NUMERICAL DEVELOPMENT AND OPTIMIZATION OF A PHASE CHANGE
MATERIAL SOLAR THERMAL STORAGE UNIT SYSTEM MODEL

A Thesis
presented to
the Faculty of California Polytechnic State University,
San Luis Obispo

In Partial Fulfillment
of the Requirements for the Degree
Master of Science in Mechanical Engineering

by
Jared Tower
March 2016

© 2016

Jared Tower

ALL RIGHTS RESERVED

COMMITTEE MEMBERSHIP

TITLE: Numerical Development and Optimization of a
Phase Change Material Solar Thermal Storage Unit
System Model

AUTHOR: Jared Tower

DATE SUBMITTED: March 2016

COMMITTEE CHAIR: Kim Shollenberger, Ph.D.
Professor of Mechanical Engineering

COMMITTEE MEMBER: Christopher Pascual, Ph.D.
Professor of Mechanical Engineering

COMMITTEE MEMBER: Scott Patton, Ph.D.
Associate Professor of Mechanical Engineering

ABSTRACT

Numerical Development and Optimization of a Phase Change Material Solar Thermal Storage Unit System Model

Jared Tower

In this work, the feasibility of a phase change material (PCM) thermal storage unit is determined, and the design is then optimized to minimize the time it takes for the PCM to completely melt. The PCM provides a large energy density due to its characteristically large heat of fusion, but has a low thermal conductivity, leading to slow transient responses. Thus, a heatsink is employed to increase the melt rate. The heatsink is analyzed using three different methods to determine which method provides the best results in the least amount of time. First, SolidWorks' simulation tool with a variable specific heat model is used to perform a thermal finite-element analysis (FEA); second, Fluent is used to perform a computational fluid dynamic (CFD) analysis with the Boussinesq approximation; third, Fluent is used to perform a CFD analysis accounting for the variable specific heat of PCM. The results are compared and used to suggest an analysis technique when working with PCM, as well as the dimensions of a heatsink to be used in thermal storage applications with PCM. A sample solar thermal system is then presented, utilizing the heatsink that was developed. Ultimately, the Fluent CFD analysis, taking into account the variable density of the PCM, was found to provide the best results, but took on average 2.5 days of computational time to solve. The SolidWorks FEA model was found to provide accurate results for conduction dominated cases with the simulation solving in under 30 minutes. It is found that a variable specific heat finite-element model can quickly provide accurate results when analyzing a conduction dominated melting process.

TABLE OF CONTENTS

	Page
LIST OF TABLES	viii
LIST OF FIGURES	x
CHAPTER	
1. INTRODUCTION	1
1.1 Relevant Research and Technology.....	3
1.1.1 Solar Hot Water	3
1.1.2 Solar Cooking	6
1.1.3 Photovoltaics.....	7
1.1.4 Thermal Storage.....	8
1.1.4.1 Phase Change Material	9
1.1.4.2 Phase Change Material with Metal Foams	16
1.1.4.3 Hot Water as a Thermal Storage System	16
1.1.5 External PCM Heat Transfer Enhancement Methods.....	18
1.1.5.1 Heat Pipes	18
1.1.5.1.1 Standard Heat Pipes	19
1.1.5.1.2 Loop Heat Pipes	20
1.1.5.2 Fins.....	22
1.2 Objective.....	24
2. SYSTEM REQUIREMENTS AND FEASIBILITY	26
2.1 Solar House Hot Water Load.....	26
2.2 Available Solar Resource.....	30

3. PHASE CHANGE MATERIAL NUMERICAL ANALYSIS	33
3.1 Variable Specific Heat Model.....	33
3.2 SolidWorks Simulation.....	44
3.2.1 Mesh Convergence	47
3.2.2 Time Step Convergence.....	49
3.2.3 SolidWorks Transient Analysis	50
3.3 Computational Fluid Dynamic Validation.....	55
3.3.1 Computational Procedure	57
3.3.2 Fluent Simulation Set-Up	60
3.3.3 Results of Replication Study.....	62
3.3.4 Validation Conclusion	64
3.4 Boussinesq Cases	64
3.4.1 Boussinesq Theory.....	64
3.4.2 Boussinesq Case Set-Up	65
3.4.3 Mesh Convergence	67
3.4.4 Time Step Convergence.....	69
3.4.5 Results.....	71
3.5 Variable Density Fluent Cases.....	75
3.6 Comparison of Results for all Cases.....	78
4. SYSTEM DESIGN	84
4.1 Solar Thermal Loop Design.....	85
4.1.1 Pump (P1)	85
4.1.2 Expansion Tank	85

4.1.3 Flat Plate Collector	86
4.1.4 Phase Change Material Thermal Storage Unit (PCM)	86
4.1.5 Solar Hot Water Storage Tank (Hot Water Storage)	86
4.1.6 Scheffler Concentrator	87
4.2 Thermal Storage Unit Design	88
4.2.1 Solar Radiation Receiver (1).....	89
4.2.2 Phase Change Material (2).....	89
4.2.3 Solar Radiation Receiver Plate (3).....	89
4.2.4 Copper Heat Exchanger Piping (4).....	89
4.3 Heating Capabilities.....	90
5. CONCLUSION AND SUMMARY	92
REFERENCES	94
APPENDICES	
A: Engineering Equation Solver (EES) Code Used in Thesis	95
A1 Required Water from Hot Water Tank EES Code	95
A2 Clear Sky Model EES Code.....	97
B: Solar Decathlon Competition Solar Data for a Typical Year	99
C: System Thermal and Fluid Analysis	100
C1 List of Variables	100
C2 Assumptions	101
C3 System of Equations.....	102
C4 Results	106
C5 EES Code	108

LIST OF TABLES

Table	Page
1.1.1. Tabulated Results of Thermograms (Sotomayor, 2014).....	10
1.1.2. Melting Temperature and Latent Heat of Wax Blends with Copper (Molefi, 2009).....	12
1.1.3. Melting Temperature and Latent Heats of PCM Blends with Graphite Particles (Mhike, 2012).....	14
1.1.4. Thermal Properties of Sasol H1 Wax and HDPE Mixtures.....	15
1.1.5. Energy of Water and 50/50 HDPE/H1 Wax Blend	17
1.1.6. Heat Pipe Operating Materials (Thermacore, Inc).....	18
2.1.1. Summary of Shower Energy Balance Results	27
2.2.1. Clear-Sky Model Predictions	31
2.2.2. Average Values for Hourly Direct Normal Radiation and Total Energy on October 17 th in Irvine, California (John Wayne Airport, NREL Solar Database)	32
3.1.1. Summary of Egolf and Manz Equation Variables (Fisher)	34
3.1.2. PCM Properties from Nayak, et al. (2006)	37
3.1.3. Summary of Tau Value for Cases Analyzed in Solid Works	38
3.1.4. Stored Energy Results at 3600 Seconds.....	40
3.2.1. Sasol H1 Wax Material Properties.....	46
3.2.2. Summary of Meshes Analyzed	48
3.2.3. Summary of Time Step Convergence Study.....	49
3.2.4. Summary of Cases Evaluated	50

3.3.1. Wax Properties	61
3.3.2. Glass Properties	61
3.4.1. Copper Heatsink Material Properties	66
3.4.2. Sasol H1 Wax Material Properties.....	66
3.4.3. Mesh Sizes Considered	67
3.4.4. Time Step Sizes Considered	69
4.3.1. Thermal Storage Unit Performance	90
C1. Equation 3 Coefficients from Grundfos Pump Curves	104
C2. Pump Selection Results	106
C3. Results from Various Inlet Temperatures	106

LIST OF FIGURES

Figure	Page
1.1.1. 2005 Cal Poly Solar House	1
1.1.2. Flat Plate Collectors	3
1.1.3. Concentrating Solar Collector.....	4
1.1.4. Batch Collector	4
1.1.5. Cooking with a Scheffler Concentrator	6
1.1.6. Price History of Photovoltaic Panels, \$/Watt (Bloomberg New Energy Finance and pv.energytrend.com)	7
1.1.7. Levelized Cost of Electricity for Various Power Sources, C/kWh (Navigant Consulting, Inc. 2010).....	8
1.1.8. HDPE and Paraffin Wax Blends Thermograms (Sotomayor, 2014)	10
1.1.9. Degradation of HDPE and Paraffin Wax Mixtures as a Function of Temperature (Sotomayor, 2014).....	11
1.1.10. Effect of Copper Particles on Thermal Conductivity (Molefi, 2009).....	12
1.1.11. Effects of Graphite on PCM Thermal Conductivity (Mhike, 2012).....	14
1.1.12. Principals of a Standard Heat Pipe (Thermacore, Inc.)	19
1.1.13. Principals of a Loop Heat Pipe (Maydanik, 2004)	20
1.1.14. (a) Matrix Type (b) Plate-Type (c) and (d) Rod-Type (Nayak, 2006).....	22
1.1.15. Results of Nayak's Simulations	23
2.1.1. Hot Water Load for the Solar Decathlon House Competition	26
3.1.1. Estimate of Melting Region (Fisher)	35
3.1.2. Simulation Domain (Nayak, 2006).....	36

3.1.3. Mesh and Boundary Conditions for First Nayak Replication.....	36
3.1.4. Predicted Enthalpy Curve for Case 1	38
3.1.5. Predicted Specific Heat Curves for Cases 1 through 3	38
3.1.6. Comparison of Chip Temperature for Variable Specific Heat Model and CFD Simulation	40
3.1.7. Convection vs. No Convection Domain	41
3.1.8. SolidWorks Mesh Used for Nayak Replication with no Convection	41
3.1.9. SolidWorks Results Compared to Nayak, et al. Results for Temperature vs. Time... ..	43
3.1.10. Zoomed View of SolidWorks Results Compared to Nayak, et al. Results (Less Convection Case)	43
3.2.1. Thermal Storage Unit Heat Sink Schematic	45
3.2.2. Specific Heat (Top) and Enthalpy (Bottom) Curves for Sasol H1 wax.....	46
3.2.3. Variable Specific Heat Model Mesh and Boundary Conditions.....	47
3.2.4. Mesh Convergence Study Results	48
3.2.5. Time Step Convergence Results	49
3.2.6. Average Temperature of PCM vs. Time for Cases 1-4	51
3.2.7. Minimum Temperature of PCM vs. Time for Cases 1-4.....	52
3.2.8. Average Temperature of PCM vs. Time for Cases 5-8	53
3.2.9. Minimum Temperature of PCM vs. Time for Cases 5-8.....	53
3.2.10. Average Temperature of PCM vs. Time for Copper and Aluminum (Case 6 and 9 Comparison).....	54

3.2.11. Minimum Temperature of PCM vs. Time for Copper and Aluminum (Case 6 and 9 Comparison)	54
3.3.1. Experimental Set-Up (Assis, 2006)	55
3.3.2. Plot of Melt Fraction vs. Time for the Experiment and Fluent Simulation (Assis, 2006)	56
3.3.3. Comparison of Experimental (a) and Numerical (b) Melting Fronts (Assis, 2006)	56
3.3.4. Fluent Computational Domain (Assis, 2006)	57
3.3.5. Mesh Used in Simulation.....	61
3.3.6. Simulated Melting Front at 2, 5, 10, 15, 20, and 25 Minutes (Starting at the Top Left and Moving Clockwise) for 10°C Above the Average Melting Temperature Case.....	62
3.3.7. Comparison of Assis (2006) Experimental and Numerical Results with Replicated Results at 6°C Above Average Melting Temperature	63
3.3.8. Comparison of Assis (2006) Numerical Results with Replicated Results at 10°C Above Average Melting Temperature	63
3.4.1. Mesh and Boundary Conditions Used for Fluent Simulations	67
3.4.2. Liquid Fraction vs. Time Boussinesq Mesh Convergence Study	68
3.4.3. Average Temperature vs. Time Boussinesq Mesh Convergence Study	68
3.4.4. Liquid Fraction vs. Time for Boussinesq Time Step Convergence	70
3.4.5. Average Temperature vs. Time for Boussinesq Time Step Convergence	70
3.4.6. Liquid Fraction vs. Time for Fin Thickness Cases	72
3.4.7. Average Temperature vs. Time for Fin Thickness Cases	72

3.4.8. Liquid Fraction vs. Time for Base Thickness Cases.....	73
3.4.9. Average Temperature vs. Time for Base Thickness Cases.....	73
3.4.10. Typical Melting Front with Velocity Vectors Superimposed for Boussinesq Cases (30, 60, 90, 120 Seconds from Left to Right).....	74
3.5.1. Liquid Fraction vs. Time for Variable Density Fin Cases	76
3.5.2. Average Temperature vs. Time for Variable Density Fin Cases	76
3.5.3. Typical Melting Front with Velocity Vectors Superimposed for Variable Density Cases (30, 60, 90, 120 Seconds from Left to Right).....	77
3.6.1. Average Temperature Comparison for 0.0625-inch Fin.....	80
3.6.2. Average Temperature Comparison for 0.125-inch Fin.....	80
3.6.3. Average Temperature Comparison for 0.1875-inch Fin.....	81
3.6.4. Average Temperature Comparison for 0.25-inch Fin.....	81
3.6.5. Liquid Fraction Comparison for 0.0625-inch Fin.....	82
3.6.6. Liquid Fraction Comparison for 0.125-inch Fin.....	82
3.6.7. Liquid Fraction Comparison for 0.1875-inch Fin.....	83
3.6.8. Liquid Fraction Comparison for 0.25-inch Fin.....	83
4.0.1. Representation of Possible System Design.....	84
4.1.1. Simplified Solar Thermal Loop Schematic.....	85
4.1.2. Scheffler Solar Concentrator Interfacing with a House (solarcooking.wikia.com).....	87
4.2.1. Thermal Storage Unit Design	88
C1. Grundfos Pump Curves	103
C2. System Schematic	105

1. INTRODUCTION

The 2015 Solar Decathlon was held October 8th through October 18th of 2015 and is designed to demonstrate the “money-saving opportunities and environmental benefits presented by clean-energy products and design solutions” (<http://www.solardecathlon.gov/>). As

Figure 1.1.1. 2005 Cal Poly Solar House

policies are created to minimize the impact of human development on the environment, these clean-energy products and design solutions will play a larger role in the normal flow of day-to-day life. Thus, the Solar Decathlon provides a venue for new and innovative renewable technologies to be displayed by making them available for the public to view and interact with along with testing their feasibility through the Solar Decathlon competitions.

Currently, the relevance of solar thermal technology in the emerging renewable energy market is under question. With innovations to photovoltaic (PV) panels and electric storage, producing electricity using the photoelectric effect is becoming less expensive and more efficient. Thus, the relevance of large scale solar thermal plants in the future energy market is questionable. One area where solar thermal has typically dominated is in domestic hot water. However, heating water using electricity generated from PV panels is becoming more prevalent. If solar thermal is going to continue to be a part of the renewable energy market, new and innovative systems are needed.

The 2015 Solar Decathlon Competition inspired the idea to develop a solar thermal storage system to be used as a heating source for domestic hot water; thus, the requirements and competitions set by the Solar Decathlon were used as the requirements for the design of the solar thermal hot water system. The following list summarizes the requirements set by the Solar Decathlon that guided the development of this thesis (US Department of Energy, 2015):

- Contest 7-3: Clothes Washer
 - One or more complete, uninterrupted, “normal” (or equivalent) cycles in an automatic clothes washer shall be used to wash a specified load of laundry.
- Contest 7-5: Dishwasher
 - During the washing cycle, the internal temperature must reach 120°F and maintain the cycle operating temperature.
- Contest 7-6: Cooking
 - A kitchen appliance must be used to vaporize 5.000 lbs. of water within a specified time period.
- Contest 8-2: Hot Water
 - Must be able to provide at least 15 gallons of hot water at 110°F in no more than 10 minutes.

1.1 Relevant Research and Technology

1.1.1 Solar Hot Water

Generating hot water using solar irradiation is a common application of solar technology. The most common application of solar hot water is in domestic hot water supplies, but it is also used in industrial applications. There are three dominant forms of solar thermal hot water systems: flat plate collectors, concentrated collectors, and batch systems. These systems are further categorized by active and passive systems, as well as open-loop and closed-loop systems.

Figure 1.1.2. Flat Plate Collectors

Figure 1.1.2 shows two flat plate collectors mounted on the roof of a house which are being used to heat water. Water or a heat transfer fluid, HTF, enters the first collector at the bottom, and passes through a

series of pipes embedded in a black absorber plate. The water is then heated as it moves up the pipes by both direct and diffuse solar radiation. The two systems can either work in parallel, or in series. Using the two collectors in series allows for higher temperatures to be reached as the hot water from the first collector is further heated as it passes through the second collector. While flat plate collectors are the most common solar thermal system used for domestic hot water, they are best used at temperatures below 140°F as the efficiency of these systems drastically drops above these temperatures. Additionally, due to their size and weight, significant modifications may need to be made to roofs in order to support them.

Another type of solar thermal system used to heat water is concentrating collectors (see Figure 1.1.3). Concentrating collectors use parabolic dishes, or mirrors, to concentrate direct solar radiation on an evacuated tube receiver. The evacuated tube collectors

Figure 1.1.3. Concentrating Solar Collector

minimize losses by limiting convection and radiation losses. Water, or the HTF, passes through the collector and is heated by the solar radiation that is concentrated on the receiving plate within the evacuated tube. These systems are used when higher temperatures are desired. The concentrated light is able to heat the HTF to much greater temperatures than flat plate collectors are capable of, and, due to the vacuum and glass chamber, concentrated collectors with evacuated tubes are typically more efficient than a flat plate collector. However, these systems are larger and more complex than flat plate

collectors, and are typically used in large scale solar thermal plants.

Figure 1.1.4. Batch Collector

Batch or Integral Collector-Storage (ICS) Systems are the simplest solar thermal systems. They work best in hot climates that do not experience freezing temperatures. A batch collector is simply a tank that is directly exposed to the sun and insulated on the other

sides. They can utilize mirrors and black paint to increase the effectiveness of the system. Figure 1.1.4 shows a batch system that employs mirrors to increase the amount of radiation the tank receives and a glass panel to minimize convection and radiation losses. As the water is heated at the bottom of the tank, it is heated and rises to the top of the tank. From there it can supply a house with domestic hot water, or be transferred into another hot water tank. These systems are inexpensive, simple, and easy to maintain, but provide low temperature fluid.

These three systems are further classified as being active or passive, and closed or open. An active system uses a pump to move water through the system, while a passive system relies on the buoyancy of hot water to move water through the system. Active systems allow for a storage tank to be placed below the collectors while a passive system needs the storage tank to be above the collector in order for the water to circulate through the system. Passive systems are less expensive and simpler than active system, but may require significant support structures for large tanks to be raised. In an open system, the water flowing through the collector comes directly from the storage tank, while a closed system has its own internal loop that contains a heat transfer fluid. The HTF is heated in the collector loop and heats water in the main storage tank through a heat exchanger system. Closed systems are mostly used when freezing temperatures present a risk to the system as the HTF can be selected to provide a freezing point higher than that of water.

1.1.2 Solar Cooking

The following discussion is based on two years of research with Dr. Pete Schwartz of Cal Poly San Luis Obispo. Solar radiation has also been used in cooking applications. This technology has seen the most growth and use in developing communities.

Figure 1.1.5. Cooking with a Scheffler Concentrator

Scheffler Solar Concentrators are

typically used for cooking. Figure 1.1.5 demonstrates cooking with these devices. The Scheffler concentrator features a parabolic dish rotated about the earth's axis of rotation. Incoming light from the sun is focused onto the focus of the parabola. This concentrating effect from the parabola allows the intensity of the radiation to be greatly increased by reducing the surface area on which the radiation is applied (up to 10,000 times depending on the quality of the parabolic dish). A receiver surface or thermal storage unit is heated by the concentrated light and then used as a cooking surface.

While cooking directly with these devices is possible, there are a few difficulties. The first difficulty is that being able to cook is completely dependent on having direct normal radiation available at the time one desires to cook. If a cloud blocks out the sun or it is too early or late in the day, the cooking surface will not be hot enough to cook with. The second difficulty is that it is difficult to control the cooking temperature. The temperature of the cook surface is directly dependent on the current amount of radiation and the surface of your cooking instrument making precise temperature control nearly

impossible. Lastly, the location of the cooking surface must be directly north or south of the Scheffler and relatively close to the reflecting dish.

1.1.3 Photovoltaics

Photovoltaic, PV, panels are the most commonly known and familiar solar technology. By using the photoelectric effect, the radiation from the sun is turned into electricity. As shown in Figure 1.1.7, the cost of generating electricity per kWh with PVs is relatively high. However, much progress has been, and is currently being made to increase the efficiency while decreasing the cost of Solar Panels. Figure 1.1.6 shows the dramatic decrease in the price per Watt of solar panels since 1977.

Figure 1.1.6. Price History of Photovoltaic Panels, \$/Watt (Bloomberg New Energy Finance and pv.energytrend.com)

Figure 1.1.7. Levelized Cost of Electricity for Various Power Sources, C/kWh (Navigant Consulting, Inc. 2010)

While flat plate collectors and concentrated panels have long been the dominant form of heating water, PV panels may be starting to emerge in this market. With the cost of electricity from PV panels rapidly dropping, some are starting to suggest utilizing heat pumps powered by PV panels to generate hot water may be more efficient and less expensive than solar thermal methods. If this decrease in price continues, there will likely be an increase in the use of PV panels in applications that have been typically reserved for solar thermal energy in cold environments.

1.1.4 Thermal Storage

Renewable energy is dependent on the Earth's cycles to be able to produce energy. Thus, it cannot be scaled up or down to meet the instant energy demand. The following sections discuss means of storing solar thermal energy.

1.1.4.1 Phase Change Material

Phase change materials (PCM) are an ideal thermal storage medium. They are characterized by a high latent heat, which allows them to store large amounts of energy when the material transitions into different physical states. Typically, the phase transition occurs when the PCM melts, changing from a solid to liquid; however, the transition can also be a solid to solid transition as the crystalline structure of the material changes.

There are many commercially available phase change materials, but they are often characterized by high costs. Waxes are an inexpensive material that exhibit a phase change when heated (imagine a candle melting) and can be used to store thermal energy. Paraffin waxes have a high latent heat, and can be combined with plastics to create a low-cost phase change material for about half the price of commercially available materials. However, PCMs often have a low thermal conductivity which leads to a slow transient response. This makes it difficult to quickly store and access the energy stored within the PCM, so work is currently being done to improve transient responses.

Sotomayor, et al. (2014) examined the thermal properties of a high density polyethylene (HDPE) and paraffin wax (PW) blend. They created samples ranging from 100% HDPE to 100% PW and used a differential scanning calorimeter to generate heating thermograms of the samples. The results were then used to determine the melting points and latent heats of the samples. Figure 1.1.8 and Table 1.1.1 present their results. Figure 1.1.8 shows two phase transitions for the more balanced mixtures. One around the melting point of the paraffin wax and the other around the melting point of the HDPE. Sotomayor found that the mixing of the PW and HDPE affected each of the phases, as indicated by the changing melting points and latent heats for the various mixtures.

Figure 1.1.8. HDPE and Paraffin Wax Blends Thermograms (Sotomayor, 2014)

Table 1.1.1. Tabulated Results of Thermograms (Sotomayor, 2014)

HDPE/PW (vol. %)	T_m^{HDPE} (°C)	ΔH_m^{HDPE} (J/g HDPE)	T_m^{PW} (°C)	ΔH_m^{PW} (J/g PW)
100/0	130.8	178.6	-	-
95/5	129.4	179.8	-	-
90/10	128.4	180.9	48.2	43.9
80/20	125.8	185.0	49.6	105.2
70/30	124.3	184.1	50.5	139.6
60/40	123.3	205.5	50.4	135.6
50/50	122.4	210.8	51.4	127.8
0/100	-	-	53.1	183.5

Table 1.1.1 shows the melting points, T_m , and corresponding latent heats for each of the mixtures studied. As the percentage of paraffin wax increased, the melting point of the first transition increased while the melting point of the second transition decreased. Both latent heats increased with the addition of more paraffin wax (up to the 50/50 ratio). Based on these results, the optimum mixture to maximize melting temperature and latent heat appears to be a 60% HDPE, 40% PW mixture by volume percentage.

This study also evaluated the degradation of the PCM blend when exposed to high temperatures using a thermogravimetric analysis to determine the mass losses. Figure 1.1.9 presents the results starting with a 100% HDPE/0% Paraffin mixture, and ending at 0% HDPE/100% Paraffin mixture. At temperatures below 200°C, the samples showed no mass loss, but at temperatures above this, losses became significant. The addition of HDPE to the blends stabilized the losses, allowing for higher operating temperatures while minimizing the losses.

Figure 1.1.9. Degradation of HDPE and Paraffin Wax Mixtures as a Function of Temperature (Sotomayor, 2014)

Molefi, et al. (2009), further evaluated HDPE/PW wax blends as a thermal storage material by expanding on Sotomayor's work through examining the thermal conductivity of the PCM. A high thermal conductivity is desirable in thermal storage material as it allows for heat to be evenly distributed throughout the material, and allows for the stored energy to be more efficiently and quickly accessed. To improve the thermal conductivity, Molefi added copper particles to the PCM in an attempt to increase the thermal conductivity.

Figure 1.1.10. Effect of Copper Particles on Thermal Conductivity (Molefi, 2009)

Table 1.1.2. Melting Temperature and Latent Heat of Wax Blends with Copper (Molefi, 2009)

HDPE/Wax/Cu (Vol. %)	T_m^{HDPE} (°C)	T_m^{PW} (°C)	ΔH (J/g)
100/0/0	134.7 ± 0.5	-	149.3 ± 9.7
60/40/0	124.1 ± 2.3	56.4 ± 0.4	153.2 ± 9.9
59/40/1	123.8 ± 0.7	56.5 ± 0.6	150.0 ± 3.5
57/40/3	122.8 ± 0.4	56.5 ± 0.4	113.7 ± 5.0
55/40/5	123.5 ± 0.2	56.9 ± 0.4	91.7 ± 14.8
50/40/10	122.8 ± 1.0	56.8 ± 0.1	77.1 ± 5.7
0/100/0	-	58.4 ± 1.2	172.2 ± 0.1

Figure 1.1.10 presents the results of their study, and shows that the addition of copper particles did have a significant effect on the thermal conductivity of the PCM blends. However, looking at Table 1.1.2, it is evident that the copper particles had a significant effect on the storage capacity of the blends. In order to see a benefit from the increased thermal conductivity of the material, more than 5% by volume of the HDPE/PW/copper blend needs to be copper. This dropped the latent heat to an average of 91.7 J/g from an average of 153.2 for a 60/40/0 blend. Thus, while the copper was able to increase the thermal conductivity, it significantly reduces the storage capacity of the material.

Mhike, et al. (2012) examined the effects of graphite, rather than copper, on phase change material. However, for their experiment they used a Low Density Polyethylene (LDPE) and paraffin wax mixture. LDPE is very similar to HDPE and the trends it exhibits should carry over to HDPE (although LDPE has a much lower latent heat and melting point than HDPE). They used two types of graphite – expanded and regular. Just as the copper did, the graphite enhanced the thermal conductivity of the sample, but it had a much smaller effect on the latent heat than the copper particles did. Figure 1.1.11 and Table 1.1.3 present the results of Mhike, et al.'s study. Table 1.1.3 shows the results when various amounts of graphite were added to the 60/40 blend. Even with 10% graphite, the latent heat only dropped by 5 J/g for e-graphite, and 6 J/g for regular graphite. Mhike's work suggests that, graphite not only has less of a negative effect on the thermal storage capacity of the phase, but also can provide much higher thermal conductivities.

Figure 1.1.11. Effects of Graphite on PCM Thermal Conductivity (Mhike, 2012)

Table 1.1.3. Melting Temperature and Latent Heats of PCM Blends with Graphite Particles (Mhike, 2012)

Sample	T_m^{LDPE} (°C)	ΔH_m^{HDPE} (J/g LDPE)	T_m^{PW} (°C)	ΔH_m^{PW} (J/g PW)
Wax	-	-	56.8	168.2
LDPE	109.1	72.7	-	-
60/40 (wax/LDPE)	98.1	15.9	54.8	85.0
95/5 (wax/LDPE)/ e-graphite	98.7	14.9	55.8	83.2
93/7 (wax/LDPE)/ e-graphite	97.6	15.2	54.4	82.1
90/10 (wax/LDPE)/ e-graphite	97.8	14.3	54.4	80.0
90/10 (wax/LDPE)/graphite	97.6	17.9	55.1	79.0
85/15 (wax/LDPE)/graphite	97.6	16.9	54.3	81.6
80/20 (wax/LDPE)/graphite	97.0	15.9	55.1	80.2
75/25 (wax/LDPE)/graphite	97.6	10.1	53.9	62.2

The phase change materials presented thus far have had relatively low operating temperatures – around 40 to 60 °C. As the melting point of the material determines the operating temperature, a higher melting point PCM is needed for solar hot water applications. Mngomezulu, et al. (2011) proposed a PCM of HDPE and a Fisher-Tropsch Paraffin Wax. The wax used in their study was H1 wax from Sasol Wax. The H1 wax has a high melting point, approximately 100°C, and a high latent heat, an average of 194 J/g. Table 1.1.4 presents the thermal properties of the HDPE/H1 wax mixtures evaluated in their study. Once again, as more paraffin wax is added to the mixture, the melting point decreases but the latent heat increases.

Table 1.1.4. Thermal Properties of Sasol H1 Wax and HDPE Mixtures

HDPE/H1 Wax (Vol. %)	T_{melting} (°C)	ΔH (J/g)
80/20	132 ± 1	154 ± 18
70/30	133 ± 2	157 ± 11
60/40	129 ± 1	164 ± 15
50/50	132 ± 1	166 ± 10
40/60	127 ± 2	173 ± 12

1.1.4.2 Phase Change Material with Metal Foams

Chintakrinda, et al. (2011) evaluated the impacts of metal foams on phase change material thermal conductivity. Metal foams allow for heat to be conducted along the high conductivity metal pathways, and then into smaller pockets of PCM. This allows for quicker melting of the PCM. However, lower porosity foams were found to have a negative impact on the system, as they prevent natural convection from occurring inside of the melted phase change material. This led to conduction dominated heat transfer, and stratified temperatures within the PCM domain. The stratification can decrease the effectiveness of a phase change thermal storage system as the PCM may not completely melt; thus, it does not reach its full storage capacity. The study showed that if metal foams are to be used as a thermal conductivity enhancer in phase change applications, high porosity foams with a large pore size should be used. Metal foams were shown to increase the performance of a phase change thermal storage system.

1.1.4.3 Hot Water as a Thermal Storage System

The primary method for storing thermal energy for hot water has been hot water tanks/heaters. Water has a high specific heat capacity, $4.2 \text{ J/g}^\circ\text{C}$ at typical operating conditions, allowing it to store a fair amount of energy in a relatively small volume – this is the principal behind the Batch Solar Collectors discussed earlier. The thermal conductivity of water, 0.6 W/m-K , is also on the same order as the thermal conductivity for most available phase change materials. Table 1.1.5 presents the amount of energy stored in 1 kg of water, and 1 kg of the 50/50 HDPE/H1 wax blend, shown in Table 1.1.4, if the water is brought to its maximum temperature, 100°C , and the wax blend is completely melted at 132°C (for the wax, the specific heat capacity is $2.1 \text{ J/g } ^\circ\text{C}$, see Table 3.2.1).

Table 1.1.5. Energy of Water and 50/50 HDPE/H1 Wax Blend

Water Energy	Wax Energy
340,000 J	400,000 J

These results show the effectiveness of water as a thermal storage medium. The wax is able to store 18% more energy than the water is able to. However, a large portion of the wax's energy is stored in the latent heat which allows the wax to maintain its high temperature of 132°C for an extended period of time as it releases heat, while the water will immediately begin to cool from its maximum temperature of 100°C. Additionally, the PCM could store more energy as it is heated past its melting point. Thus, PCM is able to store more energy than water, and maintain higher temperatures.

1.1.5 External PCM Heat Transfer Enhancement Methods

Section 1.1.4 examined the research that has been done on how to increase the thermal conductivity of a PCM itself. The following section will discuss other means of increasing heat transfer into a PCM.

1.1.5.1 Heat Pipes

Heat pipes are relatively simple systems that allow for effective heat transfer over much greater distances than what is possible through standard conduction. They can transfer heat in lengths from a few inches, to around 75 feet without a significant temperature drop. This is possible as the heat is transferred into the heat pipe through the evaporation of the heat pipe working fluid and out of the heat pipe as the working fluid condenses. The operating temperature and power range are determined by the working fluid and the size of the heat pipe (see Table 1.1.6). Two common forms of heat pipes are standard heat pipes and loop heat pipes. The following discussion further analyzes how the thermal cycles of the two types work and the pros and cons of each system.

Table 1.1.6. Heat Pipe Operating Materials (Thermacore, Inc)

<i>MEDIUM</i>	<i>MELTING PT. (° C)</i>	<i>BOILING PT. AT ATM. PRESSURE (° C)</i>	<i>USEFUL RANGE (° C)</i>
Helium	- 271	- 261	-271 to -269
Nitrogen	- 210	- 196	-203 to -160
Ammonia	- 78	- 33	-60 to 100
Acetone	- 95	57	0 to 120
Methanol	- 98	64	10 to 130
Flutec PP2	- 50	76	10 to 160
Ethanol	- 112	78	0 to 130
Water	0	100	30 to 200
Toluene	- 95	110	50 to 200
Mercury	- 39	361	250 to 650
Sodium	98	892	600 to 1200
Lithium	179	1340	1000 to 1800
Silver	960	2212	1800 to 2300

1.1.5.1.1 Standard Heat Pipes

Figure 1.1.12. Principles of a Standard Heat Pipe (Thermacore, Inc.)

Standard heat pipes are simple systems that can be used to transfer heat. They typically consist of a copper pipe, a capillary wick structure, a working fluid and some insulation. A slight vacuum is created in the pipe to reduce the boiling point of the working fluid and allow for the fluid and vapor in the system to be at saturation. If the boiling point of the working fluid is reached at the evaporator, the liquid will evaporate and raise the pressure of the heat pipe at the evaporator side without affecting the pressure at the condenser side. Thus, fluid will be drawn from the wick structure and the vapor will move towards the condenser. When the vapor reaches the condenser it will cool and condense allowing the heat to be transferred along the length of the heat pipe. The working fluid of heat pipes will be selected to give a latent heat of vaporization which minimizes the amount of fluid needed to transfer large amounts of heat. Figure 1.1.12 shows a schematic of a standard heat pipe as well as a temperature entropy graph demonstrating the heat transfer process discussed.

1.1.5.1.2 Loop Heat Pipes

Figure 1.1.13. Principles of a Loop Heat Pipe (Maydanik, 2004)

Loop heat pipes operate similarly to a standard heat pipe; however, the wick plays a more complicated role than in a standard heat pipe. Figure 1.1.13 presents a simple schematic, and the Pressure-Temperature diagram of a loop heat pipe. The following section discusses the functioning of a loop heat pipe as presented by Maydanik (2004). When the evaporator is not exposed to a heat load, the wick is saturated with liquid. Additionally, the vapor line and the condenser are filled. If a heat load is applied, the liquid in the wick will begin to evaporate. Localized heating will occur in the wick, due to its proximity to the evaporation zone and different thermal properties from the rest of the system, resulting in a higher temperature and pressure. This prevents the hotter vapor from moving to the compensation chamber as the capillary forces in the saturated wick hold the liquid in. Thus, a pressure differential is created which, once the pressure is high enough, results in the displacement of the working fluid.

This pressure differential allows for loop heat pipes to transfer heat over large distances (approximately 60 ft. horizontally and 10 ft. vertically). Looking at the Pressure-Temperature diagram, point 1 specifies the vapor conditions. Movement from 1 to 2 indicates vapor transfer into the vapor line. The vapor is slightly superheated here as it passes along the evaporator. From 3 to 4 the vapor is condensed, and then super cooled from 4 to 5. 5 to 6 shows the movement of fluid along the liquid line, and is modeled as isothermal. From 6 to 7 the liquid enters the compensation chamber and is heated. Lastly, 7 to 8 represents the transfer of the fluid back into the evaporator section wick. The total pressure drop from point 1 to 8 is the capillary pressure. The following equations govern whether or not the loop heat pipe will operate (P_C is the capillary pressure, P_v is the pressure loss in the vapor phase, P_l is the pressure loss in the liquid phase, and P_g is the pressure loss due to hydrostatic resistance):

$$\Delta P_c \geq \Delta P_v + \Delta P_l + \Delta P_g$$

$$\Delta P_g = (\rho_l - \rho_v)gl \sin \varphi$$

Where ρ_l and ρ_v are the liquid and vapor densities respectively, g is the gravitational acceleration, l is the effective length of the pipe, and φ is the angle between the heat pipe and a horizontal plane. If the critical pressure is greater than the sum of the vapor, liquid, and gravitational pressure drops, the loop heat pipe will operate properly.

1.1.5.2 Fins

Nayak, et al. (2006), evaluated the impacts of three different forms of thermal conductivity enhancers through a numeric simulation. They started with adding metal particles to a wax (the same process that was previously discussed), then added metal plate type fins within the PCM, and finally, added rod type fins within the PCM (see Figure 1.1.14). They tracked the temperature of the chip for various power loads applied and found that the metal particles, plate fins, and rod fins all lowered the chip temperature from the default case, but the matrix type had the smallest effect while the rod type had the greatest effect. Their results are shown in Figure 1.1.15.

Figure 1.1.14. (a) Matrix Type (b) Plate-Type (c) and (d) Rod-Type (Nayak, 2006)

Figure 1.1.15. Results of Nayak's Simulations

Additionally, Shatikian (2004), evaluated the impacts of varying heatsink parameters on the time it took for wax to completely melt. Shatikian altered the thickness of the fins, the height of the fins, and the spacing of the fins. The study showed that thicker fins led to a quicker melting process as the temperature within the fin was essentially isothermal, whereas thin fins were not able to maintain constant temperatures. It was also seen that decreasing the distance between fins greatly decreased the melting time. These two studies show that adding fin or rod heatsinks to PCM systems can significantly increase the performance of the PCM. Nayak found that the effects of heatsinks was greater than that of adding metal particles to the phase change material itself. The two methods could be combined to maximize the thermal conductivity and melting performance of a phase change material. It is also seen that the temperature of the chip flattens near the melting point of the PCM. This is a result of the PCM absorbing a large amount of energy at constant temperature. and shows another potential use of PCM in electronics cooling.

1.2 Objective

This thesis will use the Solar Decathlon's competitions' rules as the design guidelines for a solar thermal hot water system. The system will use concentrated sunlight to heat, and melt a phase change material to store heat, that can later be used by the house for heating purposes. Analysis of the system will be performed to characterize its performance. The primary objective of the thesis will be to evaluate the effectiveness of various analysis techniques in accurately, and quickly analyzing storing energy in a phase change process. Three techniques will be used. First, SolidWorks will be used to perform a finite element analysis of the system. Second, Fluent will be used to perform a computational fluid dynamics analysis of the system using an approximation for the change in density that occurs when a PCM increases in temperature. Third, Fluent will again be used, but this time a full variable density model will be used. The results will be analyzed to see how well they agree with each other, and to determine how effective they are as design tools for these types of heat transfer problems.

The remainder of this thesis will be structured as follows:

1. The hot water load of a solar house will be determined, as well as the available solar resource.
2. The melting of a phase change material in a copper heatsink will be studied to determine how various parameters effect the time for the PCM to melt, using three different methods:
 - a. Variable Specific Heat Model in SolidWorks
 - b. Boussinesq Model in Fluent
 - c. Variable Density Volume of Fluids Method in Fluent
3. The results of the three methods will be compared to see how well they agree, and which method provides the best results in the least amount of time.
4. An example solar thermal hot water system, using a solar concentrator and phase change thermal storage system, will be presented.

2. SYSTEM REQUIREMENTS AND FEASIBILITY

In this section, the total amount of water required for the competition is determined, as well as the corresponding amount of energy required to heat the water. The available solar resource is then determined, and compared to the required amount of energy.

2.1 Solar House Hot Water Load

The main purpose of the solar thermal system presented in this paper is to meet the hot water supply requirements for the Solar Decathlon competition, but the solar cooking system will also add to the subjective value of the home. Water for the competition will be provided by the Solar Decathlon and stored in containers on the lot allocated for the house. The solar thermal system needs to provide hot water for showers, laundry, dishwashing and the kitchen sink. Additionally, a certain “extra” amount will be required. Figure 2.1.1 summarizes the amount of water and the temperature of water that will be required for each application. Additionally, the amount of black and grey water resulting from these applications is captured. These values are based on water consumption for a typical household of three.

Figure 2.1.1. Hot Water Load for the Solar Decathlon House Competition

The amount of hot water needed to heat the water in the reservoir tank to the application temperature was then determined by performing an energy balance on the mixing of hot water from the hot water tank and cold water from the decathlon storage tank to produce the desired amount of hot water. It was assumed that the water in the cold tank would be at 65°F and the water in the hot tank would be at 140°F.

The following calculations show how the amount of hot water required for the shower from the storage tanks was determined:

$$M_{hot\ tank}C_{water}T_{hot\ tank} + M_{reservoir}C_{water}T_{reservoir} = M_{shower}C_{water}T_{shower} \quad (1)$$

$$M_{hot\ tank} + M_{reservoir} = M_{shower} \quad (2)$$

After converting the gallons of water for the shower into a mass, the only two unknowns are the mass of water required from the hot tank and reservoir. Thus, the system of equations can be solved.

Table 2.1.1 summarizes the values that were entered into the equations and the results.

Table 2.1.1. Summary of Shower Energy Balance Results

Variable:	Description:	Value:
C_{water}	Specific heat of water	4.179 kJ/kgK
$T_{hot\ tank}$	Hot Tank Temperature	60°C (140°F)
$T_{reservoir}$	Reservoir Temperature	18.33°C (65°F)
T_{shower}	Shower Temperature	43.33°C (110°F)
M_{shower}	Mass of water for shower	908.5 kg
$M_{reservoir}$	Mass of water from reservoir	363.4 kg
$M_{hot\ tank}$	Mass of water from hot tank	545.1 kg

It was determined that 545.1 kilograms of water are required to heat water to 110°F for the shower, which is equivalent to 144 gallons, over the course of the competition. Appendix A shows the Engineering Equation Solver (EES) code that was used to determine the amounts of hot water from the storage and reservoir tanks for each application. Ultimately, a total of 257.3 gallons of hot water at 140°F will be needed to meet the water requirements for the competition. This number takes into account all the sources of hot water load as presented in Figure 2.1.1.

Knowing the total amount of water required and maintaining the assumptions that hot water is at 140°F and water in the reservoir is at 65°F (a change in temperature of 41.17°C), the total amount of energy needed to heat water for the competition can be determined as follows:

$$E_{heating} = M_{water} C_{water} \Delta T_{heating} \quad (3)$$

$$E_{heating} = \left[257.3 \text{ gal} \times .003785412 \frac{m^3}{\text{gal}} \times \frac{995.7 \text{ kg}}{m^3} \right] \times 4.179 \frac{kJ}{kgK} \times 41.17^\circ C \quad (4)$$

$$E_{heating} = 169,000 \text{ kJ} = 169 \text{ MJ} \quad (5)$$

Distributing the 169 MJ of energy required for the competitions related to hot water over the 11-day competition, it is seen that an average of 15.4 MJ will be required per day. In addition to this heating load, daily losses need to be accounted for in the daily average energy load. Assuming a 10°F drop in temperature overnight of the 50-gallon storage tank, the losses can be determined:

$$E_{losses} = M_{water}C_{water}\Delta T_{cooling} \text{ (6)}$$

$$E_{losses} = \left[50 \text{ gal} \times .003785412 \frac{m^3}{gal} \times \frac{995.7 \text{ kg}}{m^3} \right] \times 4.179 \frac{kJ}{kgK} \times 5.556^\circ C \text{ (7)}$$

$$E_{losses} = 4,400 \text{ kJ} = 4.4 \text{ MJ} \text{ (8)}$$

Based on these assumptions a total of 19.8 MJ of energy will be required daily to provide an adequate amount of hot water to the Cal Poly Solar Decathlon house, or an equivalent net zero solar home. This 19.8 MJ load is consistent with values typically given by utility companies for daily energy consumption of hot water.

2.2 Available Solar Resource

All energy used in the Solar Decathlon must be obtained from solar radiation. Therefore, a solar thermal system must be able to harvest enough energy to provide 19.8 MJ of energy per day to meet the solar thermal need of the Cal Poly solar house. Additionally, if the solar thermal system is going to be used for applications other than providing domestic hot water – cooking for example – the system will need to provide more than 19.8 MJ of energy and reach temperatures greater than 250°F.

To ensure that the system will be able to meet the demand, both mathematical models and historical data were used to predict what the available solar radiation will be during the competition. According to the NASA Surface meteorology and Solar Energy Data Subset presented by the Atmospheric Science Data Center, the monthly averaged clear sky insolation on a horizontal plate will be 18.32 MJ/m²/day at the Solar Decathlon location in Irvine, California during October (see Appendix B). Additionally, the available insolation per day of the competition was calculated using the clear sky model presented by Hottel (1976). The results are shown in Table 2.2.1 and the EES code used for the calculations can be found in Appendix A. As concentrated sunlight is dependent on beam radiation, the results show the total radiation as well as beam and diffuse components. Averaging the results shows 13.51 MJ/m² per day can be expected over the course of the competition with the system receiving more than this during the first half of the competition, and less than this during the second half of the competition. These results are slightly less than what is expected based on the historical data presented by NASA.

Table 2.2.1. Clear-Sky Model Predictions

Day	Total Radiation (MJ/m²)	Beam Radiation (MJ/m²)	Diffuse Radiation (MJ/m²)
10/8/2015	17.67	14.23	3.44
10/9/2015	17.51	14.09	3.43
10/10/2015	17.35	13.94	3.41
10/11/2015	17.19	13.80	3.39
10/12/2015	17.03	13.65	3.38
10/13/2015	16.87	13.51	3.36
10/14/2015	16.71	13.37	3.35
10/15/2015	16.55	13.22	3.33
10/16/2015	16.40	13.08	3.31
10/17/2015	16.24	12.94	3.30
10/18/2015	16.08	12.80	3.28
Average:	16.87	13.51	3.36

The historical data presented by NASA was further broken down to predict the amount of beam and diffuse radiation. Using the distributions developed by Liu and Jordan (1960), 13.52 MJ/m² of beam radiation and 4.78 MJ/m² of diffuse radiation are expected. The clear-sky model prediction for beam radiation and the NASA data prediction are essentially identical, while the clear-sky model prediction for diffuse radiation has a percent difference of -29.71% from the historical data. As a result, expecting an average of 13.51 MJ/m² of beam radiation and 3.36 MJ/m² of diffuse radiation during the competition is a conservative prediction.

In addition to knowing the average daily radiation on a horizontal plate, the hourly components of direct normal (beam) radiation should be known as well to assist with simulations. Table 2.2.2 shows these values for a typical day on October 17th, based on historical data. Appendix B includes the data for all days of the competition. The measurements come from the National Renewable Energy Laboratory solar database, and

were taken at John Wayne International Airport. Concentrated solar power utilizes direct normal radiation, so these values provide a good estimate as to what the total energy available from the sun will be, and if the sun will provide enough power to heat and melt a phase change material. The data shows that the total direct normal energy for the day is 21.208 MJ/m²; thus, a one-meter square solar concentrator dish (which is relatively small) would be able to provide enough energy for an entire day's worth of hot water. Any energy collected beyond this amount could be stored in a thermal reservoir for heating even after the sun has gone down.

Table 2.2.2. Average Values for Hourly Direct Normal Radiation and Total Energy on October 17th in Irvine, California (John Wayne Airport, NREL Solar Database)

Time	Average Direct Normal Radiation (W/m²)	Total Energy (MJ/m²)
7:00	259	0.9324
8:00	529	1.9044
9:00	695	2.502
10:00	765	2.754
11:00	816	2.9376
12:00	751	2.7036
13:00	697	2.5092
14:00	566	2.0376
15:00	557	2.0052
16:00	256	0.9216
Total		21.208

The above analysis shows that a solar thermal system could be the hot water load of a typical solar house at the Solar Decathlon. Additionally, a two-meter square dish would be able to provide more than double the amount of energy needed by a house, allowing that excess energy to be stored within a phase change material.

3. PHASE CHANGE MATERIAL NUMERICAL ANALYSIS

This section will analyze the performance of a PCM within a copper heatsink. The various heatsink features will be modified to evaluate how the features impact the melting of a PCM. Three models will be used. First, SolidWorks will be used to perform a finite-element analysis (FEA) study using a variable specific heat model – the variable specific heat model will be evaluated before being used. Second, Fluent 16.2 will be used to perform a computational fluid dynamic (CFD) study of the heatsink using the Boussinesq approximation for density. Third, Fluent 16.2 will be used again to perform a CFD studying accounting for the variable density with temperature of a PCM. SolidWorks uses FEA to solve the heat conduction equation for the energy transfer through materials, while Fluent uses the Finite Volume Method to solve the Navier-Stokes and energy equations for the flow and heat transfer through materials. Thus, FEA will neglect the fluid dynamics occurring within the system. The results for heatsink performance will be evaluated, and the results of the three models will be compared.

3.1 Variable Specific Heat Model

In order for the melting process of a phase change material to be accurately modeled, it is important to capture the melting and convection characteristics during the heating and cooling processes. Thus, a CFD program capable of handling these parameters is required. However, due to the computational complexities imposed by a phase change process and natural convection, a method of modeling a phase change material without accounting for melting and convection would allow for a greater number of design iteration to be analyzed in a cost/time effective manner.

Fisher, et al. (2013), propose an energy based model that accounts for the latent heating based on a pseudo enthalpy and specific heat curve. In their paper, they cite the work of Egolf and Manz who proposed the following equations for the enthalpy of a PCM as a function of temperature during a melting process:

$$h(T) = C_{p,1}T + \eta_1, \quad \text{for } T \leq T_M$$

$$h(T) = C_{p,1}T_M + (h_2 - h_1) + C_{p,2}(T - T_M) - \eta_2, \quad \text{for } T > T_M$$

$$\text{where:} \quad \eta_n = \left(\frac{h_2 - h_1}{2} \right) e^{\frac{-2|T - T_M|}{\tau_n}}$$

Table 3.1.1 summarizes the variables found in these equations. The subscript “1” indicates the solid region, while the subscript “2” indicates the liquid region. The specific heat curve is then derived from the enthalpy curve by numerically taking the derivative of the enthalpy with respect to temperature ($C_p = dH/dT$). Figure 3.1.1 shows an experimental specific heat curve generated to determine certain material properties, while Figure 3.1.5 shows a specific heat curve generated using this method. τ_n is a time constant broken into two regions. τ_1 is the length of the region from where the specific heat curve begins to sharply increase and the peak, while τ_2 is the length of the region from the peak of the specific heat curve and when it flattens out.

Table 3.1.1. Summary of Egolf and Manz Equation Variables (Fisher)

Variable:	Description:
$C_{p,1}$	Specific heat in the solid region
T	Temperature
T_M	Melting Temperature
$C_{p,2}$	Specific heat in the liquid region
$h_2 - h_1$	Latent heat
τ_n	Estimate of melting region (see Figure 3.1.1 below)

Figure 3.1.1. Estimate of Melting Region (Fisher)

A validation of this model was performed in SolidWorks based on an electronics cooling simulation run by Nayak, et al. (2006). Figure 3.1.2 depicts the domain that was analyzed and Table 3.1.2 states the material properties. Figure 3.1.3 shows the mesh that was used, and the boundary conditions that were applied. SolidWorks only allows for triangular elements to be used, so the mesh is not as structured as would be possible in another program, a 1-mm mesh spacing was applied to match what was done by Nayak. As the tau values, which characterize the melting region, are based on experimental results which were not available for this particular PCM, three cases with different tau values were analyzed to see the effect of the size of tau (see Table 3.1.3). The following boundary conditions were applied:

1. No slip conditions at the walls
2. Natural convection with ambient at the top (10 W/mK with ambient at 20°C)
3. Insulated side walls and bottom
4. Uniform volumetric heating applied to the chip (4 W)

Figure 3.1.2. Simulation Domain (Nayak, 2006)

Figure 3.1.3. Mesh and Boundary Conditions for First Nayak Replication

Table 3.1.2. PCM Properties from Nayak, et al. (2006)

Component	Property	Value
Chip	Density	740 kg/m ³
	Thermal Conductivity	20 W/mK
	Specific Heat	1830 J/kgK
Substrate	Density	2712.9 kg/m ³
	Thermal Conductivity	179.6 W/mK
	Specific Heat	960 J/kgK
PCM	Density, Solid	810 kg/m ³
	Density, Liquid	770 kg/m ³
	Thermal Conductivity, Solid	.39 W/mK
	Thermal Conductivity, Liquid	.157 W/mK
	Specific Heat, Solid	1900 J/kgK
	Specific Heat, Liquid	2200 J/kgK
	Latent Heat	241 kJ/kg
	Melting Point	37°C

Using Egolf and Manz's equations, the enthalpy and specific heat curves were derived for the three cases analyzed (see Table 3.1.3). As mentioned earlier, the specific heat curve is dependent on the enthalpy curve, which is dependent on the experimental values of τ_1 and τ_2 . As a result, for a PCM that has unknown tau values, the two curves are a result of the "best guess" for the tau values. Case 1 was used as a baseline with both tau values equal to 10. Case 2 used the average values of tau based on the three PCM's presented by Egolf and Manz (both values are smaller than Case 1). Finally, for Case 2 the tau values were increased to 20. The three cases provide an evaluation of the impact of the tau values on the specific heat curve, and resulting temperatures. Figure 3.1.4 shows the predicted enthalpy curve for Case 1 while Figure 3.1.5 shows the specific heat curves for Cases 1 through 3. Figure 3.1.5 demonstrates that smaller tau values lead to a larger and steeper peak than do larger values of Tau.

Table 3.1.3. Summary of Tau Value for Cases Analyzed in Solid Works

Case	Tau ₁	Tau ₂
1	10	10
2	7.48	5.85
3	20	20

Figure 3.1.4 Predicted Enthalpy Curve for Case 1

Figure 3.1.5. Predicted Specific Heat Curves for Cases 1 through 3

To try and replicate the results presented by Nayak, et al., the three tau value cases (Table 3.1.3) were run in SolidWorks for the domain presented in Figure 3.1.2 with the specified boundary conditions, and material properties from Table 3.1.2. The Egolf and Manz model specific heat curve (Figure 3.1.5) was used to set a variable specific heat in the SolidWorks material. As SolidWorks Simulation does not account for natural convection within the fluid, and the Egolf and Manz model is an energy based method, the energy stored in the PCM was compared rather than the temperature differences. The stored energy value from the Nayak paper was determined by the temperature of the chip at 3600 seconds, as the natural convection normalizes the temperature of the chip at the phase change temperature, providing a good approximation of the PCM temperature. Table 3.1.4 presents the results of the SolidWorks simulations for the stored energy which was calculated using the average temperature at 3600 seconds, and the calculated enthalpy curve for each case.

Table 3.1.4 reveals that the Egolf and Manz model applied to a SolidWorks simulation provided good agreement with the results presented by Nayak with the greatest percent error being only -1.81%. The amount of stored energy increased as the tau values were increased. Figure 3.1.6 compares the temperature of the chip with time for Nayak's results and the SolidWorks variable specific heat model, and shows that the SolidWorks model over predicts the temperature of the chip. This is most likely a result of not being able to account for the natural convection within the melted PCM. Ultimately, the variable specific heat model gave decent results for the finned heatsink case, and the study took less than 10 minutes to solve – a fraction of the time it would take to solve this problem in a CFD study.

Table 3.1.4. Stored Energy Results at 3600 Seconds

Case:	Nayak, et al.	1	2	3
Stored Energy (kJ):	12.16	11.97	11.94	12.22
% Error	n/a	-1.56%	-1.81%	0.49%

Figure 3.1.6. Comparison of Chip Temperature for Variable Specific Heat Model and CFD Simulation

Nayak, et al., present the results of their simulation for a case that does not account for convection compared to a case that does account for convection. For this case, the domain was modified by removing the fins that are shown in Figure 3.1.2. Figure 3.1.7 presents the new domain and Figure 3.1.8 shows the new mesh. The same boundary conditions and material properties as in the original case were applied in this model.

Figure 3.1.7. Convection vs. No Convection Domain

Figure 3.1.8. SolidWorks Mesh Used for Nayak Replication with no Convection

Figure 3.1.9 shows the temperature results of the SolidWorks simulation compared to the cases presented by Nayak with and without convection. The figure shows that the temperatures determined in SolidWorks using the derived specific heat curve agree relatively well with the case that does not account for convection. Figure 3.1.10 shows a zoomed in view of Figure 3.1.9 and demonstrates that the discrepancy between the temperatures between the two simulation methods begins at the melting point. This could be a result of the unknown tau values for this particular phase change material, discretization error, or due to the size of the time step used for the transient simulation (2 second time steps for 3600 seconds).

Based on these results, it is evident that the Egolf and Manz model fails to accurately capture the behavior of the phase change material during a transient melting process. The error is most likely due to the fact that the model does not account for natural convection that occurs in the melted region of the phase change material which helps to keep the temperature of the system at a more uniform distribution and provides greater heat transfer due to the mixing of the melted PCM. However, Figure 3.1.9 does indicate that the model works in a transient process up until, and slightly past the melting point. This makes sense as heat transfer is solely through conduction up until the melting point, and still dominated by conduction immediately after melting begins. Thus, this model is useful in analyzing different designs to determine what combination of parameters will allow the PCM to reach its melting point the quickest. A limitation is that one cannot simultaneously optimize the conduction (pre melting) and natural convection (post melting) regimes. A separate tool is needed to evaluate what happens after the melting process begins.

Figure 3.1.9. SolidWorks Results Compared to Nayak, et al. Results for Temperature vs. Time

Figure 3.1.10. Zoomed View of SolidWorks Results Compared to Nayak, et al. Results (Less Convection Case)

3.2 SolidWorks Simulation

Based on the analysis presented earlier, it is possible to use a derived specific heat curve to analyze a PCM in SolidWorks up until the melting point. This method was applied to determine the values for the fin thickness, fin spacing, and base thickness, as presented in Figure 3.2.1, of the copper heat sink for the thermal storage unit. A mesh and time step convergence were performed, and then various design configurations were analyzed in SolidWorks. Sasol's H1 wax, which was previously discussed, was chosen as the PCM due to its high latent heat, high thermal conductivity, and its ability to withstand high temperatures without degrading. Table 3.2.1 presents the material properties used for H1 wax in this study. Figure 3.2.2 shows the curves that were generated for the enthalpy and specific heat of the material using the method discussed in Section 3.1. τ_1 was set to 5, and τ_2 was set to 10 somewhat arbitrarily as experimental data was not available, and the results of Section 3.1 show that the tau values had little impact on the end result.

For this analysis, the following boundary conditions were used:

1. The base of the heat sink was kept at a constant 400°F
2. The initial temperature was set to 70°F (room temperature)
3. The right side was treated as a line of symmetry
4. All outer surfaces were treated as insulated

400°F was chosen as a constant temperature for the base of the heatsink even though Figure 1.1.9 shows that the Sasol H1 wax will degrade if it is heated beyond this point (approximately 200°C). The temperature could be maintained at 400°F by moving the solar concentrator out of focus - removing the power input from the sun - when a receiver plate (heatsink base) reaches this temperature. The height of the fins was kept constant at 3.00 inches, and 0.25 inches of PCM were placed above the fins.

Figure 3.2.1. Thermal Storage Unit Heat Sink Schematic

Table 3.2.1. Sasol H1 Wax Material Properties

Density	Thermal Conductivity	Specific Heat	Latent Heat	Melting Point
940 kg/m ³	0.7 W/mK	2.1 J/gK	194 J/g	93.3°C (200°F)

Figure 3.2.2. Specific Heat (Top) and Enthalpy (Bottom) Curves for Sasol H1 wax

3.2.1 Mesh Convergence

The first step in the analysis was to determine an appropriate mesh spacing by performing a convergence study. Due to the thin features on the copper heat sink, a mesh half the size applied to the phase change material was used. Four meshes were analyzed before reaching convergence. Table 3.2.2 states the meshes that were used and Figure 3.2.4 shows the resulting temperature curve as a function of time with the axes positioned to show the region around the melting point. For the mesh convergence study, a 0.5 second time step was used. Figure 3.2.3 shows the mesh that was used, and where the boundary conditions were applied.

Figure 3.2.3. Variable Specific Heat Model Mesh and Boundary Conditions

Table 3.2.2. Summary of Meshes Analyzed

Mesh	PCM Mesh Spacing (in)	Heat Sink Mesh Spacing (in)
1	.125	.0625
2	.0625	.03125
3	.03125	.012625
4	.09375	.046875

Figure 3.2.4. Mesh Convergence Study Results

Ultimately, it was determined that the Mesh 2 spacing led to convergence, and that the added accuracy of the solution did not justify the additional time required to perform the analysis with the Mesh 3 spacing.

3.2.2 Time Step Convergence

After determining an appropriate mesh spacing, a time step convergence study was performed to determine the optimal time step. The mesh spacing determined in the mesh convergence study was used. Seven time steps were analyzed. A sensor was created to determine the temperature of a particular node throughout all of the different time steps that were analyzed. Figure 3.2.5 shows the results of the study with the temperature of the node presented on the y-axis and one over the time step presented on the x-axis and Table 3.2.3 presents the results numerically.

Table 3.2.3. Summary of Time Step Convergence Study

Time Step (sec)	40	20	10	1	.5	.1	.05
1/Time Step (1/sec)	.025	.05	.1	1	2	10	20
Node Temperature at Specified Time (°F)	206.796	211.833	214.03	215.543	215.448	215.375	215.355

Figure 3.2.5. Time Step Convergence Results

Ultimately, a 0.5 second time step was chosen for the design studies. Figure 3.2.5 clearly shows that the temperature value of the node levels out past a 0.5 second time step, indicating that using a smaller time step will increase the computation time without providing additional accuracy.

3.2.3 SolidWorks Transient Analysis

Once the appropriate mesh and time step were determined, various design configurations were analyzed to determine the combination of parameters that would allow the PCM to reach its melting point the quickest. Table 3.2.4 summarizes the cases that were run, and Figures 3.2.6 through 3.2.11 show the results of the studies. The results for fin spacing are not include here as the spacing of the fins had the largest effect on the transient performance of the heat sink. The smaller the distance between fins, the quicker the PCM reached the melting point. The spacing of the fins is more a function of how much copper is allowable in the heatsink due to weight and cost constraints. For all cases in this study the fin spacing was set to 0.59 inches.

Table 3.2.4. Summary of Cases Evaluated

Case	Base Thickness (in)	Fin Thickness (in)	Fin Spacing (in)
Base Variations			
1	0.0625	0.125	0.59
2	0.125	0.125	0.59
3	0.1875	0.125	0.59
4	0.25	0.125	0.59
Fin Variations			
5	0.1875	0.0625	0.59
6	0.1875	0.125	0.59
7	0.1875	0.1875	0.59
8	0.1875	0.25	0.59
Aluminum Heatsink			
9	0.1875	0.125	0.59

Figure 3.2.6 shows that as the base of the heat sink increases in thickness, the average temperature of the PCM takes longer to increase; however, the differences between cases are negligible. Figure 3.2.7 shows that the same trend exists when the minimum temperature of the PCM is evaluated. This makes sense for a constant temperature boundary condition such as the one imposed in this simulation. For cases where heat needs to be transported horizontally, as well as vertically, the base thickness would likely have a larger impact.

Figure 3.2.6. Average Temperature of PCM vs. Time for Cases 1-4

Figure 3.2.7. Minimum Temperature of PCM vs. Time for Cases 1-4

Figure 3.2.8 shows that increasing the fin thickness allows the average temperature of the PCM to increase more rapidly. This is especially apparent in Figure 3.2.9 which shows the minimum temperature in the PCM. For the 0.1875 in and 0.25 inch fins the minimum temperature exceeds the melting temperature ($\sim 200^{\circ}\text{F}$), while the other cases have not yet reached this point. The benefits of increasing the fin thickness appear to decrease after the fin thickness passes 0.1875 inches. The effects of using aluminum, instead of copper, were also compared. The results, shown in Figure 3.2.10 and 3.2.11, show that copper is a much better metal for the heatsink.

Figure 3.2.8. Average Temperature of PCM vs. Time for Cases 5-8

Figure 3.2.9. Minimum Temperature of PCM vs. Time for Cases 5-8

Figure 3.2.10. Average Temperature of PCM vs. Time for Copper and Aluminum (Case 6 and 9 Comparison)

Figure 3.2.11. Minimum Temperature of PCM vs. Time for Copper and Aluminum (Case 6 and 9 Comparison)

3.3 Computational Fluid Dynamic Validation

Before performing a computational fluid dynamic, CFD, analysis of the cases previously evaluated in SolidWorks, an existing study was replicated to prove that Fluent could be properly used to perform an analysis of a melting phase change material. E. Assis, et al. (2006) evaluated the melting of a paraffin wax within a glass sphere. Their experimental set-up is shown in Figure 3.3.1. A water bath was used to keep the outside surface of the glass sphere at a constant temperature, and the melting process was observed using a camera. In addition to the experiment, they performed a numerical analysis in Fluent and found a good agreement between their experimental and numerical results (see Figures 3.3.2 – 3.3.3).

Figure 3.3.1. Experimental Set-Up (Assis, 2006)

Figure 3.3.2. Plot of Melt Fraction vs. Time for the Experiment and Fluent Simulation (Assis, 2006)

Figure 3.3.3. Comparison of Experimental (a) and Numerical (b) Melting Fronts (Assis, 2006)

3.3.1 Computational Procedure

Figure 3.3.4 shows the model that was used to replicate the experiment, and that was re-created in this thesis to validate setting up a PCM melting problem in Fluent. An 80mm diameter sphere was used for this replication. A two-dimensional, axial symmetric model was created to reduce the computational complexity of the simulation. The Fluent simulation calculated the conduction in the solid portion of the PCM, convection in the liquid portion of the PCM, and convection in the air at the top of the model using the volume-of-fluid, VOF, in Fluent. This model tracks the volume fraction, α_n , of a fluid in a cell where $\alpha_n = 0$ means the cell is empty of the nth fluid, $\alpha_n = 1$ means the cell is filled with the nth fluid, and α_n between 0 and 1 means the cell is partially filled with the nth fluid and another fluid (a fluid to fluid interface).

Figure 3.3.4. Fluent Computational Domain (Assis, 2006)

The volume of fluid method then uses the following forms of the governing conservation equations (presented in indicial notation):

- Continuity

$$\frac{\partial \alpha_n}{\partial t} + u_i \frac{\partial \alpha_n}{\partial x_i} = 0$$

- Momentum

$$\frac{\partial}{\partial t}(\rho u_i) + \frac{\partial}{\partial x_j}(\rho u_j u_i) = \mu \frac{\partial^2 u_i}{\partial x_j \partial x_i} - \frac{\partial p}{\partial x_i} + \rho g_i + S_i$$

- Energy

$$\frac{\partial}{\partial t}(\rho \bar{h}) + \frac{\partial}{\partial x_i}(\rho u_i \bar{h}) = \frac{\partial}{\partial x_i} \left(k \frac{\partial T}{\partial x_i} \right)$$

In the energy equation, $\bar{h}$ is defined as the specific enthalpy. The specific enthalpy equals the sum of the sensible enthalpy and the enthalpy due to phase change (Latent Heat, L).

- Specific Enthalpy

$$\bar{h} = \bar{h}_{ref} + \int_{T_{ref}}^T c_p dT + \gamma L$$

Where γ is defined as the liquid fraction:

- Liquid Fraction

$$\gamma = 0 \text{ if } T < T_s, \quad \gamma = 1 \text{ if } T > T_1$$

$$\gamma = \frac{T - T_s}{T_1 - T_s} \text{ if } T_s < T < T_1$$

In the momentum equation S_i is the source term. It is defined as follows:

- Source Term

$$S_i = -A(\gamma)u_i$$

Where $A(\gamma)$ is the porosity function, defined as:

- Porosity

$$A(\gamma) = \frac{C(1 - \gamma)^2}{\gamma^3 + \varepsilon}$$

- $\varepsilon = 0.001$ (small computational constant to avoid division by zero)
- $C = 10^5$ typical (constant reflecting the morphology of the melting front)

3.3.2 Fluent Simulation Set-Up

The following discussion specifies how Fluent 16.2 was used to re-create the results that Assis (2016) found for the melting characteristics of wax in a glass sphere. Due to processing power limitations, some of the settings are different than those used by Assis. Fluent was set to use double precision computing. The study was set to a Pressure-Based solver with Absolute Velocity Formulation. A transient study was used with a time step of 0.005 seconds. Additionally, the model was simplified to be axisymmetric as can be seen in Figure 3.3.4. The volume of fluid method was turned on, and all settings were left in their default state. Air was set as the primary phase, and the wax was set as the secondary phase. The flow was assumed to be laminar, and the energy equation was turned on. The solidification and melting option was also used.

The PISO scheme was used, and the Gradient was set to Least Squares Cell Based; pressure was set to PRESTO!; Density, Momentum, and Energy were set to Second Order Upwind; Volume Fraction was set to compressive. The First-Order Implicit setting was used for the transient formulation, and non-iterative time advancement was used. Non-iterative time advancement provided accurate results while minimizing the calculation time. Tables 3.3.1 and 3.3.2 specify the material properties that were used for the wax and glass wall. The standard properties for air were used with the density of air treated as an ideal gas. Figure 3.3.5 shows the mesh that was used. The outside wall of the glass was set to a constant 39°C (10°C above the average melting temperature of the wax), and the opening was set to a pressure-outlet with a backflow temperature of 39°C for the first trial. The second trial set both temperatures to 35°C.

Table 3.3.1. Wax Properties

Property	Solid (0°C - 28°C)	Liquid (30°C)	Liquid (100°C)
Density (kg/m³)	870	760	734.3
Specific Heat (J/kg-K)	2400	1890	1890
Thermal Conductivity (W/m-K)	0.24	0.21	0.21
Viscosity (kg/m-s)	0.00342		
Molecular Weight (kg/kgmol)	300		
Standard State Enthalpy (j/kgmol)	n/a		
Reference Temperature (°C)	25		
Thermal Expansion Coefficient (1/K)	0.001		
Pure Solvent Melting Heat (J/kg)	179,000		
Solidus Temperature (°C)	28		
Liquidus Temperature (°C)	30		

Table 3.3.2. Glass Properties

Density (kg/m³)	2500
Specific Heat (J/kg-K)	800
Thermal Conductivity (W/m-K)	0.81

Figure 3.3.5. Mesh Used in Simulation

3.3.3 Results of Replication Study

Good agreement was found between the results Assis found, and what was found by the replication study. Each simulation took approximately a day to solve. Figure 3.3.6 shows the melting front for the 10°C above the average melting temperature case. It shows that the solid wax sinks to the bottom of the enclosure as the wax near the walls melts. The shapes of the solid wax are consistent with what was found by Assis as shown in Figure 3.3.3. Figure 3.3.7 compares the 6°C above the average melting temperature case with what Assis found numerically and experimentally showing that the results were accurately replicated. Figure 3.3.8 compares the 10°C above average melting temperature case. For this case the agreement between the paper's results and the replicated results was poor. However, when the opening in the sphere was closed off (which was used as an initial step in replicating the model), the results matched nearly exactly what Assis found.

Figure 3.3.6. Simulated Melting Front at 2, 5, 10, 15, 20, and 25 Minutes (Starting at the Top Left) for 10°C Above the Average Melting Temperature Case

Figure 3.3.7. Comparison of Assis (2006) Experimental and Numerical Results with Replicated Results at 6°C Above Average Melting Temperature

Figure 3.3.8. Comparison of Assis (2006) Numerical Results with Replicated Results at 10°C Above Average Melting Temperature

3.3.4 Validation Conclusion

The simulation presented by E. Assis, et al. (2006) was replicated in Fluent 16.2. The results found by Assis were accurately reproduced, showing that a simulation of a melting phase change material could accurately be performed in Fluent. This provides credibility for the CFD studies that will be discussed later on in this report.

3.4 Boussinesq Cases

After replicating the work of E. Assis, et al. (2006), CFD analysis of the cases that were run in SolidWorks could be performed in Fluent. To start off, the variable density of the phase change material was neglected, and the Boussinesq approximation was used instead to simulate the effects of buoyancy within the computational domain. The model was created using ANSYS Design Modeler, and was exactly the same as that shown in Figure 3.2.1. The same cases stated in Table 3.2.4 were run.

3.4.1 Boussinesq Theory

The theory and set-up for the Boussinesq cases follow the same principals laid out in Section 3.3.1, except the Boussinesq cases did not require the volume of fluids method to be used. The volume of fluids method was not required as an air gap was not necessary for these simulations because the density is held constant. This prevents the PCM from expanding during the melting process so the computational domain does not need to provide room for the PCM to expand as it melts and increases in temperature. Thus, the continuity equation does not need to track the volume fraction term, α_n . Additionally, the Boussinesq approximation modifies the momentum equation as follows – specifically the buoyancy force term:

- Momentum

$$\frac{\partial}{\partial t}(\rho_0 u_i) + \frac{\partial}{\partial x_j}(\rho_0 u_j u_i) = \mu \frac{\partial^2 u_i}{\partial x_j \partial x_i} - \frac{\partial p}{\partial x_i} + \rho_0 \beta (T - T_0) g_i + S_i$$

Where ρ_0 is the constant density of the fluid, T_0 is the operating temperature, and β is the thermal expansion coefficient. This approximates the buoyancy force term as follows:

- Boussinesq Buoyancy Force Approximation

$$(\rho - \rho_0)g \approx -\rho_0 \beta (T - T_0)g$$

As a result, the Boussinesq approximation allows for natural convection cells to form within a fluid without having to know the relationship between density and temperature for the fluid being modeled. However, this approximation works best for cases where $\beta(T - T_0) \ll 1$, so it will only provide accurate results for cases where the temperature difference within the fluid is small.

3.4.2 Boussinesq Case Set-Up

The Boussinesq cases were set up the same way as the Assis paper replication case was set up, just with the volume of fluids method turned off. Table 3.4.2 summarizes the material properties that were used for the PCM, Sasol Wax's H1 Fischer Tropsch Paraffin wax, and Table 3.4.1 states the material properties of the copper heatsink. The material properties for the Sasol H1 wax were obtained from Mngomezulu's (2011) and Sasol Waxes informational material available from their website. The viscosity had to be approximated by comparison to typical Sasol paraffin waxes. A separate mesh and convergence study were performed to determine the appropriate mesh size, and time step size for these cases. The

reference temperature, T_0 , was set to 100°C – the melting point of the PCM – so that the liquid wax would have a lighter density than the solid wax, replicating the effects of melting when using a temperature dependent density. The bottom wall set to a constant temperature of 400°F (204.4°C), the right wall was set to a symmetry boundary condition, and all other walls were set to insulated. The computational domain was initialized at 70°F (21.11°C).

Table 3.4.1. Copper Heatsink Material Properties

Density:	8900 kg/m ³
Specific Heat:	390 J/kg-K
Thermal Conductivity:	390 W/m-K

Table 3.4.2. Sasol H1 Wax Material Properties

Density	940 kg/m ³
Specific Heat	2100 J/kg-K
Thermal Conductivity	0.7 W/m-K
Viscosity	0.008695 kg/m-s
Thermal Expansion Coefficient	0.001 1/K
Pure Solvent Melting Heat	194200 J/kg
Solidus Temperature	100°C
Liquidus Temperature	100°C

3.4.3 Mesh Convergence

Figure 3.4.2 shows the liquid fraction of the melting PCM with respect to time, and Figure 3.4.3 shows the average temperature of the melting PCM with respect to time for the 4 mesh sizes considered (see Table 3.4.3). After running the studies convergence was found between the third and fourth mesh sizes. The change in liquid fraction verse time, and average temperature verse time was small between the third and fourth mesh, so the third mesh size, 0.03125 in, was chosen as the mesh size for the Boussinesq cases. Figure 3.4.1 shows the final mesh and boundary conditions.

Table 3.4.3. Mesh Sizes Considered

Mesh	Initial Size (in)
Mesh 1	0.0625
Mesh 2	0.046875
Mesh 3	0.03125
Mesh 4	0.02083

Figure 3.4.1. Mesh and Boundary Conditions Used for Fluent Simulations

Figure 3.4.2. Liquid Fraction vs. Time Boussinesq Mesh Convergence Study

Figure 3.4.3. Average Temperature vs. Time Boussinesq Mesh Convergence Study

3.4.4 Time Step Convergence

After performing the mesh convergence study, a time step convergence study was run to determine the appropriate time step size for the Boussinesq cases. Figure 3.4.4 shows the liquid fraction of the melting PCM with respect to time, and Figure 3.4.5 shows the average temperature of the melting PCM with respect to time for the four, time step sizes considered (see Table 3.4.4). The time step was initially set to 0.0025 seconds. This number was chosen based on what was found to be required by E. Assis (2006) in the paper that was replicated for this thesis. After running the first trial the time step size was increased to 0.005 seconds, but the results were essentially unchanged. This trend followed all the way up until 0.05 second time step increments. However, when the time step was increased to 0.1 seconds, the program failed to run as divergence was detected in the solution. Based on this convergence study, it was determined that the time step could be set to 0.05 seconds (step 4), while still obtaining results that were not affected by the size of the time step.

Table 3.4.4. Time Step Sizes Considered

Time Step	Initial Size (s)
Step 1	0.0025
Step 2	0.005
Step 3	0.01
Step 4	0.05

Figure 3.4.4. Liquid Fraction vs. Time for Boussinesq Time Step Convergence

Figure 3.4.5. Average Temperature vs. Time for Boussinesq Time Step Convergence

3.4.5 Results

Figure 3.4.6 – 3.4.10 show the results of the Boussinesq cases. The same trends that were observed in the SolidWorks cases are seen here as well. The thickness of the base is seen to have a minimal impact on the results (Figures 3.4.8 and 3.4.9), while the thickness of the fins in the heatsink have a significant impact on how quickly the PCM melts. The benefits of increasing the fin thickness decrease with each additional trial. Figures 3.4.6 and 3.4.7 show that the benefits of increasing the fin thickness to 0.25 inches from 0.1875 inches is small, so the 0.1875-inch fin may be the best choice for maximizing heat sink performance while minimizing the amount of copper required in a thermal storage unit.

Figure 3.4.10 shows how the melt front and natural convection cells vary with time. The melting of the PCM occurs mostly along the edges of the heatsink fins. Additionally, the significant natural convection cells show how buoyancy plays an important role in the melting process of a phase change material. This figure also shows one of the limitations of the Boussinesq approximation. The melt front at 120 seconds shows a significant portion of solid wax suspended in liquid wax. In reality, this portion of solid wax would sink to the bottom of the heatsink, but with the Boussinesq approximation it remains suspended in place. Ultimately, the Boussinesq cases and SolidWorks cases reveal the same trends in how performance of the heatsink is related to the parameters that have been evaluated.

Figure 3.4.6. Liquid Fraction vs. Time for Fin Thickness Cases

Figure 3.4.7. Average Temperature vs. Time for Fin Thickness Cases

Figure 3.4.8. Liquid Fraction vs. Time for Base Thickness Cases

Figure 3.4.9. Average Temperature vs. Time for Base Thickness Cases

Figure 3.4.10. Typical Melting Front with Velocity Vectors Superimposed for Boussinesq Cases (30, 60, 90, 120 Seconds from Left to Right)

3.5 Variable Density Fluent Cases

After running the Boussinesq cases, the cases were re-run using a variable density model. The base height cases were not run for the variable density cases because the variable density simulations took approximately two days to run, and the SolidWorks analysis and Boussinesq cases showed that varying the height of the base had little effect on the system performance. Additionally, the same mesh that was used for the Boussinesq cases was used for the variable density cases. All material properties were set the same, except the density of the Sasol H1 wax was set to 940 kg/m^3 up until 99.5°C , and 790 kg/m^3 at 100.5°C with a linearly varying relationship between 99.5°C and 100.5°C . The properties of Sasol H1 wax as a liquid could not be found, so the wax's density as a liquid was approximated from similar paraffin waxes.

Figures 3.5.1 and 3.5.2 show the results of the variable density simulations. The same trends seen in the Boussinesq cases and SolidWorks studies are observed for both the liquid fraction verse time, and the average temperature of the PCM verse time. Increasing the fin thickness decreases the time it takes for the PCM to completely melt, and increases the average temperature of the PCM. Just like the other two methods showed, the impact of increasing the fin thickness decreases each time the thickness is increased. This effect is most evident between the 0.1875-inch fin and 0.25-inch fin cases, validating the observations made with the Boussinesq method and the SolidWorks studies. Figure 3.5.3 shows a typical melting front for the variable density cases. It shows the same trends observed in Figure 3.4.10, except the melting front at 120 seconds shows that when portions of the solid PCM breakaway from the main portion of the wax, they sink to the bottom of the heatsink.

Figure 3.5.1. Liquid Fraction vs. Time for Variable Density Fin Cases

Figure 3.5.2. Average Temperature vs. Time for Variable Density Fin Cases

Figure 3.5.3. Typical Melting Front with Velocity Vectors Superimposed for Variable Density Cases (30, 60, 90, 120 Seconds from Left to Right)

3.6 Comparison of Results for all Cases

Figures 3.6.1 through 3.6.4 show the results of average temperature verse time for the 4 different fin sizes with all three methods used to analyze the PCM based heatsink on each graph. Figures 3.6.5 through 3.6.8 show the results of liquid temperature verse time for the 4 different fin sizes with the results of the Boussinesq and variable density cases (the liquid fraction cannot be obtained from the SolidWorks FEA analysis). All eight of these graphs show that the Boussinesq model over predicts the performance of the heat sinks. Examining Figures 3.4.10 and 3.5.3 shows that the natural convection occurring within the PCM is much greater in the Boussinesq cases than in the variable density cases. This causes more heat transfer within the heatsink, leading to quicker melt times and higher average temperatures. The over prediction of natural convection from the Boussinesq model is likely a result of not meeting the requirement for the Boussinesq approximation that $\beta(T - T_0) \ll 1$. The large initial temperature differences between the base of the heatsink, 400°F (204.44°C), and the initial temperature of the PCM, 70°F (21.11°C), lead to $\beta(T - T_0) = .1833$, so the natural convection becomes significant as soon as melting occurs (compare 30 second melt front of Figure 3.4.10 and 3.5.3).

Figures 3.6.1 through 3.6.4 show that the results from the SolidWorks studies agree very well with the results from the Fluent variable density cases. This shows that the variable specific heat model did a good job capturing how the phase change material would react for this set-up. The agreement between these two models indicates that the heat transfer within the heatsink is predominantly conduction driven. The fins allow for the heat from the base of the heatsink to be quickly dissipated into the phase change material, minimizing the necessity of natural convection to effectively melt the PCM.

While the results of the Boussinesq cases did not agree with the results of the SolidWorks cases, all three models showed the same trends. The base thickness had little effect on the time for complete melting of the PCM and average temperature of the PCM, while the fin thickness was able to significantly decrease the melting time and average temperature of the PCM. All three models revealed that the benefits of increasing the fin thickness significantly decreased when the thickness was increase from 0.1875 inches to 0.25 inches. Thus, this study has shown that a variable specific heat model can be used as a design tool to quickly compare multiple cases and design iterations before performing a full-fledged CFD analysis. With the variable specific heat model, results could be obtained within 20 minutes of computational time, while the Fluent variable density model required around 2 days on average to acquire the same results. Additionally, cases that are primarily conduction driven – such as the PCM based fin heatsink analyzed in this thesis – can find accurate temperature results using the variable specific heat FEA method. However, what is actually occurring within the PCM can only be captured by a CFD analysis.

Figure 3.6.1. Average Temperature Comparison for 0.0625-inch Fin

Figure 3.6.2. Average Temperature Comparison for 0.125-inch Fin

Figure 3.6.3. Average Temperature Comparison for 0.1875-inch Fin

Figure 3.6.4. Average Temperature Comparison for 0.25-inch Fin

Figure 3.6.5. Liquid Fraction Comparison for 0.0625-inch Fin

Figure 3.6.6. Liquid Fraction Comparison for 0.125-inch Fin

Figure 3.6.7. Liquid Fraction Comparison for 0.1875-inch Fin

Figure 3.6.8. Liquid Fraction Comparison for 0.25-inch Fin

4. SYSTEM DESIGN

Figure 4.0.1. Representation of Possible System Design

The following section discusses the design of a potential solar thermal system incorporating a phase change based thermal storage unit. Figure 4.0.1 presents a representation of how the system could be incorporated into a house. A 2 square-meter Scheffler Solar Concentrator is placed on the deck of a house and concentrates sunlight onto the thermal storage unit, which is embedded in the wall of a house. In this section, a potential solar thermal loop will be presented, a potential solar thermal storage unit will be discussed, and the heating potential of a thermal storage unit will be shown. Figure 4.1.2 shows how a Scheffler Concentrator can be used to provide a power input into a thermal storage unit.

4.1 Solar Thermal Loop Design

Figure 4.1.1. Simplified Solar Thermal Loop Schematic

Figure 4.1.1 presents the preliminary system schematic and shows five main sections which are discussed below.

4.1.1 Pump (P1)

A Grundfos UPS50-80/4 pump was selected as a potential candidate.

4.1.2 Expansion Tank

The expansion tank will allow for the heat transfer fluid to safely expand and contract as it is heated and cooled.

4.1.3 Flat Plate Collector

The flat plate collector is included in the system to preheat the heat transfer fluid before it enters the PCM thermal storage unit. This will minimize the heat load on the storage unit, and allow for additional energy to be stored. Additionally, if a day is somewhat cloudy, the flat plate collector will still allow for hot water to be generated. The thermal storage unit could run without a flat plate collector, so this is an optional element of the loop.

4.1.4 Phase Change Material Thermal Storage Unit (PCM)

The phase change material thermal storage unit is the core element of this solar thermal loop. It interfaces with a Scheffler Solar Concentrator (see Figure 4.0.1 and Figure 4.1.2), and stores the solar radiation in the phase change material within the unit. This component raises the temperature of the heat transfer fluid, and allows excess energy to be stored so water can be heated even when solar radiation is not currently available.

4.1.5 Solar Hot Water Storage Tank (Hot Water Storage)

The solar hot water tank consists of a heat exchanger, which is connected to the solar thermal loop. This particular heatsink utilizes a closed loop, but an open loop system could be used as well. The tank also includes an electric heater to provide additional heating in case the solar thermal system is unable to adequately meet the hot water heating demand. The electric heater is an optional feature of this design.

4.1.6 Scheffler Concentrator

Figure 4.1.2 shows the basic principles behind a Scheffler Solar Concentrator. The dish component of a Scheffler Concentrator is a portion of a parabola. The parabola is rotated about an axis that is parallel to the rotation of the earth, and tracks the sun as it moves through the sky. The radiation from the sun is reflected upwards to the focal point of the parabola (which is in line with the axis of rotation). This concentrates the radiation onto a much smaller area than the overall size of the dish, creating a high heat fluxes which can be used for heating a thermal mass. The shape of the parabola is adjusted throughout the year to adapt to the sun's movement at different times of the year. Thus, the Scheffler Solar Concentrator allows for a fixed object to have solar radiation concentrated onto it throughout the year.

Figure 4.1.2. Scheffler Solar Concentrator Interfacing with a House

(solarcooking.wikia.com)

4.2 Thermal Storage Unit Design

Figure 4.2.1. Thermal Storage Unit Design

Figure 4.2.1 shows the preliminary design of the thermal storage unit. This design is from a very early portion of this thesis, and does not reflect what a finalized thermal storage unit would look like. As has previously been discussed, fins play an important role in bringing heat into the phase change material, and would need to be included in the design. Additionally, as Figure 4.1.2 shows, the radiation from the Scheffler Solar Concentrator would be directed onto the bottom surface of the unit, so the receiver plate would actually be on the bottom of the storage unit. While these are significant aspects of the design, the thermal storage unit presented in Figure 4.2.1 presents the core aspects of a phase change material based thermal storage unit. The design consists of the following main features:

4.2.1 Solar Radiation Receiver (1)

A Scheffler reflector will concentrate light on the solar radiation receiver. The receiver is shaped conically to catch any light that is diffused by the reflector. The receiver will have a specially coated copper plate to absorb the solar radiation and minimize losses.

4.2.2 Phase Change Material (2)

The unit will be filled with a Fischer-Tropsch Paraffin wax and HDPE mixture. The PCM governs the operating temperature of the unit and stores all of the energy that can be used to heat fluid as it runs through the unit.

4.2.3 Solar Radiation Receiver Plate (3)

The radiation receiver plate will transfer the heat from the receiver to the PCM. It will be made of copper and have copper piping, copper fins, heat pipes, or a combination of these features attached to it. These will help to evenly spread the heat throughout the thermal storage unit, effectively storing thermal energy.

4.2.4 Copper Heat Exchanger Piping (4)

The copper piping provides a pathway for the heat transfer fluid through the storage unit, and will allow the fluid to be adequately heated.

4.3 Heating Capabilities

Appendix C presents an EES code that was developed to analyze the performance of the phase change thermal storage unit presented in this thesis. Table 4.3.1 provides a summary of the results that were obtained when various temperature of heat transfer fluid were run through just the thermal storage unit. The table shows the temperature of the water exiting the thermal storage unit, and the amount of power required to heat the water as it flows through the unit. The results show that the thermal storage unit is capable of significantly raising the temperature of a fluid as it flows through the thermal storage unit. However, heating water that enters the system from a water storage tank, or utility line, requires a significant amount of power from the system.

Table 4.3.1. Thermal Storage Unit Performance

Inlet Temperature (°F)	Outlet Temperature (°F)	Heat Dissipation (kW)	Mass Flow Rate (GPM)
50	140	11.75	1.041
60	142.7	10.94	1.051
70	145.5	10.1	1.061
80	148.3	9.248	1.069
90	151.3	8.373	1.077
100	154.3	7.482	1.084
110	157.3	6.578	1.09
120	160.5	5.662	1.095
130	163.6	4.735	1.1
140	166.8	3.801	1.104

In Section 2.1 it was determined that approximately 23.4 gallons of water would be used per day in a solar house, and that an average of 19.8 MJ of energy would be required daily. In Section 2.2 it was found that 21.208 MJ/m² of beam radiation would be available on an average day during the Solar Decathlon (see Table 2.2.2). Assuming 70% efficiency for a two-square meter Scheffler Solar Concentrator, the available energy for the system will be 29.69 MJ, so there is enough energy to meet the daily load and store excess heat in a thermal storage unit. If the thermal storage unit is designed to be able to hold 15 MJ – a little more than the daily available solar energy minus the daily water load – the thermal storage unit would be able to heat around 22.8 gallons of water from 60°F to 142.7°F. This would nearly meet the hot water load of an entire day, just by allowing the PCM to transition from its liquid state, to its solid state. Ultimately, it is shown that a phase change material based thermal storage unit would be capable of heating water, and storing excess thermal energy in solar applications.

5. CONCLUSION AND SUMMARY

In this thesis the feasibility of a solar thermal storage unit was determined, the heat transfer mechanism was optimized, and the performance of a possible solar thermal system with a thermal storage unit was presented. The amount of hot water required for a typical solar house was determined, and it was found that a two-square meter Scheffler Solar Concentrator could supply enough energy to meet the demand of the house, and provide extra energy which could be stored in a thermal storage unit. The limiting factor for the system is the ability to quickly store the energy through melting a phase change material, utilizing the latent heat.

An evaluation of others' research found that the performance of a phase change material, based on its ability to quickly melt, could be increased by adding particles of a higher conductivity material. Copper and graphene particles were shown to increase the thermal conductivity, but the copper particles decreased the latent heat of the phase change material significantly more than the graphene particles. Additionally, the melting temperature of the phase change material could be altered by adding a plastic, such as high density polyethylene, to the mixture. While these methods helped to improve the performance of the phase change material, the melting process was further enhanced by adding a finned heatsink to the system.

The effects of the finned heatsink were evaluated using three analysis techniques – a variable specific heat finite element analysis, a computational fluid dynamic analysis using the Boussinesq approximation, and lastly, a computational fluid dynamic analysis taking into account the variation of density with temperature. The results were compared, and it was found that the variable specific heat model and computational fluid dynamic

analysis with variable density agreed well. This indicated that the finned heatsink made the melting process conduction dominated. The thickness of the fins was found to have the largest effect on the melting process. A 0.1875-inch thick fin provided the best performance while still trying to minimize the amount of copper in the heatsink. The thicker fins led to an isothermal temperature distribution within the fin, which helped to maximize the transport of energy from the heatsink to the phase change material.

It was also found that the melting process could be simulated using the variable specific heat model in an FEA program, rather than a CFD program. This provides significant time saving benefits. The FEA simulations took around 20 minutes to run in this thesis, while the variable density CFD studies took approximately two days to run. As a result, designers and analysts working with phase change material can use this model to quickly evaluate numerous designs before committing to running a full CFD analysis of the system design.

After the heatsink was optimized, the heating capabilities of a solar thermal system utilizing a thermal storage unit was analyzed. It was found that a thermal storage unit could provide enough energy to nearly meet an entire day's hot water load. Thus, this thesis showed that a phase changed material based thermal storage unit could be designed and developed to increase the abilities of a solar thermal system, making it more likely for solar thermal hot water to be implemented in homes.

REFERENCES

- Assis, E., L. Katsman, G. Ziskind, and R. Letan. "Numerical and Experimental Study of Melting in a Spherical Shell." *International Journal of Heat and Mass Transfer* 50.9-10 (2007): 1790-804. Web.
- Chintakrinda, Kireeti, Randy D. Weinstein, and Amy S. Fleischer. "A Direct Comparison of Three Different Material Enhancement Methods on the Transient Thermal Response of Paraffin Phase Change Material Exposed to High Heat Fluxes." *International Journal of Thermal Sciences* 50.9 (2011): 1639-647. Web.
- Fisher, Daniel E, et al. "An Enhanced Simulation Model for Building Envelopes with Phase Change Material". ASHRAE Transactions. Volume 119 Issue 2 (2013): 1. Web.
- Hottel, Hoyt C. "A Simple Model for Estimating the Transmittance of Direct Solar Radiation through Clear Atmospheres." *Solar Energy* 18.2 (1976): 129-34. Web.
- Liu, Benjamin Y.h., and Richard C. Jordan. "The Interrelationship and Characteristic Distribution of Direct, Diffuse and Total Solar Radiation." *Solar Energy* 4.3 (1960): 1-19. Web.
- Maydanik, Yu.f. "Loop Heat Pipes." *Applied Thermal Engineering* 25.5-6 (2005): 635-57. Web.
- Mhike, W., W.w. Focke, J.p. Mofokeng, and A.s. Luyt. "Thermally Conductive Phase-change Materials for Energy Storage Based on Low-density Polyethylene, Soft Fischer-Tropsch Wax and Graphite." *Thermochimica Acta* 527 (2012): 75-82. Web.
- Mngomezulu, M.e., A.s. Luyt, and I. Krupa. "Structure and Properties of Phase-change Materials Based on High-density Polyethylene, Hard Fischer-Tropsch Paraffin Wax, and Wood Flour." *Polymer Composites Polym Compos* 32.8 (2011): 1155-163. Web.
- Molefi, J. A., A. S. Luyt, and I. Krupa. "Investigation of Thermally Conducting Phase-change Materials Based on Polyethylene/wax Blends Filled with Copper Particles." *Journal of Applied Polymer Science J. Appl. Polym. Sci.* 116 (2010): 1766-774. Web.
- Nayak, K.c., S.k. Saha, K. Srinivasan, and P. Dutta. "A Numerical Model for Heat Sinks with Phase Change Materials and Thermal Conductivity Enhancers." *International Journal of Heat and Mass Transfer* 49.11-12 (2006): 1833-844. Web.
- Shatikan, Vartan. "Melting and Solidification of a Phase-Change Material with Internal Fins". Ben-Gurion University of the Negev (Thesis in the Department of Mechanical Engineering). 2004.
- Sotomayor, M.e., I. Krupa, A. Várez, and B. Levenfeld. "Thermal and Mechanical Characterization of Injection Moulded High Density Polyethylene/paraffin Wax Blends as Phase Change Materials." *Renewable Energy* 68 (2014): 140-45. Web.
- U.S. Department of Energy. Solar Decathlon 2015 Rules. August 31, 2015. Web. http://www.solardecathlon.gov/2015/assets/pdfs/2015_rules.pdf

APPENDICES

Appendix A: Engineering Equation Solver (EES) Code Used in Thesis

A1 Required Water from Hot Water Tank EES Code

Energy Balance on Mixing Operations

System Temperatures

$$T_{\text{hottank}} = 140 \cdot \left| 0.55555556 \cdot \frac{C}{F} \right| \quad \text{Hottank Temperature [C]}$$

$$T_{\text{reservoir}} = 65 \cdot \left| 0.55555556 \cdot \frac{C}{F} \right| \quad \text{Reservoir Temperature [C]}$$

$$T_{\text{shower}} = 110 \cdot \left| 0.55555556 \cdot \frac{C}{F} \right| \quad \text{Shower Temperature [C]}$$

$$T_{\text{laundry}} = 100 \cdot \left| 0.55555556 \cdot \frac{C}{F} \right| \quad \text{Laundry Temperature [C]}$$

$$T_{\text{dishwasher}} = 120 \cdot \left| 0.55555556 \cdot \frac{C}{F} \right| \quad \text{Dishwasher Temperature [C]}$$

$$T_{\text{sink}} = 110 \cdot \left| 0.55555556 \cdot \frac{C}{F} \right| \quad \text{Sink Temperature [C]}$$

$$T_{\text{extra}} = 110 \cdot \left| 0.55555556 \cdot \frac{C}{F} \right| \quad \text{Excess Water Temperature [C]}$$

System Masses

$$M_{\text{shower}} = 240 \cdot \left| 0.003785412 \cdot \frac{m^3}{gal} \right| \cdot 1000 \quad \text{Shower Water Mass [kg]}$$

$$M_{\text{laundry}} = 110 \cdot \left| 0.003785412 \cdot \frac{m^3}{gal} \right| \cdot 1000 \quad \text{Laundry Water Mass [kg]}$$

$$M_{\text{dishwasher}} = 15 \cdot \left| 0.003785412 \cdot \frac{m^3}{gal} \right| \cdot 1000 \quad \text{Dishwasher Water Mass [kg]}$$

$$M_{\text{sink}} = 25 \cdot \left| 0.003785412 \cdot \frac{m^3}{gal} \right| \cdot 1000 \quad \text{Sink Water Mass [kg]}$$

$$M_{\text{extra}} = 60 \cdot \left| 0.003785412 \cdot \frac{m^3}{gal} \right| \cdot 1000 \quad \text{Excess Water Mass [kg]}$$

Energy Balance on Shower

$$M_{\text{hottank},s} \cdot T_{\text{hottank}} \cdot 4.179 + M_{\text{reservoir},s} \cdot T_{\text{reservoir}} \cdot 4.179 = M_{\text{shower}} \cdot T_{\text{shower}} \cdot 4.179$$

$$M_{\text{hottank},s} = M_{\text{shower}} - M_{\text{reservoir},s}$$

Energy Balance on Laundry

$$M_{\text{hottank},l} \cdot T_{\text{hottank}} \cdot 4.179 + M_{\text{reservoir},l} \cdot T_{\text{reservoir}} \cdot 4.179 = M_{\text{laundry}} \cdot T_{\text{laundry}} \cdot 4.179$$

$$M_{\text{hottank},l} = M_{\text{laundry}} - M_{\text{reservoir},l}$$

Energy Balance on Dishwasher

$$M_{\text{hottank},d} \cdot T_{\text{hottank}} \cdot 4.179 + M_{\text{reservoir},d} \cdot T_{\text{reservoir}} \cdot 4.179 = M_{\text{dishwasher}} \cdot T_{\text{dishwasher}} \cdot 4.179$$

$$M_{\text{hottank,d}} = M_{\text{dishwasher}} - M_{\text{reservoir,d}}$$

Energy Balance on Sink

$$M_{\text{hottank,sl}} \cdot T_{\text{hottank}} \cdot 4.179 + M_{\text{reservoir,sl}} \cdot T_{\text{reservoir}} \cdot 4.179 = M_{\text{sink}} \cdot T_{\text{sink}} \cdot 4.179$$

$$M_{\text{hottank,sl}} = M_{\text{sink}} - M_{\text{reservoir,sl}}$$

Energy Balance on Excess Water

$$M_{\text{hottank,ex}} \cdot T_{\text{hottank}} \cdot 4.179 + M_{\text{reservoir,ex}} \cdot T_{\text{reservoir}} \cdot 4.179 = M_{\text{extra}} \cdot T_{\text{extra}} \cdot 4.179$$

$$M_{\text{hottank,ex}} = M_{\text{extra}} - M_{\text{reservoir,ex}}$$

System Calcs

$$M_{\text{hottank}} = M_{\text{hottank,s}} + M_{\text{hottank,l}} + M_{\text{hottank,d}} + M_{\text{hottank,sl}} + M_{\text{hottank,ex}}$$

Mass of water required at 140F from the hottank

$$V_{\text{hottank}} = \frac{M_{\text{hottank}}}{1000} \cdot \left| 264.2 \cdot \frac{\text{gal}}{\text{m}^3} \right|$$

Volume of water required at 140F from the hottank [gal]

SOLUTION

Unit Settings: SI C kPa kJ mass deg

$$M_{\text{dishwasher}} = 56.78$$

$$M_{\text{hottank,d}} = 41.64$$

$$M_{\text{hottank,s}} = 545.1$$

$$M_{\text{reservoir,d}} = 15.14$$

$$M_{\text{reservoir,s}} = 363.4$$

$$M_{\text{sink}} = 94.64$$

$$T_{\text{hottank}} = 77.78 \text{ [C]}$$

$$T_{\text{shower}} = 61.11 \text{ [C]}$$

$$M_{\text{extra}} = 227.1$$

$$M_{\text{hottank,ex}} = 136.3$$

$$M_{\text{hottank,sl}} = 56.78$$

$$M_{\text{reservoir,ex}} = 90.85$$

$$M_{\text{reservoir,sl}} = 37.85$$

$$T_{\text{dishwasher}} = 66.67 \text{ [C]}$$

$$T_{\text{laundry}} = 55.56 \text{ [C]}$$

$$T_{\text{sink}} = 61.11 \text{ [C]}$$

$$M_{\text{hottank}} = 974.1$$

$$M_{\text{hottank,l}} = 194.3$$

$$M_{\text{laundry}} = 416.4$$

$$M_{\text{reservoir,l}} = 222.1$$

$$M_{\text{shower}} = 908.5$$

$$T_{\text{extra}} = 61.11 \text{ [C]}$$

$$T_{\text{reservoir}} = 36.11 \text{ [C]}$$

$$V_{\text{hottank}} = 257.3 \text{ [gal]}$$

A2 Clear Sky Model EES Code

$$\text{Lat} = 33.68$$

$$a_0 = 0.4237 - 0.00821 \cdot [6 - A]^2$$

$$r_0 = 0.97$$

$$a_1 = 0.5055 + 0.00595 \cdot [6.5 - A]^2$$

$$r_1 = 0.99$$

$$k = 0.2711 + 0.01858 \cdot [2.5 - A]^2$$

$$r_k = 1.02$$

$$A = \frac{63}{1000}$$

$$\text{HrAng}_i = 15 \cdot [i - 12] - 7.5 \quad \text{for } i = 7 \text{ to } 18$$

$$I_{o,i} = I_{\text{zero}} [\text{nDay}, \text{Lat}, \text{HrAng}_i - 7.5 \text{ [deg]}, \text{HrAng}_i + 7.5 \text{ [deg]}] \quad \text{for } i = 7 \text{ to } 18$$

$$x_i = \text{CosZenAng} [\text{Lat}, \text{nDay}, \text{HrAng}_i] \quad \text{for } i = 7 \text{ to } 18$$

$$\tau_{b,i} = a_0 \cdot r_0 + a_1 \cdot r_1 \cdot \exp \left[-k \cdot \frac{r_k}{x_i} \right] \quad \text{for } i = 7 \text{ to } 18$$

$$\tau_{d,i} = 0.271 - 0.294 \cdot \tau_{b,i} \quad \text{for } i = 7 \text{ to } 18$$

$$I_{b,i} = I_{o,i} \cdot \tau_{b,i} \quad \text{for } i = 7 \text{ to } 18$$

$$I_{d,i} = I_{o,i} \cdot \tau_{d,i} \quad \text{for } i = 7 \text{ to } 18$$

Parametric Table: Table 1

	nDay	$I_{b,7}$	$I_{b,8}$	$I_{b,9}$	$I_{b,10}$	$I_{b,11}$	$I_{b,12}$	$I_{b,13}$	$I_{b,14}$	$I_{b,16}$	$I_{b,18}$
Run 1	281	0.03922	0.3887	0.9852	1.548	1.966	2.187	2.187	1.966	1.548	0.9852
Run 2	282	0.03736	0.3784	0.9715	1.533	1.951	2.172	2.172	1.951	1.533	0.9715
Run 3	283	0.03557	0.3682	0.9579	1.518	1.936	2.156	2.156	1.936	1.518	0.9579
Run 4	284	0.03385	0.358	0.9443	1.503	1.92	2.14	2.14	1.92	1.503	0.9443
Run 5	285	0.03218	0.348	0.9307	1.488	1.904	2.124	2.124	1.904	1.488	0.9307
Run 6	286	0.03056	0.3381	0.9171	1.473	1.889	2.108	2.108	1.889	1.473	0.9171
Run 7	287	0.029	0.3284	0.9035	1.458	1.873	2.092	2.092	1.873	1.458	0.9035
Run 8	288	0.02747	0.3187	0.89	1.443	1.857	2.076	2.076	1.857	1.443	0.89
Run 9	289	0.026	0.3092	0.8765	1.428	1.841	2.06	2.06	1.841	1.428	0.8765
Run 10	290	0.02457	0.2998	0.8631	1.413	1.826	2.044	2.044	1.826	1.413	0.8631
Run 11	291	0.02318	0.2906	0.8497	1.398	1.81	2.028	2.028	1.81	1.398	0.8497

Parametric Table: Table 1

	$I_{b,17}$	$I_{b,18}$
Run 1	0.3887	0.03922
Run 2	0.3784	0.03736
Run 3	0.3682	0.03557
Run 4	0.358	0.03385
Run 5	0.348	0.03218
Run 6	0.3381	0.03056
Run 7	0.3284	0.029
Run 8	0.3187	0.02747
Run 9	0.3092	0.026
Run 10	0.2998	0.02457
Run 11	0.2906	0.02318

Appendix B: Solar Decathlon Competition Solar Data for a Typical Year

Date	Average Direct Normal Irradiation per Hour (W/m ²)										
	7:00	08:00	09:00	10:00	11:00	12:00	13:00	14:00	15:00	16:00	17:00
10/08	0	0	6	6	12	336	690	705	635	573	202
10/09	0	0	0	6	77	601	788	755	673	595	134
10/10	231	218	588	783	625	30	121	92	101	305	42
10/11	154	0	17	659	667	721	733	700	693	564	218
10/12	66	462	593	747	780	757	818	774	681	593	146
10/13	290	598	737	818	851	865	855	792	701	533	197
10/14	295	575	719	794	786	865	806	750	600	172	108
10/15	264	607	713	718	822	703	782	731	569	544	0
10/16	0	114	592	671	774	733	746	719	645	518	0
10/17	259	529	695	765	816	751	697	566	557	256	0
10/18	0	200	259	0	0	0	30	480	532	28	0

National Renewable Energy Laboratory. *1991 – 2005 Update: Typical Meteorological Year 3. Santa Ana John Wayne AP*. April 2008. National Solar Radiation Data Base.
http://rredc.nrel.gov/solar/old_data/nsrdb/1991-2005/tmy3/by_state_and_city.html

Appendix C: System Thermal and Fluid Analysis

C1 List of Variables

W = pipe spacing [ft.]	T_m = Mean Temperature [$^{\circ}\text{F}$]
D = Pipe Diameter [ft.]	q_{total} = Total Heat Transfer [Btu/hr.]
X = Coil Length [ft.]	$\dot{m}$ = Mass Flowrate [lbm/hr.]
$N_{bends, pcm}$ = Number of bends in Coil [-]	V = Velocity [ft. /hr.]
C_p = Specific Heat [Btu/lbm*F]	V_{pipe} = Velocity in Pipes [ft. /s]
P = Density [lbm/ft ³]	g = gravitational constant [ft. /s ²]
μ = Viscosity [lbm/ft.*hr.]	L_{ab} = Pipe Lengths between Sections “A” and “B” [ft.]
Pr = Prandtl Number [-]	L_{cd} = Pipe Lengths between Sections “C” and “D” [ft.]
K_{eg} = Thermal Conductivity [Btu/hr.*ft.*F]	L_{ef} = Pipe Lengths between Sections “E” and “F” [ft.]
ν = Dynamic Viscosity [ft ² /hr.]	K_{tee} = Tee Loss Coefficient [-]
K_{Cu} = Thermal Conductivity of Copper [Btu/hr.*ft.*F]	K_{180} = 180° Bend Loss Coefficient [-]
$\bar{N}_{UD}$ = Nusselt Number [-]	K_{90} = 90° Bend Loss Coefficient [-]
Re_D = Reynold’s Number [-]	ΔP_{ab} = Pressure Drop between “A” and “B” [psi]
f = Friction Factor [-]	ΔP_{bc} = Pressure Drop between “B” and “C” [psi]
U = Overall Heat Transfer Rate [Btu/hr.*ft.*F]	ΔP_{cd} = Pressure Drop between “C” and “D” [psi]
$T_{m, I}$ = Inlet Temperature [$^{\circ}\text{F}$]	ΔP_{de} = Pressure Drop between “D” and “E” [psi]
$T_{m, x}$ = Outlet Temperature [$^{\circ}\text{F}$]	ΔP_{ef} = Pressure Drop between “E” and “F” [psi]
T_s = Surface Temperature [$^{\circ}\text{F}$]	ΔP_{fa} = Pressure Drop between “F” and “A” [psi]
T_{PCM} = PCM Temperature [$^{\circ}\text{F}$]	ΔP = Total System Pressure Drop [psi]
	$Power_{fluid}$ = Power to Heat Transfer Fluid [hp]
	$Power_{mech}$ = Mechanical Power [hp]

C2 Assumptions

The following assumptions were made in solving the thermal analysis of the flow through the pipes:

1. Constant pipe surface temperature
 - a. In order to solve for the mean temperature of the fluid in the pipe at any point along the coil, the temperature of the pipe was assumed to be constant. The temperature was set at the melting point of the wax/hdpe phase change material (PCM). This is a valid assumption due to the high latent heat of the PCM, and because the temperature is constant during the phase transition. It can be assumed that the energy stored in the PCM will be enough to maintain the temperature at the melting point. While localized heating and cooling will likely occur due to the lower thermal conductivity of the PCM, the heating coil will be routed to minimize this issue.
2. Incompressible liquid and negligible viscous dissipation
3. Constant properties
4. Fully developed flow and fully developed thermal conditions
5. The Nusselt number can be determined by the Gnielinski correlation for flow through smooth tubes, and the friction factor is determined by Petukhov's correlation

$$\text{a. } \bar{N}_{UD} = \left(\frac{(f/8)(Re_D - 1000)Pr}{(1 + 12.7(f/8)^{1/4}(Pr^{2/3} - 1))} \right)^{-1} \text{ (Gnielinski Correlation)}$$

$$\text{b. } f = (.790 \ln(Re_D) - 1.64)^{-2} \text{ (Petukhov Correlation)}$$

The following assumptions were made in solving the fluid analysis of the flow through the pipes:

1. The extended Bernoulli equation is applicable
 - a. Fully developed flow
 - b. Steady state
 - c. Incompressible pipe flow
 - d. Flow along a streamline
2. Smooth pipes with friction factor determined by the Petukhov correlation:
 - a. $f = (.790 \ln(Re_D) - 1.64)^{-2}$ (Petukhov Correlation)
3. Both major and minor losses are significant

C3 System of Equations

The purpose of this analysis was to determine the outlet temperature of the heat transfer fluid after passing through the solar thermal storage unit, and the total system head of the heating loop based on the system properties. Engineering Equation Solver (EES) was used to simultaneously solve the system of equations. The following two equations determined these outputs.

$$\frac{T_{pcm} - T_{m,x}}{T_{pcm} - T_{m,i}} = \exp\left(\frac{-U\pi Dx}{\dot{m}c_p}\right) \quad (1)$$

$$Head = \frac{\Delta P}{\rho g} \quad (2)$$

Equation 1 determines the outlet temperature of the fluid while Equation 2 determines the system head based on the system pressure drop. In order to solve these equations, the mass flow rate through the heating loop, the overall heat transfer coefficient, and the total system pressure drop were determined. The pressure drop and flowrate of the system are linked, as the pump curve determines the corresponding flowrate for a given head; therefore, curve fits were made for a series of Grundfos pumps. Equation 3 was used to determine the flowrate. Figure C1 and Table C1 show the pump curves and the corresponding coefficients “*a*”, “*b*”, and “*c*” for Equation 1.

$$Head = a * \dot{m}^2 + b * \dot{m} + c \quad (3)$$

Figure C1. Grundfos Pump Curves

Table C1. Equation 3 Coefficients from Grundfos Pump Curves

Pump	a	b	c
UPS32-80 Speed 2	-0.0032	-0.1893	23.929
UPS50-80/4 Speed 1	0.0004	-0.1921	17.643
UPS40-80/4 Speed 1	-0.0016	-0.1196	18.964
UPS40-80/4 Speed 2	-0.0013	-0.0964	21.524
UPS40-80/4 Speed 3	-0.0013	-0.0464	23.524
UPS40-80/2 Speed 1	-0.0028	0.1411	25.298

The total system pressure drop was determined by analyzing the loop in 6 separate sections, as shown in Figure C2, and summing the results. Section “a” to “b” accounts for major and minor losses along the section of piping, section “b” to “c” accounts for the flat plate collectors pressure drop, section “c” to “d” accounts for the major and minor losses along the length of piping, section “d” to “e” accounts for the pressure drop through the thermal storage unit (see equation 6), section “e” to “f” accounts for the major and minor losses along the length of piping, and section “f” to “a” accounts for the pressure drop from the hot water storage tank heat exchanger. Equation 4 was used to calculate the minor losses (n specifies the number of a particular component that creates a loss), and Equation 5 was used to calculate the major losses. K for a tee was specified as 0.08, .2 for a 180° bend, and 1.5 for a 90° bend.

$$Minor Losses = .5 \frac{\rho}{g} n K V^2 \quad (4)$$

$$Major Losses = \frac{L}{D} f V^2 \quad (5)$$

See Coil Loss Kays Heat Exchanger Design (6)

Figure C2. System Schematic

With this information, the mass flowrate, in gallons per minute, was determined. This was then converted to pounds mass per hour and used to perform the heating calculations. Additionally, the overall heat transfer coefficient was determined from the convection resistance to the flowing fluid and the radial resistance of the copper piping. Equations 7 was used to join the two resistances by summing them and taking the inverse of the result. The units of the heat transfer rate are in Btu/hr.*°F. The convection coefficient was calculated using the Nusselt number and the Gnielinski Correlation (see Equations 8 and 9 respectively). Lastly the total heat transfer to the water from the PCM was calculated in Btu/hr. using Equation 10.

$$U = \left(\frac{1}{h\pi D x} + \frac{\ln(r_2/r_1)}{2\pi x k_{cu}} \right)^{-1} \quad (7)$$

$$\bar{N}_{UD} = \frac{hD}{k_{eg}} \quad (8)$$

$$\bar{N}_{UD} = \left(\frac{(f/8)(Re_D - 1000)Pr}{(1 + 12.7(f/8)^{0.5}(Pr^{2/3} - 1))} \right)^{-1} \quad (9)$$

$$q_{total} = \dot{m}c_p(T_{m,x} - T_{m,i}) \quad (10)$$

C4 Results

The analysis was initially performed assuming the heat transfer fluid entered the thermal storage unit at 140°F after passing through the flat plate collector, and that the PCM was at 212°F (100°C). Table C2 summarizes the results that were obtained.

Table C2. Pump Selection Results

Pump	Total Head (ft.)	Outlet Temperature (°F)
UPS32-80 Speed 2	23.56	159.4
UPS50-80/4 Speed 1	17.38	166.8
UPS40-80/4 Speed 1	18.8	164.4
UPS40-80/4 Speed 2	21.36	161.3
UPS40-80/4 Speed 3	23.43	159.5
UPS40-80/2 Speed 1	25.58	158.1

Based on this information, the Grundfos UPS50-80/4 pump, run at its first speed setting, is the best choice as it maximized the outlet temperature of the fluid, and minimized the total system head. The program was then further used to determine the system performance at various inlet temperatures and PCM temperatures. Table C3 presents the results of the different operating conditions that were considered with the UPS50-80/4 pump.

Table C3. Results from Various Inlet Temperatures

Inlet Temperature (°F)	Outlet Temperature (°F)	Heat Dissipation (kW)	Mass Flow Rate (GPM)
50	140	11.75	1.041
60	142.7	10.94	1.051
70	145.5	10.1	1.061
80	148.3	9.248	1.069
90	151.3	8.373	1.077
100	154.3	7.482	1.084
110	157.3	6.578	1.09
120	160.5	5.662	1.095
130	163.6	4.735	1.1
140	166.8	3.801	1.104

Ultimately the system is able to significantly increase the temperature of a heat transfer fluid. However, the heating process requires a significant amount of power, which could quickly drain a thermal storage unit of its stored energy. This would largely be dependent on the level of hot water use.

This analysis was dependent on a constant surface temperature of the copper pipes that carry the heat transfer fluid. In reality, there will likely be localized areas of cooling in the PCM. To minimize this, the system will need to be designed to effectively spread the heat from the receiver plate throughout the PCM. This can be accomplished by adding thermal pathways from the receiver plate to the various portions of the PCM. Copper plates, or heat pipes can effectively distribute the heat. Additionally, the fluid piping coils can be spaced relatively close together to prevent large areas of localized cooling.

Using an ethylene glycol mixture presents a problem if the system is shut-off for an extended period of time. The boiling point of a 50% ethylene glycol solution is 225°F, while the operating temperature of the solar thermal storage unit is 212°F (based on the melting point of the PCM). If the system is heated beyond the melting point of the PCM, it would be possible to boil the heat transfer fluid – leading to an unsafe increase in the system pressure. A safety mechanism will need to be incorporated into the system, or a heat transfer fluid with a higher boiling point than an ethylene glycol mixture is necessary.

C5 EES Code

PCM Heating Calcs

System Sizing

$$W = \frac{1.15}{12} \text{ [ft]} \text{ Pipe Spacing}$$

$$D = \frac{0.5}{12} \text{ [ft]} \text{ Pipe Diameter}$$

$$x = \frac{33.61 \cdot 7}{12} \text{ [ft]} \text{ length of pipe}$$

$$n_{\text{bends,pcm}} = 14 \text{ [-]} \text{ \# of Bends in Piping System}$$

System Properties

$$c_p = C_p \text{ [EG, T = } T_m, C = 50 \text{ \%]} \text{ [Btu/lbm}\cdot\text{F]} \text{ Specific Heat}$$

$$\rho = \rho \text{ [EG, T = } T_m, C = 50 \text{ \%]} \text{ [lbm/ft}^3\text{]} \text{ Density}$$

$$\mu = \text{Visc} \text{ [EG, T = } T_m, C = 50 \text{ \%]} \text{ [lbm/ft}\cdot\text{hr]} \text{ Viscosity}$$

$$Pr = Pr \text{ [EG, T = } T_m, C = 50 \text{ \%]} \text{ [-]} \text{ Prandtl}$$

$$k = k \text{ [EG, T = } T_m, C = 50 \text{ \%]} \text{ [Btu/hr}\cdot\text{ft}\cdot\text{F]} \text{ Thermal Conductivity}$$

$$\nu = \frac{\mu}{\rho} \text{ [F}^2\text{/hr]} \text{ Dynamic Viscosity}$$

$$K_{cu} = 204 \text{ [Btu/hr}\cdot\text{ft}\cdot\text{F]} \text{ Thermal Resistance of Copper}$$

Heat Transfer Calculations

$$\overline{N}_{u,D} = h \cdot \frac{D}{k} \text{ [-]} \text{ Nusselt Number}$$

$$\overline{N}_{u,D} = \frac{\frac{f}{8} \cdot [Re_D - 1000] \cdot Pr}{1 + 12.7 \cdot \left[\frac{f}{8} \right]^{0.5} \cdot [Pr^{(2/3)} - 1]} \text{ [-]}$$

Nusselt Number

$$Re_D = V \cdot \frac{D}{\nu} \text{ [-]} \text{ Reynolds Number}$$

$$f = [0.79 \cdot \ln(Re_D) - 1.64]^{-2} \text{ [-]} \text{ Friction Factor}$$

$$U = \left[\frac{1}{h \cdot 2 \cdot \pi \cdot 0.5 \cdot x} + \frac{0.5 \cdot \ln\left(\frac{0.3125}{0.25}\right)}{\pi \cdot x} \right]^{-1} \text{ Overall Heat Transfer Rate}$$

$$U_{\text{watts}} = \frac{U}{0.5 \cdot \pi \cdot x} \cdot \frac{0.528}{0.092903} \text{ [W/m}^2\text{K]}$$

$$T_{pcm} = T_s \cdot \left| 1.8 \cdot \frac{F}{C} \right| \quad \text{PCM Temperature}$$

$$T_m = \frac{T_{m,i} + T_{m,x}}{2} \quad \text{Mean Temperature}$$

$$Q_{total} = \dot{m} \cdot c_p \cdot [T_{m,x} - T_{m,i}] \cdot 0.00029307107 \quad [kW] \quad \text{Total Heat Transfer}$$

$$\dot{m} = \dot{m}_{total} \cdot conv_{gpm}$$

$$conv_{gpm} = 1 \quad [gal] \cdot 0.1337 \quad [ft^3/gal] \cdot \rho \cdot 60 \quad [min/hr] \quad \text{Convert to gpm from lbm/hr}$$

$$\frac{T_{pcm} - T_{m,x}}{T_{pcm} - T_{m,i}} = \exp \left[\frac{-U}{\dot{m} \cdot c_p} \right]$$

Change in Temperature from inlet to outlet

Fluid Velocity

$$V = \frac{\dot{m}}{\rho \cdot 0.25 \cdot \pi \cdot D^2} \quad [ft/hr] \quad \text{Velocity}$$

$$conv_{hr,s} = \frac{1}{3600} \quad [hr/s] \quad \text{Converts hours to seconds}$$

$$V_{pipe} = V \cdot conv_{hr,s} \quad [ft/s] \quad \text{Converts pipe velocity to ft/s}$$

Pump Calculations

Fluid Properties

$$g = 32.2 \quad [ft/s^2]$$

System Lengths

$$L_{ab} = 15 \quad [ft] \quad \text{Pipe length between a and b}$$

$$L_{cd} = 5 \quad [ft] \quad \text{Pipe length between c and d}$$

$$L_{ef} = 15 \quad [ft] \quad \text{Pipe length between e and f}$$

Pressure Drop Calculations

$$K_{tee} = 0.08 \quad [-] \quad \text{Tee loss coefficient}$$

$$k_{180} = 0.2 \quad [-] \quad \text{Loss Coefficient for 180 Deg Bend}$$

$$K_{90} = 1.5 \quad [-] \quad \text{Loss coefficient for 90 deg bend}$$

$$\delta_{P,ab} = \frac{\frac{\rho}{32.2} \cdot 2 \cdot 0.5 \cdot K_{90} \cdot V_{pipe}^2 + \frac{L_{ab}}{D} \cdot f \cdot V_{pipe}^2}{144} \quad [psi] \quad \text{Pressure drop between a and b}$$

$$\delta_{P,bc} = 1 \quad [psi] \quad \text{Pressure drop between b and c}$$

$$\bar{U}_{P,cd} = \frac{\frac{\rho}{32.2} \cdot 2 \cdot 0.5 \cdot K_{90} \cdot V_{\text{pipe}}^2 + \frac{L_{cd}}{D} \cdot f \cdot V_{\text{pipe}}^2}{144} \quad [\text{psi}] \quad \text{Pressure drop between c and d}$$

$$\bar{U}_{P,de} = 2 \cdot \frac{\rho}{32.174 \quad [\text{lbm/slugs}]} \cdot \frac{V_{\text{pipe}}^2}{2} \cdot \left[f \cdot \frac{x}{D} + n_{\text{bends,pcm}} \cdot k_{180} \right] \cdot \frac{1}{144 \quad [\text{in}^2/\text{ft}^2]} \quad [\text{psi}] \quad \text{Pressure drop between d and e}$$

$$\bar{U}_{P,ef} = \frac{\frac{\rho}{32.2} \cdot 2 \cdot 0.5 \cdot K_{90} \cdot V_{\text{pipe}}^2 + \frac{L_{ef}}{D} \cdot f \cdot V_{\text{pipe}}^2}{144} \quad [\text{psi}] \quad \text{Pressure drop between e and f}$$

$$\bar{U}_{P,fa} = 5 \quad [\text{psi}] \quad \text{Pressure drop between f and a}$$

$$\bar{U}_P = \bar{U}_{P,ab} + \bar{U}_{P,bc} + \bar{U}_{P,cd} + \bar{U}_{P,de} + \bar{U}_{P,ef} + \bar{U}_{P,fa} \quad [\text{psi}] \quad \text{Total System Pressure Drop}$$

$$\text{Head} = \frac{\bar{U}_P \cdot 144}{\frac{\rho}{32.2} \cdot g} \quad [\text{ft}] \quad \text{System Head}$$

$$\text{Head} = a \cdot \dot{m}_{\text{total}}^2 + b \cdot \dot{m}_{\text{total}} + c \quad [\text{ft}] \quad \text{Operating Point}$$

$$\text{Power}_{\text{fluid}} = \frac{\rho}{32.2} \cdot g \cdot \text{Head} \cdot \dot{m}_{\text{total}} \cdot \frac{0.0022280093 \quad [\text{ft}^3 \cdot \text{min} / \text{gpm} \cdot \text{s}]}{550} \quad [\text{hp}] \quad \text{Mechanical Power}$$

$$\text{Power}_{\text{mech}} = \frac{\text{Power}_{\text{fluid}}}{0.6} \quad [\text{hp}] \quad \text{Fluid Power}$$

$$\text{Power}_{\text{mech,kw}} = \text{Power}_{\text{mech}} \cdot 0.7457 \quad [\text{kW}] \quad \text{Mechanical Power in kW}$$

Parametric Table: Table 2

	a	b	c	T _s	T _{m,i}	T _{m,x}	Q _{total}	Head	Δp	m _{total}	Power _{mech}
	[-]	[-]	[-]	[C]	[F]	[F]	[Btu/hr]	[ft]	[psi]	[gpm]	[hp]
Run 4	-0.0013	-0.0964	21.52	100	140	161.3	4.445	21.36	9.599	1.628	0.01519
Run 5	-0.0013	-0.0464	23.52	100	140	159.5	4.636	23.43	10.53	1.851	0.01896
Run 6	-0.0028	0.1411	25.3	100	140	158.1	4.785	25.58	11.5	2.062	0.02305
Run 7	0.00006	-0.1793	17.58	100	50	140	11.75	17.4	7.967	1.041	0.00806
Run 8	0.00006	-0.1793	17.58	100	60	142.7	10.94	17.39	7.949	1.051	0.008122
Run 9	0.00006	-0.1793	17.58	100	70	145.5	10.1	17.39	7.933	1.061	0.008181
Run 10	0.00006	-0.1793	17.58	100	80	148.3	9.248	17.39	7.916	1.069	0.008229
Run 11	0.00006	-0.1793	17.58	100	90	151.3	8.373	17.39	7.898	1.077	0.008269
Run 12	0.00006	-0.1793	17.58	100	100	154.3	7.482	17.39	7.879	1.084	0.008302
Run 13	0.00006	-0.1793	17.58	100	110	157.3	6.578	17.39	7.861	1.09	0.008329
Run 14	0.00006	-0.1793	17.58	100	120	160.5	5.662	17.39	7.842	1.095	0.008349
Run 15	0.00006	-0.1793	17.58	100	130	163.6	4.735	17.38	7.822	1.1	0.008364
Run 16	0.00006	-0.1793	17.58	100	140	166.8	3.801	17.38	7.803	1.104	0.008374

Parametric Table: Table 2

	Power _{mech,kw}	Power _{fluid}	U _{watts}
	[kW]	[hp]	
Run 1	0.01432	0.01152	100.2
Run 2	0.006245	0.005024	99.13
Run 3	0.008021	0.006454	99.54
Run 4	0.01133	0.009113	99.95
Run 5	0.01414	0.01137	100.2
Run 6	0.01719	0.01383	100.3
Run 7	0.00601	0.004836	96.82
Run 8	0.006057	0.004873	97.28
Run 9	0.006101	0.004909	97.66
Run 10	0.006137	0.004938	97.98
Run 11	0.006167	0.004962	98.25
Run 12	0.006191	0.004981	98.47
Run 13	0.006211	0.004997	98.67
Run 14	0.006226	0.00501	98.84
Run 15	0.006237	0.005018	99
Run 16	0.006245	0.005024	99.13