DISTANCE ESTIMATION USING OFDM SIGNALS FOR ULTRASONIC POSITIONING

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ABSTRACT

Distance Estimation Using OFDM Signals for Ultrasonic Positioning

Kyman Huang

This paper describes a method of estimating distance via Time-of-Flight (TOF) measurement using ultrasonic Orthogonal Frequency Division Multiplexing (OFDM) signals. Using OFDM signals allows the signals and their sub-carriers to remain orthogonal to each other while continuously transmitting. This estimation method is based on the change of phase of a traveling wave as it propagates through a medium (air for ultrasonic signals). By using signals containing multiple tones, the phase change between each frequency component is slightly different. This phase difference is dependent on the distance traveled and can thus be used to estimate distance. This paper studies the impact of tone (OFDM sub-carriers) separation on accuracy, maximum distance, and computation for two-tone and three-tone systems. The effects of the transducer channel bandwidth and channel noise are accounted for to build an accurate model for a single-transmitter single-receiver system. This study found that each additional tone provides one extra independent distance measurement which improves accuracy in the presence of noise. The inclusion of an additional tone while maintaining the same overall signal strength shows improved performance with a reduction in standard deviation of estimated distance from 5.64 mm to 3.42 mm in simulation. A four-tone system is also examined to show that this effect holds for additional tones.

Keywords: Ultrasound, Time-of-Flight, Multi-tone Signals, Indoor Positioning, OFDM
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Chapter 1

INTRODUCTION

Many different indoor positioning systems have been explored due to the increasing use of indoor positioning data for indoor navigation and robotic applications [1] [2] [3]. Due to the limitations of GPS signal strengths for indoor environments, many systems using different technologies have been explored. Generally, all local indoor positioning systems are based on measuring distances from fixed anchors [4]. There are two components for these indoor positioning systems. One is a set of fixed anchors where distance measurements can be made relative to their known positions. The other is a positioning device that can receive and process a signal to determine its distance from the fixed anchors.

Due to its widespread usage and available infrastructure, Wi-Fi is a prominent method for indoor positioning [5]. Wi-Fi based indoor positioning system’s estimate distance by either measuring the intensity of a received signal or by measuring the time of flight (TOF). These systems require minimal infrastructure but suffers for location inaccuracies in the 1m – 4m accuracy range [6]. Ultra-Wideband (UWB) systems are more expensive radio systems that use very large bandwidth signals, in the hundreds of MHz, and can measure location with accuracies in the centimeter range [3].

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<th>Accuracy</th>
<th>Cost</th>
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<td>WiFi/Bluetooth</td>
<td>1m – 10m</td>
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Ultrasound systems offer a promising alternative to these systems with many advantageous features. Ultrasound signals propagate almost one million times slower than the RF signals used in other localization systems. The significance of this is covered in a future section. Additionally, they do not significantly penetrate walls and have a low cost of hardware [1]. Ultrasonic positioning systems (UPS) use signal TOF
to estimate the distance between a transmitter and receiver node. Depending on the distance estimation method, accuracies can reach the single digit centimeter level or lower [2] [8]. A review on the different methods of distance estimation is provided later.
Chapter 2

BACKGROUND

2.1 Ultrasound Signals

Ultrasound signals are sound waves with frequencies above the audible range, typically 20 kHz to several hundred kHz [1]. Ultrasound has several attractive properties for localization applications. Since it is a sound wave, ultrasound propagates at the speed of sound at approximately 343 m/s. For a more accurate speed estimation, a system can use a temperature monitor and calculate the propagation speed via equation (1), where $T$ is temperature in Celsius [9].

$$v_{\text{sound}} = 20.05\sqrt{T + 273.15} \text{ m/s}$$  \hspace{1cm} (1)

The speed of propagation is vitally important because it defines the distance resolution that a system can measure. As an example, suppose there are two localization systems (one RF, one ultrasound) that both operate at 10 MHz. From this frequency, the minimum time that can be measured is one period of the sampling frequency, or 100 ns. This minimum time interval determines the best-case accuracy for a system. Since RF signals (e.g. Wi-Fi) propagate at the speed of light, the distance resolution of such the RF-based system is:

$$d_{\text{min}} = c \cdot T_{\text{min}} = 2.998 \cdot 10^8 \text{ m/s} \cdot 100\text{ns} = 30\text{m}$$  \hspace{1cm} (2)

In comparison, ultrasound propagates at the speed of sound and thus the ultrasound system has theoretical resolution of:

$$d_{\text{min}} = v_{\text{sound}} \cdot T_{\text{min}} = 343 \text{ m/s} \cdot 100\text{ns} = 34\mu\text{m}$$  \hspace{1cm} (3)

In this example, the accuracy of the Wi-Fi localization system is limited by the sampling speed and the signal propagation speed, which is not true for the ultrasound system [10].

The other major advantage that ultrasound has is cost of equipment. Narrowband ultrasonic transducers like the one used in this study, the 40TR12B [11], can be purchased at less than $10 a set [12]. Generally, at least three transmitters are required to create a localization system. Multiple distances are estimated to pinpoint a location based on a lateration algorithm [13]. A brief overview of the topic is covered
in section 2.4 Multilateration. To determine a location, multiple accurate distance measurements must be performed. All UPSs face problems measuring distances while multiple transmitters are simultaneously transmitting due to signal interference.

2.2 Multiple Access and Orthogonality

To prevent interference between multiple ultrasound signals, the UPS must be operated in a way that ensures orthogonality between the signals. Generally, orthogonality means that for two functions, \( x \) and \( y \), the inner product must be equal to zero [14].

\[
\langle x, y \rangle = 0
\]  

(4)

Since localization is based on measuring time shifts between a transmitted and received signal, equation (4) must hold true for all possible combinations of time shifts. Satisfying this condition means that the two signals are uncorrelated and will not interfere with each other. This allows for multiple access of the ultrasonic channel, where a system can operate multiple transmitters within a limited bandwidth. There are three main methods used to ensure orthogonality between signals for localization systems.

2.2.1 Time Division Multiple Access

A very commonly used method to ensure orthogonality is Time Division Multiple Access (TMDA). TDMA is a channel access method where at any single point in time, only one transmitter node is transmitting [15]. Thus, this channel access technique separates the different signals in the time domain (Figure 1). After each transmission, there is a small additional guard time to synchronize the network [16]. Additionally, the guard time ensures that the previous transmitted signal has been received. This is necessary for ultrasonic systems because the previous signal can still be propagating through the medium and interfere with the signal of the next transmitter. This requirement arises from the slow propagation speed of the ultrasound and the possibility that one transmitter is much closer to the user than other transmitters. An example of a TDMA system is 2G mobile cellular systems.
By utilizing a TDMA scheme, a system can operate within a very narrow bandwidth because all transmitters can share the same carrier frequency. This channel access scheme has one main flaw, the amount of time it takes to update a location measurement [2]. Since TDMA is based on separation of signals in time, the transmitters cannot transmit simultaneously. Thus, the total amount of time required for a location measurement is dependent on how many separate distance measurements are needed. If a UPS using TDMA requires $N$ distance estimations for localization and each transmitter transmits for $T_{TRANSMIT}$, then the minimum time required to acquire a location is $N(T_{TRANSMIT} + T_{PROPAGATION})$.

2.2.2 Code Division Multiple Access

The second method commonly used for UPS is Code Division Multiple Access (CDMA). This channel access method achieves orthogonality by assigning each node a ‘code’ that is mutually orthogonal from all the other codes in the system. Figure 2 shows an example of an orthogonal Walsh-Hadamard set. However, this set does not satisfy the condition of orthogonality in the presence of a time shift. This leads to a large cross-correlation, indicating that this set of orthogonal codes is not suitable for CDMA localization usage.
Instead, typical CDMA systems use codes such as Gold codes and Kasami Codes which are carefully constructed to have close to zero cross-correlation [17] [18]. By assigning each user a specific code, the signals can transmit without interfering with each other [15]. If a system needs to also transmit information, a signal can be encoded using a node’s unique digital code and transmitted. The GPS system encodes information about each satellite using this method [19]. If distance measuring is the only criteria, the codes by themselves can be transmitted [15]. Since the individual signals are independent, each signal can be extracted properly even when transmitted together. One problem that can occur with CDMA is called the near-far problem [14]. The near-far problem occurs when a receiver has trouble detecting a weaker signal due to presence of a stronger signal. If two signals are transmitted at the same power but one node is much further away than the other, that signal will be much weaker than the other at the receiver. Thus, the signal-to-noise ratio (SNR) of one signal is much larger and can “drown” out the other.

2.2.3 Frequency Division Multiple Access

As the name suggest, Frequency Division Multiple Access (FDMA) is a channel access method that divides the frequency spectrum. This method splits the channel bandwidth into multiple non-overlapping
frequency bands and assigns each node a sub-band [15]. Thus, all transmitter nodes can transmit continuously like CDMA.

Figure 3 - Example Frequency Response of a System Using FDMA

Since all the nodes operate in non-overlapping frequency bands, there is no interference between the signals irrespective of any time shift. The main disadvantage of this method is the presence of guard bands, the frequency gaps that are introduced between the carrier frequency that ensures the sub-bands do not overlap. Since it is not possible to create infinitely sharp band-pass filters, the guard bands are required to properly separate the individual signals in the frequency domain. The inclusion of guard bands restricts how many transmitter nodes the system can have, especially if the communication channel is narrowband, as is the case of UPS that uses narrowband transducers.

2.3 Distance Finding

There are three main ways to measure the time shift between signals to obtain a distance estimation. The three methods are: cross-correlation, phase shift and intensity. Most of these methods require synchronization of transducers and receiver to compare the sent and received signals at the same moment in time [4]. This synchronization for a UPS can be accomplished easily by using electromagnetic waves. Electromagnetic waves travel at the speed of light and thus the time required for signal propagation is almost six orders of magnitude faster than ultrasound. Some possibilities for synchronization include RF signals
(Bluetooth), optical (blinking lights) and Infrared (IR) illumination. This requirement is due to the properties of a traveling wave. A single-tone ultrasound wave from a point source moving in the positive x direction can be described by equation (5), where \( \lambda \) is the wavelength in meters, \( d \) is the distance traveled in meters, \( f \) is the frequency of the signal in hertz and \( t \) is the time in seconds.

\[
y(d, t) = \sin\left(\frac{2\pi d}{\lambda} - 2\pi ft\right)
\]

Additionally, the amplitude of the traveling wave follows a simple inverse square relationship based on the intensity of sound as it travels through space without any reflections [20], where \( d \) is the distance the wave travels.

\[
|y(d, t)| \propto \frac{1}{d^2}
\]

The phase of a sinusoidal traveling wave is a function of both time and distance as seen in equation (5). By synchronizing the transmitter and receiver in time, the phase shift of the wave becomes only a function of distance. In Figure 4, the received and transmitted signals are synchronized in time. Both the phase shift and the amplitude reduction of the received signal seen in the example is due to distance (5)(6). Both cross-correlation and phase shift can be used to measure the phase difference between the signal at the transmitter \((d = 0)\) and the receiver to determine the distance the wave has traveled.

![Figure 4 - Effects of Distance on Phase Shift and Amplitude for Time Synchronized Signals](image-url)
2.3.1 Cross-correlation

Cross-correlation is a measure of the similarity between two signals by comparing one signal to the time-lagged versions of the other signal. Equation (7) defines the cross-correlation function [21], where ‘*’ is the notation for convolution.

$$r_{xy}(t) = \int_{-\infty}^{\infty} x(\lambda)y(\lambda - t) \, d\lambda = x(t) * y(-t)$$

(7)

The peak of the cross-correlation function represents the time shift where the two functions most closely align. For a UPS, this represents the time delay that occurs as the wave travels from the transmitter to the receiver.

![Cross-correlation example](image)

**Figure 5 - Cross-correlation to Estimate Time Delay**

Figure 5 shows a simple example of the cross-correlation between a square wave and a time-delayed version of that square wave. The received signal has a time lag of 2 s compared to the transmitted signal. The computed cross-correlation function has a peak also at −2 s, identifying the time shift between the two signals. Once the time delay $t_{\text{delay}}$ is found, the distance can be then be estimated based on equation (8) where $v_{\text{sound}}$ is the speed of sound in meters.
\[ d = v_{\text{sound}} \cdot t_{\text{delay}} \] (8)

To accurately determine the peak of the cross-correlation for bandpass signals, it is useful to extract the envelope of the signal using the Hilbert transform (Figure 6). Extracting the envelope of a signal allows for a more precise measurement of the cross-correlation peak by ignoring the higher frequency component of the cross-correlation function. Figure 6 shows the cross-correlation and envelope extraction of a signal containing two closely spaced frequencies. One major drawback to this method is the computation time. To obtain the time delay, the whole data set must be processed. Even when using a fast FFT-based cross-correlation algorithm, the computation resources are significant for applications that use large amount of data points.

Figure 6 - Envelope Extraction of Cross-correlation Function using Hilbert Transform

2.3.2 Phase Shift in Sinusoidal Signals

Another way to measure the time delay of a signal as it travels is to use the phase information of the transmitted and received signals. As seen in equation (5), the phase of a traveling wave is a function of both time and distance. By synchronizing the transmitter and receiver at some moment in time \( t \), the phase of the
received signal referenced to the transmitted signal becomes a function of distance \( d \) as shown in equation (9) where \( \lambda \) is the wavelength in meters.

\[
\phi(d) = 2\pi \frac{d}{\lambda}
\] (9)

It is important to emphasize that this is the relationship between the phase of the transmitted and received signal, as any phase measurement needs a reference to obtain useful information. Figure 7 shows an example of determining distance via phase shift. In this example, the system is using a single tone (signal with single frequency component) to find the distance. The phase of both the transmitted and received signal is found by performing a Fast Fourier Transform (FFT). The phase shift between the received and transmitted signal relates to the time delay between the two signals.

![Figure 7 - Example of Phase Measurement to Calculate Time Delay](image)

In this example, there is a phase shift of \( \pi/2 \) from the transmitted signal to the received signal. A calculation of the distance using the phase shift of this example is shown below.

\[
\phi_{TX} = 0, \quad \phi_{RX} = \frac{\pi}{2}
\] (10)

\[
\phi_D = \phi_{RX} - \phi_{TX} = \frac{\pi}{2}
\] (11)
\[ d = \frac{\phi_D \lambda}{2\pi} = \frac{\lambda}{4m} \]  

This linear relationship holds true if the phase difference does not exceed a full periodic rotation. Once the phase exceeds \(2\pi\), it is no longer possible to uniquely determine distance. This is a result ambiguity when distinguishing between a particular phase measurement \(\phi\) and a rotated version of that phase \(\phi + 2\pi n\).

For a signal with a single tone, this effectively limits the maximum distance that can be measured to \(v_{\text{sound}}T\), where \(T\) is the period of the signal in seconds. For an UPS using an ultrasound frequency of 40 kHz, the maximum distance that can be measured with a single tone is less than 1 cm. Once the distance exceeds 1 cm, the result is ambiguous. A solution to this problem involves the use of multi-tone signals, this topic is discussed in the section 3.3 Multi-Tone Signals.

As with cross-correlation, this phase shift method also requires significant computation resources for long data sequences. Obtaining phase information requires the use of an FFT which resource intensive. Another problem associated with using an FFT for phase measurements is spectral leakage. Spectral leakage for a periodic signal occurs when an FFT is applied on a finite-length signal that does not contain an integer number of periods. To prevent leakage, enough data should be collected to contain an integer number of the period. For a further discussion on this topic, see 3.4.1 Windowing and Spectral Leakage. As with most digital signal processing, the results are more accurate the more data is collected.

2.3.3 Intensity

Unlike the other two methods described above, using the magnitude of the ultrasonic signal cannot produce an accurate distance estimation. The magnitude of the voltage waveform at the ultrasonic transducer is dependent on the intensity of the ultrasonic wave. As described with equation (6), the magnitude follows an inverse square law relationship in the absence of reflections and multi-path effects. However, the intensity of the transmitted ultrasound wave can only be estimated based on the frequency response of the transducers as described by a datasheet [11]. The RMS voltage at the output can be used to calculate the sound intensity at the receiver based on the receiver’s sensitivity. Since the initial starting point of the measurement is an estimation, this method can at best only be used to get an estimation of the relative distance of one transmitter.
compared to another. An example of a system that uses signal strength based localization is a Received Signal Strength Indicator (RSSI) Wi-Fi ranging system [22].

2.4 Multilateration

Multilateration is the technique of determining a user’s location through distance measurements. If the UPS is only required to determine location in two dimensions, then at least three TOF measurements are required [3]. For applications that require all three dimensions, at least four measurements are required. This method uses circles with radii equal to the distance measurements. Each distance measurement reduces the possible positions of the user. In the simple case of two dimensions, one ToF measurement confines the solution space to a circle. With two ToF measurements, the possible solutions are reduced to the two points where the two circles intersect. Finally, three ToF measurements limits the solution to the singular point where all three circles intersect.

![Figure 8 - Example of Localization Based on ToF Measurements [3]](image)

With three measurements, a system of equations can be constructed that represents the coordinates of the fixed nodes \((a, b \ldots)\) and the receiver node \((n)\). Each equation represents a circle from a transmitter node with radius, \(r\), to the receiver.

\[
(n_x - a_x)^2 + (n_y - a_y)^2 - a_r^2 = 0 \tag{13}
\]

\[
(n_x - b_x)^2 + (n_y - b_y)^2 - b_r^2 = 0 \tag{14}
\]
\[(n_x - a_x)^2 + (n_y - c_y)^2 = 0 \quad (15)\]

Since there are three equations and only two unknowns, a general unique solution cannot be found. However, by introducing non-zero terms to the equations, a least-square solution can be found [13]. This solution minimizes the sum of introduced terms.

\[
(n_x - a_x)^2 + (n_y - a_y)^2 - a_x^2 = a_\Delta^2 \quad (16)
\]

\[
(n_x - b_x)^2 + (n_y - b_y)^2 - b_x^2 = b_\Delta^2 \quad (17)
\]

\[
(n_x - a_x)^2 + (n_y - c_y)^2 - c_x^2 = c_\Delta^2 \quad (18)
\]
3.1 System Overview

The proposed system takes advantage of the properties of OFDM signals, allowing for continuous and fast distance estimation. Additionally, the spectral efficiency of using an OFDMA system allows for scalability even when using narrowband ultrasonic transducers with $f_{BW} < 1 \text{kHz}$. To allow for measurements of acceptable distances (in the meter range), multi-tone OFDM signals are required. The effects of how the number of tones affect the distance measurement is explored later. After constructing and transmitting the OFDM signals, the receiver output is compared to the input signal. This can be done by using a separate electromagnetic based system to provide time synchronization.

A distance estimation is performed once the output signal is synchronized to the input. This system uses phase shift detection to determine a distance. The main advantage to using phase information is lower computation time. While an FFT is required to obtain the phase information of the entire spectrum, it is possible to dramatically reduce computation by only computing the DFT information for specific frequencies. This is ideal for this system, which uses pre-defined OFDM sub-carrier frequencies. Once the phase information is obtained for both the input and output, the distance can be quickly calculated from the difference in phase. That phase difference is directly proportional to the distance the wave has traveled.

A block diagram for a single-transmitter single-receiver version of the proposed system is shown in Figure 9. The ‘Time Synchronization’ line represents a synchronization method using electromagnetic waves (RF or IR). Extending this system to a full localization system simply requires the addition of other transmitters operating with a different set of sub-carrier frequencies. The processing steps are the similar with the only difference being additional DFT computations for the new frequencies and the use of a lateration algorithm.
Figure 9 - Proposed System Diagram (Time Synchronization Done Using IR or RF-based Method)

3.2 OFDM

OFDM is a transmission method where the available frequency spectrum is split into different narrow-band sub-carries [2]. These sub-carriers allow for continuous transmission from multiple sources. OFDMA is a type of channel multiple access where each node is assigned a specific subset of sub-carriers. The main advantage of using OFDM signals is spectral efficiency. In an OFDMA system, the sub-carriers are spaced as close as possible while still maintaining orthogonality. Orthogonality is maintained because the sub-carriers are equally spaced in a well-defined frequency axis. This leads to an FFT with ideally zero spectral leakage. Due to its spectral efficiency, OFDMA is advantageous in a UPS by allowing for operation of many transmitters. To reduce the effects of non-linear distortion in the receiver circuit, some sub-carriers can be left unused to space out the transmitters in the frequency domain. A further investigation into distortion is provided in section 5.3 Non-linear Distortion and OFDMA.

To create signals that are all orthogonal to each other, each sub-carrier should be a sinusoid that has an integer number of periods within the signal length [2]. Spacing them as close together as possible means that number of cycles between each sub-carrier for the signal length is exactly one (shown in Figure 10). In other words, for some period T the frequency spacing between individual tones must be 1/T. For example, if the time interval is 10 ms then the minimum frequency spacing is 1/10 ms, or 100 Hz. In this example with a 10 ms sampling duration, a sub-carrier with a frequency of 40 kHz would have an adjacent neighbor
sub-carrier at 40.1 kHz. When all sub-carriers have an integer number of cycles within some period of time, then the peak magnitude of the different sub-carriers occur at the nulls of the other sub-carriers [23].

In summary, OFDMA is a type of FDMA scheme that does not use guard-bands to separate each sub-carrier but still achieves orthogonality. Therefore, this method retains the advantageous of FDMA while also being spectrally efficient. Using OFDMA is especially useful for a system utilizing multi-tone signaling since it defines the minimum signal length where the sub-carriers are orthogonal.

3.3 Multi-Tone Signals

Multi-tone signals are signals with two or more frequency components. The necessity of using multi-tone signals stems from the need to measure larger distances. When measuring the phase shift between signals, the maximum distance that can be calculated occurs when the difference in phase exceeds $2\pi$. For a single tone signal the phase makes a complete rotation every period $T$. For a typical frequency in the ultrasonic range $f_{\text{sound}} > 40 \text{ kHz}$, this results in an impractical maximum distance in the millimeter range.

This problem can be solved by using multi-tone signals. In a simple two-tone signal, the signal becomes a function of two parts: a high frequency ‘carrier’ and a low-frequency envelope. As seen below, the frequency of the signal envelope is $(f_1 - f_2)/2$.

$$x(t) = \cos(2\pi f_1 t) + \cos(2\pi f_2 t)$$  \hspace{1cm} (19)
\[ x(t) = 2\cos((f_1 + f_2)\pi t)\cos((f_1 - f_2)\pi t) \] (20)

This result holds true for any sum of sinusoids if the frequency spacing between the tones are the same. The reason for this is based around the fundamental frequency of a sum of sinusoids, which can be found using the greatest common divisor, \( \text{GCD}(f_1, f_2 \ldots) \). This calculation produces the same result no matter the number of frequencies if the frequency spacing is the same. Figure 11 shows an example comparing a two and three tone signal with equal spacing. The overall signal strength (RMS) is the set to the same value to create a fair comparison. For a system using equal strength tones with \( N \) tones, the RMS value of the individual tones can be calculated using equation (22).

\[
V_{\text{RMS,signal}} = \sqrt{V_{\text{RMS,f_1}}^2 + V_{\text{RMS,f_2}}^2 + \ldots} = \sqrt{N \cdot V_{\text{RMS,tone}}^2} 
\] (21)

\[
V_{\text{RMS,tone}} = \frac{V_{\text{RMS,signal}}^2}{N} 
\] (22)

Figure 11 - Example of Two-tone and Three-tone Signal (Same Signal Strength)

Since the phase of a single tone signal is directly dependant of distance traveled, a similar result should hold for a signal with two or more tones. Given a signal with two frequency components, \( f_1, f_2 \), the
distance that the signal travels is derived starting with equation (23), where $\phi_o$ and $\phi_i$ are the time synchronized phase measurements of the received and transmitted signals respectively.

$$\phi_1 = \phi_{o,f_1} - \phi_{i,f_1} = \frac{2\pi d}{\lambda_1}$$  \quad (23)

$$\phi_2 = \phi_{o,f_2} - \phi_{i,f_2} = \frac{2\pi d}{\lambda_2}$$  \quad (24)

$$\Delta \phi = \phi_2 - \phi_1 = 2\pi d \left( \frac{1}{\lambda_2} - \frac{1}{\lambda_1} \right)$$  \quad (25)

By substituting $\lambda = \frac{v_{\text{sound}}}{f}$

$$\Delta \phi = \frac{2\pi d}{v_{\text{sound}}} (f_2 - f_1)$$  \quad (26)

$$d = \frac{\Delta \phi v_{\text{sound}}}{2\pi \Delta f}$$  \quad (27)

This equation shows that when using two tones, it is possible to estimate a distance based on the difference of phase change between the two sub-carrier frequencies. Unlike the case with a single tone where the phase wraps around every period of the tone, the two-tone phase difference wraps around every period of the envelope. This vastly increases the measurable distance when compared to a single-tone system. The maximum distance that can be determined occurs when $\Delta \phi$ exceeds $2\pi$, from equation (27) the corresponding distance is:

$$d_{\text{max}} = \frac{v_{\text{sound}}}{\Delta f}$$  \quad (28)

For a typical room area, say 10m x 10m, a suitable frequency spacing is 20 Hz. Using this frequency spacing, the distance that can be measured before it becomes ambiguous is $d = 17.2$ m. This is frequency spacing that is used in the preceding examples. Figure 12 shows the linear relationship between the two variables (phase and distance) using a tone separation of 20 Hz.
It is possible to uniquely determine distances in the tens of meter range with two tones. Since two tones already solve the distance problem, using three might not seem useful at first. It is more useful to think of a three-tone measurement as two separate two-tone measurements. In an ideal situation, both two-tone and three-tone measurements would result in the same distance. However, due to the noise in the environment, the different measurements will produce slightly different results. By averaging the two measurements, it is possible to reduce the effect of noise on the estimated distance. A more detailed exploration into this topic is presented in a later section 5.2 \textit{Noise and System Performance}.

The use of multi-tone signals fits easily into the concept of OFDM signals. As shown in the OFDM section, the frequency spacing between two signals should be an integer multiple to $1/T$ where $T$ is the total length of the signal. Achieving this ensures that the two signals will be orthogonal. For an example with a frequency spacing of 20 Hz, the minimum time length should be 0.05s. Any integer multiple of 0.05s ensures orthogonality between the two signals. For a two-tone signal with 20 Hz spacing, the sample length should be 0.1 s to capture a full period of the signal envelope. Thus, no changes are needed to create an OFDM signal with two tones.
3.4 Phase Measurements

3.4.1 Windowing and Spectral Leakage

As stated in the system overview, this system relies on phase information to estimate distance. To make proper phase measurements the signal must be collected within a proper window length. The window length requirement of this system is based on the OFDM signal and reducing spectral leakage. When performing a DFT, spectral leakage is a result of capturing a non-integer amount of signal cycles of a periodic signal. If this happens, the FFT will show the power of the true signal frequency spilling over into neighboring frequencies. This is a result of the frequency spacing of an FFT which is \( f_s / L \), where \( f_s \) is the sampling frequency and \( L \) is the window length in terms of samples. If the actual frequency of the signal is not a multiple of this index then the frequency cannot be properly represented by the FFT, resulting in spectral leakage.

There are different methods to reduce leakage such as applying different windowing functions to the sampled signal. However, since the tones used in the proposed UPS are spaced very closed together any window other than a rectangular window will ruin the phase measurements. Thus, the only way to properly capture the signal is to acquire enough data to obtain an integer number of cycles. Figure 13 shows a visualization of the difference between capturing an integer number of cycles and a non-integer number. Since calculation time is important for a UPS, the system should capture exactly and only one full period of the multi-tone signal. The update speed of the method presented in this paper is limited by this requirement.

![Figure 13 - Proper and Improper Windowing of Periodic Signals](image)

21
3.4.2 DFT and Geortzel Algorithm

To minimize the computation time required to obtain relevant phase information, it is possible to only compute the relevant DFT terms. For review, the DFT can be computed via:

\[ X(k) = \sum_{n=0}^{N-1} x(n) e^{-\frac{j2\pi kn}{N}} \]  \hspace{1cm} (27)

Since the signals of interest are multi-tone signals with known frequencies, one can calculate the DFT information about those specific frequencies without performing the entire sequence of summations. The relevant frequency indices can be found with:

\[ k = \frac{f_0 L}{f_s} \]  \hspace{1cm} (28)

where the frequency of interest is \( f_0 \) in Hz, the sampling frequency \( f_s \) in Hz and the window length \( L \) in number of samples. This drastically reduces computation time by only computing a few sums (based on number of tones used) instead of hundreds of thousands.

A similar method that can be used is the Goertzel algorithm [21][24]. The Goertzel algorithm takes advantage of the periodicity of the phase factors \( W_N^k = e^{\frac{j2\pi nk}{N}} \) in the DFT. The Goertzel algorithm can be used to compute individual DFT terms by implementing the DFT as a digital filter with a transfer function as shown in equation (29) [24].

\[ H(z) = \frac{1 - W_N^k z^{-1}}{1 - 2\cos(2\pi k/N)z^{-1} + z^{-2}} \]  \hspace{1cm} (29)

Figure 14 shows a Direct Form II realization of this transfer function which can be implemented on a digital signal processor. By providing the relevant frequency terms, the index \( k \) for those terms can be found and the DFT bins can be computed efficiently.
3.5 Example Distance Measurement

This section provides a simulation example of how a system using these methods will process the signal and determine a distance. For simplicity, a two-tone signal is used. Figure 15 shows a possible example of a two-tone signal will look like. There are two things to note in this image. First, the magnitude of the received signal will be very small at any significant distance. Thus, a low noise amplification circuit will be required to achieve the best performance. Second, exactly one full cycle of the envelope signal is collected to provide as fast an update rate as possible. The noise is the system is simulated by adding a white noise signal on top of the original signal. The SNR of the total signal for this example is 10 dB and the distance is set to 4m. This example distance falls into the range of distances that will produce a unique result.

Figure 15 - (a) Time Domain Waveform of Input and Output, (b) Phase of Input and Output Signal
The next step in determining a distance is to use the Goertzel algorithm or the DFT equation to calculate the phase of the frequencies of interest. In this case, the frequencies used are $f = 40.30 \ kHz$, $40.32 \ kHz$. Figure 15 displays the computed phase of the two signals. The change of phase can be determined by subtracting the output phase from the input phase.

$$\phi_1 = -0.4263 + 0.3877 = -0.0386 \ rad, \ \phi_2 = 0.7195 - 2.334 = -1.6145 \ rad$$  \hspace{1cm} (30)

$$\Delta \phi = -1.6145 + 0.0386 = -1.576 \ rad$$  \hspace{1cm} (31)

It is important to pay attention to the two-phase changes, $\phi_1$ and $\phi_2$. From equation (21)(22), the largest phase change should occur at the higher frequency. In this example, this is true and so one can directly subtract the two. However, if $\phi_2$ positive that would mean that the phase at that frequency has completely wrapped around the unit circle. In that situation, the system should take the larger angle $2\pi - |\Delta \phi|$ instead. The distance can now be directly calculated and the ‘error distance’ introduced by the channel (in this case 308 mm) can be subtracted to provide the final distance estimation.

$$d = \frac{\Delta \phi v_{sound}}{2\pi \Delta f} - d_{channel} = \frac{1.576 \cdot 343}{2\pi \cdot 20} - 0.308 = 3.994 \ m$$  \hspace{1cm} (32)
Chapter 4

CHANNEL MODEL

In an ideal situation where the ultrasound signal is transmitted perfectly, the phase relationship between the input and output is dependant only on the distance between the transmitter and receiver. However, the ultrasonic channel introduces a phase shift that must be accounted for to create an accurate distance estimation. The following sections detail the frequency response of the channel, the creation of a channel model and the effect the channel has on distance measurements.

4.1 Ultrasonic Channel Frequency Response

The ultrasound channel between the transmitter and receiver depends heavily on the transducers used. In this paper, the ultrasonic transducers used are the 40TR12B set. Based on the datasheet plots of the SPL of the transmitter and the sensitivity of the receiver, a bandpass type frequency response is expected.

![Figure 16 - (a) SPL Plot of 40TR12B Transmitter, (b) Sensitivity Plot of 40TR12B Receiver [12]](image)

The frequency response of the ultrasonic channel can be found by transmitting a signal with a flat frequency spectrum and using an FFT to find the frequency response. In this situation, a signal consisting of 100 equally spaced tones from 39 kHz – 41 kHz was used as the input signal. To ensure a proper measurement, the transmitter and receiver should be spaced apart (at least 1 ft) to prevent any electromagnetic coupling between the two. The transmitted and received signal was collected simultaneously on a Keysight MSOX2022 with a 10 MHz sampling frequency and a 100 ms sample length.
Figure 17 shows the resulting frequency response of the channel. There is a clear bandpass nature in the frequency response. As expected, the center frequency occurs at $40.3 \, kHz$ with a $3 \, dB$ bandwidth of approximately $450 \, Hz$. From the phase plot, it is seen that the phase is linear near the center frequency. It is important to note that this phase response includes both the phase response of the ultrasonic channel and the linear phase shift due to the distance between transmitter and receiver.

4.2 2\textsuperscript{nd} Order Channel Model

A simple 2\textsuperscript{nd} order band-pass model can be created using the information from the experimentally determined frequency response. The canonical form of a 2\textsuperscript{nd} order bandpass filter takes the form of:

$$H(s) = \frac{H_0 \omega_0}{Q} \frac{s}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2}$$

(33)

$Q$ represents the quality factor of the filter, or the ratio of the center frequency to the bandwidth. $\omega_0$ represents the center frequency of the filter. From the experimental results, a center frequency of $40.3 \, kHz$ and a bandwidth of $450 \, Hz$ is seen, providing a quality factor, $Q = 88$. The resulting 2\textsuperscript{nd} order filter has the transfer function shown in Figure (34).
\[ H_{2BP} = \frac{3057.9s}{s^2 + 3057.9s + 10068} \]  

(34)

Figure 18 - Comparison of 2nd Order Bandpass Model with Experimental Frequency Response

Comparing the 2nd order model with the experimental results show close magnitude matching inside the bandwidth which steadily deviates away at frequencies outside the bandwidth. The experimental results clearly show a steeper drop-off than can be achieved through a 2nd order filter. To properly analyze the matching of the model’s phase response, the phase response of the experimental results should be subtracted from the model. Figure 19(a) displays both phase responses of the experimental result, the 2nd order model, and the difference between them. The expected result of the difference in phase responses is a straight line, indicating a linear phase shift and a constant time shift. To analyze the difference between the phase results, it is useful to obtain time delay due to the difference in phase response (shown in Figure 19(b)). This is done by taking the derivative of the difference in phase responses. A constant time shift should result in a linear phase response. The data shows that the slope is relatively flat around the center frequency but slowly rises at frequencies outside the bandwidth. This is an indication that a 2nd order model is not sufficient to create an accurate channel model. It is important to create a valid model for the ultrasonic channel to properly estimate the distance ‘error’ that is introduced into the measurement because of the channel phase response.
Figure 19 - (a) Phase Comparison of 2nd Order Model, (b) Time Delay Found by Slope of Phase Response Difference

4.3 4th Order Channel Model

The 2nd order bandpass model cannot achieve a steep enough attenuation to match the experimentally found frequency response. This result is physically intuitive because the lowest order bandpass is 2nd order and the ultrasonic channel can be thought of as having two bandpass transducers. Thus, the channel should thus be at minimum a 4th order system. A 4th order bandpass model can be created by simply cascading two 2nd order filters. Using the previous 2nd order canonical form, the expected transfer function of a 4th order bandpass filter is:

$$H(s) = H_1(s) \cdot H_2(s) = \frac{H_0 \frac{\omega_1}{Q_1} s}{s^2 + \frac{\omega_1}{Q_1} s + \omega_1^2} \cdot \frac{\frac{\omega_2}{Q_2} s}{s^2 + \frac{\omega_2}{Q_2} s + \omega_2^2}$$

(35)

It is not easily apparent what the four parameters ($\omega_1, \omega_2, Q_1, Q_2$) should be to match the experimental frequency response. The four parameters were solved using a brute-force algorithm that minimizes the mean-square error between the experimental results and the magnitude response of the 4th order model. The equation for the magnitude of the model was calculated by multiplying the magnitude of the two individual 2nd order filters. The magnitude response of a standard 2nd order filter can be derived by:

From (29) substitute $k = \frac{\omega_0}{Q}, H_0 = 1$
\[ H(s) = \frac{ks}{s^2 + ks + \omega_0^2} \]  

(36)

\[ |H(s)| = \frac{jk \omega}{\omega^2 + jk \omega + \omega_0^2} \]  

(37)

\[ |H(s)| = \frac{k \omega}{\sqrt{(-\omega^2 + \omega_0^2)^2 + k^2 \omega^2}} \]  

(38)

The parameters that result in the best fitting frequency response is shown in Table 2. Figure 20 compares the resulting 4th order model with the experimental results. The magnitude response of the 4th order model shows good agreement with the experimental results. This shows that a cascade of two 2nd order models can accurately represent the magnitude response of the ultrasonic channel.

Table 2 - 4th Order Bandpass Model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>( H_1(s) )</th>
<th>( H_2(s) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Q )</td>
<td>55.94</td>
<td>57.95</td>
</tr>
<tr>
<td>( \omega_0 )</td>
<td>253.18 krad/s</td>
<td>253.14 krad/s</td>
</tr>
<tr>
<td>( f_0 )</td>
<td>40.295 kHz</td>
<td>40.290 kHz</td>
</tr>
</tbody>
</table>

Figure 20 - Comparison of 4th Order Bandpass Model with Experimental Frequency Response
Figure 21 - (a) Phase Comparison of 4th Order Model, (b) Time Delay found with Slope of Phase Response Difference

By performing a similar analysis to the 2nd order bandpass model, the difference between the phase response of the new 4th order model and the experimental results produce a very linear plot. Looking at the time delay, (Figure 21(b)) shows a more constant result than that seen in the 2nd order model (Figure 19(b)). This shows that the 4th order model can be used to model the response of the channel. The goal of modeling the ultrasonic channel is to find the scalar distance ‘error’ that will be seen by the ultrasound signal as it passes through the channel. The channel introduces a phase shift that adds an additional distance factor. To find that distance, the derivative of the phase response of the 4th order model is found. This derivative can be converted to a time delay by dividing it by $2\pi$. The distance can be determined through equation (39) where $v_{sound}$ is the speed of sound and $t_{delay}$ is the time delay.

$$d = v_{sound} \cdot t_{delay}$$  \hspace{1cm} (39)

Figure 22 shows that the time delay is most flat near the center frequency at 40.3 kHz with a value of approximately $t_{delay} = 0.9 ms$. The distance associated with that time delay near that frequency is:

$$d = v_{sound} \cdot t_{delay} = 343 \cdot 0.0009 = 308 mm$$  \hspace{1cm} (40)
This distance factor can be used to correct any measurements using these ultrasonic transducers. However, there is a question about whether this distance should be added or subtracted from the measurement. The behavior of the filter near the center frequency is a relatively flat line with a negative time factor as seen in Figure 22. A negative linear slope indicates a time delay in the system. This time delay adds on top of the delay that occurs due to the wave traveling through the channel. Thus, the additional distance factor due to the channel must be subtracted from the real measurement to account. Naturally, the distance factor will be different when using different frequencies. Operating near the center frequency allows for the largest frequency band where the time delay is constant. If a system wants to operate outside that region, the error is deterministic and can be calibrated out.

4.4 H(s) to H(z) Mapping

It is necessary to convert the analog transfer function of the channel to a digital representation to create a simulation system. There are three main methods used to convert a continuous transfer function to a digital one. The methods described in this section are used to find an analytical conversion from a continuous system to a discrete system. There are also other numerical methods that can be used that try to minimize the error between the frequency responses. The simulation model in this paper uses a first order hold approach.
to convert the channel model to a digital filter. A first order hold performs well for smooth inputs and there was no dramatic difference between the various methods for this situation.

4.4.1 Response Matching

This method creates a digital representation by exactly matching the output of an analog system with a specific input. For example, a zero-order hold method will exactly match the step response of the continuous and discrete representations. This method is done taking the Z transform of the sampled input and output. The transfer function is found by dividing the $Y(z)$ by $X(z)$. This type of conversion is useful for dealing with known inputs since it guarantees that that output of the digital representation will match the analog one for that specific input. For example, a first order hold (ramp input) conversion will perform very well for smooth inputs.

4.4.2 Impulse Invariance

The impulse invariance method tries to match the impulse response of the continuous system. This is done by sampling the continuous-time impulse response to convert it to a digital unit sample response [21]. The digital system transfer function can then be found using the Z transform.

Given a impulse response $h(t)$

$$h[n] = h(nT_s)$$

$$H(z) = Z(h[n])$$

4.4.3 Bilinear Transform

The Bilinear Transform maps the $s$ plane to the $z$ plane, converting all points on the left-hand plane of the $j\omega$ axis of the $s$ plane to locations inside the unit circle of the $z$ plane [21].
One problem using this method is that the frequency mapping is not linear, as seen in equation (44). This means that while the shape of the frequency response will be well matched, the frequencies of interest will be shifted.

\[
s = \frac{1}{T_s} \ln(z) \approx \frac{2}{T_s} \frac{1 - z^{-1}}{1 + z^{-1}} \quad (43)
\]

\[
s = j2\pi f_a \approx \frac{2}{T_s} \frac{1 - z^{-1}}{1 + z^{-1}} = \frac{2j\tan(\pi f_a)}{T_s}
\]

\[
f_a = \frac{1}{\pi} \tan^{-1} \left( \frac{\pi f_a}{f_s} \right) \quad (44)
\]

It is possible to solve this problem by pre-warping, or pre-shifting, the cutoff frequency of the analog filter before using the bilinear transform. Equation (45) shows the combined bilinear transform with frequency pre-warping, where \( f_p \) is the intended analog frequency where matching is required [25]. The bilinear transform typically results in excellent frequency response matching between the analog system and the digital representation.

\[
s = \frac{2\pi f_p}{\tan \left( \frac{\pi f_p}{f_s} \right)} \frac{1 - z^{-1}}{1 + z^{-1}} \quad (45)
\]
5.1 System Model

A system model of the signal as it travels through space is created to gauge the possible performance of using the methods described in this paper. Figure 23 shows an example of such a model using a two-tone signal. Two individual signals are summed together and then time shifted by a set value determined by the distance between the transmitter and the receiver. The time shifted signal is then passed into a model of the ultrasonic channel. The original analog 4th order model is converted to a digital representation using a first order hold method. Finally, the filtered signal is summed with white noise to simulate noise in the environment. The location where the noise is added will change how much the noise is filtered. Adding it during the last step results in the worst-case performance of the measurement system since there is no filtering of the noise from the bandpass. This model is also consistent with reception of low strength signals where the noise of the receiver pre-amplifier dominates the overall noise. This model is used to analyze the performance of the measurement system.

5.2 Noise and System Performance

The performance of the system as well as the performance impacts of using different amounts of tones is described in this section. To analyze how the number of tones effect system performance, three systems with different amounts of tones were simulated for a total of 1000 runs each to look at the statistical parameters of the system. The time shift experienced by the signal corresponds to 1 meter. This distance was chosen to fall within the region of distance measurements where a unique solution can be found. Namely, for
a 20 Hz spacing the maximum distance that can be uniquely determined is 17.2 meters (see equation 28).

These three systems all operate at a 1 MHz sampling frequency. Table 3 shows the standard deviation of these trials with a white noise level set at 10 dB SNR.

Table 3 - Standard Deviations of Distance Measurements

<table>
<thead>
<tr>
<th>Distance Estimation</th>
<th>Two Tone (mm)</th>
<th>Three Tone (mm)</th>
<th>Four Tone (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_{12}$</td>
<td>5.64</td>
<td>7.07</td>
<td>8.26</td>
</tr>
<tr>
<td>$d_{23}$</td>
<td>N/A</td>
<td>6.74</td>
<td>7.81</td>
</tr>
<tr>
<td>$d_{34}$</td>
<td>N/A</td>
<td>N/A</td>
<td>7.17</td>
</tr>
<tr>
<td>After Averaging</td>
<td>5.64</td>
<td>3.42</td>
<td>2.59</td>
</tr>
</tbody>
</table>

The table shows the standard deviations of all three measurements ($d_{12}, d_{23}, d_{34}$) and the standard deviation of the final distance calculation after averaging the distance measurements. Since each distance measurement requires the use of two tones (e.g. $d_{12}$ uses the tones $f_1, f_2$), a three-tone system results in two distinct measurements while a four-tone system can create three measurements. Technically, there is an additional measurement that can be used. For example, if there are three tones then three distances can be found ($d_{12}, d_{23}, d_{13}$). However, as seen in Figure 24, those additional distances do not provide any statistically relevant information.

![Figure 24 - (a) Correlation Between $d_{13}$ and $(d_{12}+d_{23})/2$, (b) Correlation Between $d_{14}$ and $(d_{12}+d_{23}+d_{34})/3$](image_url)
If each measurement is treated as an uncorrelated random variable, then the standard deviation of the sum of each measurement should follow equation (41) [14].

\[
\sigma_{x+y} = \sqrt{\sigma_x^2 + \sigma_y^2}
\]  

(41)

Table 4 - Comparison of Standard Deviations

<table>
<thead>
<tr>
<th>Method</th>
<th>Two Tone (mm)</th>
<th>Three Tone (mm)</th>
<th>Four Tone (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simulation</td>
<td>5.64</td>
<td>3.42</td>
<td>2.59</td>
</tr>
<tr>
<td>Equation (41)</td>
<td>5.64</td>
<td>3.99</td>
<td>3.26</td>
</tr>
</tbody>
</table>

Table 4 shows a comparison of the calculated values using equation (41) and the actual one produced by the simulation. The differences between them can be attributed to two things. First, the individual measurements are not truly uncorrelated. In the three-tone system example, three tones are used to generate two distance estimations. However, since the middle tone plays a part in both measurements, there is some correlation between \(d_{12}\) and \(d_{23}\).

![Histogram and Scatter Between \(d_{12}\) and \(d_{23}\)](image_a)

(a)

![Histogram and Scatter Between \(d_{12}\) and \(d_{34}\)](image_b)

(b)

Figure 25 - (a) Histogram and Scatter Between \(d_{12}\) and \(d_{23}\), (b) Histogram and Scatter Between \(d_{12}\) and \(d_{34}\)
Figure 25(a) shows a scatter plot between the two distance measurements, which displays some correlation between them. The correlation coefficient for those two distance measurements is $-0.475$. Since the measurements share a tone and are somewhat correlated, it should be expected that measurements that share no tones will be uncorrelated. This is exactly what is found when comparing distance measurements where each distance measurement is done using different tones (shown in Figure 25(b)).

Another thing that can be seen in Table 3 is that the standard deviations of the individual measurements slowly spreads the more tones that are used. The most probable explanation is that while the overall strength of the signal stays the same, each added tone reduces the strength of each individual tone. This leads to a reduction in performance of the individual measurements. However, using more tones and averaging the results does tighten the spread between measurements (shown in Figure 26).

![Figure 26 - Comparison of Measurement Error After 1000 Runs](image)

5.3 Non-linear Distortion and OFDMA

Intermodulation distortion occurs when a system is represented by a non-linear function. Generally, this is modeled as a polynomial of as shown in equation (42).
\[ y_{out} = \alpha_0 + \alpha_1 x + \alpha_2 x^2 + \alpha_3 x^3 + \cdots \] (42)

Third-order intermodulation (\(IM_3\)) occurs when these non-linearities allow for two out-of-band signals to generate in-band frequency components [26]. In a UPS, this is of concern because any non-linearities in the receiver circuit can create intermodulation products from the signal of one transmitter that causes distortion in another transmitter’s frequency band. To analyze this effect on an OFDMA system, the receiver can be modeled using only the third order term from equation (42).

\[ v_{out} = \alpha_1 v_{in} + \alpha_3 v_{in}^3 \] (43)

Since multiplication in the time domain equates to convolution in the frequency domain, equation (43) can be represented in the frequency domain by equation (44).

\[ V_{OUT} = \alpha_1 V_{IN} + \alpha_3 V_{IN} \ast V_{IN} \ast V_{IN} \] (44)

This convolution in the frequency domain will cause signal power to spill over into neighboring sub-carriers. To see how this convolution causes distortion between OFDM sub-carriers, an example with two composite signals is created. One signal contains three transmitters (nine tones total) with no frequency spacing between the transmitters. The other signal uses the same setup but with three unused sub-carrier tones inserted between each transmitter’s frequency band. These magnitude response of the two composite input signals are seen in Figure 27. The three sets of sub-carriers have different signal strength to show the effects of a system where one transmitter is at an arbitrary distance \(d\) away from the receiver, while the other two transmitters are at a distance \(2d\) and \(4d\) respectively.
These two composite signals are passed through a third order polynomial as described by equation (43). In this example, the sampling frequency is set at 1 MHz with $\alpha_1 = 1$ and $\alpha_3 = 0.04$. Figure 27 shows effects of IM3 on both systems. The bottom images shows how much additional strength is added to the original frequency components of the signal due to IM3. In both situations, the sub-carriers create intermodulation terms in all frequencies with a spacing of 20 Hz. In the case of the signal that is spaced out, the additional IM3 tones have lower peaks and some of the IM3 tones occur in frequencies that are not used by the system. In the case where there is no frequency spacing, the IM3 tones have higher peak and the majority of the IM3 tones stack on top of the frequencies used by the system. This example shows the advantage of spacing out the sub-carrier sets used by the transmitters. Doing so reduces the overall amount of intermodulation distortion that adds on top of the intended signal.
A look into the effects of IM3 on the phase of the signal is required as this paper uses phase information to estimate distance. The top two plots show an example of a phase shift that might be seen in an actual system. The effects of distance on phase is accounted for in this simulation by changing the amount of phase shift between tones based on the signal magnitude shown in Figure 27. For example, the middle transmitter is furthest away which is accounted for by having a larger phase shift compared to the other transmitters. After passing the signal through the third order polynomial, additional tones are seen the frequencies near the frequencies of interest. The phase change depends on how much of the signal power is a result of IM3. Figure 29 shows the difference between the original signal phase and the phase of the signal with IM3 tones ignoring the frequencies that are not used by the two systems.
Including frequency spacing between the sub-carriers sets reduces the amount of phase shift caused by IM3. From Figure 29, the corresponding distance error introduced due to IM3 is:

\[
d_{\text{error, no spacing}} = \frac{\Delta \phi_{\text{error}} v_{\text{sound}}}{2\pi \Delta f} = \frac{(-0.1673 + 0.0539) \cdot 343}{2\pi \cdot 20} = 310 \text{ mm}
\]

\[
d_{\text{error, spacing}} = \frac{\Delta \phi_{\text{error}} v_{\text{sound}}}{2\pi \Delta f} = \frac{(-0.06862 + 0.03507) \cdot 343}{2\pi \cdot 20} = 92 \text{ mm}
\]

This example shows that introducing tone spacing between sub-carrier sets leads to a significant reduction in the distance error due to IM3. The introduced error is a function of the strength of the signal and how much intermodulation distortion is caused by the receiver.
5.4 Future Work

5.4.1 Impacts of COVID-19

COVID-19 has impacted this work mainly in the restriction of facility access and materials for an initial hardware implementation of the system. This paper has found no significant deficiency in the proposed method that could make a physical implementation unusable for indoor localization. Originally, this paper would have provided real-world distance estimation results in addition to simulations. The hardware platform for these real-world measurements is shown in Figure 30. The multi-tone OFDM signals are generated with an arbitrary function generator and transmitted through the ultrasonic channel. The received signal is amplified with a low-noise amplifier to a suitable voltage level for processing. This amplifier is required as the signal magnitude will quickly decay to the low millivolt range or lower at any significant distance. The transmitted and received signal are captured simultaneously using an oscilloscope and processed with a personal computer.

Figure 30 - Initial Hardware Platform for Distance Measurements

If the hardware system finds no significant deficiencies in this method of distance estimation, a proper synchronization system based on infrared (IR) diodes would have been the next step.

5.4.2 Additional Topics to Explore

There are a few topics of interest to explore once the results of this paper are confirmed through a hardware demonstration. As stated in the previous section, the next step is to create a suitable synchronization system. There are many possible solutions based on using electromagnetic waves. Some ideas include using infrared (IR) or RF-based systems. After this is solved, an analysis of the performance requirement of the
data acquisition system and the processing should be done. This is important as it will decide the hardware components of the system. After this is done, a PCB for the receiver circuit should be designed and fabricated. Once the receiver circuit has been developed, a hardware implementation of the rest of the system can follow.
Chapter 6

CONCLUSION

This paper seeks to explore the feasibility and performance of a new ultrasound-based distance measuring method. By using an OFDMA scheme, the proposed system can maximize the spectral efficiency of the narrowband transducers while maintaining orthogonality between transmitters. Additionally, using multiple tones allows for a relatively easy and computationally efficient way to estimate distance based on phase shift. Furthermore, use of multiple tones allows for multiple measurements that are relatively uncorrelated with each other. This leads to significant performance increases in the presence of noise at the cost of bandwidth. A model of the ultrasonic channel was created to determine the phase shift that occurs from the bandpass nature of the transducers. Additionally, a study of how the number of tones affects the overall measurement system is presented. The effects of non-linear distortion and how frequency spacing reduces its effects is also discussed. Overall, the use of multi-tone signals in measuring distance using ultrasound is a promising idea that warrants further study.
WORKS CITED


