STABILITY ANALYSIS OF NEGATIVE RESISTANCE-BASED
SOURCE COMBINING POWER AMPLIFIERS

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ABSTRACT

Stability Analysis of Negative Resistance-Based Source Combining Power Amplifiers

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An investigation into the stability of negative resistance-based source combining power amplifiers is conducted in this thesis. Two different negative resistance-based source combining topologies, a series and parallel version, are considered. Stability is analyzed using a simple and intuitive broadband approach that leverages linear circuit stability criterion and two different linearization methods: linearization around the operating point and in the frequency domain. Using this approach, it is shown that conditions for self-sustained oscillation exist for both topologies. For the series combining topology, self-sustained oscillation is prevented by means of injection locking.

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1.1 Motivation

A new category of linear, high efficiency power amplifiers (PAs) for applications requiring significant peak to average power ratios (PAPRs) has been proposed by Prodanov [1], which will be referred to throughout this thesis as negative resistance-based source combining PAs. As their name suggests, the PAs are based on the property of negative resistance, which is commonly associated with the design and development of oscillators. The use of negative resistance places the proposed PAs within a unique device category that is situated between being an amplifier and being an oscillator; because of this, the topologies have been hypothesized as being possibly prone to instability in the form of self-sustained oscillation. Implementations of two different topologies of this category, parallel and series versions, by King [2] and Bendig [3], have demonstrated that negative resistance-based source combining PAs are a valid solution for enhancing the efficiency of PAs that experience signals with high PAPR, however, one topology exhibited peculiar and unexpected behavior during implementation.

The topology implemented by King was extensively simulated in the narrowband regime before being declared a viable and stable topology for PA applications. When implemented in hardware, however, the topology fell victim to “noise or other fluctuations” when the RF input drive exceeded a particular drive level. King later attributed this anomalous phenomena to “significant higher-order harmonic content,” although the cause of the harmonic content was never precisely determined by King. On the contrary, Bendig did not report observing similar peculiar phenomena as King, or any other

behavior indicative of instability, for that matter. Moreover, he concluded that his implementation was a “pivotal step in demonstrating that [the topology] is practical” for PA applications.

In this thesis, it is demonstrated both King and Bendig’s implementations of the negative resistance-based source combining PA topologies are prone to self-sustained oscillation under specific conditions. This indicates that while the topologies implemented by King and Bendig may be considered viable amplifiers in the narrowband regime, the topologies’ broadband behavior must be investigated before determining if they will become unstable during operation. For both of the topologies implemented, broadband behavior is investigated and predictions are made with regard to the conditions that cause instability by using a simple and intuitive broadband stability analysis approach based on the describing function quasi-linearization method, the general oscillation start-up condition, and the Barkhausen criterion. Conclusions regarding the viability of both topologies for PA applications are drawn: while one topology’s implementation shows potential, the other implementation is destined to oscillate.

1.2 Organization

Chapter 2 begins with a brief overview of the priorly implemented series and parallel negative resistance-based source combining PA topologies, their purpose, and their overall functionality. A summary of the fundamental principles of the describing function quasi-linearization method, the general oscillation start-up condition, and the Barkhausen criterion that compose of the stability analysis approach follows. The stability analysis of the series combining topology implemented by Bendig is presented in Chapter 3, followed by an abbreviated stability analysis of the parallel combining topology implemented by King. Conclusions regarding the stability of the negative resistance-

based source combining PAs are presented in Chapter 4, along with suggestions for future prospects related to the topic.

2.1 Negative Resistance-Based Source Combining PAs

Negative resistance-based source combining PAs address the linearity and efficiency trade-off that exists for PAs undergoing significant PAPRs [1]. Conventional single-transistor PAs are designed to operate at maximum efficiency at a single power level, usually near the maximum rated output power for the amplifier, however, if the PA is near the maximum power rating, the signal envelope becomes distorted. Moreover, when the amplifier is backed off from its maximum power rating, the efficiency of the PA decreases [4]. A widely used efficiency enhancement technique, the Doherty amplifier, remedies this problem by employing two separate amplifier stages: a “main” amplifier to amplify average power levels, and an “auxiliary” amplifier to amplify peak power levels. While they will not be discussed here, the Doherty amplifier has critical shortcomings that have motivated the research of new PA topologies [1], such as the negative resistance-based source combining PAs.

There are two versions of negative resistance-based source combining PA topologies that have been implemented to date, which are referred to as series and parallel source combining topologies. The negative resistance-based parallel source combining PA was the first to be proposed by Prodanov in 2006 [1]. The topology was inspired by the Doherty amplifier in that it uses a main and auxiliary amplifier stage to accommodate high PAPR signals and increase operating efficiency and linearity in comparison to conventional single-transistor PAs. The parallel source combining topology proposed in [1] was later implemented by King [2]. Following this implementation, Prodanov proposed a series source combining topology which was later implemented by

Bendig [3]. Both of these topologies include a negative resistance-based auxiliary amplifier stage in addition to a main amplifier stage. Sections 2.1.1 and 2.1.2 provide a simplified explanation of the series and parallel source combining topologies' general operation.

2.1.1 Series Source Combining Topology

The series combining PA topology implemented by Bendig [3] is abstracted to the Figure 1 circuit diagram, which illustrates the individual amplifier stages as power sources. The main amplifier is modeled as a current source and is in series with an auxiliary amplifier modeled as a negative resistance-based voltage source. Since the auxiliary amplifier, or rather, source, is negative resistance-based, voltage is generated from the source and applied to the circuit only when there is a current passing through it, as shown in (2.1). In other words, it only behaves as a voltage source when the main source is driving it.

Figure 1: Abstracted series source combining topology

$$\tilde{v}_{AUXILIARY} = (-R)(\tilde{i}_{MAIN}) \quad (2.1)$$

When the auxiliary source is applying voltage to the circuit, it reduces the power demand from the main source.

$$p_{MAIN} = \tilde{i}_{MAIN} \tilde{v}_{MAIN} = \tilde{i}_{MAIN} (\tilde{v}_{LOAD} - \tilde{v}_{AUXILIARY}) \quad (2.2)$$

Relating this back to amplifiers, the main amplifier is therefore the sole determiner of the power delivered to the load, as demonstrated in (2.3) through (2.5), as well as the gain and linearity of the overall amplifier.

$$\tilde{i}_{LOAD} = \tilde{i}_{MAIN} \quad (2.3)$$

$$\tilde{v}_{LOAD} = \tilde{i}_{MAIN} R_{LOAD} \quad (2.4)$$

$$p_{LOAD} = \tilde{i}_{LOAD} \tilde{v}_{LOAD} = \tilde{i}_{MAIN}^2 R_{LOAD} \quad (2.5)$$

The auxiliary amplifier's only operation is to assist the main amplifier in delivering power to the load.

$$p_{MAIN} = \tilde{i}_{MAIN}^2 (R_{LOAD} + (-R)) \quad (2.6)$$

It is important to reiterate that Figure 1 is an abstracted version of the topology implemented by Bendig [3]. The main current source and auxiliary negative resistance-based voltage source are implemented with transistors and suitably chosen RLC networks. The topology's implementation will be explained in further detail in Section 3.1.1.

2.1.2 Parallel Source Combining Topology

The parallel source combining PA topology implemented by King is abstracted to the Figure 2 circuit diagram, which illustrates the individual amplifier stages as power sources. The main amplifier is modeled as a voltage source and is in parallel with an auxiliary amplifier modeled as a negative resistance-based current source. Since the auxiliary amplifier, or rather, source, is negative resistance-based, current is generated from the source and applied to the circuit only when there is a voltage applied across it,

as shown in (2.7). In other words, it only behaves as a current source when the main source is driving it.

Figure 2: Abstracted parallel combining topology

$$\tilde{i}_{AUXILIARY} = \frac{\tilde{v}_{MAIN}}{(-R)} \quad (2.7)$$

When the auxiliary source is injecting current to the circuit, it reduces the power demand from the main source.

$$p_{MAIN} = \tilde{v}_{MAIN} \tilde{i}_{MAIN} = \tilde{v}_{MAIN} (\tilde{i}_{LOAD} - \tilde{i}_{AUXILIARY}) \quad (2.8)$$

Relating this back to amplifiers, the main amplifier is therefore the sole determiner of the power delivered to the load, as demonstrated in (2.9) through (2.11), as well as the gain and linearity of the overall amplifier.

$$\tilde{v}_{LOAD} = \tilde{v}_{MAIN} \quad (2.9)$$

$$\tilde{i}_{LOAD} = \tilde{v}_{MAIN} G_{LOAD} \quad (2.10)$$

$$p_{LOAD} = \tilde{v}_{LOAD} \tilde{i}_{LOAD} = \tilde{v}_{MAIN}^2 G_{LOAD} \quad (2.11)$$

The auxiliary amplifier's only operation is to assist the main amplifier in delivering power to the load.

$$p_{MAIN} = \tilde{v}_{MAIN}^2 (G_{LOAD} + (-G)) \quad (2.12)$$

It is important to reiterate that Figure 2 is an abstracted version of the topology implemented by King [2]. The main voltage source and auxiliary negative resistance-based current source are implemented with transistors and suitably chosen RLC networks. The topology's implementation will be explained in further detail in Section 3.2.1.

2.2 Stability Analysis Methodology

The stability analysis approach implemented to analyze the negative resistance-based source combining PAs incorporates both linear and nonlinear PA stability analysis techniques by utilizing describing function quasi-linearization method to “linearize” the otherwise nonlinear topologies, and the Barkhausen criterion and general oscillation start-up condition for linear systems to determine if the topologies will oscillate. By using this approach, the broadband behavior of the PA is accounted for and taken into consideration. Sections 2.1.1 and 2.1.2 summarize the fundamental principles of the Barkhausen criterion, general oscillation start-up condition, and describing function method that are relevant to the implemented stability analysis approach.

2.2.1 Barkhausen Criterion and General Oscillation Start-Up Condition

The Barkhausen criterion specifies the necessary gain and phase conditions for sustained oscillation for a linear system, while the general oscillation start-up condition specifies the necessary gain and phase conditions for the start-up of oscillation [5]. Both are widely used in the design of resonator-based LC oscillators, however their use is not restricted to these circuits. To demonstrate the conditions set by the Barkhausen criterion and the general oscillation start-up condition, consider the resonator-based parallel LC

oscillator shown in Figure 3, consisting of a parallel LC resonant network, a resistor to model the resonator's loss, and a transconductor operated in positive feedback, to emulate a negative resistance and compensate for the resonator's loss. The transfer function for this oscillator is represented by (2.13).

Figure 3: Resonator-based parallel LC oscillator behavioral model

$$H(s) = \frac{Y_{out}(s)}{X_{in}(s)} = \frac{\frac{s}{C_p}}{s^2 + \frac{1}{C_p}(G_1 + (-G_2))s + \frac{1}{L_p C_p}} \quad (2.13)$$

Alternatively, a generalized linear feedback model, Figure 4, with a transfer function represented by (2.14) can model the oscillator. For this model, the forward gain, $\alpha(s)$, corresponds to the transconductor's gain, g_m , and the feedback factor, $\beta(s)$, represents the transfer function formed by the parallel LC resonator and its corresponding loss.

Figure 4: Generalized linear feedback model

$$H(s) = \frac{Y_{out}(s)}{X_{in}(s)} = \frac{\alpha(s)}{1 + \alpha(s)\beta(s)} \quad (2.14)$$

If the open-loop gain, $\alpha(s)\beta(s)$, is equal to unity at a specific frequency, it follows from (2.14) that the circuit, without given an input signal, will have a non-zero output signal at that frequency; by definition, this is an oscillator [5]. A unity open-loop gain corresponds to the transconductor exactly compensating for the loss; in other words, the magnitude of the negative resistance is exactly equal to the resistance modeling the LC resonator loss. By (2.13), this implies that the poles of the circuit will lie on the imaginary axis of the s -plane, Figure 5a. For this case, if the circuit is already oscillating, the oscillation will be sustained indefinitely. Referring back to the generalized transfer function, (2.14), this case implies the satisfaction of the conditions outlined in (2.15); these are the Barkhausen criterion gain and phase conditions for sustained oscillation.

$$\begin{aligned} a) \quad & |\alpha(s)\beta(s)| = 1 \\ b) \quad & \angle \alpha(s)\beta(s) = 180^\circ(2m + 1), m \in 0, 1, 2 \dots \end{aligned} \quad (2.15)$$

Realize that in order for the circuit to sustain oscillation, it must already be oscillating. For the circuit to begin oscillating, the negative resistance must over-compensate for the loss of the LC resonator; in other words, the magnitude of the negative resistance must be greater than the resistance modeling the LC resonator loss.

By (2.13), this implies that the poles of the circuit will lie to the right of the s-plane's imaginary axis, Figure 5b. Referring back to the generalized transfer function, (2.14), this case implies satisfaction of the conditions outlined in (2.16); these are the general oscillation gain and phase conditions for oscillation start-up.

$$\begin{aligned} a) \quad & |\alpha(s)\beta(s)| > 1 \\ b) \quad & \angle\alpha(s)\beta(s) = 180^\circ(2m + 1), m \in 0, 1, 2 \dots \end{aligned} \quad (2.16)$$

Lastly, it should be noted that if the negative resistance under-compensates for the LC resonator's loss, any oscillation that may exist will be suppressed with time. By (2.13), this corresponds to the circuit's poles lying to the left of the s-plane's imaginary axis, Figure 5c.

Figure 5: S-plane pole locations and corresponding time-domain waveform for the resonator-based parallel LC oscillator

2.2.2 Describing Function

The describing function is a quasi-linearization technique for nonlinear systems that replaces a system nonlinearity with a linear gain [6]. Nonlinear systems are often

linearized by constraining the input to a particular range of magnitudes about an operating point, such that the input-output behavior is approximately linear. The describing function method eliminates this constraint [7] by performing a linearization in the frequency domain [8]. For a nonlinear system excited by a sinusoid with a particular frequency and amplitude, linearization in the frequency domain is performed by discarding all the system's output components except for the output component with frequency equal to the input, hence it is quasi-linear. The describing function for a particular nonlinear system is the collection of all the possible input-output amplitude and phase shift relationships for the fundamental frequency component [8], or rather, the complex fundamental-harmonic gain [6].

The describing function is a valid linearization technique for a nonlinear system if the output spectra is dominated by the fundamental frequency component for a given sinusoidal input. If the output of the system is dominated by harmonics of the input sinusoid, results obtained using the describing function technique will not be accurate. For RF circuits, this limitation is not critical as band-pass filters are often utilized to eliminate harmonics, thus, the output spectra is typically dominated by the fundamental frequency component.

For a specific example of deriving a describing function, consider a BJT-based differential pair, Figure 6, whose behavior is described by a nonlinear transfer characteristic, Figure 7. To determine the describing function, a cumbersome piece-wise mathematical derivation can be performed on Figure 7 [6], or, alternatively, the describing function can be determined through simulation combined with asymptotic approximation. For simplicity, the latter method is chosen.

Figure 6: BJT-based differential pair

Figure 7: Differential pair transfer characteristic

The differential pair's describing function has two asymptotes that describe the two primary regions of the transfer characteristic corresponding to when the sinusoidal input is very “small” and when the sinusoidal input is very “large” in magnitude. Furthermore, an input small in magnitude corresponds to the differential pair input-output

relationship being linear, while an input large in magnitude corresponds to the differential pair input-output relationship being nonlinear. It is assumed that the transistors have sufficient latency such that there is not a phase shift between the input and output, thus, the asymptotes and overall describing function are described strictly in terms of gain, or transconductance.

When the input drive is small in magnitude, the differential pair's describing function is bounded by its small-signal transconductance. The small-signal differential pair transconductance is determined by considering the current flowing through a collector terminal of either branch. By Figure 7, when the input is small, the current flowing through each of the collectors is approximately equal in magnitude to (2.17).

$$i_{c1} \approx \frac{I_{TAIL}}{2} \left(\frac{v_1 - v_2}{2V_T} \right) = I_C \frac{(v_{base}^{dp})}{2V_T} = \frac{1}{2} g_m^{bjt} v_{base}^{dp} \quad (2.17)$$

Therefore, the differential pair small-signal transconductance is (2.18). This indicates that if the sinusoidal input voltage applied across the differential pair input spans across the linear region of the transfer characteristic, the output current waveform will be linearly related to the input by the small-signal transconductance.

$$g_m^{dp} = \frac{\partial i_{c1}}{\partial v_{in}} = \frac{1}{2} g_m^{bjt} \quad (2.18)$$

When the input is large in magnitude, the differential pair describing function is bounded by its large-signal transconductance. Similar to the small-signal transconductance, the large-signal transconductance is determined by considering the current flowing through the collector terminals of each branch. When the input drive is large enough such that the differential pair's input-output relationship is nonlinear, the input voltage waveform commutates the tail current such that it is switched from branch to branch of the differential pair; this, in turn, corresponds to the collector currents becoming rectangular waveforms. The fundamental component of the current is (2.19).

$$I_{c_{fundamental}}^{dp} = \frac{4}{\pi} I_C = \frac{4}{\pi} \frac{I_{TAIL}}{2} = \frac{2}{\pi} I_{TAIL} \quad (2.19)$$

Thus, the large-signal transconductance, neglecting the higher order terms of the collector current, is equal to (2.20).

$$G_m^{dp} = \frac{\partial i_c}{\partial v_{in}} = \frac{I_{c_{fundamental}}^{dp}}{v_{base}^{dp}} = \frac{\frac{2}{\pi} I_{TAIL}}{v_{base}^{dp}} \quad (2.20)$$

Figure 8 illustrates the complete describing function for a BJT-based differential pair normalized by the small-signal transconductance. The small and large-signal transconductance, (2.18) and (2.20), asymptotically bound the describing function. As previously mentioned, more precise describing function data points are determined through simulation.

Figure 8: Normalized differential pair describing function

3.1 Series Source Combining Topology

Before the series source combining topology's stability is analyzed, the topology implemented by Bendig [3] is presented in Section 3.1.1 in the context of power sources, as introduced in Section 2.1.1. A feedback loop is identified in Section 3.1.2, indicating that oscillation may be possible if the gain and phase conditions outlined by the general oscillation start-up condition and the Barkhausen criterion are satisfied. In this section, it is revealed that the topology does oscillate. Sections 3.1.3 through 3.1.5 delve into the stability analysis approach briefly outlined in Section 2.2. Based off the results of the analysis, predictions are made with regard to the conditions that cause the start-up of oscillation and a decision is made regarding the viability of the topology for PA applications.

3.1.1 Topology Implementation

It was illustrated in Section 2.1.1 that the series source combining topology consists of a main current source and an auxiliary negative resistance-based voltage source, which model the main and auxiliary amplifier stages. Figure 9 is the series source combining topology implemented in this thesis, which is an adaptation of the series source combining topology implemented by Bendig [3]. A single-transistor class-AB PA implements the main current source, producing a current with a magnitude proportional to the RF input drive. The auxiliary negative resistance-based voltage source is implemented by a commutated differential pair and the LC components that its output branches. It is explained in [3] that this configuration, driven with a particular RLC phase-

shifting network, behaves as a negative resistance-based voltage source. Detailed information regarding the design and implementation of the topology is provided in [3], although it should be noted that this implementation is a narrowband system intended to operate with a 1 MHz RF input signal.

Figure 9: Series source combining topology prototype

The series source combining topology is implemented in both simulation and hardware. Figure 10 and Figure 11 provide a basic verification that the constructed prototype behaves as specified in Section 2.1.1. As the differential pair's tail current increases, the auxiliary source's output voltage increases. When the auxiliary source's output voltage increases, the magnitude of the voltage at the output of the main source

decreases, implying that the main source's output current also decreases, Figure 10. Increasing the voltage supplied by the auxiliary source does not have an effect on the voltage across the load, Figure 11. The decrease in the main driver output voltage corresponds to a reduction in power demand from the main source, while the conservation of voltage across the load corresponds to constant power delivered to the load; this is the intended operation of the topology as specified in Section 2.1.1.

Figure 10: Output voltage of main source vs. differential pair tail current

Figure 11: Load voltage vs. differential pair tail current

Figure 12 is the frequency response of the constructed RLC network between the main voltage source and the load. The frequency response has a band-pass response with a center frequency of 1 MHz and a bandwidth of approximately 500 kHz, implying that the resonant network is tuned to operate with a 1 MHz input signal and will sufficiently filter spectra outside of this band from the load.

Figure 12: RLC network's narrowband band-pass frequency response

3.1.2 Recognizing the Potential for Oscillation

A feedback loop involving the auxiliary source and the RLC network exists within the series source combining topology: the differential pair employed by the auxiliary source is commutated by the RLC network that it injects current into. The existence of a feedback loop within the topology introduces the possibility for oscillatory behavior if the gain and phase conditions described by the Barkhausen criterion and general oscillation start-up condition are satisfied. Oscillation was not reported when the prototype was first implemented in [3], however a limited number of test cases were conducted by Bendig. For instance, Bendig failed to test a complete sweep of the RF input drive's magnitude. If

Bendig had done so, he may have noticed that when the main source is disabled, as illustrated in Figure 13, the topology oscillates.

Figure 13: Transient response simulation of series source combining topology

The presence of oscillation brings the validity of the results presented in [3] and the viability of the overall topology into question:

- Did Bendig fail to report that oscillation was observed, or did he not observe it? If he didn't observe oscillation, why was it not present for his measurements?

- Now that oscillation has been observed, is the topology even useful as an amplifier? For instance, are there specific conditions that cause the topology to behave as an oscillator, implying that oscillation is predictable and introduced during certain conditions? Are the oscillation conditions mutually exclusive with “normal” operation?
- Once oscillation has begun, is there a way of suppressing it?

To attempt to answer these questions, the feedback network is translated to an equivalent linearized block diagram so that it can be analyzed for oscillatory behavior by means of the general oscillation start-up condition and the Barkhausen criterion.

3.1.3 Development of the Describing Function-Based Linearized Model

The feedback loop within the series source combining topology is translated into an equivalent linearized block diagram by first separating the topology into linear, nonlinear, active, and passive subsections that can be replaced by their corresponding linearized models as necessary. The prototype implemented is separated into three subsections that represent: the main source, the differential pair, and a RLC network,

Figure 14; Figure 15 illustrates the complete block diagram of the entire topology. Since the Barkhausen criterion and general oscillation start-up condition are the means for analyzing the topology’s potential for oscillation, the only subsections that require a linearized model are the subsections that compose the feedback loop: the differential pair and the RLC network.

Figure 14: Series source combining topology translation into block diagram

Figure 15: Series source combining topology block diagram

As discussed in Section 2.2.2, a BJT-based differential pair has a nonlinear input-output relationship while being commutated. Since the differential pair is primarily operated in a nonlinear region of operation of its transfer characteristic curve within the series source combining topology, the differential pair must be linearly modeled via a linearization method that incorporates both linear and nonlinear behavior, such as the describing function. Figure 8 illustrates the describing function for the BJT-based differential pair implemented within the series source combining topology.

Unlike the differential pair, the RLC network is a linear network. Since it is a linear network, its behavior is accurately described using a linear model, such as a Laplace domain transfer function. More simply, the behavior is represented graphically by its frequency response, which may be found via simulation. With respect to the feedback loop, the network's input port is across the differential pair's collector terminals, and the network's output port is the across the differential pair's base terminals. It was implied in Section 0, and it will later be shown in Section 3.1.4, that the broadband behavior of the RLC network is critical for predicting the topologies' potential to oscillate. Broadband behavior can be drastically different for equivalent, matched circuits (e.g., Figure 16 through Figure 18). Figure 16 through Figure 18 depict the frequency response of three output configurations, found via simulation, that present an equivalent impedance to the topology output at the fundamental frequency: a $200\ \Omega$ load, a $200\ \Omega$ to $50\ \Omega$ low-pass L-match network with $50\ \Omega$ load, and a $200\ \Omega$ to $50\ \Omega$ high-pass L-match network with $50\ \Omega$ load.

Figure 16: $|Z_{RLC}(f)|$ vs. frequency for 200 Ω load at the output

Figure 17: $|Z_{RLC}(f)|$ vs. frequency for low-pass 200 Ω to 50 Ω load L-match network at the output, terminated with 50 Ω

Figure 18: $|Z_{RLC}(f)|$ vs. frequency for high-pass 200 Ω to 50 Ω load L-match network at the output, terminated with 50 Ω

3.1.4 Application of Oscillation Conditions to the Linearized Model

The presence of self-sustained oscillation in the Figure 13 transient response simulation suggests that the series source combining topologies' feedback loop satisfies the gain and phase conditions specified by the Barkhausen criterion and general oscillation start-up condition. Since these conditions are satisfied, predictions can be made regarding the state of the topology when oscillation will occur. For instance, the build-up of oscillation will only occur if the general oscillation start-up condition, (3.1), is satisfied at a particular frequency.

$$\begin{aligned} a) \quad & G_m |Z_{RLC}(f)| > 1 \\ b) \quad & \angle (G_m Z_{RLC}(f)) = 180^\circ(2m + 1), m \in 0, 1, 2 \dots \end{aligned} \quad (3.1)$$

For the build-up of oscillation to occur, the differential pair's describing function-based transconductance, G_m , must exceed the inverse of the magnitude of the RLC network's impedance, $|Z_{RLC}(f)|$, required by the gain conditions of the general oscillation start-up condition, (3.2). In addition, the phase condition, (3.1), must also be satisfied. The phase condition is satisfied if the phase response of the RLC network, Figure 16 through Figure 18, passes through 180° , as the differential pair has a 180° input-output phase shift based on its driving RLC network [3]. By inspection of the phase response of the RLC network, it is revealed that the phase condition is satisfied at multiple frequencies for all the output configurations constructed. Moreover, the topology will oscillate at the frequency, $f_{oscillation}$, that satisfies the inequality shown in (3.2). It should be noted here that the RLC network's impedance, $|Z_{RLC}(f)|$, with the largest magnitude and appropriate phase shift is most likely to satisfy the gain criteria for the loop, however, in theory multiple modes of oscillation could exist [9].

$$G_m > \frac{1}{|Z_{RLC}(f_{oscillation})|} \quad (3.2)$$

By inspection of the differential pair's describing function, the transconductance is largest, and thus most likely to satisfy (3.2), when the differential pair is driven by a small-signal (i.e., a signal that results in a linear differential pair input-output relationship) or not driven at all. By Figure 8, in this describing function region, the describing function is asymptotically bounded by the small-signal transconductance of the differential pair, which is a function of the bias collector current.

$$g_m^{dp} = \frac{1}{2} g_m^{bjt} = \frac{1}{2} \left(\frac{I_C^{dp}}{V_T} \right) \quad (3.3)$$

Further, the bias collector current is linearly related to the differential pair tail current.

$$I_C^{dp} = \frac{1}{2} I_{TAIL} \quad (3.4)$$

The dependence of the small-signal transconductance on the differential pair's tail current implies that there is a minimum differential pair tail current to satisfy the start-up conditions for oscillation. Combining (3.1) through (3.4), the topology will oscillate if there is sufficient describing function transconductance, G_m , set by the tail current, and RLC network impedance, $|Z_{RLC}(f_{oscillation})|$, (3.5).

$$I_{TAIL} > \frac{4V_T}{|Z_{RLC}(f_{oscillation})|} \quad (3.5)$$

Table 1 summarizes the theoretical oscillation frequency, $f_{oscillation}$, and the corresponding RLC network impedance, $|Z_{RLC}(f_{oscillation})|$ for each output configuration in addition to the minimum describing function transconductance, G_m , and corresponding minimum differential pair tail current, I_{TAIL} , for satisfaction of oscillator start-up conditions given a particular RLC network impedance, $|Z_{RLC}(f_{oscillation})|$.

Table 1: Summary of theoretical oscillation frequency, RLC network impedance at the oscillation frequency, corresponding minimum differential pair describing function-based transconductance, G_m , and tail current for different output configurations

	$f_{\text{oscillation}}$, MHz	$ Z_{\text{RLC}}(f_{\text{oscillation}}) $, Ω	Min. G_m , mA/V	Min. I_{TAIL} , mA
200 Ω load	1.20	91	10.99	1.14
50 Ω , 200 to 50 Ω low-pass match	5.43	901	1.11	0.12
50 Ω , 200 to 50 Ω high-pass match	0.57	205	4.88	0.51

Since oscillation is sustained, this implies that the differential pair describing function-based transconductance, G_m , must decrease from its value during start-up conditions to satisfy the Barkhausen criterion's gain conditions for sustained oscillation. By inspection of the differential pair's describing function, Figure 8, the describing function-based transconductance, G_m , decreases by increasing the base drive to the differential pair, $v_{\text{base}}^{\text{dp}}$. When base drive is increased beyond the linear input-output relationship region, the describing function is asymptotically bounded by the large-signal transconductance of the differential pair, $G_{m\text{fundamental}}$:

$$G_{m\text{fundamental}} = \frac{\left(\frac{4}{\pi}\right)\left(\frac{1}{2}I_{\text{TAIL}}\right)}{v_{\text{base}}^{\text{dp}}} = \frac{\frac{2}{\pi}I_{\text{TAIL}}}{v_{\text{base}}^{\text{dp}}} \quad (3.6)$$

The differential pair base drive increases by two different means that both introduce a current into the RLC network that commutates the differential pair: the start-up of oscillation and increasing the current supplied from the main source. The first means allows for the oscillation to be self-sustained, while the second means provides an external method to alter the differential pair's describing function-based transconductance, G_m . The external method suggests that if the topology satisfies the general oscillation start-up condition and Barkhausen criterion, oscillation may be

suppressed, or quenched, by externally increasing the fundamental current drive so that the differential pair's describing function-based transconductance, G_m , is decreased to a level that violates the general oscillation start-up condition and the Barkhausen criterion.

3.1.5 Prediction and Suppression of Oscillation

To test the proposed claim that the observed self-sustained oscillation may be suppressed by increasing the main source's current drive, generalized design equations for the minimum required current drive from the main source and corresponding minimum required differential pair base drive to suppress oscillation are derived. The equations are derived from the inverse of the generalized oscillation start-up condition and Barkhausen criterion gain condition. That is, if there are oscillation present, there is a minimum value of differential pair describing function-based transconductance, G_m , before the gain around the feedback loop violates the gain conditions; this is denoted as the critical differential pair describing function transconductance, $G_{m_{critical}}$.

$$G_{m_{critical}} \leq \frac{1}{|Z_{RLC}(f_{oscillation})|} \quad (3.7)$$

Before oscillation are suppressed, the differential pair's base drive is a function of the RLC network impedance at the oscillation frequency, $|Z_{RLC}(f_{oscillation})|$, rather than a function of the RLC network impedance at the fundamental frequency, $|Z_{RLC}(f_{fundamental})|$; fundamental current sourced by the main driver is linearly proportional to the differential pair base drive by this impedance. Combining these relations formulates the theoretical minimum differential pair base drive design equation for oscillation suppression.

$$v_{base}^{dp} \geq \left(\frac{2}{\pi}\right) I_{TAIL} |Z_{RLC}(f_{oscillation})| \quad (3.8)$$

The design equation for the minimum fundamental current drive from the main source, (3.9), is derived by combining (3.8) and the relationship between the differential

pair base drive and the RLC network impedance when oscillation is suppressed, $|Z_{RLC}(f_{fundamental})|$.

$$i_{collector_{fundamental}}^{main} \geq \left(\frac{2}{\pi}\right) I_{TAIL} \frac{|Z_{RLC}(f_{fundamental})|}{|Z_{RLC}(f_{oscillation})|} \quad (3.9)$$

Calculated minimum current drive from the main source and differential pair base drive required to suppress oscillation are compared to simulation and measurement-based results. Simulation and measurement-based results are obtained by forcing the topology into oscillation and then increasing the RF input drive to the main source until the oscillation frequency spectra is suppressed. As mentioned Section 3.1.4, oscillation is forced by increasing the differential pair tail current while the main source is disabled until oscillation occurs; the minimum value of tail current required for oscillation is specified in Table 1.

Figure 19 through Figure 21 illustrate the frequency spectra as the main source's current drive is increased for each of the output configuration. Note that when the main source is deactivated, the topology oscillates at the frequency at which the RLC network impedance is largest in magnitude, $|Z_{RLC}(f_{oscillation})|$, as predicted in Section 3.1.4 and noted in Table 1. When the current drive from the main source is increased, the fundamental and oscillation frequency spectra are visible and the oscillation frequency spectra decreases in magnitude; past a particular drive-level, however, the oscillation frequency spectra abruptly crashes below the noise floor. The abrupt suppression of the oscillation frequency spectra is consistent with injection locking, which has been observed and studied in negative-resistance-based LC oscillators [10]. For all cases, the oscillation frequency spectra falling below the noise floor of the oscilloscope corresponds to greater than 40 dB of difference between the fundamental and oscillation frequency spectra; for all practical purposes, this corresponds to oscillation being suppressed.

Figure 19: Suppression of oscillation for 200 Ω load by increasing fundamental current drive from main source from 0 μA_{pp} to 65 μA_{pp} , 125 μA_{pp} , and 131 μA_{pp} (a) through d))

Figure 20: Suppression of oscillation for low-pass $200\ \Omega$ to $50\ \Omega$ match, $50\ \Omega$ load output configuration by increasing fundamental current drive from main source from $0\ \mu A_{pp}$ to $65\ \mu A_{pp}$, $88\ \mu A_{pp}$, and $137\ \mu A_{pp}$ (a) through d))

Figure 21: Suppression of oscillation for 50 Ω load, high-pass 200 Ω to 50 Ω match, 50 Ω load output configuration increasing fundamental current drive from main source 0 μ A_{pp} to 37 μ A_{pp}, 78 μ A_{pp}, and 84 μ A_{pp} (a) through d))

Figure 22 through Figure 24 provide a comparison between calculation, simulation, and measurement-based results for the 200 Ω load output configuration for a range¹ of differential pair tail current. Figure 25 through Figure 27 and Figure 28 through Figure 30 provide the same comparison for output configurations involving a 200 Ω to 50 Ω low-pass and high-pass impedance L-match network, respectively, and a 50 Ω load. For each output configuration tested, the minimum fundamental current from the main source and corresponding differential pair base drive required to suppress oscillation in simulation and measurement are less than the values predicted using calculations for all output configurations. These results imply that (3.8) and (3.9) are valid conditions for design; if these conditions are met, oscillation is guaranteed not to occur.

It is speculated that the prevention of oscillation through providing sufficient drive from the main current source, as well as the injection locking behavior that the topology exhibited while oscillation was suppressed, may have been the reasons why Bendig did not report observing oscillation in [3]. While Bendig reported performing a limited amount of test cases, the test cases that he did perform were in line with “normal” operating conditions for the PA topology; the auxiliary source, and therefore differential pair, *should* always be driven, to some extent, by the main source in application. If the RF input signal is large enough in magnitude for the main source to produce a current that exceeds the design equations derived, the series source combining topology implemented will behave as a PA, rather than as an oscillator.

To further reduce the required current from the main source to suppress oscillation, the broadband responses for equivalent RLC networks can be compared and chosen appropriately: if a RLC network’s broadband response contain peaks higher in magnitude compared to other equivalent networks, the topology will require a larger differential pair describing function transconductance to suppress oscillation, which correlates to a lower current drive from the main source. The output configuration with

¹ The upper limit is the maximum tail current allowed without forward biasing PN junction of the differential pair’s tail current-setting BJT

low-pass impedance L-match network required the least fundamental main source current drive as its peak impedance is approximately nine times greater than the 200 Ω load output configuration and four times higher than the 200 Ω to 50 Ω high-pass impedance L-match network output configuration.

Figure 22: Fundamental current supplied by main source vs. differential pair base drive to suppress oscillation for the 200 Ω load output configuration

Figure 23: Required fundamental current supplied by main source to suppress oscillation vs. differential pair tail current for the 200 Ω output configuration

Figure 24: Required differential pair base drive to suppress oscillation vs. differential pair tail current for the 200 Ω load output configuration

Figure 25: Fundamental current supplied by main source vs. differential pair base drive to suppress oscillation for the 200 Ω to 50 Ω low-pass L-match, 50 Ω load output configuration

Figure 26: Required fundamental current supplied by main source to suppress oscillation vs. differential pair tail current for the 200 Ω to 50 Ω low-pass L-match, 50 Ω load output configuration

Figure 27: Required differential pair base drive to suppress oscillation vs. differential pair tail current for the 200 Ω to 50 Ω low-pass L-match, 50 Ω load output configuration

Figure 28: Fundamental current supplied by main source vs. differential pair base drive to suppress oscillation for the 200 Ω to 50 Ω high-pass L-match, 50 Ω load output configuration

Figure 29: Required fundamental current supplied by main source to suppress oscillation vs. differential pair tail current for the 200 Ω to 50 Ω high-pass L-match, 50 Ω load output configuration

Figure 30: Required differential pair base drive to suppress oscillation vs. differential pair tail current for the 200 Ω to 50 Ω high-pass L-match, 50 Ω load output configuration

3.2 Parallel Source Combining Topology

In Chapter 1, it was suggested that while both of the implemented topologies behave as oscillators under specific conditions, one of the implementations demonstrated potential at being a viable power amplifier topology, while the other implementation is “destined to oscillate.” Following the suppression of oscillation for the series source combining topology, it may have been deduced that the implementation destined to oscillate is the parallel source combining topology. For this topology, an identical approach for analyzing the stability as was conducted for the series source combining topology is performed, but in an abbreviated form. Before the parallel source combining topology’s stability is analyzed, the topology implemented by King [2] is presented in the context of power sources, as introduced in Section 2.1.2. Similar to the series source combining topology, a feedback loop is identified in Section 0, indicating that oscillation may be possible if the gain and phase conditions outlined by the general oscillation start-up condition and the Barkhausen criterion are satisfied. In this section, it is revealed that the topology does self-sustain oscillation. Sections 3.2.3 and 3.2.4 delve into the abbreviated stability analysis approach, which is strictly simulation-based, which reveals the reasons why the implementation is destined to oscillate.

3.2.1 Topology Implementation

It was illustrated in Section 2.1.2 that the parallel source combining topology consists of a main voltage source and an auxiliary negative resistance-based current source, which model the main and auxiliary amplifier stages. Figure 31 is the parallel source combining topology implemented by King [2] and simulated for this thesis. A multi-finger class-AB amplifier, which drives the input of an impedance inverter, implements the main voltage source, producing a voltage with a magnitude proportional to the RF input drive. The auxiliary source is implemented by a multi-finger class-C amplifier that is

driven by an RLC network that is connected to its output. It is explained in [2] that this configuration creates a negative resistance-based current source. Detailed information regarding the design and implementation of the topology is provided in [2], although it should be noted that King's topology is a narrowband system intended to operate with a 1 MHz RF input signal.

Figure 31: Implementation of the negative-conductance parallel source combining power amplifier topology

3.2.2 Recognizing the Potential for Oscillation

A feedback loop involving the auxiliary source and the RLC network is present within the parallel source combining prototype: the class-C amplifier is driven by the RLC network that it injects current into. Using a similar argument as was posed for the series source combining topology's implementation, the existence of a feedback loop introduces the possibility for oscillatory behavior if the gain and phase conditions described by the

general oscillation start-up condition and the Barkhausen criterion are satisfied. As previously mentioned in Chapter 1, the observation of oscillation was not explicitly stated by King [2], however King did report in his study that “Measurements [for his implementation] were difficult to obtain...due to noise or other fluctuations” when the RF input drive exceeded 200 mV_p [2]. In his conclusion, he added that the observed noise was “likely” due to “harmonic content...being fed back to [the input] of the class-C amplifier once the class-C was activated, [resulting] in... higher-order harmonics being amplified” [2]; despite this claim, the cause was never precisely determined.

King performed extensive narrowband regime simulations that consistently indicated that the topology would behave as an amplifier, rather than as an oscillator. Because of this, King never suspected that the phenomena exhibited by the topology’s hardware implementation was indicative of out-of-band instability in the form of self-sustained oscillation. Figure 32 illustrates that after applying one cycle of a 1 MHz, 300 mV_p sinusoid to the input of the topology, the topology sustains an oscillation.

Figure 32: Transient response simulation of parallel source combining topology with 300 mV_p external RF input

3.2.3 Development of the Describing Function-Based Linearized Model

To translate the parallel source combining topology into an equivalent linearized block diagram, the topology is separated into three subsections, representing: the main source, the auxiliary source, and a passive RLC network, Figure 33Figure 34. Since the Barkhausen criterion is being employed to analyze the topologies' potential for oscillation,

the only subsections that require a linearized model are the subsections that compose the loop: the multi-finger class-C amplifier and the RLC network.

Figure 33: Parallel source combining topology translation into block diagram

Figure 34: Parallel source combining topology block diagram

Similar to the differential pair implemented within Bendig's auxiliary source, the class-C amplifier is described by a nonlinear transfer characteristic, *Figure 35*. Since the class-C amplifier's input-output relationship is altered for varying input drive levels, a

describing function for the multi-finger class-C power amplifier is required for the linearized model of the topology, Figure 36. Likewise to the differential pair's describing function, the class-C's describing function is derived through simulation combined with asymptotic approximation. Note that there are two asymptotes that bound the describing function: the cut-off biased g_m asymptote and the class-AB G_m asymptote. If the class-C's base drive is "small," the describing function approaches the cut-off biased g_m asymptote. Since class-C is biased in cut-off, this corresponds to the input signal to the class-C being small enough in magnitude such that it does not exceed the required magnitude to transition the BJT from the cut-off mode of operation. Alternatively, if the input signal is very "large," the describing function approaches the class-AB G_m asymptote. The portion of the class-C's input waveform that traverses the cut-off region of the transfer characteristic approaches zero as the input's magnitude increases; this corresponds to if the same set of multi-finger transistors were biased in class-AB mode of operation, rather than class-C.

Figure 35: Class-C transfer characteristic

Figure 36: The normalized multi-finger class-C describing function

The frequency response of the linear RLC network implemented by King is illustrated in Figure 37. With respect to the feedback loop, the network's input port is across the output of the main source, and the network's output port is across the base of the auxiliary source.

Figure 37: $|Z_{RLC}(f)|$ vs. frequency for King's output configuration

3.2.4 Application of Oscillation Conditions to the Linearized Model

The parallel source combining topology's critical flaw, which causes it to behave as an oscillator rather than an PA, is revealed by attempting to replicate the same steps that were taken for the series source combining topology's stability analysis in Sections 3.1.4 and 3.1.5. In the series source combining topology, it was argued that the presence of self-sustained oscillation in the topology's transient response simulation indicated that the feedback loop satisfied the gain and phase conditions specified by the Barkhausen criterion and general oscillation start-up condition at a particular frequency. Since the conditions are satisfied, predictions can be made regarding the state of the topology when oscillation will occur. For the series source combining topology, a build up of oscillation occurred when the differential pair's describing function-based transconductance, G_m , exceeded the inverse of the magnitude of the RLC network's

impedance, $|Z_{RLC}(f)|$, required by the gain conditions of the general oscillation start-up condition, (3.2), assuming that the phase conditions were also satisfied. Phase conditions were satisfied at multiple frequencies, however, the topology oscillated at the frequency in which the RLC network's impedance, $|Z_{RLC}(f)|$, with the largest magnitude and appropriate phase shift when the differential pair's describing function-based transconductance, G_m , exceeded the minimum required value for oscillation.

The same argument can be replicated for the parallel source combining topology, except in terms of the class-C amplifier's describing function-based transconductance, G_m , rather than the differential pair's. Similar to the series source combining topology, the general oscillation condition's phase condition is satisfied at multiple frequencies. Since the class-C amplifier has an inherent 0° phase shift from input to output, assuming sufficient latency, the phase condition is satisfied if the RLC network's phase response passes through 0° . Thus, the topology will oscillate as long as the class-C's describing function-based transconductance, G_m , exceeds the inverse of the magnitude of the RLC network's impedance, $|Z_{RLC}(f)|$, (3.2), required by the gain condition of the general oscillation start-up condition. For the parallel source combining topology, the circuit self-sustained oscillation at 1.46 MHz when driven by a single cycle of a 1MHz, 300 mV_p RF input signal, Figure 38, which is equal in frequency to the largest magnitude RLC network impedance, $|Z_{RLC}(f)|$, with the appropriate phase shift to satisfy the phase condition, Figure 37; this is consistent with oscillation frequency prediction and results for the series source combining topology.

Figure 38: FFT of load voltage during self-sustained oscillation

For the series source combining topology, the differential pair's describing function-based transconductance, G_m , exceeded the inverse of the RLC network's impedance, $|Z_{RLC}(f)|$, when the differential pair was driven by a small signal, so that the transconductance was at or near the maximum value. It was then demonstrated that the describing function-based transconductance, G_m , can be decreased with external drive from the main source, and therefore the RF input drive, such that the phase and gain conditions for the general start-up condition and the Barkhausen criterion are violated; this is what made the suppression of self-sustained oscillation possible.

For the parallel source combining topology, the class-C's describing function-based transconductance, G_m , exceeds the inverse of the RLC network's impedance, $|Z_{RLC}(f)|$, when the when the class-C is driven by a large signal, so that the transconductance is at or near the maximum value. External drive from the main source, and therefore the RF input drive, only further increases the describing function-based transconductance, G_m , of the class-C amplifier; this is the critical flow of the parallel source combining topology. The auxiliary amplifier's purpose is to aid the main amplifier when the main amplifier is undergoing a large RF input signal drive; for this

implementation, however, the auxiliary amplifier forces the topology into oscillation when the main amplifier is undergoing a larger RF input signal drive. Thus, the parallel source combining topology, as currently implemented, is destined to oscillate.

It is expected that if a narrowband-equivalent RLC network is used that forced the general oscillation start-up condition and the Barkhausen criterion to fail, a topology such as King's could become stabilized. Some alternative narrowband-equivalent RLC networks involving harmonic traps and impedance matching networks including: L-match, π -match, and T-match networks were attempted to stabilize King's prototype, however they were not successful in stabilizing the topology.

Under specific conditions, King and Bendig's implementations of negative resistance-based source combining PAs are prone to self-sustained oscillation. Using a stability approach based on the describing function quasi-linearization method, the general oscillation start-up condition, and the Barkhausen criterion, the conditions that lead to self-sustained oscillation can be predicted, and in some cases, prevented. The stability analysis approach revealed that there is a minimum describing function-based transconductance, G_m , corresponding to the negative resistance-based auxiliary amplifier, given a specific RLC network with an impedance that varies with frequency, $|Z_{RLC}(f)|$, that will cause the topologies to oscillate. Oscillation will only occur if the phase condition outlined by the general oscillation start-up condition and the Barkhausen criterion are also satisfied; for each topology and variation of topology tested, the phase condition is satisfied at multiple frequencies. When the minimum describing function-based transconductance, G_m , is attained, each of the topologies self-sustain oscillation with a frequency equal to the frequency in which the RLC network's impedance, $|Z_{RLC}(f)|$, was largest in magnitude, while still satisfying the phase condition.

The describing function-based transconductance, G_m , is a function of the main amplifier's drive and therefore the external RF input drive. The describing function-based transconductance, G_m , increases with increasing drive from the main amplifier for King's prototype, while for Bendig's prototype, it decreases. The impact of the main amplifier's drive on the describing function-based transconductance, G_m , is a factor in determining the conditions in which the topology will oscillate. The auxiliary amplifier implemented in King's prototype causes the prototype to sustain oscillation when the drive from the main

amplifier exceeds a particular level, while the auxiliary amplifier implemented in Bendig's prototype causes the prototype to sustain an oscillation when the main amplifier's drive is under a particular level; the minimum describing function-based transconductance, G_m , required for oscillation is attained during these conditions.

The oscillation conditions determine the viability of the topology. While King's topology will inevitably behave as an oscillator if the main amplifier's drive exceeds a particular level, Bendig's topology will behave as a PA. Since Bendig's describing function-based transconductance, G_m , decreases with increasing drive from the main amplifier, oscillation can be prevented by providing sufficient external RF input drive during operation. Prevention of oscillation is made possible by the impact of the main amplifier on the auxiliary amplifier's corresponding describing function-based transconductance, G_m ; thus, in order to design viable negative resistance-based source combining PAs, it is necessary to employ an auxiliary amplifier stage that has a describing function that decreases with increased drive. Inequalities for the required drive levels necessary to prevent oscillation from occurring in Bendig's prototype were derived, tested in simulation and hardware, and shown to guarantee that instability in the form of self-sustained oscillation will not occur if the derived inequalities are satisfied.

Future prospects related to the stability of negative resistance-based source combining PAs include investigating two topics: narrowband-equivalent RLC networks and their effect on topology stability, and single-transistor auxiliary amplifier topologies with describing functions that decrease with input drive. It is expected that if a narrowband-equivalent RLC network is used that forced the general oscillation start-up condition and the Barkhausen criterion to fail, a topology such as King's could become stabilized. Single-transistor auxiliary amplifier topologies with describing functions that

decrease with input drive, like the differential pair implemented in Bendig's prototype, could be advantageous for PA designs that are constrained by size.

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