MODELING AND TESTING POWERPLANT SUBSYSTEMS OF A SOLAR UAS

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ABSTRACT

Modeling and Testing Powerplant Subsystems of a Solar UAS

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In order to accurately conduct the preliminary and detailed design of solar powered Unmanned Aerial Systems (UAS), it is necessary to have a thorough understanding of the systems involved. In particular, it is desirable to have mathematical models and analysis tools describing the energy income and expenditure of the vehicle. Solar energy income models may include available solar irradiance, photovoltaic array power output, and maximum power point tracker efficiency. Energy expenditure models include battery charging and discharging characteristics, propulsion system efficiency, and aerodynamic efficiency. In this thesis, a series of mathematical models were developed that characterize the performance of these systems. Several of these models were then validated against test data. Testing was conducted on specific components used by a solar UAS designed and built by students at the California Polytechnic State University, San Luis Obispo, which completed a six-hour flight relying only on solar energy in May 2019. Results indicate that, while some models accurately predicted test outcomes, others still need further improvement. While these models may be useful during the preliminary and detailed design phases of a solar powered UAS, specific component testing should be conducted to converge on the most desired design solution.

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Chapter 1

INTRODUCTION

Due to advances in both solar energy and storage technology, solar powered aircraft, in which energy from the sun is collected by a photovoltaic (PV) array and converted to mechanical power required for flight, are becoming more feasible. Examples of this increasing feasibility include successful flights from Solar Impulse [1], SunSeeker Duo [2], and AtlantikSolar [3]. Solar powered Unmanned Aerial Systems (UAS) may provide longer flight endurance than other electric (or even hydrocarbon fuel) based UAS which is desirable for many missions, including surveillance and reconnaissance, aerial mapping, and search and rescue.

The energy balance of a solar UAS may be described as the relation between incoming solar energy collected and the energy expended by the air vehicle. This energy balance is a function of environmental conditions and the performance characteristics of the solar UAS powerplant. In order to accurately predict the vehicle endurance and other characteristics, a mathematical description of these powerplant subsystems is required. This thesis provides a series of test-validated mathematical models and analysis tools characterizing the powerplant performance.

1.1 Powerplant Subsystems Investigated

Six subsystems affecting the powerplant performance of a solar UAS were investigated. A simplified diagram of these subsystems, and their interactions, is depicted in Figure 1.1. Subsystems labeled 1 through 3 describe the energy income of the solar UAS. The solar irradiance (1) describes how much solar energy is available to be extracted by

the vehicle and is independent of the specific UAS characteristics, but rather is a function of the current environmental conditions, such as the local climate type classification. The Photovoltaic (PV) array (2), determines how much of the available solar energy can be collected and converted to electrical energy. The PV array power output is dependent on the solar UAS geometry, orientation with respect to the sun, and individual solar cell performance.

Figure 1.1: Overview of Powerplant Subsystems of a Solar UAS

The Maximum Power Point Tracker (MPPT) (3) ensures the PV Array is producing the maximum possible amount of power and has an associated electrical efficiency reduction.

Subsystems labeled 4 through 6 describe the energy expenditure of the solar UAS. The battery (4) stores excess electrical energy produced by the PV array to be used when solar energy is reduced or unavailable and has resulting charging and discharging electrical inefficiencies. The propulsion group (5), which consists of an Electronic Speed Controller (ESC), electric motor, and propeller, provides the propulsive force required to maintain

flight by converting the available electrical power into mechanical power. Finally, the vehicle aerodynamics (6) describe how much mechanical power is required to sustain flight and is dependent on the specific geometry of the solar UAS.

1.2 Modeling and Testing Small Solar UAS

Of particular interest are solar powered UAS classified as small UAS (sUAS), which according Federal Aviation Regulation (FAR) Part 107 is an unmanned aircraft weighing less than 55 lbs (25 kgs) that may only be flown within visual-line-of-sight (VLOS), and at a maximum altitude of 400 ft above ground level (AGL) [4]. sUAS are often more accessible to university research groups than their larger High-Altitude Long Endurance counterparts, due to their smaller size and potentially reduced complexity.

A group of students at the California Polytechnic State University, San Luis Obispo has been designing and prototyping fixed-wing solar sUAS. The most recent prototype, designated the *MK3*, was designed as a technology demonstrator and is capable of year-round daytime solar powered flight. The models presented in this thesis are validated against the *MK3* sUAS and its associated powerplant subsystem components.

1.3 *MK3* sUAS Technical Description

The *MK3* sUAS measures 3.8 meters in wingspan and has a gross takeoff weight of 3.85 kg. The main wing has an aspect ratio of 10.7 and utilizes the Eppler E174 low Reynolds number airfoil. Both the horizontal and vertical stabilizers use the symmetric Eppler E171 airfoil. Additional dimensional characteristics of the *MK3* may be seen in Figure 1.2.

Figure 1.2: The MK3 sUAS Airframe. All Dimensions in mm

Avionics, batteries, MPPTs, and the propulsion system are housed in a fiberglass fuselage pod located at the front of the vehicle. The 3-piece wing is constructed from extruded polystyrene foam, with a carbon fiber tubular spar, while a carbon fiber tailboom runs from the fuselage pod to the empennage and serves as the attachment point for the main wing. The aircraft has a wing loading of 29 N/m^2 , and a continuous power loading of 8.73 W/N . Typical cruising speeds are 8-14 m/s, with a typical average power consumption of 45 W.

The *MK3* PV array consists of 58 SunPower C60 mono-crystalline silicon solar cells integrated into the main wing which can convert up to 22.4% of the available solar energy into electrical energy, according to manufacturer specifications [5]. The PV array electrical output is regulated by two GenaSun GV-5 MPPTs. Two onboard 3-cell 1.5 amp-hour Lithium-Polymer batteries provide approximately 20 minutes of flight time in the event that available solar power is temporarily reduced or unavailable. Mechanical power for propulsion is provided by the “MAD 5010” electric motor and the “Aeronaut” 20-inch by 8-inch carbon fiber folding propeller.

Realtime telemetry and First-Person Video (FPV) footage are relayed to a ground station while a flight computer records sensor data and provides the option for autonomous flight capabilities. Aircraft instrumentation includes a pitot-static system, inertial measurement unit, magnetometer, and voltage and current sensors, which are further described in section 5.6.2. Estimates from flight testing indicate that the avionics consume 7 W of electrical power.

The *MK3* has served as a valuable learning tool for students at the California Polytechnic State University, San Luis Obispo, and a significant aid to this thesis. On May 28th, 2019 the aircraft flew for 6 hours and 2 minutes, thereby setting a solar powered flight endurance record at the California Polytechnic State University, San Luis Obispo.

1.4 Objective

The primary purpose of this thesis is to corroborate and expand upon the available literature by providing a set of individually test-validated energy income *and* expenditure models. With a few exceptions, there is a lack of available literature discussing both the design and testing aspects of solar UAS. Furthermore, while energy income models given in [6] and [7] are viable, presented data is often the aggregate results of combined models and tests. As such, more detail needs to be added at the subsystem level. The models presented in this thesis, which are best suited for the preliminary and detailed aircraft design phases, will be developed and applied in the context of solar powered sUAS, as test data from the *MK3* aircraft will be used to assess their accuracy. Finally, improvements to the subsystem models and test procedures will be discussed. In doing so, this thesis may contribute toward a more accurate solar powered aircraft design process.

Chapter 2

LITERATURE REVIEW

Ever since the first solar powered aircraft, Sunrise I, flew in 1974 vehicle performance has progressively increased. The first manned solar powered aircraft, the modified Gossamer Penguin, took flight in 1980 with a maximum distance of two miles flown [8]. Later, Eric Raymond flew the Sunseeker I solar powered aircraft across North America in a series of 21 flights in 1990. More recently, Solar Impulse 2, flown by André Borschberg, achieved a world record endurance for manned solar powered aircraft by flying from Japan to Hawaii over the course of 117 hours and 51 minutes [1]. A more detailed and thorough history of solar powered aircraft is provided in [6] .

Solar powered UAS have the potential to provide even longer flight times due to the lack of onboard pilots. The first solar powered aircraft that completed a day and night cycle flight was Alan Cocconi's "SoLong" UAS in 2008 [9]. A team of graduate students at the Swiss Federal Institute of Technology Zurich developed a solar powered UAS called "AtlantikSolar", which set the world flight endurance record for aircraft weighing less than 50 kg with an 81-hour continuous flight in 2015 [3]. The absolute world record flight endurance for unmanned aircraft was set by the Airbus Zephyr solar UAS in 2018, at 623 hours and 57 minutes [10].

A large contributor to the increasing performance of solar powered aircraft has been improvements in solar energy collection and storage technologies. Solar energy collection is largely dependent on solar cell efficiency, which describes how much of the available solar energy can be converted into electrical energy. In 1977, a mono-crystalline silicon type solar cell, which is the most common due to low cost, was 13% efficient. As of 2018, mono-crystalline cells have reached peak efficiencies of up to 26.1%. Other solar

cells, while still in developmental stages, have reached peak efficiencies of up to 47.1% [11]. Similarly, the performance of Lithium-ion batteries, which are typically used to store electrical energy for a solar aircraft, has been improving. A significant parameter in battery electrical performance is its specific energy density, which is the amount of energy the battery can store per unit mass, typically measured in Watt-hours per kilogram. A higher specific energy means that a battery can store more electrical energy with a lower given mass, and as such is desirable for solar powered aircraft. The first commercially available Lithium-ion battery cell was released by Sony in 1991 and had a specific energy density of 80 W-h/kg [12], while a modern Panasonic 18650 Lithium-ion battery cell has a specific energy density of 243 W-h/kg [13], and is widely available for commercial use. Advances in these two technologies have increased the feasibility and practicality of solar powered aircraft.

While some traditional aircraft design methods apply to solar powered aircraft, the energy collection and storage subsystems require new analysis techniques. Due to only the recent feasibility of solar powered flight, most of these new design methods remain largely theoretical in nature [14]. In the case of solar powered aircraft that have been successfully built and flown, the design methodology is typically unavailable due to proprietary restrictions [6].

An exception to this lack of design literature is the work done at the Swiss Federal Technical Institute of Technology Zurich (ETH Zurich) [6], [7], [3]. Investigations into solar powered sUAS have been published and corroborated with the development of actual solar aircraft. In 2008, André Noth designed and constructed the “Sky-Sailor” sUAS which completed a 27-hour flight. This process is documented in [6], along with an assessment of solar aircraft conceptual design methodologies applied to vehicles of different scales.

Similarly, the conceptual design and subsequent flight testing of the Atlantik Solar sUAS is given in [3].

While the work published by students at ETH Zurich will be useful to future solar aircraft designers, information is still missing. Specifically, only the aggregate results of design models and tests are presented. For example, the solar irradiance model, which is used to predict the amount of solar energy available to the aircraft, is not individually compared to test data. Rather, the combined results of the solar irradiance, photovoltaic array, and maximum power point tracker models are compared to test results. Additionally, more engineering effort has been spent on developing solar energy *income* models, as these are the novel part of this design process. The energy *expenditure* of the solar powered aircraft, which is critical for determining the vehicle energy balance during the design process, has been largely simplified in the presented design methodologies. Furthermore, while some of the energy income models are compared to test data, none of the energy expenditure models are.

This current state of literature motivates the objective for this thesis, which is to provide test-validated energy income *and* expenditure mathematical models and analytical tools for the design of solar powered UAS. Energy income models presented by André Noth and AtlantikSolar, [6] and [7], are of use, but increased model fidelity and testing may be added at the subsystem level. Additionally, the energy expenditure of the vehicle must also be investigated in order to accurately determine the energy balance of the solar UAS. The following models presented in this thesis will attempt to satisfy this objective.

Chapter 3

ENERGY INCOME MODELING METHODOLOGY

The energy income of the solar UAS may be described by three models, which are solar irradiance, Photovoltaic (PV) array, and Maximum Power Point Tracker (MPPT). The solar irradiance model is independent of UAS characteristics, while the PV array and MPPT models require the UAS geometry, orientation with respect to the sun, and solar cell performance characteristics as inputs. This chapter will discuss the approach to developing these models.

3.1 Solar Irradiance Models

Solar irradiance is the amount of power received by a unit surface from the sun and is measured in Watts per square meter. The magnitude of solar irradiance is dependent on atmospheric conditions, and directly affects the power output of the PV Array. Note that the term “radiation” describes the energy emitted by the sun while “irradiance” describes that energy interacting with a surface.

In this section, two irradiance models are presented. The first one, called the Clear Sky irradiance model, is more generic and requires less detailed information about the operating environment of the solar UAS. The second, called the Environmentally Corrected irradiance model, requires more detailed environmental inputs. A comparison of the two irradiance models as well as a discussion regarding the selection of the more suitable model for the solar UAS design process is included at the end of this section.

Solar radiation is emitted by the sun and travels through space until it encounters the edge of the Earth’s atmosphere, where it can be broken into two categories: beam and

diffuse radiation. Beam radiation is the amount of power delivered from the sun that is not scattered by the atmosphere, whereas the direction of the diffuse radiation is first changed by atmospheric scattering before reaching the solar UAS [15]. The total power delivered from the sun is the sum of the beam and diffuse irradiance and is given in equation 3.1.

$$G_N = G_b + G_d \quad (3.1)$$

where G_N is the irradiance on a surface normal to the sun, G_b is the beam irradiance, and G_d is the diffuse irradiance. An additional quantity that is required for subsequent calculations is the total irradiance reaching a horizontal surface, G_H .

3.1.1 Sun Position Angles

To determine the direction of the total irradiance, the position of the sun in relation to the solar UAS may be described by the three angles shown in Figure 3.1.

Figure 3.1: Sun Zenith, Elevation and Azimuth Angle Definitions

The zenith angle z is the position of the sun in degrees measured from vertical while the elevation angle e is its complement. The azimuth angle a is defined as positive East of true North. The value of these three sun position angles are a function of the latitude and longitude of the solar UAS, calendar date, and time of day, and may be calculated according to the method described in [15]. The details of this calculation may be viewed in Appendix A. An example of the sun position angle values was generated using the latitude and longitude of San Luis Obispo for May 28th, 2019 and is shown in Figure 3.2. Note that *Solar Noon*, during which the sun elevation angle e is at its maximum, occurs at 13:01 local time.

Figure 3.2: Example of Sun Position Angles

The elevation, and azimuth angle may now be used to determine the direction of the total irradiance, expressed in the Earth Reference Frame E , which is defined in [16] as

$$E \equiv \begin{bmatrix} North \\ East \\ Down \end{bmatrix} \quad (3.2)$$

The direction of the total irradiance may then be described by a unit vector that is calculated in [14] as

$$\hat{S}^E = \begin{bmatrix} \cos(a) \cos(e) \\ \sin(a) \cos(e) \\ -\sin(e) \end{bmatrix} \quad (3.3)$$

This sun position vector $\hat{S}^E$ points from the solar UAS to the sun and is expressed in the Earth reference frame. The components of beam and diffuse radiation may now be calculated and summed using the following irradiance models.

3.1.2 Clear Sky Irradiance Model

A model is developed in [15] that describes the solar irradiance when there are no clouds and is called the “Clear Sky” (CS) Model. This model is valid for altitudes less than 2.5 kilometers above mean sea level and is therefore suitable for sUAS. The inputs to this model are the sun position vector, UAS altitude, and climate type which must be specified from available options as “Tropical”, “Midlatitude summer”, “Subarctic summer”, or “Midlatitude winter”. For the remainder of this thesis, the “Midlatitude summer” climate type is used, as this climate is representative of the location and time of year that testing on the *MK3* sUAS was conducted. The outputs from this model are the beam and diffuse components horizontal irradiance.

The calculations required for the CS irradiance model are presented in Appendix B. The maximum daily horizontal irradiance predicted by the CS model, $G_{H,CS}$ is shown versus time of year below.

Figure 3.3: Yearly Variation of Horizontal Irradiance $G_{H,CS}$ Predicted by Clear Sky Model

Note that the maximum available horizontal irradiance occurs during the summer solstice (June 21st, 2019) and the minimum available horizontal irradiance occurs during the winter solstice (December 21st, 2019). This yearly variation of available horizontal irradiance may dictate the portion of the year in which a solar UAS may perform a given mission.

Of more interest to the solar UAS designer may be the *daily* variation of available solar irradiance, which directly impacts flight endurance, the sizing of the PV array, battery system, and air vehicle. The daily variance of solar irradiance, in addition to the components of beam and diffuse horizontal irradiance predicted by the CS irradiance model are shown in Figure 3.4. The maximum daily value of the horizontal irradiance occurs at solar noon when the sun elevation angle e is at its daily maximum. Solar noon, and therefore the maximum daily horizontal irradiance value of 924 W/m², occurs at 13:01 local time for May 28th in San Luis Obispo, CA.

Figure 3.4: Daily Variation of Horizontal Irradiance $G_{H,CS}$

3.1.3 Environmentally Corrected Irradiance Model

A more detailed model for estimating the solar irradiance is presented in [17]. This model is similar to the CS model, but accounts for variations in local weather conditions, specifically temperature, pressure, and humidity. Thus, it will be referred to as the “Environmentally Corrected “ (EC) model. Like the CS model, the EC irradiance model requires the relative sun position angles calculated in section 3.1.1, in addition to local weather conditions, as inputs. The outputs from the EC model are the beam and diffuse components horizontal irradiance. Details of the calculations used in this irradiance model may be viewed in Appendix B. Figure 3.5 shows the total horizontal component of irradiance $G_{H,EC}$ predicted by the EC irradiance model for May 28th, 2019. The arbitrary environmental conditions used in this example were 29.92 inHg of pressure, relative humidity of 60%, and an ambient temperature of 59 °F (15 °C).

Figure 3.5: Daily Variation of Horizontal Irradiance $G_{H,EC}$ Predicted by the Environmentally Corrected Irradiance Model

3.1.4 Comparison of Irradiance Models

The primary difference between the two models presented above is the method of accounting for current environmental conditions. The CS model only requires generic climate type inputs, while the EC model requires detailed information about the environment that the solar UAS is operating in. It appears that the EC model estimates higher amounts of solar irradiance than the CS model; up to 9% at the daily maximum depending on environmental conditions. The difference between the two irradiance models can be seen in Figure 3.6.

Proper engineering judgement must be applied when selecting which solar irradiance model to use during the aircraft design process. The CS model may perhaps be used during the early stages of conceptual design as it provides more conservative irradiance estimates based on limited information, while the more detailed EC model may be used during later stages when the system is more developed.

Figure 3.6: Comparison of Horizontal Irradiance Predicted by Clear Sky and Environmentally Corrected Models

The effect of clouds on horizontal irradiance is difficult to quantify, as it is dependent on many factors including cloud geometry. While a passing cloud will reduce the available horizontal beam irradiance, it may in fact increase the horizontal diffuse irradiance. The total available irradiance however will be reduced. Statistically based methods for estimating the effects of clouds on irradiance are presented in [15].

This concludes the development of Energy Income models that are *independent* of the specific solar UAS characteristics. In the following sections, Energy Income models are developed that use inputs from the Solar Irradiance model and the characteristics of the solar UAS.

3.2 Photovoltaic Array Model

The Photovoltaic (PV) array converts the incident solar irradiance into electrical energy to be used by the UAS. A PV array is composed of several solar cells, which are electrically connected individual photovoltaic devices. The total power output from the PV array is affected by three parameters: available solar irradiance, array geometry, and solar cell electrical performance characteristics. The following model captures these influences on PV array power output, requiring available irradiance (section 3.1), solar UAS orientation with respect to the sun, and solar cell specifications provided by manufacturers as inputs.

3.2.1 Photovoltaic Array Geometric Modeling

The geometry of the PV Array on a solar UAS may be complex due to the wing dihedral and curvature of the airfoil. The analysis of this complex geometry can be simplified by approximating the array as a series of planar sections following the curvature of the actual array, on which subsequent power calculations are performed.

The power output of each array section is dependent on the solar incidence angle i , which is the angle between the total incoming solar irradiance, (described by the sun position vector in equation 3.3) and the array section unit normal vector $\hat{A}$. In order to calculate the array normal vector, which is dependent on both the UAS orientation and the orientation of the array section relative to the vehicle, the aircraft body axes must first be defined.

The aircraft body axes B are a right-handed coordinate frame constrained to move with the aircraft, and are given in [16] as:

$$B \equiv \begin{bmatrix} \text{Nose} \\ \text{Right Wing} \\ \text{Down} \end{bmatrix} \quad (3.4)$$

Thus, the orientation of the solar UAS body axes may be described in the Earth reference frame E through the Euler angles roll ϕ , pitch θ , and heading ψ [16]. Before computing the array normal vectors, their orientations in relation to the body frame must first be described through the angular offsets $\Delta\phi$ and $\Delta\theta$. The roll offsets $\Delta\phi$ are caused by wing dihedral angle, while the pitch offsets $\Delta\theta$ are caused by the camber of the main wing airfoil to which the PV array is fixed. The specific geometry of the *MK3* will be used to demonstrate these angular offsets.

The PV array of the *MK3* may be approximated by six planar array sections, shown in Figure 3.7. The corresponding pitch and roll offsets for each section are given in Table 3.1.

Table 3.1: Angular Offsets of *MK3* PV Array Sections

Section Number	$\Delta\phi$ [°]	$\Delta\theta$ [°]
1	9	-1
2	9	9
3	0	-1
4	0	9
5	-9	-1
6	-9	9

The offset roll and pitch angles of each array section, ϕ_j^* and θ_j^* , may then be calculated as

$$\phi_j^* = \phi + \Delta\phi_j \quad (3.5)$$

$$\theta_j^* = \theta + \Delta\theta_j \quad (3.6)$$

where the subscript j indicates the j^{th} planar approximation of the PV Array.

Figure 3.7: Geometrically Independent Sections of the MK3 PV Array

The array normal vector for each section, expressed in the Earth reference frame, may then be calculated as presented in [14].

$$\hat{A}_j^E = \begin{bmatrix} -\sin(\psi) \sin(\phi_j^*) - \cos(\psi) \sin(\theta_j^*) \cos(\phi_j^*) \\ \cos(\psi) \sin(\phi_j^*) - \sin(\psi) \sin(\theta_j^*) \cos(\phi_j^*) \\ -\cos(\theta_j^*) \cos(\phi_j^*) \end{bmatrix} \quad (3.7)$$

Finally, the cosine of the irradiance angle, which is required for subsequent solar cell power calculations presented in the following section, may be found by taking the dot product of the array unit normal vector and the sun position vector.

$$\cos(i_j) = \hat{S}^E \cdot \hat{A}_j^E \quad (3.8)$$

The *MK3* airframe, Euler angles, array unit normal vectors, and sun position vector may be seen in Figure 3.8. Note that array sections two, four, and six are omitted for clarity.

Figure 3.8: Photovoltaic Array Geometric Modeling Applied to the *MK3* sUAS

3.2.2 Solar Cell Electrical Performance

A PV array is composed of several solar cells, electrically connected in a series or parallel configuration. In this section, the cell performance parameters specified by the manufacturer and solar incidence angle are used to predict the power output of the solar cell at various operating conditions.

The voltage produced by a solar cell is a function of the current induced by the electrical load. The cell voltage can be plotted against the induced current in what is known as an I-V curve. The shape of the I-V curve characterizes the cell performance and is dependent on environmental factors, such as available incident solar irradiance and cell temperature, in addition to the electrical characteristics of the solar cell. An example I-V curve is shown along with a P-V curve, which is the power output of the solar cell versus the voltage, in Figure 3.9. Cell power output is calculated as

$$P_{cell} = I_{cell}V_{cell} \quad (3.9)$$

where I_{cell} and V_{cell} are the current and voltage respectively at any point along the cell I-V curve.

There are several significant points along the solar cell I-V curve that characterize its electrical performance. The short-circuit current I_{SC} (the current when the voltage is 0) and the open-circuit voltage V_{OC} (the voltage when the current is 0) are both functions of available solar irradiance and cell temperature. Perhaps the most important point is maximum power point (MP). This is the combination of current I_{MP} and voltage V_{MP} that will produce the maximum given power output for the cell P_{MP} . Solar cell efficiency η_{cell} may be defined as the following:

$$\eta_{cell} = \frac{P_{cell}}{G_H A_{cell}} \quad (3.10)$$

where G_H is the available horizontal irradiance, and A_{cell} is the cell area.

These parameters are specified by solar cell manufacturers at Standard Testing Conditions (STC), which are 1000 W/m^2 incident irradiance, and a cell temperature of 25°C . The performance of the solar cell will be negatively impacted by an increase in cell temperature. Specifically, the value of short-circuit current and open-circuit voltage will decrease, resulting in a reduction of the maximum power output. Thus, manufacturers also provide an open-circuit voltage temperature coefficient $\mu_{V,OC}$ and short-circuit current temperature coefficient $\mu_{I,SC}$.

Figure 3.9: Current-Voltage (I-V) and Power-Voltage Curves for a Solar Cell

Several models are available that predict the characteristics of a solar cell. They range from relatively simple equivalent circuits to complex modeling of the physical

processes occurring within the solar cells. The model presented here from [15] relies on an equivalent circuit that replicates the I-V curve of a solar cell. Furthermore, inputs to this model can readily be found from datasheets published by manufacturers.

The I-V curve of a solar cell may be replicated using an equivalent circuit containing a current source, a diode, and two resistors, shown in Figure 3.10.

Figure 3.10: Solar Cell Equivalent Circuit

The resulting current and voltage (I_{cell} and V_{cell}) relation for this equivalent circuit is given by

$$I_{cell} = I_L - I_D - I_{SH} = I_L - I_O \left[\exp \left(\frac{V_{cell} + I_{cell} R_S}{a} \right) - 1 \right] - \frac{V_{cell} + I_{cell} R_S}{R_{SH}} \quad (3.11)$$

where I_L is the solar irradiance-generated “light current”, I_D is the diode current and I_{SH} is the shunt current. However, the diode current and shunt current may be expressed using other circuit parameters. These parameters are the diode reverse saturation current I_O , the series resistance R_S , the shunt resistance R_{SH} , and the modified ideality factor a . Note that this equation must be solved numerically. Thus, the equivalent circuit of the solar cell may be uniquely expressed by five parameters: I_L , I_O , R_S , R_{SH} , and a .

These five equivalent circuit parameters are not available from solar cell manufacturers. Instead, the parameters that are provided by manufacturers, $I_{SC,ref}$, $V_{OC,ref}$, $V_{MP,ref}$, $I_{MP,ref}$ (Figure 3.9), and $\mu_{V,OC}$, must be used to solve for the equivalent circuit values. The subscript *ref* indicates a value measured at Standard Test Conditions (STC) for solar cells.

The equivalent circuit parameters may be related to the actual cell operating parameters using known points along the I-V curve, such as the open-circuit voltage and short circuit current conditions. This results in a system of five nonlinear equations that are then solved numerically to obtain the five equivalent circuit parameters at STC ($I_{L,ref}$, $I_{O,ref}$, $R_{SH,ref}$, $R_{S,ref}$, and a_{ref}). Details of this calculation are provided in Appendix C.

In actuality, the solar cell is almost never operating at STC. Therefore, the reference equivalent circuit parameters must be converted to the operating equivalent circuit parameters. In particular, the operating light current I_L is nearly linearly proportional to the amount of solar irradiance received, which is determined by the irradiance angle i as calculated in section 3.2.1. Thus, the geometric modeling of the PV array will have a direct impact on the cell power output.

I-V curves for a solar cell, generated using the equivalent circuit model are presented in Figure 3.11.

Figure 3.11: Solar Cell Model I-V Curves at Various Operating Conditions

Note that the short-circuit current I_{SC} as well as maximum cell power P_{MP} decrease with available solar irradiance and cell temperature, therefore it is necessary to accurately predict the available solar irradiance using the geometric modeling technique presented in section 3.2.1.

In order to assess the applicability of the solar cell equivalent circuit model, a modeled I-V curve was compared to an I-V curve from a solar cell specification sheet at STC [5] in Figure 3.12.

Figure 3.12: Comparison of I-V Curves from Datasheet and Equivalent Circuit

Solar Cell Model

The error at of the maximum power point P_{MP} prediction between the datasheet and the equivalent circuit model is 0.01%.

3.2.3 Effect of Encapsulation Material

Bare solar cells are easily susceptible to damage and are therefore protected by being covered in an encapsulation material. Glass is a typical encapsulation material on ground-based PV arrays. However, for a solar UAS, a thin and flexible thermoplastic such as Ethylene-Vinyl Acetate (EVA), is much more suitable as an encapsulation material.

The encapsulation material, while increasing array durability, has a detrimental effect on the amount of solar energy absorbed, especially at larger incidence angles i . A method is presented in [15] for estimating the ratio of power output from an encapsulated PV array P_{enc} to the power output from a bare array P_{bare} as a function of incidence angle, and may be viewed in Appendix C. Inputs to this model are the optical properties of the

encapsulation material, such as its index of refraction and thickness, and the solar incidence angle. The results of this estimate can be seen in the figure below.

Figure 3.13: Effect of Solar Cell Encapsulation Material

At low incidence angles, when the solar cell is oriented more normal to the sun, the encapsulation material has little effect on the absorbed solar energy. However, at higher irradiance angles, the detrimental effect of the encapsulation material quickly becomes apparent. This effect can be seen by simulating the PV array power flux compared to the available solar irradiance over the course of an entire day in Figure 3.14. Note that the array power output is significantly diminished during dawn and dusk, as the encapsulation material reflects more of the available solar energy.

Figure 3.14: Effects of Photovoltaic Array Encapsulation Material on Power Output

3.2.4 Estimating Power Output of the Entire PV Array

The collective power output of the entire solar sUAS PV array will now be estimated using the models presented above. If the solar cells in the PV array all receive identical levels of irradiance, the power output of the PV array P_{PV} is directly proportional to the total number of solar cells in the array n .

$$P_{PV} = nP_{cell} \quad (3.12)$$

However, if the cells are not receiving the same amount of solar irradiance, equation 3.12 does not hold true. Instead, the cells receiving the lowest irradiance will act as an electrical load and reduce the total power output of the PV array. This is a foreseeable issue for a solar UAS, as the non-planar array geometry will result in varying solar cell irradiance levels, especially during dawn or dusk. Predicting the array power output with varying levels of irradiance requires detailed electrical modeling of the individual cells in the array and their interactions, as shown in [18].

For the purpose of the model presented here, the following simplifying assumption is made: the power output of a PV array with nonconstant irradiance P_{PV} , may be approximated be an equivalent array P_{PV}^* with constant cell irradiance, using an area-weighted average irradiance angle.

$$P_{PV} \approx P_{PV}^*(n, \cos(\bar{l})) \quad (3.13)$$

This assumption allows equation 3.12 to be used to estimate the total power output of the array. The constant solar cell irradiance value, which is used to predict the individual cell power output in section 3.2.2, is computed using the area-weighted average irradiance angle $\bar{l}$.

$$\cos(\bar{l}) = \frac{\sum_{j=1}^{j=k} \cos(l_j) A_j}{A_{PV}} \quad (3.14)$$

where the subscript j indicates the j^{th} planar PV array section, A_j is the area of each planar array section, k is the total number of planar sections, and A_{PV} is the total area of the PV array (section 3.2.1).

Additionally, the array temperature will have an impact on the maximum power output. A flight-test based approximation for the PV array temperature T_{PV} of a solar sUAS is given in [7] as

$$T_{PV} \approx T_{amb} + \Delta T_{max} \frac{P_{PV}}{P_{PV,max}} \quad (3.15)$$

where T_{amb} is the ambient air temperature, ΔT_{max} is a temperature constant equal to 12°C, P_{PV} is the current power output of the PV array, and $P_{PV,max}$ is the maximum possible power output of the PV array.

A phenomenon that is not captured by this model is the partial shading of the array from components of the solar UAS, such as the empennage. This would most likely have the greatest impact during dawn or dusk when shadows are more pronounced. Reference [7] provides an example of energy income modeling that accounts for this effect. An improvement to the model above would be to include the detrimental effect of partial shading on the PV array based on the methodology provided in [7].

3.3 Maximum Power Point Tracker Modeling

The shape of the I-V curve of the PV array on a solar UAS is constantly altered due to changes in the aircraft orientation with respect to the sun, available solar irradiance, and array temperature, therefore the location of the maximum power point P_{MP} along the I-V curve is also subject to change. Because of this changing maximum power point location, a device is needed that continually tracks and draws the maximum power from the array. This device is called a Maximum Power Point Tracker (MPPT).

An MPPT employs a control law to draw the maximum power from the PV array. It then converts the output voltage to a constant value to be used by other aircraft electrical loads, such as the motor, avionics, and battery. This process is shown in Figure 3.15.

Figure 3.15: MPPT Electrical Inputs and Outputs

Perhaps the most important MPPT parameter related to energy income modeling is the electrical efficiency η_{MPPT} . This efficiency may be defined as

$$\eta_{MPPT} = \frac{P_{UAS}}{P_{PV}} = \frac{I_{UAS}V_{UAS}}{I_{PV}V_{PV}} \quad (3.16)$$

where P_{UAS} is the electrical power delivered to the UAS and P_{PV} is the electrical power collected by the PV array.

The MPPT efficiency is dependent on the specific maximum power point tracking algorithm implemented. Various tracking algorithms have been developed, each with advantages and disadvantages. The most common, due to its simplicity, is called Perturb and Observe. In this method, the MPPT controller will incrementally increase the electrical current drawn from the PV array. If the measured array power output increases ($V_{PV} I_{PV}$), the controller will again incrementally increase the current. However, if the array power output decreases from the previous measurement, the controller will then decrease the current drawn from the array by the MPPT. Thus, the Perturb and Observe algorithm may be described as a “trial and error”-type process.

A block diagram of this procedure is shown in Figure 3.16 below, where k indicates the current measurement recorded by the MPPT controller, $k - 1$ indicates the previous measurement, and C is the incremental current adjustment.

Figure 3.16: “Perturb and Observe” MPPT Tracking Algorithm

The Perturb and Observe algorithm, while simple to implement in an MPPT controller, results in a loss of electrical efficiency, due to the continued oscillation around the array maximum power point P_{MP} . Other algorithms exhibit various levels of electrical efficiency reduction when attempting to track the maximum power point.

The algorithm represents a major difficulty when attempting to model the electrical efficiency of the MPPT, as manufacturers do not list the specific type of tracking method employed. Additionally, the specific circuitry and electrical components used in the MPPT, such as diodes, inductors, and MOSFETs, must be modeled to predict the device efficiency, as shown by [19].

For the sake of brevity, and to reduce the number of assumptions in proprietary tracking algorithms, it was decided that test data would suffice in determining the MPPT efficiency. Testing results from [7] indicate that this efficiency should be close to 95%. This value may be used in the solar UAS design process until testing is conducted on the specific MPPT to be used.

$$\eta_{MPPT} = 0.95 \quad (3.17)$$

This concludes the development of Energy Income modes. In the detailed models presented in this chapter, the predicted energy income is a function of solar UAS global position, orientation, PV array geometry, solar cell characteristics, and available solar irradiance. In the next chapter, models that predict the energy expenditure of the solar UAS will be described.

Chapter 4

ENERGY EXPENDITURE MODELING METHODOLOGY

While solar energy may be collected by the UAS through the processes described in Chapter 3, it is expended through the battery charging and discharging process, conversion of electrical power to mechanical power via the propulsion group, and the aerodynamic power required to maintain flight. This chapter will discuss the development of models and analysis tools that predict the energy expenditure of a solar UAS.

4.1 Battery Model

There are two main characteristics of batteries that have significant effects on the design of solar UAS. The first is the specific energy density of the battery, which is the amount of electrical energy stored by the battery per unit mass. A higher specific energy density is more desirable for a solar UAS, as it implies a reduced battery weight for a given amount of electrical energy. The second characteristic is the electrical efficiency of the battery, which describes the amount of energy lost through the charging and discharging process. In this section, a time-dependent battery model is developed from [20] that captures these characteristics. Furthermore, the inputs to this model may be derived from a manufacturer specification sheet of battery performance.

4.1.1 Specification-Based Battery Model

The voltage of a battery V may be described as a function of its instantaneous capacity Q_{inst} , and additional parameters.

$$V = V_0 - K \frac{Q_{max}}{Q_{max} - Q_{inst}} + A \exp(-BQ_{inst}) - RI \quad (4.1)$$

where,

I = battery electrical current, positive for discharging (A)

V_0 = battery constant voltage (V)

K = polarization voltage (V)

Q_{max} = maximum battery capacity (Ah)

$Q_{inst} = \int I dt$ = instantaneous discharge capacity, 0 when battery is fully charged (Ah)

A = exponential zone amplitude (V)

B = exponential zone time constant inverse (Ah)⁻¹

R = internal resistance (Ω)

Note that this model must be numerically integrated in order to find the instantaneous battery discharge capacity Q_{inst} . The applied electrical current I is specified by the user. The parameters Q_{max} and R may be obtained directly from manufacturers (if an internal resistance is not provided, it must be physically measured from the actual battery). The parameters V_0 , K , A , and B are all derived from a manufacturer-provided discharge profile, which is a plot of the battery voltage versus instantaneous capacity. A typical discharge profile is shown in Figure 4.1 below.

Figure 4.1: Typical Battery Discharge Profile

The discharge profile may be defined by three zones: an exponential zone, which occurs toward the beginning of the discharge and is characterized by a sharp decrease in cell voltage, a nominal zone, which occurs toward the middle of the discharge and is characterized by a nearly linear voltage decrease, and a sub-nominal zone which occurs at the end of the charge cycle and is characterized by a rapid voltage decrease. If the battery is operated in the sub-nominal zone for extended periods of time, permanent damage may occur in a process known as over-discharging.

In the following procedure, three points on the 0.2C discharge profile are used to derive the parameters required for equation 4.1. The “C-rate” is a measure of how fast a battery is charged or discharged and is defined as

$$C = \frac{Q_{max}}{I} \quad (4.2)$$

where Q_{max} is the maximum capacity of the battery in Amp-hours, and I is the charge or discharge current in amps. For example, if the maximum capacity of the battery is 3.5 A-h and the discharge current is 0.7 A, the battery is being discharged at 0.2C.

The three points on the manufacturer-provided 0.2C discharge curve that are required for this model are the starting conditions $(0, V_{full})$, end of the exponential zone (Q_{exp}, V_{exp}) , and end of the nominal zone (Q_{nom}, V_{nom}) . These three points may be seen in Figure 4.1 above. The parameters V_0 , K , A , and B must be derived from these three points on the 0.2C discharge curve to complete the battery model.

The exponential zone amplitude A may be found by subtracting the voltage at the end of the exponential zone V_{exp} from the full voltage V_{full} .

$$A = V_{full} - V_{exp} \quad (4.3)$$

The exponential zone time constant inverse B is given by

$$B = \frac{3}{Q_{exp}} \quad (4.4)$$

where Q_{exp} is the instantaneous capacity at the end of the exponential zone. The polarization voltage K may be given by

$$K = \frac{(V_{full} - V_{nom} + A (\exp(-B Q_{nom}) - 1)) (Q_{max} - Q_{nom})}{Q_{nom}} \quad (4.5)$$

Finally, the voltage constant V_0 is calculated as

$$V_0 = E_{full} + K + RI - A \quad (4.6)$$

This concludes the development of the battery voltage model. The following assumptions were applied from [20].

- The internal resistance is constant during charge and discharge cycles
- The model input parameters, which are derived from the 0.2C discharge profile, are assumed to be identical for the charge profile

- The capacity of the battery does not change with the amplitude of the current
- Cell temperature does not affect model behavior
- Self-discharge (internal chemical reactions that reduce charge in the absence of an electrical load) is not modeled

A limitation of this model is that the voltage at maximum discharge capacity ($Q_{inst} = Q_{max}$) can never be predicted, as this would result in a discontinuity in equation 4.1. However physically, this would represent the battery being completely discharged, which is an unrealistic situation due to the safety hazard this would pose. Therefore, the model is applicable only to normal operating regions on the discharge curve.

4.1.2 Use of Battery Model to Determine Specific Energy Density and Electrical Efficiency

The charge and discharge profiles produced by the battery model may be used to predict the specific energy density as well as the electrical efficiency of the battery. The instantaneous power output from the battery during discharge may be found by multiplying the current and voltage.

$$P = IV \quad (4.7)$$

The total energy during the discharge may be found by integrating the power with respect to time.

$$E = \int P \, dt = \int IV \, dt \quad (4.8)$$

The battery energy can then be divided by the mass m , to find the specific energy density e .

$$e = \frac{E}{m} \quad (4.9)$$

This process was modeled using a generic battery and can be seen in Figure 4.2.

Figure 4.2: Total Electrical Energy Predicted by Battery Model

The total energy at the end of the discharge cycle may then be divided by the battery mass to obtain the specific energy.

During a charge and discharge cycle, a battery exhibits a loss of energy due to the internal resistance. This energy loss is characterized by the battery electrical efficiency η_{batt} , and may be defined as the ratio between total electrical energy provided by the battery during discharge to the total energy accepted during charge [6].

$$\eta_{batt} = \frac{E_{dchg}}{E_{chg}} \quad (4.10)$$

where E_{dchg} is the energy provided during discharge and E_{chg} is the energy accepted during charge, and may be found using equation 4.8. The voltage and energy profiles for a charge and discharge cycle were computed using the battery model presented above and are shown in Figure 4.3.

Figure 4.3: Electrical Energy Loss During Charge and Discharge Cycle Predicted by Battery Model

Note that the x-axis, “Discharge Capacity”, refers to the capacity that has been *discharged* by the battery and, as such, decreases during a charge cycle and is 0 when the battery is fully charged. It can be observed that a charge and discharge cycle leads to an overall energy loss from the battery, due to the internal resistance. In the case of this generic battery electrical efficiency is calculated to be 0.91.

This concludes the development of a time-dependent battery model that can estimate the two important battery performance parameters required for solar UAS design, which are the specific energy and electrical efficiency. Additionally, voltage and current can be estimated throughout the charge and discharge cycle, allowing for more detailed design of electrical systems.

4.2 Propulsion Group Model

The propulsion group converts available electrical power into mechanical propulsive power required for flight. The group consists of an electronic speed controller (ESC), electric motor, and propeller. The ESC and motor have an associated electrical efficiency reduction while the propeller exhibits a mechanical efficiency reduction. In this section, previously published test data is used to predict the electrical efficiency of the ESC and motor, while an analysis software is used to predict the propeller efficiency.

4.2.1 Electronic Speed Controller

The electronic speed controller (ESC) converts incoming direct-current (DC) from the batteries into three-phase alternating-current (AC) to be used by the motor and directly controls the speed at which it is operating. The efficiency reduction is caused by heat produced by the ESC and by the process of converting DC electric power to AC electrical power [21]. Similar to the maximum power-point tracker, it is difficult to model the ESC efficiency short of modeling the actual circuitry contained within. However, experimental results published in [21] indicate that typically ESC efficiencies remain at 90% or above for normal electric motor operation.

4.2.2 Electric Motor

The electric motor converts the AC electrical power from the ESC into mechanical power transferred to the propeller. The associated efficiency loss may be due to heating, internal electrical resistance, and internal friction. In [6], a database of 224 brushless electric motors was compiled that examined motor power and efficiency. It was found that for a continuous maximum power output of 100 W and above, motor efficiencies of 80% or higher could be expected, with a peak efficiency of 93% for a 3000 W motor.

4.2.3 Propeller Efficiency

The propeller converts the mechanical power provided by the motor to propulsive power and has an efficiency reduction due to aerodynamic phenomena. The efficiency of a specific propeller geometry may be analyzed using a concept called Blade-Element Momentum theory (BEM).

In BEM, a propeller geometry can be discretized into a series of elements described by a local chord, twist, and radial position from the propeller's axis of rotation. Applying the conservation of momentum, the local tangential and axial flow velocity components may be found at each element. A panel code may then be used to estimate the thrust and torque at each section dependent on the airfoil used by the propeller and the local coefficients of lift and drag. The resultant forces on each section are then summed to find the total thrust and torque produced by the propeller at a given airspeed and rotational speed. Furthermore, the propeller mechanical efficiency may be estimated using BEM, and is calculated as

$$\eta_{prop} = \frac{TV}{Q\omega} \quad (4.11)$$

where T is the thrust produced by the propeller, V is the free-stream air velocity, Q is the torque required to rotate the propeller, and ω is the angular velocity of the propeller.

A program called JBlade was developed in [22] that utilizes BEM to estimate propeller efficiency in addition to other performance characteristics such as thrust, torque, and power required. The geometry of the propeller used by the *MK3* was recreated in JBlade in order to estimate its specific performance characteristics.

4.2.3.1 Propeller Geometry Measurement

JBlade requires four geometric inputs to describe the propeller blade geometry: radial position, twist angle, chord, and airfoil. In order to obtain these values, two photos of the *MK3* propeller blades were taken; one displaying the projected view of the blade from directly upstream and the other depicting a horizontal view in the propeller's plane of rotation.

The propeller photos were then digitized in order to find the upstream projected chord a and the rotation plane projected chord o . The blade pitch angle β was then found using:

$$\beta = \text{atan}\left(\frac{o}{a}\right) \quad (4.12)$$

The local blade chord C_{blade} was found by using the Pythagorean theorem.

$$C_{blade} = \sqrt{o^2 + a^2} \quad (4.13)$$

The results of the measurement process are depicted in Figure 4.4 .

Figure 4.4: Propeller Geometry Measurement Results

Note that the circular plot markers indicate the blade-wise station that was specified in JBlade. The propeller airfoil was assumed to be a symmetric NACA 4-series airfoil, with the local blade thickness and chord used to calculate the airfoil thickness-to-chord ratio. The propeller geometry, displayed in the JBlade design environment, is shown in Figure 4.5.

Figure 4.5: Measured Propeller Geometry in the JBlade Design Environment

Once the *MK3* propeller geometry was entered, the JBlade blade-element-momentum simulation was conducted to determine propeller performance characteristics. Propeller

efficiency was plotted against freestream velocity for a constant rotational speed of 3000 Revolutions per Minute (RPM) and may be seen in Figure 4.6.

Figure 4.6: MK3 Propeller Efficiency Predicted by JBlade

Jblade predicts a maximum propeller mechanical efficiency of 0.67 which occurs at an airspeed of 11 m/s and at 3000 RPM.

The total efficiency of the propulsion system may then be estimated as the combined efficiencies of the ESC η_{ESC} , electric motor η_{motor} , and propeller η_{prop} .

$$\eta_{Pgroup} = \eta_{ESC} \eta_{motor} \eta_{prop} \quad (4.14)$$

where η_{Pgroup} is the overall efficiency of the propulsion group.

4.3 Vehicle Aerodynamics Model

An additional model required to estimate the energy expenditure of a solar powered UAS is the vehicle aerodynamics. The power required for an aircraft to stay aloft, P_{level} is given as

$$P_{level} = \left(\frac{2}{\rho S_{ref}} \right)^{\frac{1}{2}} \left(\frac{W}{C_L} \right)^{\frac{3}{2}} C_D \quad (4.15)$$

where ρ is the local air density, S_{ref} is the wing reference area, W is the weight of the aircraft, and C_L and C_D are the non-dimensional lift and drag coefficients, and may be calculated using equations 4.16 and 4.17.

$$C_L = \frac{L}{\frac{1}{2} \rho V^2 S} \quad (4.16)$$

$$C_D = \frac{D}{\frac{1}{2} \rho V^2 S} \quad (4.17)$$

where L and D are the lift and drag forces created by the aircraft, ρ is the air density, V is the aerodynamic velocity, and S is the wing reference area.

The values of C_L and C_D are not independent but are related to each other through an aircraft drag polar. The drag polar is a plot of lift coefficient versus drag coefficient and is a function of the aircraft geometry. The ratio of C_L to C_D is called the Lift-to-Drag ratio and is analogous to the “aerodynamic efficiency” of the aircraft. A higher lift-to-drag ratio means the aircraft is more aerodynamically efficient, and thus requires less energy to sustain flight, therefore an accurate drag polar estimation method is required for determining the energy expenditure of the solar UAS.

In the following section, a drag build-up is used to calculate the drag polar of the *MK3* based on geometric inputs. A drag build-up is a method in which the drag of individual aircraft components, such as the wing, fuselage, etc., are estimated and then summed to predict the total drag. Two numerically based programs were used in conjunction with empirical estimates from [23] to predict the various types of drag created by the geometry. It should be noted that this model was developed, and subsequent flight testing conducted, prior to the integration of the PV array on the *MK3*.

4.3.1 Induced Drag

Induced drag is the drag created due to the production of lift. Lifting surfaces on aircraft, such as the main wing and horizontal tail, are the primary contributors of induced drag. A useful tool for estimating the induced drag of an aircraft is a Vortex Lattice Method (VLM).

A VLM is a numerical tool that discretizes a given aircraft geometry into a series of panels and can then quickly estimate the induced drag created. Furthermore, the total forces, moments, and control surface deflections required to trim the aircraft may be calculated. VLMs do not, however, predict viscous (or pressure-caused) drag or flow separation. Additionally, the following assumptions must be made according to [24]:

- The flowfield around the aircraft geometry is incompressible, inviscid, and irrotational
- The lifting surfaces that are modeled must be thin
- The angle of attack and sideslip must be small

The VLM used in this model is Athena Vortex Lattice (AVL) created by Dr. Mark Drela and Harold Youngren [25]. Figure 4.7 depicts the geometry of the *MK3* sUAS in AVL after it has been discretized into panels. AVL is then run at a specified lift coefficient value and the estimated induced drag, spanwise lift coefficient distribution, and elevator deflection angle required for trim are calculated. The spanwise lift coefficient distribution C_l , which is used in subsequent calculations, is shown in Figure 4.8 as an example of the inviscid aerodynamic information available from this VLM.

Figure 4.7: *MK3* Solar sUAS AVL Model After Panel Discretization

Since a VLM is a numerical method, the discretization of the geometry may affect the accuracy of the results. In order to increase confidence in the accuracy of the AVL predictions, a grid resolution study was conducted. In this study, the density of panels on the main wing was increased while the spanwise efficiency factor e was recorded. This process can be seen in Figure 4.9.

Figure 4.8: Section Lift Coefficient Distribution from AVL

It appears that the solution begins to converge with 144 panels present on the main wing. It is highly recommended that any users of this model familiarize themselves with Vortex Lattice Methods in general. References [25] and [24] provide sufficient detail on the subject.

Figure 4.9: Results of AVL Grid Resolution Study

4.3.2 Wing Viscous Drag

After the induced drag is calculated using AVL, the viscous drag of the wing and empennage is calculated. This process starts with estimating the 2-dimensional section viscous drag coefficient using a tool called XFOIL [26], which is a numerically-based program designed for the subsonic viscous analysis of airfoils.

In order to estimate the airfoil viscous drag at each station along the wing, the local Reynolds number Re is first calculated.

$$Re_i = \frac{\rho V c_i}{\mu} \quad (4.18)$$

where ρ is the air density, V is the aerodynamic velocity of the aircraft, c_i is the wing chord at the i^{th} spanwise colocation point (from the AVL panel discretization), and μ is the dynamic viscosity of air.

The section viscous drag coefficient of the airfoil, $C_{d,af\ell}$ is then calculated at each spanwise station across the wing by XFOIL using the local Reynolds number, the section lift coefficient C_l provided by AVL (Figure 4.8), and the specific airfoil geometry. The total drag of the wing, $C_{D,wing}$ may then be found by taking an area-weighted average of the section drag coefficients.

$$C_{D,wing} = \frac{\sum_{i=1}^n S_i C_{d,af\ell,i}}{S_{ref}} \quad (4.19)$$

where n is the total number of spanwise stations, S_i is the local strip area, $C_{d,af\ell,i}$ is the viscous airfoil drag calculated by XFOIL, and S_{ref} is the wing reference area. This process was then repeated for both the horizontal and vertical stabilizers. It should be noted that in XFOIL, flow transition to turbulence was forced at $x/c = 0.15$, as this is the chordwise

location of the leading edge of the PV array. It is assumed that this sharp step would cause a transition on the actual wing surface.

4.3.3 Parasitic Drag Build-up for Non-lifting Components

Finally, empirical results from [23] are used to estimate the drag of non-lifting surfaces, as well as lifting surface imperfections. The following section details the drag calculation performed for each component of the *MK3* sUAS, which are then summed in a process is known as a parasitic drag build-up. All drag coefficients are referenced to the wing reference area, S_{ref} .

4.3.3.1 Array Relief

Because the encapsulated SunPower C60 solar cells used on the *MK3* have a thickness of approximately 2 mm, a relief was cut into the upper surface of the main wing in an effort to maintain a smooth and accurate airfoil cross section once the array was integrated. As previously mentioned, the *MK3* was modeled and tested without the PV array, therefore the array relief geometry, shown in Figure 4.10, was exposed to the flow field around the aircraft during flight testing and was modeled accordingly.

Figure 4.10: PV Array Relief Dimensions on Main Wing (Modified E174 Airfoil)

The drag caused by this imperfection was approximated as the drag caused by a sheet metal joint exposed to the airflow in [23]. S. Hoerner provides the following approximation for the drag of sheet metal joints.

$$C_{D,relief} = 3 \left(\frac{h}{c_{ref}} \right)^{\frac{1}{3}} c_{D_0} \frac{hb}{S_{ref}} \quad (4.20)$$

where h is the depth of the relief, c_{ref} is the mean aerodynamic chord, and c_{D_0} is the independent drag coefficient of the relief boundary shape which can be obtained from the results presented in [23]. The variable b is the spanwise length of the array relief.

Figure 4.11: Drag of Sheet-Metal Joints from “Fluid Dynamic Drag”

4.3.3.2 Antenna

The antenna of the *MK3*, which transmits the vehicle telemetry, is assumed to be circular in cross section. From data provided in [23], the drag coefficient of a cylinder at Reynolds numbers encountered during flight can be assumed to be a constant value of 1.2. This is then multiplied by the antenna frontal area $S_{antenna}$ and divided by the wing reference area.

$$C_{D,antenna} = 1.2 \frac{S_{antenna}}{S_{ref}} \quad (4.21)$$

4.3.3.3 Control Surface Gaps

Large control surface gaps are present on the *MK3*, notably on the bottom surface of the ailerons. The empirically determined drag coefficient of these gaps is listed as $C_{D,gap} = 0.02$ for the lower surface of the wing. Thus, the drag due to the control surface gaps referred to the wing reference area is

$$C_{D,gap} = 0.02 \frac{A_{gap}}{S_{Ref}} \quad (4.22)$$

where A_{gap} is the area of the control surface gaps at the surface of the wing.

4.3.3.4 Wing-Body Pylon

A pylon connects the wing of the *MK3* to the carbon fiber tail boom. The shape of this pylon was approximated as a NACA 0015 airfoil. XFOIL was then used to calculate the section drag coefficient C_d at the current Reynolds number of the pylon.

$$C_{D,pylon} = C_d \frac{S_{pylon}}{S_{ref}} \quad (4.23)$$

where S_{pylon} is the area of the pylon.

4.3.3.5 Skid

As described in [23], a skid has approximately the same drag coefficient as a faired tailwheel. This drag coefficient is listed as 0.27. Both the main skid and the tail skid of the *MK3* are accounted for in this calculation, given by:

$$C_{D,skid} = 0.27 \frac{S_{skid}}{S_{ref}} \quad (4.24)$$

where S_{skid} is the frontal area of the skid.

4.3.3.6 Fuselage Pod

Next, the Reynolds number-dependent drag coefficient of the *MK3* fuselage pod was calculated according to the method given in [23]. Characteristic dimensions of the fuselage pod, which is square in cross section, are given in millimeters in Figure 4.12.

Figure 4.12: Fuselage Pod Characteristic Dimensions [mm]

The first step in calculating the fuselage parasitic drag is to find the coefficient of skin friction, C_f , by interpolating the following graph presented in [23].

Figure 4.13: Coefficient of Friction as Presented in “Fluid Dynamic Drag”

The relative grain size is the height of the surface imperfections k divided by the fuselage length l which is 500 mm for the *MK3*. The height of the surface imperfections on the *MK3* fuselage, which was fabricated using smooth fiberglass, was estimated to be $2 \cdot 10^{-6}$ m. Additionally, the boundary layer on the fuselage was assumed to be fully turbulent, despite the relatively low Reynolds numbers encountered during flight ($2.5 \cdot 10^5$ to $8.0 \cdot 10^5$). This was assumed due to the sharp edge at the front of the fuselage as well as the effect of propwash during powered flight.

Once the coefficient of friction was found, the parasitic drag coefficient of the fuselage was calculated using equation 4.25.

$$\frac{C_{D0,fuse}}{C_f} = 3 \left(\frac{l}{d} \right) + 4.5 \left(\frac{d}{l} \right)^{0.5} + 21 \left(\frac{d}{l} \right)^2 \quad (4.25)$$

where l is the fuselage length and d is the maximum fuselage diameter.

An additional type of drag present on the fuselage is pressure drag, which results from separated flow behind an object. For the *MK3* fuselage, it was assumed that the flow

at the rear of the fuselage was completely separated due to the relatively sharp tailcone sweep angle. In order to calculate the pressure drag of the fuselage, the skin friction drag of the forebody, C_{fB} is first calculated as:

$$C_{fB} = C_f \frac{S_{wet}}{S_B} \quad (4.26)$$

where C_f is the skin friction coefficient, S_{wet} is the wetted surface area of the fuselage, and S_B is the base area which in this case is assumed to be equal to the frontal area for the *MK3* fuselage pod. The drag coefficient for the pressure drag can then be calculated using the following empirical relation:

$$C_{DB,fuse} = \frac{0.029}{\sqrt{C_{fB}}} \quad (4.27)$$

The total drag of the fuselage, which is referenced to the fuselage frontal area S_0 , was found by summing the parasitic and pressure drag.

$$C_{D,fuse} = 1.1(C_{D0,fuse} + C_{DB,fuse}) \frac{S_0}{S_{wet}} \quad (4.28)$$

Finally, the drag caused by air leaking out of openings in the fuselage was accounted for by adding 10% to the total fuselage drag, as recommended in [23].

4.3.3.7 Tail Boom

The drag caused by the tail boom was approximated using the “cross-flow principle”, which describes the drag of a cylinder oriented at an angle to the flow.

$$C_{D,TB} = (C_{D,basic} \sin^3 \alpha + \pi C_f) \frac{S_{0,TB}}{S_{ref}} \quad (4.29)$$

where α is the angle of attack of the cylinder as measured from the longitudinal axis. In the case of the *MK3* sUAS, there is no rigging angle for the tail boom, therefore the tail boom angle of attack is equal to the aircraft angle of attack. The variable C_f is the friction coefficient for the tailboom, calculated in the same manner as section 4.3.3.6. The variable C_{Dbasic} is the drag of a cylinder at subcritical Reynolds numbers, which can be approximated as a constant value of 1.2. The projected area of the tail boom is $S_{0,TB}$.

4.3.3.8 Interference Drag

Interference drag results from the aerodynamic interaction of two or more objects. The total drag of the two objects will be greater than the sum of the individual drag contributions of each component due to this interaction. On aircraft, interference drag most commonly occurs at the wing-fuselage, fuselage-empennage, and empennage-empennage junctions. On the *MK3*, the wing is situated above and behind the fuselage (via the pylon). Therefore, it is assumed that the fuselage-wing interference drag is negligible. The fuselage-empennage and empennage-empennage interference drag is calculated as:

$$C_{DE-E} = \frac{n}{2} \left[17 \left(\frac{\bar{t}}{\bar{c}} \right)^4 - 0.05 \left(\frac{\bar{t}}{\bar{c}} \right)^2 \right] \frac{\bar{c}^2}{S_{ref}} \quad (4.30)$$

where n is the number of corners at the junction, $\frac{\bar{t}}{\bar{c}}$ is the average thickness-to-chord ratio of the empennage airfoil, and $\bar{c}$ is the average wing chord at the junction.

4.3.4 Total Drag Estimate

In order to find the total drag of the aircraft, the induced, wing viscous, and parasitic drag build-up contributions are summed. Table 4.1 indicates the contribution of individual components at the maximum lift to drag ratio $\left(\frac{C_L}{C_D}\right)_{max}$, which is predicted to be 16.7, occurs at a lift coefficient value of 0.78, and an aerodynamic velocity of 7.1 m/s.

Table 4.1: Contributions to Total Drag at Maximum Lift-to-Drag Ratio (For $V = 7.1$ m/s)

Component	Drag Coefficient	Percent of Total
Induced Drag	0.0208	43.3%
Wing Viscous*	0.0173	36.0%
Horizontal Viscous*	0.0026	5.4%
Vertical Viscous*	0.0020	4.2%
Array Relief	0.0011	2.3%
Antenna	0.0009	1.9%
Control Surface Gaps	0.0010	2.1%
Wing-Tailboom Pylon *	0.0002	0.4%
Skids	0.0003	0.6%
Fuselage Pod*	0.0013	2.7%
Tailboom*	0.0003	0.7%
Empennage Interference	0.0002	0.4%
Total	0.0480	

*indicates Reynolds number-dependent drag estimate

Finally, the modeled drag polar of the *MK3* sUAS was plotted using the methodology described above and by varying velocity and thus lift coefficient and calculating the resulting total drag coefficient. The results are shown in Figure 4.14 below.

Figure 4.14: Drag Polar for the MK3 sUAS Predicted by Drag Buildup Model

Due to the Reynolds number dependency, this polar is only valid for steady level flight, and for an aircraft weight of 3.25 kg. Additionally, since the model assumes no flow separation, the use of this polar is limited up to a lift coefficient value of 0.9, as this is well below the maximum section lift coefficient of the Eppler174 airfoil used on the main wing.

A plot of the power required to maintain flight versus airspeed, generated using equation 4.15 and the modeled drag polar, is shown in the figure below.

Figure 4.15: Power Curve Predicted by Drag Buildup Model

This so called “power curve” of the aircraft has significant design implications, and directly affects the energy expenditure of the solar UAS. Additionally, while lower airspeeds result in less power required to maintain flight, adverse weather conditions such as strong winds may prevent the aircraft from flying at these more desirable airspeeds.

4.3.5 Aerodynamic Model Limitations

Although this methodology does capture many geometric details of the *MK3* configuration, several features are not included. For example, several electrical wires are secured to the Outer Mold Line (OML) of the aircraft during testing that were not modeled. Details such as control horns and linkages are not captured. Perhaps the most significant omitted feature is the impact of the PV array on the airfoil aerodynamic performance. The *MK3* was modeled (and the drag polar measured through flight testing) without the PV array integrated. When the array was installed after this testing and analysis was conducted, it became apparent that, due to the imperfect manufacturing process, the cells did not sit flush with the upper surface of the airfoil. It is almost certain that this would have a

detrimental impact on airfoil performance, however further modeling and testing would be required to verify this hypothesis. It is for these reasons that this model can be expected to underpredict the drag of a solar-powered sUAS.

POWERPLANT SUBSYSTEM TESTING METHODOLOGY

A series of tests were conducted in order to assess the accuracy of the models presented in Chapter 3 and Chapter 4. Some tests examined individual models, while others investigated the aggregate results of multiple models. This chapter will only discuss test setups and procedures, while results and comparisons to models are presented in Chapter 6.

5.1 Solar Irradiance

A device called a pyranometer was used to validate the solar irradiance models presented in section 3.1. A pyranometer measures the total (beam and diffuse) irradiance, incident to a horizontal surface [15]. The specific pyranometer used was the Apogee Instruments SP-422, which was mounted to a plate with leveling set screws and a bubble level to ensure it remained horizontal during testing. The SP-422, which can digitally record irradiance data, and leveling apparatus are shown below.

Figure 5.1: Apogee Instruments SP-422 Pyranometer and Leveling Apparatus

The pyranometer was placed in a location away from shadows or reflections. Irradiance data was digitally recorded throughout an entire day at a rate of 1 sample every 30 seconds. Temperature, pressure, and humidity were obtained from a local weather station and recorded to be used as inputs to the Environmentally Corrected irradiance model (Section 3.1.3).

5.2 Photovoltaic Array Testing

Two photovoltaic (PV) arrays, composed of five solar cells electrically connected in series, were created to validate the model presented in section 3.2. The cells used were SunPower C60 Mono-crystalline silicon solar cells (used on the *MK3* sUAS). These cells are 22.4% efficient at Standard Test Conditions according to the manufacturer [5] and are flexible enough to follow the curvature of the upper surface of an airfoil. The goal of this test was to replicate the I-V curves shown in Figure 3.9, and to compare maximum array power output between the equivalent circuit model, and the actual PV array.

In order to quantify the effect of the encapsulation material on the array power output, one array was left bare while the other array was encapsulated. A K-type thermocouple was affixed to the back of the arrays to provide a temperature inputs for the model. The array was then mounted to a rigid surface along with the SP-422 Pyranometer which ensured that the array remained horizontal and provided irradiance data to compare to the models.

In order to induce current through the array, an Agilent 6063B DC Electronic Load was used, provided courtesy of the Cal Poly Space Environments Lab. The Electronic DC Load acts as a power dissipater and attempts to draw the specified DC current from the

electrical system. In the case of a solar powered UAS, the DC load simulates the electrical loads encountered during flight from the propulsion system, battery charging, avionics, etc.

To obtain the I-V curves from the physical PV arrays, the current drawn by the DC Load was increased from 0 amps to the short circuit current I_{SC} of the array in steps of 0.5 amps. Near the expected maximum power point of the array, the electrical current stepsize was decreased to 0.1 amps in order to obtain a finer resolution around this critical point of the I-V curve. The voltage at each current step was then recorded with a multimeter. The resulting current and voltage combinations created the I-V curve for the two solar arrays. A simplified diagram of this test setup is shown in Figure 5.2

Figure 5.2: Simplified Diagram of PV Array Test Setup

Prior to the start of the test, temperature, pressure, and humidity were recorded to be used as inputs in the Environmentally Corrected irradiance model. It should be noted that this test was compared to the *aggregate results* of the both the solar irradiance model and PV array model.

5.3 Maximum Power Point Tracker Testing

An important performance characteristic of the Maximum Power Point Tracker (MPPT) is the electrical efficiency η_{MPPT} , which may be defined as the ratio of the output power P_{UAS} (to the solar UAS) to the input power P_{PV} (from the PV array).

$$\eta_{MPPT} = \frac{P_{UAS}}{P_{PV}} = \frac{I_{UAS}V_{UAS}}{I_{PV}V_{PV}} \quad (5.1)$$

Here, I_{UAS} and V_{UAS} are the current and voltage provided to the solar UAS, and I_{PV} and V_{PV} are the current and voltage from the PV array. Note that I_{PV} and V_{PV} must lie on the I-V curve (as presented in section 3.2) of the array.

In order to measure the input and output power from the MPPT, the DC Load was used to simulate the electrical load of the solar UAS. The MPPT was connected to a PV array and to the DC Load. Current and voltage sensors were placed between the array and MPPT, and between the MPPT and DC Load. A simplified diagram of this test setup can be seen in Figure 5.3 below.

Figure 5.3: Simplified Diagram of MPPT Test Setup

5.4 Battery Testing

Due to time constraints, physical testing was not conducted on Lithium-ion batteries. Therefore, the battery model presented in section 4.1 is compared to information available from a manufacturer specification sheet in section 6.5. While this procedure may determine the accuracy of the battery model compared to manufacturer-provided data, testing on physical batteries should be conducted to fully assess the differences between the actual battery, manufacturer data, and model.

5.5 Propulsion Group Testing

The propulsion system of the *MK3* was tested in a low-speed wind tunnel at the California Polytechnic State University San Luis Obispo. A team of undergraduate students was placed in charge of this wind tunnel testing in order to provide testing experience, and to provide a unique learning opportunity. The propeller was mounted to a dynamometer which measured torque and thrust using load cells, propeller rotational rate, and electrical current provided to the Electronic Speed Controller (ESC) using voltage and current sensors. The test setup may be seen in Figure 5.4.

Figure 5.4: Propeller Testing Setup

A test matrix of varying airspeeds and propeller RPM values was created in order to assess propulsion system performance across a variety of potential flight conditions. Force data and electrical power data were collected from the dynamometer at each point in the test matrix.

The propeller mechanical efficiency η_{prop} may be calculated as

$$\eta_{prop} = \frac{Tu}{Q\omega} \quad (5.2)$$

where T is the thrust produced by the propeller and is measured by the dynamometer, u is the airspeed in the wind tunnel test section, Q is the torque produced by the motor as measured by the dynamometer, and ω is the rotational speed of the propeller also measured by the dynamometer.

The combined electrical efficiency of the ESC and the motor $\eta_{motor,ESC}$ may be calculated as

$$\eta_{motor,ESC} = \frac{Q\omega}{IV} \quad (5.3)$$

where I is the electrical current supplied to the motor, and V is the voltage supplied to the motor. The total efficiency of the propulsion group η_{Pgroup} may be found by multiplying the measured propeller mechanical efficiency with the combined measured motor and ESC electrical efficiency.

$$\eta_{Pgroup} = \eta_{prop} \eta_{motor,ESC} \quad (5.4)$$

5.6 Vehicle Aerodynamics Flight Testing

In order to assess the accuracy of the drag polar model presented in section 4.3, the actual drag polar of the *MK3* was measured via flight testing. These tests consisted of a series of glides in which the pilot completely removed motor power and dove the aircraft to gain airspeed until it was three meters Above Ground Level (AGL). Then pilot then

attempted to glide the aircraft at a constant altitude, using the runway as a reference. This flight test procedure is depicted in Figure 5.5 .

Figure 5.5: Flight Test Procedure for Measuring the Drag Polar of the MK3 sUAS

The total forces F on the MK3 during the constant altitude glide can be categorized as parallel to the aerodynamic velocity ($\parallel V$) or perpendicular to aerodynamic velocity ($\perp V$). These forces are shown in Figure 5.6, and can be written in the following equations as

$$\Sigma F_{\parallel V} = m\dot{V} = -D - W \sin \gamma \quad (5.5)$$

$$\Sigma F_{\perp V} = ma_n = L - W \cos \gamma \quad (5.6)$$

where m is the aircraft mass, $\dot{V}$ is the time derivative of the aerodynamic velocity, W is the weight of the aircraft, and a_n is the acceleration of the aircraft perpendicular to the aerodynamic velocity. The lift force L acts perpendicular to the aerodynamic velocity while the drag force D acts in the opposite direction of the aerodynamic velocity. The variable γ is the flight path angle measured positive from the horizon to the aerodynamic velocity vector, and α is the angle of attack of the aircraft, measured positive from the aerodynamic velocity vector to a reference line on the vehicle.

Figure 5.6: Total Forces on the MK3 During Glides

Aerodynamic velocity V , aircraft weight W , and altitude h were measured during glides. However, an estimate for the flight path angle γ , which was not directly measured during flight testing, was needed for the calculation of total force in equations 5.5 and 5.6. The value of γ was calculated using the following.

$$\gamma = \sin^{-1} \frac{\Delta h}{V \Delta t} \quad (5.7)$$

where Δh is the differential altitude calculated from each data point, and Δt is the time elapsed between data points. An important assumption in this calculation is that the aerodynamic velocity V , which was measured by the pitot-static system, is identical to the aircraft ground speed. In other words, there can be no wind for this calculation to be accurate because the altimeter is measuring altitude with respect to the Earth reference frame, while the aerodynamic velocity is measured with respect to the wind reference frame. Since flight testing was conducted on calm days this is a reasonable assumption.

An additional assumption required for this analysis was that the normal acceleration of the aircraft, a_n is small. Equation 5.6 then reduces to

$$\begin{aligned}\Sigma F_{\perp V} &= 0 = L - W \cos(\gamma) \\ \rightarrow L &= W \cos(\gamma)\end{aligned}\tag{5.8}$$

And, if the assumption is made that the flightpath angle γ is small, equation 5.8 reduces to

$$L = W\tag{5.9}$$

Note that while this is a reasonable assumption when applied to equation 5.6 and 5.8, it is not reasonable for equation 5.5. This is because the drag force is much smaller than the aircraft weight, and as a result even small deviations in flight path angle have a significant effect on the total force parallel to aerodynamic velocity.

The lift coefficient during the glide may then be calculated by modifying equation 4.16 so that

$$C_L = \frac{W}{\frac{1}{2} \rho V^2 S_{ref}}\tag{5.10}$$

In order to mitigate the effects of environmental perturbations, such as gusts, and the pilot deviating from a horizontal glide, a data reduction method was developed that relies on a curve fitting-type process using a parabolic drag polar approximation.

5.6.1 Use of the Parabolic Drag Polar to Analyze Flight Test Results

A simplifying assumption required by the following data analysis method is that the actual drag polar of the *MK3* aircraft closely resembles a parabolic approximation of the form

$$C_D^* = C_{D0} + K C_L^2\tag{5.11}$$

where C_D^* indicates the drag coefficient computed using this approximation, C_{D0} is the parasitic drag component and K is the induced drag constant. This is a reasonable

assumption because the drag polar predicted by the drag buildup model in section 4.3 closely follows this parabolic form, as shown in Figure 5.7.

Figure 5.7: Comparison of Drag Buildup Model and Parabolic Approximation

The values of C_{D0} and K that most closely approximate the drag polar developed in section 4.3 are 0.0212 and 0.0408 respectively. It should be noted that there is only a limited region of the drag polar in which this approximation may be used. For example, flow separation near the maximum lift coefficient will result in a true drag polar that deviates from the parabolic approximation at higher C_L values. It is for this reason that the parabolic drag polar will only be applied at lift coefficient values less than or equal to 0.9, as this is well below the estimated maximum lift coefficient of the aircraft.

A 1-dimensional time-dependent simulation of the level glide procedure presented in Figure 5.5 was created by integrating the total force parallel to velocity. In the following section, a superscript (*) indicates a *simulated* quantity from the parabolic drag polar. The

drag force D^* was estimated by using the parabolic drag polar model presented in equation 5.11. Equation 5.5 can then be rewritten as

$$\dot{V}^* = -\frac{D^*}{m} - g \sin \gamma \quad (5.12)$$

where D^* is the drag force due to the *parabolic drag polar approximation* and is calculated as

$$D^* = \frac{1}{2} \rho V^* C_D^* S \quad (5.13)$$

The variables V^* and $\dot{V}^*$ are the simulated velocity and acceleration profiles respectively and are obtained by integrating equation 5.12 with respect to time. The variable C_D^* is the drag coefficient calculated using the parabolic drag polar given in equation 5.11.

The simulated velocity profile V^* is then mathematically compared to the flight test velocity profile V by computing the R^2 value between the two curves. R^2 , also known as the coefficient of determination, is a measure of how well an outcome (in this case the flight test velocity profile V) is replicated by a model (in this case the simulated velocity profile V^*). An R^2 value of 1 indicates that the model perfectly replicates the outcome. For this analysis, the R^2 value was calculated as:

$$R^2 = 1 - \frac{\sum_{i=1}^N (V_i - V_i^*)^2}{\sum_{i=1}^N (V_i - \bar{V})^2} \quad (5.14)$$

where the subscript i indicates the i^{th} test sample, N is the total number of samples recorded during a glide, and $\bar{V}$ is the average flight test velocity during a glide.

For the final step in this data reduction method, an optimization algorithm optimizes the values of C_{D0} and K used by the parabolic drag polar such that the R^2 value between the simulated and test velocity profiles is maximized. In other words, the parabolic

drag polar model is adjusted until the simulated velocity profile most closely matches the flight test velocity profile. This process is repeated for each of the level glides conducted during flight testing. This method of data reduction results in a parabolic drag polar estimate that is less susceptible to environmental perturbations and to deviations from a horizontal flight path during the glides than if the raw or even filtered test data were used.

5.6.2 Drag Polar Flight-Testing Instrumentation

A custom data recorder was built in order to collect flight test data during drag polar flight testing. The data recorder, which was mounted in the fuselage pod, is shown in Figure 5.8.

Figure 5.8: Custom Flight Data Recorder Used During Drag Polar Testing

This data recorder relies on a Robotis Open CM 9.0 microcontroller (1) to receive and record test data to a micro-SD card (2) at 45 Hz. A TE Connectivity MS4515DO temperature-compensated differential pressure sensor (3) was connected to a pitot-static tube to measure airspeed V . An InvenSense MPU-6000 accelerometer (4) was used to measure longitudinal acceleration during the glides.

5.7 *MK3* Record Endurance Solar Flight

On September 28th, 2019 the *MK3* solar powered sUAS set an endurance record for solar powered aircraft at the California Polytechnic State University, San Luis Obispo, with a flight time of six hours and two minutes and gave students valuable experience dealing with real-world solar powered flight. During the flight, several thermals, which are rising columns of warm air, were used to reduce the energy expenditure of the aircraft and gain altitude. Additionally, it was noted that due to high afternoon wind speeds of up to 8 m/s, the *MK3* was not able to fly at its minimum power consumption speed of 6.3 m/s. Environmental factors such as these should be considered during future solar UAS design projects.

During the six-hour flight, test data was recorded with a commercially available data recorder at 1 Hz [27]. This data logger recorded the roll ϕ , pitch θ , and heading angle ψ of the *MK3*. Airspeed, barometric altitude, and propulsion group power consumption were also recorded using pressure, current, and voltage sensors. Due to the limited storage capacity of this data logger, only the last 1.7 hours of the flight, from 16:30 to 18:12 local time, were captured. The power output of the two PV arrays prior to the MPPTs was also recorded from commercially available power sensors [28]. A simplified diagram of the *MK3* instrumentation during the endurance flight is shown in Figure 5.9.

Figure 5.9: Simplified Diagram of Data Logging Sensors During *MK3* Endurance Flight

The environmental conditions required for the Environmentally Corrected solar irradiance model (section 3.1.3) were manually recorded every 30 minutes from a local weather station. These conditions included ambient temperature, pressure, and relative humidity.

RESULTS AND DISCUSSION

In this chapter results from the tests described in Chapter 5 will be presented. These results will then be compared to their respective powerplant subsystem models. A discussion of the model suitability as well as possible test improvements will be included where applicable.

6.1 Irradiance Subsystem Results

The horizontal irradiance recorded by the SP-422 pyranometer was compared to predictions from the two models presented in section 3.1, and can be seen in Figure 6.1 below.

Figure 6.1: Comparison of Irradiance Test Data and Models

Note that at 16:00 and 18:40 local time, passing clouds affected the irradiance recorded by the pyranometer. The maximum error throughout the entire day between the Clear Sky (CS) model and the recorded irradiance data (neglecting the passing clouds) is

8.9%, while the maximum error between the Environmentally Corrected (EC) model and the test data is 3.5%. Thus, the EC irradiance model appears to be the more accurate of the two.

However, as previously noted, the EC model requires more detailed environmental inputs than the CS model. Additionally, the CS model predicts more conservative values of the available horizontal irradiance. Due to these caveats, it is recommended that the CS model be used during the early stages of the solar UAS design process, while the EC model be used in later stages, when more is known about the vehicle and its operating environment.

6.2 Photovoltaic Array Subsystem Results

The I-V curves of the bare and encapsulated Photovoltaic (PV) arrays are compared to the equivalent circuit model presented in section 3.2.2. The equivalent circuit model relies on the value of horizontal irradiance predicted by the EC irradiance model, as well as solar cell performance information provided by the manufacturer, which is listed in Table 6.1.

Table 6.1: Inputs to Photovoltaic Array Model from Manufacturer Datasheet

Parameter	Value from Manufacturer
I_{SC}	6.24 A
V_{OC}	3.41 V
I_{MP}	5.88 A
V_{MP}	2.72 V
$\mu_{V,OC}$	-0.0018 V/°C

The results of the bare and encapsulated array tests, and comparison to the PV array model are shown in Figure 6.2 below.

Figure 6.2: Comparison of PV Array Tests and Model

As seen in the plot, it appears that the encapsulated array produces less power than the bare array. Additionally, while the equivalent circuit model accurately predicts the short circuit current I_{SC} and open circuit voltage V_{OC} of the arrays, the location of the maximum power point V_{MP} and I_{MP} does not agree with test results. The peak power values from the testing and model is summarized in Table 6.2.

Table 6.2: Array Power Outputs from Test and Equivalent Circuit Model

Source	P_{MP} [W]	% Difference from Encapsulated Array Test
Laminated Array	10.6	-
Bare Array	11.4	+7.5%
Equivalent Circuit Model	14.1	+33%

The most likely cause of the model inaccuracy is that the true location of the maximum power point is different from the location specified by the manufacturer. This may be due to lower quality solar cells, or an optimistic report of cell performance by the manufacturer. Regardless, the solar cell performance inputs to the equivalent circuit model may be adjusted more closely match test results. Table 6.3 shows the adjusted inputs to the PV Array model.

Table 6.3: Inputs to Solar Cell Model Adjusted to Match Test Data

Parameter	Adjusted Value
I_{sc}^*	6.24 A
V_{oc}^*	3.41 V
I_{MP}	5.50 A
V_{MP}	2.28 V
$\mu_{V,oc}^*$	-0.0018 V/°C

*Indicates values identical to manufacturer specifications

The results from the adjusted model I-V curve may be seen in Figure 6.3. This model adjustment process highlights an important concept regarding the design of solar UAS: while component information specified by manufacturers may be used for initial performance estimates, results from physical testing of these components and subsystems will result in a more accurate design solution.

Figure 6.3: PV Array Equivalent Circuit Model Calibrated to Test Data

The PV array test conducted does not capture the full effect of the encapsulation material on power output. As stated in [15], the detrimental effects of encapsulation material on array power output does not become apparent until large irradiance angles i are encountered, specifically during dawn and dusk (Figure 3.13). Therefore, an improvement to the PV Array test would be to obtain array I-V curves throughout an entire day. This would experimentally quantify the characteristics of the encapsulation material.

An additional improvement to this PV array test would be to better isolate the arrays from unwanted environmental disturbances such as wind. It was observed that a change in windspeed cooled the PV arrays, thus altering the shape of the I-V curve. This phenomenon was captured by the temporary fluctuation in voltage of the unlaminated PV array, which can be seen on the right side of its I-V curve in Figure 6.2. Thus, some sort of non-reflective shielding device may improve that accuracy of this test.

6.3 Comparison of Energy Income Models to Flight Test Data

The previous section made a relatively simple comparison between energy income models, and a flat, non-rotating PV array test at a single point in time during the day. While the results of this simplistic comparison are useful, a more complex time-variant analysis may be conducted using data recorded during the record endurance flight of the *MK3*.

The Euler angles recorded by the commercially available flight data logger (section 5.7), roll ϕ , pitch θ , and heading ψ , may be used in conjunction with the more accurate EC irradiance model, and adjusted PV array model (previous section) to predict the fully time and state dependent power output of the *MK3* PV array. Note that the term “energy income models” will refer to these two specific models. The PV array power output prediction may then be compared to the actual array power output recorded by the power sensors during the flight in order to assess the accuracy of the aggregate results from the energy income models. A centered-moving average filter with a semi-window length of five samples (equivalent to five seconds) was applied to the flight test data before using these models. The results of this comparison may be seen in Figure 6.4.

Note that the array power output is especially sensitive to changes in the aircraft pitch angle θ . While the energy income models appear to predict the correct trends in power output from the PV array, there are still discrepancies. A possible cause of this is that the *tracking speed* of the MPPT is not captured by the energy income models. The tracking speed refers to how quickly the MPPT can respond to changes in the I-V curve of the array and locate the new maximum power point, e.g. after a sudden change in aircraft orientation. The energy income models assume an instantaneous change in array power output, while in actuality this may take the MPPT several seconds to achieve.

Figure 6.4: Comparison of Flight Test PV Array Power Output and Energy Income Models

Of more interest may be the solar energy income comparison. This may be found by numerically integrating the measured and predicted PV array power output from the flight test and energy income models respectively. A comparison of the test and model based solar energy income may be seen in Figure 6.5.

According to this comparison, it appears that until 17:20 local time, the energy income models match the test data closely, resulting in a 1.1% accurate energy income prediction at 17:20. After 17:20, when the sun elevation angle is less than 18° , the solar energy income models begin to underpredict incoming solar energy, resulting in a 9.6% lower prediction of collected solar energy at the conclusion of the flight. Results from [7] indicate that solar energy income models are more accurate towards the middle of the day, and less accurate towards dawn or dusk, corroborating the results presented here. The increase in model prediction error is most likely due to limited accuracy of the solar cell encapsulation model presented in section 3.2.3.

As the sun elevation angle decreases with the approach of dusk, the average incidence angle i between the total solar irradiance and the PV array increases. This in turn causes the encapsulation material to reflect more of the incoming solar irradiance, reducing the array power output, which is illustrated in Figure 3.13. As mentioned in section 6.2, the simplistic PV array test methodology did not capture the full influence of the encapsulation material. Because of the sensitivity to solar incidence angle, any inaccuracies in modeling the optical characteristics of the encapsulation material will be magnified during dawn or dusk when the sun is closer to the horizon. The increase in the effect of encapsulation material during dawn and dusk underscores the need for a day-long horizontal PV array test, so that this phenomenon may be better quantified.

Figure 6.5: Comparison of Flight Test Energy Income with Energy Income Models

Despite some discrepancies, it appears that the detailed solar energy income models may be used with confidence to predict UAS energy balance during the middle of the day, when the sun elevation angle is greater than 18° . Further testing and improved modeling are required before they can be used with certainty for sun elevation angles less than 18° .

6.4 Maximum Power Point Tracker Subsystem Results

The electrical efficiency of the Maximum Power Point Tracker (MPPT) was calculated from test results and can be seen in Figure 6.6. The power output of the Photovoltaic (PV) array P_{PV} was normalized to the maximum allowable input power of the MPPT as given in [29].

Figure 6.6: Maximum Power Point Tracker Test Results

It appears that the efficiency of the MPPT, while reduced at lower PV array power levels, increases with array output power. Additionally, the constant efficiency assumption presented in [3] and [7] appears to be a sufficient approximation above array power ratios $\frac{P_{PV}}{P_{MAX}}$ of 0.4. These test results agree with data presented in [7]. Note that the electrical efficiency of the MPPT may be interpolated from the test data presented above for use during the design process.

The measured characteristics of the MPPT have significant design implications. Most notably, the array will be producing very little power during dawn and dusk, when the sun is close to the horizon. It is at these times during the solar UAS mission that the

MPPT electrical efficiency is also severely reduced. Thus, even less power from the PV array can be expected during dawn or dusk due to this added efficiency reduction.

An improvement to this test would be to investigate the time-variant characteristics of the MPPT. As discussed in section 6.3, the tracking speed of the MPPT may prevent it from finding the maximum power point of the array if irradiance levels are rapidly changing due to aircraft orientation, which may have an impact on the total power collected by the PV array.

6.5 Battery Subsystem Results

The battery model developed in section 4.1 was applied to a Panasonic NCR18650B Lithium-ion battery and compared to results given on the manufacturer-provided specification sheet. Table 6.4 lists the inputs to the battery model, which were obtained from the 0.2C discharge profile given in [13].

Table 6.4: Inputs to Battery Model from Specification Sheet 0.2C Discharge Profile

Location on Discharge Profile	Discharge Capacity Q (A-h)	Voltage V (V)
Start of Discharge $(0, V_{full})$	0.00	4.20
End of Exponential Zone (Q_{exp}, V_{exp})	2.10	3.52
End of Nominal Zone (Q_{nom}, V_{nom})	3.10	3.14

A value for the battery internal resistance is required as an input to the battery model. This value was assumed to be 50 m Ω , as this is the typical internal resistance of Panasonic 18650 Lithium-ion battery cells according to experimental results from [30]. The specification-based 0.2C discharge voltage and energy are compared to predications from the battery model in Figure 6.7.

Figure 6.7: Comparison of Battery Model and Specification Sheet 0.2C Discharge Profiles

The maximum error during the discharge between the model voltage and specification sheet voltage, excluding the sub-nominal zone, is 3.1%. The total energy released during discharged is 11.88 W-h according to the specification sheet and 11.77 W-h according to the battery model. Using a battery cell mass of 0.0475 kg obtained from the specification

sheet, the specification- and model-based specific energy densities are 250 W-h/kg and 248 W-h/kg, resulting in a 0.8% difference. Note that the specification sheet directly lists the specific energy of the cell as 243 W-h/kg. Thus, the battery model appears to accurately predict the specific energy density of the battery, which is one of the key battery performance characteristics in solar UAS design.

Similarly, the charging profile provided by the specification sheet and the model may be compared. The safest and most efficient way to charge Lithium-ion batteries is what is known as the Constant-Current Constant Voltage (CC-CV) charge cycle [12]. In this charging method, a constant value of electrical current, typically less than $0.5C$, is used to charge the battery until a specific voltage cutoff is reached (4.2 V for the NCR18650B battery [13]). This portion of the charge cycle is called “constant-current” accordingly. Once the cutoff-voltage is reached, the current is incrementally reduced such that the battery remains at the cutoff voltage for the remainder of the charge cycle, thus this portion of the cycle is termed “constant-voltage”. This CC-CV charge cycle was applied to the battery model developed in section 4.1.1 and compared to charge profiles from the manufacturer specification sheet in Figure 6.8. Note that the charging cycle progresses from right to left on the plot, since the x-axis is the capacity that has been discharged by the battery.

Figure 6.8: Comparison of Specification Sheet and Battery Model Charging Profile

The model appears to underpredict the charging voltage until the constant-voltage portion of the charge cycle is reached. Additionally, the capacity at which the constant-current portion of the charge cycle ends is different between the specification sheet and the model. The inaccuracy of the battery in model in predicting the current and voltage during the charging process may be caused by the assumption that the charge characteristics are identical to the discharge characteristics (section 4.1.1).

In order to calculate the electrical efficiency, the energy absorbed and released during charge and discharge may be calculated. The battery electrical energy is compared between the specification sheet and the model in Figure 6.9.

Figure 6.9: Comparison of Charge and Discharge Cycles from Battery Model and Specification Sheet

The battery electrical efficiency, which is defined in equation 4.10, is calculated as 0.88 according to the specification sheet, and 0.95 according to the battery model, which represents a 7% error at the end of the charge and discharge cycle.

It appears that, while the battery model accurately predicts discharge characteristics, it does not accurately predict charging characteristics, and therefore the electrical efficiency of the battery. This is most likely caused by the assumption in developing the battery model that the charging characteristics are identical to the discharging characteristics which are used as inputs to the model. Thus, while the presented battery model may be used to accurately estimate discharging voltage and specific energy density, further model improvements and testing are needed before it can accurately estimate charging current, voltage, and electrical efficiency.

6.6 Propulsion Group Subsystem Results

The propeller and combined motor and electronic speed controller (ESC) efficiency were calculated from the low-speed wind tunnel test data according to the method described in section 5.5. The propeller, combined motor/ESC, and overall propulsion group efficiency at 4000 RPM may be seen in Figure 6.10.

Figure 6.10: MK3 Propulsion Group Wind Tunnel Test Results

The propulsion group reaches a maximum efficiency of 0.63 at a velocity of 12 m/s for 4000 RPM. In order to assess the accuracy of the JBlade Blade-Element momentum propeller simulation, the propeller efficiency predicted by JBlade was compared to test results at two RPM and wind tunnel velocity combinations in the test matrix.

Figure 6.11: Comparison of Wind Tunnel Test Results and JBlade Propeller Model

The propeller efficiency predicted by JBlade appears to follow the general trend of the test data. The velocity at which the maximum propeller efficiency occurs increases with rotational speed. However, JBlade overpredicts the speed at which this will occur. The most likely cause of this error is that the propeller geometry specified in JBlade may deviate from the actual propeller geometry used in the wind tunnel testing. Additionally, JBlade predicts the same value for maximum propeller efficiency at both RPMs while the test data indicates it increases with RPM. A possible cause for this error is that the dependency of the propeller performance on local blade Reynolds number is not fully captured by JBlade.

JBlade appears to be a useful tool for the design of solar UAS, as it accurately models propeller performance trends based on geometry. However, like other subsystems, test data is needed to truly capture the propulsion group performance.

6.7 Aerodynamic Performance Results

The data reduction method detailed in section 715.6.1 was applied to results from the flight test to obtain an approximate parabolic drag polar for the *MK3*. The simulated velocity profile V^* may be seen compared to the flight test velocity profile V after the parabolic drag polar optimization procedure in Figure 6.12.

**Figure 6.12: Comparison of Simulated and Test Velocity Profiles During Level
Glide**

An equivalent parabolic drag polar was found for each of the glides conducted during the flight test. Table 6.5 summarizes the optimized values of C_{D0} and K for each glide analyzed.

Table 6.5: Summary of Optimized Parabolic Drag Polar Inputs for Each Glide

Glide Number	C_{D0}	K	R^2
1	0.0251	0.0473	0.992
2	0.0226	0.1096	0.991
3	0.0177	0.1583	0.983
4	0.0202	0.0878	0.992

In order to find an averaged parabolic drag polar across all four glides, the means of C_{D0} and K were computed. The standard deviations of C_{D0} and K values were used to estimate the error of this method and are given in Table 6.6.

Table 6.6: Mean and Standard Deviation of C_{D0} and K

	C_{D0}	K
Mean	0.0215	0.1008
Standard Deviation	0.0032	0.0463
% Standard Deviation	14.9 %	45.9%

The flight test-based parabolic drag polar, with error estimates, is compared to the drag buildup model in Figure 6.13. The drag buildup model overpredicts the maximum lift-to-drag ratio of the *MK3* solar sUAS. The most likely cause of this is that details present on the physical aircraft, such as large hatch openings or wrinkling of the hobby-grade plastic skin covering, are not captured by this aerodynamic model.

Figure 6.13: Comparison of Drag Buildup Model and Flight Test-Based Parabolic Drag Polars

Complex flow features and component interactions, which are not modeled by the drag buildup method may also account for the discrepancy between the model and test data. Table 6.7 compares the maximum lift to drag ratio between the model and test results.

Note that the parabolic drag polar approximation is only valid for lift coefficient values less than 0.44, which is the maximum valid lift coefficient value that is achieved by the set of glide tests being analyzed.

Table 6.7: Maximum Lift-to-Drag Ratios

	Panel Code/Drag Build-up	Test-Based Parabolic
$\left(\frac{C_L}{C_D}\right)_{max}$	16.6	10.9
$C_L \left(\frac{C_L}{C_D}\right)_{max}$	0.79	0.44*

*indicates maximum lift coefficient that polar is valid for

It would be inaccurate to further extrapolate the parabolic drag polar as not all glides were flown in this region. The maximum lift-to-drag ratio of the parabolic polar is 10.8 and occurs at the maximum *valid* lift coefficient of 0.44.

A phenomenon present in this method of data analysis was that the percent standard deviation was lower for C_{D0} across all glides than the percent standard deviation of K which is due to the nature of the parabolic drag polar approximation. A change in the parasitic drag coefficient C_{D0} will cause the same change in the total drag coefficient C_D^* , whereas the effect of a change in the induced drag constant K is only more apparent at higher lift coefficients due to the quadratic C_L term. Since the maximum lift coefficient achieved by all glides was 0.44, the effect of the value of K on the simulation results was much less than the effect of C_{D0} and as such, more variability is seen in induced drag constant between glides.

While this testing method appears to be suitable for measuring the parasitic drag coefficient, improvements are needed to accurately assess a broader range of the drag polar. One such improvement could be the implementation of an autopilot. This autopilot could potentially maintain a more accurate level glide than a human pilot, at the expense of adding complexity to the test. Another improvement to make would be to wait for calmer

atmospheric conditions. Although drag polar flight testing was conducted early in the morning, small atmospheric perturbations, to which a sUAS is especially susceptible, were still present.

Additionally, changes may be made to the flight-testing methodology. The level glide procedure was chosen because it allowed the pilot to maintain a constant reference to the ground during the test. However, if an autopilot were implemented, other test procedures such as a series of constant-velocity glides, that do not require a pilot reference could be explored.

Chapter 7

CONCLUSION

This thesis examined several detailed models of solar UAS powerplant subsystems. Several of these models were then compared to test data to assess their accuracy. While some models accurately predicted test results, others need further improvement. Additionally, while some tests accurately characterized the subsystem in question, the methodology of others could be improved to provide more conclusive results.

The solar irradiance models presented in section 3.1 appear to accurately predict measured solar irradiance. Specifically, the Environmentally Corrected irradiance model, which relies on current environmental conditions such as temperature, pressure, and humidity, predicted the measured solar irradiance to within 3% or less throughout the entire day. The Clear Sky irradiance model, which uses a generic climate type input, predicts the measured irradiance to within 9% throughout the entire day. However, its irradiance predictions are conservative and, due to the limited number of inputs required, is better suited for early stages in the design process. A possible improvement to this test procedure would be to collect irradiance data on days with various amounts of clouds to determine the reduction in solar power available to the UAS.

The photovoltaic (PV) array model presented in section 3.2, which relies on solar cell characteristics provided by the manufacturer as inputs, appears to predict the general trends of the PV array test. However, the test data disagreed with both the PV array model as well as the manufacturer-provided characteristics. This suggests that characteristics from the manufacturer cannot always be relied upon, and thus necessitates testing an actual PV array and adjusting the model to match results accordingly.

Since the PV array was tested only at one point in time during the day, the detrimental time-dependent effect of the encapsulation material on power output was not quantified. As such, an improvement to the PV array testing methodology would be to obtain array I-V curves of the PV array throughout an entire day. Additionally, it was observed that the power output of the PV array during the test was affected by changing environmental factors, such as a sudden breeze temporarily cooling the solar cells and artificially increasing array power output. Thus, the second major improvement to this test would be to further isolate the PV array from environmental disturbances, e.g. using a windscreen.

The fully time and state dependent PV array model was compared to test results from the *MK3* endurance flight record. This detailed energy income model, which relies on the specific array geometry on the solar UAS as an input, accurately predicted the total solar energy income during the test to within 1.1% at 17:20 local time on the day which testing was conducted. For sun elevation angles below 18°, during dawn or dusk, the accuracy of the energy income models decreased to within 9.6% of the measured test data at the conclusion of the flight. This is most due to inaccurately characterizing the optical properties of the encapsulation material, as this effect will be magnified at low sun elevation angles. This model would benefit from the improved, day-long PV array test discussed above, as this would serve to quantify the effect of the encapsulation matter on PV array performance.

The characteristics of the Maximum Power-Point Tracker (MPPT) are impractical to model, as the specific tracking algorithm and circuit components must be known. Test data indicates that at higher PV array output power, the electrical efficiency of the MPPT remains above 95%. However, at lower array power output levels, the efficiency of the MPPT is reduced to 82%. This has significant design implications, as the PV array will be

producing low amounts of power during dawn and dusk, resulting in reduced MPPT electrical efficiency. An improvement to this test would be to quantify the effect of operating temperature on the MPPT efficiency, as this may drive in-flight cooling requirements. Additionally, a test may be conducted that quantifies time-dependent electrical characteristics of the MPPT, namely how rapidly it can locate a new array maximum power point under changing irradiance conditions.

The time-dependent battery model appears to accurately predict the discharge profile of a Lithium-ion battery, including specific energy density and discharging voltage. However, the charging profile predicted by the model does not accurately replicate the manufacturer-provided characteristics. This is most likely due to the assumption that the charging characteristics are identical to the discharging characteristics, which are used as inputs to the battery model. It is likely that improved accuracy would only be achieved with a more sophisticated battery model, which may require an impractical amount of inputs or computational resources. It would also be of use to compare the battery model to actual test data rather than manufacturer-provided information. A suitable test would be to repeatedly charge and discharge a lithium-ion battery with a charger capable of digitally recording electrical current and voltage information.

Several aspects of the propulsion group model may be improved. While the propeller performance characteristics including efficiency, thrust, and power may be modeled by a program called JBlade that utilizes Blade-Element Momentum theory, models are lacking for the electronic speed controller (ESC) and electric motor subsystems. Testing-based results from [21] may be used to estimate the electrical efficiency of the ESC. However, a dynamic 3-phase induction motor model would be of use for estimating the overall performance of the motor and propulsion group. The propeller performance trends predicted by JBlade appear to agree well with wind tunnel test data. However, as

propeller geometry has a significant impact on performance, care must be taken to accurately replicate the geometry in JBlade.

The aircraft aerodynamics model utilizes a combination of empirical-based drag estimates and numerical analysis tools to predict the total drag of the vehicle. However, despite the level of detail accepted by the model, certain geometric features on the aircraft may be omitted that in fact have a significant drag contribution. Thus, the aerodynamics model can be expected to underestimate the total drag of the aircraft.

Flight test results indicate that the drag build-up model does indeed underestimate total drag of the aircraft resulting in a 34% overprediction of the maximum lift-to-drag ratio. However, this overprediction may also be due to inaccurately measuring the drag polar due to the flight test methodology, which may be improved. The methodology implemented, which relied on using the level glide characteristics of the *MK3* sUAS to measure the drag polar, appears to only result in an accurate parasitic drag estimate. Significant amounts of variation were present in the lift-dependent drag test results due to the specific test methodology. A significant improvement to this test would be the implementation of an autopilot. This would potentially allow the current test methodology to be executed more accurately, at the expense of increased avionics complexity. The implementation of an autopilot may even allow for variations on the current test methodology, such as a series of constant-speed glides to more accurately measure the aircraft drag polar.

The results presented in this thesis have provided a greater understanding of solar UAS characteristics to students at the California Polytechnic State University, San Luis Obispo, and to the author. A significant outcome is the realization that, while subsystem models can and should be used during the design stages of solar UAS, physical component testing is critical to truly estimate the overall vehicle performance. The models and test

data provided in this thesis may be used and improved upon by those further investigating the subject. In this way, this work can contribute to the ever-advancing field of solar powered UAS.

7.1 Suggestions for Future Improvements

The following improvements may be made to the models described in this thesis:

- Capturing the effects of partial shading of the photovoltaic array by components such as the empennage
- Inclusion of transient effects on maximum power point tracker model
- Development of a battery model that more accurately predicts charging characteristics
- Implementation of a 3-phase induction motor model

The following improvements may be made to the subsystem tests described in this thesis:

- Conducting a day-long photovoltaic array test to more accurately determine the effects of encapsulation material on array power output
- Better isolating the photovoltaic array from environmental disturbances such as wind
- Testing physical lithium-ion batteries
- Accurately capturing the transient effects of the MPPT during rapidly changing irradiance conditions
- Adopting a new drag polar flight-testing methodology, potentially with the use of an autopilot, to reduce the variability of the results

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APPENDICES

Appendix A

Sun Position Calculations

The position of the sun relative to the solar UAS may be calculated using the method described in [15]. In order to calculate the zenith and azimuth angles, the year fraction B must first be calculated in degrees:

$$B = \frac{(n - 1)360}{365} \quad (\text{A. 1})$$

where n is the integer value of the day of the year, counting up from January 1st. Next, solar declination angle δ , which is the position of the sun at solar noon, is calculated using the following empirical relation:

$$\delta = 0.006918 - 0.399912 \cos B + 0.0256 \sin B - 0.00675 \cos 2B + 0.000908 \sin 2B \\ - 0.002679 \cos 3B + 0.00148 \sin 3B \quad (\text{A. 2})$$

According to [15], the error from this approximation is less than 0.035° .

In order to calculate the azimuth and zenith angles, the solar time must first be calculated. Solar time is local time shifted such that solar noon coincides with the sun's maximum elevation angle. The solar time is calculated by applying two corrections to the local time: the first is the difference in the longitude of the vehicle and the standard meridian of the local time zone multiplied by the amount of time required for the Earth to rotate 1 degree (240 seconds) . The second correction is an empirically derived equation that accounts for perturbation in the Earth's rotation. The two corrections are shown below.

$$t_{solar} = t_{local} + 240(Lon_{st} - Lon_{UAS}) + E \quad (\text{A. 3})$$

where t_{local} is the local time, Lon_{st} is the standard Meridian for the local time zone, and Lon_{UAS} is the current longitude of the UAS. E is the correction factor for perturbations in the Earth rotational rate and is measured in seconds. This correction factor is given by:

$$E = 13752(0.000075 + 0.001868 \cos B - 0.032077 \sin B - 0.014615 \cos 2B - 0.04089 \sin 2B) \quad (A. 4)$$

An additional seasonal affect that must be accounted for is Daylight Savings time, which occurs between March 10th and November 3rd in North America. If daylight savings time is observed, an additional 3600 seconds (1 hour) must be subtracted from the solar time t_{solar} calculated in equation A.3. In all, the corrections applied between local and solar time may be greater than 1 hour.

Next, the sun hour angle ω , which is the angular displacement of the sun East or West of the local meridian, is calculated. Since it is known that the Earth rotates at 15° per hour, ω is calculated in degrees as

$$\omega = \left(\frac{t_{solar}}{3600} - 12 \right) 15 \quad (A. 5)$$

where t_{solar} is the solar time in seconds as calculated in equation A.3. Thus, ω will be 0° at solar noon, negative in the morning, and positive in the afternoon.

With the hour angle ω , and declination angle δ defined, the solar zenith, elevation, and azimuth angles may be calculated in degrees as follows:

$$z = \cos^{-1}(\cos Lat_{UAS} \cos \delta \cos \omega + \sin Lat_{UAS} \sin \delta) \quad (A. 6)$$

$$e = 90 - z \quad (A. 7)$$

$$a = \frac{\omega}{|\omega|} \cos^{-1} \left(\frac{\cos(z) \sin(Lat_{UAS}) - \sin \delta}{\sin(z) \cos(Lat_{UAS})} \right) \quad (A. 8)$$

Appendix B

Irradiance Model Calculations

B.1 Clear Sky Irradiance Model

The solar constant G_{SC} is the time-averaged value of irradiance that reaches the edge of the Earth's atmosphere and has been measured as 1367 Watts per square meter. However, due to the eccentricity of the Earth's orbit, the solar irradiance at any given day of the year G_{ON} is non-constant. The value of G_{ON} , which is required for the following irradiance models, can be calculated in Watts per meters squared as

$$G_{ON} = G_{SC} (1.00011 + 0.034221 \cos B + 0.001280 \sin B + 0.000719 \cos 2B + 0.000077 \sin 2B) \quad (B.1)$$

where B is the year fraction in degrees.

Next, the atmospheric beam transmittance τ_b is calculated. Transmittance is defined as the fraction of energy that passes through a body, in this case the atmosphere. The beam transmittance is calculated as,

$$\tau_b = a_0 + a_1 \exp\left(-\frac{k}{\cos(z)}\right) \quad (B.2)$$

where z is the sun zenith angle, and a_0 , a_1 , and k are constants dependent on climate type, and are given by:

$$a_0 = r_0(0.4237 - 0.008216 (6 - h)^2) \quad (B.3)$$

$$a_1 = r_1(0.5055 + 0.00595(6.5 - h)^2) \quad (B.4)$$

$$k = r_k(0.2711 + 0.01858(2.5 - h)^2) \quad (B.5)$$

h is the aircraft altitude in kilometers, and r_0, r_1 and r_k are correction factors for the current climate type and can be found by referencing Table B.1.

Table B.1: Correction Factors for Various Climate Types

Climate Type	r_0	r_1	r_k
Tropical	0.95	0.98	1.01
Midlatitude summer	0.97	0.99	1.02
Subarctic summer	0.99	0.99	1.01
Midlatitude winter	1.03	1.01	1.00

Once the atmospheric beam transmittance τ_b is calculated, the atmospheric diffuse transmittance τ_d can be calculated. The empirical relation for the diffuse transmittance is given by:

$$\tau_d = 0.271 - 0.294\tau_b \quad (\text{B. 6})$$

Thus, with the beam and diffusive transmittance calculated, the beam and diffuse irradiance G_b and G_d may be calculated by multiplying the yearly variation of solar irradiance with the respective transmittance coefficients:

$$G_b = G_{on}\tau_b \quad (\text{B. 7})$$

$$G_d = G_{on}\tau_d \quad (\text{B. 8})$$

And finally, the total irradiance on a surface normal to the sun can be calculated as the sum of beam and diffuse irradiance.

$$G_{N,CS} = G_b + G_d \quad (\text{B. 9})$$

$G_{N,CS}$ is the normal irradiance computed using the Clear Sky model. It may be desired to estimate the total irradiance on a surface that is horizontal in order to standardize experiments related to photovoltaic array performance. This is done by multiplying the total irradiance by the cosine of the zenith angle.

$$G_{H,CS} = G_{N,CS} \cos z \quad (\text{B. 10})$$

Here, $G_{H,CS}$ is the irradiance on a horizontal surface calculated using the Clear Sky model.

B.2 Environmentally Corrected Irradiance Model

The primary difference between the Clear Sky irradiance model and the Environmentally Corrected one is the calculation of the beam transmittance, τ_b which is given as

$$\tau_b = 0.98 \exp \left(\frac{0.00146P}{K_t \sin e} - 0.075 \left(\frac{W}{\sin e} \right)^{0.4} \right) \quad (\text{B. 11})$$

Where:

- P = measured atmospheric pressure [kPa]
- K_t = turbidity coefficient, $0 \leq K_t \leq 1.0$. where $K_t = 1.0$ for clean air, and $K_t \leq 0.5$ for extremely turbulent or polluted air ($K_t = 1.0$ recommended)
- e = Sun elevation angle
- W = Precipitable water in the atmosphere [mm]

The calculation for the precipitable water in the atmosphere is given as:

$$W = 0.14e_a P + 2.1 \quad (\text{B. 12})$$

where e_a is the actual vapor pressure of the air in kPa, and can be given by:

$$e_a = RH P_{sat} \quad (\text{B. 13})$$

RH is the relative humidity of the environment as a fraction (i.e. 0.80 instead of 80%) and

P_{sat} is the saturation pressure of air and is calculated using the following equation:

$$P_{sat} = \frac{\exp\left(77.345 + 0.0057T - \frac{7235}{T}\right)}{T^{8.2}} \quad (\text{B. 14})$$

where T is the local environmental air temperature in °K.

Once the beam transmissivity has been found, the diffuse transmissivity, τ_b can be found using the following relation.

$$\tau_d = \begin{cases} 0.35 - 0.36\tau_b & \tau_b \geq 0.15 \\ 0.18 + 0.82\tau_b & \tau_b < 0.15 \end{cases} \quad (\text{B. 15})$$

The beam, diffuse, and horizontal components of irradiance are found in the same manner as the Clear Sky model.

Appendix C

Solar Cell Performance Calculations

C.1 Solar Cell Equivalent Circuit

An equivalent circuit that replicates the I-V curve of a solar cell is given in Figure 3.10. This circuit relies on five parameters, which are the solar generated light current I_L , diode reverse saturation current I_O , the series resistance R_S , the shunt resistance R_{SH} , and the modified ideality factor a , which is calculated as

$$a = \frac{nkT_C N_s}{q} \quad (C.1)$$

In equation C. 1, k is the Boltzmann constant ($1.381\text{e-}12$ [J/K]), T_C is the array temperature in degrees Kelvin, N_s is the number of solar cells in the PV array that are connected in series, and q is the electronic charge ($1.602\text{e-}19$ coulomb). The parameter n is the diode ideality factor (equal to 1 for an ideal diode, and between 1 and 2 for real diodes). Thus, the five parameters required to model the array I-V curve are I_L , I_O , R_S , R_{SH} , and a .

However, these parameters are not provided by manufacturers. Instead, five parameters that are typically provided must be used to solve for I_L , I_O , R_S , R_{SH} , and a . The five provided parameters are $I_{SC,ref}$, $V_{OC,ref}$, $V_{MP,ref}$, $I_{MP,ref}$ (refer to Figure 3.9), and $\mu_{V,OC}$, where $\mu_{V,OC}$ is the open-circuit voltage temperature coefficient, given in Volts per degree Celsius above Standard Test Conditions (STC). This temperature coefficient describes the relation of open-circuit voltage with cell temperature. The subscript *ref* denotes cell characteristics recorded at reference STC, which for PV arrays are $1000 \frac{W}{m^2}$ incident radiation, a cell temperature of 25 degrees Celsius, and a spectral solar radiation distribution corresponding to a zenith angle (z) of 48.2 degrees.

The five provided parameters are used in conjunction with various array conditions, such as open circuit voltage measurement, to solve for the 5 unknown parameters and generate the array I-V curve. The array conditions and resulting equations are shown in Table C.1.

Table C.1: Equations used in 5-Parameter PV Array Model

Array Condition	Resulting Equation
Short Circuit	$I_{SC,ref} = I_{L,ref} - I_{O,ref} \left[\exp \left(\frac{I_{SC,ref} R_{S,ref}}{a_{ref}} \right) - 1 \right] - \frac{I_{SC,ref} R_{S,ref}}{R_{SH,ref}} \quad (C.2)$
Open Circuit	$I_{L,ref} = I_{O,ref} \left[\exp \left(\frac{V_{OC,ref}}{a_{ref}} \right) - 1 \right] - \frac{V_{OC,ref}}{R_{SH,ref}} \quad (C.3)$
Maximum Power Point	$I_{MP,ref} = I_{L,ref} - I_{O,ref} \left[\exp \left(\frac{V_{MP,ref} + I_{MP,ref} R_{S,ref}}{a_{ref}} \right) - 1 \right] - \frac{V_{MP,ref} + I_{MP,ref} R_{S,ref}}{R_{SH,ref}} \quad (C.4)$
$\frac{\partial P}{\partial V} = 0$ (at MPP)	$\frac{I_{MP,ref}}{V_{MP,ref}} = \frac{\frac{I_{O,ref}}{a_{ref}} \exp \left(\frac{V_{MP,ref} + I_{MP,ref} R_{S,ref}}{a_{ref}} \right) + \frac{1}{R_{SH,ref}}}{1 + \frac{I_{O,ref} R_{S,ref}}{a_{ref}} \exp \left(\frac{V_{MP,ref} + I_{MP,ref} R_{S,ref}}{a_{ref}} \right) + \frac{R_{S,ref}}{R_{SH,ref}}} \quad (C.5)$
Known Temp. Coefficient	$\mu_{V,OC} = \frac{V_{OC}(T_C) - V_{OC}(T_{C,ref})}{T_C - T_{C,ref}} \quad (C.6)$

In equation C.6, which results from the known cell temperature coefficient, $\mu_{V,OC}$, $T_{C,ref}$ is the cell temperature at STC (25 °C), and T_C is an arbitrary cell temperature within ± 10 °C of the reference temperature.

The resulting system of five equations is non-linear, and numerical methods must be used to solve it. Recall that the known parameters, given by the solar cell manufacturer, are $I_{SC,ref}$, $V_{OC,ref}$, $I_{MP,ref}$, $V_{MP,ref}$, and $\mu_{V,OC}$. Note that, if a PV array consists of multiple solar cells wired in series, the values $V_{OC,ref}$, $V_{MP,ref}$, and $\mu_{V,OC}$ for a single solar cell must be multiplied by the number of solar cells in the array.

Once the system of equations is solved and the five reference parameters are found, the effect of environmental operating conditions on each parameter must be accounted for.

The modified ideality factor a is a function of cell temperature, and is given by

$$\frac{a}{a_{ref}} = \frac{T_C}{T_{C,ref}} \quad (C.7)$$

The light current, I_O is a function of the cell temperature, short-circuit current temperature coefficient $\mu_{I,SC}$ and the effective absorbed solar radiation G .

$$I_L = \frac{G}{G_{ref}} [I_{L,ref} + \mu_{I,SC}(T_C - T_{C,ref})] \quad (C.8)$$

Where $\frac{G}{G_{ref}}$ is the ratio of solar radiation absorbed by the PV array at a given operating condition to the radiation absorbed at STC and is further explained in section C.2.

The diode reverse saturation current I_O is a function of array temperature and material composition. It is given by:

$$\frac{I_O}{I_{O,ref}} = \left(\frac{T_C}{T_{C,ref}} \right)^3 \exp \left(\frac{E_{g,ref}}{kT_{C,ref}} - \frac{E_g}{kT_C} \right) \quad (C.9)$$

$$\frac{E_g}{E_{g,ref}} = 1 - C(T_C - T_{C,ref}) \quad (C.10)$$

where E_g is the material bandgap energy, that is, the energy required to remove a valence electron (the fundamental process behind photovoltaic energy production). $E_{g,ref} = 1.12 \text{ eV} = (1.794 \times 10^{-19} \text{ J})$ and C is a material temperature constant, equal to $0.0002677 \left[\frac{1}{^\circ\text{C}} \right]$ for silicon. k is the Boltzmann constant ($1.381 \times 10^{-23} \frac{\text{J}}{^\circ\text{K}}$).

The shunt resistance is a function of the absorbed solar radiation, and is given by

$$\frac{R_{SH}}{R_{SH,ref}} = \frac{G_{ref}}{G} \quad (\text{C. 11})$$

Lastly, the series resistance is assumed to be independent of both temperature and absorbed radiation.

$$R_S = R_{S,ref} \quad (\text{C. 12})$$

C.2 Effective Absorbed Solar Radiation

The ratio of effective absorbed solar radiation is the ratio of solar radiation absorbed by the PV array during any given operating condition to solar radiation absorbed by the PV array during STC. This ratio is dependent on the components of beam and diffuse radiation (G_b and G_d), the spectral distribution of the solar radiation, the array orientation, and the specific encapsulation material used on the array. Solar cells are typically covered with a thin protective material in a process called encapsulation. This encapsulation material has a detrimental impact on the solar energy absorbed by the PV array, especially at higher incidence angles i . This effect is further described in C.3.

The cosine of the irradiance angle for the array $\cos(i)$ must first be calculated according to methods presented in section 3.2.1. Similarly, the cosine of the array elevation

angle β must be calculated by taking the dot product of the array unit normal vector $\hat{A}$ and a vertical vector expressed in the Earth reference frame.

$$\cos(\beta) = \hat{A} \cdot \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} \quad (\text{C. 13})$$

The ratio of effective solar radiation absorbed is then given by:

$$\frac{G}{G_{ref}} = M \left(\frac{G_b}{G_{ref}} K_b \frac{\cos(i)}{\cos z} + \frac{G_d}{G_{ref}} K_d \frac{1 + \cos \beta}{2} \right) \quad (\text{C. 14})$$

where G_b and G_d are the beam and diffuse radiation respectively and may be calculated using the methods presented in section 3.1. The parameters K_b and K_d are called incidence angle modifiers and describe the effect of the encapsulation material on the ratio of effective absorbed solar radiation. The calculation for the incidence angle modifiers is presented in section C.3 . The variable M is the air mass modifier, which accounts for changes in the spectral distribution of solar radiation as a function of zenith angle z .

$$M = \sum_{i=0}^4 b_i \left(\frac{1}{\cos(z)} \right)^i \quad (\text{C. 15})$$

b_i are constants for different array materials, and for monocrystalline silicon cells are given in [15] as

$$b = [0.935823, 0.054289, -0.008677, 0.000527, -0.000011] \quad (\text{C. 16})$$

C.3 Incidence Angle Modifiers

The incidence angle modifiers describe the effect of the encapsulation material on the amount of beam and diffuse components of solar irradiance absorbed. The beam incidence angle modifier K_b may be calculated as

$$K_b = \frac{\exp\left(-\frac{KL}{\cos(\theta_r)}\right) \left[1 + \frac{1}{2} \left(\frac{\sin^2(\theta_r - i)}{\sin^2(\theta_r + i)} + \frac{\tan^2(\theta_r - i)}{\tan^2(\theta_r + i)}\right)\right]}{\exp(-KL) \left[1 + \left(\frac{n-1}{n+1}\right)^2\right]} \quad (\text{C. 17})$$

where:

- K = encapsulation material extinction coefficient [m^{-1}]
- L = encapsulation material thickness [m]
- n = encapsulation material refractive index
- θ_r = refraction angle

Typical values for K and n for glass are 4 m^{-1} and 1.526. The refraction angle may be calculated using Snell's law as

$$\theta_r = \sin^{-1}\left(\frac{1}{n \sin(i)}\right) \quad (\text{C. 18})$$

The diffuse incidence angle modifier K_d may be calculated using equation C.17. However, the equivalent array elevation angle β_{eq} must be substituted for the incidence angle i according to results presented in [15].

$$\beta_{eq} = 59.7 - 0.1388\beta - 0.001497\beta^2 \quad (\text{C. 17})$$