USE OF MANIFOLDS IN THE INSERTION OF BALLISTIC CYCLER
TRAJECTORIES

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ABSTRACT

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Today, Mars is one of the most interesting and important destinations for humankind and copious methods have been proposed to accomplish these future missions. One of the more fascinating methods is the Earth-Mars cycler trajectory which is a trajectory that accomplishes repeat access to Earth and Mars with little to no fuel-burning maneuvers. This would allow fast travel to and from Mars, as well as grant the possibility of multiple missions using the same main vehicle.

Insertion from Earth-orbit onto the cycler trajectory has not been thoroughly explored and the only existing method so far is a Hohmann-esque transfer via direct burn. The use of manifolds from gravitational equilibrium points has not been considered for low energy transfer to the cycler trajectory. This work is primarily focused on closing this gap and analyzing the feasibility of this maneuver.

To accomplish this, a study of the cycler trajectory – and the S1L1-B class specifically – was completed. The required gravity assist maneuvers at each planet was analyzed through V_∞ matching and the entire trajectory was generated over the required inertial period. This method allowed for the generation of 2 cycler trajectories of the inbound and outbound classes, which combine to allow for a reduction in the amount of time the astronauts spend in space.

The Earth-Sun L2 point is analyzed as a potential hub for the maneuver and a halo orbit about this libration point is optimized for low energy transfer from an Earth parking orbit. The associated invariant manifold is then optimized for launch date and distance to the first trajectory on the cycler in order to burn from a trajectory on the manifold to the cycler trajectory.

The comparisons of this work lie in the required ΔV to perform each maneuver compared to a direct burn onto the cycler trajectory. These values are compared and the practicality of this maneuver is drawn from these comparisons. It was found that the total required ΔV for the manifold method is larger than a direct burn from Earth orbit. However, this considers the trajectory from Earth to the halo orbit and if this is removed from consideration the ΔV is significantly reduced.

It was shown that the feasibility of this method relies heavily on the starting position of the cycler vehicle. If the vehicle begins in Earth-orbit, a direct burn is preferred, however, if the vehicle began in a halo orbit (say it was assembled there) the manifold maneuver is largely preferable.

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Chapter 1

INTRODUCTION

1.1 The Cyclor Trajectory

For decades, Mars has been the source of human fascination and is currently the next destination for today's astronauts. Currently, the most pressing issue with getting to the red planet lies in it's sheer difficulty. This is especially apparent when considering crewed missions and the associated ΔV cost. Several remedies have been proposed for this issue and among the most popular is the Earth-Mars cyclor, first introduced by Buzz Aldrin in 1985[9]. Through gravity assist maneuvers and repeat visits to Earth and Mars, this type of trajectory would allow several several crews over many years to visit Mars with only one main transport vehicle and little to no ΔV cost throughout its course. Many cyclors since the introduction of the "Aldrin Cyclor" have been proposed with the most notable (and practical) being the S1L1-B cyclor[14].

Though these trajectories are very effective in reducing overall mission cost and lifetime ΔV , cyclor insertion is still expensive in many respects. The vehicle in question would likely be very large to accommodate a long lifetime, a habitat for the crew, as well as multiple types of cargo; which would mean launching from the ground directly into transfer would be impractical. Furthermore, the insertion of cyclor trajectories has not been thoroughly examined, and would thusly required a direct burn transfer. Therefore, a rectification to the cyclor insertion is necessary in order to remedy some of these potential issues.

1.2 Potential Manifold Solutions

This work explores the potential of using manifolds from halo orbits in the Earth-Sun system in order to reduce the mission cost, effectively making the cycler trajectory a more practical option for crewed missions to Mars. Under the assumption that the vehicle would start in a predetermined parking orbit, the vehicle could do a series of relatively small burns to enter a halo orbit around Earth-Sun L2, perturb its orbit slightly to enter onto a trajectory in the associated manifold, and then perform a final burn to enter the desired first leg of the cycler trajectory (after some time in the manifold).

By greatly reducing the ΔV cost of the first transfer (compared to a normal Hohmann transfer) the practicality of a large vehicle designed for a large mission lifetime is substantially increased. Furthermore, because the velocities of objects on manifolds are very slow compared to the celestial speed of our solar system, the supplementary advantage of additional time to perform maintenance or further planning may be available if necessary.

1.3 Scope

The scope of this work is aimed solely at determining the validity of this type of transfer as well as explore the dynamics of cycler trajectories in order to gain a better understanding of them. This work will be limited to S1L1-B cycler trajectories, and only the dynamics around Earth-Sun L2 will be considered during the manifold phase. Further work will be discussed in the appropriately named section. An analysis will also be made about the total ΔV mission savings.

Chapter 2

CYCLER TRAJECTORY EXAMINATION

In this chapter, the underlying concepts, background, and various computation methods of cycler trajectories will be succinctly analyzed in order to become more familiar with their properties before they are analyzed in greater detail later in this work.

2.1 Background on Cycler Trajectories

In the early 1980s, famous astronaut and engineer Buzz Aldrin began working on his concept of a Lunar transport system that would allow routine measures, such as maintenance, supplies, or crewed missions, to be a more repeatable process to the Moon[1]. Eventually dubbed the "Lunar Cycler" this process was deemed unfit for further analysis because of the largely increased transfer time between the Earth and Moon. Later in 1985, Aldrin began extending his work however, to the Earth-Mars system as a more practical approach to his solution. He then created what is now called the "Aldrin Cycler"[1].

The concept of a cycler can be simply summarized as: *any trajectory with repeated access to two or more bodies in a system*. Many cycler trajectories have been studied (or proposed) with some of the more notable being,

- Earth-Moon or Lunar Cyclers
- Earth-Mars Cyclers
- Earth-Venus Cyclers

For the purposes of this project, the Earth-Mars cycler will be the focal point of study as it is the most studied and the most exciting in terms of future human developments

in space. Many variations of this trajectory exist and thusly the computation and shape of them differs widely. A few of the methods and visualizations of their shapes will be analyzed later in the chapter; additionally, a detailed computation of these can be found in Appendix A.

Since the formulation of the Aldrin cyler, many other Earth-Mars cyler trajectories have been discovered and analyzed. In a paper by a team at the University of Texas in Austin[18], many different classes of cyclers are analyzed and compared for the purposes of generating a solution space. Though many of these trajectories are impractical for human exploration, they do allow for further understanding of what is required to generate and optimize them.

Following these developments, a team from Purdue University was able to take these concepts and polish the cyler trajectory for human exploration purposes, generating the S1L1-B cyler[13]. This specific cyler will be used throughout this work as a running example. Further investigative or applicative work of the cyler has been sparse for many years since 2002 when the S1L1-B was generated; however, these trajectories still prove vital in the quest for human exploration to mars.

2.2 Gravity Assist Maneuvers

The fundamental process by which cyler trajectories operate, especially those in the ballistic class, are *gravity assist* maneuvers, otherwise known as *flybys*. Using the gravitational forces of the planet alone, it is possible to perform a rotation of the heliocentric (sun-centered) orbital trajectory. This is extremely useful in applications that require very complicated patched-conic trajectories, as well as those that require the spacecraft to move large distances. Some of the most notable missions that made use of the flyby are Voyager I & II[5], Cassini[2], and New Horizons[4]. Cyler trajectories take full advantage of this maneuver to allow for repeat access between

Figure 2.1: Flyby visualization for general system

Earth and Mars.

The rotation of the orbit occurs as a rotation of the velocity vector of the spacecraft on the incoming trajectory towards the planet. The rotation of the vector can only occur in the region known as the sphere of influence (SOI), which is the approximate distance from the planet that its gravity will still have an effect on the spacecraft. Once inside the SOI, the velocity vector rotates by an angle, δ , known as the turn angle. A diagram to visualize the effects and principles of a flyby are given in Fig. 2.1.

The turn angle is determined by a few parameters; the first of which is called the hyperbolic excess velocity, or v_{∞} . Although usually represented as a magnitude,

the vector is simply the difference in the velocity of the spacecraft, $\vec{v}$, to the velocity of the planet, $\vec{v}_{planet}$, in the heliocentric reference frame. The next parameter is the flyby radius, r_p which is the closest distance to the planet that the spacecraft will experience. The last parameter is the standard gravitation parameter, μ_p of the planet itself. This is simply the product of the gravitational constant, G , with the mass of the planet. The derivation will be spared here, but two equations for the turn angle are represented by the following :

$$\delta = 2 \arcsin \frac{\mu_p}{\mu_p + r_p v_\infty^2} \quad (2.1)$$

$$\delta = \arctan 2 \left(\frac{\|\vec{v}_{in} \times \vec{v}_{out}\|}{\|\vec{v}_{in}\| \|\vec{v}_{out}\|}, \frac{\vec{v}_{in} \cdot \vec{v}_{out}}{\|\vec{v}_{in}\| \|\vec{v}_{out}\|} \right) \quad (2.2)$$

where, $\arctan 2$ is the multi-value inverse tangent function that allows for quadrant correction. It is also possible to rearrange both equations to solve for r_p ; more on that very soon. If the desired $\vec{v}_{in}$ and $\vec{v}_{out}$ are known, the flyby is only ballistic (does not require an extra maneuver) if the incoming and outgoing magnitudes of the hyperbolic excess velocity are equivalent. That is to say

$$\|\vec{v}_{\infty out}\| - \|\vec{v}_{\infty in}\| = 0 \quad (2.3)$$

Sometimes, the turn angle is not sufficient to allow for this, as the required r_p is smaller than the radius of the planet. In this case, extra fuel must be spent to make up for the total ΔV required/desired. This is known as a powered flyby and will actively be avoided for the purposes of this project. Furthermore, a method developed for this work for determining ballistic flybys will be presented in Chapter 4.

2.3 Lambert's Problem

One of the oldest and classic problems in orbital mechanics is Lambert's problem. The problem statement is simple: given two orbital position vectors and a transfer time between them, can we find the velocity vectors at each position? Many, *many*

solutions have been developed for this problem with varying levels of speed and robustness. However, they all rely on an iterative method as the exact solution can not be found analytically.

For the direction of this work, I have chosen to use a method developed by Izzo & Gooding, with elements borrowed from a method by Lancaster & Blanchard [11]. This method was chosen because it is very fast and is also capable of *multi-rev* solutions, which are solutions that span more than one orbital period.

Other than multi-rev solutions, there are a few other characteristic inputs that can determine the shape of the solution. Chiefly, these are the *short & long way* and *left & right branch* solutions. Short and long way simply determine which direction around the sun the orbital path will take, while left and right branch determines the shape of the multi-rev solution. A diagram illustrating how these parameters affect the solution is given in Fig. 2.2. The start and end points are both positions of Earth but only the right branch multi-rev solution matches Earth's orbit.

The most important use of the solution to Lambert's problem for this thesis is to be able to determine the encounter times of Earth and Mars to connect the flyby maneuvers necessary to complete the cycler trajectory, as well as the shape of the cycler trajectory itself. Using the ephemeris model of the solar system, the position and velocity vectors of each planet are known at any time; which, when coupled with a chosen time of flight, a trajectory between Earth and Mars can be determined.

2.4 Cycler Classification

Before examining a cycler's computation, its nomenclature will first be momentarily discussed. There are two ways to classify a cycler trajectory, depending on its characteristics. The first belongs to cyclers that contain no intermediate or "taxi" gravity assists or flybys and simply go from one planet to the next. This designation takes

Figure 2.2: Shape of Lambert's solution with varying parameters

the form nPr , where n is the number of synodic periods until repetition, P is the period (transfer) type, and r is the number of revolutions (rounded to the nearest integer) in that period. The three period "types" are 'S', 'L', and 'U', for the short, long, and unique period classes respectively. For example, the "Aldrin Cyclor", which will be discussed later, is designated to the 1L1 class.

More recently identified cyclers now involve one or more of what may be called a 'taxi Earth flyby' which concerns extra, intermediate, flybys at Earth, thus creating a new trajectory between Earth and itself. These are mainly used in order to achieve ballistic status which will be discussed later. These classes are defined by their transfer between Earth and Mars, followed by Mars to Earth, nominated as *Leg 1* and *Leg 2*. In general, the designation of a specific cycler takes the form $P_1r_1P_2r_2$, where P and r share their previous descriptions and the subscripts denote the first and second legs respectively. An example of this type of trajectory is the S1L1 cycler which will be used throughout this project as the working example.

Beyond this, cyclers can also be defined by how fast they operate between planets, known as *outbound* and *inbound*. An outbound cycler is one that has a 'quick' trajectory from Earth to Mars while an inbound is quick from Mars to Earth. Both inbound and outbound trajectories are necessary for a feasible crewed mission to Mars as the expedition there and back would need to be short due to space environment concerns. This work will discuss the insertion into both inbound and outbound trajectories since they are both necessary.

As a final appendage, cyclers can also be defined as *powered* or *ballistic*. As briefly discussed with the flybys, this designation simply implies whether or not extra ΔV is required at each gravity assist in order to continue on the proper trajectory. For the direction of this work, a ballistic cycler will be analyzed in order to remain consistent with the overall goal of reducing mission ΔV .

2.5 The "Aldrin Cyclers"

When the concept of these types of trajectories was first being considered, many assumptions had to be introduced in order to simplify the problem as a proof of concept rather than a practicality. Buzz Aldrin pioneered this work and his assumptions can be summarized as follows:[19]

- The Earth-Mars synodic period, S , is exactly $2\frac{1}{7}$ years
- The orbits of Earth, Mars, and the cycler lie in the same plane
- The orbits of Earth and Mars are circular
- Gravity assist maneuvers can only occur around Earth
- Gravity assist maneuvers are instantaneous rotations
- ΔV maneuvers may be required during gravity assist maneuvers

The Aldrin cycler has been explored in some detail and a few methods exist for calculating the trajectory. The following method that was advanced for this work (see Appendix A) will be briefly elucidated here. Under the assumption that at $t = 0$ the position of Earth can be written in flat ($x - y$ plane only), Cartesian coordinates, the circular assumption allows the position of Earth at any time, t , to be written as $[\cos(2\pi t), \sin(2\pi t)]$. Thus, the required position of Earth at encounter events can be written as:

$$R_{t=0} = [1, 0] \tag{2.4}$$

$$R_{t=nS} = [\cos(2\pi nS), \sin(2\pi nS)] \tag{2.5}$$

where, n carries the designation from the nPr cycler class and represents the number of periods until the next Earth encounter occurs. The position of Earth at any encounter

can be solved for depending on the amount of periods to repeat are desired; in this case, $n = 1$. This automatically designates the shape of the orbit which can be found easily through a multi-rev Lambert’s solver, where $t = 0$ and $t = 2\frac{1}{7}$ are the Earth encounter times. The required positions of Mars at its encounter times can then be calculated by analyzing where the cycler trajectory and Mars’ orbit intersect.

A visual representation of the Aldrin cycler can be seen in Fig. 2.3. The numbers, 1 – 4, in each Earth and Mars correspond to the chronological order of the event of a Martian encounter or Earth flyby. For the sake of brevity in the following explanation, the positions of both planets will be referred to by this number. The vehicle begins at Earth 0 and performs a transfer to Mars 1 in about 147 days (142 sols). No gravity assist occurs at Mars and the vehicle continues for another 633 days (614 sols) until Earth 2 and performs a flyby as well as any additional required ΔV maneuver. This process is dubbed *Transfer 1* and takes a total of 779 days (748 sols) or $2\frac{1}{7}$ years. The vehicle then repeats this process indefinitely, beginning with *Transfer 2*, where the spacecraft moves from Earth 2 to Mars 3 and so on.

Though this specific cycler is useful as a case study for verifying the possibility of this type of transfer, the corpulent assumptions do not allow for a real-world application. This necessitates a new cycler that can operate in an environment that does not require these limiting assumptions. Furthermore, this cycler always requires extra burns at each flyby in order to obtain the proper turn angle. This is an undesirable result that drastically increases mission cost.

2.6 The Ballistic S1L1 Cycler

The S1L1 class represents a short, 1-rev period to Earth, followed by a long, 1-rev period to Earth, (since we are using the $P_1r_1P_2r_2$ designation). As a quick note, the short trajectory to Earth does not contain a Martian encounter. This cycler was first

Figure 2.3: The Aldrin Cyclers over two transfers

introduced in the circular/co-planar model but can also be carried to the ephemeris model of the solar system as was shown by McConaghy et al.[13]. For our purposes, both the circular and ephemeris model solutions will be introduced, but the focus will lie more heavily on the ephemeris model solution. Furthermore, this cycler was chosen as a case study principally because of its capacity to perform ballistically, i.e. there are no corrective ΔV maneuvers over the mission lifetime.

2.6.1 The Circular Solution

This solution refined here by the author draws heavily from the process used previously for the Aldrin Cycler (see Appendix A), but was first introduced by McConaghy et al.[13]. Since an intermediate Earth encounter has been added, the encounter time positions can now be written as

$$R_{t=0} = [1, 0] \quad (2.6)$$

$$R_{t=\tau} = [\cos(2\pi\tau), \sin(2\pi\tau)] \quad (2.7)$$

$$R_{t=nS} = [\cos(2\pi nS), \sin(2\pi nS)] \quad (2.8)$$

where τ is the intermediate Earth encounter time. It is important to note that in this case n is now equal to 2 as this is a multi-period solution. Furthermore, the time between 0 and τ is referred to as *Leg 1*, while *Leg 2* refers to the time between τ and nS . Unlike the Aldrin Cycler solution, all possible values of $\tau \in (S, 2S)$ are evaluated, thus removing some of the simplicity and restrictions from the method. The transfer from $\tau \rightarrow 2S$ is simply an Earth-Earth trajectory and is not required to cross the orbital path of Mars.

The required extra ΔV at each flyby of Earth is calculated and solutions with values outside of a certain tolerance are removed from consideration. Trajectories which allow for no ΔV maneuvers are set aside for further consideration and are put in the ballistic class. As an example, the results of a ballistic solution found for

$\tau = 2.8276$ years are as follows:

$$R_{t=0} = [1, 0]$$

$$R_{t=2.8276} = [0.4690, -0.8832]$$

$$R_{t=4.2857} = [-0.2225, 0.9749]$$

It is important to reiterate that these are the positions of Earth at the cycler vehicles encounter times with Earth. The encounter positions and times of Mars are calculated later just as with the Aldrin Cycler. A diagram of the S1L1 trajectory, similar to the Aldrin cycler, is given in Fig. 2.4. For the sake of clarity, the trajectory will be briefly explained in detail. The vehicle begins on *Transfer 1* at Earth 0 and begins a transfer to Mars 1. No flyby occurs at Mars and the vehicle continues to Earth 2 where a flyby is performed and the trajectory is shifted. The vehicle then moves to Earth 3 and performs a final flyby after which the entire process recurs in *Transfer 2* and so on.

2.6.2 The Ephemeris Model Solution

In order to perform this analysis, some of the previously limiting assumptions, such as circular/co-planar orbits, had to be relaxed. The new assumptions, or rather constraints, used can be summarized as follows:

- Earth and Mars orbits are modeled using JPL's 405 Ephemerides[7] model
- Gravity assist maneuvers can occur at both Earth and Mars
- Gravity assist maneuvers are modeled as instantaneous rotations

Because this model no longer contains some of the niceties from the previous solutions, the approach to the Aldrin cycler or S1L1 circular solution cannot be taken.

Figure 2.4: The S1L1 Cycler first 2 transfers (Circular/co-planar model)

Instead, a system of inequalities is optimized in order to converge to a solution. It is important to note that unlike the Aldrin cyler, each 1st leg is now different from the last, so all legs must be solved for ad infinitum (or at least until the positions of Earth and Mars inertially return to their initial states). This return time is ~ 33 years and will be hereafter referred to as the *Inertial Period*. A method presented by McConaghy *et al.*[14] describes the use of the SNOPT optimizer to generate a trajectory of this caliber, and a self-developed method of verification and generation of these trajectories will be presented later. The SNOPT optimizer stands for Sparse Nonlinear OPTimizer and uses a quasi-newton method of optimization[8].

Because every leg of this solution varies, the simplistic and concise nomenclature of encounter times and positions of Earth cannot be used to represent the trajectory. Instead, over the course of the entire inertial period, all encounter times of both planets are listed (rather than only those at Earth), as well as the flyby altitude and corresponding v_∞ . The positions of Mars are added since gravity assist maneuvers are now allowed for this planet. This list is known as the *cycler itinerary*. An example itinerary for an outbound cycler, which will be used extensively throughout this work, is given in Table 2.1. It is important to note that this itinerary was provided by T. Troy McConaghy[15] (along with an inbound example) and was not calculated by the author. However, these will be used as a basis for developing new ephemeris solutions later on.

A visualization of the first portion of this trajectory can be seen in Fig. 2.5. Since flybys are allowed at Mars, *Leg 1* is differentiated into *Leg 1* and *Leg 1.5* to account for the fact that although the trajectory changes slightly, they are still portions of the same leg.

Table 2.1: Example Outbound Cyclor Vehicle Itinerary (T. Troy McConaghy)

<i>Outbound Cyclor Vehicle (Troy McConaghy)</i>							
Encounter	Fly-By Planet	Year	Month	Day	V_{∞}	Closest Approach Distance	Leg, TOF
#	<i>P</i>	yyyy	mm	dd	km/s	km	days
0	Earth	2005	8	13	4.01	--	--
1	Mars	2006	2	27	3.02	4816	198
2	Earth	2008	6	9	6.89	20130	833
3	Earth	2009	12	3	6.9	31110	541
4	Mars	2010	6	6	4.31	17710	186
5	Earth	2012	8	24	6.42	26490	809
6	Earth	2014	2	14	6.43	41520	539
7	Mars	2014	7	3	7.14	12190	138
8	Earth	2016	12	9	4.01	27730	890
9	Earth	2018	5	22	4.03	19920	530
10	Mars	2018	9	15	6.47	11580	115
11	Earth	2021	4	6	4.61	22990	934
12	Earth	2022	9	20	4.59	14780	532
13	Mars	2023	5	1	2.77	7601	223
14	Earth	2025	7	2	7.08	23860	793
15	Earth	2026	12	26	7.09	35120	542
16	Mars	2027	6	14	5.27	13840	170
17	Earth	2029	9	21	5.8	26850	830
18	Earth	2031	3	12	5.8	37520	537
19	Mars	2031	7	15	7.85	8802	125
20	Earth	2034	1	15	4.21	24870	915
21	Earth	2035	6	28	4.2	2756	529
22	Mars	2035	11	12	5.87	1770	137
23	Earth	2038	5	5	7.23	--	906

Figure 2.5: The Ballistic S1L1 Outbound Cyclers first two legs (Ephemeris Model)

Chapter 3

MANIFOLD DYNAMICS EXAMINATION

This chapter will briefly discuss the background and of the dynamics of manifolds as well as the 3-body dynamics required to access the manifold solutions necessary for this work. The concepts required to evaluate these solutions will also be analyzed.

3.1 Background of Manifold Dynamics

For most, if not all of spaceflight history, the trajectories of spacecraft have been determined through the use of patched conics and planetary flybys. Despite the usefulness and rigidity of these 2-body methods, they can require precise timing, a relatively large amount of fuel consumption, and can be inflexible in handling more complicated mission requirements. In order to allow for more complicated mission trajectories and requirements, the dynamics of multi-body systems can be utilized. The concepts involved with multi-body dynamics and manifolds is very complicated and will be discussed in detail here.

The dynamics of multi-body systems, and 3-body systems in particular, stem from the *circular restricted 3-body problem*, which describes the motion of an object around 2 other large bodies. Though this problem has been known for around 200 years, the formulation of the equations of motion were not well-described until 1967[20], and one of the first missions to utilize them was the ISEE-3 which didn't launch until 1978[6].

One of the most notable missions to date that will make use of these dynamics is the James Webb Space Telescope (JWST)[12]. Through the utilization of halo orbits around the Earth-Sun L2 point and corresponding approach trajectories on a

manifold, the telescope is capable of communicating with Earth, unimpeded, and can arrive with very low ΔV cost with respect to the distance travelled. The fundamentals of this mission are helpful in understanding halo orbit insertion, which will be discussed later.

3.2 The Circular Restricted 3-Body Problem

Before the dynamics of manifold trajectories are discussed in any form, it is necessary to introduce the underlying physics of the matter. The circular restricted 3-body problem (CR3BP) describes the effects that two separate large bodies, dubbed the *primary* and *secondary*, have on a much smaller third body, called the *tertiary* body[3]. As a side note, some call the larger bodies the smaller and larger primary, but for our purposes, we will use the previous nomenclature. The geometry used to represent this problem is given in Fig. 3.1. It is important to note that for most systems, the barycenter of the two objects will be contained within, and usually very close to the center of, the primary body.

The dynamics of the CR3BP primarily take place in what is called the *fixed frame*, or sometimes called the *synodic frame*. This frame does not move with respect to the secondary and allows the trajectory with respect to the secondary become more apparent. It also allows the equations of motion to greatly simplify. For the most part, trajectories under the influence of CR3BP dynamics will be plotted in this frame.

In order to make the dynamics of this problem a little more manageable, the equations are derived using a unit system known as *canonical units*. These are units uniquely defined by a parameter in the system that we want to normalize, such as the distance between two objects for example. For the purposes of CR3BP, the masses of the primary and secondary bodies in the system are used to define the desired

Figure 3.1: Circular Restricted 3-Body Problem Geometry

canonical units. The parameter in question is given through the following equation:

$$\mu^* = \frac{m_2}{m_1 + m_2} \quad (3.1)$$

where m_1 and m_2 are the masses of the primary and secondary bodies respectively.

The assumptions necessary for these dynamics to work are listed as follows:

- All bodies will be modeled as point masses
- There are no orbital perturbations present (gravity is only acting force)
- The primary and secondary bodies have circular orbits around the barycenter of the system
- $m_3 \lll m_2 < m_1$

The diagram in Fig. 3.1 gives an idea of how masses and distances can be defined in terms of mass units [MU] and distance units [DU]. After normalizing the system with μ^* , the properties of the system are as follows.

$$m_1 + m_2 = 1 \quad [\text{MU}] \quad (3.2)$$

$$d_1 + d_2 = 1 \quad [\text{DU}] \quad (3.3)$$

where, 1 DU is equivalent to the distance between the primary and secondary in km, and 1 MU is equivalent to the mass of the entire system in kg. Furthermore, the derived units are given below.

$$[\text{TU}] = \sqrt{DU^3/\mu} \quad (3.4)$$

$$[\text{VU}] = DU/TU \quad (3.5)$$

where μ is the standard gravitational parameter of the primary body in km^3s^{-2} . It is also important to note that the barycenter of the system is fixed to be at the point $(0, 0)$ in the x-y plane.

To derive the equations of motion, the gravitational effects of the objects are modeled as point forces. A detailed derivation can be found in reference [3]. Summing the forces over all directions gives the *neatly* summarized accelerations as

$$\ddot{x} = 2\dot{y} + x - (1 - \mu^*)\frac{x + \mu^*}{r_1^3} - \mu^*\frac{x - 1 - \mu^*}{r_2^3} \quad (3.6)$$

$$\ddot{y} = -2\dot{x} + y - (1 - \mu^*)\frac{y}{r_1^3} - \mu^*\frac{y}{r_2^3} \quad (3.7)$$

$$\ddot{z} = -(1 - \mu^*)\frac{z}{r_1^3} - \mu^*\frac{z}{r_2^3} \quad (3.8)$$

where,

$$r_1 = \sqrt{(x + \mu^*)^2 + y^2 + z^2} \quad (3.9)$$

$$r_2 = \sqrt{(x - 1 + \mu^*)^2 + y^2 + z^2} \quad (3.10)$$

Due to the nature of these equations, there exist no analytic methods of integrating them over time. To mitigate this, many numerical methods exist to describe the motion of an object under these dynamical laws. For the sake of conciseness, such a method will be observed when discussing the computation of halo orbits. It is also very important to note that all equations of motion when dealing with the CR3BP are in the *synodic frame*, which means that the position of the planet will be fixed according to the equations of motion.

3.2.1 Libration Points

One of the underlying and most prevalent effects of CR3BP are the gravitational equilibrium points, termed *Lagrange* points or *libration* points. In each system, there exists 5 libration points where no acceleration of a third body (assumed to lie in

Figure 3.2: Libration point locations for generic system

the x-y plane) occurs. The positions of these points can be found by setting the accelerations in x and y to zero, as well as any velocities. This gives the following:

$$x - (1 - \mu^*) \frac{x + \mu^*}{r_1^3} - \mu^* \frac{x - 1 - \mu^*}{r_2^3} = 0 \quad (3.11)$$

$$y - (1 - \mu^*) \frac{y}{r_1^3} - \mu^* \frac{y}{r_2^3} = 0 \quad (3.12)$$

In order to solve for all solutions to x and y, one must solve a quintic equation. This gives the 3 co-linear points, L1, L2, & L3, and the 2 triangular points, L4 & L5. In general, L1 lies between the primary and secondary, L2 lies beyond the secondary, L3 is on the far side of the primary, L4 lies at an upper equilateral triangle to the primary and secondary, and similarly L5 lies at the lower triangle. A diagram for a generic system of libration points is given in Fig. 3.2.

For reference in a physical application, if a point mass were to be placed at any of these libration points with no initial velocity, it would remain in that exact spot

relative to the primary and secondary body. These points can also be orbited in a special trajectory called a *halo orbit* – sometimes called a periodic orbit around a libration point – and can even be approached/departed in a sort of free ride known as a *manifold* trajectory. It also is interesting to mention that the co-linear points are unstable and slight perturbations can effect the position of the point mass heavily at the points. However, this instability will not be considered for the purposes of this work.

3.3 Halo Orbits

As previously mentioned, halo orbits are perfectly periodic trajectories about a libration point, similar to a traditional orbit around a body. There are a few key differences however, that make these trajectories extremely unique. These are also not the only orbital paths around libration points; which includes lissajous and Lyapunov orbits, but these will not be discussed here. In order to spare an ample amount of derivation, the computation of these trajectories will not be presented here. However, a detailed method for their computation can be found in a paper written by Megan Rund[17]. The calculation begins with an approximate, analytic solution, which is then iterated upon through a numerical error method. This mechanism provides a sufficiently adequate solution of the halo orbit and has been deemed suitable for the purposes of this work.

Halo orbits do not form a traditional orbital shape – such as an ellipse – and instead display a saddle geometry that curves around the libration point. There are two main factors that determine the shape of the halo orbit: first is its out-of-plane amplitude, hereafter referred to as *z-Amplitude* or A_z ; the next is its class, either *northern* or *southern*, which determines the orientation of its curvature around the libration point. The classes are related through the symmetry $z \rightarrow -z$ and are purely

Figure 3.3: Various halo orbits about Earth-Sun L1

a consequence of symmetry[10]. An example of various halo orbit shapes and sizes can be found in Fig. 3.3.

3.4 Manifolds

Due to the dynamics of the CR3BP, there are a few interesting anomalies that can occur; namely with the trajectories that spacecraft can take. Manifolds are a collection of trajectories that approach or depart a particular libration point or libration orbit. Any object that can somehow maneuver onto a trajectory in a manifold will continue to follow this trajectory until libration point/orbit insertion, or until the influence of the CR3BP is no longer active. Due to the nature of the system, the shape of these

manifolds can vary widely.

Before discussing shape, however, it is necessary to discuss the various categories of manifolds. Manifolds can belong to the *stable* or *unstable* denomination which denotes whether the trajectory approaches or departs a libration orbit (or libration point) respectively. In many cases, the stable manifold is the most explored since the libration orbit itself is the desired result. However, the unstable manifold can be used to send a spacecraft beyond the system in question. The unstable manifold will prove critical to the success of this thesis later on.

The invariance of manifolds is also an important factor to be considered. *Invariant* manifolds are those that will always approach or depart a halo orbit, while the *variant* variety cannot be mapped to a periodic orbit around a libration point[16]. For the purposes of this work, only the invariant manifolds will be used. To visualize the shape of the invariant manifolds, a plot of the stable and unstable variety from an L1 halo orbit are given in Fig. 3.4. It is clear from the diagram that far from the halo orbit, the trajectories on the manifold follow a semi-regular pattern and form a tube shape that is common to most manifolds. However, very near the halo orbit, the dynamics are very complicated and do not form any recognizable shape. This should not prove to be an issue however, as the dynamics in this region are predictable.

Since the manifold is a collection of trajectories, only a single trajectory within the manifold can be chosen for a specific mission purpose. A method for choosing one of these trajectories, relevant to this work, will be discussed in great detail in Chapter 5.

Figure 3.4: Manifolds approaching and departing L1 halo

Chapter 4

CYCLERS

The cycler trajectory can be a complicated path to understand, and an even more complicated one to find; that is, in the Ephemeris model of the solar system. One of the most pressing questions that one may have is why the cycler trajectory is calculated in the ephemeris model, when the manifolds have to be calculated using a circular model of the solar system.

Although the CR3BP obviously requires the assumption of a circular orbit of the secondary body, the dynamics of the CR3BP have proven to be adaptable to the accurate model of the solar system. Another point, is that the circular solution of the cycler trajectory simply can't be used for practical application; as was discovered by McConaghy *et al.*[13], when adapting a cycler trajectory to the ephemeris model, the circular solution cannot even be used as an initial guess for the solution as the algorithm can never converge. This de-legitimizes the circular solution as a valid feasibility study.

Because the goal of this work is to prove feasibility of the manifold-to-cycler system, it is necessary to obtain the most reasonably accurate model in order to determine a valid solution space. With that in mind, this chapter will discuss the fundamentals of obtaining a cycler trajectory in the ephemeris model of the solar system.

4.1 Verifying The Ephemeris S1L1-B Cycler

Before discussing the method developed for generating the cycler trajectory, a method for verifying existing cycler trajectories will be presented here. This is because the

underlying concepts of a cycler’s verification feed into its generation; a fact that will become clear later. In order to demonstrate this process, the itinerary presented in Chapter 2 in Table 2.1 will be used.

4.1.1 Pork-Chop Plots

The principle method for which verification will occur relies on an extension to the *pork-chop plot*. These are plots that demonstrate the V_∞ (hyperbolic excess velocity) contours plotted against the launch and arrival dates for an interplanetary flight. Sometimes, instead of V_∞ the characteristic energy (C_3) is used instead. As a visual example, Fig. 4.1 provides a pork-chop plot for the first leg of the particular cycler of study; that being the transfer from Earth to Mars. Since the launch and arrival dates are known, I extended the search space to include 13 days before and after each of these dates in order to generate a plot with more than one solution.

Notice that the V_∞ at both Earth and Mars are considered, as well as the time of flight. This allows for anyone designing a mission profile to quickly choose the transfer that is best for them simply by looking for the most desired intersection of the contours. For the purposes of this work however, it is necessary to generate 2 pork chop plots for any 1 Leg in order to guarantee that the flyby is ballistic. After the spacecraft travels from Earth to Mars, it must travel back to Earth using only a ballistic flyby, so another plot is generated to show the possibilities of the flight back; which is given in Fig. 4.2.

Since the V_∞ contours at Mars are presented in both charts, one can simply find where they are the same and this is where the possibility of a ballistic flyby occurs. It is important to keep in mind that the arrival date of Mars must be the same as its departure date when determining this flyby. The same process can be carried out for the Earth-Earth transfer that occurs afterwards.

Figure 4.1: Pork-chop plot for Earth-Mars transfer

Figure 4.2: Pork-chop plot for Mars-Earth transfer

V_∞ Surface Plots

Unfortunately, the previous process is relatively under-detailed and only truly allows for a visual marker into the actual flyby that is needed. Scouring various methods for a way to accomplish this more favorably proved fruitless, thus it was decided that developing a new method for determining the flyby date (launch and arrival dates) was necessary. This method works by consolidating the pork-chop plots into a surface map that only shows the possible ballistic flyby dates; this will be hereafter referred to as a V_∞ *surface plot*.

The method for generating the pork-chop plots already obtains matrices of all possible V_∞ values for approaching and departing a planet (depending on departure and arrival dates), these values can be consolidated into two surface plots and overlaid graphically. The effect of this graphing method allows the intersection contour of the two surface plots to be visualized as well as directly calculated through mesh intersection algorithms. An example of a surface plot generated using the previously shown pork-chop plots is given in Fig. 4.3.

The approaching and departing surfaces contain all possible values of the V_∞ based the dates at which they occur. In this case, for the approaching plane, the departure dates of Earth and the arrival dates of Mars are considered, whereas with the departing plane, the departure dates from Mars and the arrival dates at Earth are considered. The importance here is that the arrival and departure dates of Mars should be the same (or at least within 24 hours to account for the flyby time); this allows the two surfaces to be plotted atop one another. The intersection of these planes, the magenta dashed line, gives the corresponding dates at which a ballistic flyby is possible. These intersections or *contours* will prove important when discussing the cycler timeline.

Figure 4.3: V_∞ surface plot for Earth-Mars-Earth trajectory

Figure 4.4: Generated timeline from known itinerary

Timeline

By generating a plot of this sort for every flyby in the cyclor itinerary, all ballistic flyby contours can be generated for the cyclor trajectory. Each one can then be laid on an x-y coordinate frame where each axis represents time in days past earliest launch from Earth. A line of $x = y$ can then be drawn through all of the contours, termed the *timeline* from now forward. Wherever this timeline intersects with the contour, that is the date at which a ballistic flyby should occur. The goal is the make sure that the timeline crosses the path of every single contour in the the x-y time-plane we have created. To illustrate this, a plot of this process can be found in Fig. 4.4.

Some will notice that there are some contours that do not intersect the timeline

– labeled as "*Non-Intersecting*" in the figure. This is due to the fact that only entire days are considered and the time it takes to complete a flyby is not. This causes some curves to come very close to, but not directly cross the timeline. To mitigate this, the closest point on the contour to the timeline is chosen to act as the flyby date. For the interested, an illustrative plot of some of the intersections of contours and the timeline can be found in Appendix A.

When comparing the found flyby dates to the original itinerary, found in Table 2.1, one can see that they are quite similar, with some admitted variation. This is mostly attributed to the fact that the itinerary was originally generated through an optimization, whereas this replica was generated through flyby contours. However, both are technically valid and it is now possible to extend this method of verification to generating an entirely new cycler trajectory, albeit of the same shape.

4.2 Cycler Generation

Though not entirely the focus of this work, in order to prove the feasibility of cycler insertion via manifolds, it is necessary to first prove that a proper cycler trajectory can be generated in the first place. Though there have been a few works illustrating this possibility, their methods were unadaptable for this work as powerful optimization software was used and the original authors of the method had not worked on the problem in over a decade[15]. It was also found that cyclers can be generated via an adaptation of the previously derived verification method.

4.2.1 Transfer Times

The method designed for this work involves using known cycler itineraries to obtain a general time line to search around. This is down by finding the average transfer times between each of the three transfers, those being Earth-Mars, Mars-Earth, and Earth-

Table 4.1: Transfer times for inbound and outbound cyclers

Trajectory	Outbound [days]	Inbound [days]
Earth-Mars	161 (115,231)	868 (788,938)
Mars-Earth	862 (788,930)	159 (108,225)
Earth-Earth	536 (529,542)	536 (529,542)

Earth. Using the itineraries provided by T. McConaghy[15], the average transfer times were determined for the outbound and inbound cyclers, which can be found in Table 4.1.

There are a few interesting things of note here. Firstly, the Earth-Mars outbound trajectory has very similar characteristics to the Mars-Earth inbound trajectory. The same relationship is true for Mars-Earth outbound vs. Earth-Mars inbound. This makes sense due to the fact that although the trajectories have the same shape, they are designed for the exact opposite purpose. Notably, the Earth-Earth trajectories not only have similar characteristics between inbound and outbound trajectories, but they are the exact same. Furthermore, their transfer time range is much tighter than the other transfers, with a maximum range of only 13 days.

4.2.2 Relative Positions

The ranges of these transfer times are now used as the basis for how broad a search must be performed by the V_∞ surface plots, with a bit of leeway for outliers that may occur. It is always possible to start the search from any launch and arrival dates, but these must be chosen wisely in order to insure that this method converges to a solution quickly. Because the first Earth-Mars transfer has to maintain a certain shape, the relative positions of Earth and Mars at the launch date are relatively constrained. That is to say that in order to exclude solutions from the space that have unorthodox transfer times, the relative angular positions of Earth and Mars must be maintained

Table 4.2: Relative angular positions of Earth and Mars

	Radians [<i>rad</i>]	Degrees [$^{\circ}$]
Outbound (Earth-Mars)	4.5392 (4.3309,4.7507)	260.08 (248.14,272.20)
Inbound (Mars-Earth)	3.5866 (3.4007,3.6737)	205.50 (194.84,210.49)

within a certain range. To find this range, a similar analysis to the previous was performed but instead found the average relative angular position of Earth to Mars just before the Earth-Mars transfer. This does not give insight into the complete cyler, but only the possible launch dates from Earth at the start of any given cyler.

The relative angular positions of Earth and Mars can be found through their angular offset from a chosen reference direction, Υ , with direction $[1, 0, 0]$ relative to the sun. The angular position of a planet, ϕ_p can be found via the following:

$$\phi_p = \omega + \theta - 2\pi m \quad (4.1)$$

where, ω is the argument of periapse of the orbit and θ is the true anomaly of the current planet position. Also, $m = 0, 1$ and the entire term is meant to wrap the angle, ϕ_p to $[0, 2\pi)$. Using Eq. 4.1, the relative angular positions of Earth and Mars, ϕ , can be found at any time by simply subtracting to two. As a quick note, Earth's angular position is always subtracted from Mars', so a positive angle represents Mars ahead of Earth while a negative represents the opposite. A summary of the average relative angles for Earth-Mars outbound and Mars-Earth inbound trajectories are given in Table 4.2.

4.2.3 The Search

We now have all of the data and tools necessary to begin searching for and computing S1L1-B cyler trajectories of the inbound and outbound classes. Though this is most certainly a brute force method for calculating a cyler trajectory, most of the

complications that come along with an optimization are remedied. As a quick note, the flyby altitude, r_p , for each flyby will not be calculated until the entire cyclical trajectory is generated. This is mostly due to the fact that the flyby can occur at any of the possible times (which will have varying r_p and these times cannot be completely determined until the entire trajectory is known.

To get a general idea of how these itineraries are generated, the following is a list of steps taken to determine them.

1. Choose an arbitrary launch date from Earth
2. Calculate the relative angular position, ϕ , to Mars and adjust launch date until within known bounds (Table 4.2)
3. Using the known transfer time bounds (Table 4.1) for the desired Earth-Mars trajectory, generate a V_∞ surface plot for all possible arrival and departure dates around the chosen launch date
4. Choose an arrival date from the generated possibilities and repeat the process for a Mars-Earth transfer, then for an Earth-Earth and so on
5. Repeat until no possible arrival/departure dates can be found and adjust the initial arrival date at Mars
6. Repeat entire process until a feasible itinerary is found; generating a timeline and looking for intersections
7. Calculate all flyby altitudes, r_p , and v_∞ s for the trajectory
8. If a flyby crashes into a planet, start the process over entirely

One may see this as an extremely arduous and painstaking process. However, this method is only meant to generate a few cyclical trajectories to prove the concept as

well as form a basis for the rest of the work. Because most of the visualization of the process is contained within the previous sections of this chapter, that will not be presented here, however, a few steps of the process and a few extra itineraries can be found in Appendix A.

However, to demonstrate that this process does work, an itinerary and visual representation of an outbound cyler in Table 4.3 & Fig. 4.5 as well as an inbound cyler in Table 4.4 & Fig. 4.6 are provided. Notice that only given the first 3 legs are given in the visual representation; this is due to the fact that although there are slight variations in the shape of each similar leg, plotting every leg would not be useful in understanding the repetition of the cyler trajectory. Furthermore, the third leg is sufficient in demonstrating this repetition since it looks similar in shape and scope to the first leg.

As a quick note, it is necessary to mention that a visual representation of any of the flybys will not be presented here. This is because the flybys are modeled as instantaneous rotations of the velocity vector and will therefore not take the shape of the flyby diagram, Fig. 2.1 in Chapter 4. However, this diagram should be satisfactory in demonstrating the shape that a flyby trajectory takes.

4.3 Insertion ΔV

With that being said, we now have cyler trajectories ready to be inserted onto with the help of manifolds. It is important to note that only the first leg of each cyler is necessary for insertion since the cyler is self-repeating. However the first leg of a cyler had to be found before it could be utilized in any appropriate manner. Before moving on to manifolds however, a quick number will be generated for comparison purposes later. Since the interest in this mission lies with the possibility of reducing the total ΔV required, it is first vital to know what a normal burn would require.

Table 4.3: Generated outbound cyclor trajectory itinerary

<i>Outbound Cyclor Vehicle 1</i>							
Encounter	Fly-By Planet	Year	Month	Day	V_{∞}	Closest Approach Distance	Leg, TOF
#	P	yyyy	mm	dd	km/s	km	days
1	Earth	2007	9	30	4.52	--	--
2	Mars	2008	5	19	3	6600	231
3	Earth	2010	7	16	7.05	25000	788
4	Earth	2012	1	9	7.06	37500	542
5	Mars	2012	6	17	5.89	9800	160
6	Earth	2014	10	11	5.33	25500	846
7	Earth	2016	3	29	5.31	35200	535
8	Mars	2016	7	25	7.87	9600	118
9	Earth	2019	2	4	3.99	22800	924
10	Earth	2020	7	17	3.98	4270	530
11	Mars	2020	12	18	4.36	5150	154
12	Earth	2023	5	25	6.08	20050	887
13	Earth	2024	11	13	6.1	27300	538
14	Mars	2025	6	1	3.71	16100	200
15	Earth	2027	8	8	6.72	25900	799
16	Earth	2029	1	30	6.73	41300	541
17	Mars	2029	6	27	6.62	16700	148
18	Earth	2031	11	14	4.46	32000	871
19	Earth	2033	4	29	4.47	24900	532
20	Mars	2033	8	18	7.58	7100	111
21	Earth	2036	3	6	4.72	22300	930
22	Earth	2037	8	20	4.74	600	532
23	Mars	2038	1	22	5.66	1450	155
24	Earth	2040	5	11	11.05	--	840

Figure 4.5: First 3 legs of generated outbound cycler

Table 4.4: Generated inbound cycler trajectory itinerary

<i>Inbound Cycler Vehicle 2</i>							
Encounter	Fly-By Planet	Year	Month	Day	V_{∞}	Closest Approach Distance	Leg, TOF
#	P	yyyy	mm	dd	km/s	km	days
1	Earth	2007	5	29	4.29	--	--
2	Mars	2009	10	17	5.83	4500	872
3	Earth	2010	3	25	6.94	40600	159
4	Earth	2011	9	18	6.95	23400	542
5	Mars	2013	11	17	3.25	12000	791
6	Earth	2014	6	15	5.81	23700	210
7	Earth	2015	12	4	5.79	18300	537
8	Mars	2018	5	27	4.6	6960	905
9	Earth	2018	10	15	3.78	8150	141
10	Earth	2020	3	26	3.79	26100	529
11	Mars	2022	9	28	7.72	12500	916
12	Earth	2023	1	27	6.05	43300	121
13	Earth	2024	7	18	6.07	11200	538
14	Mars	2026	11	3	5.57	2100	838
15	Earth	2027	4	17	7.16	31000	165
16	Earth	2028	10	10	7.15	24400	542
17	Mars	2030	12	8	2.76	9600	788
18	Earth	2031	7	21	4.66	17300	225
19	Earth	2033	1	3	4.66	21800	532
20	Mars	2035	7	26	6.34	9400	934
21	Earth	2035	11	19	3.96	19000	116
22	Earth	2037	5	2	3.95	26200	530
23	Mars	2039	10	14	7.08	41800	895
24	Earth	2040	4	7	3.4	--	176

Figure 4.6: First 3 legs of generated inbound cycler

In order to transition from an Earth-centered orbit to a Sun-centered one, if it is first assumed that the parking orbit of the spacecraft is such that the velocity of the spacecraft relative to the sun is indistinguishable from the velocity of the Earth relative to the sun, the spacecraft would only have to move at V_∞ km/s faster than the Earth in order to transition onto the cycler; so the only number to consider is the V_∞ required at the first leg. As an example, when using the itinerary found in Table 2.1, the ΔV value would be 4.01 km/s. Other values will be presented and compared when discussing test cases in Chapter 6. However, this is not always the case as the shape of the parking orbit can vary widely. In the next chapter, a method to find a parking orbit will be discussed, and the associated ΔV to burn from the parking orbit onto the cycler trajectory directly will be analyzed.

Chapter 5

MANIFOLDS

The basic principles involved with this section involve the characteristics of the trajectories on the manifolds that will allow the spacecraft to move from Earth onto the cycler trajectory, while attempting to reduce the overall ΔV involved in the process. This method will then be compared against a normal Lambert's solution transfer.

5.1 Halo Orbit Insertion

In order to simplify the following problem, the launch environment and its accompanying aspects will not be analyzed or considered in this work. This leads to the assumption that the spacecraft will start in some parking orbit around Earth and perform a burn from there. The main reasons for this exclusion are that the size, weight, & other attributes of the spacecraft are largely unknown and most interplanetary spacecraft execute their final burns from parking orbits nevertheless. This allows the ΔV to be optimized in a much less binding environment; however, the various shapes of parking orbits will be considered both in their performance as well as their practicality.

Before optimizing the shape of the halo orbit, it is first necessary to make sure we can find the trajectory on a stable manifold that would reduce the overall distance from the planet. For pragmatic reasons, the trajectory closest to Earth is the most 'do-able' in terms of the spacecraft arriving at the associated parking orbit. A code developed by Megan Rund[17] was adapted for this work in order to find this trajectory. By giving the function the characteristics of the halo orbit and the propagation time in [TU], the following steps are taken to find the closest trajectory to Earth.

1. The points in the halo orbit are broadened to allow for quick computation of the various trajectories in the manifold
2. The invariant-stable manifold trajectories are calculated from each point
3. The closest trajectory in the manifold is then found through testing each trajectory
4. The points on the halo are refined to their original quantity for a fine search algorithm
5. The trajectories on the manifold are then calculated for points close to and on either side of the recently found closest trajectory
6. The closest trajectory of that set is found and is used as the output closest trajectory to Earth
7. The necessary parking orbit to reduce the overall ΔV to burn onto the manifold is then calculated

The outputs of the function give the closest trajectory on the manifold as well as the parking orbit and ΔV necessary to burn onto the manifold. To visualize the outputs of this process, as southern class halo around L2 with $A_z = 476083$ km was calculated and utilized for the trajectory optimizing code. Fig. 5.1 gives a representation of the entire trajectory from Earth to halo, while Fig. 5.2 gives a visualization of the parking orbit and its departure. A larger and more detailed figure will be given for the optimized trajectory later on.

For the curious, the outputs and associated parameters of this test are as follows: Parking Orbit Altitude (h_p) = 330.5385 km, Total Time of Flight (TOF) = 197.0438 days, $\Delta V = 3.1629$ km/s. One may notice that the time of flight is very large. This is because relative to the fixed frame, a trajectory on the manifold moves very slowly. A

Figure 5.1: Example optimized trajectory for given halo orbit

Figure 5.2: Parking orbit to trajectory in manifold

Figure 5.3: Trajectory in manifold in sun-centered inertial frame

relatively unexciting result can be obtained when switching from this fixed frame to the inertial frame. A representation of how the earth, the L2 point, and the spacecraft on the trajectory move over time in the inertial frame can be seen in Fig. 5.3.

5.1.1 Halo Orbit Optimization

Now that we have the ability to optimize (albeit via brute-force) a trajectory given a certain halo orbit, the halo orbit shape itself can be optimized in order to further reduce the required ΔV . In order to do this, two separate optimization methods that perform a broad search followed by a fine search were set up. This was done using MatLab's *fminsearch* optimization function. Though a bit of a departure from the

previous method of optimization, the author settled on the fact that the nature of halo orbits and their stable manifolds are extremely unpredictable by human standards; thus, attempting to find the proper size of halo orbit would have required an unprecedented amount of trial and error. Allowing an optimization method to 'take care of the dirty work' per se is a much more viable option for this case.

The *fminsearch* function is a derivative-free method optimizer that operates by giving the user the choice of an initial guess of the solution which is then iterated upon. This allows an optimized solution to reach a local minima. To mitigate this and allow for a global minimum to be obtained, the optimization is run at various initial values which are then compared later. Although this is a multi-dimensional optimizer, the only input will be the size of the halo or A_z . The classes of the halo orbit were not considered, as it does not matter in this case. The default parameter chosen was the southern class.

In order to attain the range of optimized solutions, five separate optimizations with values of $A_z = \{3E5 \rightarrow 7E5 \mid 1E5\}$ km were produced. These values were derived from known possible values for the size of halo orbits around L2 without pushing the limits of what is possible. The optimization was also only allowed to run for 20 iterations to view the trend of the optimization and avoid optimizing on values with very low change. An example of the optimization iterations for $A_{z0} = 7E5$ is given in Fig. 5.4. In conjunction an organization of the results is given in Table 5.1.

The obvious trend that most will notice is that increasing size of the halo orbit generally decreases the magnitude of the ΔV . While this trend may seem favorable, it is also important to look at both the amount of time that the spacecraft will spend on this trajectory, as well as the parking orbit that is associated with each trajectory. Though the focus of this work lies in reducing the ΔV for cycler trajectory insertion, it is vital not to ignore practicality along the way. For the purposes of this work, any

Figure 5.4: Optimization example for $A_z = 7 * 10^5$ km

Table 5.1: Optimzation results for various halo orbit regimes

A_{z0}	A_z	ΔV	h_p	TOF
$[10^5 km]$	$[10^5 km]$	$[km/s]$	$[km]$	$[days]$
3	2.999707	3.18390	235	197
4	4.197894	2.75254	2413	31
5	5.183945	3.08510	353	169
6	6.300286	2.31501	5799	738
7	7.746880	1.58907	11042	596

transfer that takes more than a year, or any parking orbit higher than lower MEO will deem the manifold transfer unfit for consideration. The time constraint is chosen based on the fact that the cycler trajectory will already need to be operable for a significant amount of time (~ 33 years), and any longer than that is beyond what most spacecraft are conceivably capable of. The parking orbit constraint is derived from the basic fact that LEO is the most accessible orbital regime around Earth much higher than lower MEO can be costly; even if the cycler spacecraft were to be built in orbit, LEO to lower MEO is still the most viable option.

Though a strict weighting system was not considered, one of the most obvious choices for the halo orbit lies in the 400,000 km regime with a reasonable parking orbit, relatively low ΔV , and a very favorable transfer time. In an attempt to find the best solution in this locality, the value $A_z = 4.197894E5$ km was plugged back into the optimization and run until the default tolerances of the optimization were met ($1E-4$). Though not much of a change, the ΔV cost was lowered to 2.7513 km/s. Furthermore the new parameters are $A_z = 4.197279E5$ km, $TOF = 172$ days, and $h_p = 2430$ km. Despite the efforts of the optimizer, the change in TOF compared to the ΔV saved was displeasing, and therefore it was decided that the original halo orbit was sufficient.

A diagram of the halo orbit and trajectory are given in Fig. 5.5. This halo orbit will now act as the departure point of our unstable manifolds required for cycler trajectory insertion. This trajectory will also be shown in the inertial frame at a later time when discussing the spacecraft's entire trajectory. Furthermore, a view of the new parking orbit is also available in Fig. 5.6.

As a quick note, it is possible to launch directly from Earth into the stable manifold, removing the need entirely for a parking orbit around Earth. This allows the possibility of saving some of the required ΔV to arrive at the halo orbit. This will

Figure 5.5: Optimized halo orbit and stable manifold trajectory

however not be considered for the purposes of further calculation, but may be important to keep in mind when considering the final ΔV values for each method.

Similarly, it is possible to burn directly from the parking orbit onto the cycler trajectory via direct inject; considerably changing the assumed ΔV to burn onto the cycler (when using the nominal V_∞ value at Earth). Using the parking orbit presented for this particular manifold, the largest and smallest required ΔV to burn onto the cycler are 4.52 km/s and 4.01 km/s respectively, with an average of 4.26 km/s. These values will be set aside for now and considered further when discussing the total ΔV via the manifold method.

Figure 5.6: Optimized solution parking orbit and trajectory

5.2 Halo Orbit Departure

Now that the spacecraft is on the halo orbit, it is possible to slightly perturb the periodic trajectory and enter onto a trajectory in the unstable manifold. This will ultimately allow the spacecraft to perform cycler trajectory insertion.

In order to perform this maneuver, a process similar to the halo orbit insertion method was developed. The central difference between the two however, is that it calculates which trajectory passes closest to the first leg of the cycler, rather than Earth itself. Also, the halo orbit will no longer need to be optimized or even calculated, since that was taken care of previously. However, the unstable manifold will be optimized via similar methods to the halo orbit; this optimization process will be much more refined however. This is primarily due to the fact the trends of the unstable manifolds are far more predictable and adaptable.

It is also extremely important to mention that the circular assumption of Earth's orbit for the purposes of the CR3BP will continue to be in effect until the transfer to the first leg of the cycler occurs. While this may seem dubious at first, since the orbit of Earth is far more circular than Mars, this does not hinder or breach the dynamics involved in the transition between operating environments. To illustrate this fact, the orbits of Earth and Mars in the circular and ephemeris models can be seen in Fig. 5.7. It is clear that Earth's orbit is 'circular enough' to allow for this assumption to hold. Moreover, this assumption allows all operations to be contained in the $x - y$ plane for the duration of this maneuver.

5.2.1 Time of Departure

Unfortunately, with this system the arrival date at Mars is very specific, and thus the first leg of the cycler has a specific shape and location. In order to obtain the

Figure 5.7: Earth and Mars circular vs. ephemeris orbit shapes

best result possible, it is necessary to make sure that the departure date of the halo orbit is sufficient in getting a trajectory on the unstable manifold adequately near that first leg. This first necessitates a test for the relationship between transfer times and distances travelled on the unstable manifold. Since we are using the invariant manifold and circular solar model, we can do this process at any departure time and simply shift the departure date via a frame rotation as necessary, a process that will be demonstrated later. For the purposes of the following demonstrations and explanations, the outbound cycler found in Table 2.1 will be used as the example first leg of a cycler trajectory.

Following the self-imposed constraint of keeping these transfers to under a year, the remaining 334 days or about 6 TU (subtracting 31 days for the first transfer) of the year were allowed to propagate the unstable manifold. For the purposes of visualization, both the inertial frame and fixed frame are plotted in Fig. 5.8 and Fig. 5.9 respectively.

From the diagrams one can see that while some trajectories on the manifold do not venture too far from the halo orbit, others are able to veer outwards and decently far away from Earth. One may also see that the manifold does not 'lag' behind the orbit of Earth, essentially meaning that the remaining 334 days is plenty of time to cross the path of the cycler from any halo departure date. Finally, it is also noticeable that these trajectories seem to exhibit much more predictable behavior; something that will prove useful when performing frame rotations.

A key characteristic of these trajectories is that no matter what date is chosen to depart the halo, the shape of the manifold remains consistent, and the frame is simply shifted in angle around the z -axis. This allows for a much smoother process when choosing the desired trajectory on the unstable manifold. By simply overlaying the first leg of the cycler trajectory onto the manifold space, it is possible to test

Figure 5.8: Unstable manifold over 6 TU in inertial frame

Figure 5.9: Unstable Manifold over 6 TU in fixed frame

for trajectories on the manifold that intersect (or get very close to) the cycler. The departure date of the halo can then be shifted (via an angular rotation of the plot itself) and tested further until an optimal solution is found. An algorithm that uses exactly this approach was developed and takes the following steps:

1. The number of points in a given halo orbit are broadened, similar to the halo orbit optimization
2. The corresponding trajectories on the unstable manifold are calculated and transformed into the inertial frame with initial angular rotation, θ_z of 0 rad
3. The first leg of a cycler trajectory is then compared to each trajectory on the manifold to find the distances, d_c , between them
4. Similarly, the required ΔV to burn onto the cycler trajectory is also calculated through simple vector comparison
 - (a) Note that this ΔV can be deceptive. Since only the velocity vectors are compared, this value is only true if the cycler trajectory and manifold trajectory intersect. We will keep this in mind for later.
5. The d_c and ΔV values are used to develop a weighted score for the overall performance of the maneuver
6. The manifold is then rotated in frame by some $\Delta\theta_z$ (via the optimizer's choice) and the process is repeated until tolerance is met on the optimizer.
7. The output of the optimizer is the θ_z that produces the lowest score. This angle is then chosen as the basis for a fine search optimization
8. The optimization is re-run (if necessary) until the best value for θ_z is found

Table 5.2: Table of example output values for frame rotation

θ_z [rad]	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$
ΔV [km/s]	3.38	3.63	3.64	3.57	3.62	3.62
d_c [10^4 km]	0.57	3.29	4.01	4.05	3.55	3.75
<i>score</i> [-]	1.41	3.39	3.90	3.97	3.57	3.71

Before discussing the optimizer, the generally expected outputs that the system expects for various θ_z values will first be illustrated. This is for the purposes of understanding the score associated with each value. To demonstrate this process, values of $\theta_z = \{0 \rightarrow \frac{5\pi}{3}\}$ with $\Delta\theta_z = \frac{\pi}{6}$ were used. In order to determine the most optimum trajectory on the manifold, the ΔV and d_c values are compared and weighted to give a score to be minimized. This score is given by the following equation:

$$score = \frac{w_1 d_c}{10^3} + w_2 \Delta V \quad (5.1)$$

where $w_1 = 0.7$ and $w_2 = 0.3$ are the weights of importance given to each parameter. The rational for weighting the distance higher than the magnitude of the burn is due to the fact that getting a trajectory on the manifold as close as possible to the cycler trajectory allows for a direct burn and does not require a Lambert's solution to patch the conics. Furthermore, this greatly simplifies the metrics necessary to determine the best trajectory.

A table of associated values with each angle are provided in Table 5.2. To spare the reader of an influx of figures, Fig. 5.10 displays an example result ($\theta_z = \frac{2\pi}{3}$) from this list. This is meant solely for visual purposes and is aimed at showing the first leg of the cycler trajectory as well as the departure region in the inertial frame; a much more detailed figure of the final, optimized trajectory will be presented later.

To perform the optimization, a slightly different algorithm than that used in optimizing the halo orbit was used. Matlab's *fminbnd* function was used because of

Figure 5.10: Example of unstable manifold trajectory

its capacity to allow the user to input bounds for the the optimizer to search between, rather than a single value to be iterated upon. This means that the initial guess for the optimized value does not have to be a 'good guess'.

The optimizer implemented the aforementioned algorithm by testing various values for θ_z and optimizing the associated score. The bounds used as the input were $\theta_z = (0, 2\pi)$. As a slight fault of the optimizing function, there is a tenancy to not test the limits of these bounds, therefore, a second, similar optimization was run simultaneously with $\theta_z = (-\pi, \pi)$. The results of each optimization can be visualized in Fig. 5.11 and Fig. 5.12 respectively.

Both optimizations performed competently, arriving at similar solutions for θ_z ; with the best value of 0.741596 rads ($\sim \pi/4$) and a score of 1.0639. This is already far better than most of the scores in our table of examples, so this method seems promising so far. For the inquisitive, the distance, $d_c = 1500$ km and the $\Delta V = 3.51$ km/s.

To perform the second search, the halo orbit is refined to its original resolution and

Figure 5.11: First halo optimization | $\theta_z = (0, 2\pi)$

Figure 5.12: First halo optimization | $\theta_z = (-\pi, \pi)$

the known solution is used to find any solutions in that region that may perform more preferentially. As a side note, it was found that a second, fine search optimization, was not necessary to refine the input angle and lower the score since the tolerances of the previous optimization are so strict. With that being said, the value for θ_z was plugged into the algorithm at full manifold fidelity and run a final time. Staggeringly, the score dropped to a value of 0.91, with $d_c = 132$ km and $\Delta V = 3.02$ km/s.

Our total ΔV for the entire trajectory from parking orbit to cyler insertion is now 5.77 km/s. Unfortunately, this is not lower than the value of 4.01 km/s that we were trying to overcome, however the feasibility of this maneuver has been demonstrated. Furthermore, in the following chapter, various results of a few more manifold and cyler combinations will be demonstrated, which will hopefully give insight in to whether or not this maneuver can be used to save ΔV . It is also worthy to note that the first leg of a cyler trajectory is not necessarily the only start available for the trajectory. Any leg that starts at Earth can be used as the initial point and many of these V_∞ values are much higher than our value of 5.77 km/s. This means that depending on the launch date, this maneuver may be useful.

For visual purposes Fig. 5.13 displays a plot of the entire trajectory from parking orbit to cyler insertion in the synodic frame. It was chosen not to show *just* the unstable manifold portion here in the synodic frame as it is mostly unhelpful. To remedy this, a figure of the unstable portion of the trajectory in the inertial frame is given in Fig. 5.14. More plots similar to these will be given in the following chapter discussing various results.

Figure 5.13: Entire trajectory from parking orbit to cycler insertion in synodic frame

Figure 5.14: Unstable manifold trajectory to cyclor insertion in inertial frame

Chapter 6

DISCUSSION OF VARIOUS TEST CASES

This chapter will discuss and analyze some of the results obtained using the methods discussed throughout the entire work for various testing cases. These results will be compared on a few parameters such as feasibility, desirability, and complexity.

All known cycler trajectories, both provided to the author and generated by the author will be used, and all unstable manifolds will be calculated and optimized just as before. However, the parking to halo orbit will not be reanalyzed as this was already determined to be the most desirable case for halo insertion. It should be pointed out that the processes investigated in previous sections are mostly automated so that will not be reanalyzed here for each case. Instead the final result and its properties/consequences will be discussed.

6.1 Nominal Cases

For the first 3 cases, the algorithm developed in the previous chapter for 3 other cycler trajectories was run, those being the inbound cycler provided by T. McConaghy[15], and the outbound & inbound cyclers that were generated in Chapter 4. While many of these results are similar and relatively unexciting, they do provide insight into the consistency of these transfers.

All three cases involve the S1L1-B cycler trajectory, simply for the various launch dates known. In the last chapter, an outbound cycler (T. McConaghy) was used as a working example and will hereafter be referred to as Outbound-1. This chapter will go over the remaining results of the Inbound-1 vehicle (see Appendix A for itinerary) as well as the generated outbound and inbound trajectories, which will be called

Figure 6.1: Inbound vehicle 1 results

Outbound-2 and Inbound-2.

Purely for visual purposes, the plots of each of these trajectories can be found in Fig. 6.1, Fig. 6.2, and Fig. 6.3, respectively. The results are not given here in the synodic frame as they mostly look extremely similar, however these can be found in Appendix B. Furthermore, a summary of the results of each vehicle can be seen in Table 6.1

Interestingly, even between the inbound and outbound cycler trajectories, the characteristics of the manifold-to-cycler trajectory remain very similar throughout the solution space. The main difference lies in the launch date (θ_z) which is relatively obvious, but all other parameters linger within the same range. This is likely due

Figure 6.2: Outbound vehicle 2 results

Table 6.1: Manifold to cycler insertion results for various vehicles

	θ_z [rad]	ΔV [km/s]	ΔV_{total} [km/s]	V_∞ [km/s]	d_c [km]	TOF [days]
Outbound-1	0.7632	3.02	5.77	4.01	132	298
Inbound-1	-0.4909	3.15	5.90	3.33	286	248
Outbound-2	3.0434	2.97	5.72	4.52	841	265
Inbound-2	5.4978	3.09	5.84	4.29	137	332

Figure 6.3: Inbound vehicle 2 results

to the fact that the manifold is unchanging, even with launch date; and likewise the shape of the cycler trajectory is relatively similar, even though we are using the ephemeris model. Similar results between each are to be expected, but the affinity between the inbound and outbound cyclers is quite surprising.

As for the total ΔV required for each trajectory, no value can compete with the initial V_∞ at Earth. While unfortunate in the sense that this means the mission is impractical to begin in this manner, the feasibility has still been demonstrated. While unstable manifolds can be used to burn onto cycler trajectories for less fuel than a normal burn from Earth, the burn from the parking orbit to the halo orbit is too large to allow for any sort of consideration.

A more feasible aspect of this method may lie in the possibility that the cycler vehicle can somehow start in a halo orbit or be built there via many smaller missions to the libration point. This is not so far fetched since many future missions have been proposed that involve building a vehicle in space rather than on the ground, not to mention that the International Space Station (ISS) was built in this manner. In the case where only the halo orbit is considered, this method is most certainly viable for saving on ΔV cost and should be considered for future missions.

Chapter 7

CONCLUSIONS

Mars is still one of the most fascinating destinations for this era's astronauts, and the cycler trajectory provides a very practical and elegant solution to getting astronauts there and back as efficiently as possible. Because getting the Mars is still expensive in terms of ΔV , a solution was proposed that would allow the use of manifolds to reduce mission cost.

While a few cycler trajectories were analyzed, the S1L1-B was chosen because of its capacity to perform ballistically as well as its ability to transfer to the ephemeris model of the solar system. The verification of cycler trajectories in the ephemeris model was demonstrated, which proved useful for generating further cycler trajectories without the use of an optimizer. It was also shown that both the inbound and outbound cycler trajectories are feasible and necessary for agile transport to and from Mars.

This work most importantly explored the possibility of the potential use of trajectories on a manifold to connect to a cycler trajectory, with a secondary emphasis on reducing mission cost. The effects of the CR3BP continue to provide interesting solutions to many orbital problems and the potential connection to cycler trajectories can provide more insight into their vast amounts of uses.

While the maneuver from manifold to cycler is technically possible, the necessity to burn from a parking orbit around Earth to a halo orbit requires an excess amount of ΔV not present in a direct burn sequence. This reduces the practicality of this mission type as the secondary goal was the reduce the ΔV cost associated with a cycler trajectory insertion burn. However, it was demonstrated that if the vehicle were to begin in the halo orbit automatically and burn from there, a significant amount of

fuel can be saved.

Overall, this work was effective in establishing the ground work for the connection between manifolds and cycler trajectories. While only the Earth-Sun L2 point and the Earth-Mars cycler were considered, the feasibility of this mission type was demonstrated. However, much more analysis should be done in order to optimize the solutions further as well as expand the solution space to other libration points and cycler trajectories. Further work is also discussed in the following chapter.

Chapter 8

FUTURE WORK

Due to the sheer scope of this project, there were many aspects that could not be considered. This is largely due to the fact that two very distinct concepts – those being cycler trajectories and manifolds – were analyzed in such a way that described a possible connection between the two, rather than an in-depth analysis on either. As a result, there are many things that can be done to improve not only the results of this work, but also the nature of it.

8.1 Cycler Trajectories

One of the most obvious areas to expand upon is determining which leg of a generated cycler trajectory is the best to leave on. Because the cycler can technically start at any trajectory that begins at Earth, any of these are valid departure points. This can allow for another optimization parameter when constructing the manifold.

Another area to expand upon would be a more rapid approach to generating the cycler trajectory in the first place. While robust, the method developed in this work is slow-paced and does not allow for much in the way of optimization for certain parameters. Adjusting for this may allow for the cycler to be optimized for the manifold itself, rather than the other way around.

The last area of improvement to the cycler trajectory would be to expand the solution space to include other types of cyclers rather than only the S1L1-B. It may be shown that just because the cycler is ballistic, doesn't mean that it will carry the best results.

8.2 Manifolds

A clear improvement in the way of manifolds is expanding to the other libration points, especially the Earth-Sun L1 point, but more generally to include the Earth-Moon libration points as well. This could give insight as to how various libration points operate, especially in terms of relative velocity to the cycler trajectory, which is the most important aspect of cycler insertion. One might even be audacious enough to use halo to halo manifold transfers from Earth to Mars rather than the flyby system of the cycler trajectory.

Another extension would be to change the size of the halo orbit within the optimization, that is to say, optimize the stable and unstable manifolds at the same time. This may allow for a very desirable trajectory to appear that was previously hidden from the solution space.

The final advancement would be to drastically increase the fidelity of the model and allow for much more accurate representations of the concepts to be calculated and shown. This can be done very simply through increasing the calculation accuracy as well as the amount of usable points on the cycler and manifold vectors.

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APPENDICES

Appendix A

CYCLER TRAJECTORY EXTRAS

A.1 Aldrin Cyclor Computation

I first begin by finding the period of Earth's Orbit through

$$P_e = 2\pi \sqrt{\frac{a_e^3}{\mu_s}}$$

where a_e is the semi-major axis of Earth and μ_s is the standard gravitational parameter of the sun. I then find the period of Mars' orbit, P_m through the definition of the synodic period, S .

$$\frac{1}{S} = \frac{1}{P_e} - \frac{1}{P_m}$$

where, $S = 2\frac{1}{7}$. By setting $n = 1$ rev, I find the positions of Earth at the start and finish of one cycle as

$$R_0 = a_e[1, 0, 0]$$

$$R_{2\frac{1}{7}} = a_e[\cos(2\pi nS), \sin(2\pi nS), 0]$$

Using the Izzo-Gooding Lambert's solution method, a multi-rev solution for the corresponding velocity vectors is found, namely V_0 and $V_{2\frac{1}{7}}$. R_0 and V_0 are then propagated through an ODE solver, where the accelerations is defined by

$$A = \frac{-\mu_s}{\|R\|^3 R}$$

This outputs the position and velocity of the cyclor vehicle, R_c and V_c respectively, over the duration of the trajectory. The encounter position of Mars, R_m is then analyzed graphically.

Figure A.1: Aldrin cyclers results for first trajectory

My results can be summarized as follows with $a_e = 149598023$:

$$R_0 = a_e[1, 0, 0]$$

$$R_{2\frac{1}{7}} = a_e[0.6235, 0.7818, 0]$$

$$R_m = a_e[-1.0163, 0.2505, 0]$$

These and the shape of the cycloidal are also graphically represented in Fig. A.1.

A.2 S1L1-B Cycloidal Computation (Circular Model)

The computation starts the same as the Aldrin except that now $n = 2$. Now I create a vector of various values of $\tau \in \{S, 2S\}$ which represents the intermediate encounter

time with Earth. I then calculate the position of Earth at each of these encounter times through

$$R_\tau = a_e[\cos(2\pi\tau), \sin(2\pi\tau), 0]$$

I then calculate two separate Lambert's solutions for each τ from $0 \rightarrow \tau$ and $\tau \rightarrow 4\frac{2}{7}$. Both use a multi-rev solution but the first transfer is left branch and long way, while the second is right branch and short way. This generates the velocity vectors, V_0 , V_{τ_1} , V_{τ_2} , and $V_{4\frac{2}{7}}$. Where V_{τ_1} and V_{τ_2} occur at the same time.

In order to find the ballistic trajectory the v_∞ at τ_1 and τ_2 must be calculated through the following equation:

$$v_\infty = \|V_\tau - V_e\|$$

where V_e is the velocity of Earth at the corresponding point and can be calculated by

$$V_e = \hat{u}_t \sqrt{\mu \left(\frac{2}{\|R_\tau\|} - \frac{1}{a_e} \right)}$$

where $\hat{u}_t$ is the unit vector perpendicular to, and in plane with, the position vector, R_τ . I then find the minimum of $\|v_{\infty 1} - v_{\infty 2}\|$. At the time, τ where this is zero, a ballistic trajectory occurs. My results for the S1L1-B are as follows:

$$R_0 = a_e[1, 0, 0]$$

$$R_{2.8277} = a_e[0.4688, -0.8833, 0]$$

$$R_{4\frac{2}{7}} = a_e[-0.2225, 0.9749, 0]$$

$$R_m = a_e[-0.5192, 1.4292, 0]$$

A diagram of the shape of the trajectory can also be found in Fig. A.2.

A.3 Extra Itineraries

The first itinerary given in Table A.1 is the inbound itinerary given to me by Troy McConaghy and used a basis for the inbound cycler trajectory shape.

Figure A.2: S1L1-B results for legs 1 and 2

Table A.1: Example Inbound Cyclers Itinerary (T. Troy McConaghy)

<i>Inbound Cyclers Vehicle (Troy McConaghy)</i>							
Encounter	Fly-By Planet	Year	Month	Day	V_{∞}	Closest Approach Distance	Leg, TOF
#	P	yyyy	mm	dd	km/s	km	days
0	Earth	2005	4	1	3.33	--	--
1	Mars	2007	10	5	7.25	12140	918
2	Earth	2008	2	15	6.22	44630	133
3	Earth	2009	8	7	6.23	19040	539
4	Mars	2011	11	5	4.78	6710	820
5	Earth	2012	4	30	7.05	29830	177
6	Earth	2013	10	24	7.05	24830	542
7	Mars	2016	1	8	2.75	9870	805
8	Earth	2016	8	13	4.15	13410	218
9	Earth	2018	1	25	4.17	22550	530
10	Mars	2020	8	20	7.19	10440	938
11	Earth	2020	12	6	4.69	31100	108
12	Earth	2022	5	22	4.67	16020	532
13	Mars	2024	10	16	6.64	3854	877
14	Earth	2025	3	11	6.72	40750	146
15	Earth	2026	9	3	6.71	22390	541
16	Mars	2028	11	11	3.79	10580	800
17	Earth	2029	5	29	6.19	25610	199
18	Earth	2030	11	18	6.18	30640	539
19	Mars	2033	4	7	3.77	15350	870
20	Earth	2033	9	16	3.87	10950	162
21	Earth	2035	2	27	3.84	24270	529
22	Mars	2037	9	8	7.71	678	924
23	Earth	2037	12	13	9.12	--	96

Table A.2: Second generated outbound cyclers trajectory itinerary

<i>Outbound Cycler Vehicle 3</i>							
Encounter	Fly-By Planet	Year	Month	Day	V_{∞}	Closest Approach Distance	Leg, TOF
#	<i>P</i>	yyyy	mm	dd	km/s	km	days
1	Earth	2005	9	9	5.32	--	--
2	Mars	2006	3	3	3	9960	175
3	Earth	2008	6	9	6.92	20500	830
4	Earth	2009	12	3	6.93	31200	542
5	Mars	2010	6	6	4.31	17800	185
6	Earth	2012	8	24	6.42	26500	809
7	Earth	2014	2	14	6.43	41500	540
8	Mars	2014	7	3	7.14	12200	138
9	Earth	2016	12	9	4.01	27700	890
10	Earth	2018	5	22	4.03	19900	530
11	Mars	2018	9	15	6.47	11600	115
12	Earth	2021	4	6	4.61	23000	934
13	Earth	2022	9	20	4.59	14800	532
14	Mars	2023	5	1	2.77	7590	223
15	Earth	2025	7	2	7.08	23900	793
16	Earth	2026	12	26	7.09	35200	542
17	Mars	2027	6	14	5.26	13800	170
18	Earth	2029	9	21	5.78	26800	830
19	Earth	2031	3	12	5.78	39000	537
20	Mars	2031	7	15	7.7	10600	125
21	Earth	2034	1	15	3.78	23000	915
22	Earth	2035	6	28	3.76	9630	529
23	Mars	2035	11	13	4.68	15700	138
24	Earth	2038	5	8	5.54	--	907

This next itinerary in Table A.2 was found by me by adjusting the launch date of the itinerary in Table 2.1 slightly and running my algorithm. Interestingly, the itinerary matches the previous very well for a significant portion of the middle of the trajectory.

Figure A.3: Intersection of V_∞ contour on timeline

A.4 Timeline Characteristics

As mentioned in the main portion of this work, the details of the timeline are simply the V_∞ contours overlayed onto a dual-axis timeline. A line of $x = y$ is placed on the graph and any intersection points with the contours are valid flyby dates. Furthermore, the cyclor is only complete when all flybys are valid. Fig. A.3 displays a zoomed-in visual of the timeline for one of the intersections to gain a better understanding of this graph.

Sometimes, the contour will intersect with more than one point on the timeline. This means that either flyby is perfectly valid to continue the trajectory, but only one can be used to continue the remaining computation of the cyclor trajectory. An

Figure A.4: Double intersection of V_∞ contour on timeline

example of this contour can be found in Fig. A.4.

Appendix B

MANIFOLD EXTRAS

B.1 Synodic Frame Test Case Plots

The manifold trajectories for the test case cycler trajectories are given in Fig. B.1, Fig. B.2, and Fig. B.3. These may provide some insight onto how the shape of the unstable manifold trajectory can change depending on the parameters of the cycler trajectory.

Figure B.1: Trajectory in synodic frame for Inbound-1 vehicle

Figure B.2: Trajectory in synodic frame for Outbound-2 vehicle

Figure B.3: Trajectory in synodic frame for Inbound-2 vehicle