AUTONOMOUS FORMATION FLYING AND PROXIMITY OPERATIONS USING
DIFFERENTIAL DRAG ON THE MARS ATMOSPHERE

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ABSTRACT

Autonomous Formation Flying and Proximity Operations Using Differential Drag on the Mars Atmosphere

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Due to mass and volume constraints on planetary missions, the development of control techniques that do not require fuel are of big interest. For those planets that have a dense enough atmosphere, aerodynamic drag can play an important role. The use of atmospheric differential drag for formation keeping was first proposed by Carolina L. Leonard in 1986, and has been proven to work in Earth atmosphere by many missions. Moreover, atmospheric drag has been used in the Mars atmosphere as aerobraking technique to decelerate landing vehicles, and to circularize the orbit of the spacecraft. Still, no literature was available related to formation flying on Mars.

To analyze the use of differential drag on the Mars atmosphere, the researcher accessed the two high resolution models available: NASA's Mars-GRAM and ESA's Mars Climate Database. These models allowed the simulation of conditions that a spacecraft would experience while in orbit around the planet. To explore the feasibility, the researcher first conducted a study where Mars atmosphere density was compared to Earth atmosphere, determining its applicability. Then, a simulation using MATLAB® was conducted, using a Keplerian two-body problem including the effects of Mars zonal harmonics (i.e. J2) and drag perturbations. Two 6U CubeSat were used in the simulation with deployable drag plates of different sizes, giving the possibility of having five differential drag scenarios as means of formation control.

The conclusions showed that, although with some limitations, the use of differential drag as means of autonomous formation flying and proximity operations control is feasible using proven techniques previously validated in Low Earth Orbit. Lyapunov control was selected as the control strategy, where three different methods were evaluated and compared.

Keywords: Mars, Formation Flying, Proximity Operations, Differential Drag, Lyapunov, Control Theory, Guidance, Navigation, Perturbations.

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1 INTRODUCTION

Since 1957, humanity has been launching exploratory missions to better understand the solar system and collect data to characterize it. From the wide variety of environments, many scientists are focused in exploring planets with conditions similar to Earth to develop life. Because its similarities with our planet, it is not surprising that Mars has become one of the most active planets to send exploratory missions to in the last two decades.

The exploration of Mars started in 1960 with many failures, and it was in 1964 when NASA's Mariner 4 mission got the first close-up pictures from the surface of the red planet. Since then, different countries conducted more than 30 missions, and others are being developed (NASA GSFC, 2016). Some of the missions were orbiters and others were rovers, but the returning of samples is still pending to be accomplished, where it is highly probable that docking with a Mars orbiter have to be done to return the samples back to Earth (ESA, 2015).

The development of algorithms for autonomous maneuvers of deep space missions are a key factor to achieve future goals due to many reasons: the minimum time required to communicate with a spacecraft orbiting Mars is between 4 and 24 minutes each way; the operational cost of a small satellite flying in formation can become dominant of total project lifecycle cost (Collins, Dawson, & Wertz, 1996); the usage of the Deep Space Network cannot be monopolized by one mission; and, the precision of measurements required for rendezvous and docking maneuvers cannot be obtained using Earth-based radiometric tracking. To support navigation capabilities in Mars orbits, a Multi-Spacecraft Autonomous Positioning System (MAPS) using existing and future commercial orbiters is in development (Anzalone, Becker, Crump, & Heater, 2015). Using this concept, it might be possible to obtain the precision required to perform such operations. Also, missions like Deep Space 1 (DS-1), back in 1998, already successfully demonstrated autonomous, onboard cruise navigation, using optical-only measurements (Bhaskaran, et al., 2000) (Riedel, et al., 2000).

Furthermore, due to mass and volume constraints on planetary missions, the development of control techniques that do not require fuel are of big interest, such solar sails (McInnes, 1999). For those planets that have dense enough atmosphere, aerodynamic drag can play an important role. The use of atmospheric differential drag for formation keeping was first proposed in 1986 (Leonard, 1986) an improved in 1989 (Leonard, Hollister, & Bergmann, Orbital Formation-Keeping with Differential Drag, 1989), and has been proven to work in Earth atmosphere by many missions, like the ORBCOMM constellation (Maclay & Tuttle, 2015), and the JC2Sat (Kumar, Ng, Yoshihara, & De Ruiter, 2007). Also, atmospheric drag has been used in the Mars atmosphere as aerobraking in order to decelerate the landing vehicles (Withers, 2013) or trying to circularize the orbit of the spacecraft (Lyons, Beerer, Esposito, Johnston, & Willcockson, 1999).

For mission analysis and design, it is of big importance knowing the characteristics of the atmosphere where the spacecraft will perform the maneuvers. Two Mars atmosphere models were available at the moment of writing this thesis: The Mars Global Reference Atmospheric Model (Mars-GRAM), and the Mars Climate Database (MCD). These models have all the required information to calculate the aerodynamic drag that a spacecraft will face during its mission.

1.1 Pre-Study Feasibility Analysis

Atmospheric density, ρ , is one of the driven parameters in calculating the force due to atmospheric drag acting upon a spacecraft. For this reason, a feasibility analysis was conducted using information obtained from the Mars Climate Database, to determine if the atmospheric density of Mars would allow the use of formation flying techniques. In order to compare with available studies conducted for Earth, the atmospheric density of both planets was compared. Four different years with different solar cycles were plotted in Figure 1, where it can be seen that the atmospheric density has variations of almost one order of magnitude for Earth atmosphere, and there is not such huge difference with Mars atmosphere. When planning a mission, the worst case scenario has to be considered, which corresponds to the lowest density. It can also be seen in Figure

1 that the Mars atmosphere at around 400 kilometers of altitude for year 2014, corresponds to the density found at 600 kilometers of altitude on Earth atmosphere for year 2010. For the purpose of this study 200 kilometers in altitude above Mars mean level was taken as a starting point for the analysis. With this value it was possible to calculate the differential drag that can be obtained between two spacecraft when switching between their biggest and smallest area exposed to atmospheric drag.

Figure 1. Atmospheric density of Mars and Earth for different solar cycles

The results obtained after performing the feasibility analysis (Villa, 2016), demonstrates that it is possible to use differential drag as means of formation flying control on the Mars atmosphere.

1.2 Formation Flying Approaches

Different concepts can be associated with formation flying techniques and how can they be applied. Thus, the definition has not been universally agreed. In a general sense, formation flying

requires controlling the relative position or geometry between or among spacecraft. To achieve this goal different approaches can be used (Alfriend, Vadali, Gurfil, How, & Breger, 2010).

1.2.1 Leader/Follower

In this type of formation technique, one spacecraft acts as the leader (also referred as target), while one or more spacecraft perform maneuvers to adjust their positions relative to the leader, acting as followers (also referred as chaser). This approach allows any change commanded to the leader parameters to be followed automatically by the other spacecraft. However, when using differential drag as the only mean of control (i.e. thrusters are not used), it will be necessary that the roles can be switched over time, because if a follower is already in a lower orbit than the leader, there is no way to control it to raise its orbit.

1.2.2 Absolute Tracking

Although unfeasible using only aerodynamic drag as means of control, there are techniques which will command each spacecraft to maintain the orbital parameters to stay in a particular orbit. By doing so, a specific desirable geometry can be obtained. The advantage of this approach is that no cooperation is required between the spacecraft involved, while the disadvantage is that it requires a propulsion system. This method has been proposed by Microcosm (Wertz, Collins, Dawson, Koenigsmann, & Potterveld, 1997), and proven to work in 2006 on the TacSat-2 mission. Despite the fact that only one spacecraft was used, the concept is the same.

1.3 Statement of the Problem

Spacecraft formations can be achieved if the spacecraft involved on it can change their relative position and velocity. As proposed earlier (Leonard, Hollister, & Bergmann, Orbital Formation-Keeping with Differential Drag, 1989), the low-cost alternative to achieve this task is by using the aerodynamic drag. Drag forces will affect the motion of the spacecraft creating a negative acceleration that is function of the atmospheric winds; the relative velocity of the spacecraft with respect to the atmosphere; the areas of the spacecraft exposed to the velocity vector;

the spacecraft mass; and the atmospheric density. Since most of these parameters are not controllable by the spacecraft, it is necessary for the latest to have the ability to change the area exposed to drag, hence changing the magnitude of the drag force acting against it. The following equation represents the aerodynamic drag acceleration:

$$a_d = -\frac{1}{2}\rho v_{rel}^2(BC)\hat{\mathbf{v}}_{rel} \quad (1)$$

where v_{rel} is the relative velocity of the spacecraft with respect to the angular velocity of the atmosphere, BC is the ballistic coefficient, and $\hat{\mathbf{v}}_{rel}$ is the unit vector in the direction of the relative velocity. Then, the equation used to calculate the ballistic coefficient:

$$BC = C_d \frac{A}{m} \quad (2)$$

where C_d is the drag coefficient, A is the spacecraft area, and m the spacecraft mass. C_d is possibly the most difficult value to calculate since it depends not just on the geometry and orientation of each structure of the spacecraft, but also on the flow characteristics between the surface of the spacecraft and the neutral particles in the atmosphere. Although, a value of $C_d = 2.2$ is often used to calculate BC with reasonable accuracy (Gaposchkin & Coster, 1988). Also, for the purpose of this study the change in area is performed by changes on spacecraft attitude as if it was instantaneous. In other words, the spacecraft can face the velocity vector direction generating either maximum or minimum aerodynamic drag, with no intermediate values.

Moreover, in order for two or more spacecraft to keep a desired formation autonomously, it is necessary for them to perform other important tasks. First they have to be able to know their location exactly and the location of the target, which is going to be called navigation information problem. Second, the use of differential atmospheric drag techniques will require knowing the attitude of the target spacecraft to calculate the difference in drag, which will be called relative atmospheric drag problem. And finally, the perturbations the spacecraft will experience, due to atmospheric drag and the oblateness of Mars, will modify their future trajectory into the orbit, which is going to be called perturbations problem.

The navigation information problem requires each spacecraft being capable of determining its own location. In addition, the follower will need to know the leader location. Knowing this information is vital to solve the problem of calculating the relative position and velocity. Additionally, if the follower is already in a lower orbit than the leader it will be necessary to reorganize the formation, changing the roles of the spacecraft involved.

The relative atmospheric drag problem requires the follower to know the attitude information of the leader, to calculate the magnitude of the relative acceleration caused by the differential aerodynamic drag. It is usually assumed that atmospheric density is constant for control and guidance purposes (Leonard, et al., 1989), although variable density models are available. Furthermore, since the drag forces acting are difficult to calculate, maneuvering the spacecraft using differential drag is challenging.

Finally, the perturbations problem requires having a precise formulation of the equations of motion to create a realistic behavior, to determine the stability of the control laws. Since the oblateness of Mars is greater than the oblateness of Earth, it can be expected that the zonal harmonics on Mars become a big impact on the equations of motion. Moreover, for the follower to know how a particular action will affect its trajectory relative to the leader, it will be necessary to have a set of equations to express a spacecraft motion relative to a second spacecraft.

All these problems derive in an additional problem: a method to transmit some information among the fleet, which will be named inter-spacecraft communication problem. This problem cannot to be diminished, particularly if proximity operations have to be performed. If traditional methods of relay communications using Earth-based ground stations have to be used, the distance from Mars to Earth would be prohibitive to accomplish this task, as the one-way communication time might be orders of magnitude greater than the time between two successive maneuvers.

A solution to these problems is presented in the following chapters, with some limitations and constraints.

1.4 Background and Need

Although extensive literature exists about some of the problems intended to solve by this study, no literature could be found to solve all the four formation flying problems presented on the previous section for Mars atmosphere as a whole. Hence, existing literature was reviewed to reuse previous works as parts for making a complete solution. Moreover, most of the previous work was formulated for a different application or a different situation, so there exists a gap between what has been done and what it is needed on this particular study.

1.4.1 Navigation Information Problem

When orbiting in Low Earth Orbit (LEO) it is possible to obtain precise navigation information just by using a Global Positioning System (GPS) receiver. But, for a spacecraft that goes beyond the GPS fleet range, another solution is needed. For interplanetary missions, NASA Marshall Space Flight Center has patented a technology called Multi-Spacecraft Autonomous Positioning System (MAPS). This system uses the existing and routinely used inter-spacecraft Martian communication network, composed by the Mars Reconnaissance Orbiter, Mars Odyssey, and MAVEN. It embeds headers in communication links, which are used with onboard navigation estimation routines to generate ranging measurements. As the spacecraft communicate with each other, the overall navigation accuracy will increase. Simulations have demonstrated the possibilities of relaxing the need of using the Deep Space Network (DSN) for updating navigation information states (Anzalone, et al., 2015).

As part of the research on creating algorithms for an autonomous satellite constellation maintenance system (Collins, Dawson, & Wertz, 1996), the problem of obtaining an estimate position of the spacecraft has been faced and solved by using the Microcosm Autonomous Navigation System (MANS) (Wertz, 2003). This navigation system uses the position of the Sun and the Moon and the apparent angular diameter of the Earth, calculated by the sensors, to estimate position autonomously from any external source. NASA's DS-1 spacecraft demonstrated

autonomous navigation in interplanetary orbit using MANS. Further studies have demonstrated that the use of a star tracker can substantially improve the accuracy of measurements, even more for low planetary orbits where the angular size of the planet provides a sensitive measure of the distance. When obtaining the velocity information there is no direct measurement available, it is calculated using equations of motion and a sequence of positions.

These two solutions already developed can provide, alone or combined, solutions to the navigation information problem that each spacecraft has to face. Although MAPS has not been fully implemented in Mars yet, it is an active development. On the other hand, MANS has already been proven to work for interplanetary missions, so it would be the preferred solution to use to solve this problem. Either way, this study investigates the frequency and amount of information needed to be transmitted between spacecraft in the formation.

1.4.2 Relative Atmospheric Drag Problem

Investigations conducted in the University of Florida by Pérez and Bevilacqua (Pérez & Bevilacqua, 2016) present the equation to calculate the magnitude of the relative acceleration due to differential aerodynamic drag between a follower and leader. The equation shown there requires not just the ballistic coefficient of the follower, but also the ballistic coefficient of the leader. Moreover, although the atmospheric density is considered constant, the presented equations will be used in this study as a base of analysis.

One of the key factors presented by David Geller (Geller, 2007) on his study about when autonomy is required, is the environment modeling error, where atmospheric density variations generate the largest deviations. This may become more critical when using differential atmospheric as the only parameter to control a formation.

Furthermore, previous experimental missions have used a miniaturized instrument to measure the composition of the atmosphere. The instrument used is called Wind-Ion-Neutral Composition Suite (WINCS) and was developed by NASA Goddard Space Flight Center, and the

Naval Research Laboratory. It was part of the CubeSat Investigation Atmospheric Density Response to Extreme Driving (CADRE), sent to the International Space Station onboard of the Cygnus OA-4 delivery (Cutler, Ridley, & Nicholas, 2011).

For the purpose of this study, in order to simulate a more realistic scenario, more precise atmospheric densities were used when computing the orbital dynamics that describes the spacecraft trajectories. Despite of that, further analysis done during this study demonstrate if using more accurate atmospheric densities in the control block helps reducing instabilities, determining the necessity of including any means of obtaining precise measurements of the atmospheric density.

1.4.3 Perturbations Problem

Researchers from the Space Systems Laboratory of the Massachusetts Institute of Technology (MIT) presented a solution (Schweighart & Sedwick, 2002) that changed the way formation flying was calculating the relative motion between the spacecraft. Previous to this work, researchers turned into Hill's equations, also known as Clohessy-Wiltshire equations, to describe this relative motion. Although Hill's equations are very simple to implement, and they have been successfully used to describe relative motion during rendezvous maneuvers before, they treat the Earth as perfectly spherical. Rather than that, Earth and Mars are both oblate spheroid, so modeling errors are introduced due to planet's spherical harmonics. The improvement made on their work is a set of constant-coefficient linear differential equations that can be solved analytically, including perturbations due to J_2 . Additional effects are depicted in this publication that will be described in more detail during the Literature Review chapter.

In 2008, researchers from the Naval Postgraduate School presented a solution (Bevilacqua & Romano, Rendezvous Maneuvers of Multiple Spacecraft Using Differential Drag Under J_2 Perturbation, 2008) where the equations proposed by Schweighart & Sedwick were used to describe the relative motion of two or more spacecraft performing rendezvous maneuvers using differential

drag. Although their work is similar to this thesis, the application was specifically for Earth atmosphere and focused in close proximity operations only.

The available literature found, only contains equations of relative motion including J_2 perturbations for Earth orbits. Further analysis made on this thesis determine the feasibility of using the same equations to describe relative motion for Mars orbits, where perturbations due to J_2 effects are greater.

1.4.4 Inter-Spacecraft Communications Problem

Inter-spacecraft communications was studied by David Geller (Geller, 2007). In that study the researcher enumerated four key parameters that determined at what relative distance between two spacecraft that are flying in formation, autonomous systems must take over. One of the parameters was the time delay between successive ground-command maneuvers, where it has been mentioned before what are the times required for commands going between Earth and Mars.

During the description of the navigation information problem, it was presented the Multi-Spacecraft Autonomous Positioning System, which already makes use of the growing inter-spacecraft communication network around Mars. Although it can be used for calculating navigation information, it cannot be assumed that the network capacity is enough to transmit position information of all spacecraft trying to perform formation flying. For such reason, this study does not assume its use as a means of extra information interchange.

However, NASA recently launched a mission composed by two CubeSat that was developed to demonstrate swarm communications autonomy during a two week's mission. The Network & Operation Demonstration Satellite (NODeS) mission consists of two 1.5-unit CubeSat carrying an adapted hardware and software of the Edison Demonstration of Smallsat Networks (EDSN). The mission is intended to demonstrate advanced communications, including inter-satellite ad-hoc data communication network, for extremely flexible data distribution.

Multiple other solutions have been proposed, some based on laser communications, while others based on radiofrequency communications. Whatever the solution is, it will be assumed for the purpose of this thesis that inter-spacecraft autonomous communications are feasible, even when more than two spacecraft could be needed to prevent occultation from happening. However, it has to be noted that the aberration due to communication delays has to be considered as part of the problem. As with optics, aberration correction is required when the propagation delay of the transmitted information makes it being different to when it was transmitted. In other words, at the moment a follower receives the position and velocity information transmitted by the leader, it has already changed, thus has to be corrected to accurately calculate the relative results.

1.5 Purpose of the Study

The purpose of this study was to analyze the use of existing differential drag techniques as means of formation flying control for Earth atmosphere, to be applied for the same purpose on Mars atmosphere, considering perturbations due to second order zonal harmonics (J_2).

Saving the amount of fuel used is critical for any mission (Alfriend, Vadali, Gurfil, How, & Breger, 2010). By one hand it is expensive because it adds mass. By the other hand, it is finite, so it might determine the mission duration. Moreover, for interplanetary missions it is still more critical, because carrying massive amounts of fuel far away is even worst. The use of differential drag as a means of formation flying helps reducing the use of fuel for certain tasks, including circularizing the orbit.

To analyze the use of differential drag on the Mars atmosphere, the researcher accessed the two high resolution Mars atmosphere models available: NASA's Mars-GRAM, and ESA's Mars Climate Database. These models allowed the simulation of conditions that a spacecraft would experience while in an orbit around Mars. To explore the feasibility, the researcher first conducted a study where Mars atmosphere density was compared to Earth atmosphere, determining its applicability (see section 1.1). Then, a simulation using MATLAB® was conducted, using

Schweighart-Sedwick equations, including effects of Mars zonal harmonics (i.e. J_2). The spacecraft simulated were two 6U CubeSat, with deployable panels, giving the possibility of having three differential drag scenarios between them as means of formation control. The initial orbit around Mars used was varying to obtain different trends, with a maximum of 200 km altitude.

By learning the possibilities of using existing differential drag techniques on Mars atmosphere, it was demonstrated that these techniques can be used to save fuel for future missions that may require different formation flying approaches. By extension, this study can be applied to other planets that contain atmosphere, or even natural satellites with atmosphere, like Titan.

1.6 Research Questions

Below are the research questions this study intent to answer. These questions were obtained from the previous sections of the Introduction, and have been taken into considerations during the whole process of this study. The reader will find them reviewed once over time during the following sections, and answered at the end of this report.

Research question 1: *What would be the amount of information required to be exchanged between two or more spacecraft who are performing a Formation Flying using the algorithms presented in this study? Moreover, what would be the frequency that information has to be exchanged depending on the type of maneuvers they are performing?*

Research question 2: *When a follower spacecraft calculates the differential acceleration due to atmospheric drag, is it required to have precise information about the atmospheric density of the leader, or it can be presumed to be constant as it is usually assumed?*

Research question 3: *What is the relationship between atmospheric density, area exposed to aerodynamic drag, and the time required to achieve a formation? Is there a maximum difference in altitude that could be deterministic of the ability to perform formation flying between two spacecraft? How this relates to their drag areas?*

Research question 4: *How long a mission can last before re-entering, depending on the mass to area ratio of the spacecraft involved in this study? How that can be extrapolated to other missions?*

Research question 5: *What is the total delta-v that two spacecraft can save by just using differential aerodynamic drag as means of formation flying control?*

1.7 Significance to the Field

Even though Mars aerobraking techniques already exist and have been used in previous missions and proven to work, no previous work has been done implementing formation flying around the red planet. This study provides an extensive analysis and application of existing techniques to Low Mars Orbits, where atmosphere density seems to be permissive, proposing means of control to achieve formations in a finite time. It also presents the limitations that can be expected in a mission using this approach.

1.8 Definitions

This section introduces terminology and concepts used throughout the study.

1.8.1 Chief, Leader, Target

All these terms can be alternatively used during the text to identify the reference orbit that one or more spacecraft are using to calculate relative motion. It is important to note that the reference orbit may be a physical spacecraft or just a reference ideal orbit.

1.8.2 Deputy, Follower, Chaser

All these terms can be alternatively used during the text to identify a spacecraft who is performing the active control in order to perform maneuvers to maintain a desired relative position with the chief, leader, or target.

1.8.3 Satellite vs. Spacecraft

In order to prevent confusions, only planet's moons are considered satellites for this study. All the called "artificial satellites" are named Spacecraft during the text.

1.8.4 Martian Solar Longitude (Ls)

It is the angle between Mars and the Sun, measured from the northern hemisphere. The value of solar longitude $L_s=0$ corresponds to the spring equinox of Mars, while $L_s=90$ corresponds to the summer solstice, $L_s=180$ corresponds to the autumn equinox, and $L_s=270$ to the winter solstice.

1.9 Limitations

This study has been carried out making the following assumptions that could derive on limitations of its application:

1. The attitude dynamics and attitude control of the spacecraft involved in the simulations are not considered. Attitude is assumed to be stable during the trajectory, while the study is focused on obtaining control algorithms to analyze the atmospheric density.
2. The maneuverability of the spacecraft (i.e. its attitude change), has been considered instantaneous, without possible values in between maximum drag and minimum drag. Moreover, the shape of the spacecraft used has three different areas that could be used to generate different drag coefficients, but only the two extreme values where used.
3. Control implementation that uses only atmospheric differential drag are limited to produce changes in the orbital plane. Hence, no cross-track or out-of-plane maneuvers can be produced.
4. It has been assumed that the follower was able to obtain information about the leader, referent to atmospheric density, attitude, and navigation, at any point and instantaneously during the simulation. In a more realistic case there could be periods

of time where the information is not available, which was considered beyond of the scope of this thesis.

5. A numerical integration method has been used to propagate the orbits. The implementation has been done such that it provides more control on the time-steps and intermediate variables during the simulation. This can make the results to differ if the implementation is done using higher order propagation methods.

2 LITERATURE REVIEW

2.1 Introduction

The literature review will address four areas related to autonomous formation flying and proximity operations on the Mars atmosphere. In the first section, research studies related to how to autonomous navigation information can be obtained will be addressed. The second section will focus on research studies about how the relative differential drag is used in formation flying solutions. A third section will include those research studies related to atmospheric perturbations included when describing the relative motion of two or more spacecraft flying in formation. Finally, the fourth section will discuss research related to inter-spacecraft communications systems available, or how they can be carried out.

2.2 Navigation Information

Navigation information problems are related to how a spacecraft can precisely know its location and velocity autonomously. Solving for this problem is critical for fully automated systems, where minimum to no human-in-the-loop is expected. One possible solution to the problem is the use of on-board sensors, although expensive.

When a spacecraft travels away from the GPS coverage, it is a common practice to use ground-based state updates to keep the guidance navigation and control subsystem informed about its navigation information. Having the algorithms already developed for GPS-like systems, the implementation of a network for interplanetary missions is becoming a reality. Although it is not broadly used, NASA has already reserved a slot in their communication frame to perform ranging measurements when communicating in between different spacecraft.

The purpose of this study was to present the Multi-Spacecraft Autonomous Positioning System (MAPS), and what future steps the Agency is taking to validate the concept in a LEO mission. As the number of spacecraft needing simultaneous operations grows, the long duration observations that Earth-based radiometric measurement becomes more complex. This paper

presented an overview of MAPS architecture from concept design and simulation to hardware-in-the-loop testing. Also, development plans for a CubeSat-based in-space demonstration were presented. MAPS provides a unique architecture for inter-spacecraft autonomous navigation capability, providing a software-centric approach to state estimation and providing several opportunities for future capability growth through technology infusion and aligned with published roadmaps (Anzalone, Becker, Crump, & Heater, 2015).

The study takes advantage of a solar system-wide network of space data relays, spacecraft, and ground assets to provide a navigation capability. It also offers the capability of being scalable and expandable. Multiple orbiters in Martian orbit, Mars Odyssey, Mars Reconnaissance Orbiter, and MAVEN composed the network by the time the study was presented. Although these spacecraft do not have the MAVEN protocol, the implementation could potentially provide navigation, reducing any reliance on human operators to generate high accuracy orbit determination on the ground.

The LEO demonstration mission is planned to be deployed from the International Space Station (ISS), where the two CubeSat would employ differential drag to establish and maintain the desired formation for MAPS testing. In the meanwhile, a hardware-in-the-loop (HIL) setup was designed and implemented, consisting in three main elements: a simulation coordinator to track the truth state of the simulation agents, a timing coordinator to maintain and control timing delays within the active simulation scenario, and a series of independent flat satellites to simulate the hardware and software of an orbital or ground asset.

Moreover, the test scenario includes a series of flight software tasks to fully emulate the CubeSat onboard capabilities. As a result, the study presented the results of the HIL implementation, obtaining reasonable timing stability, which is critical for obtaining ranging, and thus navigation information. Future work included the description of different testing scenarios, characterization of the latency at multiple levels, and comparison between implementations using a pre-emptive and non-pre-emptive operating system.

In conclusion, this study expanded the MAPS concept from simulations and software-based tests to a hardware-based implementation, while proposing a concept of operations for a LEO mission. The development of MAPS algorithms and support libraries also promises implementation across a variety of mission and payloads to demonstrate and finally implement a solar-system wide autonomous positioning system.

Although the proposed solution is very promising, the study does not cover important aspects like the network traffic usage, and how a real implementation timeline would be. Moreover, it will require a software upgrade for already flying Mars orbiters, which suppose a big risk for those missions, and it is not clear if NASA is going to take that risk.

Autonomous navigation can be defined as the capability of a spacecraft to determine, without outside intervention, precise navigation information: position and velocity. This capability is becoming a more essential requirement for future missions, while the number of simultaneous interplanetary missions continues to grow. Moreover, for small satellites, operational costs can start playing a big role in the project lifecycle if they do not have autonomous navigation. In addition, autonomous station-keeping can be defined as the capability of a spacecraft to automatically maintain all its orbital parameters.

The purpose of this study was to extend the capabilities of the Microcosm Autonomous Navigation System (MANS), demonstrated on the TAOS mission in 1994, and the first fully autonomous Orbit Control Kit (OCK), demonstrated on UoSAT-12 in 1999. While MANS used the Sun/Moon/Earth reference set for fully autonomous orbit determination, OCK used a GPS receiver for navigation source. With the miniaturization and processing capabilities now available, the study proposes an extension autonomous navigation to use stars and a central planet, providing an exceptionally robust navigation solution for planetary missions with both singularity-free deterministic and high accuracy Kalman Filter solutions available. Moreover, OCK can make use of the obtained solution to provide autonomous absolute orbit control independent of any external data or commands (Wertz, 2003).

Microcosm has been working on autonomous navigation algorithms since the late 1980's. One of the interplanetary missions where this system was successfully tested was the NASA's DS-1 spacecraft. Commonly the system would use the Moon and Sun as inertial references, because they are easy to identify and unambiguous. However, the study proposes the use of modern star and planet sensing, enabling a better approach. The results of that approach were exceptionally robust in low planetary orbits because the angular size of the planet provides a sensitive measure of distance and, therefore, each measurement set provides a non-singular deterministic solution of the spacecraft position in inertial space. This presents an important advancement for missions going to distant planets, where the communication with ground may not be presented all the time. The accuracy presented is of the order of a hundred meters, to few kilometers. However, one of the problems mentioned is that there is no way to measure the velocity, which has to be calculated using the equations of motion and a sequence of position states.

The study continues focused in cost savings when autonomous orbit-control is used, which requires the use of a propulsion system. The concept of station-keeping is behind the field of study of this thesis, because it requires impulsive maneuvers, so it will not be discussed further. However, the autonomous navigation system presented on the study, which has been already demonstrated to work for interplanetary missions, is the perfect solution to the navigation problem presented on this thesis.

The research literature for the navigation information problem demonstrates that autonomous navigation is critical for planetary exploration. Fortunately, state-of-the-art techniques already have presented solutions for it.

2.3 Relative Atmospheric Drag

The relative atmospheric drag problem becomes of special interest when the relative position and velocity of two spacecraft has to be calculated. The real problem behind this is that density estimation errors are the largest component of error when calculating the drag acceleration.

Thus, if no state updates are received from the leader spacecraft, the error will accumulate over time and the relative position and velocity calculated will be absolutely incorrect. However, the architecture presented on this paper requires periodic state updates, so the problem may be diminished. Nevertheless, a literature review was conducted in order to understand how the relative atmospheric drag was calculated in previous studies.

Since the proposed use of drag force for controlling the relative motion between two spacecraft (Leonard, Hollister, & Bergmann, Orbital Formation-Keeping with Differential Drag, 1989), many studies about how to take advantage of this technique have been published. In addition, formation flying developments, together with small satellite advancements, are a promising technique for replacing large complex spacecraft.

The purpose of this paper was presenting the use of Artificial Neural Networks (ANN) predictors capable of forecasting the atmospheric density along the future orbit of a spacecraft, used for designing rendezvous reference trajectories created specifically for differential drag-based rendezvous maneuvering (Pérez & Bevilacqua, 2016).

The experiment took place in a simulated 400 kilometers Low Earth Orbit, where there is enough atmospheric density for generating differential drag accelerations of sufficient magnitude. Two spacecraft with a mass of 10 kilograms were used, each one with the ability to deploy or retract a set of drag surfaces with areas of 2.8409 m^2 and 0.3409 m^2 respectively. The local vertical/local horizontal (LVLH) reference frame was used for representing spacecraft relative motion, where x axis points from the center to the Earth to the target spacecraft; z axis points in the direction of the orbit angular momentum; and y axis completes the right-handed frame. An initial relative separation in the x and y axis was defined, and an adaptive Lyapunov controller was implemented to force non-linear equations to track the reference trajectory.

The method used in the study focused on improving the creation of reference trajectory for drag maneuvering, implementing a neural network forecasted density to obtain more precise values than those provided by estimations based on empirical data. Moreover, other methods were pointed

out as viable, like the use of models such as NRLMSISE-00, which requires past and future averaged values of solar and geomagnetic indices. Also, the study stated that these indices, that can be obtained from the National Oceanic and Atmospheric Administration (NOAA). However, the empirical model will be impacted by the errors of the forecast averaged values. Furthermore, the study indicated that highly accurate density estimations can be obtained almost in real time using the High-Accuracy Satellite Drag Model (HASDM), but its access is restricted. Finally, the use of onboard sensors was also suggested as an option.

As part of the study, the authors expressed the magnitude of the relative acceleration created by differential aerodynamic drag, where the action of the acceleration was only considered to affect the y axis of the LVLH frame, defined, as:

$$a_d = \frac{1}{2}\rho(BC_C - BC_T)v_{rel}^2 \quad (3)$$

where BC_C and BC_T are the ballistic coefficients of the chaser and the target respectively. It can be observed that the atmosphere density, ρ , was considered equal for the target and the chaser orbits.

Furthermore, the study presented a solution to the control law using an adaptive Lyapunov controller to force the nonlinear model to track the reference trajectory. Then, a linear quadratic regulator was used to stabilize the Schweighart-Sedwick dynamics, resulting in a control law that was then applied to the drag plates of both spacecraft.

The results of the study presented a comparison between the tracking errors when using a reference trajectory created using a constant density, and trajectories created with an Artificial Neural Network. Also, the number and frequency of control signals during time were compared. The metrics obtained from the comparison demonstrated that the use of predicted density from ANN improves the control signal, and the rendezvous time was reduced.

In conclusion, this study provided a solid reference about the importance of atmospheric density values when implementing a control law. However, the use of neural networks predictor still requires density inputs to be propagated. On the one hand, these densities can be obtained using

atmospheric density models, which would require either sending them from ground or implementing a model onboard according to the mission. On the other hand, these densities can be obtained using onboard sensors. In either case, the effects of bias in the estimated or measured densities might increase the error or predictions made by the ANN, which was presented as a future work on that study. It should be pointed that the study was focused on rendezvous maneuvers only, which allowed the authors to consider the same atmospheric density for the chaser and target on the equations. Additionally, atmospheric winds were not considered on the study, which in the atmosphere of Low Mars Orbits would be of big importance.

Development of miniaturized, low-cost sensors, to obtain real time information about the surrounding total atmospheric density of a spacecraft, has already been done on a CubeSat mission, and waiting to be deployed from the International Space Station to qualify it in an operational environment. Moreover, a previous version the instrument has been already deployed onboard the STP-Sat3 spacecraft (Nicholas, Herrero, Stephan, & Finne, 2015). The sensor is an in situ instrument consisting of an extremely compact suite of instruments to measure density, temperature, neutral wind, plasma drift, and the composition of neutrals and ions. The Winds-Ions-Neutral Composition Suite (WINCS) instrument was developed for ionosphere-thermosphere investigations in orbit between 120 and 750 kilometers altitude. The true benefit presented about WINCS is the ability to measure space weather parameters with an extremely small and low-cost instrument that could be flown on almost any spacecraft (Englert, et al., 2012).

Even though the instrument may not be ready to go to an interplanetary mission, for the purpose of this study, it has been considered feasible the inclusion of the sensor to obtain the atmospheric density of Mars.

2.4 Perturbations

Depending on the altitude of at which a spacecraft is flying, it will face different types and magnitudes of perturbations. Some perturbations are more related to the environment, like solar

radiation pressure, atmospheric drag and lift, magnetic field, etc.; while others are related to a presence of other massive objects. Considering only a theoretical two-body problem will differ from reality due to other bodies with representative mass generate influence. There are additional forces that are not considered in Keplerian motion. For instance, if the oblateness of the planet the spacecraft is orbiting is ignored, it will be impossible to predict its position over a long period of time. The fact that Mars is an oblate spheroid makes the use of the simplified gravitational potential equation, Eq. (4), becomes useless for precise tracking (Bate, Mueller, & White, 1971).

$$\phi = \frac{\mu}{r} \quad (4)$$

The inclusion of zonal harmonics on the equations of motion normally is limited to the effects of the second degree (J_2) only, because its magnitude is around one thousand times greater than the next harmonic. This simplification is particularly effective for most large celestial bodies, including Mars.

Historically researchers have been using Hill's equations, also known as Clohessy-Wiltshire equations, to describe relative motion between two or more spacecraft. In 2002, researchers from the Space Systems Laboratory of the Massachusetts Institute of Technology (MIT) presented a solution that made an important contribution on the inclusion of the effects of planet's oblateness, specifically J_2 harmonics.

The purpose of this study was to include the time average of the gradient of the J_2 potential when calculating the in-plane relative motion of two spacecraft. Moreover, when calculating the cross-track motion, mean variations in the orbital elements and spherical trigonometry were used. The authors presented a set of constant-coefficient linear differential equations that can be solved analytically (Schweighart & Sedwick, 2002).

In order to obtain the equations, the authors started deriving the analytic equation of motion for an orbiting spacecraft including the perturbing acceleration due to the second harmonic potential, J_2 . Then, they introduced a reference orbit with constant radius into the derivation,

defining the relative position of the spacecraft as the subtraction of the spacecraft orbit minus the reference orbit, denoting it as $\mathbf{x}$. The local vertical/local horizontal (LVLH) reference frame was used to define the coordinate system, where the $\hat{\mathbf{x}}$ unit vector points from the center to the Earth to the target spacecraft; the $\hat{\mathbf{z}}$ unit vector points in the direction of the orbit angular momentum; and $\hat{\mathbf{y}}$ unit vector completes the right-handed frame.

Furthermore, to keep the spacecraft and reference orbit in close proximity, corrections to the orbital period of the reference orbit were included into the equation. Also, corrections to the reference orbit for nodal drift, to prevent the reference orbit and the spacecraft to drift apart due to separation of the longitude of the ascending node. Finally, because the time averaging of the gradient of the J_2 potential causes errors in the cross-track motion, corrections were also added to the resulting equations.

The resulting linear constant-coefficient differential equations of motion, relating the motion of a spacecraft to a circular reference orbit obtained in the study, are presented below:

$$\begin{aligned}
\ddot{x} &= 2(nc)\dot{y} + (5c^2 - 2)n^2x - 3n^2J_2 \left(\frac{R_e^2}{r_{ref}} \right) \\
&\quad \left\{ \frac{1}{2} - \left[\frac{3 \sin^2 i_{ref} \sin^2(kt)}{2} \right] - \left[\frac{1 + 3 \cos 2i_{ref}}{8} \right] \right\} \\
\ddot{y} &= -2(nc)\dot{x} - 3n^2J_2 \left(\frac{R_e^2}{r_{ref}} \right) \sin^2 i_{ref} \sin(kt) \cos(kt) \\
\ddot{z} &= -q^2z + 2lq \cos(qt + \phi)
\end{aligned} \tag{5}$$

Additionally, after solving these differential equations, the authors presented the final closed form solution as follows:

$$\begin{aligned}
x &= (x_0 - \alpha) \cos(nt \sqrt{1-s}) + \frac{\sqrt{1-s}}{2\sqrt{1+s}} y_0 \sin(nt \sqrt{1-s}) + \alpha \cos(2kt) \\
y &= -\frac{2\sqrt{1+s}}{\sqrt{1-s}} (x_0 - \alpha) \sin(nt \sqrt{1-s}) + y_0 \cos(nt \sqrt{1-s}) + \beta \sin(2kt) \\
z &= (lt + m) \sin(qt + \phi)
\end{aligned} \tag{6}$$

with,

$$\begin{aligned}
\alpha &= -\frac{3J_2 R_e^2 n^2 (3k - 2n\sqrt{1+s})}{4kr_{ref} [n^2(1-s) - 4k^2]} \sin^2 i_{ref} \\
\beta &= -\frac{3J_2 R_e^2 n^2 [2k(2k - 3n\sqrt{1+s}) + n^2(3 + 5s)]}{4kr_{ref} 2k[n^2(1-s) - 4k^2]} \sin^2 i_{ref} \\
\dot{y}_0 &= -2x_0 n\sqrt{1+s} + -\frac{3J_2 R_e^2 n^2}{4kr_{ref}} \sin^2 i_{ref} \\
\dot{x}_0 &= y_0 n \left(\frac{1-s}{2\sqrt{1+s}} \right)
\end{aligned} \tag{7}$$

Complementing the study, the authors also provided the equations to describe the relative motion between two spacecraft, which are presented as crucial for formation flying design. To start deriving relative motion, authors defined the relationship:

$$\mathbf{x}_1 - \mathbf{x}_2 = \Delta \mathbf{x} \tag{8}$$

where $\mathbf{x}_1$ is the relative position of the first spacecraft with respect to the reference orbit, $\mathbf{x}_2$ is the relative position of the second spacecraft with respect to the reference orbit, and $\Delta \mathbf{x}$ represents the relative motion of the first spacecraft with respect to the second.

Finally, the authors presented the resulting differential equations of motion in $\hat{x}\hat{y}\hat{z}$ coordinates:

$$\begin{aligned}
\Delta \ddot{x} - 2(nc)\Delta \dot{y} - (5c^2 - 2)n^2 \Delta x &= 0 \\
\Delta \ddot{y} - 2(nc)\Delta \dot{x} &= 0 \\
\Delta \ddot{z} + q^2 \Delta z &= 2lq \cos(qt + \phi)
\end{aligned} \tag{9}$$

Some additional relationships needed to use Eqn.(5), Eqn.(6), Eqn.(7) and Eqn.(9) were also given, where the only required inputs for these equations are: the J_2 constant of the planet, R_e , and μ , parameters of the reference orbit r_{ref} and i_{ref} , the inclination of the second spacecraft i_{sc2} , and the initial conditions Δx_0 , Δy_0 , Δz_0 , and $\Delta \dot{z}_0$. Those relationships are presented below:

$$\begin{aligned}
s &= \frac{3J_2 R_e^2}{8r_{ref}^2} (1 + 3 \cos 2i_{ref}) \\
c &= \sqrt{1+s}
\end{aligned} \tag{10}$$

$$n = \sqrt{\frac{\mu}{r_{ref}^3}}$$

$$k = nc + \frac{3nJ_2R_e^2}{2R_{ref}^2} \cos^2 i_{ref}$$

$$i_{sc1} = \frac{\Delta \dot{z}_0}{kr_{ref}} + i_{sc2}$$

$$\Delta \Omega_0 = \frac{\Delta z_0}{r_{ref} \sin i_{ref}}$$

$$\gamma_0 = \cot^{-1} \left[\frac{\cot i_{sc2} \sin i_{sc1} - \cos i_{sc1} \cos \Delta \Omega_0}{\sin \Delta \Omega_0} \right]$$

$$\Phi_0 = \cos^{-1} [\cos i_{sc1} \cos i_{sc2} + \sin i_{sc1} \sin i_{sc2} \cos \Delta \Omega_0]$$

$$\dot{\Omega}_{sc1} = \frac{3nJ_2R_e^2}{2r_{ref}^2} \cos i_{sc1}$$

$$\dot{\Omega}_{sc2} = \frac{3nJ_2R_e^2}{2r_{ref}^2} \cos i_{sc2}$$

$$q = nc - (\cos \gamma_0 \sin \gamma_0 \cot \Delta \Omega_0 - \sin^2 \gamma_0 \cos i_{sc1}) (\dot{\Omega}_{sc1} - \dot{\Omega}_{sc2}) - \dot{\Omega}_{sc1} \cos i_{sc1}$$

$$l = -r_{ref} \frac{\sin i_{sc1} \sin i_{sc2} \sin \Delta \Omega_0}{\sin \Phi_0} (\dot{\Omega}_{sc1} - \dot{\Omega}_{sc2})$$

The new solutions presented on the study are still dependents on initial conditions. These initial conditions are specified as the initial position and velocity of the spacecraft with respect to the reference orbit or to another spacecraft in the formation.

Finally, numerical results were presented from a simulation of two spacecraft placed in a free-orbit ellipse, with a projection of 100 m circle in the radial direction. The maximum error obtained was approximately 30 cm in the in-track direction, which represents only 0.3% of the 100 meters free-orbit-ellipse. The authors emphasized the large improvement of the results with respect to Hill's equations, which have periodic errors that will grow without bounds.

Although the presented equations, including perturbing accelerations due to J_2 , provided excellent results, they cannot be used completely when differential drag is the only means of

control. This is because no corrections can be made for the cross-track motion. Anyway, they are still valid and can be applied for the purpose of this thesis. Indeed, as they are constant-coefficient linear differential equations, numerical propagation methods can be easily implemented, which is desirable for using onboard of a spacecraft.

Schweighart & Sedwick's proposed equations represented a great opportunity to develop new theories around them, so much so that many new studies of differential drag are based on the new set of equations. It is clear also that perturbations are a big problem when two or more spacecraft need to fly in formation. Still, they include J_2 perturbations, but control algorithms were out of the scope.

This study introduced a novel control method for autonomous orbit stabilization and rendezvous. By exploiting the differences in drag among spacecraft in the formation, relative accelerations are generated, and therefore their relative orbits are controlled. Moreover, this paper made use of differential drag to perform rendezvous maneuvers in low Earth orbits using the Schweighart-Sedwick equations. For instance, these maneuvers were proposed for autonomous on-orbit assembly. The main contribution of this work with respect to the method proposed by Leonard et al. (Leonard, Hollister, & Bergmann, Orbital Formation-Keeping with Differential Drag, 1989), where no J_2 effects were considered, was the elimination of residual distance at the end of the maneuvers, and the elimination of an optimization method for solving. Instead the proposed method is based on an analytical expression (Bevilacqua & Romano, Rendezvous Maneuvers of Multiple Spacecraft Using Differential Drag Under J_2 Perturbation, 2008).

The dynamic model presented on the paper is a linearized dynamic model of a spacecraft moving with respect to a circular reference orbit, including the J_2 effects. The equations used by the authors are presented below. It has to be mentioned that they are similar to Eqns.(5), but they have not included the correction made by Schweighart and Sedwick to the orbital period or the reference orbit, thus the mean motion, n , was replaced by the target's circular-orbit angular velocity, ω . However, the respective control inputs were included for all axes.

$$\begin{aligned}
\ddot{x} &= 2(\omega c)\dot{y} + (5c^2 - 2)\omega^2 x - 3\omega^2 J_2 \left(\frac{R_e^2}{r_{ref}} \right) \\
&\quad \left\{ \frac{1}{2} - \left[\frac{3 \sin^2 i_{ref} \sin^2(kt)}{2} \right] - \left[\frac{1 + 3 \cos 2i_{ref}}{8} \right] \right\} + U_x \\
\ddot{y} &= -2(\omega c)\dot{x} - 3\omega^2 J_2 \left(\frac{R_e^2}{r_{ref}} \right) \sin^2 i_{ref} \sin(kt) \cos(kt) + U_y \\
\ddot{z} &= -q^2 z + 2lq \cos(qt + \phi) + U_z
\end{aligned} \tag{11}$$

$$k = \omega c + \frac{3nJ_2 R_e^2}{2R_{ref}^2} \cos^2 i_{ref}$$

where relations defined in Eqns.(10) also apply. The coordinate system used is also LVLH, with the same definition as before. Since no correction was made to the angular rate, the angular velocity of the coordinate system with respect to the inertial frame is ωc .

This paper also presented the relative dynamics between one spacecraft on the formation and the leader spacecraft, including the relative control acceleration components for each axis, as:

$$\begin{aligned}
\Delta \ddot{x} &= 2(\omega c)\Delta \dot{y} - (5c^2 - 2)\omega^2 \Delta x + u_x \\
\Delta \ddot{y} &= -2(\omega c)\Delta \dot{x} + u_y \\
\Delta \ddot{z} &= -q^2 \Delta z + 2lq \cos(qt + \phi) + u_z
\end{aligned} \tag{12}$$

where $[u_x \ u_y \ u_z]$ indicate the components of the relative acceleration between the two spacecraft due to a control action. As drag by changing the cross-sectional surface area S is the only means of control that a spacecraft has, this acceleration was considered as the control vector with the only nonzero component u_y .

The equation to calculate the acceleration due to atmospheric drag was presented as:

$$\mathbf{a} = \left(-\frac{1}{2} \rho \frac{SC_D}{m} V^2 \right) \hat{V} \tag{13}$$

where V was defined as the spacecraft velocity vector with respect to Earth's atmosphere, and $\hat{V}$ as the spacecraft velocity unit vector relative to Earth's atmosphere.

Several assumptions were made on the paper. Most of them are common assumptions made for formation flying studies, being the most relevant that the problem was confined to the x - y plane.

Therefore, for each follower spacecraft, the state vector is $[\Delta x \ \Delta y \ \Delta \dot{x} \ \Delta \dot{y}]^T$ with the final condition $[\Delta x \ \Delta y \ \Delta \dot{x} \ \Delta \dot{y}]^T = [0 \ 0 \ 0 \ 0]^T$. Then, the control of the dynamics along the z axis, which is oscillatory and independent from the one on the x - y plane (thus non controllable using only drag), was considered to be beyond of the scope of their work.

After applying a state vector transformation to separate the mean secular motion from the oscillatory part (Bevilacqua, Romano, & Curti, 2010), the state vector for the follower was transformed into a new intermediate state vector, as:

$$\begin{bmatrix} z'_1 \\ z'_2 \\ z'_3 \\ z'_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -\frac{A}{A^2 - B} & 0 \\ -\frac{BA}{A^2 - B} & 0 & 0 & -\frac{B}{A^2 - B} \\ 0 & 0 & -\frac{A^2}{2(A^2 - B)^{3/2}} & 0 \\ -\frac{A^2 B}{2(A^2 - B)^{3/2}} & 0 & 0 & -\frac{A^3}{2(A^2 - B)^{3/2}} \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta \dot{x} \\ \Delta \dot{y} \end{bmatrix} \quad (14)$$

$$a = 2(\omega c)$$

$$b = (5c^2 - 2)\omega^2$$

where $A^2 - B$ is an always-positive constant.

Furthermore, the paper presented the rate change of the intermediate state vector, considering only a control component along y , as:

$$\begin{bmatrix} \dot{z}'_1 \\ \dot{z}'_2 \\ \dot{z}'_3 \\ \dot{z}'_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -(A^2 - B) & 0 \end{bmatrix} \begin{bmatrix} z'_1 \\ z'_2 \\ z'_3 \\ z'_4 \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{B}{A^2 - B} \\ 0 \\ -\frac{A^3}{2(A^2 - B)^{3/2}} \end{bmatrix} u_y \quad (15)$$

which was described by the authors as the combination of a double integrator and a harmonic oscillator. The closed-form solution for constant control acceleration, u_y , of Eqn.(15) was then given and expressed as:

$$\begin{aligned}
z'_1 &= -\frac{B}{A^2 - B} u_y \frac{t^2}{2} + z'_{2_0} t + z'_{1_0} \\
z'_2 &= -\frac{B}{A^2 - B} u_y t + z'_{2_0} \\
z'_3 &= \left(z'_{3_0} - \frac{A^3 u_y}{2(A^2 - B)^{5/2}} \right) \cos \left[\left(\sqrt{A^2 - B} \right) t \right] + \frac{z'_{4_0}}{\sqrt{A^2 - B}} \sin \left[\left(\sqrt{A^2 - B} \right) t \right] \\
&\quad + \frac{A^3 u_y}{2(A^2 - B)^{5/2}} \\
z'_4 &= z'_{4_0} \cos \left[\left(\sqrt{A^2 - B} \right) t \right] - \sqrt{A^2 - B} \left(z'_{3_0} - \frac{A^3 u_y}{2(A^2 - B)^{5/2}} \right) \sin \left[\left(\sqrt{A^2 - B} \right) t \right]
\end{aligned} \tag{16}$$

For convinience, the authors defined the following relationships to represent the final transformed state vector:

$$z_1 = z'_1 \quad z_2 = z'_2 \quad z_3 = z'_3 \quad z_4 = \frac{z'_4}{\sqrt{A^2 - B}} \tag{17}$$

where the variables z_1 and z_2 represent the forced double integrator, and the variables z_3 and z_4 represented the forced harmonic oscillator.

Finally, the authors of this work presented two control algorithms to accomplish the two proposed control stages: Relative Orbit Stabilization, and Rendezvous to the Target. These presented algorithms were based on the phase plane analysis of each stage, given by the forced double integrator and the forced harmonic oscillator. A series of conditions were presented depending on the location of the spacecraft with respect to the phase plane. Even though the analysis was made separately, the authors state that the control sequence has to take into account the conditions related to each of the two phase planes at the same time.

Concluding, the solution proposed by this paper provides an unprecedent progress since formation flying using differential drag started to be considered. The solution provides not only a method to allow spacecraft rendezvous without propellant expediture, but also a closed-form analytical solution, validated through numerical simulations.

3 METHODS

3.1 Introduction

Formation flying techniques using differential atmospheric drag have been evolving since 1996, when Leonard et al. presented the first approach. It is common to think about its application on Earth atmosphere, because that was the initial conception. This thesis intends to go further analyzing and validating its usage on the Martian atmosphere, discovering also its limitations. However, many problems, that might not be important on Low Earth Orbits, are presented when a spacecraft embraces in a planetary mission. This study has categorized them in four main areas: Navigation Information problems; Relative Atmospheric Drag problems; Perturbation problems; and, finally, Inter-Spacecraft Communication problems.

Furthermore, many research questions catch the attention of the researcher, and were previously presented on this study. The most important derived questions are repeated below:

1. What is the relationship between atmospheric density, area exposed to aerodynamic drag, and the time required to achieve a formation? Is there a maximum difference in altitude that could be deterministic of the ability to perform formation flying between two spacecraft? How this relates to their drag areas?
2. How long a mission can last before re-entering, depending on the mass to area ratio of the spacecraft involved in this study? How that can be extrapolated to other missions?
3. What is the total delta-v that two spacecraft can save by just using differential aerodynamic drag as means of formation flying control?

The simulations performed in this study were designed with all the research questions in mind, having the advantage of count on access to the two existing high-resolution atmospheric models of Mars. After determined the feasibility of taking advantage of the thin atmosphere density of the red planet, numerical simulations were performed to validate the use of the equations of

motion developed by Schweighart & Sedwick, and the control techniques developed by Bevilacqua & Romano, presented in Section 2.

After having an operational environment, the resulting state vectors of the orbital dynamics and attitude data were stored in the format described by the Navigation and Ancillary Information Facility (NAIF), mainly for validation and visualization purposes, and also for future use. Several plots were generated during this process, to help visualizing and explaining the results obtained, presented in Section 4.

3.2 Background

3.2.1 Reference Frames

In the present work and related simulation environment, two different reference frames were used: one inertial frame to reference the orbit described by the spacecraft with respect to Mars; and one time varying frame to reference one spacecraft position relative to other. Both frames are shown in Figure 2.

Figure 2. Mars Centered Inertial Frame (MCI) and LVLH reference frame

3.2.1.1 Mars Centered Inertial Frame (MCI)

The inertial frame used to define the orbit around the planet, has been defined locally with its origin at the center of Mars, where the mean equatorial plane defines the fundamental reference X - Y plane. The X axis was defined by the intersection, at vernal equinox, of the equatorial plane (reference plane) and the ecliptic plane. The Z axis was made coincident with the rotational axis of

Mars, and the Y axis completes the frame with accordance to the right-handed system. A visual representation of the MCI frame can be seen in Figure 2, centered on Mars.

Any location given in spherical coordinates with right ascension, α , declination, δ , and distance from the origin of the reference frame, r , can be described in the Cartesian coordinates X , Y , Z , by the transformation given in Eqn.(18). The declination is measured positive from the reference plane to the north, and negative to the south (-90° to 90°), while the right ascension is measured positive to the East in the reference plane, starting from the X axis (0° to 360°).

$$\underline{r} = \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = r \begin{pmatrix} \cos \alpha \cos \delta \\ \sin \alpha \cos \delta \\ \sin \delta \end{pmatrix} \quad (18)$$

3.2.1.2 Local Vertical / Local Horizontal Frame

The preferred coordinate system to use when relative motion is involved is defined by the Local Vertical Local Horizontal, which is centered and moving with the object. For this study, the $\hat{x}$ axis was defined always pointing out from the spacecraft along Mars' radius vector to the spacecraft, as it moves through the orbit. The $\hat{z}$ axis was defined normal to the orbital plane. The $\hat{y}$ axis completes the set using the right-hand rule, pointing to the velocity direction (but not necessarily parallel) (Vallado, 2013). A visual representation of the LVLH frame can be observed in Figure 2, centered on the spacecraft.

These notations are defined based on this coordinate system: *radial* displacements occurs when a spacecraft moves parallel to the position vector ($\hat{x}$ axis), *along-track* displacement occurs when the spacecraft moves normal to the position vector (along the $\hat{y}$ axis), and *cross-track* when it moves normal to the orbital plane (along the $\hat{z}$ axis) (Vallado, 2013).

3.2.1.3 Notation

To identify position, velocity, and acceleration of the spacecraft that is orbiting Mars on the inertial frame, the following notation was used:

$$\begin{aligned}
\underline{r}(t) &= x(t)\hat{X} + y(t)\hat{Y} + z(t)\hat{Z} \\
\underline{v}(t) = \dot{\underline{r}} &= \frac{dx(t)}{dt}\hat{X} + \frac{dy(t)}{dt}\hat{Y} + \frac{dz(t)}{dt}\hat{Z} = \dot{x}(t)\hat{X} + \dot{y}(t)\hat{Y} + \dot{z}(t)\hat{Z} \\
\underline{a}(t) = \ddot{\underline{r}} &= \frac{d^2x(t)}{dt^2}\hat{X} + \frac{d^2y(t)}{dt^2}\hat{Y} + \frac{d^2z(t)}{dt^2}\hat{Z} = \ddot{x}(t)\hat{X} + \ddot{y}(t)\hat{Y} + \ddot{z}(t)\hat{Z}
\end{aligned} \tag{19}$$

where the vectors $\hat{X}$, $\hat{Y}$ and $\hat{Z}$, are the direction unit vectors, and the components x , y , and z , are the time varying components of the movement.

Furthermore, similar notation was used when representing position, velocity, and acceleration of the spacecraft in the LVLH frame, where the only difference is that the unit vectors were identified as $\hat{x}$, $\hat{y}$ and $\hat{z}$.

3.2.2 Orbit Determination

To solve the problem of determining the location of an object in orbit over time, three propagation methods can be used: analytical methods, numerical integration, and a mixture of both called semi-analytical methods (Vetter, 2007). The first group, also known as general perturbation methods, “superimpose the approximated effects of the most prominent perturbations on the Keplerian equations of motion, leading to efficient propagation methods capable of predicting satellite states at any arbitrary time in the future or past, at the cost of lacking the small variations caused by more complex perturbations”. The following group, also known as special perturbation methods, numerically integrates the orbit dynamic equations, which involves more precise force and perturbation models. These methods provide a substantial improvement in accuracy, at the expenses of higher computational load. The semi-analytical methods solve the 2-body problem analytically, and then numerically integrate the perturbations.

It has to be noted that numerical integration methods are sensitive to the time step size between propagations, while analytical methods are not. Even though, for the purpose of this study numerical integration methods were used because they have the advantage of defining the step size,

which is useful for benchmarking and post processing. The solution developed was based on the widely used MATLAB® ode45 solver, including perturbations due to J_2 and atmospheric drag.

3.2.3 Two-Body Problem

3.2.3.1 Assumptions

The most elementary orbit description is made by the two-body problem. The following assumptions are essential for this purpose (Vallado, 2013):

1. The spacecraft's mass is negligible small compared to the attracting body
2. The reference frame is inertial. This removes the derivatives of the reference frame when differentiating vectors. For the purpose of this thesis the MCI frame was used.
3. The spacecraft and attracting body are spherically symmetrical and of uniform density, allowing to treat the problem as point masses.
4. No other forces than gravity between the spacecraft and attracting body are present.

3.2.3.2 Kepler's Laws

Johann Kepler (1571-1630) proposed three laws of planetary motion, in order to summarize data collected by Tycho Brahe in the early 1600s.

The author of this work has no intentions of going deeper on basic concepts of astrodynamics that can be found in many literatures. Thus, his three laws are summarized as in Vallado (Vallado, 2013):

1. The orbit of each planet is an ellipse (or any other conic section) with the Sun at one focus.
2. The line joining the planet to Sun sweeps out equal areas in equal times.
3. The square of the period of a planet is proportional to the cube of its mean distance to the Sun.

These laws are still considered an accurate description of the motion of any planet, any satellite, and any spacecraft.

3.2.3.3 Kepler's Orbital Motion

The significance of Kepler's second law is that it relates elapsed time to the angular displacement of a body within an orbit. This is important because the velocity does not change linearly with time, unless a perfect circular orbit is examined. Velocity changes faster at periapsis than at apoapsis.

When an elliptical orbit is described, as the one shown in Figure 3, the angle found when a line is traced from the center of the ellipse to the intersection of an imaginary circular orbit of radius a , and a line extrapolated perpendicularly to the semi-major axis through the spacecraft position, is called the eccentric anomaly, E . Using trigonometry it is easy to relate the eccentric anomaly with the true anomaly, v (Vallado, 2013):

$$\begin{aligned} E &= \arcsin\left(\frac{\sin v \sqrt{1 - e^2}}{1 + e \cos v}\right) \\ v &= \arcsin\left(\frac{\sin E \sqrt{1 - e^2}}{1 + e \cos E}\right) \end{aligned} \tag{20}$$

where v is the true anomaly, and e is the eccentricity of the orbit.

Figure 3. Interpretation of eccentric anomaly angle, E

Then, to describe the motion of the spacecraft around the orbit, Kepler defined the mean anomaly, M , that corresponds to uniform angular motion on a circle of radius a , as:

$$M = E - e \sin E = n(t - t_0) \quad (21)$$

$$n = \sqrt{\frac{\mu}{a^3}}$$

where n is called mean motion, or mean angular velocity, which is the mean angular rate of the orbital motion, and the expression $(t - t_0)$ corresponds to the change in time (Vallado, 2013).

3.2.3.4 Newton's Solution to the Two-Body Problem

Starting from Newton's law of gravitation, it is possible to determine the gravitational force acting onto a spacecraft as:

$$g_{sc} = -\frac{G m_{Mars} m_{sc}}{r^2} \quad (22)$$

where g_{sc} is the magnitude of the gravitational force, G in a universal gravity constant, m_{Mars} and m_{sc} are the masses of Mars and the spacecraft, respectively, and r is the distance between the spacecraft and the center of Mars. Moreover, Newton's law states that the gravitational force is pointing along the line intersecting the two objects centers of mass, thus:

$$\underline{g}_{sc} = -\frac{G m_{Mars} m_{sc}}{r^2} \left(\frac{\underline{r}}{r}\right) = -\frac{G m_{Mars} m_{sc}}{r^3} \underline{r} \quad (23)$$

where $\underline{r}$ is the position vector of the spacecraft relative to the center of Mars.

Making use of Newton's second law of motion $\underline{F} = m \underline{a} = m \ddot{\underline{r}}$, and assuming the mass of Mars stays constant, it is possible to define the gravitational parameter for Mars, $\mu_{Mars} = G m_{Mars}$. Then, the solution to the Two-Body problem is as follows:

$$\ddot{\underline{r}}_{body} = \frac{\underline{g}_{sc}}{m_{sc}} = \frac{G m_{Mars}}{r^3} \underline{r} = \frac{\mu_{Mars}}{r^3} \underline{r} \quad (24)$$

3.2.3.5 Gravitational Perturbation due to the Oblateness of Mars

When a spacecraft is traveling near Mars' surface, the assumption made for the two-body problem about the spherical symmetry of the attracting body cannot be made. In reality Mars is not a sphere, but rather an oblate spheroid, with a polar radius of 3376.2 kilometers, and an equatorial radius of 3396.2 kilometers, giving a difference of 20 kilometers between each other. Although it

is almost the same value as for Earth (21 kilometers), the size of Mars is nearly one half, making the oblateness even bigger, and thus the perturbation due to zonal harmonics greater.

The solution to take the perturbation into account was considering the acceleration generated by the planet's shape, defined by Vallado as (Vallado, 2013):

$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix}_{J_2} = \frac{3J_2\mu R_{Mars}^2}{2r^5} \begin{bmatrix} \left(\frac{5z^2}{r^2} - 1\right)x \\ \left(\frac{5z^2}{r^2} - 1\right)y \\ \left(\frac{5z^2}{r^2} - 3\right)z \end{bmatrix} \quad (25)$$

Although higher order terms of the perturbations exist and could be calculated, like J_3 and J_4 , the effect that they have is small compared to J_2 . Therefore, for the purpose of this study those coefficients were not included.

3.2.3.6 Aerodynamic Drag

Another perturbation that will affect a spacecraft flying in conditions where atmosphere density exists is the negative acceleration due to aerodynamic drag (see Eqn. (3)). This perturbation was also not included in the two-body problem, and it is of big importance for this work.

In order to obtain the atmospheric density at different times and locations around Mars, the Mars Climate Database (Lewis, et al., 1999) will be used.

3.2.3.7 Orbit Integration

After solving the equation of motion Eqn.(24), the accelerating perturbations due to zonal harmonics Eqn.(25), and the accelerating perturbation due to atmospheric drag Eqn.(3), a step-wise integration can be computed as:

$$\ddot{\underline{r}} = -\frac{\mu}{r^3}\underline{r} + \underline{P}_{J_2} + \underline{P}_{atm} \quad (26)$$

where $\underline{r}$ and $\ddot{\underline{r}}$ are the inertial position and acceleration vectors, and $\underline{P}_{J_2}$ and $\underline{P}_{atm}$ are vectors of the perturbations due to J_2 and drag per unit mass acting towards the spacecraft, respectively.

3.2.4 Mars Facts

Knowing more about the red planet is essential to plan missions to continue discovering it. For orbiters and rovers, it is important to know the conditions they will face during their missions, and in the end, to estimate mission duration. For manned missions it would be important to be well prepared, because mission duration cannot be decided.

Mars is the third largest planet in the Solar System, with just 11% of the mass of the Earth. The distance to the Sun varies from 1.38 AU when it is at perihelion, to 1.67 AU when it is at aphelion.

Just like Earth, due to the tilt of its axis of rotation, Mars has seasons. With them, the polar ice caps grow and recede through the Martian year. The time that takes Mars to go around the sun, called orbit period, is 686.98 Earth days, with an orbit eccentricity of 0.0934.

“NASA’s Mars Science Laboratory mission, with its rover Curiosity, is examining Martian rocks and soil at Gale Crater, looking for minerals that formed in water, signs of subsurface water, and carbon-based molecules called organics, the chemical building blocks of life”, NASA’s Solar System Exploration web page says (NASA Planetary Science Division, 2016).

The Martian atmosphere is composed mostly by carbon dioxide (CO_2), representing a bit more than 95% of its total volume. Today Mars has no magnetic field like Earth. However, scientists have obtained from NASA’s Mars Global Surveyor orbiter, information that presents areas highly magnetized in the southern hemisphere.

3.2.5 Martian Date and Local Time

In section 3.2.3.3 the three anomalies used when describing orbital motion were presented: true anomaly, eccentric anomaly, and mean anomaly, that can be calculated using Eqns.(20)-(21). The convention used on this work is such that $v = 0$ at perihelion for Mars.

3.2.5.1 Aerocentric Solar Longitude

The Aerocentric Solar Longitude, L_s , is the measurement of the position of Mars on his orbit around the Sun. The convention used is such that $L_s = 0$ at the vernal equinox. The solar longitude is related directly with the true anomaly by the following equation (Forget, Millour, & Lewis, 2015):

$$v = L_s - L_{s_{peri}} \quad (27)$$

where $L_{s_{peri}}$ is the perihelion date, which corresponds to the angle at which the perihelion is located (see Table 2).

Table 1. Mars quick facts

Mean distance from the Sun	227.94 million km
Orbit period	686.98 Earth days
Orbit eccentricity	0.0934
Orbit inclination to ecliptic	1.8 degrees
Inclination of equator to orbit	25.1919 degrees
Rotation period	24.62 hours
Successive sunrises	24.66 hours
Equatorial radius	3,397 km
Mass	0.10744 of Earth's mass
Density	3.934 g/cm ³ (0.714 Earth's density)
Surface gravity	0.38 of Earth's gravity
Atmosphere primary components	Carbon dioxide, nitrogen, argon
Temperature range	-87 to -5 °C (-125 to 23 °F)
Known moons	2 (Deimos, Phobos)
Rings	0

Credit: (NASA Planetary Science Division, 2016)

3.2.5.2 Martian Sol Number

A Martian sol number, D_s , is the reciprocal to Earth day, and represents the number of revolutions elapsed since beginning of a Martian year (when $L_s = 0$). The duration of one complete revolution of Mars around its rotational axis takes 88775.245 seconds, that is, the duration of one D_s . Then, the number of sols in one Martian year, N_s , is 668.6.

To calculate the Martian sol number given a Julian Date, the following equation was used (Forget, et al., 2015):

$$D_s = (JD - JD_{ref}) \frac{86400}{88755.245} \pmod{N_s} \quad (28)$$

where JD is the given Julian Date for which the Martian date has to be calculated, and JD_{ref} is a reference Julian Date at which $L_s = 0$, for example on December 19, 1975 4:00:00, corresponding to Julian date 2442765.667.

Table 2. Mars constants

Nomenclature	Value	Description
N_s	88775.244	Number of seconds in a sol (Martian day)
L_{speri}	250.99	Perihelion date (in degrees)
t_{peri}	485.35	Perihelion date (in sols)
a	1.52368	Semi-major axis of Martian orbit (in AU)
ϵ	25.1919	Obliquity of equator to orbit (in degrees)
J_2	1960.45 (10^{-6})*	Mars gravitational harmonic J_2 coefficient
Ω	7.08822 (10^{-5})†	Mars rotation rate (in rad/s)

Credit: (Forget, Millour, & Lewis, 2015)

* Credit: (Williams, 2016)

† Credit: (Turcotte, Shcherbakov, Malamud, & Kucinskis, 2002)

3.2.5.3 Computing Solar Longitude from Sol Number

The Mars Climate Database contains data referenced using L_s , thus it is important to obtain the correct value corresponding to the simulation reference time used. The procedure is a sequence of steps, as follows:

Step 1: Calculate the mean anomaly, M , in radians, using the following equation:

$$M = 2\pi \frac{D_s - t_{peri}}{N_s} \quad (29)$$

Step 2: Calculate the eccentric anomaly E , in radians, using Eqn.(21), which must be solved using an iterative procedure. For the purpose of this study, a simple Newton iterative integration was used with a tolerance of 10^{-7} .

Step 3: Calculate the true anomaly, v , using Eqn.(20) and E from Step 2.

Step 4: Calculate L_s , in degrees, using the following equation:

$$L_s = \left(v \frac{180}{\pi} + L_{s_{peri}} \right) (mod\ 360) \quad (30)$$

3.3 Tools

3.3.1 MATLAB®

All the equations of motion, control algorithms, and calculations required to evaluate the proposed solution were coded in a set of MATLAB functions and scripts. MATLAB provides an environment suitable to perform very complex simulations, and it is widely used by the scientific community. The version used was R2015b, with a student license, including the Aerospace Toolbox.

3.3.2 Mars Climate Database

The Mars Climate Database (MCD) offers a great opportunity for applications other than just atmospheric study. Being able to access to high resolution empirical data can be used in spacecraft mission planning, like formation flying capabilities analysis. Other uses also include the development of density profiles for aerobreaking or aerocapture, or to analyze the effects of dust storms in a particular spot of Mars.

3.3.2.1 Getting Access

The database was created thanks to the funds provided, in part, by the European Space Agency (ESA). Although the database was released free for public use, ESA has required the distributors to ask for an official letter requiring access to it. The letter consists on an agreement of no redistribution, either in part or totally, and not to make any business use of it, either directly or indirectly. Moreover, no redistribution was allowed. Interested parties on getting access to it should contact them at:

Laboratoire de Météorologie Dynamique, IPSL
Université Pierre et Marie Curie, BP 99
Tour 45-55, 2ème étage, Bureau 215
4 Place Jussieu, 75252 Paris Cedex 05
FRANCE
<http://www-mars.lmd.jussieu.fr/mars/access.html>

3.3.2.2 Database Setup

The database is a series of data files in a particular format that were generated with a Mars General Circulation Model (GCM), which is able to generate accurate simulations of weather and climate. Therefore, the only setup is decompressing the files into a folder. Then, a wrapper to access the data is provided for MATLAB, among others. The wrapper consists of three files, one is a MATLAB function that has to be called by the code implementing the MCD, and the other two files are in Fortran format, and they require to be compiled. No binary executables are provided with the distribution, so a Fortran compiler was required.

An additional set of software libraries is needed, called NetCDF. It can be obtained from their website at: <http://www.unidata.ucar.edu/packages/netcdf/>. At the moment of writing this report the latest version available was 4.4.3, therefore that was the version used. The package provides libraries for C and Fortran. Unfortunately, Fortran libraries did not come compiled, so they had to be compiled following the steps provided on the NetCDF website.

It has to be mentioned that the NetCDF libraries are required in order to compile the MCD wrapper for MATLAB access.

3.3.3 SPICE

NASA's Navigation and Ancillary Information Facility (NAIF), has created a set of tools called SPICE to assist scientists in the interpretation of data obtained from space-based instruments aboard robotic planetary spacecraft (NASA's NAIF, 2016). It was originally created for archiving purposes, but nowadays is used for many scientists in all mission phases, included mission planning.

The SPICE Toolkit is free and open for public use, and can be obtained from the NASA's NAIF website at: <http://naif.jpl.nasa.gov/naif/>. At the moment of writing this report, the latest version available was N65, therefore it was the version used.

The purpose of using SPICE is to store the data generated on MATLAB in a standard format for post-processing and results visualization.

3.4 Setting

Two spacecraft were considered on the study, with the same mass and dimensions, one acting as a leader and the other as follower. It was considered that each spacecraft had the ability of deploying or retracting panels, to increase or reduce the area exposed to the velocity direction, instantaneously when commanded. This assumption should not modify the results of the simulation, because in reality it should take in the order of seconds to rotate, or at the most minutes, while the effects of drag can be observed after a long time. Moreover, it was not considered the stabilization of the attitude control, assuming that they have a control algorithm to do so, which was considered out of the scope of this work. The characteristics of the spacecraft were based on a 6U CubeSat mission type with deployable appendages according to the orbit where they were located, and are shown in Table 3.

The orbit considered for the study was, initially, a 165 kilometer altitude circular with the orbital parameters shown in Table 4. Furthermore, additional orbits were used during the analysis to determine the validity of the models at higher altitudes. Those orbits will be presented in the next chapter. The leader was considered to be at the given position right the beginning of the simulation, while the follower location was considered relative to the location of the leader. The relative location of the follower with respect to the leader spacecraft can be seen in Table 5.

Figure 4. System architecture

Table 3. Spacecraft characteristics for initial analysis

Parameter	Value
Mass, kg	8
Maximum effective area exposed to drag, m ²	0.247511
Minimum effective area exposed to drag, m ²	0.0366
Drag coefficient	2.2

The architecture of the environment created to simulate the conditions of the two spacecraft under study was composed of three parts, as can be observed in Figure 4: simulation engine, spacecraft model, and supporting tools. The simulation engine was created to perform the orbit propagation of the leader, used to feed the follower with target information, as if it was received from the communication link between both spacecraft. Moreover, the simulation engine was keeping attitude and navigation information for both spacecraft, used to feed the simulated attitude and navigation sensors of the follower. The spacecraft model was considered to be all what it is required to run onboard of the follower spacecraft. Finally, the supporting tools were the external tools used for the simulation, which were the Mars Climate Database, and SPICE. This architecture

allows one to easily distinguish components and where future implementations and improvements can be added.

On the spacecraft model, the attitude sensors and the navigation sensors were not simulated. Instead, two functions were included to complete the blocks that a real spacecraft would have. Upon request of the control block, one of the functions gives the information of where the spacecraft is located (navigation), and the other function gives the information of the attitude and deployment status of the drag plates. Similarly, a function was created to simulate the actuators that would be commanded by the spacecraft when it requires a change in attitude. Again, the function is only provided with the idea of making the mode as real as possible, but no additional functionality was added. Finally, the communication channel was not modeled nor simulated, considering that the follower received the information by the proper mechanisms and the information was stored in a format the control block can access and understand, exactly as it would be in a real scenario.

Table 4. Orbital parameters for the target's initial conditions

Parameter	Value
Inclination (deg)	51.94116
Semi-major axis (km)	3562.2
Right ascension of the ascending node (deg)	206.3578
Argument of perigee (deg)	101.0712
True anomaly (deg)	108.0480
Eccentricity	0

Table 5. Relative position of the leader in LVLH frame

Parameter	Value
x (km)	-1
y (km)	-2
$\dot{x}$ (km/sec)	8.0187×10^{-7}
$\dot{y}$ (km/sec)	1.40×10^{-3}

3.5 Procedure

3.5.1 Guidance Algorithm

The state space representation of the spacecraft trajectory in the x-y plane as proposed by Schweighart and Sedwick, Eqns.(9) and implemented by Bevilacqua et al. (2008) in Eqns.(12). Assuming only control on the x-y plane, and the control command acting on the y axis only, Eqns. (12) take the form:

$$\begin{aligned}\Delta\ddot{x} &= a\Delta\dot{y} - b\Delta\dot{x} \\ \Delta\ddot{y} &= -a\Delta\dot{x} + u_y\end{aligned}\tag{31}$$

Furthermore, in state vector format they can be represented as follows:

$$\dot{\underline{x}} = \underline{A}\underline{x} + \underline{B}u\tag{32}$$

with matrices $\underline{A}$, $\underline{B}$, and the state vector $\underline{x}$ defined as:

$$\underline{A} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ b & 0 & 0 & a \\ 0 & 0 & -a & 0 \end{bmatrix}, \underline{B} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \underline{x} = \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta\dot{x} \\ \Delta\dot{y} \end{bmatrix}\tag{33}$$

with the variables a and b defined as:

$$\begin{aligned}a &= 2nc \\ b &= (5c^2 - 2)n^2\end{aligned}\tag{34}$$

Then, the square matrix $\underline{A}$ is called a *Hurwitz matrix* if all eigenvalues of $\underline{A}$ have strictly negative real part, which is not the case for the linearization obtained by Schweighart & Sedwick, thus the model is not stable and cannot be used as the target equations of motion. Because of that, Bevilacqua et al. presented a state vector transformation (Bevilacqua, Romano, & Curti, 2010) resulting in the previously discussed Eqn.(14), which was redefined in Eqn.(35) to update it with the new variable names.

$$\underline{z} = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -\frac{a}{a^2-b} & 0 \\ -\frac{ab}{a^2-b} & 0 & 0 & -\frac{b}{a^2-b} \\ 0 & 0 & -\frac{a^2}{2(a^2-b)^{3/2}} & 0 \\ -\frac{a^2b}{2(a^2-b)^{3/2}} & 0 & 0 & -\frac{a^3}{2(a^2-b)^{3/2}} \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta \dot{x} \\ \Delta \dot{y} \end{bmatrix} \quad (35)$$

where the inverse transform of Eqn.(35) is:

$$\begin{bmatrix} \Delta x \\ \Delta y \\ \Delta \dot{x} \\ \Delta \dot{y} \end{bmatrix} = \begin{bmatrix} 0 & -\frac{a}{b} & 0 & -2\frac{\sqrt{a^2-b}}{a^2} \\ 1 & 0 & 2\frac{\sqrt{a^2-b}}{a} & 0 \\ 0 & 0 & -2\frac{\sqrt{(a^2-b)^3}}{a^2} & 0 \\ 0 & 1 & 0 & 2\frac{\sqrt{a^2-b}}{a} \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} \quad (36)$$

Similarly to the process done to obtain Eqn.(16), the closed form solution to the state space system presented in Eqn.(35) is:

$$\begin{aligned} z_1 &= -\frac{bt^2}{2(a^2-b)}u_y + z_{20}t + z_{10} \\ z_2 &= -\frac{bt}{a^2-b}u_y + z_{20} \\ z_3 &= \left(z_{30} - \frac{a^3u_y}{2(a^2-b)^{5/2}}\right)\cos\left[\left(\sqrt{a^2-b}\right)t\right] + \frac{z_{40}}{\sqrt{a^2-b}}\sin\left[\left(\sqrt{a^2-b}\right)t\right] \\ &\quad + \frac{a^3u_y}{2(a^2-b)^{5/2}} \\ z_4 &= z_{40}\cos\left[\left(\sqrt{a^2-b}\right)t\right] - \sqrt{a^2-b}\left(z_{30} - \frac{a^3u_y}{2(a^2-b)^{5/2}}\right)\sin\left[\left(\sqrt{a^2-b}\right)t\right] \end{aligned} \quad (37)$$

Furthermore, the control sequence presented by Bevilacqua et al. (2008) was composed by two stages, as discussed in Section 2.4 of this report: a relative orbit stabilization, and the rendezvous to the target, were at the end of the maneuvers the spacecraft achieves zero relative position and velocity. Using the transformed system from Eqn.(35), in order to satisfy the

rendezvous condition $\underline{z} = [z_1 \ z_2 \ z_3 \ z_4]^T$ must reach zero, which will make the original state vector being zero.

As demonstrated in Leonard's thesis (Leonard, 1986), the trajectory on the phase plane $z_1 z_2$ during the first phase will drive the z_1 and z_2 components to zero, i.e. the origin of the phase plane. In that process, the spacecraft starts on a state α where a differential acceleration between the spacecraft is induced. Due to the nature of the scenario, the relative acceleration can be either maximum positive, maximum negative, or zero. The sign of the acceleration is commanded to change at three future states, β, γ, δ , which will drive z_1 and z_2 to zero (see Figure 5).

The second phase is considered the rendezvous maneuver to the target, and will drive the variables z_3 and z_4 to zero. In the process, the values of z_1 and z_2 will deviate from the origin; however, applying the negative and positive accelerations for exactly the same amount of time, the z_1 and z_2 variables will again go to zero. In this other process, the spacecraft will start from a state ϵ , where no differential acceleration is applied until the spacecraft reaches the state ς (see Figure 6). It was demonstrated before (Bevilacqua, Romano, & Curti, 2010) how to determine the waiting time that is used to ensure that "the system follows a trajectory in which the time interval for positive and negative relative accelerations is the same". Then, once in state ς , relative acceleration is induced, and finally at states η and θ , the sign of the acceleration is commanded to change, as in phase one. After another given time, the trajectory will end up in the center of both phase planes. Once this trajectory is found, the analytical control is anti-transformed to the x-y plane, and the guidance trajectory is generated.

Once the trajectory is obtained, the problem will be reduced to design a feedback control law for the attitude of the spacecraft, forcing them to follow the two-phase guidance explained before, including the nonlinearities and navigation errors.

Figure 5. z1-z2 phase plane (image credit: Bevilacqua, R.)

Figure 6. z3-z4 phase plane (image credit: Bevilacqua, R.)

3.5.2 Linear and Nonlinear Models

A linear reference trajectory is first created using the Schweighart-Sedwick equations. But, as explained before, due to the fact that matrix $\underline{A}$ of Eqn.(33) is not a Hurwitz matrix (i.e. unstable), a Linear Quadratic Regulator (LQR) feedback controller was developed to make it stable. The resulting reference model for the desired trajectory is described by (Pérez & Bevilacqua, 2011):

$$\begin{aligned}
 \dot{\underline{x}}_d &= \underline{A}_d \underline{x}_d + \underline{B} \underline{u}_d \\
 \underline{A}_d &= \underline{A} - \underline{B} \underline{K} \\
 \underline{x}_d &= [x_d \quad y_d \quad \dot{x}_d \quad \dot{y}_d]^T \\
 \underline{u}_d &= \underline{K} \underline{x}_t
 \end{aligned} \tag{38}$$

where $\underline{A}$ and $\underline{B}$ were defined in Eqn.(33), and $\underline{K}$ is a constant gain matrix obtained by solving the LQR problem, that will make $\underline{A}_d$ becoming a Hurwitz, and $\underline{x}_t$ is the reference trajectory. The new system can be regulated, where the term $\underline{u}_d$ goes away, or forced to follow a reference trajectory. Applying Bryson's rule to determine the initial values for $\underline{Q}_{LQR}$ matrix and R_{LQR} value, the following data were obtained:

$$\underline{Q}_{LQR} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (39)$$

$$R_{LQR} = 10^{18}$$

where the values of $\underline{Q}_{LQR}$ were unknown, so the unit was used, and the value of R_{LQR} , which according to Bryson's rule should be based on the reciprocal of the maximum acceptable value for u^2 , and due to the fact that the maximum value of acceleration due to atmospheric drag obtained during the feasibility analysis was in the order of 10^{-10} , the value used initially was 10^{18} .

Then, when describing the dynamics of the relative motion between the leader and follower, the nonlinearities due to J_2 , and atmospheric drag will be present. Thus, a Lyapunov control was used, which was previously proposed and demonstrated to work by Pérez et al. (Pérez & Bevilacqua, 2011) as a solution to this problem. The equations presented by them, including the nonlinearities of the motion are:

$$\begin{aligned} \dot{\underline{x}} &= f(\underline{x}) + \underline{B}\underline{u} \\ \underline{x} &= [x \quad y \quad \dot{x} \quad \dot{y}]^T \\ \underline{u} &= [0 \quad 0 \quad 0 \quad u_y]^T \\ u_y &= \begin{cases} a_D \\ 0 \\ -a_D \end{cases} \end{aligned} \quad (40)$$

It has to be noted again that the control variable $\underline{u}$, which is the relative acceleration due to atmospheric drag, was only considered to act against the y axis, and can take three possible values depending on the attitude of the follower with respect to the leader. The difference will be calculated depending on the ballistic coefficient of each spacecraft, and since both have the same physical characteristics, ultimately will depend on their attitude. The equation used to calculate the magnitude of the relative acceleration due to relative attitude was already presented on Eqn.(3).

When developing the Lyapunov controller, the Lyapunov function was considered to be only function of the follower spacecraft position in the inertial frame, as stated below (Pérez & Bevilacqua, 2011):

$$f(\underline{r}) = \frac{-\mu r}{r^3}$$

$$\underline{r} = [x_{in} \quad y_{in} \quad z_{in}]^T \quad (41)$$

where $\underline{r}$ is the spacecraft position vector in the MCI frame, r is the magnitude of the position, and μ is the gravitational parameter of Mars.

3.5.3 Lyapunov Controller Design

A Lyapunov function was defined as follows:

$$V = \underline{e}^T \underline{P} \underline{e}$$

$$\underline{e} = \underline{x} - \underline{x}_d$$

$$\dot{\underline{e}} = \dot{\underline{x}} - \dot{\underline{x}}_d \quad (42)$$

where $\underline{e}$ is the error defined by the difference between the current state, $\underline{x}$, and the desired state, $\underline{x}_d$, and $\underline{P}$ is a positive definite matrix. Then, the first order time derivative of the Lyapunov function is as follows (Pérez & Bevilacqua, 2016):

$$\dot{V} = \underline{e}^T (\underline{A}_d^T + \underline{P} \underline{A}_d) \underline{e} + 2 \underline{e}^T \underline{P} (f(\underline{x}) - \underline{A}_d \underline{x} + \underline{B} a_D \hat{u} - \underline{B} u_d) \quad (43)$$

where the matrix $\underline{A}_d$ was obtained in Eqn.(38) after solving the LQR presented before. This matrix has to be a Hurwitz, so a symmetric positive definite matrix $\underline{Q}$ can be found such that it satisfies the Eqn.(43), reason why a stable reference model is needed. The equation for $\underline{Q}$ should satisfy (Pérez & Bevilacqua, 2011):

$$-\underline{Q} = \underline{A}_d^T \underline{P} + \underline{P} \underline{A}_d \quad (44)$$

such that the first order time derivative of the Lyapunov function results in:

$$\dot{V} = -\underline{e}^T \underline{Q} \underline{e} + 2\Delta \quad (45)$$

where Δ is given by:

$$\begin{aligned}
\Delta &= \beta \hat{u} - \delta \\
\beta &= \underline{e}^T \underline{P} \underline{B} a_D \\
\delta &= \underline{e}^T \underline{P} (\underline{A}_d \underline{x} - f(\underline{x}) + \underline{B} u_d) \\
\hat{u} &= \begin{cases} 1 \\ 0 \\ -1 \end{cases}
\end{aligned} \tag{46}$$

where the three states of $\hat{u}$ represent the command to send to the spacecraft to change its attitude, being 1 to represent maximum drag on the follower, 0 to represent no differential drag (i.e. same attitude), and -1 for the leader with maximum and the follower minimum drag.

It is worth to mention that in order to solve Eqn.(44) using MATLAB®, the matrix $\underline{A}_d$ has to be transposed, because MATLAB® solves the Lyapunov equation of the form shown below:

$$\underline{A}_d \underline{P} + \underline{P} \underline{A}_d^T + \underline{Q} = 0 \tag{47}$$

In order to guarantee global asymptotic stability, the first time derivative of the Lyapunov function must be negative, so the value of Δ has to be negative or zero. This will make the tracking error, $\underline{e}$, converge to zero, because the first term of the right hand side of Eqn.(45) is already negative. Moreover, the magnitude β is linearly dependent on a_D , thus on the magnitude of the atmospheric density. As presented by Pérez et al. (Pérez & Bevilacqua, 2011), if ρ is too small the value of δ can be greater than β , making the system unstable. That is, there is no enough control authority. Finally, the term $\beta \hat{u}$ has to be negative, so the control variable can be used to ensure that, making the attitude controller variable to be:

$$\hat{u} = -\text{sign}(\beta) = -\text{sign}(\underline{e}^T \underline{P} \underline{B}) \tag{48}$$

Numerical simulations carried out for this thesis using real values for the atmospheric density, obtained from the Mars Climate Database, demonstrate the application of these procedures. Due to the nature of the problem, in order to prevent high frequency drag plates deployment or attitude change commands, the control algorithm was executed every 10 minutes.

3.6 Data Analysis

The main purpose of this section was focused in organizing all the obtained data in a way they help answering the research questions presented in Section 1.6. The results obtained are then presented in Section 4 and discussed in Section 5.

3.6.1 Data Required to Answer Research Question 1

The research question 1 was: What would be the amount of information required to be exchanged between two or more spacecraft who are performing a Formation Flying using the algorithms presented in this study? Moreover, what would be the frequency that information has to be exchanged depending on the type of maneuvers they are performing?

To answer this question, it was important to keep track of the information “exchanged” between the leader and the follower spacecraft. This information was comprised by two parts: first, information sent from the target to the leader containing the spacecraft location at that specific time; second, the information sent by the follower to command the attitude of the leader so the desired differential drag could be achieved. On the other hand, the frequency of data exchange was analyzed considering scenarios with different control times, and then analyzing the frequency and number of the commanded changes in attitude.

3.6.2 Data Required to Answer Research Question 2

The research question 2 was: When a follower spacecraft calculates the differential acceleration due to atmospheric drag, is it required to have precise information about the atmospheric density of the leader, or it can be presumed to be constant as it is usually assumed?

To answer this question, it was necessary to modify the implementation of the algorithm, discarding the information received from the leader about its measured atmospheric density, and then compare the results obtained using both techniques.

3.6.3 Data Required to Answer Research Question 3

The research question 3 was: What is the relationship between atmospheric density, area exposed to aerodynamic drag, and the time required to achieve a formation? Is there a maximum difference in altitude that could be deterministic of the ability to perform formation flying between two spacecraft? How this relates to their drag areas?

This question requires most of the data collected during the simulation to relate all the parameters together. In order of being able to answer it, different area sizes were simulated trying to correlate the spacecraft area exposed with the rendezvous time. Moreover, different initial altitudes were used to obtain data, used to better understand the relationship between ballistic coefficient and rendezvous time.

3.6.4 Data Required to Answer Research Question 4

The research question 4 was: How long a mission can last before re-entering, depending on the mass to area ratio of the spacecraft involved in this study? How that can be extrapolated to other missions?

Answering this question requires a different simulation be executed, where the spacecraft faces the drag plates deployed in the direction of the velocity during certain amount of time. The conclusions obtained from the results are of big importance, because they will give a general idea of the required mass to area ratio required to stay in orbit, and that can be extrapolated to be used in other missions.

3.6.5 Data Required to Answer Research Question 5

The research question 5 was: What is the total delta-v that two spacecraft can save by just using differential aerodynamic drag as means of formation flying control?

To answer this final question, which is indirectly representing the cost benefit of implementing this study, data from all simulations were used, where the accumulated acceleration due to differential drag was accounted for the leader and follower spacecraft.

3.7 Simulation Scenarios

Five different scenarios are presented on this section. The main difference between each configuration are, the altitude, and the physical configuration required for the spacecraft to cope the differences in atmospheric density found at different altitudes. Moreover, three different control techniques were considered for each scenario:

1. A Lyapunov control law that forces the nonlinear system to follows a pre-generated guidance trajectory
2. A Lyapunov control law that forces the nonlinear system to follow a trajectory generated by a reference model, which was stabilized using a linear quadratic regulator and is regulated
3. A Lyapunov function that forces the nonlinear system to follow a trajectory generated by a reference model that was forced to follow a pre-generated guidance trajectory

Furthermore, all simulations where conducted using the same parametric tool, developed in MATLAB®, and validated using data from many of the cited works of this study. The simulation step was set to 30 seconds between calls to ode45, and 10 minutes between calls to the control algorithm. The number of control actuations was recorded in order to evaluate each case with respect to the effort required to achieve the final rendezvous. The simulation was considered to encounter the “rendezvous condition” when the relative distance of the spacecraft was less than 10 meters, and when the relative velocity was less than 10 centimeters per second.

3.7.1 Scenario 1: μ Cube-k165 Mission

The follower spacecraft was commanded to rendezvous from its initial position, using each of the three control techniques presented before. The mission was a collaborative scenario, where the follower and the leader worked as a team to accomplish the objective of the mission. In that

sense, the follower was sending commands to the leader to collaborate with the maneuvers required to obtain the desired differential drag.

3.7.1.1 Spacecraft Details and Initial Conditions

Two spacecraft with same physical characteristics were simulated, starting on a circular orbit at 165 kilometers altitude over Mars zero elevation. The orbital parameters of the initial location of the leader spacecraft were previously presented in Table 4. The relative position of the follower was previously presented in Table 5. The reader should recall that each spacecraft was considered to have the ability of deploying or retracting a series of panels to generate the desired differential drag. An ideal physical characteristic of the spacecraft is shown in Figure 7, where the deployable are assumed to be retracted instantaneously (or the attitude changed instantaneously), with the drag panels not affecting on the velocity direction.

Figure 7. Conceptual Leader-Follower flight configuration

It is worth mentioning that the purpose of this work is not designing the spacecraft, but just analyzing the feasibility, and developing the tools, to simulate more realistic scenarios.

3.7.2 Scenario 2: α Cube-k165 Mission

The only difference with the previous mission is in the physical configuration of the spacecraft, where the idea was to verify how the difference in area exposed to drag at the same

altitude was correlated to the differential drag. All the other orbital parameters and relative position between the spacecraft in the formation were kept the same.

3.7.2.1 Spacecraft Details and Initial Conditions

After designing the trajectory for the maneuver, the spacecraft requirements obtained for this particular mission, at the mentioned initial altitude, can be seen in Table 7.

Table 6. Spacecraft characteristics for α Cube-k165 Mission

Parameter	Value
Mass, kg	8
Maximum effective area exposed to drag, m ²	0.45235836
Minimum effective area exposed to drag, m ²	0.0366
Drag coefficient	2.2

3.7.3 Scenario 3: β Cube-k175 Mission

This mission has an initial altitude 10 kilometers higher than the previous case, while applying the same three Lyapunov control schemes. All the other orbital parameters and relative position between the spacecraft in the formation were kept the same.

3.7.3.1 Spacecraft Details and Initial Conditions

After designing the trajectory for the maneuver, the spacecraft requirements obtained for this particular mission, at the mentioned initial altitude, can be seen in Table 7.

Table 7. Spacecraft characteristics for β Cube-k175 Mission

Parameter	Value
Mass, kg	8
Maximum effective area exposed to drag, m ²	0.6785
Minimum effective area exposed to drag, m ²	0.0366
Drag coefficient	2.2

3.7.4 Scenario 4: γ Cube-k185 Mission

The only difference with the previous case, was that this mission had an initial altitude 20 kilometers higher than the first case. Again, all other orbital parameters and relative position between the spacecraft in the formation were kept the same.

3.7.4.1 Spacecraft Details and Initial Conditions

After designing the trajectory for the maneuver, the spacecraft requirements obtained for this particular mission, at the mentioned initial altitude, can be seen in Table 8.

Table 8. Spacecraft characteristics for γ Cube-k185 Mission

Parameter	Value
Mass, kg	8
Maximum effective area exposed to drag, m ²	1.3571
Minimum effective area exposed to drag, m ²	0.0549
Drag coefficient	2.2

3.7.5 Scenario 5: λ Cube-k200 Mission

This mission had an initial altitude 35 kilometers higher than the first case, achieving the desired altitude of 200 kilometers. All the orbital parameters and relative position between the spacecraft in the formation were kept the same for simplicity.

3.7.5.1 Spacecraft Details and Initial Conditions

After designing the trajectory for the maneuver, the spacecraft requirements obtained for this particular mission, at the mentioned initial altitude, can be seen in Table 9.

Table 9. Spacecraft characteristics for λ Cube-k200 Mission

Parameter	Value
Mass, kg	8
Maximum effective area exposed to drag, m ²	2.7142
Minimum effective area exposed to drag, m ²	0.1464
Drag coefficient	2.2

3.7.6 A Different γ Cube-k185 Mission: Same Atmospheric Density

This case was a variation of scenario 4, with the only difference that the same atmospheric density was used (for leader and follower) when calculating the atmospheric differential drag magnitude for control purposes. Also, since the spacecraft that is commanding the formation is the follower, its atmospheric information was used.

It has to be mentioned that this behavior does not modify the calculations made, when computing the acceleration disturbances due to atmospheric drag, to propagate the trajectory of the spacecraft. When calculating the trajectory using the nonlinear orbit propagator, each spacecraft uses the corresponding atmospheric density obtained from the Mars Climate Database according to its location. Although, a simplification was made assuming the same atmospheric density for the 60 seconds propagation time in order to reduce the number of queries to the database from the ode45 integrator.

The spacecraft details and initial conditions for this derived mission are the same as those presented in section 3.7.4.1.

3.7.7 Maximum Flight Time

One important part of a mission is its duration in orbit, and using drag plates with big areas considerably decreases the flight time of the spacecraft. To determine what would be the maximum flight time of a spacecraft at certain altitudes, depending on their ballistic coefficient, a simulation was performed using atmospheric density obtained from the MCD. For the study, the spacecraft was considered to fly during one orbit exposing its maximum area to drag, and the subsequent orbit exposing its minimum area, and so on. Four different altitudes were considered, with the five different spacecraft used on previous cases.

Another approach to face the problem was considering a set of generic spacecraft with a range of differential ballistic coefficients, trying to rendezvous from some given initial conditions, and starting at different altitudes. The range of differential ballistic coefficients was from $0.05\text{m}^2/\text{kg}$

to 1.00m²/kg with increments of 0.05m²/kg. The range of altitudes was from 165km to 220km with increments of 5 kilometers. Moreover, the control law used was based on a Lyapunov approach, where the nonlinear system was forced to follow a specific guidance generated based on the ballistic coefficient and the initial altitude. The script would capture the total rendezvous time until the two spacecraft were closer than a given tolerance. Even when this solution will not provide a direct answer to the total flight time, it still will provide useful information about feasibility of performing a rendezvous maneuver based on the differential BC and the initial altitude.

3.7.8 Required Δv for the Maneuver

To answer how much Δv will be saved using differential drag, the differential acceleration among the spacecraft has to be computed for the simulation time. For the whole rendezvous maneuver, the total Δv will be as follows:

$$\Delta v = \int_0^T \text{abs}(a_{Drel}) dt \quad (49)$$

where abs indicate the absolute value of the relative acceleration due to atmospheric drag to account for positive and negative values, depending on which spacecraft is performing the maneuver.

Furthermore, the control loop is executed as a discrete sequence of events with a predefined period, P . The number of control periods per mission, N , is proportional to the mission duration time. Then, the total Δv can be calculated as:

$$\Delta v = \sum_{i=1}^N \int_0^P \text{abs}(a_{Drel_i}) dt \quad (50)$$

Since the commanded attitude of each spacecraft is held constant between control periods, and the atmospheric density is considered to be constant during the same period, Eqn.(50) can be simplified as follows:

$$\Delta v = \sum_{i=1}^N \text{abs}(a_{Drel_i}) \int_0^P dt \quad (51)$$

Finally, the total Δv for the maneuver can be estimated as:

$$\Delta v = \sum_{i=1}^N (a_{Drel_i} P) \quad (52)$$

A negative a_{Drel} corresponds to the leader facing the velocity direction with its drag plates deployed, while the follower has the drag plates retracted. A positive a_{Drel} is when the follower has its drag plates deployed, while the leader has them retracted. A zero a_{Drel} can be obtained when both spacecraft have their plates either deployed or retracted at the same time.

It is possible to calculate the contribution that each spacecraft makes to the required Δv for the maneuver, considering the negative values as for the leader contribution, and positive values as for the follower contribution.

4 RESULTS

Five different scenarios were simulated with the main purpose of collecting enough data to answer the research questions presented in previous chapters. The following sections of this chapter will present the results obtained from those simulations. It is worth mentioning that the results presented may look similar, but the conditions experienced during the maneuvers were very different in each case. Raw data is also presented in Appendices with the output returned by MATLAB® at the end of each simulation.

4.1 Scenario 1: μ Cube-k165

4.1.1 Linear Guidance Trajectory

A guidance trajectory was generated using the equations of relative motion developed by Schweighart and Sedwick, previously presented in Chapter 3. The expected trajectory for the formation is first obtained in the z -plane, and then converted to the x - y plane, as explained in Chapter 3. The linear guidance trajectory was generated to be used by two of the three control strategies. The first strategy forces the nonlinear system to follow directly the guidance trajectory, while the second strategy forces the nonlinear system to follow a reference trajectory that is following the generated guidance. This section presents the linear guidance trajectory obtained for the first mission, μ Cube-k165.

4.1.1.1 Phase Plane

The resulting trajectory in the z_1 - z_2 plane is shown in Figure 8. From the start point, denoted in the plot by the red reference mark, a negative referential drag was applied until the trajectory intersects the optimal trajectory, where the sign of the differential drag was changed. From that point, the trajectory is driven toward a stable relative orbit, obtained when the trajectory intersects the origin of the z_1 - z_2 plane. At that moment, the second phase starts.

Figure 8. Guidance trajectory in the z_1 - z_2 plane for scenario 1

Furthermore, comparing the trajectory obtained on the z_1 - z_2 phase plane for the nonlinear dynamics applying Lyapunov strategy #1, and the guidance trajectory generated, it can be seen in Figure 9 how they differ. In this figure it is evident how the nonlinear terms affect the expected trajectory.

Figure 9. Phase plane with nonlinear trajectory and guidance for μ Cube-k165

It is worth mentioning here that the nonlinear trajectory is following the pre-generated guidance trajectory, thus no differentiation between first stage and second stage can be made.

The same trajectory, but as seen in the z_3 - z_4/d plane is presented in Figure 10. In this plot it is easier to see the second phase, that starts at the point demarked with the circle. During that phase the oscillation of the relative orbit is canceled.

Figure 10. Guidance trajectory in the z_2 - z_4/d plane for scenario 1

As before, the same trajectory obtained when forcing the nonlinear system to follow the guidance trajectory, but in the z_3 - z_4/d plane, is presented in Figure 11. Here it is noticeable how different is the nonlinear trajectory with respect to the linear guidance obtained in Figure 10.

4.1.1.2 x-y Plane

Following, the obtained linear guidance is converted to the x-y plane, which can be seen in Figure 11. In this plot the changes in the sign of the differential drag applied were demarked with a star. These drag sign changes correspond to those made during phase 1 and phase 2 presented before.

Figure 11. Phase plane for the nonlinear trajectory (z_3 - z_4/d)

Figure 12. Guidance trajectory in the x-y plane for scenario 1

To better see where the changes in drag were applied during the second phase in the x-y plane, a zoom was made in the rectangle demarked and presented in Figure 13. As it can be seen, the trajectory does not end at the origin, because the algorithm ends when the tolerance is met.

Figure 13. Guidance trajectory in the x-y plane for scenarios 1 (zoomed)

4.1.2 Lyapunov Control Strategy #1

The Lyapunov control using the first approach, consisting on forcing the nonlinear system to follow the guidance trajectory obtained with the linear model, is presented in Figure 14.

Figure 14. Lyapunov strategy #1 for $\mu\text{Cube-k165}$

Similarly, as when comparing these trajectories in the z_1 - z_2 plane, the nonlinearities are evident when tracking the desired linear guidance trajectory. Moreover, an overshoot occurred when the trajectory was going down in the x_2 axis. The cause of it was the gain chosen for the LQR used to obtain the desired A matrix, which should be tuned depending on the initial altitude of the two spacecraft participating in the formation.

Then, the normalized tracking error can be seen in Figure 15, where the error in kilometers from the desired trajectory over time is presented. Here, the initial point starts exactly where the guidance trajectory starts, thus the error is zero. Then, near the end of the first day the error went very low, moment at which the overshoot occurred.

Figure 15. Normalized tracking error for Lyapunov strategy #1 of μ Cube-k165

It is worth recalling that the tolerance set to consider “rendezvous condition” not only depends on the relative position between the spacecraft, but also in the relative velocity. After this condition is met, the tracking algorithm was stopped, resulting in the time presented in these plots.

Furthermore, the commands sent to control the drag plates of both spacecraft can be seen in Figure 16. Due to the initial conditions, it can be seen that the leader was commanded to have maximum drag during most of the first day. Also, at some point a high frequency drag plates

changes occur, that correspond to the start of the close proximity operations, until achieving the rendezvous condition. It is worth noting that the control cycle is executed every 10 minutes.

Figure 16. Lyapunov control commands for strategy #1

4.1.3 Lyapunov Control Strategy #2

The Lyapunov control using the second strategy, which was following a trajectory generated by a reference linear model that was regulated, is presented in Figure 17. Here, no guidance trajectory is required, so no phase plane is involved in this analysis.

Here, the nonlinear trajectory differs more with the linear reference trajectory, when compared to the previous strategy. In this case, the LQR has more implications than before, because the desired A matrix obtained with it was used to generate the regulated reference trajectory, and used in the Lyapunov controller. Again, tuning the gain used on the Linear Quadratic Regulator can help improving the reference trajectory generated. It is worth noting at this point that, for the purpose of this study, the gain was kept constant for all altitudes in order to determine the implications of this assumption.

Figure 17. Lyapunov strategy #2 for $\mu\text{Cube-k165}$

Furthermore, Figure 18 and Figure 19 show the normalized tracking error and the control commands obtained when applying Lyapunov strategy #2 to the mission $\mu\text{Cube-k165}$.

Figure 18. Normalized tracking error for Lyapunov strategy #2

Figure 19. Lyapunov control commands for strategy #2

A similar result was obtained, where the overshoot was evident in the normalized tracking error, and where the high frequency commands started when the close proximity operations started.

4.1.4 Lyapunov Control Strategy #3

Finally, the application of the third strategy, where the nonlinear system was forced to follow a reference trajectory that is tracking the guidance trajectory, is presented in Figure 20, Figure 21, and Figure 22 for the μ Cube-k165 mission.

In the x-y plane plot, presented in Figure 20, it can be seen how the desired trajectory (i.e. reference trajectory) follows the guidance trajectory. However, the reference trajectory is already generating an overshoot that was “copied” by the nonlinear dynamics. One more time, the LQR gain chosen was the responsible of this overshoot.

Figure 20. Lyapunov strategy #3 for μ Cube-k165

The normalized tracking error plot, Figure 21, shows how the nonlinear trajectory followed very well the reference trajectory at the beginning, but was bigger later because of the generated overshoot. Anyway, the maximum normalized error was always smaller than in the two previous strategies.

Figure 21. Normalized tracking error for Lyapunov strategy #3

Finally, the control commands generated for both spacecraft is presented in Figure 22 below, where it can be seen that it is very similar to the previous cases. However, the number of control command generated over the number of control cycles may be very different. A summary of these results is presented in the next section.

Figure 22. Lyapunov control commands for strategy #3

4.1.5 Summary of Results for μ Cube-k165

When generating the guidance trajectory, which is based on the linear model, the expected final relative position and velocity, and the final expected rendezvous time can be obtained. These results are presented in Table 10. Also, the constant atmospheric density used to generate the guidance is presented in that table.

It is worth mentioning that the guidance trajectory had a bigger tolerance when considering the rendezvous condition, reason why the expected final values may seem larger than the final obtained distances. However, after finishing with the obtained values, the guidance will set the origin as the desired trajectory, making this error negligible.

Table 10. Expected values at the end of scenario 1

Parameter	Value
Expected final position, $\mathbf{x}$ (m)	-0.095724
Expected final position, $\mathbf{y}$ (m)	-41.615532
Expected final velocity, $\dot{\mathbf{x}}$ (m/sec)	-0.000001
Expected final velocity, $\dot{\mathbf{y}}$ (m/sec)	0.000140
Expected rendezvous time	1 day 8 hrs. 5 min
Constant atmospheric density (kg/m ³)	3.5888e-11

Moreover, a summary with all the results obtained from the three strategies for μ Cube-k165 can be seen in Table 11. For simplicity, the numbers for rendezvous time were rounded to the closest hour.

Table 11. Summary of Lyapunov control for the μ Cube-k165

Parameter	Strategy #1	Strategy #2	Strategy #3
Leader final altitude (km)	158.86	157.21	157.62
Final position, $\mathbf{x}$ (m)	-6.653407	9.206999	5.079313
Final position, $\mathbf{y}$ (m)	-3.458631	1.358436	6.511395
Final velocity, $\dot{\mathbf{x}}$ (m/sec)	0.026319	-0.032801	-0.028841
Final velocity, $\dot{\mathbf{y}}$ (m/sec)	0.008463	-0.020702	-0.006603
Rendezvous time	2 days, 9 hrs.	2 days, 11 hrs.	2 days, 20 hrs.
Number of control cycles	342	352	407
Number of switching	117	96	129

4.2 Scenario 2: α Cube-k165

A very similar analysis can be made with all the following scenarios, thus only a summary of the expected results from the guidance, and the results obtained from the three control strategies applied to α Cube-k165 are presented in Table 12 and Table 13 respectively. All the plots obtained for the mission can be seen in Appendix A.

Table 12. Expected values at the end of scenario 2

Parameter	Value
Expected final position, $\mathbf{x}$ (m)	27.201980
Expected final position, $\mathbf{y}$ (m)	-161.022997
Expected final velocity, $\dot{\mathbf{x}}$ (m/sec)	-0.058897
Expected final velocity, $\dot{\mathbf{y}}$ (m/sec)	-0.053375
Expected rendezvous time	16 hrs. 48 min
Constant atmospheric density (kg/m ³)	3.5888e-11

Table 13. Summary of Lyapunov control for the α Cube-k165

Parameter	Strategy #1	Strategy #2	Strategy #3
Leader final altitude (km)	153.20	155.85	157.62
Final position, $\mathbf{x}$ (m)	-8.677710	-6.137511	-4.823749
Final position, $\mathbf{y}$ (m)	-1.076632	-1.359124	0.694004
Final velocity, $\dot{\mathbf{x}}$ (m/sec)	0.040175	0.041024	0.069360
Final velocity, $\dot{\mathbf{y}}$ (m/sec)	0.018738	-0.001329	0.004512
Rendezvous time	1 day 13 hrs.	1 day 2 hrs.	1 day 15 hrs.
Number of control cycles	222	155	236
Number of switching	77	55	100
Mean atm. Density (kg/m ³)	4.3100e-11	4.3100e-11	4.3100e-11

4.3 Scenario 3: β Cube-k175

A summary of the expected results from the guidance, and the results obtained from the three control strategies applied to β Cube-k175 can be seen in Table 14 and Table 15 respectively.

The plots obtained for the mission can be seen in Appendix B.

Table 14. Expected values at the end of scenario 3

Parameter	Value
Expected final position, $\mathbf{x}$ (m)	-0.692823
Expected final position, $\mathbf{y}$ (m)	-1.078131
Expected final velocity, $\dot{\mathbf{x}}$ (m/sec)	-0.000001
Expected final velocity, $\dot{\mathbf{y}}$ (m/sec)	0.001010
Expected rendezvous time	19 hrs. 41 min
Constant atmospheric density (kg/m ³)	1.9742e-11

Table 15. Summary of Lyapunov control for the β Cube-k175

Parameter	Strategy #1	Strategy #2	Strategy #3
Leader final altitude (km)	166.88	170.79	165.64
Final position, $\mathbf{x}$ (m)	7.287855	-1.284184	-8.809386
Final position, $\mathbf{y}$ (m)	-2.069088	-8.850526	4.176981
Final velocity, $\dot{\mathbf{x}}$ (m/sec)	-0.038959	-0.008116	0.012178
Final velocity, $\dot{\mathbf{y}}$ (m/sec)	0.001060	0.016232	0.025980
Rendezvous time	1 day 9 hrs.	1 day 10 hrs.	2 days 2 hrs.
Number of control cycles	199	205	299
Number of switching	56	77	132
Mean atm. Density (kg/m ³)	2.1144e-11	2.1144e-11	2.1144e-11

4.4 Scenario 4: γ Cube-k185

A summary of the expected results from the guidance, and the results obtained from the three control strategies applied to γ Cube-k185 can be seen in Table 16 and Table 17 respectively.

The plots obtained for the mission can be seen in Appendix C.

Table 16. Expected values at the end of scenario 4

Parameter	Value
Expected final position, $\mathbf{x}$ (m)	25.880016
Expected final position, $\mathbf{y}$ (m)	-102.021996
Expected final velocity, $\dot{\mathbf{x}}$ (m/sec)	-0.036022
Expected final velocity, $\dot{\mathbf{y}}$ (m/sec)	-0.050165
Expected rendezvous time	16 hrs. 47 min
Constant atmospheric density (kg/m ³)	1.1396e-11

Table 17. Summary of Lyapunov control for the γ Cube-k185

Parameter	Strategy #1	Strategy #2	Strategy #3
Leader final altitude (km)	175.42	175.59	175.48
Final position, $\mathbf{x}$ (m)	7.953339	2.440972	-4.200407
Final position, $\mathbf{y}$ (m)	-0.147008	8.136063	-4.491792
Final velocity, $\dot{\mathbf{x}}$ (m/sec)	-0.026216	-0.020915	0.039486
Final velocity, $\dot{\mathbf{y}}$ (m/sec)	-0.000994	-0.003871	0.007803
Rendezvous time	1 day 15 hrs.	1 day 7 hrs.	3 days
Number of control cycles	232	189	433
Number of switching	98	65	252
Mean atm. Density (kg/m ³)	1.0800e-11	1.0800e-11	1.0800e-11

4.5 Scenario 5: λ Cube-k200

A summary of the expected results from the guidance, and the results obtained from the three control strategies applied to λ Cube-k200 can be seen in Table 18 and Table 19 respectively. The plots obtained for the mission can be seen in Appendix D.

Table 18. Expected values at the end of scenario 5

Parameter	Value
Expected final position, $\mathbf{x}$ (m)	-272.492318
Expected final position, $\mathbf{y}$ (m)	-138.731142
Expected final velocity, $\dot{\mathbf{x}}$ (m/sec)	-0.066828
Expected final velocity, $\dot{\mathbf{y}}$ (m/sec)	0.523788
Expected rendezvous time	18 hrs. 43 min
Constant atmospheric density (kg/m ³)	5.3169e-12

Table 19. Summary of Lyapunov control for the λ Cube-k200

Parameter	Strategy #1	Strategy #2	Strategy #3
Leader final altitude (km)	189.27	193.01	187.36
Final position, $\mathbf{x}$ (m)	-7.970681	-6.159779	-1.380615
Final position, $\mathbf{y}$ (m)	-4.467939	-5.997449	9.056392
Final velocity, $\dot{\mathbf{x}}$ (m/sec)	0.006483	0.043271	0.018435
Final velocity, $\dot{\mathbf{y}}$ (m/sec)	0.010689	0.001639	-0.004709
Rendezvous time	2 days 11 hrs.	1 day 16 hrs.	3 days 9 hrs.
Number of control cycles	356	244	486
Number of switching	132	52	192
Mean atm. Density (kg/m ³)	4.3278e-12	4.3278e-12	4.3278e-12

4.6 Information Exchanged

The information required by the follower to keep track of the relative dynamics with respect to the leader, was described in Table 20, with an estimated size in bits. Additionally, an overhead to the data was added in order to account the fact that a protocol will be necessary, and other handshaking might be necessary as well. The overhead was estimated to be the same size of the information exchanged per request.

Table 20. Information required to be exchanged per control loop

Information	Size (bits)
Position vector, $\mathbf{r}_x, \mathbf{r}_y, \mathbf{r}_z$ (km)	96 (3x32)
Velocity vector, $\mathbf{v}_x, \mathbf{v}_y, \mathbf{v}_z$ (km/sec)	96 (3x32)
Atmospheric density	32
Status information (panels, attitude, etc.)	32
Overhead (100%)	256
Total	512

For each one of the cases, the exchanged data and frequency was recorded. For this analysis, the best control strategy of each scenario was considered. The communication rate is dependent on the control loop rate, which is the moment when the follower requires to know the current navigation information of the leader. Table 21 summarizes the information exchanged, where the total size is assuming the 100% overhead.

Table 21. Information exchange summary

Case	Control rate	Maneuver duration	Control cycles	Total size (bytes)
μ Cube-k165 #2	300 seconds	2 days 11 hrs.	352	22,528
α Cube-k165 #1	300 seconds	1 day 13 hrs.	222	14,208
β Cube-k175 #1	300 seconds	1 day 9 hrs.	199	12,736
γ Cube-k185 #2	300 seconds	1 day 7 hrs.	189	12,096
λ Cube-k200 #2	300 seconds	1 day 16 hrs.	244	15,616

From this analysis, it is easy to see that the required amount of information at the required rate (i.e. 512 bits over 300 seconds) is not a limiting requirement, and easily achievable.

4.7 Using Same Atmospheric Density

When calculating the differential drag between the spacecraft, an additional case was created to use the leader atmospheric density information instead of the leader provided information, as mentioned before in section 3.7.6. The information obtained with the simulation was summarized in Table 22.

Table 22. Simulation results for the γ Cube-k185-II Mission

Parameter	With same ρ_L & ρ_F	Previous results
Leader final altitude (km)	177.96	175.42
Final position, $\mathbf{x}$ (m)	3.947257	7.953339
Final position, $\mathbf{y}$ (m)	8.827137	-0.147008
Final velocity, $\dot{\mathbf{x}}$ (m/sec)	-0.022835	-0.026216
Final velocity, $\dot{\mathbf{y}}$ (m/sec)	-0.006697	-0.000994
Rendezvous time	1 day 13 hrs.	1 day 15 hrs.
Number of control cycles	220	232
Number of switching	88	98
Mean atm. Density (kg/m ³)	1.0800e-11	1.0800e-11

The results presented shown that, in this particular case, it was even better considering the atmospheric density being the same for the follower and the leader. Although this is not the case all the time, the differences in the number of switching commands required for one or the other case does not present a significant change.

Moreover, since the instrument required to have a precise measurement of the atmospheric density can be very expensive, the necessity of reducing the number of switching commands should be evaluated at mission level.

4.8 Maximum Flight Time

A computation of the maximum flight time depending on the initial altitude and area exposed to atmospheric drag was conducted. The procedure used to collect the information was previously presented on section 3.7.7. The results obtained are shown in Table 23.

From the results, it can be seen that, depending on the ballistic coefficient, it might be impossible for some configurations to stay in orbit the time required to perform a rendezvous, while other configurations will give enough time to do it.

Table 23. Maximum flight time results (in days)

Spacecraft	Ballistic coefficient (m^2/kg)		Orbit altitude (km)			
	max	min	165	175	185	200
$\mu\text{Cube-k165}$	0.0680655	0.010065	5.16	11.63	24.30	69.56
$\alpha\text{Cube-k165}$	0.124399	0.010065	3.69	7.21	14.23	39.79
$\beta\text{Cube-k175}$	0.186588	0.010065	3.01	5.04	10.35	28.22
$\gamma\text{Cube-k185}$	0.373203	0.0150975	2.00	3.28	5.75	15.10
$\lambda\text{Cube-k200}$	0.746405	0.0402600	1.26	2.14	3.33	7.69

A more in-depth analysis was made using general information, with the intention of generating a set of curves, while applying the control strategy #2. These simulations provided information about rendezvous time over differential ballistic coefficient, at different altitudes. After taking data from 120 simulations with increments of 5 kilometers in altitude and $0.05 \text{ m}^2/\text{kg}$ in differential ballistic coefficient, the curves seen in Figure 23 were generated.

Figure 23. Rendezvous time over ballistic coefficient for different altitudes

The fact that makes the time not decreasing monotonically, is due to the overshoot experienced, and the settling time required for each case to achieve the rendezvous condition, that depends on the nonlinearities of the problem.

4.9 Δv due to Differential Drag

The information about the Δv generated due to differential drag maneuvers was recorded on each formation flying simulation, as previously mentioned in section 3.7.8. Again, the values considered were those corresponding to the best control strategy among the results. The data collected are shown in Table 24.

Table 24. Total Δv contribution to the mission, in m/s

Mission	Δv Leader	Δv Follower	Δv Total	Ratio L/F	Ratio
μ Cube-k165 strategy #1	9.5150	6.7378	16.2528	1.41	
α Cube-k165 strategy #2	8.4185	5.8384	14.2569	1.44	
β Cube-k175 strategy #1	6.9173	4.9356	11.8529	1.40	
γ Cube-k185 strategy #2	6.8409	4.5905	11.4314	1.49	
λ Cube-k200 strategy #2	6.6580	4.1814	10.8394	1.59	

It can be seen that the contribution of the leader to the total delta-v was always bigger than the contribution of the follower. The reason was due to the given initial conditions, that were more unfavorable for the leader. In a non-collaborative scenario, this would not be the case, because the leader should have to keep its position as unchanged as possible.

Although missions presented in this study do not provide any other means of propulsion, the concept of “saving” delta-v can be considered as so for missions that can take advantage of the differential drag technique while performing formation flying maneuvers, instead of using fuel.

5 DISCUSSION

The use of differential drag at Low Earth Orbit (LEO) has been widely studied by many authors, trying to obtain a propellant free relative motion control. With the growing interest on Mars, and the growing interest on formation flying missions, an analysis of the combination was proposed as purpose of this thesis.

Many questions were presented during the development of this work (see section 1.6). A simulation environment was set using the state-of-the-art techniques and tools, and the most reliable source of information about the Martian atmosphere, to generate data to help the researcher answering those questions.

Mars is well known for having a thin atmosphere, and with way less gravity than on Earth. But the global question was: Is it possible to use the same techniques we are using on Earth atmosphere, to be applied to Mars?

Data obtained during simulations were presented on chapter 4. From there, a simple first finding is that Mars *has enough dense atmosphere to perform formation flying*, using the same techniques used for Earth atmosphere.

However, it can be seen from the plots that nonlinearities and uncertainties make the reality differ from the linear simulations. One of the most important differences with respect to the linear model used to generate the guidance is that it uses a constant atmospheric density to calculate the trajectory, ignoring the constant fluctuation the atmosphere will suffer during a long term control period (i.e. in the order of days). During the study it was observed that, in some cases, the trajectories generated using a constant density atmosphere influenced negatively in the final trajectory. This problem was already detected and studied by Bevilacqua et al. (2016), where they proposed the use of a neural network for density forecast when generating the guidance trajectory.

Furthermore, three different Lyapunov strategies were analyzed. The conclusions for each one are described below:

Lyapunov strategy #1 was: “force the nonlinear system to follow the pre-generated guidance trajectory”. This strategy was the preferred solution in two of the five cases. The benefit with respect to the other two strategies, was the ratio of the number of switching commands to the total number of control cycles.

Lyapunov strategy #2 was: “force the nonlinear system to follow a linearized model of the problem that is regulated (no guidance trajectory is used)”. This strategy was preferred in the other three cases, where the number of control commands was the lowest. Moreover, in the two cases where strategy #1 was preferred, strategy #2 was close enough to consider it viable too.

Lyapunov strategy #3 was: “force the nonlinear system to follow a linearized model that is following a pre-generated guidance trajectory”. This approach was the worst with respect to performance, having resulted in the highest number of control commands over time.

A summary of the best obtained values for each scenario is presented in Table 25 below, where a combination between the total decay, a total number of switching commands to control cycles ratio, and the rendezvous time was considered.

Table 25. Summary of Lyapunov control for all missions

Parameter	μ Cube-k165 strategy #2	α Cube-k165 strategy #2	β Cube-k175 strategy #1	γ Cube-k185 strategy #2	λ Cube-k200 strategy #2
Leader altitude decay (km)	7.79	9.15	8.12	9.41	6.99
Rendezvous time	2 days 11 hrs.	1 day 2 hrs.	1 day 9 hrs.	1 day 7 hrs.	1 day 16 hrs.
Number of control cycles	352	155	199	189	244
Number of switching	96	55	56	65	52

In other aspect, assuming that the navigation information problem presented in section 1.4.1 can be solved by means of inter-spacecraft communication links, the results from simulations show the amount of data transfer required to be exchanged between the spacecraft. The number of bytes required will depend mostly on formation altitude and ballistic coefficient of each spacecraft. The simulations demonstrate that the amount of data required to be interchanged are not a limitation

to perform autonomous formation flying, with minimum to no man-in-the-loop, due to its low requirements. Anyway, further analysis should be done to determine if the considered 100% overhead for protocol is realistic.

Additionally, when considering the same value of atmospheric density for both spacecraft when calculating the control loop, it can be seen that the difference obtained in rendezvous time is not substantial. Even though in the case presented the result was beneficial, it is not always the case. Nevertheless, if the leader spacecraft is already sending navigation information to the follower, and it has one of the atmospheric density sensors previously discussed, it is worthy to include that information to calculate the differential drag maneuvers in a more precise way.

Also, when the perturbations problem presented in section 1.4.3 is considered, the data obtained from simulations shown that the Schweighart and Sedwick equations are valid to generate reference trajectories, due to the fact that the parameter that varies the most, is also the most difficult to obtain: the atmospheric density. However, the inclusion of J_2 perturbations is of great value to this work, because J_2 harmonics are greater in Mars compared to Earth.

Finally, when talking about the inter-spacecraft communication problem presented in section 1.4.4, it was already mentioned that the solutions proposed in the literature review are promising and considered as valid solutions to this problem. However, it is worth mentioning that aberration correction due to communication delays has to be considered.

Now, the following paragraphs intend to explicitly answer each of the research questions presented before:

1. Answer: The amount of information required to be exchanged between two spacecraft intending to perform a rendezvous maneuver will depend on the control loop period being implemented. However, Table 21 presents a summary of the amount of information exchanged and the frequency of communication between the follower and the leader for the cases developed in this study. It is possible extrapolating that information to calculate the amount of information required by different scenarios.

2. Answer: The differential drag computations required by the control algorithm are sensitive to the value of atmospheric density used for both leader and target. Still, the control algorithm is enough stable to solve the problem anyway, at a cost of slightly more time to find the solution.
3. Answer: The relationship between the area exposed to drag and the atmospheric density can be seen in Table 23, where the flight time was presented. It can be seen that for low Martian orbits there is impossible to have big areas exposed to drag if the objective is stay in orbit, otherwise the spacecraft will decay really fast. Respect to the maximum separation, the algorithm here presented allows using any relative distance between the spacecraft. However, the reader has to take into consideration the assumptions made when obtaining the linear solutions.
4. Answer: The flight time of a spacecraft will be highly impacted by the amount of area exposed to drag. The results presented in Table 23 demonstrate how short can be a mission if the λ Cube-k200 spacecraft exposes its 2.71 m^2 drag panels to the velocity direction. Anyway, the information presented in the mentioned table was filled with the ballistic coefficient of the spacecraft, so the information can be extrapolated to other missions with the same or similar ballistic coefficient. Moreover, the curves presented in Figure 23, can give an idea of a different delta ballistic coefficient to rendezvous time. It is important to note that the rendezvous time is not always decreasing for a given altitude because the control strategy generates an overshoot, which ends up taking more time for the spacecraft to rendezvous.
5. Answer: The total Δv it can be saved by using differential drag as a means of formation control for the four missions analyzed on this study was presented in Table 24.

5.1 Limitations

5.1.1 Collaborative Environment

In this work a collaborative environment was presented, where both the leader and the follower take an active role to achieve the mission goal. In an environment with more than one follower this is not desirable, because the leader has to be a reference for many spacecraft, so it cannot be changing his relative position with respect to only one of them. To solve this, the analysis could have contained a constant drag for the leader, while the follower changes its configuration to have more or less relative drag, generating the resulting positive and negative differential drag values required by the control algorithm.

5.1.2 Mars Climate Database Access

While developing the algorithm, it was found that the access to the MCD was a bottle-neck because it loads several big files of data for every query. The improvement that was the simplest to make was to calculate the future locations of the spacecraft, including J_2 disturbances, but without drag disturbances, and obtaining a block of data from the database. This solution helped to improve the run time by one order of magnitude, while creating the limitation of not obtaining the true real value during the simulation.

5.1.3 Initial Parameters

When calculating the initial relative velocity from the given initial orbital elements of one spacecraft and the relative location of the other, errors are introduced to the initial conditions. These error are mostly because nonlinear terms, nor J_2 disturbances, are considered in the equations used for that purpose. The main impact of these errors will derive in a drift on the x_2 plane after some orbits.

5.1.4 Constant Atmospheric Density Between Simulation Steps

It was already mentioned that a constant atmospheric density is used in between two propagation steps. This might not be a problem if the steps are kept small, but that impacts on the

computational time required to run each simulation. The results presented on this work were produced using a step time of 120 seconds, which at a velocity of approximately 3.2 kilometers per second (based on the altitude being considered on the study), the difference in atmospheric density could be substantial. Even though, no deep analysis was made in order to correlate the difference in atmospheric density and simulation step size, assuming no big impact on the results.

5.1.5 No Attitude Dynamics

It was already mentioned that no attitude dynamics were included in this work. Even when they should not impact the results obtained here, it would be worth considering the real time requirements for a spacecraft having drag plates to act changing its attitude.

5.2 Recommendations for Future Research

5.2.1 Non-Collaborative Environment

A non-collaborative environment is of big interest, because future spacecraft formation missions will be comprised of more than two actors. Moreover, when a single spacecraft wants to follow a desired trajectory to get to a specific location, there is no other spacecraft to interact with. Although that is a particular case of this study, having the possibility of simulating a mission in a non-collaborative environment would help to solve that problem.

5.2.2 Mars Climate Database Access Improvement

A better solution than the workaround implemented for this work, would be creating a MATLAB® component to have a persistent access to a function that is accessing the database. Nevertheless, the time that it would require was prohibitive at the moment of realizing this study.

5.2.3 Improvement to the ODE45 Solver. Better Atmospheric Data

On this study, the atmospheric density was considered constant during simulation steps, as mentioned before. However, a good improvement (having already a better MCD access) will be using calls to ode45 with time-varying parameters, which is the case of atmospheric density. If the

access to the database is good enough, using these calls should improve considerably the simulation results, without impacting too much the performance.

5.2.4 No Minimum-Error Cost Function

The cost function implemented on the Lyapunov approach was such that the error, defined as the difference between the actual state and the desired state, stays as small as possible. Designing a control law with other constraints in mind will open more possibilities to find different trajectories and maybe better solutions to the problem.

5.3 Final Conclusions

Performing autonomous formation flying and proximity operation on a Mars atmosphere is possible, although with some limitations. Probably the most important is that even when the density of the atmosphere seems to be comparable to the values required to perform formation flying on Earth, at the altitudes where it might be feasible on Mars, the atmospheric density changes very fast, becoming dense enough on matter of days to bring the spacecraft down to the surface of the planet. This derives in the requirement of having spacecraft with very low ballistic coefficient, which might not be useful when drag has to be used as a means of control.

Additionally, applying the initial research made by Leonard in 1986 seems to be viable on Mars when implementing the modifications done by Bevilacqua, et al. in 2008, where they started to use the Schweighart and Sedwick equations of motion to describe the relative position of two spacecraft under the influence of J2 perturbations.

Having open access to the available Mars climate databases helped on creating a reliable testing environment, where the simulation for one Martian day took between 30 to 90 seconds to run. Moreover, some of the tools provided by the NAIF group at NASA JPL were extremely valuable when analyzing interplanetary data. Less but not least, MATLAB® has provided an excellent flexible environment to create the simulations.

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APPENDICES

APPENDIX A: ADDITIONAL RESULTS FOR SCENARIO 2

This section presents all the plots obtained from the simulations ran for the α Cube-k165 mission. Moreover, the outputs obtained on the MATLAB® console are provided.

1. Linear Guidance Trajectory

The following figure shows the trajectory obtained when executing the algorithm for guidance trajectory generation, based on the linearized relative motion equations.

Figure 24. Linear guidance trajectory for α Cube-k165 (x_1 - x_2 plane)

2. Lyapunov Control Strategy

The following figures show the system response to the control imparted by the three Lyapunov strategies described for this study. For each strategy, three plots are presented, representing the trajectory of the nonlinear system, the normalized tracking error, and the control commands generated by the controller.

Figure 25. Nonlinear trajectory when applying Lyapunov strategy #1

Figure 26. Normalized tracking error for Lyapunov strategy #1 of $\alpha\text{Cube-k165}$

Figure 27. Lyapunov control commands generated for strategy #1

Figure 28. Nonlinear trajectory when applying Lyapunov strategy #2

Figure 29. Normalized tracking error for Lyapunov strategy #2 of α Cube-k165

Figure 30. Lyapunov control commands generated for strategy #2

Figure 31. Nonlinear trajectory when applying Lyapunov strategy #3

Figure 32. Normalized tracking error for Lyapunov strategy #3 of α Cube-k165

Figure 33. Lyapunov control commands generated for strategy #3

3. Phase Plane Plots

Figure 34. Phase plane trajectory for strategy #2 of α Cube-k165 (z_1 - z_2)

Figure 35. Phase plane trajectory for Lyapunov strategy #2 (z_3 - z_4/d)

4. Raw Outputs from MATLAB®

The output received when the script that generates the guidance trajectory was executed for the α Cube-k165 mission, is presented below:

```
=====
Formation Flying guidance for Mars
-----
Mission name:          alphaCube-k165
Total rendezvous time: 0 days, 16 hours, 48 minutes, and 49 seconds
-----
Leader initial altitude:          165.00 [km]
Maximum area:                     0.4523584 [m^2]
Minimum area:                     0.0366000 [m^2]
Delta BC max:                     0.103940 [m^2/kg]
Constant density value:           3.5888e-11 [kg/m^3]
Final relative pos. in x:         27.201980 [m]
Final relative pos. in y:        -161.022997 [m]
Final relative vel. in x:         -0.058897 [m/s]
Final relative vel. in y:         -0.053375 [m/s]
Final distance to target:         163.3045 [m]
=====
```

The output received when the script that generates the control strategy #1 was executed for the α Cube-k165 mission, is presented below:

```
=====
Formation Flying mission on Mars
-----
Mission name:          alphaCube-k165
Total rendezvous time: 1 days, 12 hours, 56 minutes, and 0 seconds
Number of control cycles: 222
Number of control switch: 77
-----
Leader initial altitude:          165.00 [km]
Leader final altitude:            153.20 [km]
Maximum area:                    0.4523584 [m^2]
Minimum area:                    0.0366000 [m^2]
Delta BC max:                    0.103940 [m^2/kg]
Mean density leader:              4.3100e-11 [kg/m^3]
Mean density follower:            4.0067e-11 [kg/m^3]
Final relative pos. in x:         -8.677710 [m]
Final relative pos. in y:        -1.076632 [m]
Final relative vel. in x:         0.040175 [m/s]
Final relative vel. in y:         0.018738 [m/s]
Final distance to target:         8.7442 [m]
Delta-v total:                    20.7018 [m/s]
Delta-v follower:                 9.0746 [m/s]
Delta-v leader:                   11.6272 [m/s]
-----
Simulation time step:             120
Control algorithm time step:      600
Control strategy ID:              2
Tolerance in position:            10 [m]
Tolerance in velocity:            0.1 [m/s]
Tolerance met:                    1
Total run time:                   48.735 seconds
=====
```

The output received when the script that generates the control strategy #2 was executed for the α Cube-k165 mission, is presented below:

```
=====
Formation Flying mission on Mars
-----
Mission name:          alphaCube-k165
Total rendezvous time: 1 days, 1 hours, 48 minutes, and 0 seconds
Number of control cycles: 155
Number of control switch: 55
-----
Leader initial altitude:          165.00 [km]
Leader final altitude:            155.85 [km]
Maximum area:                    0.4523584 [m^2]
Minimum area:                    0.0366000 [m^2]
Delta BC max:                    0.103940 [m^2/kg]
Mean density leader:              4.3100e-11 [kg/m^3]
Mean density follower:            4.0067e-11 [kg/m^3]
Final relative pos. in x:         -6.137511 [m]
Final relative pos. in y:        -1.359124 [m]
Final relative vel. in x:         0.041024 [m/s]
Final relative vel. in y:        -0.001329 [m/s]
Final distance to target:         6.2862 [m]
Delta-v total:                    14.2569 [m/s]
Delta-v follower:                 5.8384 [m/s]
Delta-v leader:                   8.4185 [m/s]
-----
Simulation time step:             120
Control algorithm time step:      600
Control strategy ID:              3
Tolerance in position:            10 [m]
Tolerance in velocity:            0.1 [m/s]
Tolerance met:                    1
Total run time:                   34.9665 seconds
=====
```

The output received when the script that generates the control strategy #3 was executed for the α Cube-k165 mission, is presented below:

```
=====
Formation Flying mission on Mars
-----
Mission name:          alphaCube-k165
Total rendezvous time: 1 days, 15 hours, 20 minutes, and 0 seconds
Number of control cycles: 236
Number of control switch: 100
-----
Leader initial altitude:          165.00 [km]
Leader final altitude:            157.62 [km]
Maximum area:                    0.4523584 [m^2]
Minimum area:                    0.0366000 [m^2]
Delta BC max:                    0.103940 [m^2/kg]
Mean density leader:              4.3100e-11 [kg/m^3]
Mean density follower:            4.0067e-11 [kg/m^3]
Final relative pos. in x:         -4.823749 [m]
Final relative pos. in y:         0.694004 [m]
Final relative vel. in x:         0.069360 [m/s]
Final relative vel. in y:         0.004512 [m/s]
Final distance to target:         4.8734 [m]
Delta-v total:                    22.4417 [m/s]
Delta-v follower:                 9.8916 [m/s]
Delta-v leader:                   12.5501 [m/s]
-----
Simulation time step:             120
Control algorithm time step:      600
Control strategy ID:              4
Tolerance in position:            10 [m]
Tolerance in velocity:            0.1 [m/s]
Tolerance met:                    1
Total run time:                   52.1492 seconds
=====
```

APPENDIX B: ADDITIONAL RESULTS FOR SCENARIO 3

This section presents all the plots obtained from the simulations ran for the β Cube-k175 mission. Moreover, the outputs obtained on the MATLAB® console are provided.

1. Linear Guidance Trajectory

The following figure shows the trajectory obtained when executing the algorithm for guidance trajectory generation, based on the linearized relative motion equations.

Figure 36. Linear guidance trajectory for β Cube-k175 (x_1 - x_2 plane)

2. Lyapunov Control Strategy

The following figures show the system response to the control imparted by the three Lyapunov strategies described for this study. For each strategy, three plots are presented, representing the trajectory of the nonlinear system, the normalized tracking error, and the control commands generated by the controller.

Figure 37. Nonlinear trajectory when applying Lyapunov strategy #1

Figure 38. Normalized tracking error for Lyapunov strategy #1 of β Cube-k175

Figure 39. Lyapunov control commands generated for strategy #1

Figure 40. Nonlinear trajectory when applying Lyapunov strategy #2

Figure 41. Normalized tracking error for Lyapunov strategy #2 of β Cube-k175

Figure 42. Lyapunov control commands generated for strategy #2

Figure 43. Nonlinear trajectory when applying Lyapunov strategy #3

Figure 44. Normalized tracking error for Lyapunov strategy #3 of β Cube-k175

Figure 45. Lyapunov control commands generated for strategy #3

3. Phase Plane Plots

Figure 46. Phase plane trajectory for strategy #2 of β Cube-k175

Figure 47. Phase plane trajectory for strategy #2 (z_3 - z_4/d)

4. Raw Outputs from MATLAB®

The output received when the script that generates the guidance trajectory was executed for the α Cube-k165 mission, is presented below:

```
=====
Formation Flying guidance for Mars
-----
Mission name:          betaCube-k175
Total rendezvous time: 0 days, 19 hours, 41 minutes, and 13.8739 seconds
-----
Leader initial altitude:          175.00 [km]
Maximum area:                     0.6785375 [m^2]
Minimum area:                     0.0366000 [m^2]
Delta BC max:                    0.160484 [m^2/kg]
Constant density value:          1.9742e-11 [kg/m^3]
Final relative pos. in x:        -0.692823 [m]
Final relative pos. in y:        -1.078131 [m]
Final relative vel. in x:        -0.000000 [m/s]
Final relative vel. in y:        0.001010 [m/s]
Final distance to target:        1.2816 [m]
=====
```

The output received when the script that generates the control strategy #1 was executed for the β Cube-k175 mission, is presented below:

```
=====
Formation Flying mission on Mars
-----
Mission name:          betaCube-k175
Total rendezvous time: 1 days, 9 hours, 2 minutes, and 0 seconds
Number of control cycles: 199
Number of control switch: 56
-----
Leader initial altitude:          175.00 [km]
Leader final altitude:            166.88 [km]
Maximum area:                    0.6785375 [m^2]
Minimum area:                    0.0366000 [m^2]
Delta BC max:                    0.160484 [m^2/kg]
Mean density leader:              2.1144e-11 [kg/m^3]
Mean density follower:            1.9755e-11 [kg/m^3]
Final relative pos. in x:         7.287855 [m]
Final relative pos. in y:        -2.069088 [m]
Final relative vel. in x:        -0.038959 [m/s]
Final relative vel. in y:         0.001060 [m/s]
Final distance to target:         7.5759 [m]
Delta-v total:                    11.8529 [m/s]
Delta-v follower:                 4.9356 [m/s]
Delta-v leader:                   6.9173 [m/s]
-----
Simulation time step:             120
Control algorithm time step:      600
Control strategy ID:              2
Tolerance in position:            10 [m]
Tolerance in velocity:            0.1 [m/s]
Tolerance met:                    1
Total run time:                   46.2122 seconds
=====
```

The output received when the script that generates the control strategy #2 was executed for the β Cube-k175 mission, is presented below:

```
=====
Formation Flying mission on Mars
-----
Mission name:          betaCube-k175
Total rendezvous time: 1 days, 10 hours, 6 minutes, and 0 seconds
Number of control cycles: 205
Number of control switch: 77
-----
Leader initial altitude:          175.00 [km]
Leader final altitude:            170.79 [km]
Maximum area:                    0.6785375 [m^2]
Minimum area:                    0.0366000 [m^2]
Delta BC max:                    0.160484 [m^2/kg]
Mean density leader:              2.1144e-11 [kg/m^3]
Mean density follower:            1.9755e-11 [kg/m^3]
Final relative pos. in x:         -1.284184 [m]
Final relative pos. in y:         -8.850526 [m]
Final relative vel. in x:         -0.008116 [m/s]
Final relative vel. in y:         0.016232 [m/s]
Final distance to target:         8.9432 [m]
Delta-v total:                    12.2775 [m/s]
Delta-v follower:                 4.9669 [m/s]
Delta-v leader:                   7.3107 [m/s]
-----
Simulation time step:             120
Control algorithm time step:      600
Control strategy ID:              3
Tolerance in position:            10 [m]
Tolerance in velocity:            0.1 [m/s]
Tolerance met:                    1
Total run time:                   52.6391 seconds
=====
```

The output received when the script that generates the control strategy #3 was executed for the β Cube-k175 mission, is presented below:

```
=====
Formation Flying mission on Mars
-----
Mission name:          betaCube-k175
Total rendezvous time: 2 days, 1 hours, 46 minutes, and 0 seconds
Number of control cycles: 299
Number of control switch: 132
-----
Leader initial altitude:          175.00 [km]
Leader final altitude:            165.64 [km]
Maximum area:                     0.6785375 [m^2]
Minimum area:                     0.0366000 [m^2]
Delta BC max:                     0.160484 [m^2/kg]
Mean density leader:              2.1144e-11 [kg/m^3]
Mean density follower:            1.9755e-11 [kg/m^3]
Final relative pos. in x:         -8.809386 [m]
Final relative pos. in y:         4.176981 [m]
Final relative vel. in x:         0.012178 [m/s]
Final relative vel. in y:         0.025980 [m/s]
Final distance to target:         9.7495 [m]
Delta-v total:                    20.5319 [m/s]
Delta-v follower:                 8.9408 [m/s]
Delta-v leader:                   11.5911 [m/s]
-----
Simulation time step:             120
Control algorithm time step:      600
Control strategy ID:              4
Tolerance in position:            10 [m]
Tolerance in velocity:            0.1 [m/s]
Tolerance met:                    1
Total run time:                   71.5835 seconds
=====
```

APPENDIX C: ADDITIONAL RESULTS FOR SCENARIO 4

This section presents all the plots obtained from the simulations ran for the γ Cube-k185 mission. Moreover, the outputs obtained on the MATLAB® console are provided.

1. Linear Guidance Trajectory

The following figure shows the trajectory obtained when executing the algorithm for guidance trajectory generation, based on the linearized relative motion equations.

Figure 48. Linear guidance trajectory for γ Cube-k185 (x_1 - x_2 plane)

2. Lyapunov Control Strategy

The following figures show the system response to the control imparted by the three Lyapunov strategies described for this study. For each strategy, three plots are presented, representing the trajectory of the nonlinear system, the normalized tracking error, and the control commands generated by the controller.

Figure 49. Nonlinear trajectory when applying Lyapunov strategy #1

Figure 50. Normalized tracking error for Lyapunov strategy #1 of γ Cube-k185

Figure 51. Lyapunov control commands generated for strategy #1

Figure 52. Nonlinear trajectory when applying Lyapunov strategy #2

Figure 53. Normalized tracking error for Lyapunov strategy #2 of γ Cube-k185

Figure 54. Lyapunov control commands generated for strategy #2

Figure 55. Nonlinear trajectory when applying Lyapunov strategy #3

Figure 56. Normalized tracking error for Lyapunov strategy #3 of γ Cube-k185

Figure 57. Lyapunov control commands generated for strategy #3

3. Phase Plane Plots

Figure 58. Phase plane trajectory for strategy #2 of γ Cube-k185

Figure 59. Phase plane trajectory for strategy #2 (z_3 - z_4/d)

4. Raw Outputs from MATLAB®

The output received when the script that generates the guidance trajectory was executed for the α Cube-k165 mission, is presented below:

```
=====
Formation Flying guidance for Mars
-----
Mission name:          gammaCube-k185
Total rendezvous time: 0 days, 16 hours, 47 minutes, and 25.6297 seconds
-----
Leader initial altitude:          185.00 [km]
Maximum area:                    1.3570751 [m^2]
Minimum area:                    0.0549000 [m^2]
Delta BC max:                   0.325544 [m^2/kg]
Constant density value:         1.1396e-11 [kg/m^3]
Final relative pos. in x:        25.880016 [m]
Final relative pos. in y:        -102.021996 [m]
Final relative vel. in x:        -0.036022 [m/s]
Final relative vel. in y:        -0.050165 [m/s]
Final distance to target:        105.2533 [m]
=====
```

The output received when the script that generates the control strategy #1 was executed for the γ Cube-k185 mission, is presented below:

```
=====
Formation Flying mission on Mars
-----
Mission name:          gammaCube-k185
Total rendezvous time: 1 days, 14 hours, 40 minutes, and 0 seconds
Number of control cycles: 232
Number of control switch: 98
-----
Leader initial altitude:          185.00 [km]
Leader final altitude:            175.42 [km]
Maximum area:                    1.3570751 [m^2]
Minimum area:                    0.0549000 [m^2]
Delta BC max:                    0.325544 [m^2/kg]
Mean density leader:              1.0800e-11 [kg/m^3]
Mean density follower:            1.0151e-11 [kg/m^3]
Final relative pos. in x:         7.953339 [m]
Final relative pos. in y:        -0.147008 [m]
Final relative vel. in x:        -0.026216 [m/s]
Final relative vel. in y:        -0.000994 [m/s]
Final distance to target:         7.9547 [m]
Delta-v total:                    13.4371 [m/s]
Delta-v follower:                 5.5929 [m/s]
Delta-v leader:                   7.8442 [m/s]
-----
Simulation time step:             120
Control algorithm time step:     600
Control strategy ID:              2
Tolerance in position:            10 [m]
Tolerance in velocity:            0.1 [m/s]
Tolerance met:                    1
Total run time:                   54.9972 seconds
=====
```

The output received when the script that generates the control strategy #2 was executed for the γ Cube-k185 mission, is presented below:

```
=====
Formation Flying mission on Mars
-----
Mission name:          gammaCube-k185
Total rendezvous time: 1 days, 7 hours, 30 minutes, and 0 seconds
Number of control cycles: 189
Number of control switch: 65
-----
Leader initial altitude:          185.00 [km]
Leader final altitude:            175.59 [km]
Maximum area:                    1.3570751 [m^2]
Minimum area:                    0.0549000 [m^2]
Delta BC max:                    0.325544 [m^2/kg]
Mean density leader:              1.0800e-11 [kg/m^3]
Mean density follower:            1.0151e-11 [kg/m^3]
Final relative pos. in x:         2.440972 [m]
Final relative pos. in y:         8.136063 [m]
Final relative vel. in x:         -0.020915 [m/s]
Final relative vel. in y:         -0.003871 [m/s]
Final distance to target:         8.4943 [m]
Delta-v total:                    11.4314 [m/s]
Delta-v follower:                 4.5905 [m/s]
Delta-v leader:                   6.8409 [m/s]
-----
Simulation time step:             120
Control algorithm time step:      600
Control strategy ID:              3
Tolerance in position:            10 [m]
Tolerance in velocity:            0.1 [m/s]
Tolerance met:                    1
Total run time:                   44.2825 seconds
=====
```


The output received when the script that generates the control strategy #3 was executed for the λ Cube-k185 mission, is presented below:

```
=====
Formation Flying mission on Mars
-----
Mission name:          gammaCube-k185
Total rendezvous time: 3 days, 0 hours, 2 minutes, and 0 seconds
Number of control cycles: 433
Number of control switch: 252
-----
Leader initial altitude:          185.00 [km]
Leader final altitude:            175.48 [km]
Maximum area:                    1.3570751 [m^2]
Minimum area:                    0.0549000 [m^2]
Delta BC max:                    0.325544 [m^2/kg]
Mean density leader:              1.0800e-11 [kg/m^3]
Mean density follower:            1.0151e-11 [kg/m^3]
Final relative pos. in x:         -4.200407 [m]
Final relative pos. in y:         -4.491792 [m]
Final relative vel. in x:         0.039486 [m/s]
Final relative vel. in y:         0.007803 [m/s]
Final distance to target:         6.1498 [m]
Delta-v total:                    30.4436 [m/s]
Delta-v follower:                 14.0472 [m/s]
Delta-v leader:                   16.3964 [m/s]
-----
Simulation time step:             120
Control algorithm time step:      600
Control strategy ID:              4
Tolerance in position:            10 [m]
Tolerance in velocity:            0.1 [m/s]
Tolerance met:                    1
Total run time:                   103.573 seconds
-----
Rendezvous time:                  72.0333 Earth hours
=====
```

APPENDIX D: ADDITIONAL RESULTS FOR SCENARIO 5

This section presents all the plots obtained from the simulations ran for the λ Cube-k200 mission. Moreover, the outputs obtained on the MATLAB® console are provided.

1. Linear Guidance Trajectory

The following figure shows the trajectory obtained when executing the algorithm for guidance trajectory generation, based on the linearized relative motion equations.

Figure 60. Linear guidance trajectory for λ Cube-k200 (x_1 - x_2 plane)

2. Lyapunov Control Strategy

The following figures show the system response to the control imparted by the three Lyapunov strategies described for this study. For each strategy, three plots are presented, representing the trajectory of the nonlinear system, the normalized tracking error, and the control commands generated by the controller.

Figure 61. Nonlinear trajectory when applying Lyapunov strategy #1

Figure 62. Normalized tracking error for Lyapunov strategy #1 of $\lambda\text{Cube-k200}$

Figure 63. Lyapunov control commands generated for strategy #1

Figure 64. Nonlinear trajectory when applying Lyapunov strategy #2

Figure 65. Normalized tracking error for Lyapunov strategy #2 of λ Cube-k200

Figure 66. Lyapunov control commands generated for strategy #2

Figure 67. Nonlinear trajectory when applying Lyapunov strategy #3

Figure 68. Normalized tracking error for Lyapunov strategy #3 of λ Cube-k200

Figure 69. Lyapunov control commands generated for strategy #3

3. Phase Plane Plots

Figure 70. Phase plane trajectory for strategy #2 of γ Cube-k185

Figure 71. Phase plane trajectory for strategy #2 (z_3 - z_4/d)

4. Raw Outputs from MATLAB®

The output received when the script that generates the guidance trajectory was executed for the α Cube-k165 mission, is presented below:

```
=====
Formation Flying guidance for Mars
-----
Mission name:          lambdaCube-k200
Total rendezvous time: 0 days, 18 hours, 43 minutes, and 28.0907 seconds
-----
Leader initial altitude:          200.00 [km]
Maximum area:                     2.7141502 [m^2]
Minimum area:                     0.1464000 [m^2]
Delta BC max:                    0.641938 [m^2/kg]
Constant density value:          5.3169e-12 [kg/m^3]
Final relative pos. in x:        -272.492318 [m]
Final relative pos. in y:        -138.731142 [m]
Final relative vel. in x:        -0.066828 [m/s]
Final relative vel. in y:        0.523788 [m/s]
Final distance to target:        305.7751 [m]
=====
```


The output received when the script that generates the control strategy #1 was executed for the λ Cube-k200 mission, is presented below:

```
=====
Formation Flying mission on Mars
-----
Mission name:          lambdaCube-k200
Total rendezvous time: 2 days, 11 hours, 12 minutes, and 0 seconds
Number of control cycles: 356
Number of control switch: 132
-----
Leader initial altitude:          200.00 [km]
Leader final altitude:            189.27 [km]
Maximum area:                    2.7141502 [m^2]
Minimum area:                    0.1464000 [m^2]
Delta BC max:                    0.641938 [m^2/kg]
Mean density leader:              4.3278e-12 [kg/m^3]
Mean density follower:            4.0910e-12 [kg/m^3]
Final relative pos. in x:         -7.970681 [m]
Final relative pos. in y:         -4.467939 [m]
Final relative vel. in x:         0.006483 [m/s]
Final relative vel. in y:         0.010689 [m/s]
Final distance to target:         9.1375 [m]
Delta-v total:                    17.5954 [m/s]
Delta-v follower:                 7.3483 [m/s]
Delta-v leader:                   10.2471 [m/s]
-----
Simulation time step:             120
Control algorithm time step:     600
Control strategy ID:              2
Tolerance in position:            10 [m]
Tolerance in velocity:            0.1 [m/s]
Tolerance met:                    1
Total run time:                   81.0281 seconds
=====
```

The output received when the script that generates the control strategy #2 was executed for the λ Cube-k200 mission, is presented below:

```
=====
Formation Flying mission on Mars
-----
Mission name:          lambdaCube-k200
Total rendezvous time: 1 days, 16 hours, 32 minutes, and 0 seconds
Number of control cycles: 244
Number of control switch: 52
-----
Leader initial altitude:          200.00 [km]
Leader final altitude:            193.01 [km]
Maximum area:                    2.7141502 [m^2]
Minimum area:                    0.1464000 [m^2]
Delta BC max:                    0.641938 [m^2/kg]
Mean density leader:              4.3278e-12 [kg/m^3]
Mean density follower:            4.0910e-12 [kg/m^3]
Final relative pos. in x:         -6.159779 [m]
Final relative pos. in y:         -5.997449 [m]
Final relative vel. in x:         0.043271 [m/s]
Final relative vel. in y:         0.001639 [m/s]
Final distance to target:         8.5972 [m]
Delta-v total:                    10.8394 [m/s]
Delta-v follower:                 4.1814 [m/s]
Delta-v leader:                   6.6580 [m/s]
-----
Simulation time step:             120
Control algorithm time step:      600
Control strategy ID:              3
Tolerance in position:            10 [m]
Tolerance in velocity:            0.1 [m/s]
Tolerance met:                    1
Total run time:                   57.5279 seconds
=====
```

The output received when the script that generates the control strategy #3 was executed for the λ Cube-k200 mission, is presented below:

```
=====
Formation Flying mission on Mars
-----
Mission name:          lambdaCube-k200
Total rendezvous time: 3 days, 8 hours, 52 minutes, and 0 seconds
Number of control cycles: 486
Number of control switch: 192
-----
Leader initial altitude:          200.00 [km]
Leader final altitude:            187.36 [km]
Maximum area:                    2.7141502 [m^2]
Minimum area:                    0.1464000 [m^2]
Delta BC max:                    0.641938 [m^2/kg]
Mean density leader:              4.3278e-12 [kg/m^3]
Mean density follower:            4.0910e-12 [kg/m^3]
Final relative pos. in x:         -1.380615 [m]
Final relative pos. in y:          9.056392 [m]
Final relative vel. in x:          0.018435 [m/s]
Final relative vel. in y:         -0.004709 [m/s]
Final distance to target:          9.1610 [m]
Delta-v total:                    26.2527 [m/s]
Delta-v follower:                 11.8775 [m/s]
Delta-v leader:                   14.3752 [m/s]
-----
Simulation time step:             120
Control algorithm time step:      600
Control strategy ID:              4
Tolerance in position:            10 [m]
Tolerance in velocity:            0.1 [m/s]
Tolerance met:                    1
Total run time:                   111.127 seconds
=====
```