes a novel method of using mechanical hinges and snap-fit joints in deployable rigid-origami structures. A schematic drawing of the proposed hinge design is shown in Figure 1.2 and fully dimensioned drawings of the hinge design are included in Appendix A. This design allows the structure to be unfolded and snap into a locked position.
once fully deployed. Since the hinge locks into place automatically, this design is highly scalable across larger structures with many fold lines. These hinges also allow for two disassembly mechanisms. The first is to simply remove the snap-fit hooks from the hinge leaf. The second is by manufacturing the snap-fit hooks using shape memory polymers (SMP) or shape memory alloys (SMA). These materials will deform and return to a remembered shape under certain stimuli, most commonly through temperature changes [4]. In the case of this design the remembered shape would be a deformation such that the hooks can move past each other without touching. Therefore, even if the hinge connections are inaccessible, the structure can still be dissembled by heating the snap-fit elements. To evaluate the feasibility and performance of the design, prototypes were created using three-dimensional (3D) printing technology and subjected to flexural load test per ASTM D3043-17 [5].

Figure 1.2. Schematic drawing of hinge design with snap-fit locking hooks.
1.2. Statement of Purpose

The purpose of this thesis is to investigate the feasibility of using mechanical hinges and snap-fit hooks in rigid origami structures and to increase their efficacy as load bearing connections. This thesis has used various tools to create a framework for the future design of such systems. Load test has been performed to validate and improve upon the analysis provided.

1.3. Research Objectives

This project has the following four research objectives:

1. Assess the performance of mechanical hinges with snap-fit locking mechanisms under quasi-static loading conditions that would be typical for small-scale residential structures.

2. Create a design procedure that can be performed by hand for stress analysis of the hinge assembly under static structural loads.

3. Assess the feasibility of using 3D printed parts and discuss other fabrication methods.

4. Compare the calculated capacity from the design procedure to the finite element analysis and test results and evaluate the effectiveness of each modeling approach.
2.1. Origami Engineering: Theory and Applications

*Origami* is the ancient Japanese art of paper folding [6]. Although origami has existed for generations, it has recently become the source of inspiration for mathematicians, physicist, and engineers that are using it to create novel designs [7]. This section will review the wide breadth of multidisciplinary research in origami engineering.

**2.1.1. Origami Engineering Background**

Origami patterns can generally be defined as ether rigidly foldable (RF) or not rigidly foldable (Not RF), where an origami crease pattern is rigidly foldable if it can be folded without causing the panels of the crease pattern to deform [8], [9]. Not RF patterns can be categorized by the degree of deformation that occurs during the folding process and rigidly foldable patterns can be subcategorized as either thick or thin, where thin means the thickness of the panel is negligible [8]. A categorization tree for the different origami models is shown in Figure 2.1.

![Figure 2.1. Categorization tree for origami models [8].](image_url)
For most engineering applications, it is desirable to use thick rigidly foldable origami models, because the panels can be manufactured from materials that are stiffer and more suitable for engineering applications. For instance, deployable building structures are typically designed with plywood or corrugated plastic panels [3].

Thick rigidly foldable models require thickness accommodation techniques to allow for continuous rotation to occur. Lang et al. [10] evaluated seven thickness accommodation techniques and compared them based on their required manufacturing processes. Three of those thickness accommodation techniques are shown above in Figure 2.2. The hinge shift technique was used in this design as it offers two main advantages when used for applications in deployable structures:

1. It allows for a constant thickness of panels and minimizes the risk of stress concentration at tapered panel edges.
2. It avoids gaps in the structure which could create points of low stiffness and gaps in waterproofing.

![Diagram of thickness accommodation techniques](image)

**Figure 2.2.** Three thickness accommodation techniques for thick origami [10].
The simplest fold to highlight thickness accommodation techniques is a *degree-four vertex* (D4V): a single vertex with four folds surrounding it [10]. A schematic of a thin (zero-thickness) D4V is shown in Figure 2.3, where $\alpha$ represents sector angles (the angles between adjacent folds) and $\gamma$ represents the dihedral angles (the angles between adjacent faces). A schematic of a thick counterpart to the degree-four vertex is shown in Figure 2.4, where the offset distances are labeled as $d_1$-$d_4$. The offset distance is defined as the distance from the rotational axis of the hinge to the plane of zero-thickness.

![Figure 2.3. A schematic of a thin (zero-thickness) degree-four vertex [10].](image)

![Figure 2.4. A schematic of a thick panel counterpart to the degree-four vertex [10].](image)
Chen et al. [11] showed the kinematic motion for degree-four, -five, and -six vertex origami patterns for both thick and thin origami can be kinematically equivalent by modeling their motion as over-constrained spatial linkages. Degree-six vertex linkages can be particularly useful for engineering applications as certain crease patterns can be folded flat and into stacks without any voids [12]. An example of one such degree-six vertex crease pattern known as the Yoshimura fold is shown in Figure 2.5.

Figure 2.5. Yoshimura crease pattern tessellation.

2.1.2. Origami Engineering Applications

Origami inspired design has a countless number of applications in different engineering disciplines including aerospace, biomedical, manufacturing, optical engineering, and structural engineering [7]. The application of origami engineering to deployable building structures is discussed in depth in Section 2.3.

In aerospace applications, rigid origami can be used for the deployment of compact solar arrays and solar sails. Zirbel et al. [13] developed a mathematical model for a thick rigidly foldable origami flasher design for deployable solar arrays, shown in Figure 2.6. It is particularly important in this application that the solar panels do not bend or flex to avoid
damaging the solar cells and for this reason it is as an excellent example of rigid origami as defined earlier in this chapter.

In biomedical engineering, there is a growing desire to use origami engineering for the delivery of devices in areas that are difficult to reach [7]. One example is deployable stent grafts that can be unfolded in blocked or weakened arteries for minimum invasive surgeries as shown in Figure 2.7. These stent grafts are typically manufactured from shape-memory alloys that respond to temperature so that once they are inserted into the body, they will deploy without the need of actuation [14].

In manufacturing engineering, origami-based design can reduce the number of separate processes required during assembly. For example, products can be designed to start from
a flat sheet and sequentially folded into the final product. Not only does this simplify the manufacturing process but it can also result in flat foldability for storage [7].

Optical engineers have been able to use origami to determine effective ways to pack long-focal-length optics into smaller spaces. So called optigami, can reduce the size and weight of optical devices like telescopes and cameras [15].

Although this list of applications is not exhaustive, it encapsulates the diversity of design solutions that can benefit from origami engineering. For origami engineering to continue to grow and develop it is important to take inspiration from these different applications and apply them to a host of design problems.

2.1.3. Hinge Mechanisms in Origami Engineering

All origami designs require hinges to allow for the kinematic rotation of the rigid panels. These hinges can include mechanical hinges, thin membranes, and strain joints [10]. The fundamental design issue with hinges in origami engineering is that an extremely low rotational stiffness is desired during folding and deployment while a high rotational stiffness is desired once the design is deployed [8]. This section will focus on hinge designs that incorporate stiffness accommodation techniques that create rotational stiffness once deployed.

Yellowhorse [8] modeled and assessed the following three stiffness accommodation techniques for large thin plates in deployable space structures: 1) partially actuated stiffeners 2) yielded stiffener plates and 3) expanded stiffeners. Although these methods are promising for space applications, the applicability to civil structures is limited because the ratio of deployed area to panel thickness is much less in civil structures. In addition, no bending moment capacities were published for these designs, so it is difficult to evaluate their stiffness relative to other designs.
Tape springs hinges are another stiffness accommodation technique that use a thin curved surface that resembles a steel tape gauge to automatically lock once deployed [16]. An example of a tape spring hinge created by Jeong et al. is shown in Figure 2.8 [17]. Tape hinges are mostly used for space applications and are designed for comparatively smaller loads. For example, the largest bending moment capacity from the design presented by Han et al. was 9040 N-mm (80 lb.-in) [16].

Vacuumatics have also been used to create hinges in deployable structures by creating a pressurized gap between the rigid panels. The pressure differential between the sealed gaps of the rigid panels and atmospheric pressure is what creates the rotational stiffness of the system [18]. Although this design looks promising no flexural load test have yet been to be performed to evaluate the performance of the design. Additionally, actuation of the internal pressure of the hinges could present difficulties with scalability as multiples hinges would need to be controlled simultaneously.

Wagner et al. [19] performed a comparison between three flexural hinges manufactured using multimaterial additive manufacturing. The three flexural hinges studied were an
aramid hinge, a polyamide hinge, and a photopolymer hinge. The aramid hinge design was fabricated using a fused deposition modeling (FDM) 3D printer. The hinge regions were printed using continuous aramid fibers impregnated by a polyamide matrix. The polyamide hinge also uses FDM 3D printing but was fabricated solely from polyamide. Finally, the photopolymer hinge was fabricated from a flexible thermoset using an inkjet 3D printer. The aramid hinge demonstrated the highest flexural strength with a maximum applied bending moment of 5 N-mm (0.04 lb.-in.) This hinge is ideal for origami designs that require small scale application because the aramid fibers can be as small as the diameter of the FDM 3D printing nozzle which is typically 0.4 mm.

Notably, the designs discussed in this section have little flexural strength when compared to typical design demands for residential buildings. For this reason, this thesis proposes a novel hinge design that aims to exceed the flexural strength of previous designs.

2.2. Disaster Relief Shelters and Deployable Structures

Disaster relief (DR) shelters serve a critical role in the response to large scale disasters [20]. This section will begin by describing key design considerations for DR shelters and then review existing designs for deployable DR shelters. Although not all DR shelters are necessarily deployable structures, they are the most promising solution for creating constructible and space-effective designs.

2.2.1. Disaster Relief Background and Design Considerations

Prior to discussing specific design considerations, a few categories of shelter/housing will be defined. The three categories of shelters that are the most pertinent to DR are emergency shelters, temporary shelters, and temporary housing [20]. Emergency shelters are typically used for only a few nights and offer immediate support following a disaster [20], [21]. Temporary shelters are typically tents or temporary mass shelters such as
gymnasiums or auditoriums that are used for a few weeks [20], [22]. Due to the short-term use of these structures, cost-efficiency and deployment speed are two of the highest priorities [20]. Finally, temporary housing is defined as structures that are used for time periods that extend from six months to three years following. Temporary housing includes rental homes and prefabricated units [20]. Depending on the design, deployable structures can either be defined as temporary shelters or temporary housing.

Three important technical design considerations for the design of DR shelters are: 1) ease of construction and deconstruction, 2) materials and insulation and 3) classification and design for hazards. For DR shelters to be easy to construct and collapse, they should be designed from lightweight materials and require only a few pieces to assemble [20]. Deployable structures are simple to construct and can typically unfold as a single continuous structure. When choosing materials for deployable structures, cost, weight, and local familiarity with the material should all be considered. The design should also include noise, temperature, and weather insulation [20].

2.2.2. Existing Deployable Shelter Designs

This section will discuss three classifications of deployable structure designs: 1) scissor linkages, 2) inflatable structures, and 3) thick rigid origami. The efficacy of each design as a DR shelter will be evaluated based on the design considerations from the previous section.

Scissor linkage structures consist of scissor-like elements (SLE) made of thin beam members connected at pivot points [23]. SLE’s can be configured together to form a complete structure as shown in Figure 2.9. Scissor linkages, like many other designs, require the difficult optimization of their weight, transformability, and stiffness [23].
One major drawback to scissor linkages is that the structure requires a flexible membrane to wrap the SLE’s to provide shelter [24]. For this reason, scissor linkages can only function as emergency shelters or temporary shelters but cannot provide insulation for semi-permanent usage.

![Diagram of scissor linkage design with bistable configuration](image)

**Figure 2.9.** Scissor linkage design with bistable configuration [23].

Inflatable or pressure-deployable structures typically use very lightweight materials in their design to create lightweight, constructible structures [25], [26]. The structure shown in Figure 2.10 uses origami inspired bistability so that once it has been deployed it no longer needs to be pressurized, and the door can be opened with no impact to the stiffness of the structure [26]. Although this design presents a very constructible and lightweight design it is incompatible with stiffer and heavier materials that would offer more structural resiliency and greater insulation.
The last category of structures that are the primary focus of this thesis, is thick rigid origami structures. These structures utilize thick RF crease patterns to create simple rectangular structures. Typically, the rigid panels are made from plywood, or sheet metal but can also utilize corrugated plastic [3]. One specific origami design that has a long history of usage within the U.S. military is the accordion style design [3]. One example design of an accordion style shelter is shown in Figure 2.11. Although these structures have a long history of use, the mechanical hinge design was cited as one key issue with many of the designs for accordion shelters [3]. This is one primary motivation behind the research presented in this thesis.

![Inflatable bistable origami structure](image)

**Figure 2.10.** Inflatable bistable origami structure [26].

Many rigid origami designs are reliant on a partially deployed state to create bistability [27]. For example, the accordion shelter shown in Figure 2.10 is only partially deployed to create more stiffness in the design. The auto-locking mechanism in the hinge design
proposed in this study avoids the need to deploy designs only partially and therefore it can offer an increase to the total deployed volume.

One drawback to using stiffer material like plywood or sheet metal in deployable structure design is that they are comparatively heavier than the thin and light materials used in soft walled tent-like structures. A removable lever arm design was proposed by Quaglia et al. [28] which still allows for designs to utilize heavier and stiffer materials without foregoing constructability.

Figure 2.11. Accordion shelter design used by the U.S. military [3].

2.3. Snap-Fit Joints

Snap-fit joints have a long history of usage in the automotive and electronics industries, but they are limited in their usage as force resisting elements in building structures [29].
Snap fit joints can be generally described as a component with a hook that is deflected during the mating process and then catches in an undercut over another component to lock its position in place [30]. In the design presented in this thesis, a snap-fit locking mechanism is used to allow for the hinges to have an effective rotational stiffness of zero during deployment until the hinges have completed the mating process.

Although snap-fit joints are rarely used in building structures, Robeller et al. [29] created snap-fit joints in wood components through CNC machining and concluded that snap-fit connections have potential to offer rigid structural connections between wood components.

2.3.1. Active Disassembly

Typically, there are two ways to disengage snap-fit joints. The first method is to design the hook with a slope in each direction, as shown in Figure 2.12, such that the hook can be deflected by pulling in either direction [31]. Although this allows for easy disassembly it creates a very small load capacity for the connection. The second method is to disengage the hook manually and pull the components apart [30].

![Snap-fit hook with bidirectional design](image)

**Figure 2.12.** Snap-fit hook with bidirectional design [31].
Both methods are limited in how they can be implemented throughout a design however a novel approach called active disassembly is overcoming some of those limitations. Active disassembly can generally be defined as the practice of using specialized disassembly mechanisms within the product's design [4]. One way to achieve this behavior in snap-fit components is by manufacturing the hook from a shape-memory polymer (SMP) or shape-memory alloy (SMA) [4]. These materials are called smart materials because they achieve shape recovery when exposed to an external stimulus, typically a heat field [4], [32]. The design proposed in Carrel et al. [4] used SMP hooks that were then heated above the glass transition temperature ($T_g$) to release from the undercut as shown in Figure 2.13.

![Figure 2.13. Disengagement procedure of snap-fit hook printed from SMP [4].](image)

Figure 2.13. Disengagement procedure of snap-fit hook printed from SMP [4].
CHAPTER 3. HINGE FABRICATION AND MATERIAL CHARACTERIZATION

3.1. Hinge Fabrication

Fused deposition modeling (FDM) 3D printing creates objects by extruding a material from a metal nozzle that is heated to a temperature exceeding the melting temperature of the filament. The material is extruded onto a flat build plate and a 3D object is created by successively stacking layers of extruded material [33]. A schematic of the basic components of a FDM 3D printer is shown in Figure 3.1.

![Figure 3.1. Schematic drawing of basic components of a FDM 3D printer [33].](image)

All specimens tested in this thesis were fabricated using a Creality Ender 3 3D printer, which is a consumer-level FDM 3D printer. An image of the 3D printer used is shown in Figure 3.2. The only modification made to the stock 3D printer was the addition of a glass
build plate to improve the adhesion of the extruded filament to the print bed and provide a smoother finish surface.

Figure 3.2. Creality Ender 3 FDM 3D Printer used for fabrication of test specimens.

Two 3D printing filaments produced by the manufacturer eSUN were used to manufacture the test specimens. In all test, the hinge leaves and hinge pins were fabricated using eSUN’s PLA+ filament. Polylactic acid (PLA) is a bioplastic manufactured from renewable resources like corn and is a common and inexpensive 3D printing filament [33]. The removable snap-fit hooks were tested using both eSUN’s PLA+ filament and eSUN’s e4D-1 filament. The e4D-1 filament is a PLA based SMP filament, meaning that it can recover
a remembered shape when exposed to a temperature greater than is glass transition
temperature. Note that PLA plastics are susceptible to degradation of mechanical
properties when exposed to ultraviolet light, high temperatures, or high relative humidity
[34]. Therefore, it is important to provide waterproofing and protection from UV exposure
where PLA parts are implemented.

3.2. Material Characterization

To characterize the material properties of the filaments, two test procedures were
performed. Ultimate Tensile Strength (UTS) test were conducted per ASTM D638-14 [35]
to determine the elastic modulus, and the tensile stress and strain at break for both
filaments. And to evaluate the shape recovery function in the e4D-1 SMP filament a
heating and cooling cycle was applied to the snap-fit hooks and the curvature in the hooks
were measured at each phase.

3.2.1. Tensile Testing

The layered nature of FDM 3D printing creates orthotropic material properties in printed
parts [36]. To describe the orthotropic properties of 3D printed components, the following
coordinate system will be used: the XY plane is parallel to the flat 3D printing build plate
and the Z axis is orthogonal to the build plate. A diagram of the loading direction relative
to the strong axis/print direction is shown in Figure 3.3. To determine the mechanical
properties of the two 3D printing filaments of interest, four sets of Ultimate Tensile Strength
(UTS) test were conducted per ASTM D638-14 [35]. For each filament material, one set
of tests was conducted loading specimens in the XY direction, and one set was conducted
loading specimens in the Z direction. Each of the four sets of tests consisted of three test
specimens (i.e., 12 specimens in total). The UTS tests were performed on an Instron 5969
Universal Testing Machine equipped with a 50 kN load cell. The strain was measured
using an Instron static axial clip-on extensometer.
Figure 3.3. Loading direction of UTS tests relative to the strong axis/print direction.

The mean tensile stress at break, strain at break, and corresponding standard deviations for the four sets of tests are shown in Table 3.1. The elastic modulus for each test was calculated per ASTM D638-14 [35] and the effective elastic modulus was taken as the mean elastic modulus between the six tests (including both XY and Z directions) performed on each filament. Representative stress-strain curves for the PLA+ and e4D-1 filament are shown in Figure 3.4 and Figure 3.5, respectively. Note that the reported values for stresses and strains in are the true stress and strain calculated per ASTM D638-14 [35] and have accounted for the change of area and elongation of the gauge length during testing.
<table>
<thead>
<tr>
<th>Material</th>
<th>Print Direction</th>
<th>Tensile Stress at Break (psi)</th>
<th>Strain at Break (%)</th>
<th>Effective Elastic Modulus (psi)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Mean</td>
<td>Standard Deviation</td>
<td>Mean</td>
</tr>
<tr>
<td>PLA+</td>
<td>X-Y</td>
<td>4.45E+03</td>
<td>4.36E+01</td>
<td>2.21</td>
</tr>
<tr>
<td></td>
<td>Z</td>
<td>2.76E+03</td>
<td>2.35E+02</td>
<td>1.19</td>
</tr>
<tr>
<td>e4D-1</td>
<td>X-Y</td>
<td>5.09E+03</td>
<td>1.23E+02</td>
<td>2.38</td>
</tr>
<tr>
<td></td>
<td>Z</td>
<td>3.46E+03</td>
<td>2.70E+02</td>
<td>1.49</td>
</tr>
</tbody>
</table>

**Figure 3.4.** Tensile stress-strain curves of 3D printed PLA+ filament.
The six fractured PLA+ coupons specimens are shown in Figure 3.6. Little to no necking was observed in both print directions for the PLA+ and e4D-1 filaments. A zoomed-in image of the brittle fracture for the XY specimens is shown in Figure 3.7. In this direction the fractures occurred across individual extrusion lines creating the rough cleavage on the fracture face. A zoomed-in image of the brittle fracture for the Z specimens is shown in Figure 3.8 where the fractures occurred along layer lines. In the Z direction the limiting factor to the ultimate tensile strength is the adhesion between print layers which is why the tensile stress at break in the Z direction is nearly half of that in the XY print direction. Therefore, all hinge parts were designed and printed such that the largest loaded would occur primarily be in the XY direction.

**Figure 3.5.** Tensile stress-strain curves of 3D printed e4D-1 filament.
Figure 3.6. Fractured UTS PLA+ coupon specimens. a) XY print direction b) Z print direction.

Figure 3.7. Zoomed in image of fracture face for XY specimens. a) e4D-1 b) PLA+.
3.2.2. Shape Memory Properties Characterization

Figure 3.9 shows the basic steps of the shape memory process in the snap-fit hooks as it pertains to the deployment and deconstruction of a structure. To quantify the degree of shape retention and recovery at each step in this process the cycle shown in Figure 2 was performed on eight individual snap-fit hooks. The snap-fit hooks were initially fabricated with the hook deflected and then to achieve their target temporary shape, they were placed into a hot water bath at the glass transition temperature of 140 °F (60 °C). The specimens were removed from the water bath and immediately placed into a mold with a curvature of zero and left to cool. Once removed from the mold, an image processing script was used to determine the curvature of the eight hooks in their temporary shape.

The hooks were then reheated to the glass transition temperature to stimulate the shape memory function and left to cool. The curvature of the hooks was measured again using the same image processing script as shown in Figure 3.10. Although only eight snap-fit hooks were tested for the full deformation and recovery cycle, all the e4D-1 snap-fit hooks
tested under flexural loading in Chapter 6 were initially printed in the deformed shape and then heated and molded to the temporary zero curvature shape.

**Figure 3.9.** Diagram of shape memory polymer's temporary states during a structure's lifetime.

**Figure 3.10.** Calculation of curvature of individual snap-fit hook using image processing.
The mean curvature, standard deviation, and recovery ratio for both shape-memory states are shown in Table 3.2. The recovery ratio is defined per equation (3.1).

<table>
<thead>
<tr>
<th>State</th>
<th>Mean Curvature (in⁻¹)</th>
<th>Standard Deviation (in⁻¹)</th>
<th>Target Curvature (in⁻¹)</th>
<th>Recovery Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temporary Shape</td>
<td>1.965E-02</td>
<td>4.126E-03</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>Recovered Shape</td>
<td>1.470E-01</td>
<td>9.080E-03</td>
<td>1.481E-01</td>
<td>99.21%</td>
</tr>
</tbody>
</table>

Recovery Ratio = \( \frac{K_{\text{target}} - K_{\text{actual}}}{K_{\text{target}}} \) \hspace{1cm} (3.1)

Where:

\( K_{\text{target}} \) = Target curvature of snap-fit hook after reheating.

\( K_{\text{actual}} \) = Measured curvature of snap-fit hook after reheating.
CHAPTER 4. DESIGN AND ANALYSIS METHODOLOGY

This chapter will provide a methodology for the design and analysis of mechanical hinges with snap-fit locking mechanisms. This procedure is based on an Allowable Stress Design (ASD) philosophy. ASD was used because rigid deployable structures will likely be designed with timber components as they provide a lightweight and inexpensive structural solution [3]. This chapter will only consider strength design and not deflection. Although deflection is a necessary consideration for all structures to avoid cracking of non-structural finish materials and for public perception of safety it is less of a design priority for temporary deployable structures [20].

The design procedure in this chapter only considers out-of-plane flexure from gravity and wind loading as shown in Figure 4.1. To use the hinges in diaphragms and shear walls this design procedure must be expanded to also consider in-plane shear loading.

Figure 4.1. Arbitrary out-of-plane loading of hinges.
A detailed analysis is shown in Appendix B that considers typical loading conditions for a residential structure per ASCE 7-16 [37]. The hinge design from that example is used as the prototype design for the finite element simulation and experimental testing.

For clarification on the components of the design discussed in this chapter, see the schematic drawing of the hinge assembly in Figure 1.2.

For out-of-plane loading, the primary design demand of interest is $M_t$, the applied moment on an individual hinge about its rotational axis. For determinate systems, $M_t$ can easily be determined using structural analysis. The design procedure is complicated with indeterminate systems as it is necessary to assume a stiffness of the hinges prior to designing them. Therefore, for simplicity, this study will only consider determinate systems.

Depending on the direction of the applied moment, two separate load cases can be considered as shown in Figure 4.2. Load Case 1 induces flexure in the snap-fit hooks, while Load Case 2 causes the rigid panels to be compressed.

![Figure 4.2](image)

**Figure 4.2.** Schematic of four-point bending loading of hinges. a) Load Case 1, b) Load Case 2.
Additionally, Load Case 1 includes two subcategories: Load Case 1a considers snap-fit hooks that are manufactured from PLA+ filament and Load Case 1b considers snap-fit hooks that are manufactured from e4D-1 filament. Load Case 2 only considers snap fit hooks printed from PLA+ filament. This is because the hinge leaf and panels were assumed to resist most of the flexural loading, hence the filament material of the snaps was not of particular importance.

4.1. Load Case 1

This subsection will explain the design procedure for Load Case 1. This section exclusively discusses the design of the snap-fit hooks. The design of the hinge pin and hinge knuckle will be discussed in Section 4.3. Figure 4.3 shows the generalized dimensions of a snap-fit hook for reference.

\begin{figure}
\centering
\includegraphics[width=0.5\textwidth]{snap-fit-hook-dimensions.png}
\caption{Dimensions of snap-fit hook. a) profile view, b) section view.}
\end{figure}

Several assumptions are made at the start of this analysis to allow for a closed form solution for determining the stresses in the hinge assembly. The accuracy of these assumptions is later validated based on finite element simulation and load testing. The four main assumptions are the following:

1. The snap-fit hooks resist the entire tensile load.
2. The hinge knuckles resist the entire compressive load.

3. The hinge pin and knuckle are under a uniform pressure and therefore the equivalent force acts at the center of the hinge knuckle.

4. The entire snap-fit hook is under a triangular compressive stress distribution and therefore the equivalent force $P_{h,1}$ acts at a distance of $b_s/3$ from the edge of the hook.

The equivalent tensile and compressive loads can be shown as an equivalent force couple per Figure 4.4. Based on moment equilibrium, the force $P_{h,1}$ can be calculated using equation (4.1) and the effective distance, $d_{eff,1}$, can be calculated using equation (4.2):

$$P_{h,1} = \frac{M_a}{d_{eff,1}}$$  \hspace{1cm} (4.1)
Where:

\[ P_{h,1} = \text{Equivalent point load of force couple for Load Case 1.} \]

\[ M_a = \text{Applied moment on an individual hinge.} \]

\[ d_{\text{eff},1} = \text{Effective distance between point loads of force couple for Load Case 1.} \]

\[ d_{\text{eff},1} = \frac{2b_s}{3} + r_h + t_k \]  \hspace{1cm} (4.2)

Where:

\[ b_s = \text{Width of snap-fit hook.} \]

\[ r_h = \text{Inner radius of hinge knuckle.} \]

\[ t_h = \text{Thickness of hinge knuckle.} \]

The snap-fit hooks shall be designed to keep the maximum combined tensile and bending stress in the hook less than the allowable tensile stress. The maximum combined tensile stress in the snap-fit hooks can be calculated using superposition. The bending stress created from the eccentricity of the applied load, \( P_{h,1} \), can be separated into two separate moments as shown in Figure 4.5. The corresponding bending moments can be calculated using equations (4.3) and (4.4):

![Figure 4.5](image)

**Figure 4.5.** Bending moment from eccentricity of applied load. a) \( M_{s,1} \) b) \( M_{s,2} \).
\[ M_{s,1} = \frac{P_{h,1}}{N_s} \left( \frac{b_s}{6} \right) \quad (4.3) \]

\[ M_{s,2} = \frac{P_{h,1}}{N_s} \left( \frac{y_s}{2} + \frac{t_s}{2} \right) \quad (4.4) \]

Where:

\[ N_s \quad = \quad \text{Number of primary snap-fit hooks on one side of the hinge.} \]

\[ y_s \quad = \quad \text{Depth of snap-fit undercut.} \]

\[ t_s \quad = \quad \text{Thickness of snap-fit hook.} \]

The maximum bending stress, from \( M_{s,1} \) can be calculated using equation (4.5):

\[ f_{b,s,1} = \frac{M_{s,1} \left( \frac{b_s}{2} \right)}{I_{s,1}} \quad (4.5) \]

Where, the moment of inertia of an individual snap-fit hook in the strong axis direction is given per equation (4.6):

\[ I_{s,1} = \frac{1}{12} t_s b_s^3 \quad (4.6) \]

Substituting equations (4.3) and (4.6) into (4.5), the bending stress from \( M_{s,1} \) can be simplified as shown in equation (4.7):

\[ f_{b,s,1} = \frac{P_{h,1}}{N_s t_s b_s} \quad (4.7) \]
Similarly, the maximum bending stress, from $M_{s,2}$ and moment of inertia about the weak axis can be calculated per equation (4.8) and (4.9):

$$f_{b,s,2} = \frac{M_{s,2} \left( \frac{t_s}{2} \right)}{I_{s,2}} \quad (4.8)$$

$$l_{s,2} = \frac{1}{12} b_s t_s^3 \quad (4.9)$$

Substituting equation (4.4) and (4.9) into (4.8) the bending stress from $M_{s,2}$ can be simplified as shown in equation (4.10):

$$f_{b,s,2} = \frac{3P_{h,1}}{N_s b_s t_s^2} (y_s + t_s) \quad (4.10)$$

The stress due to pure tension can be calculated per equation (4.11) and the total combined tensile stress can be calculated per equation (4.12):

$$f_{t,s} = \frac{P_{h,1}}{N_s b_s t_s} \quad (4.11)$$

$$f_{s,\text{total}} = f_{b,s,1} + f_{b,s,2} + f_{t,s} \quad (4.12)$$

To account for stress concentrations around the fillet edges the stress concentration factor, $K_c$, can be approximated based on Figure 4.6. The adjusted combined stress, accounting for stress concentration, can be calculated per equation (4.13):

$$f'_{s,\text{total}} = K_c \left( f_{b,s,1} + f_{b,s,2} + f_{t,s} \right) \quad (4.13)$$
Figure 4.6. Stress concentration factor in filleted plate [38] and equivalent dimensions on snap-fit hook.

Note that stress concentration factor is only a very rough approximation of the increase in stress at the filleted corner. The relationship shown in Figure 4.6 assumes that the plate is infinitely long and subjected to pure axial loading, neither of which are true for the snap-fit-hook.

The adjusted bending stress should be checked against the allowable bending stress per equation (4.14):

\[
F'_{s} = \frac{F_s}{\Omega} > f'_{s,total}
\]  \hspace{1cm} (4.14)

Where:

\( F'_{s} \) = Allowable stress in the snap-fit hook.

\( F_s \) = Yield stress of filament.

\( \Omega \) = Factor of safety. Typically, 1.67 for yielding failure [39].
Additionally, the hooks should be designed such that during deployment the deflection and locking of the hooks is fully elastic. The length of the hook, \( l \), should satisfy equation (4.15), modified from Bayer [30]:

\[
l \geq \sqrt{\frac{0.746t_y\gamma_s}{\varepsilon}} \quad (4.15)
\]

Where:

\( l \quad = \quad \) Length of snap-fit hook.

\( \varepsilon \quad = \quad \) Permissible flexural strain. Typically, 2\% for plastic parts [30].

4.2. Load Case 2

This subsection will explain the design procedure for Load Case 2 and is divided into the following subsections:

4.2.1. Design of Hinge Knuckles

4.2.1. Design of Rigid Panels

4.2.3. Design of Hinge Pin

Like the last section, several assumptions are made at the start of this analysis to allow for a closed form solution. The four main assumptions are the following:

1. The rigid panel resolves the entire compressive load.

2. The contribution of the snap-fit hooks to the tensile capacity is negligible and therefore it is assumed the hinge knuckle resolves the entire tensile load.

3. The entire panel is under a triangular compressive stress distribution and therefore the equivalent compressive force \( P_{h,2} \) acts at a distance of \( t_p/3 \) from the top of the panel.
4. The hinge pin and knuckle are under a uniform pressure and therefore the equivalent force acts at the center of the hinge knuckle.

Again, the equivalent tensile and compressive load, $P_{h,2}$, can be shown as an equivalent force couple per Figure 4.7.

**Figure 4.7.** Cross section of hinge under out of plane loading. a) dimensions b) stress distribution diagram c) equivalent force diagram

Based on moment equilibrium, the force $P_{h,2}$ can be calculated using equation (4.16) and $d_{eff,2}$ can be calculated using equation (4.17).

$$P_{h,2} = \frac{M_a}{d_{eff}}$$  \hspace{1cm} (4.16)

Where:

$P_{h,2}$ = Equivalent point load of force couple for Load Case 2.
\[ d_{\text{eff},2} = \text{Effective distance between point loads of force couple for Load Case 2.} \]

\[ d_{\text{eff},2} = 2t_p/3 + r_h + t_h \quad (4.17) \]

Where:

\[ t_p = \text{Thickness of rigid panel.} \]

\[ t_h = \text{Thickness of hinge.} \]

**4.2.1. Design of Hinge Knuckle**

The hinge knuckle shall be designed such that the maximum combined tensile and bending stress in the hinge knuckle is less than the allowable stress. A moment diagram of the hinge knuckle is shown in Figure 4.8 assuming that the hinge is completely fixed at the screw locations. The maximum moment in the knuckle, \( M_k \), can be calculated using equation (4.18) and the maximum combined tensile and bending stress can be calculated using equation (4.19) where the moment of inertia, \( I_k \), can be calculated per equation (4.20):

\[ M_k = \frac{P_{h,2}}{N_k} \left( r_h + \frac{t_k}{2} \right) \quad (4.18) \]

**Figure 4.8.** Moment diagram and cross section of hinge knuckles.
Where:

\( N_k \) = Number of knuckles on one hinge.

\[
f_{k,total} = \frac{M_k \left( \frac{t_k}{2} \right)}{l_k} + \frac{P_{h,2}}{N_k t_k b_k}
\]  \hspace{1cm} (4.19)

Where:

\( b_k \) = Width of individual hinge knuckle.

\[
l_k = \frac{1}{12} b_k t_k^3
\]  \hspace{1cm} (4.20)

Substituting into equation (4.19) the bending stress in the hinge knuckle can be written as shown in equation (4.21):

\[
f_{k,total} = \frac{6P_{h,2} \left( r_h + \frac{t_k}{2} \right)}{N_k b_k t_k^2} + \frac{P_{h,2}}{N_k t_k b_k}
\]  \hspace{1cm} (4.21)

The stress concentration factor, \( K_c \), can again be approximated per Figure 4.6 based on the equivalent plate dimensions of the hinge leaf shown in Figure 4.9. Recall that stress concentration factor is only a very rough approximation of the increase in stress at the filleted corners. The adjusting bending stress is given per equation (4.22):

\[
f'_{k,total} = K_c \left( \frac{6P_{h,2} \left( r_h + \frac{t_h}{2} \right)}{b_k t_h^2} + \frac{P_{h,2}}{t_k b_k} \right)
\]  \hspace{1cm} (4.22)
The adjusting bending stress should satisfy the allowable stress inequality per equation (4.23):

\[ F'_{k} = \frac{F_{k}}{\Omega} > f'_{k,total} \]  

Where:

- \( F'_{k} \) = Allowable stress in the knuckle.
- \( F_{k} \) = Yield stress of filament.
- \( \Omega \) = Factor of safety. Typically, 1.67 for yielding failure [39].

Figure 4.9. Equivalent plate dimensions on hinge leaf for calculation of stress concentration factor.
4.2.2. Design of Rigid Panel

The maximum stress in the panel should be less than the maximum allowable compressive stress of the panel. The average compressive stress, \( f_{p,c} \) in the panel can be calculated per equation (4.24) and the maximum compressive stress at the extreme compressive fibers of the panel, \( f'_{p,c} \), can be calculated per equation (4.25):

\[
f_{p,c} = \frac{P_{h,2}}{t_p b_p}
\] (4.24)

Where:

\( b_p \) = Total tributary width of panel-to-panel contact.

\[
f'_{p,c} = \frac{3P_{h,2}}{2t_p b_p}
\] (4.25)

The calculated stress should satisfy the stress inequality per equation (4.26).

\[
F'_{p} = \frac{F_p}{\Omega} > f'_{p,c}
\] (4.26)

Where:

\( F'_{p} \) = Allowable stress in the panel.

\( \Omega \) = Factor of safety. Typically, 1.67 for crushing failure [39].

\( F_p \) = Compressive yield stress of panel. Compressive yield stress of plywood is approximately 775 psi [40].
4.2.3. Design of Hinge Pin

The final step in the design process is to design the hinge pin for shear loading. The maximum shear force applied to the hinge pin, \( V_p \), can be calculated using equation (4.27) and the maximum shear stress in the hinge pin, \( f'_{v,p} \), can be calculated using equation (4.28):

\[
V_p = \frac{P_{h,2}}{N_K}
\]  
(4.27)

\[
f'_{v,p} = \frac{4V_p}{3A_p}
\]  
(4.28)

Where:

\( A_p \) = Area of the hinge pin.

The calculated shear stress should satisfy the stress inequality per equation (4.29).

\[
F'_{v,p} = \frac{F_{v,p}}{\Omega} > f'_{v,p}
\]  
(4.29)

Where:

\( F'_{v,p} \) = Allowable shear stress in the hinge pin.

\( F_{v,p} \) = Maximum shear strength of filament. Shear strength of PLA+ 3D printed parts is approximately 4750 psi [41].

\( \Omega \) = Factor of safety. Typically, 2.0 for rupture failure [39].
4.3. Design of Fasteners

This design procedure omits a detailed methodology for the design of the fasteners because it is explained in detail in Section 12 of the *NDS* [42] and Veerman [43]. In general, the connection design should conform to the following three guidelines:

1. A minimum of one fastener should be aligned with the center of each knuckle to ensure a direct load transfer from the knuckle to the fastener.

2. The required number of fasteners should be determined per Section 12 of the *NDS* [42]. The lateral capacity of the fasteners can be determined per Table 12.3.1A of the *NDS* [42] or for screws into plywood a shear capacity can be taken per Table 1 of the *Fastener Loads for Plywood - Screws* [44]. The total applied shear load can be taken as the greater of $P_{h,1}$ and $P_{h,2}$.

3. The spacing and edge requirements for the plywood panels and plastic hinge should conform to section 12.5 of the *NDS* [42] and Veerman [43], respectively. Generally, all holes in plastic parts should be spaced at twice the thickness of the hinge leaf or the diameter of the screw hole, whichever is largest.
Finite element (FE) simulations were conducted using Abaqus software to further analyze the hinge design. A total of three FE simulations were created corresponding to the three load cases of interest (i.e., Load Case 1a, 1b, and 2). See Figure 4.2 for clarification on loading direction for each load case. In addition, a convergence study was performed for Load Case 1a to validate the mesh density and the same mesh was used for the remaining load cases.

5.1. Model Development

In ABAQUS, the PLA+ and e4D-1 parts were defined as linear isotropic solids with an effective elastic modulus of 3.16E+05 and 3.70E+05 psi, respectively, based on the experimental measurements presented in Chapter 3. Although 3D printed parts have orthotropic properties, the linear elastic behavior is similar in all directions as shown in Figure 3.4 and Figure 3.5. Therefore, the parts were modeled as isotropic to reduce computation time and simplify the analysis. The Poisson's ratio for both filaments was defined as 0.33 per the material characterization of 3D printed PLA parts performed by Li et al. [45]

The plywood panels were defined as a linear orthotropic solid based on the material properties for Scots pine plywood using the strain-energy method shown in Table 2 of Gerrard [46].

To simulate a simply supported span the exterior bottom edge of one panel was constrained from displacement in the X, Y, and Z direction \((U_1 = U_2 = U_3 = 0)\) and the exterior bottom edge on the opposite panel was constrained from displacement in the Y and Z direction \((U_2 = U_3 = 0)\). A line load was applied at 4.75 inches from either support...
across the entire width of the panel, as shown in Figure 4.8. The total span from one support to the other is 16 inches.

Figure 5.1. Boundary conditions and loading for Load Case 1.

The contact interaction between the plywood panels was modeled as node-to-surface contact. This formulation was used because, at sharp corners, it typically provides more accurate results [47]. To account for small imperfections in the sawn face of the panel, a 0.05-inch chamfer was added to the edges of the panel.

All other contact interactions within the simulation were set as surface-to-surface with the tangential behavior modeled as frictionless and the normal behavior modeled as hard contact. The hard contact formulation doesn’t allow penetration of the slave surface into the master surface and doesn’t allow the transfer of tensile stress across the master and slave surfaces [48].
Tie constraints were set between the faces of the bolt holes of the hinge leaf and the rigid panels, so no displacement can occur between the panel and hinge leaf at the bolt/screw locations.

For Load Case 1b, the snap-fit hooks were modeled with a curvature of .01965 in\(^{-1}\) per Table 3.2 to account for the imperfection in the shape-memory molding process.

### 5.2. Mesh Development and Mesh Convergence

All parts were meshed as hexahedral elements with full integration enabled and with a linear geometric order (C3D8: 8-node linear brick). Hexahedral elements were used because they tend to provide more accurate stress predictions for contact simulations [49]. A linear geometric order was used to limit the required computation time and reduced integration was disabled to increase the accuracy of the stiffness formulation. Zero-stiffness elements can be created in linear reduced-integration elements when deformed under an applied moment however ABAQUS addresses this by assigning a small amount of artificial “hourglass stiffness” [50]. The artificial “hourglass stiffness” can give acceptable results if a reasonably fine mesh is used [50] however, reduced integration was disabled so a slightly coarser mesh could be used. An image of the complete meshed assembly is shown in Figure 5.2.

The hinge leaf, snap fit hooks, and hinge pin were meshed with an approximate element size of 0.075 in., 0.065 in., and 0.05 in., respectively. The panels were meshed with an approximate global element size of 0.35 in. and a local element size of 0.1 in. at the connection to the hinge leaf and panel to panel interactions. Figure 5.3. shows the FE mesh for the hinge leaf and snap-fit hook.
Figure 5.2. Complete assembly of meshed parts.

Figure 5.3. Zoomed in image of meshed hinge leaf and snap-fit hook.
A convergence study was performed based on Load Case 1a. The hinge assembly was loaded with an applied moment, $M_a$, of 30 lb-in and the maximum displacement was measured at the center of the panel. The simulation was performed on six meshing densities and Figure 5.4 shows the relationship between the number of elements in the hinge assembly and the maximum displacement. Convergence occurs with approximately 90,000 elements. The final mesh used for all simulations has 108,000 elements in the total assembly. Of the total number of elements, 0.966% are distorted. Abaqus defines distorted elements as elements where the angle between isoperimetric lines is less than 45 degrees or greater than 135 degrees.

![Figure 5.4. Convergence relationship between number of elements in hinge assembly and maximum displacement for Load Case 1a ($M_a = 30 \text{ lb-in}$).](image)

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CHAPTER 6. MECHANICAL TESTING

For each load case of interest (i.e., 1a, 1b, and 2) two four-point bending experiments were performed based on the procedures described in ASTM D3043-17 [5]. Method B (four-point loading) was used because it is commonly used to determine the effects of features which can be placed entirely between the load points such as finger joints, veneer joints and gaps [5]. A schematic drawing of the test setup is shown in Figure 4.6. For all tests, \( L \) was 16 in. and \( a \) was 4.75 in.

\[ M_a = \frac{1}{2} Pa \]  

(6.1)

Where:

\( M_a \) = Applied bending moment.

Figure 6.1. Schematic drawing of four-point bending load test.
\( P \) = Total applied load including the weight of the rollers and plate.

\( a \) = distance from support to adjacent load.

\( L \) = span.

A GeoJac digital load actuator (GeoTac) equipped with a 1,000 lb. S-type load cell was used to apply the load to the specimens. The load actuator can provide 1.5 in. of linear motion. The load was applied directly to a 0.5 in. steel plate that distributed the load to two 0.5 in. diameter steel rollers. 1 in. diameter steel roller supports were placed at either side of the specimen and bearing plates were omitted because the magnitude of loading was not large enough to create localized crushing of the plywood at the supports or load points. An image of the test setup is shown in Figure 6.2.
To measure the displacement at the center of the panel a linear variable differential transformer (LVDT) displacement sensor was used. The sensor can measure a maximum of 6 in. of linear displacement. The tip of the LVDT sensor was placed at a measured offset from the center of the panel. The displacement was not measured directly at the center of the panel because the load frame for the GeoJac made it inaccessible. The displacement at the center of the panel was approximated assuming that the plywood panels are perfectly rigid and remain linear during loading. Another adjustment was made to account for the displacement caused by the gravity load applied from the weight of the steel roller and plate. The adjustment was performed by using a best fit linear curve created based on the first 0.1 in. of measured displacement for each specimen.

All test specimens were loaded at a constant rate of 0.1 in./min. with a sampling rate of 0.01 sec. ASTM D3043-17 [5] states that the load shall be applied within a range of 25% of the rate calculated per equation (6.2). For specimens where the hooks did not become disconnected after approximately 1.25 in. of deflection, the panels were unloaded at the same rate.

\[
N = \left(\frac{za}{3d}\right)(3L - 4a) \quad \text{(6.2)}
\]

Where:

\( N \) = Rate of motion.

\( z \) = unit rate of fiber strain (0.0015 in/in-min. is recommended)

\( d \) = depth of beam
The plywood panels are 0.5 in. thick grade BC 5-ply pine plywood. The hinges are secured to the panels with three #6 machine screws with flat plate washers. Two of the test specimens are shown in Figure 6.3.

**Figure 6.3.** Hinge-connected panel load test specimens.
CHAPTER 7. RESULTS AND DISCUSSION

The flexural behavior of the hinge design is presented and discussed in this chapter. Section 7.1 gives the results of the mechanical load tests. Section 7.2 compares the moment-displacement curves from the FE simulation and mechanical load tests. It also includes a discussion and a comparison of failure modes and stress concentrations between the FE models and mechanical load tests. Finally, Section 7.3 compares the stress analysis proposed in Chapter 4 is to the results of the FE simulation.

7.1. Mechanical Testing Results of Hinge-Connected Panels

Moment-displacement curves for Load Case 1a and 2 are shown in Figure 7.1 and Figure 4.8. The specimens were subjected to one loading and unloading cycle to observe the degree of non-linear deformation. Note that in Load Case 1b, the snap-fit hooks became disconnected for both specimens (see Figure 7.5) and therefore did not undergo unloading. A summary of the load test results is shown in Table 3.2. The hinges were designed to resist an applied moment of approximately 72 lb-in (see design analysis in Appendix B) and for each load case the maximum moment exceeded the design moment.

<table>
<thead>
<tr>
<th>Load Case</th>
<th>Mean Max. Moment (lb-in)</th>
<th>Mean Displacement at Max. Moment (in)</th>
<th>Mean Inelastic Deformation (in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1a</td>
<td>176.33</td>
<td>1.64</td>
<td>0.59</td>
</tr>
<tr>
<td>1b</td>
<td>95.45</td>
<td>0.96</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>156.00</td>
<td>1.47</td>
<td>0.61</td>
</tr>
</tbody>
</table>

Table 7.1. Summary of flexural load test results.
Figure 7.1. Non-linear moment-deflection curve for Load Case 1a (PLA+ hinge).

Figure 7.2. Non-linear moment-deflection curve for Load Case 2 (PLA+ hinge).
Several hinge specimens were fabricated on a different 3D printer than the one described in Chapter 3, but the load results are not included in this chapter due to defects in the part. The maximum applied moment was half of that listed in Table 3.2 and premature delamination of print layers was observed in most specimens as shown in Figure 7.3. The printer was under extruding filament and therefore there were gaps and voids in the part. This highlights the high potential for quality control (QC) issues in FDM 3D printing. Computed tomography scanning can be used as a QC method to compare the printed part to the 3D model [51]. These defects could also be avoided by performing calibration test on each 3D printer and adjusting the print settings accordingly. The same print settings were used across both printers and therefore small differences in the extrusion rate of the printers could have caused dimensional inaccuracies and mechanical differences between the parts.

Figure 7.3. Delamination of print layers during flexural loading.

The hinges could also be fabricated through another manufacturing process such as injection molding, which could help to decrease defects and QC issues however may also
increase costs [52]. Additionally, new advances in metal 3D printing could allow for the part to be printed from stainless steel which would drastically increase the stiffness of the design [53]. In the case of a metal-based design, a SMA would be used in lieu of an SMP to achieve shape recovery properties.

### 7.2. Finite Element Simulation Results

Moment-deflection curves for all the load test and FE simulations are shown in Figure 7.4, Figure 7.5, and Figure 7.6. The ‘x’ mark at the end of the curve denotes a failure, where the snap-fit hooks become disconnected. For all load cases the FE simulation demonstrated excellent agreement with the load test results in the linear-elastic range.

![Moment-deflection curve for Load Case 1a (PLA+ hinge specimen)](image)

**Figure 7.4.** Moment-deflection curve for Load Case 1a (PLA+ hinge specimen)
Figure 7.5. Moment-deflection curve for Load Case 1b (PLA+ hinge with e4D-1 snap-fit hooks).

Figure 7.6. Moment-deflection curve for Load Case 2 (PLA+ hinge specimen)
For load case 1, it was assumed that a failure to converge to a solution indicated that the snap-fit hooks became disconnected. This assumption is reasonable as the load step prior to the convergence failure showed minimal contact between the snap-fit hooks as shown in Figure 7.7. A dynamic simulation could be used instead of a static simulation to accurately model the detachment of the snap-fit hooks.

**Figure 7.7.** Zoomed-in contour of deformation (in.) in snap-fit hooks prior to failure for Load Case 1a ($M_a = 129$ lb-in).

For Load Case 1a, Figure 7.4 shows the snap-fit hooks unhooking in the FE simulation at an applied moment of 129 lb-in however the experimental specimens never unhooked. The experimental specimens instead showed yielding, flexural cracking, and delamination in small portions of the hinge leaf as shown below in Figure 7.8. The simulation did show high stress concentration at this area in the model, as shown in Figure 7.9 and Figure
7.10, but failed to capture the yielding and delamination because non-linear material properties were not used.

Figure 7.8. Failure of hinge leaf specimens in Load Case 1a. a) Yielding and flexural cracking, b) delamination.
Figure 7.9. Contour of maximum principal stress (psi) of hinge assembly under Load Case 1a ($M_a = 129$ lb-in)

Figure 7.10. A zoomed-in view of the contour of maximum principal stress (psi) at high stress corners under Load Case 1a ($M_a = 129$ lb-in)
The FE simulation of Load Case 1b demonstrated a good agreement with the mechanical testing as shown in Figure 7.5. The mean moment at failure for the experimental specimens was 84.03 lb.-in. and the FE simulation calculated the moment at failure as 87.12 lb.-in which is a percent difference of only 3.7%. No visible yielding of the parts was observed during or after testing however the experimental test exhibited an additional 0.35 in. of displacement at unhooking compared to the FE results. It should be noted that the moment capacity was much less for Load Case 1b compared to 1a and this is likely due to the small curvature in the e4D-1 hooks after molding.

Figure 7.11. Contour of maximum principal stress (psi) under Load Case 2 with hinge assembly hidden ($M_a = 166 \text{ lb-in}$).
Load Case 2 exhibited the largest degree of uncertainty between the specimens and the simulations. This is likely because the flexural load is resisted primarily by the interaction of the two rigid panels compressing together. The compression between the panels can be seen in Figure 7.11. The simulation assumed the two panels are perfectly flat with a small .05 in. chamfer at the edges however the sawcut edges of the plywood panels will have small imperfections and inaccuracies. These small inaccuracies can result in large differences in the gross area of the plywood that is in contact during loading which resultantly has large effects on the flexural stiffness of the system. The plywood panels were also modeled with linear orthotropic material properties, so the non-linear response observed in the test specimens at an applied moment of approximately 110 lb.-in. was not captured by the simulation as seen in Figure 7.6.

7.3. Stress Prediction Comparison

Using the analysis procedure proposed in Chapter 4, the maximum stresses were calculated in certain regions of the hinge design and compared to the FE simulation. The maximum stress in the hinge knuckles and snap-fit hooks for Load Case 1a are shown in Figure 7.12 and Figure 7.13 based on both calculation method.

The hand calculation method failed to predict the non-linear stress-moment relationship in the snap-fit hooks. This occurred because the assumption was made in the analysis procedure that the resultant force, $P_{h,1}$, acts at the center of the undercut as shown in Figure 4.5 b. As the hooks deflect the location of the resultant force shifts closer to the edge of the hook which increases the bending moment, $M_{s,2}$. Despite this assumption the hand calculation method still gives a good approximation for the stresses in the hinge knuckle and snap-fit hooks for load case 1a.
Figure 7.12. Maximum stress in hinge knuckle for Load Case 1a.

Figure 7.13. Maximum stress in snap-fit hook for Load Case 1a.
The maximum stress in the hinge knuckles and rigid panels for Load Case 2 are shown in Figure 7.14 and Figure 7.15. The maximum principal stress in the panels was probed in the FE simulation at the panel edge, where the panel-to-panel contact interaction occurred. Like Load Case 1, the hand calculation method gives a good approximation of the stresses in both cases. The standard deviation and maximum percent difference for the four stress predictions are shown in Table 7.2.

![Figure 7.14. Maximum stress in hinge knuckle for Load Case 2.](image)
Table 7.2. Percent difference of stress predictions between hand calculation method and FE simulation.

<table>
<thead>
<tr>
<th>Load Case</th>
<th>Region</th>
<th>Maximum Percent Difference in Predicted Stress</th>
</tr>
</thead>
<tbody>
<tr>
<td>1a</td>
<td>Knuckle</td>
<td>11.57%</td>
</tr>
<tr>
<td></td>
<td>Snap-fit Hook</td>
<td>30.11%</td>
</tr>
<tr>
<td>2</td>
<td>Knuckle</td>
<td>1.82%</td>
</tr>
<tr>
<td></td>
<td>Panel</td>
<td>10.52%</td>
</tr>
</tbody>
</table>
CHAPTER 8. CONCLUSIONS AND FUTURE RESEARCH

8.1. Conclusions

Origami engineering is a growing area of study with countless applications including in the design of rigid deployable structures. These structures can create temporary housing for refugees and victims of natural disasters. For deployable structures to be successful, improvements must be made to the stiffness of the hinge mechanisms. The novel design proposed in this thesis paper uses mechanical hinges and snap-fit hooks to create a stiff locked state in deployable structures. This mechanism can be reversed with two techniques: 1) the snap fit hooks can simply be removed from the hinge leaf; and 2) if the hooks are inaccessible, they can be made using shape memory polymers/alloys and heated to recover a remembered shape.

The experimental testing of 3D printed prototypes demonstrated that the design could resist out-of-plane flexural loading typical for small scale residential structures. The smallest moment capacity occurred in the e4D-1 snap-fit hooks (Load Case 1b) where the mean maximum moment between the two specimens was 95.45 lb.-in which still exceeded the design value of 72 lb.-in. for the applied moment. The largest moment capacity occurred in the Load Case 1a where the mean maximum moment was 176.33 lb.-in.

FE simulations conducted using ABAQUS software showed agreement with the load test results especially within the linear elastic range of loading. The FE highlighted key stress concentrations within the design where improvement should be made. Additionally, the stress predictions from the closed form analysis method generally agreed with the FE simulation results.
8.2. Future Research

8.2.1. Design Improvements
To facilitate the future application of this design, future research can focus on exploring methods to limit the deflection of the hinge under loading. These methods could potentially include applying stiffer materials, utilizing more reliable manufacturing techniques, and implementing supports throughout the parts. If PLA parts are implemented, methods to protect the part from hydrolysis and UV exposure should be considered.

8.2.2. Modeling
To improve the results of the FE simulation non-linear orthotropic material properties could be used instead of linear isotropic properties for the 3D printed parts. Non-linear orthotropic material properties would likely capture the failure behavior of the hinge specimens. However, the layered nature of FDM 3D printing would still not be captured and therefore delamination of print layers would still not be simulated correctly. Non-linear material properties could also be used in the plywood panels to capture the non-linear crushing of plywood that occurred in Load Case 2.

8.2.3. Shear Design and Performance
To utilize the hinge design in shear walls and diaphragms the analysis procedure should be expanded to include shear loading. FE simulations could be used to validate the analysis procedure.

8.2.4. Shape Memory Implementation
To utilize shape memory material in the hinge design research must be performed to evaluate methods of heating the snap-fit hooks in efficient way. The heating element should be designed to limit heat spreading to rest of the hinge assembly as the stiffness of PLA is highly temperature dependent.
REFERENCES


[34] Copinet, A., Bertrand, C., Govindin, S., Coma, V., & Couturier, Y. (2004). Effects of ultraviolet light (315 nm), temperature and relative humidity on the degradation of


Appendix A. Design Drawings

Figure A.1. Total dimensions of hinge assembly
Figure A.2. Hinge leaf dimension, top view.
Figure A.3. Hinge leaf dimension, bottom view.
Figure A.4. Hinge leaf dimension, side view.

Figure A.5. Snap-fit hook, 3D view.
Figure A.6. Snap-fit hook, top view.
Figure A.7. Hinge pin, side view.
Appendix B. Analysis Example

B.1. Loading Criteria

For the design of the hinge assembly the loading criteria shown in Table 3.2 was considered. The dead load was approximated based on the weight of plywood, insulation, timber studs and rafters, and lightweight elastomer waterproofing. The roof live load is the minimum load for canopy roofs per ASCE 7-16.[37] The maximum wind pressure was calculated per ASCE 7-16 [37] assuming a ten-foot building height, risk category II, basic wind speed of 92 mph, and exposure category B.

<table>
<thead>
<tr>
<th>Load Type</th>
<th>Load (psf)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dead (D)</td>
<td>10</td>
</tr>
<tr>
<td>Live Roof (Lr)</td>
<td>10</td>
</tr>
<tr>
<td>Wind (W)</td>
<td>12</td>
</tr>
</tbody>
</table>

Load combination (3) per Chapter 2 of ASCE 7-16 [37] gives the maximum loading of 20 psf. The applied moment ($M_a$) was calculated assuming a stud and rafter spacing of 16 in. on center and a hinge spacing of 16 in. on center along fold lines. The maximum moment at the center of the hinge was calculated as shown below in equation (3.1) assuming a simply supported span.

\[ M_a = \frac{\omega W_T l^2}{8} \]  

\[ M_a = \frac{(20 \text{ psf})(1 \text{ ft}^2/144 \text{ in}^2)(16 \text{ in})(16 \text{ in})^2}{8} = 72 \text{ lb in} \]
Where:

\( \omega \) = Maximum distributed load (psi)

\( W_T \) = Tributary width/spacing of hinges (in.)

\( L \) = Span/spacing of supports (in.)

**B.2. Design of Snap-Fit Hooks**

Combing equations (4.1) and (4.2) the total resultant force across all hooks, was calculated as shown below in B.2 See Appendix A for reference dimensions.

\[
P_{h,1} = \frac{M_a}{2b_s/3 + \rho_h + t_k} \tag{B.2}
\]

\[
P_{h,1} = \frac{72 \text{ lb in.}}{2(0.575 \text{ in.}) + 0.15 \text{ in} + 0.225 \text{ in}} = 94 \text{ lb}
\]

The bending stress \( f_{b,s,1} \) and \( f_{b,s,2} \) were calculated per equation (4.7) and (4.10). The total uncorrected bending stress was calculated per equation (4.13):

\[
f_{b,s,1} = \frac{P_{h,1}}{N_s t_s b_s} \tag{4.7}
\]

\[
f_{b,s,1} = \frac{94 \text{ lb in}}{4(0.26 \text{ in})(0.575 \text{ in})} = 157 \text{ psi}
\]

\[
f_{b,s,2} = \frac{3P_{h,1}}{N_s b_s t_s^2 (y_s + t_s)} \tag{4.10}
\]
The stress concentration factor was determined to be approximately 1.62 per Figure 2.9.

The corrected stress in the snap was calculated in equation (4.13):

\[
K_{c}(f_{s,total}) = f_{s,total} + f_{b,s,2} + \frac{P_{h,1}}{N_{s}b_{s}t_{s}}
\]

\[
f_{s,total} = f_{b,s,1} + f_{b,s,2} + \frac{94 \text{ lb}}{4(0.575 \text{ in})(0.26 \text{ in})} = 1616 \text{ psi}
\]

\[
f_{s,total} = \frac{3(94 \text{ lb in})}{4(0.575 \text{ in})(0.26 \text{ in})^2}(0.2 \text{ in} + 0.26 \text{ in}) = 832 \text{ psi}
\]

\[
f_{s,total} = f_{b,s,1} + f_{b,s,2} + f_{t,s}
\]

\[
\text{Figure B.1. Stress concentration in snap-fit hook.}
\]

\[
f'_{s,total} = K_{c}(f_{s,total})
\]

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\[ f'_{s, total} = 1.62(1616 \text{ psi}) = 2619 \text{ psi} \]

The corrected stress was then checked to satisfy the stress inequality per equation (4.14). The tensile strength was taken as the lesser of the PLA+ and e4D-1 filament per Table 3.1.

\[ F'_s = \frac{F_s}{\Omega} > f'_{s, \text{total}} \quad (4.14) \]

\[ 4450 \text{ psi} / 1.67 > 2619 \text{ psi} \]

\[ 2664 \text{ psi} > 2619 \text{ psi} \quad O.K. \]

Additionally, the length of the hook, \( l \), should satisfy equation (4.15), modified from Bayer [30]:

\[ l \geq \sqrt{\frac{0.746t_s y_s}{\varepsilon}} \quad (4.15) \]

\[ 1.4 \text{ in} \geq \sqrt{\frac{0.746(0.26 \text{ in})(0.2 \text{ in})}{.02}} \]

\[ 1.4 \text{ in} \geq 1.39 \text{ in} \quad O.K. \]
B.3. Design Hinge of Knuckle

Substituting equation (4.17) into (4.16) the equivalent point load of force couple for Load Case 2 was calculated per equation (B.4):

\[ P_{h,2} = \frac{M_a}{2t_p/3 + r_h + t_h} \]  

(B.4)

\[ P_{h,2} = \frac{72 \text{ lb in}}{\frac{2(0.5 \text{ in})}{3} + 0.15 \text{ in} + 0.15 \text{ in}} = 113 \text{ lb} \]

The bending stress in the knuckle was then calculated per equation (4.21):

\[ f_{k,total} = \frac{6P_{h,2} \left( r_h + \frac{t_k}{2} \right)}{N_k b_k t_k^2} + \frac{P_{h,2}}{N_k t_k b_k} \]  

(4.21)

\[ f_{k,total} = \frac{6(113 \text{ lb})(0.15 + 0.225 \text{ in}/2)}{3(0.875 \text{ in})(0.225)^2} + \frac{113 \text{ lb}}{3(0.225)(0.875 \text{ in})} = 1529 \text{ psi} \]

The stress concentration factor was determined to be approximately 1.7 per Figure A.1.

The corrected stress in the snap was calculated in equation (B.5) and the corrected stress was then checked to satisfy the stress inequality per equation (4.23):
Figure B.2. Stress concentration in hinge knuckle.

\[ f'_{k,total} = K_c (f_{k,total}) \]  \hspace{1cm} (B.5)

\[ f'_{k,total} = 1.7(1529 \text{ psi}) = 2600 \text{ psi} \]

\[ F'_k = \frac{F_k}{\Omega} > f'_{k,total} \]  \hspace{1cm} (4.23)

\[ \frac{4450 \text{ psi}}{1.67} > 2600 \text{ psi} \]

\[ 2664 \text{ psi} > 2600 \text{ psi} \quad O.K. \]
B.4. Design of Rigid Panel

The maximum compressive stress in the panel was calculated per equation (4.25) Note that total tributary width of panel-to-panel contact was that used in the mechanical testing described in Chapter 6.

\[
f'_{p,c} = \frac{3P_{h,2}}{2t_p b_p}
\]

(4.25)

\[
f'_{p,c} = \frac{3(113 \text{ lb})}{2(0.5 \text{ in})(1.75 \text{ in})} = 387 \text{ psi}
\]

The maximum compressive stress in the panel was then checked to satisfy the stress inequality per equation (4.26). Recall, the compressive yield stress of plywood is approximately 775 psi [40].

\[
F'_{p} = \frac{F_p}{\Omega} > f'_{p,c}
\]

(4.26)

\[
\frac{775 \text{ psi}}{1.67} > 387 \text{ psi}
\]

\[
464 \text{ psi} > 387 \text{ psi} \quad \text{O.K.}
\]
B.5. Design of Hinge Pin

Substituting equation (4.27) into (4.28), the maximum shear stress in the hinge pin was calculated per equation (B.6):

\[
f'_{v,p} = \frac{4 \left( \frac{P_{h,2}}{N_K} \right)}{3A_p}
\]

\[
f'_{v,p} = \frac{4(\frac{113 \text{ lb}}{3})}{3(\pi)(0.15 \text{ in})^2} = 710 \text{ psi}
\]

The maximum shear stress in the hinge pin was then checked against the allowable stress per equation (4.29). Recall the shear strength of PLA filament is 4750 psi [41] and the factor of safety is 2.0 for rupture failure [39].

\[
F'_{v,p} = \frac{F_{v,p}}{\Omega} > f'_{v,p}
\]

\[
\frac{4750 \text{ psi}}{2} > 710 \text{ psi}
\]

\[
2375 \text{ psi} > 710 \text{ psi} \quad 0.K.
\]

B.6. Design of Fasteners

Per Table 1 of Fastener Loads for Plywood - Screws [44] the shear strength of one #6 screw into 0.5 in. plywood is 415 lb. As shown in the design drawings, one screw was aligned with each hinge knuckle (a total of 3 screws). The total screw capacity was calculated per equation (B.8) modified from Table 11.3.1 of the NDS [42].

\[
Z' = nZ C_D
\]
\[ Z' = (3)(415 \text{ lb})(1.25)(1)(1) = 1556 \text{ lb} \]

Where:

\[ Z' \] = Total shear capacity of fasteners (lb.)

\[ n \] = number of fasteners

\[ Z \] = Unfactored shear capacity of each fastener (lb.)

\[ C_D \] = Load duration factor per section 2.3.1 of the NDS [42]. The load duration factor is 1.25 for transient load combinations including live loads.

The total demand on the connection, \( z \), is taken as the greater of \( P_{h,1} \) and \( P_{h,2} \). The screws were checked against the design demand given per equation (B.8):

\[ Z' > z \] (B.8)

\[ 1556 \text{ lb} > 113 \text{ lb} \quad O.K. \]

The spacing of the fasteners was checked to also satisfy the following minimum requirements:

1. For plastic parts, the distance from the edge of a hole to the edge of the part should be the greater of twice the thickness of the diameter of the hole per Veerman [43]. In this case, twice the thickness is 0.3 in., and the minimum edge distance is 0.44 in.

2. The edge distance in plywood per section 12.5.1.3 of the NDS [42] shall be 1.5 times the diameter of the fastener. #6 screws have a diameter of 0.13 in., so the minimum edge distance is 0.26 in., and the actual edge distance is 0.375 in.