EXPERIMENTAL INVESTIGATION INTO UTILIZING SYNTHETIC JET ACTUATORS TO SUPPRESS BI-MODAL WAKE BEHAVIOR BEHIND AN AHMED BODY

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ABSTRACT

Experimental Investigation into Utilizing Synthetic Jet Actuators to Suppress Bi-Modal Wake Behavior Behind an Ahmed Body

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Testing done on the flat-back Ahmed Body and other bluff bodies has shown the existence of a bi-stable reflectional symmetry-breaking wake at Reynolds numbers ranging from 340 to 2.41 x 10^6. Several methods of flow control, both active and passive, have been used to improve the efficiency of the Ahmed body but their effect on the bi-stable nature of the wake has not been investigated. This work details the experimental investigation done to determine if piezoelectrically driven synthetic jet actuators are capable of suppressing the bi-stable wake effects observed behind the Ahmed Body. The synthetic jets were designed and manufactured to have a maximum total coefficient of momentum of 1.0E-3 with a frequency range up to 2000 Hz or $F^+ = 17.25$. The piezoelectric actuators used were bimorph bending disks with no center shim and were driven by a square waveform. Pressure data was collected from 25 pressure ports on the rear of the model at 625 Hz for 600 seconds per run and filtered using a lowpass filter at 35 Hz to remove interference. Center of Pressure probability distributions and Principle Component Analysis were used to identify wake shapes and modes. Results with no jet actuation showed good agreement with previously published work on the Ahmed Body. It was found that the actuation frequency had an effect on the ability of the synthetic jets to affect the wake. Actuating at $F^+ = 1$ (116 Hz) showed a bi-stable
wake with an even distribution between wake modes. Higher actuation frequencies showed either a skewed distribution with a weakening of the bi-stable effects ($4 < F^+ < 8$) or a complete removal of the bi-stable distribution ($8 < F^+ < 12$). Frequencies higher than $F^+ = 12$ did not show any effect on the bi-stable distribution. There was a negative correlation between actuation frequency and average wake pressure; it is theorized that the synthetic jets enhance mixing in the shear layer around the recirculation bubble in the wake to decrease average pressure.
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Chapter 1

BACKGROUND

This chapter covers previous work done with the Ahmed Body reference model and the pertinent flow features in the wake. It also covers active flow control methods used previously and details the analytical model used to design synthetic jet actuators.

1.1 The Ahmed Body

First developed in 1984, the Ahmed Body was designed to simulate a basic ground vehicle bluff body for wind tunnel testing.\cite{1} The model is capable of simulating the flow features around two types of bluff body road vehicles, a passenger car when the model has a rear slant angle or a truck when there is no rear slant angle. Recently, research has been done to investigate the unsteady features of the wake and it has been shown that the wake exhibits a bi-stable symmetry-breaking flow field.

1.1.1 Time-Averaged Flow Features

The time-averaged flow features are shown in Figure 1.1 for a slant-back Ahmed body model. The large wake behind the model contains several dynamic phenomena: the side shear layer turns into a longitudinal vortex, two “horseshoe” vortices are stacked on each other inside the separation bubble, and there is a recirculation zone on the slanted face. The separation on the slanted face is dependent on the angle of the slant. At slant angles less than the critical angle of 25° the flow stays attached, but past 25° the flow separates and significantly increases the drag force.\cite{1}
It is important to note that the time-averaged wake ends up being symmetrical. This is not the case when looking at the unsteady features of the wake.

Figure 1.1: Time-averaged flow structures in the wake of a slant-backed Ahmed Body[1]

1.1.2 Unsteady Flow Features

The unsteady wake behind the Ahmed Body is made up of three distinct structures: the bi-stable “switching” in the horizontal direction, the lateral shear layer shedding vortices dependent on the current bi-stable mode, and the top/bottom shear layers interacting to shed vortices in the vertical direction.[15] Figure 1.2 represents how the vortices are shed in both the horizontal and vertical directions off the rear of the Ahmed Body for one of the bi-stable modes.

In the vertical direction, the vortex being shed is constantly switching between the top and bottom of the body because of the interaction of the top and bottom shear layers. This shedding is fairly constant and not disrupted by the bi-
stable shedding the in horizontal direction. Sufficient ground clearance is required to have vortices shed off the top and bottom because if there is not enough underbody flow the vortices will only shed off the top of the body. In the horizontal direction, the vortex being shed keeps to one side because of the bi-stability effects, and then switches to the other side when the bi-stable mode changes. Grandemange et all found that for a Reynolds number of $9.2 \times 10^4$ (based on model height) the vortex shedding for both the vertical and horizontal directions correspond to a Strouhal number of 0.17 when normalized by the height and the width respectively.\cite{15} For the current work, the bi-stable symmetry breaking wake component is of interest and will be detailed further in the next section.

![Figure 1.2: Vortex shedding in the vertical direction (top) and lateral direction (bottom) in the wake of a flat-back Ahmed Body\cite{15}](image)

Additionally, there is an overall pulsing or “breathing” of the recirculation bubble behind the model. This effect can be seen independent of the vortex shedding of the sides of the model and is most detectable in pressure ports along the centerline of the rear of the model. Duell et all found that for a Reynolds number (based on model length) of $7.5 \times 10^5$ with a square-back Ahmed Body the pulsing occurs at a Strouhal number of 0.069.\cite{9}
1.1.3 Bi-Stable Wake Effects

Experimental investigations have shown the existence of a bi-stable, reflectional symmetry breaking wake behind bluff bodies such as the Ahmed body or a full-size road vehicle.\textsuperscript{[5][15][16][37][39]} This effect has been shown to exist at a wide range of Reynolds numbers from $Re = 340$ to $Re = 2.41 \times 10^6$ and the shifting between states is random.\textsuperscript{[14]}

There are two stable states, usually denoted P and N, and one temporary unstable state, TS. The unstable state does give a symmetric wake profile but lasts significantly less time than the stable states. The switching between stable states takes place on very long-time scales, $T_s \sim 10^3 \frac{h}{U_\infty}$; it is also possible for the wake to switch from state P or N to TS then back to the same stable state as before. Because of the significant length of the bi-stable switching, it is necessary to collect data over long time periods to achieve consistent characterization of the phenomenon; an acquisition time of 450 s was determined to be sufficient to “obtain a reasonable equipresence of the two bi-stable positions.”\textsuperscript{[39]}

Figure 1.3 shows the pressure distribution on the rear of the model for the three states. It can be seen that the P and N states are very similar to each other even though they are in opposite orientations; this similarity leads to the symmetric time-averaged wake.
Figure 1.3: Contour map of Coefficient of Pressure for the three wake states, P (left), TS (center) and N (right)[37]

Ideally, the wake flow would show no inherent preference for either of the two stable states, but the amount of time spent in each state or the existence of the bi-stable wake can be affected by several factors. Bi-stability only occurs when the body is separated from the ground plane by a specific critical ground clearance of $C^* = 0.10$[15] ($C^*$ is the ground clearance normalized by the height of the model); below this value the wake will stay in the symmetric state instead. Higher values of ground clearance do not affect the degree of asymmetry or the frequency of the oscillations. It is worth noting that ground effect is not necessary to have bi-stable flow features in the wake as bi-stability has been observed even when the model is placed far from the ground plane.

Figure 1.4 shows the probability distribution of the pressure gradient on the rear of the model as the ground clearance increases. For the purposes of this work, any ground clearance sufficient to cause bi-stable effects will be satisfactory. The underbody flow can also be affected by the model supports; therefore, every attempt should be made to keep the supports as small and unobtrusive as possible.
Figure 1.4: Wake pressure gradient distribution with increasing ground clearance\textsuperscript{[15]}

The yaw angle of the model can also have a pronounced effect on the wake state selection. Yaw angles of greater than 1 degree can push the wake to prefer a single side, however it was found that the pressure distribution values did not change significantly.\textsuperscript{[39]} Theoretically, the wake should spend 50\% of the time on each side, but realistically there are too many small variations that prevent a perfect split from happening. Proportions as extreme as 75\% on one side and 25\% on the other can still be considered bi-stable.\textsuperscript{[15]}

Two main impacts of the bi-stable flow effects are: the location of the shed lateral vortices change over time, and the pressure on the rear of the model is lower in a bi-stable mode than the symmetric mode. If the shed vortices did not change sides, i.e. the bi-stable effects were removed, then it would be easier to implement flow control features that enhance the efficiency of the vehicle. With the onset of autonomous cars, it would be beneficial to reduce the amount of wake variation as the fluctuations due to the shed vortices are not insignificant on full scale vehicles. Additionally, it has been estimated that the drag due to the wake being non-
symmetric when it is in a bi-stable state can account for 4-9% of the total drag of the model.\cite{16} Therefore suppressing the bi-stable effects can give several direct improvements for road vehicle efficiency as long as the suppression doesn’t create other adverse flow effects.

1.1.4 Flow Control

Several studies have been conducted on using passive flow control methods to increase the base pressure on the Ahmed Body model to reduce drag. These efforts have largely been successful at the small scale and some have gone on to be used on full size vehicles (“boat-tails” on semi-trucks). In recent years more effort has been placed on using active methods of flow control to reduce the amount of separation over the slant-back Ahmed Body model\cite{3}\cite{8}\cite{27}\cite{29} or push the vortices generated farther from the rear of the model,\cite{6} but there has not been much research done to show the effect of these active methods on the bi-stable nature of the wake.

Table 1.1 shows some of the flow control methods that have been tested, and their corresponding effects on the drag and wake of the model. Only one, the control cylinder from Grandemange, specifically targeted the bi-stable nature of the wake to attempt to improve the base pressure. While many of these methods are viable to investigate their effect on the bi-stable nature of the wake, synthetic jets were chosen because of their versatility, compactness, and ability to be integrated with a previously constructed Ahmed Body model.
### Table 1.1: Methods of Flow Control

<table>
<thead>
<tr>
<th>Flow Control Method</th>
<th>Slant Angle</th>
<th>Drag Effect</th>
<th>Wake Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steady Blowing Jets</td>
<td>0 deg</td>
<td>-10%</td>
<td>Not Reported[40]</td>
</tr>
<tr>
<td>Steady Blowing &amp; Suction Jets</td>
<td>25 deg</td>
<td>-13%</td>
<td>Reduced Separation[8]</td>
</tr>
<tr>
<td>Static Flaps</td>
<td>25 deg</td>
<td>Not reported</td>
<td>21% Pressure Reduction[36]</td>
</tr>
<tr>
<td>Steady Blowing Microjets</td>
<td>25 deg</td>
<td>-10.6%</td>
<td>Reduced Separation[3][27]</td>
</tr>
<tr>
<td>Pulsed Blowing</td>
<td>0 deg</td>
<td>Not reported</td>
<td>Wake Mode Stabilization[37]</td>
</tr>
<tr>
<td>Porous Underbody Layers</td>
<td>0 deg</td>
<td>-20%</td>
<td>Reduced Vortex Intensity[7]</td>
</tr>
<tr>
<td>Base Cavity</td>
<td>0 deg</td>
<td>-20%</td>
<td>Increased Wake Pressure[20]</td>
</tr>
<tr>
<td>Synthetic Jets</td>
<td>25 deg</td>
<td>-4.3%*</td>
<td>Reduced Separation[29]</td>
</tr>
<tr>
<td>External Control Cylinders</td>
<td>0 deg</td>
<td>Not reported</td>
<td>5% Wake Pressure Increase[15]</td>
</tr>
<tr>
<td>Boat Tail</td>
<td>0 deg</td>
<td>-30%</td>
<td>Reduced turbulence intensity[21]</td>
</tr>
</tbody>
</table>

*Note that the drag reduction was only realized from one condition; the jets were operated at a specific frequency and angle while other conditions resulted in a net increase in drag.

1.2 Synthetic Jets

Synthetic jets (SJs) are a specialized application of jet flow. In general jet flow is a class of fluid flow where one fluid mixes with another fluid either at rest or in motion. SJs are formed by repeated ingestion and expulsion of the working fluid driven by a flexible diaphragm through a nozzle, which forms a train of vortex rings downstream from the nozzle (Figure 1.5). SJs fall under a specialized form of jet flow because they do not require a change in mass, meaning there is zero-net mass-flux (ZNMF) through the nozzle. While SJs do not impart mass to the system, they do add a net-positive amount of momentum and vorticity. Effectively, SJs are a way to convert electrical input energy into mechanical energy in the flow to energize shear or boundary layers.
1.2.1 Synthetic Jet Analytical Model

The model and equations developed by Krishnan will be used to inform design decisions and the relevant points are included here. Figure 1.6 shows the basic model of the SJ cavity with a moving actuator expelling a cylindrical “slug” of fluid. It is assumed that the volume of the expelled fluid is equal to the volume displaced by the flexible diaphragm, and the expelled fluid has the same cross section as the exit orifice.

Figure 1.5: Fluid ingestion (left) and expulsion with vortex ring formation (right) of a synthetic jet.
This allows for the definition of the nondimensional stroke ratio and the jet Reynolds number; both have been identified as influential design parameters. To solve for these two parameters, first apply the conservation of volume equation to the expelled fluid and the movement of the actuators giving:

\[ \alpha \frac{\pi D^2}{4} \Delta = \frac{\pi d^2}{4} L \]  

(1)

where \( \alpha \) is the volume fraction that a piston would achieve from displacing the membrane a distance of \( \Delta \). The volume fraction can be defined as:

\[ \alpha = \frac{2\pi \int_0^{D/2} y(r) r \, dr}{\left( \frac{\pi D^2}{4} \right) \Delta} \]

(2)

with an approximate actuator deflection profile of:

\[ y(r) = \frac{\Delta}{2} \left[ 1 - \frac{r^2}{R^2} + \frac{r^2}{R^2} \ln \frac{r}{R} \right] \]

(3)
This deflection profile assumes the actuator is a circular plate clamped on the edges subject to a uniform load. Eq. 2 yields an alpha value of 0.25 and is a sufficient approximation for the current work.

Rearranging Eq. 1 to solve for \( \frac{L}{d} \) gives a nondimensional stroke length of:

\[
\frac{L}{d} = \alpha \frac{D^2}{d^3} \quad (4)
\]

The stroke length can be calculated given the input parameters of the cavity diameter, the orifice diameter, and the maximum actuator deflection. These three parameters can be changed to suit the design specifications, requirements, and the specific application for the SJ.

The jet velocity, needed to calculate the Reynolds number, can be defined based on the momentum flux:\(^{[31]}\)

\[
U_{jet} = \sqrt{2} \frac{L}{T} = \sqrt{2} fL \quad (5)
\]

This shows that the jet velocity and Reynolds number are directly proportional to both the actuation frequency and the stroke length. Substituting Eq. 4 into Eq. 5 and solving for the velocity shows that the jet velocity is directly proportional to the maximum actuator displacement, the cavity diameter squared, and actuation frequency while being inversely proportional to the orifice diameter squared:

\[
U_{jet} = \sqrt{2} f \alpha \frac{D^2}{d^2} \quad (6)
\]

Fundamentally, this makes sense as increasing the available fluid and forcing it through a smaller opening should result in a higher jet velocity. Therefore to obtain the highest jet velocity, it is advantageous to have the largest diameter
cavity possible with the smallest orifice. With the velocity defined, jet Reynolds number can also be determined:

\[
Re_{jet} = \frac{U_{jet}d}{\nu} = \frac{\sqrt{2} f \alpha \Delta d^2}{\nu d} \tag{7}
\]

It should be noted that the analytical model presented here is good for achieving an approximation of the performance of the synthetic jets, but the actual relationship between frequency and jet velocity is not a linear dependence.\(^{11}\) It is possible to use Lumped Element Modeling (LEM) to determine a more accurate estimate of the jet velocity\(^{2}\)\(^{10}\) at all actuation frequencies; but this process can be extremely intensive and is usually a project on its own. For the purposes of this work, the approximation provided by Eq. 6 will be sufficient to design the SJs because the SJs do not need to be perfectly optimized to achieve bi-modal suppression. Any velocity calculated using Eq. 6 will be rounded to two significant digits, and should be considered an estimation, not an exact value.

One more physical parameter must be included in the model of the SJ: the channel length (Figure 1.7). This parameter is important because changing the channel length affects the Helmholtz frequency of the cavity. The Helmholtz frequency (Eq. 8) indicates where resonance will affect the jet flow and if it is too close to the actuator’s natural frequency there could be irreparable damage done to the actuator. Therefore, it is beneficial to select a channel length that moves the Helmholtz frequency away from the natural frequency of the actuator, but still allows the jet flow to take advantage of the increased velocity from the resonance induced in the cavity.
Figure 1.7: Channel length in a synthetic jet

\[ f_{Hlm} = \frac{a}{2\pi} \sqrt{\frac{A_{exit}}{V_{cav}(Ch + 0.3D_H)}} \] \hspace{1cm} (8)

\[ D_H = \frac{4A_{exit}}{P_{e_{exit}}} \] \hspace{1cm} (9)

Eq. 8 shows that a smaller cavity volume and channel length will raise the cavity resonance frequency, but this must be balanced with the resulting decrease in jet velocity as shown from Eq. 6. Similarly, increasing the orifice area will increase the cavity resonance frequency at the expense of a lower jet velocity. As such, the cavity resonance is the first design constraint considered here. Figure 1.8 shows two cases where the cavity resonance and the actuator resonance are designed to be separate (left) or complement each other (right). With the proper hardware and safety considerations it is possible to keep the two resonances close to each other to improve performance but doing so increases the risk of damage to the actuators. It was determined that the optimal ratios are \( H/D = 0.6 \) and \( Ch/D = 2.1 \) for the cavity height to cavity diameter and channel length to cavity diameter ratios, respectively.\[^{12}^\]
Two non-dimensional parameters have been identified to evaluate and compare different applications of synthetic jets: $F^+$ the normalized actuation frequency (Eq. 10) and $c_\mu$ the coefficient of momentum (Eq. 11). The normalized frequency can be calculated by:

$$F^+ = \frac{fh}{U_\infty} \quad (10)$$

In this case the frequency is normalized by the height of the model, $h$, and the freestream velocity $U_\infty$. It has been shown that operating at integer values of $F^+$ can yield significant results.\[17\] Taking this into account, specific actuation frequencies will be chosen to give integer $F^+$ values.

The coefficient of momentum is a ratio between the momentum added by the SJs and the momentum of the freestream flow:

$$c_\mu = \frac{\rho_{jet} U_{jet}^2 \eta_{jet} A_{exit}}{\frac{1}{2} \rho_\infty U_{\infty}^2 A_{ref}} \quad (11)$$

Generally, these values will be on the order of 1.0E-3 because the jets are so small compared to the freestream flow. Observant readers may note the lack of a $\frac{1}{2}$ term.
in the numerator; this was by convention.\[13\] The root-mean-square value of the jet velocity will be used instead of an integral of the velocity profile as it is next to impossible to measure an exact velocity distribution coming out of the jet orifice. 

1.2.2 Piezoelectric Actuators

It is possible to drive SJs with a number of different actuators: piezoelectric, electromagnetic, acoustic, or mechanical. It is not necessary to cover the background on all types of actuators, as they all work essentially the same. Only a brief overview of the selected actuator, piezoelectric bending disks, will be given here.

Piezoelectric materials are dielectric materials that produce an electric charge when a mechanical stress is applied or reversely, can produce a mechanical deformation in the presence of an electric charge. First discovered by Pierre and Jacques Curie, these materials have come to serve a wide range of industries and applications from the production and detection of sound to microbalances.

Dielectric materials change shape on the atomic level when exposed to an electric field. “The cations get displaced in the direction of the electric field, and the anions get displaced in the opposite direction, resulting in net deformation of the material.”\[38\] There are 32 classes of dielectric crystals, which are further split into 11 centrosymmetric and 21 non-centrosymmetric. A centrosymmetric crystal has a center of symmetry whereas a non-centrosymmetric does not. When a centrosymmetric crystal is placed in an electric field the movement of the cations and anions effectively cancel each other out, while a non-centrosymmetric crystal will have a net deformation in the lattice. This deformation is directly proportional
to the strength of the electric field. The centrosymmetric crystals cause the electrostrictive effect, while the non-centrosymmetric crystals cause the piezoelectric effect. Materials made up of non-centrosymmetric crystals are categorized as piezoelectric materials.

Somepiezoelectric materials can be subcategorized as pyroelectric materials, which allow for spontaneous polarization. If the material can have its polarity reoriented spontaneously with the application of an electric field, then it is a ferroelectric material. This small sub-subcategory of piezoelectric materials is the best for actuator applications because it gives the greatest displacement per electric field strength.

The most common piezoelectric material is Lead Zirconate Titanate (PZT).\textsuperscript{[38]} PZT parts are made by first compressing a mixture of PZT and organic binder material under pressures that can reach 69 MPa at a 2:1 compaction ratio.\textsuperscript{[19]} Then the compacted part is air fired at 600 Celsius to burn away the organic binder. Silicon carbine or diamond abrasives are then used to reduce the part to its final shape. Next a thin layer of vacuum sputtered nickel is deposited to create the electrodes on either side of the part and fired again to bond the electrode to the ceramic. The last step in the process is polarization; while the temperature is elevated the part is exposed to a high DC electric field for a fixed period of time. The temperature, field strength, and exposure time all affect the final part’s capacity to convert electricity into mechanical strain.

Piezoelectric actuators come in many shapes and sizes; for the specific application of synthetic jets, piezoelectric bending disks are used because of
axisymmetric deformation about the center. This helps simplify modeling and results in higher jet output velocity over linear benders. There are two types of bending disks, unimorphs and bimorphs (Figure 1.9). Unimorphs are single pieces of piezoceramic, usually attached to a metal shim for rigidity, and are generally easier and cheaper to produce. The tradeoff is that with only a single piezoceramic piece, unimorphs have lower deflection values. Bimorphs are two pieces of piezoelectric either bonded together directly or bonded to a center shim. They are polarized so that when one expands, the other contracts in unison. This gives much higher deflection values than unimorphs, but bimorphs are more expensive to produce. The center shim can add rigidity but also limits the deflection.

![Unimorph, Bimorph With Shim, Bimorph Without Shim](image)

**Figure 1.9: Types of piezoelectric bending disks**

### 1.3 Research Aims

The goals of the current work are as follows:

1. Design and manufacture synthetic jets powered by piezoelectric actuators
2. Test the synthetic jets on the wake of the Ahmed Body
3. Determine the effect of the synthetic jets on the bi-stable aspect of the Ahmed Body wake
Chapter 2
EXPERIMENTAL SET-UP

This chapter covers the testing apparatus, the Cal Poly Low Speed Wind Tunnel, the model used for testing, the design and manufacture of the synthetic jets, the instrumentation used to collect data, and the test matrix for all data collected.

2.1 Cal Poly Low Speed Wind Tunnel

The Cal Poly Low Speed Wind Tunnel is an open circuit indraft tunnel with a nominal test section of 0.91 m by 1.22 m by 4.27 m. The inlet has an area of 2.74 m by 3.66 m giving an area contraction ratio of 9:1. The 1.37 m diameter fan is powered by a 93 kW motor giving a top speed through the test section of approximately 50 m/s. Test section dynamic pressure was found by subtracting the total pressure measured at the inlet by the static pressure measured from a pressure ring around the front of the test section. The turbulence intensity has been measured to be less than one percent. Flow uniformity and boundary layer measurements were performed by Cal Poly students and are shown in Figures 2.1 and 2.2. Across the whole test section, flow uniformity measurements showed the test section velocity varies less than one percent ahead of the placement for the Ahmed model. Boundary layer measurements indicated a thicker than expected boundary layer as compared to a flat plate approximation due to upstream disruptions at the connection between the test section and the inlet. However, it will be shown that the bi-stability phenomenon was still detectable despite the thicker boundary layer.
Figure 2.1: Velocity distribution across the test section ahead of the model

Figure 2.2: Ground plane boundary layer ahead of the model
2.2 Ahmed Body Wind Tunnel Model

Previous testing at Cal Poly necessitated the manufacture of an Ahmed body wind tunnel model. The model design is shown in Figure 2.3 and resulted in a 0.588:1 scale compared to the original body dimensions given by Ahmed (Figure 2.4). This gives a blockage ratio of 3.76%, which is well below the commonly used limit of 5% for bluff bodies.\[4\] The model was made out of cast acrylic and laser-cut to ensure dimensional accuracy. The body was split into two sections to allow testing of different rear sections such as a slant-back, or flow control variations. The nose cone was milled out of Medium Density Fiberboard (MDF), spray painted matte black, and sanded for smoothness.

Figure 2.3: CAD model of the test article
Figure 2.4: Dimensions of the original Ahmed Body\textsuperscript{[1]} (dim. in mm)

The model attaches to the tunnel floor by four threaded rods through the cylindrical supports under the model. The threaded rods are covered by acrylic tubes of diameter 14.6 mm, so the rods are shielded from the flow and the supports are comparable to the original Ahmed geometry. This attachment scheme allows the model to be translated vertically to test different ground clearances. In this work, a single ground clearance of 25 mm or $C^* = 0.15$ was tested; this is above the critical value for bi-stable wake behavior.\textsuperscript{[15]}

The original model did not allow space for synthetic jets to be inserted on the rear face, so a new rear section was manufactured. For consistency with the rest of the model, the new rear section was also made out of cast acrylic and laser-cut for dimensional accuracy. The laser cutter used was a Universal Laser Systems PLS6.105D with an accuracy of 0.076 mm, which was deemed sufficient for the current testing at hand. The new section was assembled by clamping two acrylic
pieces to a piece of 90-degree angle iron and chemically welded using solvent based acrylic glue. Figure 2.5 shows the diagram for the new rear face; this includes the pressure port layout and space allocation for the actuator banks to be installed. The pressure ports were made by laser-cutting holes of 1.4 mm diameter then inserting stainless-steel tubes with 1.5 mm diameter and a length of 38 mm and gluing them on the inside surface. The stainless-steel tubes were flush with the rear face of the model.

Figure 2.5: Pressure port layout on the rear of the model (dim. in mm)

2.3 Synthetic Jet Design

The design of the entire synthetic jet apparatus was split into three parts: locating the jets on the model, selecting the piezoelectric actuator, and designing the actuator bank to hold the piezoelectric disks.
2.3.1 Jet Location

Other studies showed that it was possible to influence the wake flow using constant jets located near the separation point on a slant-backed model\cite{8} or microjets along the edges of the rear face.\cite{27} The goal of the jets is to energize the shear layer coming off the edges of the model, so it is logical to place the jets along the edges of the rear face. It is important then to keep the jets as close to the edges as possible for maximum effect. It was decided that the jets would operate perpendicular to the rear face, instead of at an angle, to reduce complexity in the actuator bank design. As cost was a significant factor in this project, jets were not able to be placed at each edge, and instead it was chosen that the jets would be placed only on the left and right edges of the rear face. This was because the bistable behavior has been observed as a left-to-right phenomenon and not top-to-bottom.

2.3.2 Piezoelectric Actuator Selection

A bi-morph actuator driven by a square wave yielded the highest jet output velocity as shown in Table 2.1.\cite{26} A bi-morph actuator is two pieces of piezoelectric material bonded together with opposite polarity so that in the presence of an electric current one side extends and the other contracts. If the current is reversed, then the bending motion is also reversed. In an attempt to reduce costs a unimorph actuator was tested but the results showed that it was not able to produce a comparable velocity to the bi-morph actuator.
Table 2.1: Synthetic Jet Velocities (m/s)\textsuperscript{26}

<table>
<thead>
<tr>
<th>Waveform/Actuator</th>
<th>Sine</th>
<th>Saw-tooth</th>
<th>Square</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bimorph</td>
<td>7 ± 2</td>
<td>35 ± 6</td>
<td>36 ± 5</td>
</tr>
<tr>
<td>Thunder</td>
<td>5 ± 2</td>
<td>26 ± 4</td>
<td>27 ± 4</td>
</tr>
<tr>
<td>RFD</td>
<td>6 ± 2</td>
<td>28 ± 5</td>
<td>32 ± 3</td>
</tr>
</tbody>
</table>

Based on cost, manufacturing lead time, and reliability Piezo.com was chosen as the supplier for the piezoelectric actuators. This limited the diameter of the actuator to three choices: 12.7, 31.8, 63.5 mm. It is important to note that these actuators do not come electrically insulated; this means that the whole top surface is one terminal and the whole bottom surface is the other. Because of this the actuator banks could not be made out of metal as the actuators could not be insulated completely. Due to the varying sizes for possible actuators, three actuator banks were designed, and the evaluation detailed in the following section.

2.3.3 Actuator Bank Design

The main design factors for the actuator banks were actuator size, actuator/jet number, cavity depth, jet orifice type, and channel length. It was not necessary to optimize each individual factor as the goal of this work was to determine if synthetic jets were capable of modifying the wake; therefore, only one successful design was needed, even if that was not the optimal design.

In order to place the jet orifice as close to the edges as possible the actuators were placed perpendicular to the jet-flow. Figure 2.6 shows the modified arrangement used for this work. A similar arrangement was used by Goodfellow to better fit within the size constraints of a wing.\textsuperscript{13} The modified arrangement is not expected to have a significant effect on the jet output or the vortex ring development downstream of the jet.
Figure 2.6: Modified synthetic jet layout

A design for each actuator size was developed and considered; Table 2.2 lists the pertinent dimensions and predicted values and Figure 2.7 shows a comparison of each design. The following is a breakdown of the three designs and an explanation of why the final design was chosen. Note that each actuator bank also has a corresponding top piece that would cover the piezoelectric actuators for protection, but as their design did not affect the actuator bank design they are not discussed here.

<table>
<thead>
<tr>
<th>Table 2.2: Actuator Bank Design Specifications</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Design Property</strong></td>
</tr>
<tr>
<td>Actuator Size (mm)</td>
</tr>
<tr>
<td>Cavity Diameter (mm)</td>
</tr>
<tr>
<td>Cavity Depth (mm)</td>
</tr>
<tr>
<td>Cavity Vol (mm$^3$)</td>
</tr>
<tr>
<td>Max Deflection (mm)</td>
</tr>
<tr>
<td>Displacement Vol. (mm$^3$)</td>
</tr>
<tr>
<td>Exit Orifice Diameter (mm)</td>
</tr>
<tr>
<td>Actuator Resonant Freq. (Hz)</td>
</tr>
<tr>
<td>Channel Length (mm)</td>
</tr>
<tr>
<td>Helmholtz Freq. (Hz)</td>
</tr>
<tr>
<td>Number of Jets per Side</td>
</tr>
<tr>
<td>Nondimensional Stroke Length</td>
</tr>
<tr>
<td>Estimated Jet Velocity* (m/s)</td>
</tr>
<tr>
<td>Estimated Single Jet $C_{\mu}$</td>
</tr>
<tr>
<td>Estimated Total $C_{\mu}$</td>
</tr>
</tbody>
</table>

*Velocity was estimated at the Helmholtz Frequency
### 2.3.3.1 Actuator Option 1 – 12.7 mm

The main advantage of the smallest actuator is that it allows for the most jets because more actuators can be packed in the limited space. However, this also comes with the challenge of smaller areas between the actuators and very small cavity volumes. With a maximum displacement of 0.019 mm for each actuator, this leads to a maximum volume displacement of 1 mm³. As such, the ratio of cavity volume to displaced volume is 39, which is quite high. This means that there is not much air being displaced relative to the whole cavity. While shrinking the cavity volume would be a solution to this, it becomes impractical to manufacture a cavity with a depth smaller than 0.05 mm with the tools currently available. Additionally, the estimated jet velocity is too low to provide a large enough momentum transfer relative to the ambient flow.
2.3.3.2 Actuator Option 2 – 31.8 mm

The midsize actuator strikes a good balance between the low velocity of the small actuator, and the relatively few jets of the larger actuator. The actuator resonant frequency is close enough to the cavity resonant frequency that the actual velocity in the range ~500 to 1000 Hz may be higher than the estimated velocity. The range of $c_\mu$ is comparable to other tests using synthetic jets,$^{[17]} [22] [24] [29]$ and with four jets per actuator bank, the added momentum will be spread across the whole length of the sidewall.

2.3.3.3 Actuator Option 3 – 63.5 mm

It is unlikely that the jet velocity will reach 1015 m/s, as estimated by the analytical model. This would be a speed of Mach 2.95, and the model was strictly to be used for incompressible flow. While the estimated value is clearly too high, it is safe to assume that the 63.5 mm actuators will be able to achieve a higher jet velocity than the 31.8 mm actuators; previous work at Cal Poly showed output velocities on the order of 30 m/s for a 53 x 0.75 mm slot orifice using the 63.5 mm actuators.$^{[24]}$ With a much smaller exit orifice the velocity would increase accordingly, likely on the order of 150 to 200 m/s. However, the larger actuators have significant problems with the amount of electrical power they need to operate. It was found that the equipment used, which will also be used in the present work, was not able to generate enough current to operate the actuators over a wide range of frequencies.$^{[24]}$ The amplifier is current limited to 300 mA and that occurred around 100 Hz for a bank of six actuators. As such, this electrical limit would
severely impact the range of frequencies to be tested and may not achieve velocities higher than the 63.5 mm actuator.

The 31.8 mm actuator was chosen because it will be able to provide a wide range of test frequencies while also giving an acceptable range of $c_\mu$.

2.4 Actuator Bank Manufacture, Assembly, and Installation

Three options were considered for the manufacturing of the actuator banks: milling out of aluminum, 3D printing via Fused Deposition Modeling (FDM), and 3D printing via PolyJet Resin Printing (PRP). Aluminum was ruled out because it would have necessitated insulating the piezoelectric actuators completely from the actuator bank. If any part of the actuator touched the actuator bank, then it could short circuit the system and cause significant damage. FDM was used to prototype the actuator bank design using a Lulzbot Taz 6, but it was found that FDM was not able to reach the accuracy required to achieve consistent dimensions across both banks. Thus, PRP was chosen because of its high accuracy and repeatability even for small parts. The actuator banks were printed by Sculpteo.com using an Objet30 Pro printer which has an estimated accuracy of 0.6% per dimension using VeroWhite Opaque PolyJet Resin. The finished parts were inspected, measured, and found to have an accuracy of +/- 0.2 mm across all measurable dimensions, such as the cavity diameter, jet orifice spacing, and the length, width, and thickness. Due to the small diameter of the jet orifice it was necessary to finish the hole with a 0.50 mm drill bit. The relative softness of the material meant that the drill bit could be spun by hand to remove the material. To ensure the jet orifice was dimensionally accurate, material was removed until the 0.50 mm drill bit could
easily slide into and out of the orifice, but a 0.60 mm drill bit still could not fit. This means the final dimension of the orifice was 0.55 +/- 0.05 mm.

The piezoelectric actuators were first cleaned with isopropyl alcohol and then 16-gauge wires were soldered onto the top and bottom of each actuator. For consistency with the polarity of the piezoelectric actuators, the side with the red band was considered the negative and the bare side was considered the positive (Figure 2.8).

![Piezoelectric actuators with lead wires attached](image)

**Figure 2.8: Piezoelectric actuators with lead wires attached**

It was crucial to maintain directional consistency across all of the actuators to ensure uniform performance. Each piezoelectric actuator was placed with the negative side facing into the cavity and then sealed with liquid rubber gasket across the top edges. This created an airtight seal so that when the piezoelectric actuator would vibrate no air would be lost. Preliminary testing showed that creating an airtight seal was extremely important to the performance of the synthetic jet. Figure 2.9 shows the installed actuators in the actuator banks and the jet exit orifice. There was a small manufacturing defect on the exit orifice due to the printing process, but this is not expected to cause a significant effect.
Finally, the top cover of the actuator banks were screwed on using M2 x 0.4 flat head hex drive screws, washers, and M2 x 0.4 nuts. The top covers added protection for the delicate piezoelectric actuators as well as added rigidity to the actuator bank structure. It is important to note that when the actuator banks were installed in the model the front line of screws were removed; they were not necessary, nor fit, because the edge of the actuator banks mounted flush with the rear of the model. The screws were used when doing bench testing of the synthetic jets. As the jet orifices were difficult to see, the hole tended to blend in with the rest of the actuator bank, strips of tape were added between each jet orifice to make it easier to locate them. The tape did not interfere with the operation of the jets. Figure 2.10 shows the completed assembly.
Installing the assembled actuator banks into the rear section of the model was straightforward; the banks were slid into the slots cut out of the rear panel. A simple press fit was found to be sufficient to hold the actuator banks in place. Several strips of aluminum tape were added to ensure the actuator banks maintained their position throughout testing. Figure 2.11 shows the actuators installed in the rear section of the model. The model was inspected periodically throughout testing to check that the actuators were still working and positioned properly.

Figure 2.11: Actuator banks installed in the rear tray

2.5 Instrumentation

Pressures were measured using a Scanivalve ZOC 33/64Px -64 miniature pressure scanner which had 32 high-resolution ports available; 25 for the pressure ports on the rear of the model, five for the pressure ports on the front of the model, one for the total pressure port at the tunnel inlet, and one for the static ring in front of the test section. The high-resolution ports have an accuracy of +/- 5.5 Pa over a 7 kPa range. The Scanivalve has a maximum scanning rate of 45 kHz and was
connected to the ERAD4000 Remote A/D which has a maximum data throughput of 625 Hz per channel. It is important to note that the Scanivalve does not measure all of the pressure ports simultaneously, it scans through them sequentially. In this experiment, there was a delay of 25 microseconds between port measurements; there was no indication that this phase difference had an influence on the data, and it was considered small enough to be disregarded. The power and ethernet cables for the Scanivalve and ERAD were routed through the legs of the model and through the wind tunnel floor to the data collection computer. Data was collected using a LabView 2015 VI and a LabView M DAQ card. The collection frequency was set to 625 Hz and only limited by the ERAD. The DAQ card read data from the ERAD sent over a TCP/IP connection. For speed the data was written to .csv files and later converted to .m files for use in Matlab.

Pressure tubing was 1.37 mm diameter vinyl tubing from Scanivalve (Part Number Y-129 URTH-063) and routed from the pressure ports to a collection block then to the Scanivalve. The tubing attached to the rear pressure ports was kept to a minimum so the effect of filtering from the tubing would be reduced. The tubing measured 200 mm from the pressure port to the collection block and then 100 mm to the Scanivalve. The tubing to the rest of the ports was longer as there was no need to collect high frequency data from those locations. The tubing for the total pressure port and the static ring were routed up one of the legs of the model and checked to ensure they did not pinch or otherwise effect the measurement. Figure 2.12 shows the assembled Ahmed body model installed in the wind tunnel.
2.6 Test Conditions

All tests were performed at a Reynolds number of $3.2 \times 10^6$ based on the height (as is common for studies looking at the bi-stable wake) of the model which gives an approximate freestream speed of 20 m/s. This Reynolds number is within the range tested by Grandemange et. al. and is the same as used in previous work at Cal Poly, therefore the bi-stable effects should be clearly visible at this speed. Across all testing days the air density varied from 1.18 to 1.20 kg/m$^3$, a difference of 1.6%. This percentage is low enough to assume that the density change did not affect the results. Table 2.3 shows the actuation frequencies tested, normalized frequency values (Eq. 10), normalized frequency uncertainty (Eq. 12), estimated jet velocity (Eq. 6), and the estimated momentum coefficient (Eq. 11). The frequencies were chosen with three objectives in mind: to test as broad a range as possible, to select integer values of $F^+$, and to select some frequencies near each other to test if
small changes in frequency have any effects. The uncertainty for the $F^+$ values were calculated using the percentage error equation below:

$$
\varepsilon_{F^+} = F^+ \left( \frac{\varepsilon_f}{f} + \frac{\varepsilon_h}{h} + \frac{\varepsilon_{U_\infty}}{U_\infty} \right) 
$$

(12)

Where $\varepsilon_f = 0.01$ Hz as the error in the output of the waveform generator, $\varepsilon_h = 0.5$ mm as the estimated error in measuring height of the model, $\varepsilon_{U_\infty} = 0.05 \frac{m}{s}$ as the estimated error in wind tunnel velocity.
Chapter 3

DATA ANALYSIS METHODS

This chapter details the analyses used to interpret the collected wind tunnel data. These include Welch’s modified periodogram, probability density functions for the Center of Pressure location and pressure gradient, and principal component analysis to determine the wake features.

3.1 Frequency Analysis

To determine data quality and investigate the possibility of the piezoelectric actuators affecting the wake shedding frequency, a frequency analysis using Welch’s modified periodogram method was employed. This method allowed for insight relating to the wake shedding frequency and the interference caused by the piezoelectric actuators. The following is a short explanation of Welch’s method and the rationale behind the inputs chosen for the method.

Welch’s method works by first splitting the signal into segments, estimating the PSD for each segment and then averaging all of the estimates together. It can be shown that Welch’s method is an asymptotically consistent estimator of the PSD because for an infinitely long signal the variance tends to zero.\textsuperscript{[34]} For long signals, such as data collected at 625 Hz over 10 minutes, Welch’s method is appropriate in terms of its accuracy due to low variance and the speed in which it can be implemented. The following derivation and implementation is used from Spors.\textsuperscript{[33]}

To start, the signal $x[k]$ is split into $L$ overlapping segments $x_l[k]$ of length $N$ with $0 \leq l \leq L-1$ starting at multiples of the step size $M \in 1, 2,…N$. The window, $w[k]$ of length $N$ is then applied to each segment. A Hanning window was
used for this analysis because of its versatility. The Fourier transform $X_l(e^{j\Omega})$ of the
windowed $l$-th segment is:

$$X_l(e^{j\Omega}) = \sum_{k=0}^{N-1} x[k + l \cdot M] w[k] e^{-j\Omega k}$$  \hspace{1cm} (13)

Normally, $N - M$ samples overlap, or it can be represented as the ratio of $\frac{N - M}{N} \cdot 100\%$; 50% segment overlap was used in this analysis. Introducing $X_l(e^{j\Omega})$ into the
definition of the periodogram gives the periodogram of the $l$-th segment as:

$$\hat{\Phi}_{xx,l}(e^{j\Omega}) = \frac{1}{N} |X_l(e^{j\Omega})|^2$$  \hspace{1cm} (14)

Lastly, averaging all of the segment’s periodograms (eq) gives the estimated PSD:

$$\hat{\Phi}_{xx}(e^{j\Omega}) = \frac{1}{L} \sum_{l=0}^{L-1} \hat{\Phi}_{xx,l}(e^{j\Omega})$$  \hspace{1cm} (15)

If the summed length of the segments exceeds the total length of the original signal, the final segment will need to be zero-padded to complete the algorithm.

Several variations on the implementation of this method were tested including windowing the segments with different windows, Hann, Hamming, Kaiser, and changing the overlap ratio. None of the different windows had a significant effect on the results and only extreme values on the overlap ratio, less than 10% or greater than 90% affected the results. As such, the Hamming window and 50% overlap ratio were deemed acceptable for the present work. A simplified
flowchart of Welch’s method is presented in Figure 3.1.
3.2 Data Quality Assessment

Several possible interference effects needed to be investigated before utilizing the data to achieve meaningful results: the Variable Frequency Drive (VFD) powering the motor for the wind tunnel fan may have given off electrical interference, the piezoelectric actuators may have given off both electromagnetic and vibrational interference, and the Scanivalve itself may have had issues aliasing the pressure signal because of the high sample rate. Each of these interference effects were investigated using the frequency analysis detailed in Section 3.1 and adjustments were made where appropriate.

To determine if the VFD was generating enough electrical interference to significantly affect the pressure data, several tests were conducted comparing the PSD for the tunnel off, the tunnel on without the model installed, and the baseline condition of the model installed without the actuators powered. Any interference would likely show up at exactly 60 Hz or fractions thereof because the electrical power used by the VFD is at 60 Hz. The off and uninstalled conditions showed no
effects at all. The baseline installed condition (Figure 3.2) did show very small spikes at approximately 70 Hz intervals. These spikes are inconsequential relative to the rest of the data and can be disregarded.

![Figure 3.2: PSD plot for baseline (0 Hz) condition](image)

Finding the interference caused by the piezoelectric actuators and the aliasing of the Scanivalve was considerably more complicated. An abbreviated version of the investigation conducted will be discussed here but more information is presented in Appendix A: Scanivalve Interference Investigation. Tests were conducted comparing the PSDs when the piezoelectric actuators were on at selected frequencies and compared when the tunnel is on verses off. Figure 3.3 shows the comparison between the actuators vibrating at 1000 Hz and 1500 Hz when the tunnel is on; there are clear interference spikes caused by the actuators. While this could be interpreted as physical phenomenon, it is more likely that the spikes are simply interference because no previous tests have found physical phenomenon above the wake shedding at 10 – 20 Hz. How this interference is generated (electromagnetic, vibrational, acoustic, pneumatic, aliasing) is not conclusively
known at this time but the exact cause is not necessary to remove the interference. By testing the piezoelectric actuators at multiple frequencies, it was shown that the interference spikes large enough to affect the data only occur higher than 35 Hz on the PSDs. This means that it is possible to remove the main effects of the interference by using a lowpass filter at 35 Hz.

![PSD Plot](image)

**Figure 3.3: Unfiltered PSD plot for 1000 and 1500 Hz actuation conditions**

Filtering the data using a lowpass filter removed the interference from the piezoelectric actuators, but several caveats must be noted to this approach. Firstly, it is possible that actual physical phenomenon was removed along with the interference. It may be unlikely, but this possibility still needs to be considered; if the synthetic jets were causing any high frequency vortex shedding on the edge of the model the filtering would remove the ability to detect this effect. Secondly, filtering at a relatively low frequency of 35 Hz removes all of the possible information gained from sampling at higher frequencies. This makes it inefficient to sample at higher frequencies; for any future testing using piezoelectric actuators
it is recommended that the highest sampling frequency is only twice the lowest interference frequency. This will help limit data files sizes and simplify data processing. Lastly, applying the filter itself significantly affected some of the results by changing or removing trends. Due to this effect, certain frequencies will be excluded from the results because it is impossible to determine if the changes were due to removing the interference, or the filter removed real trends. The effects of the filter will be discussed further in Section 4.2.

3.3 Base Pressure Map

With 25 pressure ports spread across the rear of the model it was possible to interpolate between the points to determine a complete pressure map for the rear of the model. The main goal of the interpolation is to look for large scale flow structures, i.e. which side the wake is shedding from, and attempt to see the vortex shedding and pulsing of the wake. A pressure map can be created for each timestep that data was recorded, and the images played together to create a video of the map moving through time. While the videos were helpful for pressure visualization, there is no good medium for displaying them in this report.

3.4 Center of Pressure

The location of the Center of Pressure (Cp) on the base of the model is a good predictor for which side the wake is shedding from. It was shown that the Cp on the back of the model will fall into three locations, two stable and one transient.[36] The transient location is more centrally located, but the wake is unlikely to stay centered on the model. The instantaneous Cp location was calculated by the following equation:
\[ C_{p_x} = \frac{\sum_{n=1}^{N} (c_{p_n} \cdot x_n)}{\sum_{n=1}^{N} c_{p_n}} \]  
(16)

\[ C_{p_y} = \frac{\sum_{n=1}^{N} (c_{p_n} \cdot y_n)}{\sum_{n=1}^{N} c_{p_n}} \]  
(17)

where the coefficient of pressure, \( c_p \), for each port is defined as:

\[ c_p = \frac{P - P_\infty}{\frac{1}{2} \rho_\infty U_\infty^2} \]  
(18)

When graphed using a scatterplot of all of the \( C_p \) locations, the trend in \( C_p \) becomes apparent. Figure 3.4 shows the two stable locations, left and right, but there is no third transient location. While this type of plot is helpful for seeing global trends, it can be difficult to make comparison between different data sets using scatterplots. Therefore, the data was reduced to a single \( x \) or \( y \) axis and the probability density function of the location was graphed.

![Scatterplot of \( C_p \) location on the rear of the model for the 0 Hz condition](image)

**Figure 3.4:** Scatterplot of \( C_p \) location on the rear of the model for the 0 Hz condition
This allows for clear comparisons between different conditions. Figure 3.5 (left) shows the baseline condition for the $C_p_x$ data and Figure 3.5 (right) shows the baseline for the $C_p_y$ data. This approach is good for looking at the surface effects of the jets and being able to efficiently compare frequency conditions. It should be noted that this method is equivalent to looking at the data on the surface, other methods will be used to look deeper into the effects.

![Graphs of $C_p$ distribution](image)

**Figure 3.5: $C_p$ distribution in the horizontal direction (left) and vertical direction (right)**

### 3.5 Pressure Gradient

The pressure gradient across the base of the model was calculated using the four ports labeled in Figure 3.6. These ports were chosen because they give a reasonable spread across the base and they are slightly removed from the vortex shedding on the edges of the model thus reducing error and variation in the results. These locations are similar to those used by Bonnavion. The gradient was calculated using the following equations:
Figure 3.6: Ports selected for pressure gradient calculation

\[ g_x = \frac{1}{2} \left( \frac{c_p(x_B, y_B) - c_p(x_A, y_A)}{x_B - x_A} + \frac{c_p(x_D, y_D) - c_p(x_C, y_C)}{x_D - x_C} \right) \]  

(19)

\[ g_y = \frac{1}{2} \left( \frac{c_p(x_A, y_A) - c_p(x_C, y_C)}{y_A - y_C} + \frac{c_p(x_B, y_B) - c_p(x_D, y_D)}{y_B - y_D} \right) \]  

(20)

The Cartesian components will then be converted to the polar components \((g^*, \Phi)\) referred to as magnitude and orientation. The pressure gradient will allow for a deeper look into effects the jets may have on the wake structure and since the plots can be represented at probability density functions, they allow for direct comparison between the effects of different actuation frequencies. Additionally, the gradient makes it possible to look at large scale effects of the jets with relatively few points of data. With 25 pressure ports on the back of the model the interpolated surface pressure maps should be fairly accurate in terms of flow structures but the gradient gives results without needing any interpolation.

3.6 Principle Component Analysis

The process of Principle Component Analysis (PCA) is used to reduce the dimensionality of a complex data set while retaining as much information as
possible. It can also be referred to as Proper Orthogonal Decomposition (POD) and has been widely used in aerospace applications as well as for the specific application of investigating the wake of an Ahmed Body.\textsuperscript{[23][30][35]} The derivation presented here is often referred to as the standard derivation from Jolliffe\textsuperscript{[18]} but there are other ways to reach the same conclusions, like the method presented by Ly.\textsuperscript{[25]} Figure 3.7 shows a data set first presented on its original variables $X_1$ and $X_2$ then graphed according to its two principle components, $Z_1$ and $Z_2$. It can be seen in the original plot that both of the variables are highly correlated and there is variation in each axis; this is contrasted with the principle components graph where there is only significant variation in one axis allowing for a reduction in dimensionality.

![Figure 3.7: A data set graphed along its original axis (left) and its first two principle components (right)\textsuperscript{[18]}](image)

To start, assume that $x$ is a vector of $p$ random variables. For the purposes of this work, each entry in $p$ will be a time series of pressure data taken from a pressure port on the rear of the model. Next a linear function $\alpha' x$ of the elements
of \( \mathbf{x} \) having maximum variance where \( \mathbf{a}'_1 \) is a vector of the same length as \( p \), and \( \cdot \) denotes transpose is determined:

\[
\mathbf{a}'_1 \mathbf{x} = a_{11}x_1 + a_{12}x_2 + \cdots + a_{1p}x_p = \sum_{j=1}^{p} a_{1j}x_j
\]  

Then a second linear function \( \mathbf{a}'_2 \mathbf{x} \) that is uncorrelated from \( \mathbf{a}'_1 \mathbf{x} \) with maximum variance can be found. The process repeats until \( p \) principle components are found. Generally, not all of the principle components are useful in later analysis as only the first few contain the majority of the variation in the data. This is how PCA lowers dimensionality; instead of dealing with 25 separate functions with 25 variables per function, each describing the variation seen by one pressure port, PCA shows where a majority of the variation is coming from and focuses the analysis there. In order to reach the form of the principle components useful in analyses, further derivation must be done.

Consider the first linear function \( \mathbf{a}'_1 \mathbf{x} \); the vector \( \mathbf{a}_1 \) maximizes \( \text{var}[\mathbf{a}'_1 \mathbf{x}] = \mathbf{a}'_1 \Sigma \mathbf{a}_1 \). Since the maximum will not be achieved for a finite \( \mathbf{a}_1 \), a normalization constraint of \( \mathbf{a}'_1 \mathbf{a}_1 = 1 \) must be imposed. Then Lagrange multipliers, \( \lambda \), can be introduced to maximize the variance subject to the constraint:

\[
\mathbf{a}'_1 \Sigma \mathbf{a}_1 - \lambda (\mathbf{a}'_1 \mathbf{a}_1 - 1)
\]

and differentiation with respect to \( \mathbf{a}_1 \) gives:

\[
\Sigma \mathbf{a}_1 - \lambda \mathbf{a}_1 = 0 \quad \text{or} \quad (\Sigma - \lambda \mathbf{I}_p) \mathbf{a}_1 = 0
\]

where \( \mathbf{I}_p \) is the \((p \times p)\) identity matrix. Thus, the principle components can be found by solving the eigenvalue-eigenvector problem because \( \lambda_k \) will be the \( k \)-th eigenvalue of \( \Sigma \), and \( \mathbf{a}_k \) will be the corresponding eigenvector. It can be shown that
PCA returns the eigenbasis of the covariance matrix $\Sigma$ of the original data. Since all of the variables returned by PCA are not of equal importance, it is necessary to have a way to determine the amount of variation contained in each principle component. By definition, the largest eigenvalue will correspond to the first principle component, the second largest to the second principle component and so on.

It is possible to rank the eigenvalues by how much variation they contain using the below equation:

$$E_k = \frac{\lambda_k}{\sum_{n=1}^{p} \lambda_n} \cdot 100\%$$  \hspace{1cm} (24)

This equation determines the amount of variation, or energy, contained in each principle component by dividing its relative eigenvalue by the sum of all of the eigenvalues. By mapping the eigenvectors, in this case a vector with 25 entries, to their corresponding pressure port it is possible to create principle component maps showing the wake modes responsible for the variation in the wake. (Figure 3.8) Note, when interpreting PCA maps the magnitude of the coefficients represents the relative strength of the variation and the sign represents the phase. Meaning, coefficients of 0.28 and -0.30 are relatively the same strength, but out of phase with each other. Figure 3.9 contains a flowchart for the process of decomposing data into its principle components.
Figure 3.8: First PCA mode for the baseline condition

Figure 3.9: Flowchart for the PCA process
Chapter 4

RESULTS

This chapter presents the results of the previous analyses. It shows that the collected data was repeatable and compared favorably to previous results. It was determined that the SJs did significantly affect the wake flow and the effects are dependent on the actuation frequency.

4.1 Repeatability

Extensive testing was conducted on the baseline 0 Hz actuation condition to ensure that the results compared adequately to previously published results. On every day of testing at least two baseline condition tests were run; one at the start of the day and one at the end to determine if the bi-stable effects could be observed. If the initial run showed no bi-stable effects, the model was adjusted until the bi-stable effects were observed. This involved rotating the model slightly in the yaw direction or checking the seams on the model to make sure they were not disrupting the airflow.

Figure 4.1 shows a time series of pressure data taken from two ports on the rear of the model. The back and forth switching is clear from the data and matches previous experimental investigations.\textsuperscript{[15][36][37]}
Figure 4.1: $c_p$ data taken from Port 1 at the baseline condition

Figure 4.2 shows a 2D probability plot of the $C_p$ location on the rear of the model compared previously published results.$^{[36]}$ Varon reported three distinct states but here only the two bi-stable wake positions are observed. This is likely because the wake spent so little time in the transient symmetric state that the probability was small compared to the bi-stable state probability.

Figure 4.2: $C_p$ probability distribution for Varon$^{[36]}$ (left) and the results of the current work (right)
Figure 4.3 shows the first four PCA wake modes observed in the current experiment, and those reported by Volpe. There is good agreement on multiple aspects: the wake structure of each mode is very similar, the energy percent for each mode is comparable, and the values for the principle components are within a reasonable margin of each other. All of these results taken together show that the baseline condition was able to replicate the work previously done and the analysis methods were implemented correctly. The first mode represents the bi-stable shifting of the wake; the two sides have coefficients of equal magnitude but opposite signs meaning they are similar strength but out of phase with each other. The second and third modes relate to the pumping of the recirculation bubble behind the model. The fourth mode represents the top to bottom wake shedding. More modes could be shown here, there are 25 in total because there are 25 pressure ports, but past the 4th or 5th mode the small amount of energy contained in each mode makes them unnecessary.
Figure 4.3: First four PCA wake modes for the baseline condition from Volpe\textsuperscript{[39]} (left) compared to the current results (right)
To determine repeatability, the baseline condition tests can be examined together. Figure 4.4 shows the horizontal Cp probability distribution for a sampling of the baseline tests. There is a clear bias toward one side of the model; an even 50/50 distribution was not achieved. However, an even distribution is not required, and the results show that this bias persisted through every test, so it was repeatable. The reasons for the bias can be attributed to either the model being at a slight angle to the oncoming flow, or small imperfections in the model that disrupt the airflow in unnatural ways. The seam where the rear tray connection to the main body of the model was especially troublesome to keep smooth, even with silicone sealant. Regardless, the results showed the same distribution for each test and the slightly skewed distribution is sufficient for the current work.

![Figure 4.4: Cp distribution in the horizontal direction for multiple data runs at the baseline condition (0 Hz)](image)

**4.2 Filter Effects**

Figure 4.5 shows the Cp probability distribution for the baseline and 1000 Hz actuation condition both before and after the 35 Hz lowpass filter is applied to
the data. The baseline condition does not change significantly after the filter is applied; the trends remain the same. However, the distribution in the 1000 Hz data is significantly changed. It is returned almost to the baseline bi-stable distribution, but with less steep peaks. Because of this radical change after the filter is applied, it is difficult to make conclusions about the data since it is unclear if the original distribution is a physical phenomenon, or simply caused by interference. Table 4.1 shows the results of a qualitative assessment of which frequencies were affected by the filter.

Figure 4.5: Cp distribution before and after 35 Hz lowpass filter is applied
Table 4.1: Frequencies Affected by Filtering

<table>
<thead>
<tr>
<th>Freq. (Hz)</th>
<th>F*</th>
<th>Filter Effect</th>
<th>Post-Filter Cp Distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>None</td>
<td>Bi-stable</td>
</tr>
<tr>
<td>10</td>
<td>0.08</td>
<td>None</td>
<td>Bi-stable</td>
</tr>
<tr>
<td>50</td>
<td>0.43</td>
<td>None</td>
<td>Bi-stable</td>
</tr>
<tr>
<td>116</td>
<td>1.00</td>
<td>None</td>
<td>Bi-stable</td>
</tr>
<tr>
<td>200</td>
<td>1.72</td>
<td>None</td>
<td>Bi-stable</td>
</tr>
<tr>
<td>400</td>
<td>3.44</td>
<td>None</td>
<td>Bi-stable</td>
</tr>
<tr>
<td>500</td>
<td>4.31</td>
<td>Minimal</td>
<td>Skewed, two peaks</td>
</tr>
<tr>
<td>550</td>
<td>4.74</td>
<td>None</td>
<td>Skewed single peak</td>
</tr>
<tr>
<td>581</td>
<td>5.00</td>
<td>None</td>
<td>Skewed single peak</td>
</tr>
<tr>
<td>600</td>
<td>5.17</td>
<td>None</td>
<td>Skewed single peak</td>
</tr>
<tr>
<td>698</td>
<td>6.00</td>
<td>Significant</td>
<td>Bi-stable</td>
</tr>
<tr>
<td>800</td>
<td>6.90</td>
<td>Significant</td>
<td>Bi-stable</td>
</tr>
<tr>
<td>814</td>
<td>7.00</td>
<td>Significant</td>
<td>Bi-stable</td>
</tr>
<tr>
<td>930</td>
<td>8.00</td>
<td>Significant</td>
<td>Bi-stable</td>
</tr>
<tr>
<td>1000</td>
<td>8.62</td>
<td>Significant</td>
<td>Bi-stable</td>
</tr>
<tr>
<td>1050</td>
<td>9.05</td>
<td>Significant</td>
<td>Bi-stable</td>
</tr>
<tr>
<td>1150</td>
<td>9.91</td>
<td>Minimal</td>
<td>Bi-stable</td>
</tr>
<tr>
<td>1250</td>
<td>10.78</td>
<td>None</td>
<td>Broad, multi-peak</td>
</tr>
<tr>
<td>1280</td>
<td>11.03</td>
<td>None</td>
<td>Broad, multi-peak</td>
</tr>
<tr>
<td>1400</td>
<td>12.06</td>
<td>Significant</td>
<td>Bi-stable</td>
</tr>
<tr>
<td>1450</td>
<td>12.50</td>
<td>Significant</td>
<td>Bi-stable</td>
</tr>
<tr>
<td>1500</td>
<td>12.93</td>
<td>Significant</td>
<td>Bi-stable</td>
</tr>
<tr>
<td>1550</td>
<td>13.36</td>
<td>Minimal</td>
<td>Bi-stable</td>
</tr>
<tr>
<td>1750</td>
<td>15.08</td>
<td>Minimal</td>
<td>Bi-stable</td>
</tr>
<tr>
<td>2000</td>
<td>17.24</td>
<td>None</td>
<td>Bi-stable</td>
</tr>
</tbody>
</table>

Figures 4.6, 4.7 and 4.8 show the three categories of filter effects: none, minimal, and significant. The minimal category is defined as not changing the trends the data is already showing but there are some changes in the values of the probability curve. The significant category is defined as completely erasing or adding in new trends to the data. This mostly shows up as changing from a broadband or single peak distribution into the two peaked bi-stable distribution. If a data set is significantly affected by the filter it is impossible to make concrete conclusions regarding the effectiveness of the synthetic jets with respect to the Cp
distributions. This is because the data is too contaminated with interference. Moving forward, only the conditions with minimal or no filter effects will be discussed with regard to the Cp distributions and the principle component analysis. However, it is important to note that the filtering did not significantly affect the average pressure recorded from each port. Therefore, it is still possible to use all of the data when looking at average pressure trends in the wake. More discussion on this topic is added in Appendix A: Scanivalve Interference Investigation.

![Figure 4.6: Horizontal Cp distributions for 200 Hz both before (red) and after filtering (blue) showing no filter effects](image)
Figure 4.7: Horizontal Cp distributions for 500 Hz both before (red) and after filtering (blue) showing minimal filter effects

Figure 4.8: Horizontal Cp distributions for 1500 Hz both before (red) and after filtering (blue) showing significant filter effects

4.3 Synthetic Jet Wake Effects

No jet frequency was able to significantly change the wake shedding frequency off the sides of the model, or the frequency of the pulsing of the wake bubble behind the model. Figure 4.9 shows the change in shedding frequency over
the whole range of frequencies tested; there are small variations between 930 to 1280 Hz, but these are small enough to be within error. It should be noted that the error bars here represent the error in the analysis method. Each “bin” in the periodogram was 0.6 Hz wide therefore each frequency in that range would be counted as a single value.

Figure 4.9: Wake shedding frequency for two columns of ports, left (green) and center (blue) as the SJ actuation frequency increases

There was a clear correlation of decreasing the average wake pressure as the jet actuation frequency increased (Figure 4.10). This is consistent with Park’s results and indicates that the overall drag is increasing as the actuation frequency is increasing. It is interesting that this decrease in wake pressure was observed no matter the Cp distribution in the model’s wake. This would indicate either that suppressing the bi-stable effects is independent of the average pressure in the wake or that the gains of suppressing the bi-stable effects are not enough to overcome the pressure decrease due to the jets. The error bars on the pressure
measurements represent twice the standard deviation of all average pressures recorded at that actuation frequency.

**Figure 4.10: Average wake pressure on the rear of the model**

4.3.1 $F^+ = 0$ to 4 ($0$ to $400$ Hz)

Within this range, only 166 Hz showed any deviation from the baseline results. With a $c_\mu$ of only 4.6E-5, it was unlikely that operating the jets at 116 Hz would have a large effect on the wake, however an interesting result appeared. While the baseline distribution was consistently skewed, the 116 Hz distribution was consistently an even split between the two bi-stable wake positions. Figure 4.11 shows two data sets with the jets operated at 116 Hz compared to the jets being off. Other frequencies tested near 116 Hz, 50 and 200 Hz, did not show this even distribution. If the jets simply being on stabilized the flow into two bi-stable modes, then other frequencies would show this same distribution, but 166 Hz was the only frequency that consistently produced even distributions. There was not enough
momentum transfer from the jets to remove the bi-stable effects, but the added mixing in the wake was enough to overcome any effects of imperfections in the model, or the model being angled relative to the freestream flow. It may be possible to use low power or low frequency jets to stabilize the wake in this way even when the flow is at a significant crosswind, but further experiments would need to be done to verify this. As no other frequency showed this even distribution, it is likely that the frequency required to equalize the bi-stable wake has a very narrow window of working properly. In this case, the actuation frequency was a more important variable that the overall amount of momentum transfer.

![Figure 4.11: Horizontal Cp distribution for 116 Hz actuation](image)

This evening of the distribution is confirmed by the gradient orientation. (Figure 4.12) There is a clear decrease in the time spent at $-\pi/\pi$ and an increase in the center value. PCA modes are not shown here as there is no meaningful difference between the slightly skewed baseline distribution and the even distribution.
4.3.2 $F^+ = 4 \text{ to } 8 \ (500 \text{ to } 930 \ Hz)$

Within this range, all frequencies that were not affected by filtering the data, 500, 550, 581, 600, showed a similar skewed single peaked $C_p$ distribution. The wake was certainly forced to one side, but it was not always the same side. Figure 4.13 shows two separate runs at 581 Hz compared to the baseline condition. Both show nearly identical distributions but mirrored across the centerline of the model. The choice of side was shown to be consistent for runs where the model was not taken out of the tunnel, but the side could change after an installation. Therefore, it is likely that the side choice is not dependent on the jets themselves; instead the side is chosen by external disturbances on the model or the angle relative to the flow. In this way the jets do force a specific wake distribution but not to the degree that it will always be the same regardless of other factors.
The strength of the pressure gradient probability distribution (Figure 4.14 top) for 581 Hz shows that the magnitude of the wakes are different with the second run spending much more time at a higher strength than the baseline or first run. This shows that when the wake is primarily on the right side of the model there is a higher-pressure gradient across the rear of the model. The orientation (Figure 4.14 bottom) confirms that the wakes are on opposite sides of the model during the two runs. Additionally, the orientation shows more of the bi-stable effects with two distinct peaks, although there is a clear bias. From this it appears that the bi-stable effects are not completely removed but are suppressed.
Turning to the PCA results for this frequency range, there is a clear increase in the percent of energy contained in the second mode when operating the jets at 581 Hz compared to the baseline condition. (Figure 4.15) There is a corresponding decrease in the first mode to compensate for the energy gain in the second mode, with the rest of the modes relatively unchanged.
Figure 4.16 shows a comparison of PCA maps between two runs at 581 Hz and the baseline condition. The first mode for 581 Hz still shows the clear bias to one side but it is interesting that both runs show the bias to the same side. The overall shape of the 1st mode agrees with the Cp distribution and the Cp gradient; the bi-stable effects have been suppressed but not altogether removed. Both runs at 581 Hz are the same until the 4th mode where the orientation is flipped similar to the Cp distribution. There was little to no change in the third mode while there was a phase change in the second mode compared to the baseline condition. Since both the 2nd and 3rd mode relate to the pumping of the recirculation zone behind the model it is surprising that they can be affected independently of each other.
Figure 4.16: PCA maps for the baseline condition (left), 581 Hz run 1 (center) and 581 Hz run 2 (right)

The second mode was energized by the synthetic jets which would indicate that the jets are able to affect the recirculation zone in some way at this frequency. With the corresponding decrease in average pressure it is possible that the jets are energizing the mixing layer between the freestream flow and the vortices being
shed of the sides of the model. Khalighi found that increasing the pressure in the wake increased the wake size therefore it is possible that decreasing the pressure in the wake also decreases the wake size.

4.3.3 $F^+ = 8 \text{ to } 12$ (1000 to 1280 Hz)

This range of frequencies was significantly affected by interference and filtering effects. The two conditions not effected, 1250 and 1280 Hz both showed a significantly different Cp distribution compared to the baseline condition. (Figure 4.17) Instead of the bi-stable distribution, these two frequencies elicit a broad multi-peaked distribution. It appears as if there are almost three peaks or locations the wake prefers to be in. It is possible this third, central peak is the stable symmetric wake mode that was not apparent earlier. However, it is clear that the jets have not stabilized the wake into the symmetric mode even if it now spends more time in the center.

Figure 4.17: Horizontal Cp distribution for 1250 Hz (blue) and 1280 Hz (red)
The strength and orientation of the gradient (Figure 4.18) shows similar results. The increase in the strength indicates that there is a stronger pull across the back of the model due to the wake moving rapidly. Instead of equalizing into a single symmetric state, the wake moves quickly back and forth across the model. The orientation shows that there is a preferred phase for the wake, but the phase still changes as the wake moves.

Figure 4.18: Pressure gradient magnitude (top) and orientation (bottom) of 1250 Hz (blue) and 1280 Hz (red)
Similar to the previous range, in this range the second PCA mode showed an increase in energy relative to the baseline. (Figure 4.19) The first mode also showed a decrease, but the 3rd through 5th modes also showed a decrease outside an amount that can be explained by experimental variation (two standard deviations).

![Graph showing PCA mode energy for 1250 Hz (blue) and 1280 Hz (red)](image)

**Figure 4.19: PCA mode energy for 1250 Hz (blue) and 1280 Hz (red)**

For 1250 and 1280 Hz the first two principle component modes significantly changed the amount of energy they contain because the first and second modes have switched places. Figure 4.20 shows a comparison between the baseline condition and actuating at 1250 and 1280 Hz. This shows that the mode associated with the recirculation zone pumping behind the model now accounts for the most amount of variation in the data because it is the strongest PCA mode.
Therefore, either the synthetic jets have increased the energy of the formerly second mode, or they have decreased the strength of the bi-stable effects to a point where they do not account for a majority of the variation.

Looking at the 4th mode, which in the baseline condition are associated with the top-to-bottom vortex shedding, it has also been affected when actuating at 1250 Hz. The shape of the new mode (Figure 4.21) indicates that there is more random variation than was present before. It is possible that the jets are inducing additional mixing between the recirculation bubble and the freestream flow which decreases the strength of the vortices being shed, but this is not able to be confirmed with the current pressure data on the base of the model.
Figure 4.21: Modes three to five for the baseline (left) 1250 Hz (center) and 1280 Hz (right)

4.3.4 $F^+ > 12$ (1400 to 2000 Hz)

Frequencies in this range did not deviate from the baseline bi-stable $C_p$ distribution. There were some slight changes to the distributions (Figure 4.22), but this can be explained by the randomness of the bi-stable switching. It is unlikely the synthetic jets affected the $C_p$ distributions when actuated at frequencies in this range.
Figure 4.22: Horizontal Cp distribution for 1750 Hz (blue) and 2000 Hz (red)

Similarly, the orientation of the gradient showed the bi-stable distribution, but there were some residual effects. (Figure 4.23) The strength of the gradient was significantly stronger. This can be explained by the decrease in pressure in the wake; the low pressure creates more suction across the rear of the model. The decrease in pressure is likely due to a shortening of the recirculation zone because of additional mixing from the synthetic jets. This shows that the jets can affect global properties of the wake, such as the average pressure, without affecting the bi-stable wake effects.
Figure 4.23: Pressure gradient magnitude (top) and orientation (bottom) of 1750 Hz (blue) and 2000 Hz (red)

The PCA modes for this frequency range are not shown here because they did not significantly deviate from the baseline condition.
Chapter 5

CONCLUSIONS

Wind tunnel tests of a flat-back Ahmed Body model with synthetic jets show that it is possible to affect the wake distribution of the Ahmed Body model using synthetic jets. It was found that the actuation frequency is as important to the resultant wake distribution as the amount of momentum imparted from the jets.

Using a normalized frequency, $F^+$, of one resulted in a bi-stable wake distribution that spent equal amounts of time in each bi-stable wake mode. In this case, low energy input resulted in a predictable output and may be useful in other conditions such as crosswinds.

For the range of $F^+$ from 4 to 8, it was shown that the jets do force the wake to prefer one side, giving a skewed $C_p$ distribution. The direction of skew was independent of the jets and was the result of external factors. PCA analysis showed that the bi-stable mode started to weaken but was still the primary cause of variation in the data.

For $F^+$ values from 8 to 12, the bi-stable effects were removed completely from the $C_p$ distributions, but not entirely from the pressure gradient distributions. The magnitude of the gradient was increased significantly leading to the theory that the wake moves significantly more in this range. PCA analysis showed that the mode associated with the pumping of the recirculation zone behind the Ahmed Body was responsible for most of the variation in the data and that the bi-stable mode was now second. This means that the jets increased the strength of the recirculation while decreasing the bi-stable effects.
Values of $F^*$ higher than 12 did not show any change to the bi-stable effects but did show an increase in pressure gradient magnitude. This indicates that the average pressure in the wake can be affected independently of the bi-stable effects.

However, simply changing the wake distribution alone is not enough to improve the efficiency of the model; the wake pressure decreases, and thus drag increases, with all actuation frequencies tested. Therefore, to positively improve the efficiency of the model the synthetic jets must be designed with consideration both to removing the bi-stable wake effects and preserving the pressure in the wake.

There are three categories of additional work that could yield significant results: testing different conditions such as crosswinds, implementing a controller for the synthetic jets, or gathering more data about the wake structure downstream of the model when the jets are active.

This work showed that the synthetic jets were capable of shifting the $C_p$ when there was no angle of attack in the oncoming flow. However, full size bluff bodies such as semi-trucks or passenger vans experience significantly varied conditions such as crosswinds. More testing should be done to determine if the synthetic jets are still capable of shifting the $C_p$ under these different conditions.

While the synthetic jets were capable of shifting the $C_p$, it is possible they could be more effective if driven by a controller. Since the bi-stable mode does not change on short time scales, it is feasible to measure the pressure distribution on the rear of the model, calculate the position of the bi-stable mode, and then adjust the jets accordingly. For example, if the wake was found to be shedding from the left side, the jets only on the right side could be activated, or both sets could be
activated with different frequencies. This would allow for more customization to jet’s response to the wake conditions, as well as the implementation of optimization algorithms for the actuation frequency.

Lastly, more testing could be done on the downstream wake of the model when the jets are active. Obtaining pressure data downstream of the model would allow of the identification of effects on large wake structures that the jets may have caused; methods such as hot-wire anemometry or Particle Image Velocimetry (PIV) would be particularly well suited to this task.
References


[38] Vijaya, M. S. *PIEZOELECTRIC MATERIALS AND DEVICES: Applications in Engineering and Medical Sciences*. CRC Press, 2017.


Appendix A: Scanivalve Interference Investigation

The purpose of this appendix is to provide additional background on the work done to investigate the issues the Scanivalve pressure transducer had while collecting data.

First, to establish a noise baseline within the system, data was collected with the tunnel turned off, the VFD powered down, and any electronics in the room except for the collection equipment turned off. Figure A.1 shows the noise in the collection system when there as minimal inputs as possible. This means that all further PSDs should be expected to have a minimum error of 0.003 db/Hz.

![Figure A.1: PSD for the noise baseline in the Scanivalve system](image-url)

As was shown in Section 3.2, there is considerable interference in the data from an unknown source. To attempt to determine the source of the interference and the best way to mitigate any ill effects, several tests were run by varying the actuation frequency of the synthetic jets. The results of the PSD’s are shown in figures A.2, A.3, A.4 and A.5. This shows there are non-negligible interference
spikes and that the frequency of interference is dependent on the actuation frequency of the synthetic jets. Additionally, all of the interference spikes that may have an influence on the data are above the threshold of 35 Hz used for the lowpass filter. Multiple frequencies were tested for the filter, from 20 to 50 Hz, and they all showed the same capability to remove the interference.

Figure A.2: PSD for 0 Hz actuation

![Figure A.2: PSD for 0 Hz actuation](image)

Figure A.3: PSD for 581 Hz actuation

![Figure A.3: PSD for 581 Hz actuation](image)
Now that it has been proven that the piezoelectric actuators are the source of the interference, the next step is to determine the method of energy transmission. After a meeting with engineers at Scanivalve, the following list of possibilities was compiled.
1. Mechanical Interference – The vibration from the actuators is traveling through the wind tunnel model to the transducer inside the pressure scanner. The transducer inside the model is insulated against vibration but it may still have some effect. This can be tested by running the collection system while separated from the piezoelectric actuators.

2. Acoustic Interference – The actuators make a significant amount of noise, so it is possible the sound wave is strong enough to vibrate the transducer and cause interference. This can be tested by placing the transducer inside a soundproof box.

3. Electrical Radiation – This is most likely to occur through the cables. During previous testing the cabling for the actuators was coiled and that may have amplified the electrical field generated by the current passing through the cables. To test for this interference the cabling can be moved to different positions and routes to see the effect.

4. Electrical Conducted – The main path of electrically conducted interference would be through the Earth ground on the Scanivalve system. The Scanivalve uses the Earth ground for the shielding on its cabling. There are two options for remedying this type of interference: removing the Earth ground connections with a cheater plug or filtering the power with a power conditioner.

Due to time and testing constraints, only a few possible configurations were tested (Table A.1). The following are the results of the testing for the cases shown in Table A.1; all testing was done at an actuation frequency of 1000 Hz with data collected for 60 seconds at a rate of 625 Hz.
Table A.1: Interference Test Cases

<table>
<thead>
<tr>
<th>Name</th>
<th>Description</th>
<th>Interference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Separate Tables</td>
<td>Scanivalve was placed on a separate table from the piezoelectric actuators, effectively mechanically isolated.</td>
<td>Mechanical</td>
</tr>
<tr>
<td>Same Table – Far</td>
<td>Scanivalve was placed ~450 mm away from the actuators</td>
<td>Mechanical</td>
</tr>
<tr>
<td>Same Table - Near</td>
<td>Scanivalve was placed ~5 mm away from the actuators</td>
<td>Mechanical</td>
</tr>
<tr>
<td>Scanivalve on Top</td>
<td>Scanivalve was placed on top of the housing for the piezoelectric actuators</td>
<td>Mechanical</td>
</tr>
<tr>
<td>Wrapped Wires</td>
<td>The cabling for the actuators was wrapped around the Scanivalve system while it was mechanically separated from the actuators</td>
<td>Electrical Radiation</td>
</tr>
</tbody>
</table>

With the Scanivalve completely isolated from the actuators (Figure A.6) there are still noticeable interference spikes (Figure A.7). These are much thinner than the ones presenting with the tunnel running, which suggests these are closer to the actual interference frequency for this actuation frequency. The magnitude of the spikes is quite low, but they still rise above the random noise level.

Figure A.6: Set-up for Separate Tables Test Case
Moving the Scanivalve closer to the actuators, (Figure A.8) shows that the interference gets many orders of magnitude stronger (Figure A.9). This is true for all three of the test cases where the Scanivalve is on the same table as the actuators so only the Same Table – Near results are shown here. These interference spikes are much greater in magnitude than the ones seen during other testing. This may mean the Scanivalve was better insulated within the model during testing than the position it was placed during this test, or that running the wind tunnel somehow dampens the strength of the interference. The former is much more likely.
The final test case involved wrapping the cabling for the actuators around the Scanivalve system (Figure A.10) to determine if the electric field generated by the wires was enough to impact the measurements. The Scanivalve was again physically isolated from the actuators and the results (Figure A.11) are similar to the Separate Tables test case. There was an increase in the magnitude of the PSD...
for the Wrapped Wires case, but it does not come close to the amount seen in the Same Table – Near case. This indicates that the mechanical vibration has significantly more effect on the measurement interference than the electrical radiation.

![Figure A.10: Set-up for Wrapped Wires Test Case](image)

**Figure A.10: Set-up for Wrapped Wires Test Case**

![Figure A.11: PSD results for the Wrapped Wires Test Case](image)

**Figure A.11: PSD results for the Wrapped Wires Test Case**