

DENSITY-BASED CLUSTERING OF HIGH-DIMENSIONAL DNA
FINGERPRINTS FOR LIBRARY-DEPENDENT MICROBIAL
SOURCE TRACKING

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ABSTRACT

Density-Based Clustering of High-Dimensional DNA Fingerprints for Library-Dependent Microbial Source Tracking

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As part of an ongoing multidisciplinary effort at California Polytechnic State University, biologists and computer scientists have developed a new Library-dependent Microbial Source Tracking method for identifying the host animals causing fecal contamination in local water sources. The Cal Poly Library of Pyroprints (CPLOP) is a database which stores *E. coli* representations of fecal samples from known hosts acquired from a novel method developed by the biologists called Pyroprinting. The research group considers *E. coli* samples whose Pyroprints match above a certain threshold to be part of the same bacterial strain. If an environmental sample from an unknown host matches one of the strains in CPLOP, then it is likely that the host of the unknown sample is the same species as one of the hosts that the strain was previously found in. Clustering is a computer science technique for finding groups of related data (i.e. strains) in a data set. In this thesis, we evaluate the use of density-based clustering for identifying strains in CPLOP. Density-based clustering finds clusters of points which have a minimum number of other points within a given radius. We contribute a clustering algorithm based on the original DBSCAN algorithm which removes points from the search space after they have been seen once. We also present a new method for comparing pyroprints which is algebraically related to the current method. The method has mathematical properties which make it possible to use Pyroprints in a spatial index we designed especially for Pyroprints, which can be utilized by the DBSCAN algorithm to speed up clustering.

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CHAPTER 1

INTRODUCTION

Clean sources of water are important for preventing the spread of diseases, and for maintaining the environment. One of the undesirable contaminants often found in bodies of water is feces[17]. The bacteria found along with fecal matter can make animals, including humans, sick. Environmental and Resource agencies are interested in finding the sources of fecal contamination. If the origin of contamination can be ascertained, then actions can be taken to remove or reduce the amount of fecal matter in the water.

The study of identifying and discriminating fecal bacteria in the environment is called Microbial Source Tracking (MST). MST techniques usually look for fecal indicator bacteria (FIB) in environmental samples[19]. These FIB are bacteria found in the digestive tracts of animals, called hosts, that sometimes leave the animals along with fecal matter. Investigators can look at the quantity of FIB in an environmental sample to estimate the amount of fecal contamination in the sampled environment[17]. Additionally, they can examine individual bacteria found in the sample and try to determine what host species they came from[17]. Generally, investigators are not interested in finding the exact individual host from which the bacterium came, but instead are satisfied with knowing the species of the host.

At the intersection of the fields of Computer Science and Biology lies a method of MST called library-dependent MST[19]. Library-dependent MST works off the assumption that there exist subgroups of an FIB species, called strains, which are only found in certain host species[17]. The method starts with the collection of a large number of bacteria samples from fecal matter from a known host species. Representations of these bacteria are stored in a database along with the information

about their provenance. Once the database is established, MST is performed by comparing environmental samples from unknown hosts to the samples stored in the database with the hope of finding a match. If matching bacteria are found, then researchers can look up the host species of the matching sample in the database and use that as evidence that the unknown sample came from the same host species.

Simply determining that two bacteria are of the same species is not necessarily enough to determine that two samples of FIB came from the same host species. In fact, a common FIB species used for MST, *Escherichia coli* (*E. coli*), is found in the guts and fecal matter of many host animals[17]. Instead, researchers must determine that the bacteria both came from the same subgroup of the species. The idea is that the more closely related two individuals are, the more recently they came from a common ancestor and thus the more likely they came from the same host species. Any meaningful subdivision of bacteria beyond species like this is called a strain. MST research groups formally define their notion of a strain by picking a metric of similarity and a threshold at which they are confident that bacteria are as closely related as their metric can determine; maximizing the probability that the bacteria came from a common ancestor and thus the same host.

Many methods can be used to measure the similarity between two bacteria cultures. The methods can be classified by whether they look at the phenotypes of the bacteria or the genotypes. Phenotypic comparison looks at appearance or behavior of the bacteria, for example their reaction to a certain chemical[17]. Genotypic comparison on the other hand looks at the actual DNA of the bacteria. Comparing genotypes can be more expensive but is generally more discerning[19], as similar appearance or behavior can be produced by different sequences of DNA.

As part of an ongoing multidisciplinary effort at California Polytechnic State University, biologists and computer scientists have developed a new library-dependent

MST method. The Cal Poly Library of Pyroprints (CPLOP) is a database developed by the computer scientists to support a novel, cost-effective genotype representation method for bacteria comparison developed by the biologists called Pyroprinting. CPLOP uses *E. coli* as its FIB, storing representations acquired by running this Pyroprinting method on *E. coli* samples. To increase discrimination ability of the method, each *E. coli* sample's DNA is pyroprinted in two separate locations. The research group considers *E. coli* samples which match in both locations with a similarity above a certain threshold to be part of the same strain. If that strain has only been seen in one host species then they have good evidence that environmental samples matching bacteria in that strain are also from the same host species.[20]

The number of samples stored in the database for library-dependent MST influences the confidence in matches found with unknown samples. As the size of the database increases, so does the time to search for matches. In the case of the naive search method, an unknown sample must be compared to every sample in the database. At the time of this writing, MST in CPLOP[13] still relied on this method. If the notion of strains were to be stored in the database, then unknown samples could be compared to groups of bacteria in the database instead of every individual bacterium, speeding up the search. Storing strains in the database would also facilitate other kinds of research. Longitudinal studies, such as that performed by Emily Neal[15], look at strains observed in an individual over time and look for changes or patterns. Transference studies, such as that performed by Josh Dillard[7], look at samples from different hosts for strains found in both hosts.

The desire for a method of identifying strains from the set of bacteria in the database is clear. Clustering is a computer science technique for finding groups of related data (i.e. strains) in a data set. There are different definitions of which data is part of the same group, or cluster. This is similar to the variation in definition of a strain in biology. Thus the choice of clustering method must match the biologist's

idea of what constitutes a strain for their research.

In previous work for CPLOP, Aldrin Montana developed a method of clustering called OHClust![14]. OHClust! is based on agglomerative hierarchical clustering and utilizes information about the provenance of samples to allow efficient addition to the known clusters through incremental updates. Unfortunately, OHClust! cannot run on the CPLOP servers which have meager memory resources. In addition, in his evaluation of the clusters identified by OHClust!, Montana was unable to determine if the clusters matched the biologist’s notion of strains.[14]

The work presented in this thesis provides an alternate solution for identifying strains which hopes to address the limitations of OHClust!. It evaluates density-based clustering for the use in identifying *E. coli* strains from pyroprints. Density-based clustering, as defined by DBSCAN[9] finds clusters where points have a minimum number of other points within a given radius. The DBSCAN algorithm can find clusters like this very efficiently, especially if the points are stored in a spatial index.

The contributions of this thesis are the following:

- **A modified DBSCAN algorithm:** This is the first time density-based clusters have been tried with CPLOP. The original algorithm for finding density-based clusters was DBSCAN. Many researchers have provided modified versions of this algorithm suited for different purposes. This thesis modifies DBSCAN, allowing for the removal of points from the search space after they have been seen once. This modification results in a 2x speedup for our use case.
- **A faster method for comparing pyroprints:** Prior to this work, pyroprints in CPLOP were compared with Pearson Correlation. Pearson correlation ignores certain differences in the pyroprints which are due to inconsistencies in the physical pyroprinting process. This thesis presents a new method for comparing pyroprints which is algebraically related to the old method, allowing

reuse of previous statistical analysis. In addition we precomputed some intermediate values which speeds up comparisons with both the old method and the new method. The new method was chosen such that it has mathematical properties which make it possible to treat the pyroprints as if they were points in multidimensional space.

- **A spatial index that works with *E. coli* pyroprints:** The DBSCAN algorithm can take advantage of storing data in a special way to increase the efficiency. If the points being clustered are points in space, then they can be stored based off their position in space. This is called a spatial index. Spatial indexes can be searched quickly for points near another point. This thesis presents a spatial index tailored for the needs of the data in CPLOP. It supports dense, high-dimensional data, by partitioning the search space with multidimensional planes as well as storing bounding volumes for groups of data. It also supports multiple regions for each data point with separate search radii.
- **Better evaluation metrics of clusters for CPLOP:** Finally, this thesis contributes an evaluation of the clusters generated by this work as well as those generated by OHClust!. This evaluation was more thorough than that provided in Montana's Thesis, and is able to quantify how well the clusters match the Biologist's notion of a strain. Unlike Montana's evaluation, this evaluation goes beyond measuring the similarity of clusters to those produced by another clustering method.

The rest of this document is organized as follows. Chapter 2 provides detailed background information about both the biology and computer science sides of the problem context. Chapter 3 provides a detailed explanation of the design for the solution presented along with this thesis, as well as rationale for design decisions. Chapter 4 provides relevant details about the solution implementation along with

the benefits or limitations of particular implementation choices. Chapter 5 outlines the evaluation performed on the solution followed by the results of the tests and analysis of those results. Finally, Chapter 6 concludes the paper and suggests areas of improvement as well as ideas for other related avenues of research.

CHAPTER 2

BACKGROUND

This chapter provides context for the work presented with this thesis. Faculty from the Biology department at California Polytechnic University (Cal Poly) in conjunction with Computer Scientists from Cal Poly have developed a new method of microbial source tracking. This method is library-dependent and utilizes a novel DNA fingerprinting technique, called pyroprinting, developed at Cal Poly. The Biology department has a desire to identify bacterial strains in the library. The work presented in this thesis fulfills that desire using clustering and spatial indexes. It is an alternate solution to OHClust! created by Aldrin Montana[14].

2.1 Microbial Source Tracking (MST)

The pyroprinting technique was developed by Biologists at Cal Poly as a tool for microbial source tracking (MST)[5]. MST is the study of identifying and discriminating bacteria in the environment. Usually, the bacteria in question are found in fecal matter contaminating environmental resources such as bodies of water. MST techniques usually look for fecal indicator bacteria (FIB) in environmental samples[19]. These FIB are bacterial species found in the digestive tracts of animals, called hosts, that sometimes leave the animals along with fecal matter. Investigators can look at the quantity of an FIB in an environmental sample to estimate the amount of fecal contamination in the sampled environment[17]. Additionally, they can examine individual bacteria found in the sample and try to determine what host species they came from[17]. Generally, investigators are not interested in finding the exact individual host from which the bacterium came from, but instead are satisfied with knowing the

species of the host. The waste from a single animal has a minimal impact on the cleanliness of a body of water, while the waste from a whole group can have a large impact.

2.1.1 Library-Dependent MST

The MST method developed by the Biologists at Cal Poly is classified as library-dependent. Library-dependent MST lies at the intersection of the fields of Computer Science and Biology. Library-dependent MST works off of the assumption that there exist subgroups of an FIB species which are only found in certain host species[17]. The method starts with the collection of a large number of bacterial samples from fecal matter of a known host species. Representations of these bacteria are stored in a database along with the information about the samples provenance. Once the database is established, MST is performed by comparing environmental samples from unknown hosts to the samples stored in the database with the hope of finding a match. If matching bacteria are found, then researchers can look up the host species of the matching sample in the database and use that as evidence that the unknown sample came from the same host species.

Library-dependent MST is typically organized as follows:

1. **Collection:** The first step is to collect microbial samples from fecal matter of known origin. When collected, the host species of the fecal matter is recorded. Additional information such as location where the fecal matter was found, and date of the collection can be recorded if desired for additional analysis opportunities, but is not necessary for MST.
2. **Isolation:** A sample of fecal matter contains many individual bacteria. Library-dependent MST uses individual bacterial cultures, or isolates, taken from these samples. The process used by the biologists at Cal Poly to isolate *E. coli*

bacteria from the rest of the sample is described by Black et al.[5]. Multiple isolates can be taken from the same sample. The isolates can be cultured and frozen to replicate them for future experiments.

3. **Digital Representation:** After a single bacterium is isolated, researchers must then obtain a digital representation of the isolate. These representations are meant to be compared to each other in order to determine whether the isolates they match can be distinguished from each other. There are many different methods of obtaining these representations, and different types of representations that can result. Some examples are discussed in Section 2.1.2. The pyroprinting technique developed at Cal Poly is one of these methods.
4. **Addition to Library:** Next, the DNA representation is added to the library along with metadata about the isolate it represents. The metadata includes the information recorded in step 1, especially the host species of the origin sample. Additionally, information about the isolate, and parameters for the representation can be stored for bookkeeping purposes, and to ensure consistency.
5. **Forensics:** Once the library has grown sufficiently large (through multiple iterations of the previous steps), the system can then be used for MST. An environmental sample is collected and processed according to steps 2-3 to produce an *E. coli* isolate from an unknown host. The resulting representation of this isolate is then compared against the representations of isolates in the library. If it matches any isolates in the library, then researchers look up the metadata of those isolates. It is up to the researcher to come to a conclusion based on the amount and variety of host information from matching isolates about what host the environmental isolate likely came from.

2.1.2 Representations

Many methods can be used to measure the similarity between two bacteria. The methods can be classified by whether they look at the phenotypes of the bacteria or the genotypes. Comparing genotypes can be more expensive but is generally more discerning[19], as similar appearance or behavior can be produced by different sequences of DNA.

Phenotypic comparison looks at appearance/behavior of the bacteria. The idea is that bacteria from the same host species have adapted their behavior for survival in the particular environment encountered in the guts of the host species. Example representations that capture these traits include, results from biochemical tests, antibiotic resistance, and profiles of the proteins found on the outer membrane.[17]

Genotypic comparison on the other hand looks at the DNA sequence from the bacteria. The idea is that the more closely the DNA of two individuals are, the more recently they came from a common ancestor and thus the more likely they came from the same host species. A naive representation would simply be the full DNA sequence of the bacteria, however that would be expensive and impractical for both creation and comparison of the representations. Instead, researchers take shortcuts like measuring the size differences of DNA fragments related to a specific location (ribotyping), or by sequencing only a small part of the DNA which is highly variable[19].

2.2 Pyroprinting

Pyroprinting is a novel method of bacterial representation developed at Cal Poly by Michael W. Black, Jennifer VanderKelen, Anya Goodman, and Christopher L. Kitts[5]. They created the technique to address the problem they saw of researchers needing to choose between representations with good discrimination, and represen-

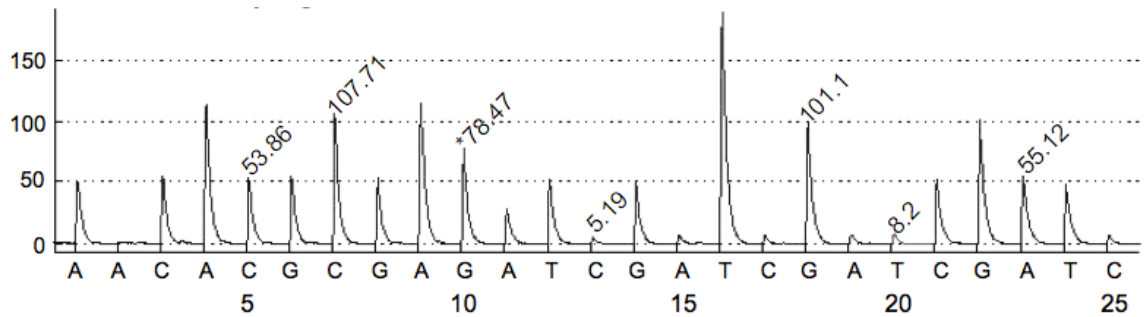


Figure 2.1: **A sample pyrogram, the result of pyrosequencing.** Values represent the heights of peaks.

tations that were cheap/convenient to generate. Pyroprinting bridges the gap by providing both features. They are currently using pyroprints for MST with *E. coli*.

2.2.1 Pyrosequencing

Cal Poly’s pyroprinting method is based off of the DNA sequencing method called pyrosequencing. The method is popular because it is a cheap and efficient way to sequence short DNA fragments.

Pyrosequencing works on short sequences of DNA called regions. The region of DNA is determined by primers. A primer is a short fragment of DNA that binds to a specific complimentary DNA sequence, opening up the double helix. Two primers are needed, one to bind to the start of the region of interest and the other binding to the end. Once the primers are in place, DNA is copied from only that region. Pyrosequencing needs lots of copies of this region, so the region is repeatedly copied, or amplified, using the Polymerase Chain Reaction (PCR)[5].

Pyrosequencing is performed by a pyrosequencer machine which takes an amplified region of DNA and outputs a vector of data called a pyrogram as shown in Figure 2.1. This pyrogram can be used to reconstruct the sequence of nucleotides present in the sequenced region.

Pyrosequencing works by building a copy of the DNA strand and measuring the light given off by the resulting chemical reactions. Along with the DNA, the researchers provide a primer. The primer binds to a specific part of the DNA, and the DNA is copied sequentially starting from that location. The DNA is copied in stages called dispensations. One type of nucleotide is added per dispensation. Machines can usually run for around 100 dispensations. The machine records the amount of light emitted which is proportional to the number of nucleotides added to the copy during that dispensation. The resulting pyrogram is a graph of light emitted at each stage of the dispensation.

2.2.2 Gene Repetition and ITS regions

In order to be able to use pyrosequencing results to compare pyrograms between different isolates, the same region needs to be sequenced every time. This means the same primers need to be used every time. Primers are specific to specific DNA which means the sequenced region has to start with the same DNA sequence for every bacteria in the species. These regions exist in the DNA, and are called conserved regions. They are regions of DNA that are important genes which if mutated would result in the death of the cell, and thus those mutations would not be passed on to progeny. However, if only the conserved region is sequenced, all isolates would get the same representation. Ideally, the sequenced region would be highly variable, one which has little or no effect on the life of the bacteria. Regions between genes, or Internal Transcribed Spacers (ITS), as shown in Figure 2.2 fit this criteria. The way to get a representation from a region that is highly variable but is in the same place for different isolates is start pyrosequencing at the end of a conserved gene, then continue sequencing into the ITS region. The research group is interested in 2 ITS regions: ITS1 and ITS2. In the remainder of the thesis, each ITS region is referred to as the genes which surround it: ITS1 = 16-23 and ITS2 = 23-5.

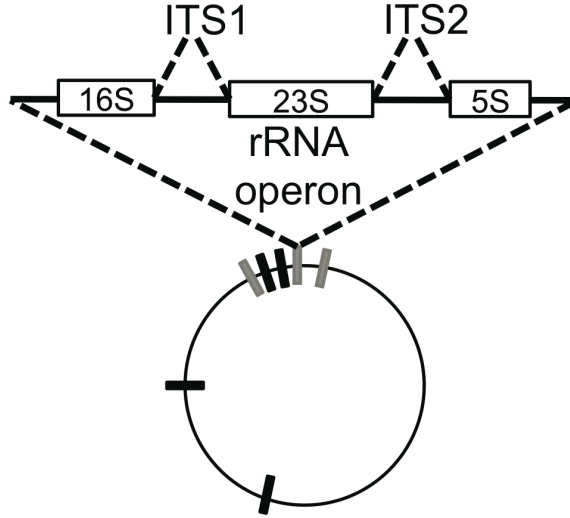


Figure 2.2: The rRNA operons in *E. coli* are present in seven copies, each of which has two Internal Transcribed Spacer (ITS) regions. Thus PCR amplification of an ITS region will generate a population of mixed products to be pyrosequenced for generating a pyroprint. ITS1 and ITS2 are also referred to as the genes surrounding them: ITS1 = 16-23 and ITS2 = 23-5. Figure taken from [5].

E. coli, as well as many other bacteria, have multiple copies of certain genes located throughout their DNA for redundancy. Each copy is located at a specific location, called a locus (multiple loci). These replications mean that primers binding to these genes will actually bind to multiple different parts of the DNA. This means PCR will not replicate a single DNA region, but instead, in the case of *E. coli*, 7 different regions, as shown in Figure 2.2. While the conserved genes are identical in each of these loci, the ITS regions can differ between the loci. This means a single bacterial isolate can have different versions of the ITS mixed together when input to the pyrosequencer.

2.2.3 Pyroprints

Pyroprints are what the biologists at Cal Poly call the result of pyrosequencing a region of DNA with multiple loci. Because pyroprinting uses pyrosequencing, the output of pyroprinting is a pyrogram. These pyrograms have different properties from pyrograms of true DNA sequences, and so are named pyroprints to differentiate the two. Specifically, pyroprints cannot be used to determine the DNA sequence of the region. This is because a pyroprint represents multiple variations (loci) of a given region. There is no way of knowing with this method whether all loci are the same, each is different, or something in between. Therefore, when examining a dispensation of a pyroprint, it is impossible to determine the distribution of added nucleotides among the loci. This property is why the biologists needed novel software and algorithms to perform MST with pyroprints.

In order to digitize a pyroprint, the biologists decided to take the maximum peak height from each dispensation. Other options were peak width and peak area. Formally, the result is a vector \vec{p} of length D

$$\vec{p} = (p_1, p_2, \dots, p_{D-1}, p_D)$$

where D is the number of dispensations of the pyroprint and each p_i is a positive real number.

To increase discrimination ability of the method, each *E. coli* sample's DNA is pyroprinted in two separate locations, 16S-23S and 23S-5S. In order to be considered indistinguishable, the representations of two isolates must match for both regions. Formally, an isolate's representation is:

$$I = (\vec{p}_{16-23}, \vec{p}_{23-5})$$

2.2.4 Pearson Correlation

Because pyroprints don't translate to a specific DNA sequence, determining if a pair of pyroprints represents the same DNA is more difficult. Pyrograms have to be compared to pyrograms instead, and variations in hardware, PCR, and technician all result in different offsets and scales for each pyrogram. Comparisons require a metric that normalizes these variations. The metric used for comparing two pyroprints \vec{x} and \vec{y} is the Pearson Correlation ρ

$$\rho(\vec{x}, \vec{y}) = \frac{1}{D} \sum_{i=1}^D \frac{(x_i - \mu_x)(y_i - \mu_y)}{\sigma_x \sigma_y}$$

where D is the number of dispensation, and μ_x and σ_x are the mean and standard deviation of the values of \vec{x} at every dispensation respectively.

$$\mu_x = \frac{1}{D} \sum_{i=1}^D x_i$$
$$\sigma_x = \sqrt{\frac{1}{D} \sum_{i=1}^D (x_i - \mu_x)^2}$$

Pearson correlation returns a value between -1 and 1, where 1 is a perfect match, -1 is perfectly inverted and 0 means the pair is unrelated. For the purposes of MST, the research team is only interested in (non-inverted) matches. Because pyroprints are non-negative, Pearson correlations between them are always ≥ 0 .

2.2.5 Statistical Analysis and Strain Definition

Due to noise in the pyrosequencing process, and by variations in the ratio of loci produced by PCR, two pyroprints will never match exactly. As such, the research team needed to determine a threshold of Pearson correlation above which 2 pyroprints are considered a match. Diana Shealy, a statistics student at Cal Poly, performed statistical analysis to determine thresholds for each ITS region[18].

In order to get a set of pyroprints which are supposed to match, the biologists made repetitive pyroprints of multiple isolates. They then took pairwise Pearson correlation values of pairs of pyroprints from the same isolate. This set of Pearson correlations was a sample of the distribution of Pearson correlations between pyroprints with the same DNA in the ITS regions.

First, Shealy analyzed the effect of dispensation count on the Pearson correlation values. Pyrogram values get more noisy in later dispensations, because in the pyrosequencer machines, chemicals and proteins are not completely removed between each dispensation. So on one hand, using more dispensations decreased the Pearson value for isolates that are supposed to match. On the other hand, using less dispensations decreased the ability of Pearson correlation to capture differences between pyroprints from truly different sources. The research group decided to only use 95 and 93 dispensations of the pyrogram for the ITS regions 16-23 and 23-5 respectively.

Shealy then fit a beta distribution to the sample distribution. Figure 2.3 shows the beta distribution fit to the pairwise Pearson correlations of pyroprints of the ITS region 16-23. From the beta distribution, Shealy calculated the percentages of false negatives at various Pearson correlation thresholds. Thresholds and false negatives for pyroprints of 16-23 are shown in Figure 2.3.

The choice of threshold affects the definition of a strain for the research group. The chance for false negatives (from higher thresholds) needed to be balanced with the chance for false positives (from lower thresholds). Shealy noted that:

“With pyroprints, we are more concerned with false positives, two pyroprints that are said to be from the same *E. coli* strain when in fact they are not. We are less concerned with false negatives, or stating two pyroprints are from different *E. coli* strains when they are actually from the same strain. The reason we are less concerned with false negatives is that hopefully any false negatives will be caught by a clustering algorithm and the false negative will then be appropriately classified.”[18]

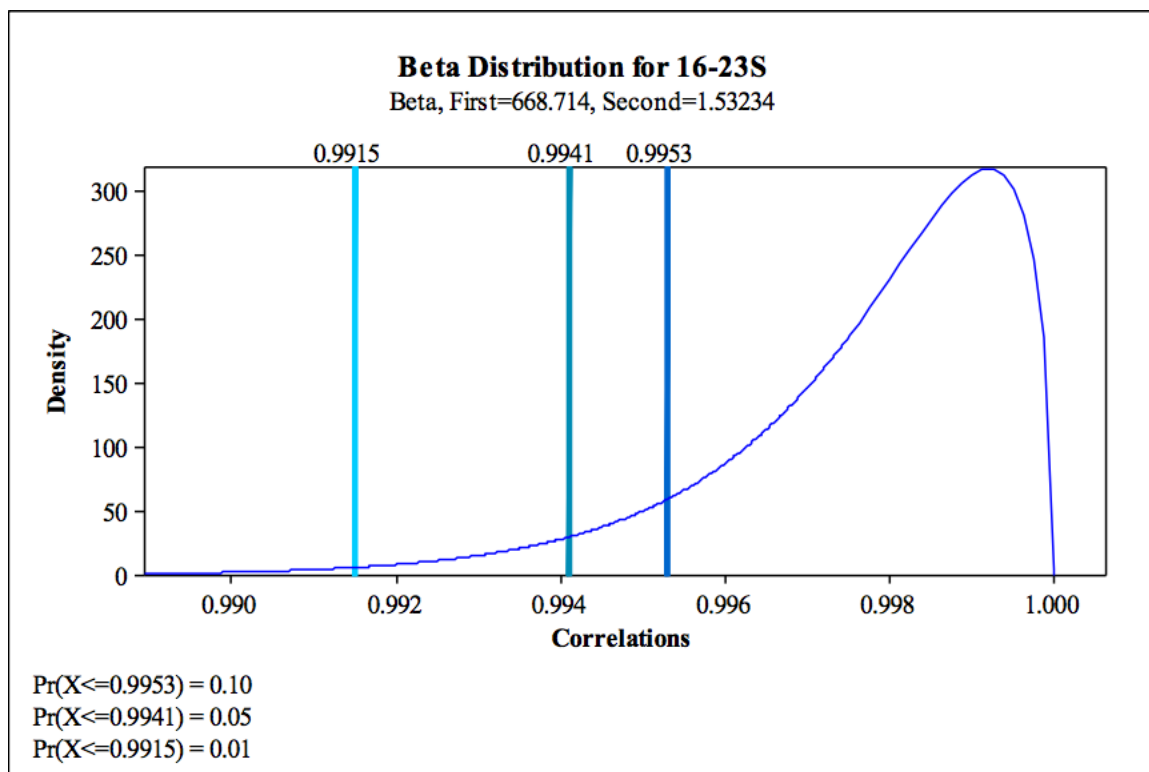


Figure 2.3: **Beta distribution for pyroprints from the same isolate for ITS region 16-23.** Taken from [18].

Based on this analysis, the biologists chose two thresholds for pyroprinting. Two isolates with a Pearson correlation above the α threshold 0.995 in both ITS regions are considered definitely similar. Two isolates with a Pearson correlation below the β threshold 0.99 in either ITS region are considered definitely dissimilar. Isolates with a Pearson correlation between the α and β thresholds may or may not be similar.[14]

2.3 CPLOP: Cal Poly Library of Pyroprints

The MST library created by the Computer Science and Biology departments at Cal Poly is called the Cal Poly Library of Pyroprints¹ (CPLOP)[20]. CPLOP started as a prototype in a class at Cal Poly. After the class, Kevin Web, one of the students

¹www.cplp.org

from the class, completed CPLOP as his senior project. Later, Jan Soliman made extensive upgrades to CPLOP dubbed CPLOP 2.0[20].

CPLOP stores pyroprints and the provenance metadata associated with them in a relational database. The work of this thesis integrates with CPLOP. It takes its input from the pyroprint database, and stores its output there.

The primary use for CPLOP is microbial source tracking. MST is discussed in depth in Section 2.1. Other uses for CPLOP include facilitating longitudinal and transference studies. Longitudinal studies, such as that performed by Emily Neal[15], look at strains observed in an individual over time and look for changes or patterns. Transference studies, such as that performed by Josh Dillard[7], look at samples from different hosts for strains found in both hosts. Currently all use cases can and have been performed with the help of CPLOP. However, they are not convenient.

The first inconvenience is that MST tasks are slow. The forensics step of library-dependent MST (see step 5 in Section 2.1.1) requires finding all isolates with representations matching that of an isolate of unknown origin. The naive method of finding these matches compares the representation against every representation in the database. Pearson correlation is an expensive computation, especially with over 90 dimensions, and there are thousands of isolate representations in CPLOP. At the time of this writing, the latest MST algorithms used for CPLOP[13], still relied on this naive matching.

The inconvenience for longitudinal and transference studies is that strains are not integrated into CPLOP. Every time a researcher wants to look into a certain strain or a group of strains which are often seen together, the biologists must ask the computer scientists to run a clustering algorithm for them.

It is clear that storing strains in CPLOP would allow biologists to access them for longitudinal and transference studies without having to wait for computer scientists.

Additionally, storing the strains in a relational database (like CPLOP) would allow researchers to more easily view the isolates and metadata from strains with relational queries. Finally, storing strains in CPLOP would also enable faster MST forensics. Strains could have single representation[20] and unknown isolates would only need to be compared to each strain instead of each isolate, resulting in many fewer expensive Pearson correlation calculations.

2.4 Clustering

With the desire for storing strains of *E. coli* in CPLOP established, we now discuss methods for identifying them. Section 2.2.5 discusses the definition of a strain for pyroprinted *E. coli* arrived at through statistical analysis. Isolates whose representations match above a certain threshold are considered members of the same strain. However, this threshold has false negatives, so strains must also include some isolates for which some pairwise comparisons are below the threshold.

Data clustering is the computer science task of grouping similar data together into clusters. It takes as input a set of data points, in this case pyroprints. The result of the task is a set of clusters and an N to 1 mapping from the input points to the resulting clusters. There are many different notions of a cluster and algorithms to create them.

2.4.1 OHClust!: Ontological Hierarchical Clustering

In previous work, Aldrin Montana created a clustering algorithm called OHClust! which addressed the need for strain identification[14]. OHClust! is based on agglomerative hierarchical clustering. Agglomerative clustering starts with every point as its own cluster. Then at every step it combines the two closest clusters. This process ends when all clusters have been combined into a single cluster. The process

of combining clusters at each step produces a hierarchical view of the clusters. The hierarchy is cut at the desired level to determine the final clusters. This algorithm has $O(N^2)$ steps, and depending on the link type used to measure cluster distance, the overall algorithm can be $O(N^3)$. OHClust! uses average-link which uses the average of all pairwise comparisons between two clusters as the distance between the clusters[14]. Average link agglomerative clustering is one of the like types that gives the overall algorithm a complexity of $O(N^3)$. OHClust! saw vast improvements in performance over standard average-link agglomerative clustering, but is still bounded at $O(N^3)$ [14].

The results and future work sections in Montana’s thesis talk about some of the limitations and issues of OHClust!. The big issue he mentions is a need for more testing and verification of his algorithm. Montana could not prove that the resulting clusters from OHClust! were similar to those of standard average-link agglomerative clustering, the algorithm previously used by the research team. Without another method of determine cluster validity, the correctness of OHClust! could not be determined[14].

Another limitation of Montana’s approach is the lack of integration with CPLOP. Currently, the biologists require assistance from the computer scientists to perform the clustering. This is because of performance problems with OHClust!. OHClust! needs more RAM than CPLOP’s servers can provide. Another limitation is that the runtime of OHClust! is very long. The work presented in this thesis tries to address these issues by focusing on performance.

2.4.2 DBSCAN: Density-Based Spatial Clustering of Applications with Noise

The work presented in this thesis uses a density-based notion of clusters and a modified DBSCAN algorithm to identify strains. The notion of a cluster used by this work

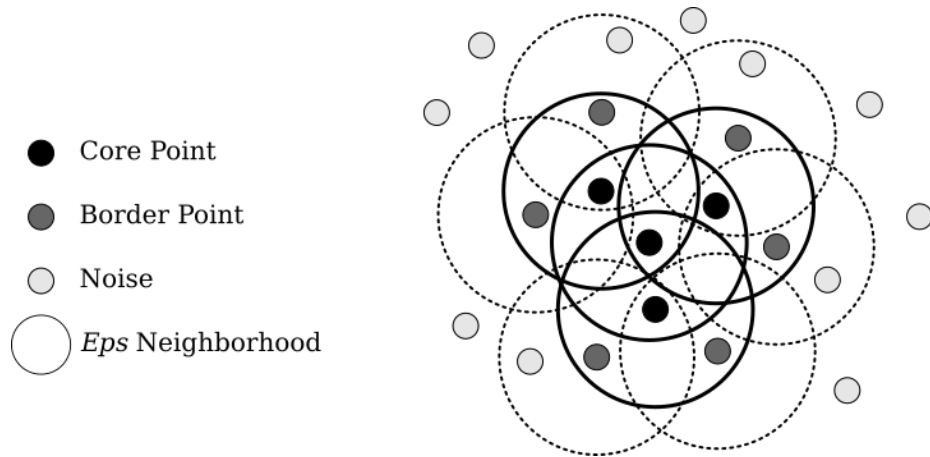


Figure 2.4: **Example density-based cluster with $MinPts=3$**

is defined by DBSCAN[9]. An example of a density-based cluster is shown in Figure 2.4. The DBSCAN definition of a cluster has two parameters, $MinPts$ and Eps . It defines a neighbor of a point to be another point within a distance of Eps . Points are classified as either a core point, a border point, or noise. A core point is defined as a point with at least $MinPts$ neighbors. A border point is defined as a point with less than $MinPts$ neighbors but within Eps of a core point. All other points are defined as noise. A DBSCAN cluster is defined as a group of neighboring core points and the group of border points that neighbor that core.

Pseudocode for the algorithm provided with the original DBSCAN paper is shown in Figures 2.5 and 2.6. The algorithm takes values for $MinPts$ and Eps as input and uses that to classify every point into clusters. The algorithm runs in $O(N \log N)$ given an $O(\log N)$ `RegionQuery()`[9].

Since the original paper came out, a few extensions of DBSCAN have been published. The OPTICS algorithm[1] provides a hierarchical clustering for density-based clusters. It generalizes away the Eps argument and only needs the $MinPts$ parameter. Another algorithm, IncrementalDBSCAN[8] significantly speeds up clustering if most of the points have previously been clustered.

```

function DBSCAN(setOfPoints, eps, minPts)                                ▷ setOfPoints are
UNCLASSIFIED
    clusterId ← nextId()
    for point ∈ setOfPoints do
        if point.clId = UNCLASSIFIED then
            seeds ← RegionQuery(setOfPoints, point, eps)
            if seeds.size < minPts then                                    ▷ not core point
                point.clId ← NOISE
            else                                                            ▷ all points in seeds are density-reachable from Point
                ExpandCluster(setOfPoints, point, seeds, clusterId, eps, minPts)
                clusterId ← nextId()
            end if
        end if
    end for
end function

```

Figure 2.5: Pseudocode for the DBSCAN algorithm.

2.5 Spatial Indexes

The clustering algorithm DBSCAN, described in Section 2.4.2, performs better with a $O(\log N)$ neighbor lookup. Such lookups require the points to be organized in an index optimized for the types of queries. In this case the queries are range-based which implies the use of a spatial index. Spatial indexes organize data to optimize for queries spatial in nature. An example of a query on a spatial index is finding all points within a geometric shape. If the shape is a sphere with radius Eps centered at one of the points, then the resulting points are all points within a distance Eps of the point at the center, ie. its neighbors.

```

function EXPANDCLUSTER(setOfPoints, point, seeds, clId, eps, minPts)
    seeds.clId  $\leftarrow$  clId
    seeds.delete(point)
    while seeds  $\neq$   $\emptyset$  do
        currentP  $\leftarrow$  seeds.first()
        result  $\leftarrow$  RegionQuery(currentP, eps)
        if result.size  $\geq$  minPts then
            for resultP  $\in$  result do
                if resultP.clId = UNCLASSIFIED||NOISE then
                    if resultP.clId = UNCLASSIFIED then
                        seeds.append(resultP)
                    end if
                resultP.clId  $\leftarrow$  clId
            end if
        end for
        end if
        seeds.delete(currentP)
    end while
end function

```

Figure 2.6: Pseudocode for the DBSCAN algorithm.

Spatial indexes are a special type of search tree. A search tree, as depicted in Figure 2.7 is composed of nodes. Most nodes have other nodes as children (inner nodes), but some nodes only contain data (leaf nodes). The nodes are arranged in a way such that when searching for certain data, only some children of each node need to be checked. In this case of the example search tree in Figure 2.7, the letter in a node comes alphabetically after all letters stored in the subtree as its left child, and

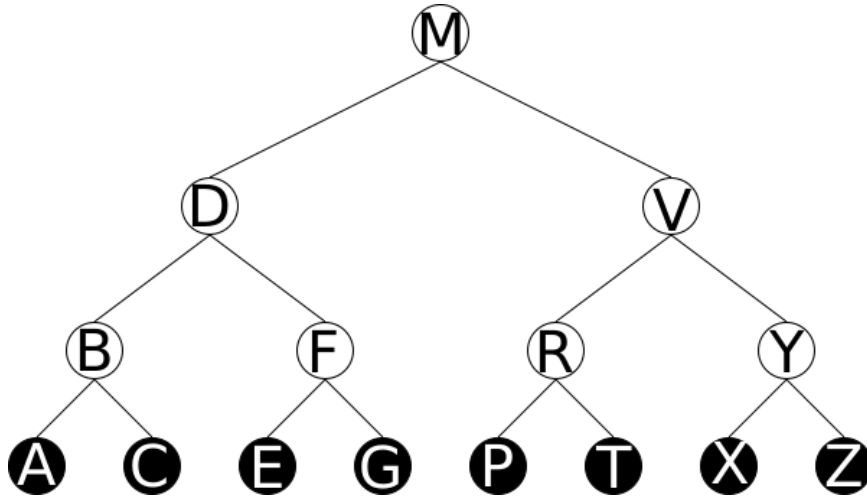


Figure 2.7: **Binary Search Tree example.** White nodes are inner nodes. Black nodes are leaf nodes.

alphabetically before all letters stored in the subtree as its right child.

A search is called a traversal and runs in $O(\log N)$ time. A traversal of the tree in Figure 2.7 searching for the letter P proceeds as follows. Traversals start at the root node of the tree, in this case M . At each step, the search letter is compared to the letter in the node. P is compared to M , and since P comes after M in the alphabet, the traversal moves to the node's right child, V . Again, P is compared to V , and since P comes before V in the alphabet, the traversal moves to the node's left child, R . Then, P is compared to R and since P comes before R in the alphabet, the traversal moves to the node's left child, P . Thus P is found after only 3 comparisons, as opposed to 8 comparisons, one for each leaf node.

Most spatial indexes organize points by their location in a Euclidean space. This allows for efficient queries based on the Euclidean distance $d(\vec{x}, \vec{y})$ between points.

$$d(\vec{x}, \vec{y}) = \sqrt{\sum_{i=1}^D (x_i - y_i)^2}$$

The math for Euclidean space scales to any positive number D dimensions.

Pyroprints as described in Section 2.2.3 can be thought of as ~ 100 dimensional

points in Euclidean space. However Euclidean distances between these points would not be comparable to Pearson correlation. Section 3.3.1 explains how we can convert pyroprints into different ~ 100 dimensional points where euclidean distances between these points are comparable to Pearson correlation of the original pyroprints. This allows us to store the points in normal spatial indexes in order to efficiently find points within a certain Pearson correlation threshold.

When choosing spatial indexes for such data, the curse of dimensionality comes in. Many spatial indexes degenerate into linear searches (searching every point) when the points have more than a few dimensions. The following sections describe some indexes designed specifically for high-dimensional data, while giving examples of some simpler ones.

2.5.1 Space Partitioning

The first class of spatial indexes is space partitioning. The general case is Binary Space Partitioning (BSP). BSP organizes data in a binary tree splitting the tree in half with plane (or hyperplane with more dimensions) each time. As implied by the name, each node of the tree "partitions" the entire space.

The KD-Tree, depicted in Figure 2.8, is a special case of BSP that partitions space with axis-perpendicular planes. At each node of the tree it partitions a single dimension. The dimension used for a node cycles as you move down the tree. KD-Trees are affected by the curse of dimensionality because partitioning each dimension x number of times requires a tree $x \cdot D$ deep where D is the number of dimensions [3].

The PK-Tree is based off the KD-Tree but addresses the issue with a large number of dimensions. It works like a KD-tree but with unnecessary nodes eliminated. Given some mild constraints on the data, the expected height of the tree can be bounded[21].

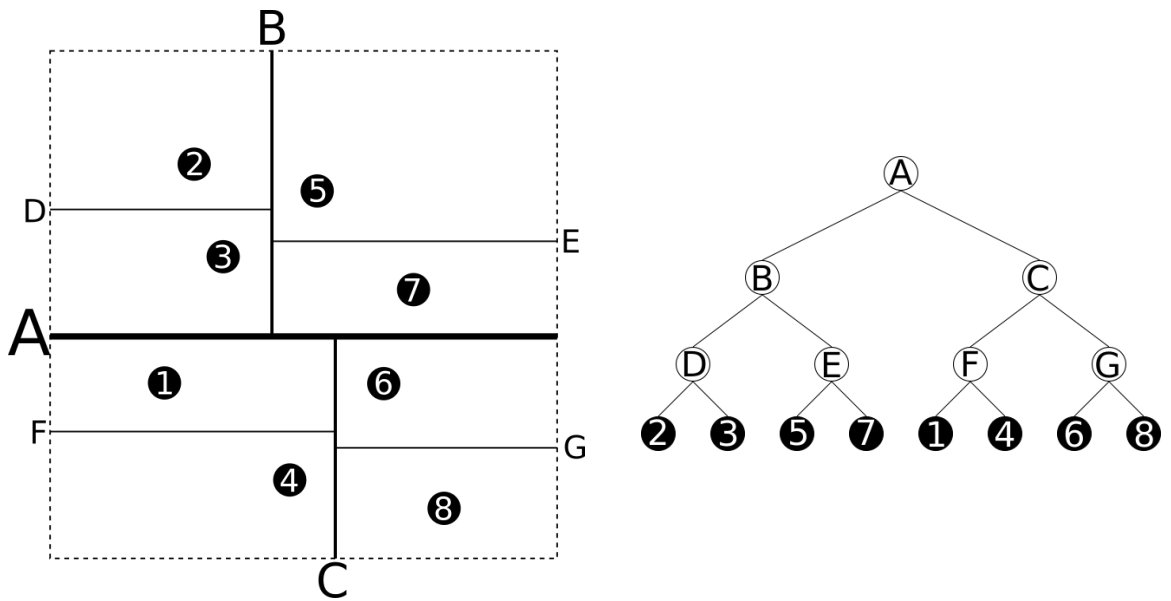


Figure 2.8: **Example KD-Tree.**

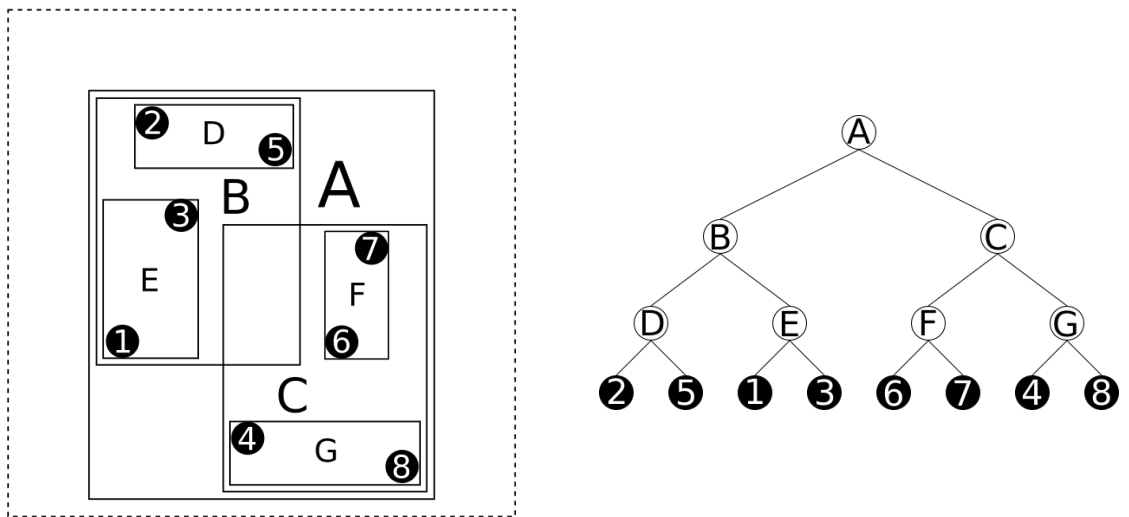


Figure 2.9: **Example R-Tree.**

2.5.2 Bounding Volume Hierarchy

Another class of spatial indexes is a Bounding Volume Hierarchy (BVH). Instead of partitioning space, a BVH organizes data within bounding volumes. Each node in the tree is a volume that bounds all volumes and points in its sub tree. Depending on the shapes used, sibling nodes can have overlapping volumes even if the points they contain are disjoint.

The R-Tree, depicted in Figure 2.9, is a popular BVH that uses only rectangular volumes. The construction of an R-Tree allows for lots of overlap. R-Trees are subject to the curse of dimensionality because overlapping nodes becomes a bigger and bigger problem with more and more dimensions[10].

The R*-Tree is based on the R-Tree which addresses the issue of overlap. The construction of an R*-Tree tries to minimize overlap by reinserting points when a node gets too full[2]. The X-Tree is based on the R*-Tree, and addresses the issue of overlap a step further. X-Trees use supernodes which allow problematic points to be stored in internal nodes instead forcing them into a leaf node which would cause overlap.[4]

An alternative to the R-Tree family is the SR-Tree which uses the intersection of spheres and rectangles for the volumes of its nodes. This is based off the idea that volumes that better fit the points they contain are less likely to overlap with each other[11]. Another alternative is the TV-Tree. TV-Trees address the dimensionality issue by using only a few of the dimensions. They use the additional dimensions only when absolutely needed[12].

2.5.3 Hybrid Spatial Index

Another class of spatial index combines space partitioning and bounding volume hierarchy. An example of this is the Hybrid-Tree. It is based on the KD-Tree but the partitions can overlap requiring the subspaces to be treated as bounding volumes for searching[6].

CHAPTER 3

DESIGN

This chapter describes the algorithms contributed along with this thesis. The algorithms are the primary contribution of this thesis. The first section outlines the goals for the solution. Next is an overview of the solution design. Following that is a detailed look at each part of the solution.

3.1 Motivation And Goals

Chapter 2 describes the origin of the Cal Poly Library of Pyroprints (CPLOP), a tool for a new method of Microbial Source Tracking (MST) developed by the biology department at Cal Poly. Section 2.3 explains the benefits of identifying strains of *E. coli* among the isolates stored in CPLOP.

The algorithms we contributed identify and analyze strains from the pyroprints stored in the CPLOP. The goals for the solution were (1) to allow for faster MST, (2) to allow strain analysis in an easy to understand form, and (3) to scale with the database as it is incrementally updated with new pyroprints.

The ultimate purpose of this thesis is to provide an alternative solution to that provided by Aldrin Montana in his thesis[14]. His solution, OHClust!, described in Section 2.4.1, was motivated by the same original need, and had similar goals. Unfortunately, the meager computational resources allocated to the CPLOP server could not run OHClust!. As such, a secondary goal for the design of this thesis was to have light resource requirements.

3.2 Overview

We started our design with the desire to try density-based clustering for strain identification to see how it compares to an agglomerative approach like OHClust! when applied to the data collected by the biologists at Cal Poly stored in Cal Poly Library Of Pyroprints. Many of the design decisions revolve around ensuring that we can leverage the performance benefits of density-based clustering.

3.2.1 Speeding Up MST

The first goal for our solution was to allow for faster MST. This is very useful, because the naive method of matching isolates for MST, comparing an unknown isolate to every isolate in the database, scales poorly with the growing library of isolates.

As described in Section 4.8 of Jan Soliman's thesis[20], it is possible to use identified strains to speed up the MST process. By using a representative isolate for each strain, matching an unknown isolate only requires looking at each strain representative instead of every isolate.

This thesis contributes to this speed up by identifying strains and storing them in CPLOP. Section 4.7.2 of Soliman's thesis[20] describes the relational data model for storing strains in CPLOP.

3.2.2 Allowing Strain Analysis

The second goal for our solution was to provide analysis of strains in an easy to understand form. The purpose of analyzing bacterial strains in the library is to find similarities between bacterial representations. These similarities signify pairs of isolates that have some identical sequences of DNA and likely share a recent common ancestor. This allows the biologists to look for patterns in the metadata of related

isolates. With metadata such as time and location of the samples, the biologists can then ask questions like "where has this strain of bacteria been seen?" and "how has that changed over time?"

OHClust! coupled the identification of strains with their analysis. As such, when Montana was unable to verify the validity of the identified strains, the analysis could not be used either. Hoping to avoid a similar fate, and for the sake of simplicity, we decided to decouple strain identification and analysis in our solution. We designed a tool that could be used to analyze strains found by any clustering method. Our solution is to store identified strains in CPLOP, allowing researchers to use SQL queries on CPLOP to analyze those strains.

3.2.3 Scaling With a Growing Database

The third goal for our solution is to grow with the database. The biologists are continually gathering new samples and sequencing them into pyroprints which are added to the database. Montana's solution for this goal was to make OHClust! incremental. The time to cluster a few new pyroprints in OHClust! is much faster than clustering everything from scratch. For our solution however, we decided not to use an incremental algorithm. This is a simpler solution that we thought would still scale with the database due to the efficiency of the clustering algorithm we chose, DBSCAN.

DBSCAN has a runtime complexity of $O(N \log N)$ compared to agglomerative clustering which has complexity of up to $O(N^3)$ (as in the case of OHClust!) depending on the intercluster distance metric used. The $O(N \log N)$ complexity of density-based clustering algorithms depends on the use of spatial indexes with $O(\log N)$ range queries.

Spatial indexes generally operate on points in euclidean space, so in order to use

a spatial index to store our data, we had to find the relationship between Pearson correlation and Euclidean distance. In Section 3.3.1 we derive the following relationship:

$$\rho(\vec{x}, \vec{y}) = 1 - \frac{d(\vec{z}_x, \vec{z}_y)^2}{2D}$$

where \vec{x} and \vec{y} are pyroprints, ρ is the Pearson correlation, d is Euclidean distance, \vec{z}_x and \vec{z}_y are the z-scores of \vec{x} and \vec{y} , and D is the number of dimensions.

Using this formula, we converted our α and β thresholds, described in Section 2.2.5, from Pearson correlation to Euclidean distance of Z-scores. Our solution pre-computes the z-scores for each pyroprint. Then during clustering, it calculates the Euclidean distance between pairs of pyroprints and compares the value to the converted thresholds.

The next step was to decide on the spatial index to use for our clustering. Spatial indexes are notoriously bad at scaling to high-dimensional spaces. Unfortunately, the 93-95 dispensations of our pyroprints is well within the classification of high-dimensional. Common aspects of spatial indexes that we needed to avoid for high-dimensional data were having overlapping nodes, and splitting dimensions independently.

The best spatial indexes designed for high-dimensional data we could find are described in Section 2.5. They were all designed to store that data persistently on disk. Because all of our data should fit in RAM for the foreseeable future, we decided not to persist our index. As such, we did not use the previous solutions and instead designed our own. The index we designed was tailored for our data and the query pattern of the DBSCAN algorithm. It is described in Section 3.4

3.3 Isolate Comparison

There are two components to comparing isolates. First is the comparison of pyroprints from the same DNA region. Second is the way comparisons from all DNA regions are used to come to a decision on similarity.

3.3.1 Euclidean Distance of Pyroprint Z-Scores

Spatial indexes generally operate on data existing in Euclidean space. The pyroprints could be thought of as points in an D -dimensional space where D is the number of dispensations. The distance between two points in Euclidean space can't be converted to the Pearson correlation between two points which was previously used by those working with CPLOP. The calculation of the Pearson correlation uses the average and standard deviation of the dimensions in a point, but Euclidean distance discards the information needed to find the average and standard deviation. There is a type of spatial indexes called a m -tree which can operate on arbitrary metrics[16]. Unfortunately, the Pearson correlation does not fit the criteria of a proper metric which is required by m -trees. Specifically, Pearson correlation violates the triangle inequality property.

The reason the biologists use Pearson correlation, as described in Section 2.2.4, is because it statistically normalizes the data. We needed Euclidean distance for use in a spatial index, and we needed to use statistical normalization for comparison validity. Since the biologists had already determined meaningful thresholds for Pearson correlation, we hoped to be able to leverage that by finding a relationship between Pearson correlation and Euclidean distance with statistical normalization. The following is a derivation of that relationship.

We start with the equations for Pearson correlation ρ and Euclidean distance d :

$$\rho(\vec{x}, \vec{y}) = \frac{1}{D} \sum_{i=1}^D \frac{(x_i - \mu_x)(y_i - \mu_y)}{\sigma_x \sigma_y}$$

$$d(\vec{x}, \vec{y}) = \sqrt{\sum_{i=1}^D (x_i - y_i)^2}$$

where D is the number of dimensions, and μ_x and σ_x are the mean and standard deviation respectively.

$$\mu_x = \frac{1}{D} \sum_{i=1}^D x_i$$

$$\sigma_x = \sqrt{\frac{1}{D} \sum_{i=1}^D (x_i - \mu_x)^2}$$

We notice that the inner terms of Pearson correlation are z-score normalizations,

$$z(x_i) = \frac{x_i - \mu_x}{\sigma_x}$$

and plug in $z(x_i)$ into the distance equation for x and y and simplify.

$$\begin{aligned} d(\vec{z}_x, \vec{z}_y) &= \sqrt{\sum_{i=1}^D \left(\frac{x_i - \mu_x}{\sigma_x} - \frac{y_i - \mu_y}{\sigma_y} \right)^2} \\ &= \sqrt{\sum_{i=1}^D \left(\frac{x_i - \mu_x}{\sigma_x} \right)^2 - 2 \left(\frac{x_i - \mu_x}{\sigma_x} \right) \left(\frac{y_i - \mu_y}{\sigma_y} \right) + \left(\frac{y_i - \mu_y}{\sigma_y} \right)^2} \\ &= \sqrt{\left(\sum_{i=1}^D \left(\frac{x_i - \mu_x}{\sigma_x} \right)^2 \right) + \left(\sum_{i=1}^D \left(\frac{y_i - \mu_y}{\sigma_y} \right)^2 \right) - 2 \sum_{i=1}^D \left(\frac{x_i - \mu_x}{\sigma_x} \right) \left(\frac{y_i - \mu_y}{\sigma_y} \right)} \\ &= \sqrt{\frac{\sum (x_i - \mu_x)^2}{\sigma_x^2} + \frac{\sum (y_i - \mu_y)^2}{\sigma_y^2} - 2 \sum_{i=1}^D \frac{(x_i - \mu_x)(y_i - \mu_y)}{\sigma_x \sigma_y}} \end{aligned}$$

We notice the the sums look like parts of the equations for σ and ρ . Solving the equations for the sums gives us:

$$\sum_{i=1}^D (x_i - \mu_x)^2 = D \cdot \sigma_x^2$$

$$\sum_{i=1}^D \frac{(x_i - \mu_x)(y_i - \mu_y)}{\sigma_x \sigma_y} = D \cdot \rho(\vec{x}, \vec{y})$$

ITS region	23-5		16-23	
	α	β	α	β
$\rho(\vec{x}, \vec{y})$	0.995	0.99	0.995	0.99
D	93		95	
$d(\vec{z}_x, \vec{z}_y)$	0.9644	1.3682	0.9747	1.3784

Table 3.1: **Converted Thresholds.**

and we substitute those back into our $d(\vec{z}_x, \vec{z}_y)$ equation and simplify.

$$\begin{aligned}
d(\vec{z}_x, \vec{z}_y) &= \sqrt{\frac{D \cdot \sigma_x^2}{\sigma_x^2} + \frac{D \cdot \sigma_y^2}{\sigma_y^2} - 2D \cdot \rho(\vec{x}, \vec{y})} \\
&= \sqrt{2D - 2D \cdot \rho(\vec{x}, \vec{y})} \\
&= \sqrt{2D(1 - \rho(\vec{x}, \vec{y}))}
\end{aligned}$$

Thus the relationship between ρ and d is:

$$d(\vec{z}_x, \vec{z}_y) = \sqrt{2D(1 - \rho(\vec{x}, \vec{y}))}$$

or

$$\rho(\vec{x}, \vec{y}) = 1 - \frac{d(\vec{z}_x, \vec{z}_y)^2}{2D}$$

Using this formula, we converted our α and β thresholds from Pearson correlation to Euclidean distance of Z-scores, shown in Table 3.1. Because the conversion depends on number of dimensions D , the threshold for distance is different for each region despite being the same for Pearson correlation.

Our solution precomputes the z-scores for each pyroprint. Then during clustering, it calculates the Euclidean distance between pairs of pyroprints and compares the value to the converted thresholds.

3.3.2 Considering Multiple DNA Regions

The biologists realized that pyroprints from a single region of DNA was not enough to be able to determine if two isolates are the same or not. So for each isolate there are actually two pyroprints, one from each of two different regions. Isolates are considered equal only if the pyroprints from both regions match under the threshold. Because the two regions have different thresholds, the distances can't be combined into one distance, to be compared against a single threshold. Instead, decisions on similarity need to take into account the distances between every region.

The first idea we considered was to cluster isolates twice, once for each region, then to take intersections of the cross product of both sets of clusters. Isolates that were in the same cluster for each region are in the same cluster of the combined regions. This design would have a couple benefits. First, it would be simple to implement because the comparison for two pyroprints is just distance. Second it could provide additional analysis opportunities for the biologists, who could look for patterns within a single region, or find relationships between strains based on which other strains share one of the region clusters. The primary drawback of this design is additional computation. At first glance, clustering twice would be twice as much work. But on top of that the clustering itself becomes more expensive because the range queries on a single region return a significant portion of our data which results in worst case performance complexity $O(N)$ from the spatial index, making the full algorithm $O(N^2)$ instead of $O(N \log N)$.

The second idea we considered was to combine the regions and do a single clustering of both regions at the same time. The benefits and drawbacks of this approach are the inverse of those of the first approach. Performance is better because the spatial index has more information on which to trim branches of the tree. Design complexity is higher because calculations and decisions during the query need to take into ac-

count two independent regions. This also means that the query algorithm takes two radii as input, one for each region, instead of the normal one radius.

We decided to use the second approach for this work. While the opportunity for extra analysis would be nice, it wasn't specifically asked for, while good performance was. Additionally, the analysis would likely be performed infrequently, and thus the extra computation would be wasted most of the time. Perhaps in the future the ability to make such analyses could be added separately from the basic strain identification.

3.4 Spatial Index

All of the design for strain identification revolved around the spatial index. The data was transformed into z-scores and the thresholds converted for Euclidean distance so that a spatial index could be used. Density-based clustering gets its performance from leveraging a spatial index.

Our primary concern with choosing a spatial index is the fact that our data is high-dimensional. Most spatial indexes scale poorly with dimensions. Common aspects of spatial indexes that we needed to avoid for high-dimensional data were having overlapping nodes, and splitting dimensions independently.

The best spatial indexes designed for high-dimensional data we could find are described in Section 2.5. They were all designed to store that data persistently on disk. Because all of our data should fit in RAM for the foreseeable future, we decided not to persist our index. As such, we did not use the previous solutions and instead designed our own.

Because we decided not to have a persistent index, it must be built from scratch whenever we want to cluster the data. Therefore, we must choose a spatial index which can be built quickly. Specifically, the complexity should not be worse than the

complexity of the clustering, $O(N \log N)$, otherwise building the tree would dominate the total runtime as the dataset scales.

Designing our own spatial index allows us to make some optimizations for its use in DBSCAN as well as for the specific characteristics of our data.

3.4.1 Optimizations for use with DBSCAN

In order to be used by DBSCAN, the spatial index needed to provide a range query. A range query takes a query point, and a range as input and returns all data points within range of the query point. The query can be thought of as a hypersphere with its center at the query point with a radius of the query range. All points that lie inside that hypersphere are returned.

The DBSCAN algorithm queries the spatial index in a unique pattern. First, all queries are centered at a point in the database, as opposed to an arbitrary point with all queries having the same range. Second, each point is only queried once, and nearby points are processed soon after each other.

Our spatial index query takes some shortcuts by realizing that once a point and all of its neighbors have been processed, DBSCAN will never make another query that would return that point. In addition, we modified the DBSCAN algorithm to keep track of the neighbors seen for each unprocessed point. This means that it will never need to see any points more than once. These modifications are described in Section 3.5.1.

By only committing to returning points that have not been previously returned we can get away with only searching points that haven't been returned yet. By removing points from the spatial index after they are returned, future queries will have fewer points to search through.

	[0.0, 0.25)	[0.25, 0.5)	[0.5, 0.75)	[0.75, 1.0)	[1.0, 1.25)	[1.25, ∞)
23-5	37	45	7	0	3	1
16-23	46	39	6	1	0	2

Table 3.2: **Number of dimensions with a standard deviation in various ranges for each ITS region.** The deviations are calculated between z-scores, not the original pyroprints.

3.4.2 Data Characteristics

Our data has some characteristics that make it difficult to use in a spatial index. The points have very little spread in most dimensions. As shown in Table 3.2, standard deviations between points in a single dimension are less than 0.5 for most dimensions. Compared to the range of the query of 0.96 or 0.97 for the α thresholds of 23-5 and 16-23 respectively, the difference between points in a single dimension usually won't be enough to exclude a point from the query results. Unfortunately, we can't ignore any single dimension because those small differences add up over ~ 100 dimensions. Unfortunately, this means that when traversing the tree, the algorithm will branch to multiple children to a greater depth, until many dimensions have been looked at.

In order to keep the tree depth low, our design splits points based on multiple dimensions per level. This is common in spatial indexes such as the popular quadtree and octree, however, that approach increases the number of children of each node exponentially (2^D) where D is the number of dimensions split at that level. Splitting all ~ 100 dimensions in one node would be impractical with 2^{90} children. We could instead split only a few different dimensions at each level of the tree, but the tree would still have the same number of leaves after splitting each dimension once. Since 2^{90} leaves is more than the number of data points we could ever possibly hope to add to the database, the majority of those leaves would be wasted. In addition, the range

query would intersect many of these children, causing the algorithm to traverse down exponentially many paths. This is somewhat offset by the reduced depth of the tree, but it is not ideal. The $O(\log N)$ performance of spatial data queries assumes that at the majority of nodes, only one child is further traversed.

What we decided to do instead was split multiple dimensions dependently. That is to split once but have that one split cover multiple dimensions. This means that the split plane won't be axis aligned. The idea is that by changing the angle of the split plane the average distance from points on either side to the split plane will increase. This increase depends on a correlation between dimensions i.e. points with low values (relative to other points) in one dimension, also tend to have low values (relative to other points) in another dimension. Inverse correlations work too, as long as most points with low values in one dimension have high values in another. Fortunately, the data in our library does exhibit correlations between some dimensions.

3.4.3 Construction

This section describes the remaining details of the design of the spatial index. Since the tree is not persistent, we can construct the tree with knowledge of all the points. Instead of constructing the tree by adding one point at a time and splitting nodes when they get too big (like many bounding volume hierarchy indexes) or picking arbitrary split criteria ahead of time (like many spatial partitioning indexes), our design looks at all points in a (sub)tree then partitions them.

As discussed in Section 3.4.2, our spatial index partitions points by finding a plane that splits the data in multiple correlated dimensions. The dimensions are chosen by picking the single dimension in which the points are most spread apart, as determined by standard deviation, and then choosing a number of other dimensions based on a combination of how correlated they are to the first dimension and how spread those

dimensions are. Specifically it chooses the N dimensions with the highest values for $|\rho| \cdot \sigma$, where ρ is the Pearson correlation of data values for a dimension with the values for the dimension with the most spread, σ is the standard deviation for a dimension, and N is a parameter to the construction algorithm.

Each dimension is only split once per path down the tree. The true distance to a node is not simply the distance to the split plane of that node, but actually the distance to the intersection of the planes of all the ancestors of the node. Instead of finding those intersections and calculating the distance to the complex shape that results, we can combine the distances to individual planes using Euclidean distance $d_{combined} = \sqrt{d_1^2 + d_2^2}$. In order to combine distances from multiple split planes during queries (the split planes from a node and all of its ancestors), each of those planes must be perpendicular to all of the others. The simplest way to guarantee this is to enforce that no plane shares any dimension with another. Dimensions will be reused in the planes of nodes that aren't direct ancestors or descendants as distances to those planes will not be combined during queries.

After picking the dimensions, our construction algorithm chooses a hyper-plane using linear regression between each dimension and the dimension with the most spread. The formula determines the slope $b = \rho \cdot \frac{\sigma_x}{\sigma_y}$ and intercept $i = s_x - b \cdot s_y$ of the regression line $a \cdot x + b \cdot y + i = 0$ where x is the value of a point in the dimension with greatest spread, y is the value of the point in the other correlated dimension, and $a := 1$. The intercept i determines the position of the split plane, and is calculated for a point s on the plane. We tried two options for s , the mean of each dimension, and the median point in relation to the split plane. The mean is less work to calculate, thus results in faster construction. The median point results in a balanced tree, thus faster queries. Since the performance of our spatial index is dominated by the queries and not the construction, we chose to use the median point.

The algorithm finds an equation of a hyper-plane by combining these linear regression equations into a form $a \cdot x + b \cdot y + \dots + c \cdot z + i = 0$ where x is the dimension with the most spread (again with $a := 1$) and each subsequent term is a correlated dimension multiplied by the slope ($b \cdot y$) of the linear regression with the first dimension. The intercept i is simply the sum of the intercepts from the linear regression for each correlated dimension.

In order to partition the points based on the hyper-plane, we needed a function that, given a point $\langle x_0, y_0, \dots, z_0 \rangle$ and the equation for the plane ($a \cdot x + b \cdot y + \dots + c \cdot z + i = 0$), would return which side of the plane it was on. We used the equation for signed distance from a plane for this purpose as well as for the spatial query algorithm. The formula for the distance is

$$d = \frac{a \cdot x_0 + b \cdot y_0 + \dots + c \cdot z_0 + i}{\sqrt{a^2 + b^2 + \dots + c^2}}$$

The result of this equation is signed, and that sign determines which side of the plane a point is on. For the purpose of partitioning, only the sign matters, so we can ignore the denominator. Since the plane is only in a subset of the dimensions, the "point" we input to the function is made up of only those dimensions from the real data point, in the same order as they are used in the plane equation.

3.4.4 Storage

After deciding which points go in which child of a tree node, the algorithm has to encode spatial information about those children for the query to use. The two most common methods of doing this are (1) storing the plane used to split the data *i.e.* spatial partitioning, and (2) storing a bounding volume for each child fit to the data points within *i.e.* bounding volume hierarchies. The benefits of storing the split plane are that it generally takes less storage space, and it guarantees no overlap between children which can be a problem with bounding volumes. Bounding volumes come

in many shapes and sizes each with different characteristics, however in general, they take more storage space and the better they fit the data, the more complex the comparisons or construction.

We tried both methods for our data, storing the plane as used during construction and storing axis-aligned bounding boxes for each child. We chose axis aligned bounding boxes for the simplicity of construction and comparison. However, axis-aligned boxes generally suffer from empty corners and in our case, since the points aren't split by axis-aligned planes, those corners could cross over the split plane, making overlapping children probable. Comparison against bounding spheres would also be fairly simple and would avoid empty corners, however they lose that simplicity when scaled in different dimensions, becoming ellipses. Since our data is very skewed, and since by the nature of spatial partitioning, the data would become more skewed in lower levels in the tree, using uniform spheres could result in good fit in the most spread dimensions but lots of extra space on the sides of other dimensions. These hypotheses were not tested, but the choice of axis aligned bounding boxes was mostly driven by simplicity of construction. Constructing a minimum bounding sphere is not nearly as simple as constructing a minimum axis aligned bounding box

Comparing a query range against split planes was more effective than axis aligned bounding boxes at higher levels in the tree closer to the root. However, deeper into the tree, the effectiveness of the methods swapped, and the axis aligned bounding boxes were better at pruning children. This complementary behaviour led to our decision to use both methods of storage. We considered storing only planes at higher levels and only bounding boxes at lower levels, however we theorized that extra discrimination and thus less branching at each level was more important than reduced node storage and comparison time. This could be something to try in future work.

All of the previous design details involve only a single ITS region. The addition of

more regions does not affect the design of a single region. While we made an effort to retain a binary split for a single region by combining multiple dimensions in a single split plane, we don't do this for multiple regions. Because of the inability to combine distances from multiple regions described in Section 3.3.2, we instead decided to add more children.

For example, with two regions we would have two split planes. Each split plane uses whichever dimensions it wants (there is no connection between any dimensions from one region and another). They don't even need to use the same number of dimensions, each plane is completely independent. For two regions there will be 4 children like a quad tree. One child is for points with a negative distance to the planes for both regions. Another child is for points with a positive distance to both. The other two children are for points with a positive distance to one region's plane and a negative one to the other.

3.4.5 Algorithms

Pseudocode for our tree construction algorithm is shown in Figure 3.1. It's a recursive algorithm that constructs and returns subtrees of the spatial index. The base case is determined by the number of points passed to the algorithm. If the number of points (isolates) passed to the algorithm is less than POINTS_PER_LEAF variable, it constructs a new leaf node with those points and returns it. In the general case the algorithm partitions the points and calls the algorithm recursively to create children nodes used to construct an inner node which is then returned. Points are partitioned according to the scheme described in Section 3.4.3 with a plane perpendicular to the linear correlation between a number of dimensions defined by DIMS_PER_SPLIT.

Figures 3.2, 3.3 and 3.4 contain pseudocode for the range query algorithm. This algorithm is also recursive, returning the set of previously unseen points within *radius*

```

function CONSTRUCTTREE(points, unusedDims)
  if points.size ≤ POINTS_PER_LEAF then
    return LeafNode(points)
  else
    splitPlanes ← ∅
    for region ∈ REGIONS do
      mainDim ← dimension with greatest standard deviation
      correlatedDims ← unused dimensions correlated to mainDim
      splitDims ← mainDim ∪ DIMS_PER_SPLIT from correlatedDims
      splitPlanes ← splitPlanes ∪ calculateSplitPlane(splitDims)
    end for
    children ← ∅
    for each side of Region1's split plane do
      for each side of Region2's split plane do
        partition points using split planes
        children ← children ∪ ConstructTree(partition, unusedDims)
      end for
    end for
    return InnerNode(children, splitPlanes)
  end if
end function

```

Figure 3.1: Pseudocode for the algorithm that constructs our spatial index. POINTS_PER_LEAF is the number of points that can be stored in a leaf node. DIMS_PER_SPLIT is the number of dimensions that are used for each partition.

of *center*. The base case is when the algorithm is called on a leaf node. In this case, the algorithm simply compares all points stored in that leaf node to the query and

```

function RANGEQUERY(node, center, radii)
  result  $\leftarrow \emptyset$ 
  switch typeof(node) do
    case leafNode
      for point  $\in$  leafNode.points do
        if PointInSpheres(point, center, radii) then
          result  $\leftarrow$  result  $\cup$  point
          remove point from leafNode.points
        end if
      end for
    end case
    case innerNode
      for childNode  $\in$  innerNode.children do
        if NodeIntersectsSpheres(innerNode, center, radii) then
          result  $\leftarrow$  result  $\cup$  RangeQuery(childNode, center, radius)
        end if
      end for
    end case
  end switch
  return result
end function

```

Figure 3.2: Pseudocode for our range query algorithm.

returns those that match. In the general case, the algorithm is called on an inner node. The algorithm first determines which nodes might possibly contain points that match the query. Then it recursively calls the algorithm on each of those children and combines the results.

```

function POINTINSPHERES(point, center, radii)
  for region  $\in$  REGIONS do
    radius  $\leftarrow$  radii[region]
    if distance(center[region], point[region])  $\geq$  radius then
      return False
    end if
  end for
  return True
end function

```

Figure 3.3: Pseudocode for our range query algorithm.

```

function NODEINTERSECTSPHERES(node, center, radii)
  for region  $\in$  REGIONS do
    planeDist  $\leftarrow$  PlaneDistance(center[region], child.planes[region])
    bboxDist  $\leftarrow$  BBoxDistance(center[region], child.bboxes[region])
    if max(planeDist, bboxDist)  $\geq$  radii[region] then
      return False
    end if
  end for
  return True
end function

```

Figure 3.4: Pseudocode for our range query algorithm.

3.5 Clustering

The primary reason we chose to use density-based clustering was that it is different from agglomerative clustering. Of course, there are other clustering methods which

are different from agglomerative clustering, but density-based clustering offers better performance and cluster determinism independent of input order. We had to decide on a specific density-based clustering algorithm from among the options described in Section 2.4.2.

A hierarchical clustering algorithm could provide extra information for strain analysis. If isolates in one strain are more similar to each other than isolates in another strain, a dendrogram would reflect that. However, generating that dendrogram would come at the cost of performance. The hierarchical density-based clustering algorithm, OPTICS, is more demanding of the spatial index. It requires finding the distance to the $minPts$ th closest neighbor for each point instead of just the number of points within the range. Nearest neighbor searches can actually be faster than radius searches. However, to get the benefit of hierarchical clusters, the max search radius would need to be bigger than the radius we would use for normal DBSCAN, ending up with a net decrease in performance. Finally, finding the nearest neighbors requires sorting points based on their distance to the query point. However, as mentioned in 3.3.2, we cannot combine the distances from multiple DNA regions into one distance, meaning we would have to cluster each region separately. Because these new analysis opportunities were not specifically requested and performance was, we decided OPTICS was not worth the tradeoff and to use DBSCAN instead. Perhaps in the future, the team could add a standalone tool using OPTICS on individual regions for infrequent analysis.

Due to the inherent performance gains from switching to a lower algorithmic complexity, we decided that using an incremental algorithm was not worth the added responsibility of persisting internal algorithm information and maintaining a list of new or removed data points between clustering runs. An algorithm with complexity $O(N \log N)$ can scale well with a growing dataset. Upgrading to an incremental version could be a good avenue for future work.

3.5.1 Modified DBSCAN Algorithm

In order to take advantage of the shortcut discussed in Section 3.4.1, we needed to modify the original DBSCAN algorithm. Our spatial index query deletes points as it returns them, meaning it only returns points that have not been previously seen. In some cases, this works with the unmodified DBSCAN algorithm, however we had to modify the algorithm to handle the edge cases.

When a point is processed by the standard DBSCAN algorithm, the spatial index is queried for the *Eps*-neighbors of the point. If the number of neighbors for a point is at least *minPts* then the point and its neighbors are added to the current cluster (creating a new cluster if there isn't one). Then, the algorithm adds each neighbor to a *seeds* list. Once a point is done being processed, another point is taken from the *seeds* list and that point is processed. If the *seeds* list is empty, then the current cluster is completed and the next point to be processed is taken from the set of all points. When a point is processed, there are two possibilities: Either it is processed in the outer DBSCAN() function, or in the inner ExpandCluster() function.

If a point is processed in the inner ExpandCluster() function, then by definition at least one of its neighbors has been seen and the point is being added to a cluster. We could compensate for this by simply adding one to the points neighbor count, but there are two edge cases here. First it is possible that (1) the point could have other neighbors in the same cluster which were already processed. Second, (2) some of the point's other neighbors could have been seen by another point in the cluster. In both cases, the neighbor in question would have been removed from the spatial index before this point was processed. Neither of these edge cases affect whether or not the point will be added to the cluster, but the count of neighbors does affect whether the point's previously unseen neighbors will also be added to the cluster.

If instead a point is processed in the outer DBSCAN() function, it generally means

that the point is not a neighbor of any points that have previously been seen; otherwise it would have likely been added to *seeds* and queried from `ExpandCluster()` instead. This means that the set of previously unseen neighbors is the same as the set of all neighbors. The edge case here is that (3) one of the point's neighbors could have been seen by a non-core point. When this happens, the neighbor would have been removed from the spatial index but not added to *seeds* (where it would have queried the point instead of the other way around).

The first modification we made is that the algorithm now maintains for each unprocessed point, a list, *seenNeighbors* of all the processed points who have seen that point. Then, when a point is processed, the number of points in *seenNeighbors* is added to the number of previously unseen neighbors returned by the spatial query. This combined count is used to determine whether the point is a core point, and if the combined count is at least *minPts*, the unseen neighbors are added to *seeds* like normal, as well as all of the already seen neighbors (which may or may not already be in that cluster). Then the algorithm adds the processed point to the *seenNeighbors* for each of the previously unseen points. By itself, this modification doesn't fix any of the edge cases, as it only accounts for the one neighbor which first saw each point, but it serves as a platform to address them.

The next modification is that when a point is processed, in addition to querying the spatial index, the point is compared to every other point in *seeds*. For each seed point within *eps* of the point being processed, the algorithm adds both the seed point and the point being processed to each other's *seenNeighbor*. This modification mostly solves edge cases 1 and 2. However, the modification itself has an edge case where (4) one of the point's neighbors could have been seen by a non-core point. When this happens, the neighbor would have been removed from the spatial index but not added to *seeds* (where it would have been found during this modified point processing).

Edge cases 3 and 4 are actually the same problem surfacing in different spots, and are related to another concern that removing points from the spatial index could result in some points never being processed. This could happen if a point was seen by a non-core point in which case it would not have been added to *seeds*, but was still deleted from the spatial index. If the spatial index is the source of unprocessed points used by the outer DBSCAN() function, then the point will never be queried.

We considered two solutions to this problem. The first solution was to decouple the removal of points in the spatial index from the query, and instead only remove points if the point being processed turns out to be a core-point (and thus the removed points will be added to *seeds*). The second solution was to keep points that have been seen by non-core points in a *noise* list like *seeds* but separate. Then when points are being processed the algorithm would compare the point to each of the points in *noise*, in addition to those in *seeds*. We chose to use the *noise* list because it removes points from the spatial index as soon as possible, hopefully speeding up future queries.

Pseudocode for our modified DBSCAN algorithm is in Figures 3.5 and 3.6. For reference, the pseudocode for the original DBSCAN can be found in Figure 2.5 and 2.6.

```

function MODIFIEDDBSCAN(setOfPoints, eps, minPts)
    ▷ setOfPoints are UNCLASSIFIED

    while setOfPoints ≠ ∅ do
        point ← setOfPoints.pop()
        neighbors ← RegionQuery(setOfPoints, point, eps)
        if neighbors.size ≥ minPts then    ▷ all neighbors are density-reachable
            clId ← nextId()
            point.clId ← clId
            neighbors.clId ← clId
            noise ← ∅
            seeds ← neighbors
        else                                ▷ not core point
            clId ← -1                        ▷ not currently expanding a cluster
            noise ← neighbors
            seeds ← ∅
        end if
        ExpandNeighbors(setOfPoints, point, seeds, noise, clId, eps, minPts)
    end while
end function

```

Figure 3.5: Pseudocode for our modified DBSCAN algorithm.

```

function EXPANDNEIGHBORS(unseen, point, seeds, noise, clID, eps, minPts)
  update seenNeighbors
  while seeds  $\neq \emptyset$  || noise  $\neq \emptyset$  do
    if seeds  $\neq \emptyset$  then
      point  $\leftarrow$  seeds.pop()
    else
      clId  $\leftarrow$  -1 ▷ not currently expanding a cluster
      point  $\leftarrow$  noise.pop()
    end if
    neighbors  $\leftarrow$  RegionQuery(unseen, point, eps)
    update seenNeighbors
    if neighbors.size + seenNeighbors[point].size  $\geq$  minPts then
      if clID = -1 then
        clId  $\leftarrow$  nextId()
        point.clId  $\leftarrow$  clId
      end if
      move any seenNeighbors[point] from noise to seeds
      seeds = seeds  $\cup$  neighbors
      neighbors.clId  $\leftarrow$  clId
      seenNeighbors[point]  $\leftarrow$  clId
    end if
  end while
end function

```

Figure 3.6: Pseudocode for our modified DBSCAN algorithm.

CHAPTER 4

IMPLEMENTATION

This chapter describes the implementation of the work contributed with this thesis. It is intended to serve as a reference for researchers using this work for future work. The code is accessible from a github repository¹.

4.1 Language and Libraries

We implemented the work in Python, and utilized a few third party libraries. We ran the evaluations with version 3.4.3, but the code should be compatible with any version 3.X of python. We chose Python originally just to prototype the solution. We were happy with the performance of the libraries though, so when we went to create the final implementation we stuck with the same language.

A couple of the packages we used are from the open source library SciPy². SciPy is a collection of software packages for math, science, and engineering. We used version 0.14.0. For evaluating clusters, we used the Statistics package (stats). The Statistics package provides objects representing statistical distributions and methods for analysing and comparing random samples or distributions. We used NumPy for storing and comparing pyroprints. NumPy provides N-dimensional array objects and efficient vector operations on those arrays. NumPy is maintained by SciPy but is available standalone, with it's own versions. We used version 1.8.1.

In order to interface with CPLOP, which is implemented as a MySQL database, we used the MySQL Connector provided by MySQL³. MySQL Connector, like MySQL

¹<https://github.com/ejohns32/Thesis-Code>

²<http://www.scipy.org/install.html>

³<http://dev.mysql.com/downloads/connector/python/>

is open source. MySQL connector handles connecting to and querying a MySQL database. It also handles data conversion into standard Python types. We used version 2.0.3.

Python and all of the libraries we used are available together through Anaconda. Anaconda is a package manager and Python distribution available from Continuum Analytics⁴. This is what we used, although newer versions will come with up to date Python and libraries that may be incompatible with this code.

4.2 Overview

This section describes how the different modules of the implementation interface with each other. It does this in the context of a typical run of the program.

First the implementation loads settings from a configuration file. These settings are described in the sections detailing each module. The configuration file includes descriptions of each ITS region which are represented by `Region` objects.

In the `pyroprinting` module, the program queries pyroprints from CPLOP using the `mysql.connector` library. The pyroprints are then organized and stored as `Isolate` objects. The `Isolate` class provides an `isWithinRadiiOf()` function which takes another `Isolate` and a radius for each `Region` returning a boolean value.

In the `spatial` and `fullsearch` modules, the implementation takes a list of `Isolates` and constructs a `SpatialIndex` or `FullSearchIndex` respectively. `SpatialIndex` and `FullSearchIndex` both provide a `getNeighborsOf()` and `popNeighborsOf()` function, which both take an `Isolate` and a radius for each `region`. The functions return the set of `Isolates` for which the `isWithinRadiiOf()` function returns true.

The `dbscan` module contains two functions for creating clusters: `dbscan()` and

⁴<https://www.continuum.io/downloads>

`popDBSCAN()` which both take an `Index` and DBSCAN's `eps` and `minPts` parameters. The functions call the `getNeighborsOf()` and `popNeighborsOf()` functions of the passed `Index` respectively to determine clusters. The functions both return a list of clusters, each cluster represented as a set of `Isolates`.

The `clusterEval` module is optional. It provides functions which evaluate one or more lists of clusters as returned by the functions in the `dbscan` module.

4.3 Data Types

The `Region` class, defined in module `pyroprinting`, contains information about a single ITS region. Instances are created from the configuration file. The class contains fields for the name of the ITS region, the dispensation count, a Pearson correlation similarity threshold (which is converted to zScore distance and used as `eps` for DBSCAN), and parameters for the beta distribution of Pearson correlations between pyroprint regions of the same strain.

The `Isolate` class, defined in module `pyroprinting`, aggregates one pyroprint from each ITS region. Pyroprints are stored as `numpy.arrays` of zScores. The class provides functions for comparing its pyroprints to those of other isolates, namely `isWithinRadiiOf()`. `Isolates` are obtained by querying CPLOP for pyroprints. The query we used is in Figure 4.1. The query grabs the latest pyroprint from each region. After the query, we also filter out isolates which don't have pyroprints for both regions.

The `BoundingBox` class, defined in module `spatial`, represents an N-dimensional axis aligned bounding box, where N is the number of dispensations for the ITS region it encloses. It is stored as two `numpy.arrays`, one contains the minimum values for each dispensation of every pyroprint contained in the bounding box, and the other contains the maximum values for each dispensation.

```

SELECT p1.isoID , p1.appliedRegion , position , zHeight
FROM zScores
INNER JOIN pyroprints p1 USING(pyroID)
LEFT JOIN pyroprints p2 ON p1.isoID = p2.isoID
    AND p1.appliedRegion = p2.appliedRegion
    AND p2.isErroneous IS FALSE AND p1.pyroID < p2.pyroID
WHERE p1.isErroneous IS FALSE AND p2.isoID IS NULL
ORDER BY isoID , appliedRegion , position ;

```

Figure 4.1: **SQL query for extracting isolates from CPLOP.** It queries for the latest pyroprint from each region for every isolate, excluding pyroprints determined to be erroneous.

The `MultiDimPlane` class, defined in module `spatial`, represents an N-dimensional plane which is non-axis aligned in M dimensions, where $M \leq N$, the total number of dispensations for the ITS region. It is represented as a list of the dimensions which it splits and plane coefficients for each of those dimensions.

Comparisons between `Isolates`, `BoundingBoxes`, and `MultDimPlanes` are implemented using operations on `numpy.arrays`. Element by element arithmetic uses overloaded operators (+, -, *, and /). More complex calculations require combining arithmetic operations and sometimes also element by element `numpy.maximum()`, `numpy.minimum()` and `numpy.absolute()` functions. Various comparisons are implemented as reductions (with `numpy.linalg.norm()` and `numpy.sum()`) after calculations, or element by element comparisons (like `numpy.greater_equal()` and `numpy.less_equal()`) followed by a boolean reduction (like `numpy.all()`).

4.4 Indexes

The `SpatialIndex` class, defined in module `spatial`, is an implementation of the data structure designed for this thesis and described in Section 3.4. It provides a `getNeighborsOf()` and `popNeighborsOf()` function, which both take an `Isolate` and a radius for each `region`. The functions return the set of `Isolates` for which the `isWithinRadiiOf()` function returns true. An instance is constructed from a list of `Isolates`. The construction algorithm uses some settings from the configuration file which affect when leaf nodes will be built, or how many dimensions will be used for `MultiDimPlanes`. A `SpatialIndex` is composed of `Nodes` which are themselves composed of `SpatialFilters` and either other `Nodes` or `Isolates`.

The `SpatialFilter` class is a generic representation of methods for determining if any points in subtree might match the neighbor query. These methods are associated with `Nodes` in the `SpatialIndex`, and are specific to a single ITS region, meaning there can be one `SpatialFilter` per filter method per ITS region per `Node`. Currently we have 2 methods and 2 ITS regions for a total of 4 `SpatialFilters` per `Node`. The `SpatialFilter` class is an abstract class providing basic functionality for connecting `SpatialFilters` from one `Node` to the corresponding (same ITS region and method) `SpatialFilters` from the `Node`'s parent and children. It is intended to be extended for each filtering method with an `intersectsQuery()` function for checking if a query intersects with the filter, and an `update() / aggregate()` function for updating the filter when points are removed from the subtrees of the `LeafNode / InnerNode` the filter is associated with.

The first filtering method is represented by the `PlanePartitionFilter` class which extends the `SpatialFilter` class. These filters represent a method of filtering using the `MultiDimPlanes` used to partition points during construction. Its `intersectsQuery()` function calculates the distance to the `MultiDimPlane` associ-

ated with the filter, and those of all of filters ancestors. The implementation uses the cached result from parent to speed up its calculation. Then it caches its result for use by its children. Its `update()` and `aggregrate()` are no-ops because updating the `MultiDimPlane` implies updating the partition. We would have liked to implement repartitioning the points to rebalance the spatial index, however we decided to leave this for future work.

The second filtering method uses the `BoundingBoxFilter` class which also extends the `SpatialFilter` class. These filters represent a bounding volume hierarchy with axis aligned bounding boxes. Its `intersectsQuery()` function calculates the distance to the `BoundingBox` associated with the filter. Its `update()` function updates its `BoundingBox` to fit the reduced set of points contained by the `LeafNode` it is associated with. The `aggregrate()` function updates its `BoundingBox` to fit those of its children filters.

`Node` is an abstract class which implements the common functionality of `InnerNodes` and `LeafNodes`. The `Node` class manages the `SpatialFilters` and logic for determining if the subtree starting at the `Node` might contain points matching the query. It provides a `intersectsQuery()` function which is used by the `InnerNode` class to determine if the query traversal can ignore the `Node`. The `intersectsQuery()` calls the `intersectsQuery()` function of each of its `SpatialFilters`, and takes advantage of short circuiting to avoid the expensive calculations of the `SpatialFilters` high-dimensional pyroprint comparisons.

The `InnerNode` class extends `Node` and represents a node of the search tree which contains other `Nodes` instead of points. This class implements the search traversal logic which decides which of its children `Nodes` to continue searching.

The `LeafNode` class extends `Node` and represents a terminating node in the search tree. It contains `Isolates` (or any other object with a `isWithinRadiiOf()` function).

This class contains the logic which ultimately decides if a point will be returned and deletes the points it does return.

The `FullSearchIndex` class, defined in module `fullsearch`, is an implementation of a naive index. Neighbor lookups search every isolate which is $O(N)$ instead of the $O(N \log N)$ of the spatial index. This naive implementation has the same interface as `SpatialIndex`, constructed with a list of `Isolates` and exposing `getNeighborsOf()` and `popNeighborsOf()` functions. This implementation serves as a benchmark for performance testing `SpatialIndex`.

The `PrecomputedIndex` class, defined in module `fullsearch`, is another implementation of an index. This index is constructed with a map from every isolate to the set of isolates which are neighbors of that isolate. The neighbor lookup is $O(1)$, however the precomputation is as expensive as running a neighbor lookup once for every isolate in one of the other indexes. Because the precomputation only needs to be ran once, this index is useful for trying out different clustering parameters quickly.

4.5 DBSCAN

The `dbscan` module contains two functions for creating clusters: `dbscan()` and `popDBSCAN()` which both take an `Index` and DBSCAN's *eps* and *minPts* parameters. The DBSCAN parameters are loaded from the configuration file. The functions call the `getNeighborsOf()` and `popNeighborsOf()` functions of the passed `Index` respectively to determine clusters. The functions both return a list of clusters, each cluster represented as a set of `Isolates`.

The `dbscan()` function is an implementation of the pseudocode for the original DBSCAN algorithm described in Section 2.4.2 and shown in Figures 2.5 and 2.6. A fairly minor change is that unlike the pseudocode which encodes clusters as member variables of the data points themselves, this implementation collects points into sets

representing clusters. This actually results in a difference in the output in that border points can be found in multiple clusters. The original DBSCAN algorithm simply used the first cluster discovered.

The `popDBSCAN()` function is an implementation of the pseudocode developed in the design of this thesis and found in Figures 3.5 and 3.6. As described in Section 3.5.1, the design of this algorithm works with an `Index` which removes the points it returns. Like the `dbscan()` function, this function collects points into sets representing clusters and can cluster border points into multiple clusters.

4.6 Evaluation

The `clusterEval` module is optional. It provides functions which evaluate one or more lists of clusters as returned by the functions in the `dbscan` module. Some of the evaluation methods use statistical methods. We used the `kstest()` and `ks_2samp()` functions from the `scipy.stats` module to compare samples of pairwise Pearson correlations from isolates in the same cluster. The `kstest()` also takes a continuous distribution function as input. We used the `beta` distribution from the model to represent the beta distribution fitted to the data in CPLOP in previous work. We also queried CPLOP for a more current sample of correct pairwise Pearson correlations among replicated pyroprints. The query we used is in Figure 4.2.

```

SELECT r.appliedRegion ,
        PearsonMatch(p1.pyroID , p2.pyroID , r.pearsonDispLength)
FROM Pyroprints AS p1
INNER JOIN Pyroprints AS p2 USING('isoID ' , 'appliedRegion ')
INNER JOIN Regions AS r USING('appliedRegion ')
WHERE (
        SELECT COUNT(*)
        FROM Pyroprints AS pyro
        WHERE pyro.isoID = p1.isoID
                AND pyro.appliedRegion = p1.appliedRegion
                AND pyro.pyroID > p1.pyroID
                AND pyro.isErroneous IS FALSE
        ) < MAX_REPLICATES
AND p1.pyroID < p2.pyroID
AND p1.isErroneous IS FALSE
AND p2.isErroneous IS FALSE
ORDER BY p1.isoID , p1.appliedRegion , p1.pyroID , p2.pyroID ;

```

Figure 4.2: **SQL query for finding the distribution of pairwise Pearson correlations from replicates in CPLOP.** Where *MAX_REPLICATES* limits the number of replicates that can be taken from a single isolate to prevent bias.

CHAPTER 5

EVALUATION

Evaluation is a big part of the contribution of our work. The strains our implementation identifies are only useful to the biologists if we can show that they are meaningful. One of the original goals of this work was to provide an alternative method of strain identification for comparison against previous methods. This chapter describes the comparisons we performed between our design, Montana’s OHClust!, and average-linkage agglomerative hierarchical clustering (hereafter referred to as just agglomerative clustering).

5.1 Evaluation Plan

This section describes the comparative tests we ran to evaluate DBSCAN against other clustering methods for our use in strain identification. We performed a few different tests for our evaluation to better characterize the similarities and differences between clustering methods as well as to try to identify a ”most correct” method. The results for these tests are shown and discussed in Section 5.5

5.1.1 Performance Evaluation

The first evaluation we made was a performance comparison. One of the goals for this work was for it to scale well with a growing database. To determine how well our spatial index and modified DBSCAN algorithm scales, we compared it to the standard DBSCAN without a spatial index with different database sizes. In order to isolate the affects of each contribution, we also ran standard DBSCAN with our spatial index (without removal of points) and our modified DBSCAN algorithm without a spatial

index. We created databases of sizes 1000 - 6000 with increments of 1000 by using random subsets of our current database. The same random subset was used for each method for a given database size to so that the randomness of the sampling didn't affect the relative results.

We measured total runtime, as well as peak memory usage for each test. Both runtime and peak memory usage were measured with the unix "time" command with the "-v" (or "-l" depending on the operating system) argument and recording the "user" and "maximum resident set size" respectively.

5.1.2 Strain Correctness

The next tests we performed evaluated the correctness of the clusters produced by each method. We consider these the most important evaluations. Similarity evaluations like Montana performed are only as reliable as the method being compared against. In addition similarity tests are only useful if the result is that the clustering methods identify the same strains. If the methods produce different clusters, then we need a way to determine which clusters are better.

Threshold Correctness

The first correctness evaluation we ran was a rough measure of cluster correctness. The biologists consider pairs of pyroprints with a distance less than the α threshold to be "definitely similar" and pyroprints with a distance greater than the β threshold to be "definitely different". It follows that a pair of isolates are "definitely similar" if the pyroprints for all regions are "definitely similar" or "definitely dissimilar" if the pyroprints for any regions are "definitely dissimilar".

In this test we evaluated the clustering methods by how they clustered isolate pairs that are "definitely similar" and "definitely dissimilar". Methods are scored

according to a threshold correctness score

$$TC = \frac{sameDS + diffDD}{totDS + totDD}$$

where *sameDS* is the count of pairs of isolates which are "definitely similar" and were put in the same cluster, *diffDD* is the count of pairs of isolates which are "definitely dissimilar" and were put in different clusters, *totDS* is the count of all pairs of isolates which are "definitely similar" and *totDD* is the count of all pairs of isolates which are "definitely dissimilar."

The closer a clustering method's *TC* score is to 1, the better it matches the biologists idea of "definitely similar" and "definitely dissimilar" isolates. However, not all pairs of isolates are either "definitely similar" or "definitely dissimilar." As such, this measure is incomplete in that it doesn't take into consideration every pair of isolates. To give an indication of how incomplete the measure is, we also provided the fraction of all isolate pairs which are neither "definitely similar" or "definitely dissimilar."

In addition, we calculated the ratios $\frac{sameDS}{totDS}$ and $\frac{diffDD}{totDD}$ to get a better idea of why a clustering method with a *TC* score not close to 1 is incorrect. It will also be able to capture incorrectness if one of the terms is small relative to the others and thus has little weight in the overall *TC* calculation.

Pearson Correlation Distribution Correctness

Then we calculated two-sample goodness of fit tests against an expected distribution. We calculated the two-sample Kolmogorov-Smirnov statistic for the distribution of Pearson correlations between isolates in the same clusters. For each clustering method we tested, we ran the evaluation separately for each DNA region.

A two-sample goodness of fit test takes the distributions of two observed samples

and calculates the likelihood of the two samples coming from the same underlying distribution. It doesn't indicate what the distributions of either sample are, just if they are the same or not. The samples we used for each clustering method being compared were the Pearson correlations between every pair of isolates which the method placed in the same cluster.

The equation for the two-sample Kolmogorov-Smirnov statistic is:

$$KS = \max_w |F_X(w) - F_Y(w)|$$

where F_X and F_Y are the cumulative distributions of the samples from X and Y respectively. In our case, X is the set of Pearson correlations drawn from the expected distribution and Y is the set of Pearson correlations drawn from clusters produced by each clustering method. The null hypothesis is that the two samples come from the same underlying distribution and it is rejected at the level α if

$$KS > c(\alpha) \sqrt{\frac{m+n}{m \cdot n}}$$

where $c(\alpha)$ is the critical value for the two-sample Kolmogorov-Smirnov test at the rejection threshold α , and m and n are the total numbers of distances in X and Y respectively.

The expected distribution we used for the test is similar to the distribution that Statistics student Diana Shealy used to fit a beta distribution[18]. Because we were comparing a sample of our data to another sample of our data, there was a concern that such a comparison would be invalid for evaluating correctness. The expected distribution came from isolates which were repeatedly pyroprinted, and when clustering, we only used a single pyroprint for each isolate (for each region). That does mean that there are some isolates that were used in both. However, since the distributions we were comparing were of Pearson correlations between pairs of data points, we just needed to make sure that we didn't reuse the same pairs for the expected and actual

distributions. Since all of the Pearson correlations used for the expected distribution were between pairs of pyroprints of the same isolate, and all of the Pearson correlations used for clustering were between pairs pyroprints from different isolates, we are guaranteed that none of the Pearson correlations were used in both.

To obtain samples from a set of clusters, we used Pearson correlations between all pairs of isolates which were placed in the same clusters. The Pearson correlations came from isolate pairs from every cluster, just without the pairs in which the isolates were in different clusters. We tested isolates from every cluster together to get an overall picture of the clusters.

5.1.3 Similarity to Current Method

The last set of evaluations we ran calculated the similarity of a set of clusters to the clusters produced by the current clustering method used by the research team. The idea is that if a clustering method produces clusters similar to those of another method which is already considered acceptable, then the new method is also acceptable. Alternatively, if two clustering methods differ, these tests can be useful by measuring in what ways the two methods are not similar.

Cluster Similarity Indexes

Montana compared OHClust! to agglomerative clustering using similarity indexes. These indexes take two sets of clusters X and Y and calculated a value indicating how similar they are. The indexes use counts of the number of isolate pairs (o_i, o_j)

which are either in the same cluster or different clusters in each set as follows:

$$n_{11} = |S_{11}|, \text{ where } S_{11} = \{(o_i, o_j) : o_i, o_j \in X_k, \quad o_i, o_j \in Y_m\}$$

$$n_{10} = |S_{10}|, \text{ where } S_{10} = \{(o_i, o_j) : o_i, o_j \in X_k, \quad o_i \in Y_m, o_j \in Y_n\}$$

$$n_{01} = |S_{01}|, \text{ where } S_{01} = \{(o_i, o_j) : o_i \in X_k, o_j \in X_l, \quad o_i, o_j \in Y_m\}$$

$$n_{00} = |S_{00}|, \text{ where } S_{00} = \{(o_i, o_j) : o_i \in X_k, o_j \in X_l, \quad o_i \in Y_m, o_j \in Y_n\}$$

The similarity indexes we used were the Rand Index, the Jaccard Index, and another similar index as follows

$$Rand = \frac{n_{00} + n_{11}}{n_{00} + n_{10} + n_{01} + n_{11}}$$

$$Jaccard = \frac{n_{11}}{n_{10} + n_{01} + n_{11}}$$

$$Different = \frac{n_{00}}{n_{00} + n_{10} + n_{01}}$$

For each index, values closer to 1 mean the clustering methods are more similar with a value of 0 meaning they are completely different and a value of 1 meaning they are the same. The Rand Index gives a complete view of how similar 2 sets of clusters are. The Jaccard Index ignores pairs of isolates which are in different clusters for both methods and thus indicates how similar the methods are at classifying isolates as the same strain. The third index ignores pairs of isolates which are in the same cluster for both methods and thus indicates how similar the methods are at classifying isolates as different strains.

Pearson Correlation Distribution Similarity

The next evaluation we ran was a two sample goodness of fit test. We calculated the two-sample Kolmogorov-Smirnov statistic for the distribution of Pearson correlations between isolates in the same clusters. We compared distributions from both DBSCAN and OHClust! to the distribution from agglomerative clustering.

This test was similar to two-sample goodness of fit test for similarity used in Section 5.1.2, except instead of comparing the distribution from a single clustering method to an expected distribution, it compares two sets of clusters against each other. We ran the test for each of the clustering methods under comparison using the two-sample Kolmogorov-Smirnov statistic.

Because there isn't an accepted way of mapping clusters from two different sets of clusters, we couldn't test if a single cluster from one clustering method had a similar distribution to a single cluster from the other clustering method. Instead, we tried to assess the overall quality of the clusters by using the same Pearson correlation sample used in the goodness of fit test described in Section 5.1.2. Again, every test was run separately for each DNA region.

5.2 Isolate Sets

The sets of isolates used for each clustering method were not exactly the same. We used Montana's implementation of OHClust! and Agglomerative clustering, which uses a different method for querying isolates from CPLOP than the implementation of DBSCAN presented with this thesis. OHClust! and Agglomerative clustering were run on a set of isolates which included 230 isolates that the isolate set used for DBSCAN did not have.

The query we used, shown in Figure 4.1, gets the most recent pyroprint which has not been marked as erroneous from each ITS region for each isolate. After the query, we filter out isolates which do not have pyroprints for both ITS regions. The query Montana used also filters isolates without pyroprints for both ITS regions, however it does not filter pyroprints which are marked as erroneous.

For the most part, both queries get the same isolates. However for some isolates, all of the pyroprints for an ITS region are marked as erroneous. In this case, the

implementation we provide will not return that isolate, but Montana’s implementation will return it. All but 29 of the extra isolates were because of this difference in queries. The 201 isolates clustered with erroneous pyroprints likely won’t affect the clusters. Because of their erroneous nature, those isolates likely won’t match anything enough to be added to any clusters and thus would be noise.

Of the 29 extra isolates which have pyroprints for both ITS regions not marked as erroneous, 26 of them didn’t actually have any data in the pyroprints, just metadata. We aren’t certain how they were compared to other isolates, but we assume that it would default to a Pearson correlation of 0, and thus not be added to any clusters.

After removing those 230 isolates from the clusters produced by OHClust! and Agglomerative clustering, the two sets still have 6093 isolates in common, so we consider differences caused by clustering with extra isolates to be negligible.

5.3 Clustering Parameters

This section describes the parameters used for each clustering method which affect the clusters produced.

5.3.1 DBSCAN

DBSCAN has two parameters: *Eps*, which is the distance at which two points are considered neighbors, and *MinPts*, which is the minimum number of neighbors a point must have before they are all considered a cluster. For our purpose, we actually need two *Eps* parameters, one for each region. We used the α Pearson Correlation threshold of 0.995 for *Eps* for both regions because it is a semantic value meaning that the two isolates are definitely similar. Technically, we used distances to cluster thus the thresholds we used were the converted values found in Table 3.1.

For *MinPts*, we tried two values, 1 and 3, for all of the tests. For the purpose of evaluation, we consider these two separate clustering methods denoted DBSCAN 1 and DBSCAN 3. With a *MinPts* value of 1, DBSCAN produces the same clusters as minimum/single-link Agglomerative clustering (not to be confused with average-link Agglomerative clustering which is the method currently used by the research group). A value of 2 also produces the same clusters except that the minimum cluster size is 3 as opposed to 2. At *MinPts* = 3, DBSCAN starts producing "density-based" clusters, and as such, is the second value we chose to evaluate.

5.3.2 Agglomerative Clustering and OHClust!

Agglomerative Clustering doesn't technically need parameters, but since it produces a dendrogram, in order to get clusters we need to pick a cutoff value for cluster similarity. We used the β Pearson Correlation threshold of 0.99 for the cutoff of the average pairwise Pearson correlation between points of two clusters

OHClust! is based on Agglomerative clustering and thus needs a cutoff value as well. We used the same cutoff of 0.99 for OHClust!. In addition, OHClust! uses an ontology to guide the order of comparisons. The ontology definition we used is shown in Figure 5.1.

5.4 Clusters Statistics

We start by providing descriptive statistics on the number and sizes of the clusters produced by each method. Table 5.1 shows the number of isolates which were not clustered (noise), the number of clusters, and statistics about the size of the clusters produced by each method. The DBSCAN methods leave the most isolates unclustered and have fewer number of clusters than the average-link Agglomerative-based methods. The DBSCAN methods have larger maximum cluster size and mean cluster


```

Samples.dateCollected ();;
Isolates.userName ();;
Samples.location ();;
Samples.hostID ();;

```

Figure 5.1: **The ontology definition used by OHClust! to generate clusters for the evaluation.** The format for defining an ontology for OHClust! is described in Section 4.2.1 of Montana’s thesis[14].

	noise	number	min_{size}	max_{size}	μ_{size}	σ_{size}
DBSCAN 1	1849	495	2	473	8.57	33.8
DBSCAN 3	2672	162	4	399	21.2	49.3
OHClust!	1550	1076	2	62	4.22	5.21
Agglomerative	1547	1054	2	65	4.31	5.47

Table 5.1: **Various statistics about the clusters produced by each method.**

size, compared to the other methods. The difference between these statistics is minor between OHClust! and Agglomerative clustering. However, between DBSCAN 1 and DBSCAN 3, there is a bigger difference. DBSCAN 3 has many more unclustered isolates, much fewer number of clusters, and a much higher mean cluster size.

Table 5.2 gives a better indication of the distribution of cluster sizes for each method by counting the number of clusters with sizes in different ranges. It seems that all of the extra clusters that the Agglomerative-based methods found have sizes less than 16. Also, the DBSCAN methods have more clusters with sizes greater than or equal to 64. There is little difference between the distribution of cluster sizes produced by OHClust! and Agglomerative clustering. The DBSCAN methods are even more similar, with the only large difference being that DBSCAN 3 does not produce any clusters with size less than 4. The 331 clusters found by DBSCAN 1 in

	[2, 4)	[4, 8)	[8, 16)	[16, 32)	[32, 64)	[64, 128)	[128, ∞)
DBSCAN 1	331	90	38	17	10	4	5
DBSCAN 3	0	86	38	18	10	5	5
OHClust!	693	282	78	12	11	0	0
Agglomerative	668	279	82	15	9	1	0

Table 5.2: **Distribution of cluster sizes for each clustering method.**

this range account for almost all of the difference in total number of clusters between DBSCAN 1 and DBSCAN 3.

An additional factor to consider is that, unlike Agglomerative clustering, density-based clustering has the possibility of clustering one isolate in multiple clusters. This can only occur for border points because if a point with enough neighbors to be considered a core point is found between two clusters, then clusters will combine into one. With a *MinPts* value of 1, all points are either noise or core points thus there cannot be points in multiple clusters. DBSCAN 3 however, clustered 9 isolates in multiple clusters for the full dataset. This is a small enough number that we don't think it will have a noticeable effect on the results. It is unclear what the interpretation of an isolate in multiple clusters would be when clusters are considered strains for our use case. It could suggest that the two strains should be combined into a single strain. Alternatively, the fact that it is possible could mean density-based clustering is not a good match for identifying strains in pyroprinted isolates.

5.5 Evaluation Results

This section presents the results from each of the evaluations we ran and gives an analysis of these results. The evaluations are described in Section 5.1

index	cluster	1000	2000	3000	4000	5000	6000
spatial	PopDBSCAN	4.22	12.39	23.57	37.77	56.37	76.72
spatial	DBSCAN	6.36	19.91	38.09	61.10	89.21	122.43
fullsearch	PopDBSCAN	15.86	62.41	140.82	252.48	387.40	570.37
fullsearch	DBSCAN	31.28	126.17	283.96	503.86	784.58	1147.72

Table 5.3: **Running times of clustering methods for increasing dataset sizes in seconds.** Times do not include time to query isolates from CPLOP and convert them to python objects. For spatial index runs, the times do include the time to build the index.

5.5.1 Performance Evaluation

One of the big goals for the work implemented with this thesis was to be fast and use a small amount of RAM. As described in Section 5.1.1, we compared various combinations of indexes and dbscan implementations on datasets of increasing sizes. The results are shown in Tables 5.3 and 5.4. For reference, OHClust! and Agglomerative clustering took approximately 1 and 1.5 hours to run respectively and used around 4GB of RAM. However, OHClust! and Agglomerative clustering were implemented in another language and were run on a different machine, so performance comparisons between them and the implementation presented in this thesis are not fair.

Table 5.3 shows the runtime in seconds of each implementation combination for each dataset size. The times do not include the time to query isolates from CPLOP and convert them to python objects (~ 60 seconds for 6000 isolates). For combinations including the spatial index, the times do include the time to build the index (~ 10 seconds for 6000 isolates). The fullsearch index is a naive index which represents the worst case performance of the spatial index by searching every isolate. Theoretically, a spatial index could perform on the order of 100x faster than fullsearch on a dataset of our size. Unfortunately, what we see is a 5-10x speedup. The speedup factor does

index	cluster	1000	2000	3000	4000	5000	6000
spatial	popdbscan	30.1	35.5	40.2	46.0	50.5	55.8
spatial	dbscan	29.7	35.6	40.2	45.8	50.8	58.2
fullsearch	popdbscan	28.9	33.8	38.9	43.9	49.1	54.6
fullsearch	dbscan	29.0	33.9	38.8	43.5	48.7	53.6

Table 5.4: **RAM usage of clustering methods for increasing dataset sizes in MB.**

increase as the the database grows, so we did achieve an implementation that scales better with a growing database. However there is certainly room for improvement.

We implemented the DBSCAN method based on the original DBSCAN algorithm[9] as shown in Figures 2.5 and 2.6. PopDBSCAN is an implementation of the modified algorithm we designed for this thesis. The results show a consistent 2x speedup of PopDBSCAN over DBSCAN for when using the fullsearch index, and a consistent 1.5x speedup when using the spatial index. It is possible that with better balancing after deletion, a different spatial index could see higher speedups.

Table 5.4 shows the maximum RAM usage of each implementation combination for each dataset size. The results show that the fullsearch index consistently uses around 2MB less than the spatial index. The results show little difference in RAM usage between PopDBSCAN and DBSCAN. For every implementation combination, there seems to be a consistent difference of around 5MB between results from increasing dataset sizes, implying that the RAM usage scales linearly with the dataset size. Most importantly, the RAM usage for our current dataset size of ~ 6000 is far below the 3GB of RAM available on CPLOP’s server, which also needs to run MySQL and Apache web server.

		1D plane	10D plane
		94.6	97.2
95D bbox	39.8	46.0	44.1
10D bbox	69.3	—	80.5

Table 5.5: **Time in seconds to perform a neighbor query for all isolates with different spatial storage types.** The dataset used contained 3000 isolates.

Performance Tuning

The spatial index we designed has a specific partitioning scheme and supports multiple ITS regions. It supports combinations of different types of spatial storage for the points in the index. Table 5.5 shows the relative performance results of different spatial storage options. The 10D plane is a non-axis-aligned plane like those used in BSP-trees, and is what is used to partition the points in the index. The 1D plane is an axis-aligned plane like those used in KD-trees. The 95D bbox is a normal axis-aligned bounding-box with separate bounds for every dimension (93 for ITS region 23-5) like used in R-trees. The 10D bbox is a non-axis-aligned bounding-box made up of perpendicular 10D planes (the same ones used to partition the index). Using a normal axis-aligned bounding-box by itself gave us the fastest neighbor queries so that is the spatial storage we used during the prior performance testing.

5.5.2 Strain Correctness

This section describes the results from tests attempting to evaluate the correctness the clusters produced by DBSCAN, OHClust! and Agglomerative clustering. The problem with evaluating the absolute correctness of clusters however is in defining what it means for clusters to be correct. We present results for 2 tests which evaluate two different notions of cluster correctness for their use as strains of pyroprinted *E. coli* as discussed in Section 5.1.2.

	$\alpha Correct$	$\beta Correct$	Average	TC
DBSCAN 1	1.00000	0.989815	0.99491	0.98984
DBSCAN 3	0.98982	0.993463	0.99164	0.99345
OHClust!	0.32368	0.999983	0.66183	0.99809
Agglomerative	0.35644	0.999981	0.67821	0.99818

Table 5.6: **Correctness of clustering methods with respect to the α and β thresholds.** The percentage of pairs matching above α which were placed in the same cluster, and the percentage of pairs matching below β which were placed in the different clusters. Only 0.4% of the comparisons were between the thresholds.

Threshold Correctness

The first test considered the correctness of clusters based on their adherence to the notion of isolate pairs which are definitely similar or dissimilar based on the Pearson correlation thresholds of α and β respectively. The metrics we used look at the Pearson correlations between pairs of isolates and whether or not the clustering method placed the pair in the same cluster. Table 5.6 shows the results of this evaluation.

$\alpha Correct$ denotes the percentage of isolate pairs with a Pearson correlation above the α threshold which were placed in the same cluster. This tries to measure of the maximality, or usefulness of the identified strains. The more pairs of isolates which are considered the same strain, the more information the biologists get. This metric attempts to expose clustering methods which are too conservative in the attempt to avoid false positives. The DBSCAN methods score highly in this metric, while OHClust! and Agglomerative have scores around 33%.

$\beta Correct$ denotes the percentage of isolate pairs with a Pearson correlation below the β threshold which were placed in different clusters. This tries to measure the amount of false positives in strains. This metric should expose clustering methods which are too inclusive in the attempt to maximize $\alpha Correct$. All of the clustering

methods scored high on this, although OHClust! and Agglomerative clustering scored higher.

Threshold correctness, denoted TC , combines $\alpha Correct$ and $\beta Correct$ weighted based on the number of isolate pairs with corresponding Pearson correlations. Because the majority of isolate pairs in the dataset have a Pearson correlation below β , the TC is highly biased and very similar to $\beta Correct$.

As an unbiased alternative to the TC score, we also calculated the average of $\alpha Correct$ and $\beta Correct$. The relative usefulness of each score in determining the correctness of the strains depends on the research group's value of strain maximality/usefulness vs a low chance of false positives. The absolute usefulness of these scores depends on the research group's confidence in the α and β thresholds.

Pearson Correlation Distribution Correctness

The second test we performed for cluster correctness was a two-sample Kolmogorov-Smirnov goodness of fit test. As described in Section 5.1.2, we performed a two-sample Kolmogorov-Smirnov goodness of fit test. This tests whether two samples came from the same distribution. We used pairwise Pearson Correlations from isolates within the same clusters as samples for each clustering method. Then we used to goodness of fit test to see if the methods produce clusters with the same Pearson correlation distribution as the expected distribution. The results of these tests are shown in Table 5.7.

The Kolmogorov-Smirnov test computes a statistic, denoted $KSstat$. This statistic is then converted to a $pValue$, or the probability of a false negative, *i.e* rejecting the null hypothesis that two samples were drawn from the same distribution when in fact they were. The results show that none of the methods have more than a remote chance of matching the expected distribution. The probabilities are so small that relative

ITS region	23-5		16-23	
	KSstat	pValue	KSstat	pValue
DBSCAN 1	0.496	0.00000	0.484	4.7e-174
DBSCAN 3	0.435	0.00000	0.374	6.1e-104
OHClust!	0.109	2.0e-31	0.121	7.27e-11
Agglomerative	0.117	2.9e-36	0.141	1.12e-14

Table 5.7: **Results from 2-Sample Kolmogorov-Smirnov Goodness of Fit against distribution of Pearson Correlations between replicated isolates.**

comparisons between the scores of the clustering methods are likely not meaningful.

More analysis is required to determine if the Pearson correlations we used as the expected sample do in fact represent the distribution of Pearson correlations of isolates from the same strain. This would be good area for future work, in addition to finding other methods for evaluating cluster correctness.

5.5.3 Similarity to Current Method

This section describes the results from evaluating the similarity of the clusters produced by DBSCAN and OHClust! to average-link Agglomerative clustering. As discussed in Section 5.1.3, Agglomerative clustering is the method currently being used by the research group. If another method were similar enough to Agglomerative clustering, than the research group would consider it a suitable replacement. If the method is different than Agglomerative clustering, these tests can also show in what ways the methods are not similar.

Cluster Similarity Indexes

Table 5.8 shows various cluster similarity indexes. All of the indexes look at which pairs of isolates a clustering method classified as part the same cluster, or different

	Jaccard	Different	Average	Rand
DBSCAN 1	0.07442	0.985084	0.52975	0.985102
DBSCAN 3	0.09427	0.988740	0.54150	0.988753
OHClust!	0.46700	0.999113	0.73306	0.999114

Table 5.8: **Similarity Metrics of clusters compared to those of Agglomerative clustering.**

clusters. If the classification of a pair of isolates is the same for both methods being compared, then it is counted towards the similarity index score.

The Rand index gives the percentage of all isolate pairs which were classified the same. Every method scores well with the Rand index, but this score is biased because the majority of isolate pairs from every clustering method are classified as different clusters because of isolates which aren't placed any cluster (noise). As such, disagreements in which pairs are classified as the same cluster are not reflected in the Rand index.

An alternative metric, the Jaccard index, tries to solve this problem. This index looks at the percentage of isolate pairs that either method classified as the same cluster which both methods classified as the same cluster. None of the clustering methods scored well on this index. Both DBSCAN methods have less than 10% agreement with Agglomerative clustering on which isolate pairs are in the same cluster. Most of this is likely because the DBSCAN methods had larger clusters and thus had many more isolate pairs classified as the same cluster than Agglomerative clustering.

We also looked at another index which is similar to the Jaccard index except it looks at isolate pairs which either method classified as different clusters. These scores are very close to the Rand index scores which seems to confirm the idea that the Rand index was dominated by isolate pairs that both clustering methods classified as different.

ITS region	23-5		16-23	
	KSstat	pValue	KSstat	pValue
DBSCAN 1	0.551	0.0000	0.577	0.0000
DBSCAN 3	0.462	0.0000	0.492	0.0000
OHClust!	0.013	0.0563	0.021	9.2e-5

Table 5.9: **Results from 2-Sample Kolmogorov-Smirnov Goodness of Fit against Agglomerative clustering.**

As an alternate score to the Rand index which reflects disagreements of both kinds of classification, we looked at the average of the difference index and the Jaccard index. The relative usefulness of all these scores depend on how we weight the importance of agreement for each type of classification. For this evaluation, the research group wants the methods to be similar in both types of classifications, thus the average of the different and Jaccard indexes is the single most useful score. OHClust! is the most similar method to Agglomerative clustering. However, just as Montana discovered in his thesis[14], it is not similar enough to be considered a suitable replacement.

Pearson Correlation Distribution Similarity

This section discusses the results from another evaluation of the similarity between cluster methods and average-link Agglomerative clustering. This was another two-sample Kolmogorov-Smirnov goodness of fit test. Similar to the distribution test for correctness, we used all pairwise Pearson correlations between isolates in the same cluster as the samples in the goodness of fit test. Instead of comparing each method to an expected sample of pairwise Pearson correlations drawn from a set of replicate pyroprints from the same isolates though, we compared them to the samples from Agglomerative clustering. Table 5.9 shows the results of this test for each ITS region.

The Kolmogorov-Smirnov test computes a statistic, denoted **KSstat**. This statistic

is then converted to a **pValue**, or the probability of a false negative, *i.e* rejecting the null hypothesis that two samples were drawn from the same distribution when in fact they were. The results show that the DBSCAN methods produce clusters with a different distribution than Agglomerative clustering with practically no chance of a false negative for both ITS regions. OHClust! on the other hand, while it rejects the null hypothesis for the 16-23 region, it cannot reject the null hypothesis for the 23-5 region with a confidence of 95%. Unfortunately, the research team would need it to fail to reject for both ITS regions in order to consider the methods similar.

CHAPTER 6

CONCLUSION

The work described in this thesis provides the Cal Poly Library Of Pyroprints (CPOLP) research group with an alternative method of strain identification. Identifying strains speeds up Microbial Source Tracking comparisons and allows researchers to perform longitudinal and transference studies.

The contributions of this thesis are the following:

- **A modified DBSCAN algorithm:** We presented a clustering method based on DBSCAN which allows for the removal of points from the search space after they have been seen once. For the CPLOP dataset, This modification results in up to a 2x speedup over the original DBSCAN algorithm.
- **A faster method for comparing pyroprints:** This thesis presented a new method for comparing pyroprints, Euclidean distance of Z-Scores. This new method is algebraically related to Pearson correlation (the current method), allowing reuse of previous statistical analysis. We precomputed Z-Score values for every pyroprint which speeds up comparisons with both the current method and the new method. We showed that we can store pyroprint Z-Scores in a spatial index to speed up clustering.
- **A spatial index that works with *E. coli* pyroprints:** This thesis presented a spatial index tailored for the characteristics of the CPLOP dataset. It supports dense, high-dimensional data by partitioning the search space with multidimensional planes and storing bounding volumes for subgroups of data. The index also supports multiple regions for each data point with separate search radii.

- **Better evaluation metrics of clusters for CPLOP:** Finally, this thesis contributed methods for evaluating the correctness of identified strains. We ran these methods on clusters generated by this work as well as those generated by previous clustering methods OHClust! and Agglomerative clustering. We were able to characterize some of the differences between all of the clustering methods that have been tried to this date for identifying strains of *E. coli* pyroprints.

6.1 Future Work

There are many avenues of future work. First, defining strains in CPLOP is still an open problem. Next, researchers could try to improve the performance of our design/implementation. Another option would be to try clustering in a way that creates extra information for the biologists to analyse. Other work could try to provide alternatives to this work that are more correct. Finally, researchers could look into other applications for the algorithms developed for this thesis.

6.1.1 *E. coli* Strains

The research group wants to answer the question: which clustering method gives the best definition of a strain for pyroprinted *E. coli*? This thesis attempted to answer part of the question by evaluating if density-based clusters are a suitable replacement for the current clustering method. Answering this question will require more analysis of the results presented in this thesis, and may also require further testing.

The research group also wants to determine the usefulness of identified strains for MST. Future work could look at the purity of strains, the amount of agreement on host species between isolates in the same strains. If a strain has been seen in many different host species, then researchers won't have a way to know which of those host

species an unknown isolate matching that strain came from, and the strain would not be useful for MST.

6.1.2 Performance

The first area for future work is improving the performance of the algorithms presented in this thesis. This is the area that we have the most ideas for, and because of the genericity of our implementation, many of them would be easy to try.

One option would be to try persisting data and try incremental clustering. We are already persisting the clusters in CPLOP, so we are partway there, but the limiting factor to clustering is the neighbor lookups. Researchers could look into persisting the spatial index or even persisting a map of isolates to their neighbors. Our current tree construction algorithm runs fairly fast and it's the querying that takes most of the runtime. Researchers could also try caching all pairwise distance computations between pyroprints, however since this would not all fit in RAM on our current server, they would need to keep most of it in disk and load it dynamically which could offset the benefits of precomputing. Another option would be to try using another clustering algorithm like incDBSCAN which performs less neighbor lookups in the first place, given some previously discovered clusters.

Other ideas to try would be using different filtering methods for determining if the isolates in a subtree possibly match a neighbor query. This would include trying using only the split plane at higher levels in the tree and only the bounding box in lower levels of the tree. Or researchers could try having some nodes only filter on a single region, perhaps alternating between regions on successive levels. Finally, they could of course try different filtering methods such as bounding spheres. Our spatial index implementation is generic enough to support all of these possibilities. The neighbor query will work unmodified; all that the researchers need to do is implement any new

filters and modify the construction of the index to add whichever filters they want to each node in the tree.

Another idea to look at is using different partitioning methods to construct the spatial index. The tree implementation is generic in how many children are at each level and the information on how the parent was split is encoded in the children themselves, so it should support many different partitioning schemes.

An idea that the implementation does not generically support is rebalancing the tree after points are removed. Currently empty nodes are removed from the tree and bounding volumes updated to fit the data of only the remaining points, which would be fine if the queries were balanced. However the query pattern of DBSCAN will query points near each other one after another and when points are removed in this fashion, it will result in a lopsided tree. The solution would be to repartition the points every once in a while, perhaps more often in subtrees.

Finally, the implementation itself has lots of room for improvement from a performance aspect. The current implementation introduces lots of overhead in the tree traversal, some of which could be better implemented. An example would be using array indexing instead of hashmap lookups for getting the pyroprint from a particular region. However, traversal is currently a small part of the runtime which is dominated by pyroprint and filter comparisons. The library we used for these comparisons is highly optimized, but perhaps there are faster libraries out there for our purposes. In addition, rewriting the codebase in a faster language could also help.

6.1.3 Additional Analysis Opportunities

Another avenue of future work would be to provide more information for the biologists to analyze. Strains give the biologists a lot of opportunities for analysis because they can look for patterns between isolates in the same strain. We have some ideas for

finding other relationships between isolates, which the biologists might be interested in.

The first idea is to cluster isolates twice, once for each region. Isolates that were in the same cluster for both regions will also be is the same strain. This design would allow the biologists to look for patterns within isolates that are indistinguishable in one ITS region but not the other, implying some relationship but not as strongly as the full strain. Additionally, it finds relationships between pairs of strains where their isolates match in one of the ITS regions.

A hierarchical clustering algorithm would provide new information for strain analysis. It could be used to look for strains that are close to each other, or to look for subgroups within a strain. Additionally it could be used to see if different strains have different amounts of spread. Perhaps some clusters are formed at higher thresholds while others would form if the threshold was just a little bit more lax.

6.1.4 Correctness

Another avenue of future research is in trying to find a method the produces clusters that better match the Biologist's notion of strains for pyroprints.

First of all, a method of evaluating the correctness of clusters is needed. This paper presented two possible methods, but more analysis is needed to determine how well those methods are at determining strain correctness. Testing other correctness measures could also be useful.

One option is the same idea that inspired this thesis. There are other clustering methods which have not yet been tried with pyroprints and evaluated on their similarity to the Biologist's notion of strains. A method that would likely do very well on our pairwise Pearson correlation distribution evaluation is distribution-based clustering.

Another idea to consider is slightly changing what is considered correct by trying out different metrics for comparing isolates/pyroprints. Perhaps there are other ways to normalize the pyroprints that better account for the noise in the pyroprinting process. An example of a different but similar normalization would be to normalize the pyroprint by the mean and standard deviation of the first ~ 10 dispensations which sequence conserved DNA (so every isolate and every locus will have the same DNA), instead of the mean and standard deviation of every dispensation.

6.1.5 Applications

The last avenue of future research would be to determine how well the algorithms presented in this thesis apply to other applications.

The modified DBSCAN algorithm should easily work for normal DBSCAN applications. All it needs is a spatial index with the ability to quickly remove points. A limitation of the modified DBSCAN algorithm though is that if there are lots of noise in the dataset and most of the points would be part of the same cluster if $minPts = 1$, then the performance could degrade to $O(N^2)$ even with a $O(N \log N)$ neighbor lookup.

Other applications for the spatial index developed for this thesis are less obvious. The multi-region aspect of the index was designed for the unique problem of spatially indexing pyroprints from the same isolate. The design works with a single region though in which case the index is basically a hybrid Binary Space Partitioning tree and Bounding Volume Hierarchy. This could potentially be applicable to many of the same uses as other spatial indexes, especially those with high-dimensional data.

BIBLIOGRAPHY

- [1] Mihael Ankerst, Markus M Breunig, Hans-Peter Kriegel, and Jörg Sander. Optics: Ordering points to identify the clustering structure. *ACM Sigmod Record*, 28(2):49–60, 1999.
- [2] Norbert Beckmann, Hans-Peter Kriegel, Ralf Schneider, and Bernhard Seeger. The r*-tree: an efficient and robust access method for points and rectangles. *ACM SIGMOD Record*, 19(2):322–331, 1990.
- [3] Jon Louis Bentley. Multidimensional binary search trees used for associative searching. *Communications of the ACM*, 18(9):509–517, 1975.
- [4] Stefan Berchtold, Daniel A Keim, and Hans-Peter Kriegel. The x-tree: An index structure for high-dimensional data. *Readings in multimedia computing and networking*, 451, 2001.
- [5] Michael W Black, Jennifer VanderKelen, Aldrin Montana, Alexander Dekhtyar, Emily Neal, Anya Goodman, and Christopher L Kitts. Pyroprinting: A rapid and flexible genotypic fingerprinting method for typing bacterial strains. *Journal of microbiological methods*, 105:121–129, 2014.
- [6] Kaushik Chakrabarti and Sharad Mehrotra. The hybrid tree: An index structure for high dimensional feature spaces. In *Data Engineering, 1999. Proceedings., 15th International Conference on*, pages 440–447. IEEE, 1999.
- [7] J. Dillard, J. Kent, D. Britton, T. Branck, A. Frey, P. McCreesh, J. VanderKelen, C. Kitts, and M. Black. E. coli strain demographics and transmission in cattle, Jan 2013.

- [8] Martin Ester, Hans-Peter Kriegel, Jörg Sander, Michael Wimmer, and Xiaowei Xu. Incremental clustering for mining in a data warehousing environment. In *VLDB*, volume 98, pages 323–333. Citeseer, 1998.
- [9] Martin Ester, Hans-Peter Kriegel, Jörg Sander, and Xiaowei Xu. A density-based algorithm for discovering clusters in large spatial databases with noise. In *Kdd*, volume 96, pages 226–231, 1996.
- [10] Antonin Guttman. R-trees: a dynamic index structure for spatial searching. *ACM Sigmod Record*, 14(2):47–57, 1984.
- [11] Norio Katayama and Shin’ichi Satoh. The sr-tree: An index structure for high-dimensional nearest neighbor queries. *ACM SIGMOD Record*, 26(2):369–380, 1997.
- [12] King-Ip Lin, Hosagrahar V Jagadish, and Christos Faloutsos. The tv-tree: An index structure for high-dimensional data. *The VLDB Journal*, 3(4):517–542, 1994.
- [13] Jeffrey D. McGovern, Alexander Dekhtyar, Chris Kitts, Jennifer VanderKellen Michael Black, and Anya Goodman. Leveraging the k-nearest neighbors classification algorithm for microbial source tracking using a bacterial dna fingerprint library. In *Proceedings of the Second Workshop on Semantic Data Analytics and Bioinformatics (SDAB 2015)*, Washington, DC, Nov 2015.
- [14] Aldrin Montana. Algorithms for library-based microbial source tracking. Master’s thesis, California Polytechnic State University, San Luis Obispo, California, 2013.
- [15] Emily R Neal. *Escherichia coli* strain diversity in humans: effects of sampling effort and methodology. Master’s thesis, California Polytechnic State University, San Luis Obispo, 2013.

- [16] M Patella, P Ciaccia, and P Zezula. M-tree: An efficient access method for similarity search in metric spaces. In *Proc. of the International Conference on Very Large Databases (VLDB)*. Athens, Greece, pages 1241–1253, 1997.
- [17] Troy M Scott, Joan B Rose, Tracie M Jenkins, Samuel R Farrah, and Jerzy Lukasik. Microbial source tracking: current methodology and future directions. *Applied and Environmental Microbiology*, 68(12):5796–5803, 2002.
- [18] Diana Shealy. Exploration of pyroprinting for environmental forensics. Technical report, California Polytechnic State University, San Luis Obispo, California, June 2012.
- [19] Joyce M Simpson, Jorge W Santo Domingo, and Donald J Reasoner. Microbial source tracking: state of the science. *Environmental science & technology*, 36(24):5279–5288, 2002.
- [20] Jan Lorenz Soliman. Cplop: Cal poly library of pyroprints. Master’s thesis, California Polytechnic State University, San Luis Obispo, California, 2013.
- [21] Wei Wang, Jiong Yang, and Richard Muntz. Pk-tree: a spatial index structure for high dimensional point data. *Information organization and databases*, pages 281–293, 2000.