THE FEASIBILITY AND APPLICATION OF OBSERVING SMALL LEO
SATELLITES WITH AMATEUR TELESCOPES

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ABSTRACT

The Feasibility and Application of Observing Small LEO Satellites with Amateur Telescopes

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This thesis demonstrates that any individual can provide relevant observational data to further research efforts within the Aerospace community, through the use of amateur telescopes. A Meade LX200 12 in. telescope and Lumenera Skynyx 2.0 camera were utilized to observe small LEO satellites, using a well-documented point-and-wait staring method. Over a period of three months, a total of 186 observation attempts were made resulting in 97 successful captures. From the gathered data, three possible aerospace applications were analyzed: validation of a satellite brightness prediction model, angles-only orbit determination including extended Kalman filtering, and temporal error growth in TLE-based orbit propagation. Further investigations include a preliminary optimization using MATLAB's `fmincon` function (informed by the previous analyses) to determine an optimal telescope size for performing LEO observations.

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LIST OF ACRONYMS

ACF	Advanced Coma Free
AER	Azimuth-Elevation-Range
ATE	Along-Track Error
CCD	Charge-Couple Device
COE	Common Orbital Elements
CTE	Cross-Track Error
DDW	Digital Dome Works
EKF	Extended Kalman Filter
FOV	Field of View
GEO	Geostationary Earth Orbit
GUI	Graphical User Interface
HEO	Highly Eccentric Orbit
HPOP	High Precision Orbit Propagator
LEO	Low Earth Orbit
LS	Least Squares Differential Corrector
MATLAB	Matrix Laboratory
NORAD	North American Aerospace Defense Command
POE	Precision Orbit Ephemerides
RAAN	Right Ascension of the Ascending Node
SGP4	Simplified General Perturbations-4
SPA	Solar Phase Angle
STK	Systems Tool Kit
TLE	Two Line Element set
USAF	United States Air Force

USB Universal Serial Bus

LIST OF SYMBOLS

AM	apparent motion of satellite within field of view of telescope (deg/s)
BG	image background noise level
D	cross-track error distance
D_T	telescope diameter
DMD	desired magnitude difference for telescope limiting magnitude
ET	exposure time
F	intensity flux
I	intensity value for given pixel(s)
L_C	characteristic length
M	apparent magnitude
N	number of pixels for a given area
PS	plate scale of image
R	range
SFR	telescope focal ratio
TLM	telescope limiting magnitude
X	drop in apparent magnitude due to atmospheric extinction
a	semi-major axis
b	"y-intercept" of equation describing direction of satellite path
d	satellite diameter
e	eccentricity
el	elevation
h	satellite altitude
i	inclination
tE	time error between observation and prediction
m_A	slope, i.e. direction of movement, of observed satellite path
m_P	slope, i.e. direction of movement, of predicted satellite path
ma	satellite mass
n_T	number of telescopes in an optical fence
pix_1	center pixel location of satellite position 1
pix_2	center pixel location of satellite position 2
q	line of sight vector
$\vec{r}$	position vector
v	satellite velocity
x	atmospheric extinction coefficient
Δ	separation between star and satellite position 1
Ω	right ascension of the ascending node
α	right ascension
β	azimuth
δ	declination

μ	gravitational parameter
ν	true anomaly
ω	argument of perigee
θ	transformation angle from CCD pixel frame to RA/dec frame

CHAPTER 1

INTRODUCTION

As of June 25th, 2013, the sum total of spacecraft, rocket bodies, and debris objects tracked by the US Strategic Command’s Joint Functional Component Command for Space numbers at 16,824[25]. These satellites are commonly described by, and propagated with the use of, a two-line element set (TLE). TLE’s are generated for a specific object from position-velocity state vectors—often created utilizing some form of recursive filtering—and contain information about the orbit of the object. The necessary state vectors for such orbit determination are provided by ground-based observatories through the use of either laser ranging, radar, or optical sensors[29]. Due to the ever-growing number of orbiting objects, established sensor networks such as the US Space Surveillance Network, Russian Space Surveillance System, and International Scientific Optical Network, face increased scheduling constraints. Because of these limits it follows that certain objects are given lower-priority status, e.g. cubesats and other university-built satellites. Where commercial satellites and high-risk debris are frequently targeted by laser ranging and radar observatories, respectively, low-priority satellites may be ideal targets for optical sensors—recently, amateur astronomers have observed many Low Earth Orbit (LEO) satellites, with thirteen observers providing over 21,500 observations in the 2010-11 year[32]. Figure 1.1 displays each of the tracked objects in LEO—the region of space below an altitude of about 2000 km—represented as white dots.

The primary aim of this paper is to investigate the feasibility and potential

Figure 1.1: Visualization of satellites and debris actively tracked in LEO[20]

utility of using amateur, commercial-sized telescopes to observe the aforementioned low-priority, LEO satellites. If such observations can be accomplished by small, cheap telescopes, more operational time can be freed up for other observatories to focus upon targets of higher interest. When considering the potential growth of hazardous debris described by the “Kessler Syndrome”—an exponential increase in orbital debris brought about by collisions not unlike the 2009 collision of Iridium 33 and Cosmos 2251—this may prove to be an invaluable tool[13]. Additional goals of this paper include the following four investigations that, if successful, could provide data relevant to research within the Aerospace community: preliminary angles-only orbit determination, photometric analysis of satellites for use in validation of a brightness prediction model, astrometrical analysis to provide a measurement of temporal error growth in TLE-based orbit propagation, and an optimization of an ideal telescope for LEO observations (informed by the previous two analyses).

CHAPTER 2

BACKGROUND AND LITERATURE REVIEW

Amidst the search for an engaging and worthwhile thesis topic, advice was received in the form of challenge: find a topic that suits my interests and skills developed from both my aerospace and astronomy classwork. After some deliberation, it was clear that coupling my knowledge of observational astronomy and orbital mechanics would provide an exciting research path: the discipline of satellite observation. In pursuing this discipline—a type of “applied astronomy”—no shortage of academic precursors or historical precedent was found. Presented in this chapter is a review of the research and academic literature found to be influential or informative or both.

2.1 Satellite Observation

Telescopic observations of satellites have been performed since the first spacecraft were launched into orbit—the Asiago Astrophysical Observatory observed over a dozen satellites from Sep. 1960 to Aug. 1962, photographed using a moving film technique with a telescope aperture of 0.4-1 m diameter[14]. Satellites such as Echo 1 and the first-generation, Corona spy satellites—at the time known as the “Discoverer” class spacecraft—were targeted by the observatory. A lower limit of 3.5 apparent magnitude was achieved with a stationary camera, whereas a tracking method attained a lower limit of 7-8 magnitude by providing a sustained exposure along a 7° arc of a satellite’s path (see 2.1.2 for explanation

Figure 2.1: Echo 1 Spacecraft launched on August 12, 1960[12]

of apparent magnitude). These restrictive magnitude and exposure limitations were accompanied by additional drawbacks to the system including significant along- and cross-track errors (see 4.2.3). As would be expected, fifty years of technological progress has greatly improved satellite observations.

The improvement of telescopes and telescope control, the development of charge-couple devices (CCD) to replace film, and increased accuracy of orbit propagation methods, have combined to yield satellite observations that are more simple and successful. The 1 m Zimmerwald optical telescope operated by the University of Bern, routinely searched for debris in Geostationary Earth Orbit (GEO)—altitude of 35,786 km to provide a geosynchronous orbit—as radar observations are limited to 1 m or larger objects in GEO[23]. With a one-second exposure, the telescope was able to detect objects up to an apparent magnitude between 17.0 and 19.5 (corresponding to object sizes of 0.58 to 0.18 m, for an albedo of 0.08). For optical observations of LEO objects, the United States Air Force’s (USAF) Starfire Optical Range operates a 3.5 m diameter telescope capa-

ble of seeing objects as small as ~ 0.24 m at a range of ~ 1600 km[10]. For some additional material on the history and development of telescopes and satellites observations, this author suggests *Artificial Satellite Observing...* and *Observational Astronomy*[17][3]. More recent research has considered optical observations of LEO objects, which must confront the additional challenge of accounting for the high apparent motion.

Of particular influence was the research of USAF Captain Matthew Schmunk presented in his thesis, “Initial Determination of Low Earth Orbits Using Commercial Telescopes”[24]. Included in Schmunk’s motivations for his research was a characterization of commercial telescopes as “inexpensive, mobile, and easily supported”, therefore making admittedly less-powerful commercial systems viable for satellite surveillance. With the use of a Meade LX200 GPS telescope and a wide-field digital camera, Schmunk pursued the imaging of satellites for use in angles-only orbit determination. A MATLAB Graphical User Interface (GUI) was developed for satellite target selection, orbital pass and brightness prediction, and tracking—the Simplified General Perturbations-4 (SGP4) model was used for pass prediction. SGP4, initially released in its first form in 1980, uses mean orbital elements (generally from TLE’s) with the “incorporation of resonances, third-body forces, atmospheric drag, and other perturbations” to propagate orbits [29]. Due to the telescope’s small Field of View (FOV), the main optics were not used—Schmunk utilized a digital camera with a 5.7° by 4.73° FOV attached to the side of the telescope[24]. Although Schmunk does not elaborate on this decision, it is safe to assume there may have been significant errors arising from inaccuracies in either the telescope’s pointing or the propagation scheme, that would prevent the use of a smaller FOV. At a frame-rate of 5 frames-per-second (fps), the system was limited by an apparent magnitude limit of ~ 6 . The paper concludes with recommendations for future research including

the following: improvements in the camera system, an operations optimization problems of 2 or more cameras, and automatic image processing.

Similar papers included USAF Captain Michael E. Graff's, "Development of a Remotely Operated Autonomous Tracking System," concerning a closed-loop control system for active satellite tracking, and United States Navy Commander Michael L. Thrall's, "Orbit Determination of Highly Eccentric Orbits Using a Raven Telescope," regarding the use of the Raven telescope to obtain angles-only data of Highly Eccentric Orbit (HEO) objects[10][26]. Both theses are of a certain relevance as they concern the use of smaller telescopes (Graff: 10 in. Meade LX200, Thrall: 14.6 in. Raven). Graff joins Schmunk in recommending the development of multiple telescope systems used in parallel for satellite observations. In addition to using a commercial-sized telescope, Thrall utilized Analytical Graphic Incorporated's Systems Tool Kit (STK) program as well as implementing the popular Kalman Filter method for orbit determination. These tools will be revisited in Chapter 4.

2.1.1 Orbit Determination - Astrometry

Observational Astronomy defines astrometry as a "general term...used to describe methods by which the positions of stars may be determined"[3]. But in the case of satellite observations, it is the positions of the satellites that are sought, from which orbital information can be derived. One of the earliest examples of orbit determination was accomplished by Edmond Halley in a famous 1705 publication, which successfully predicted the next appearance of Halley's comet through the use of Isaac Newton's graphical approximation method[29]. Many additional methods have been developed post-Newton, including methods utilizing only pointing angles (optical observations) and those benefiting from supplemen-

tary range and range-rate information (provided by radar and laser observations). For cases when range data is available, Gauss’ solution to Lambert’s problem—an analytical solution based upon generalized Euler formulas—or Gibbs’ geometrical solution can be used with a minimum of two observation sets. But because this paper concerns itself with optical observations, angles-only methods will be of greater importance.

Pointing angles describe the apparent angular position of an object and are often given as azimuth, β , and elevation, el , or right ascension, α , and declination, δ . Azimuth and elevation describe a position based on the observer’s local frame of reference—azimuth measured clockwise from North, elevation measured up from the horizon. Right ascension and declination are independent of the local frame, as they are based upon the equatorial coordinate system—right ascension measured east from the vernal equinox direction, declination measured positive north of the equator and negative south of the equator (see Fig. 2.2). With three sets of these angles obtained with the use of an optical sensor, angles-only orbit determination methods can be used. The earliest of these is Laplace’s method, developed by Pierre-Simon de Laplace (1749-1827) and “remains valid for interplanetary observations”[29]. Another option is that of Gauss’ method, which assumes two-body motion to enable conservation of angular momentum and the use of Lagrange coefficients to determine the state vector at the time of the second observation set[8]. Gauss’ method can be further improved through an iterative process involving universal Kepler’s equations—it is best used for observations of LEO satellites separated by no more than five to ten minutes. The last method, published by Escobal in 1965, is referred to as the Double R-iteration and works well with more widely-spaced observations—observations separated by as much as a full day[29]. Beginning with an initial bounded estimate, the algorithm advances by making additional intermediate guesses from

Figure 2.2: Visualization of Right Ascension and Declination within the Celestial Sphere [11]

which differential corrections are made. For a much more in-depth explanation of the ins and outs of orbit determination, see *Theory of Orbit Determination* by Andrea Milani and Giovanni F. Gronchi[16].

Once position-velocity state vectors have been found through use of the above orbit determination methods, the common orbital elements (COE's) can be calculated. Also known as Keplerian elements, COE's describe the qualities of the orbit through parameters including, but not limited to, the following: semi-major axis, a , eccentricity, e , inclination, i , right ascension of the ascending node, Ω , argument of perigee, ω , and true anomaly, ν [29]. Mean COE's are usually bundled into the TLE's mentioned in the introduction of this paper. These bundles of orbital information—which include additional parameters such as the mean anomaly and B^* terms—serve as the initial values for SGP4 propagation. But as one would expect, the accuracy of the TLE-based SGP4 propagation isn't perfect (~ 1 km at epoch) and error accrues over time[30]. Figures 2.3 and 2.4 show the

Figure 2.3: Radial, transverse, and cross-track residuals for Stella satellite over 7 days.

temporal error growth of TLE vs. Precision Orbit Ephemerides (POE) for the Stella satellite over the course of seven days[15].

2.1.2 Satellite Apparent Brightness - Photometry

Distinct from the problem of astrometry, photometry focuses on analyzing the brightness of a target object. The most simple and straightforward type of photometry is known as differential photometry, which “compares sources sufficiently close together on the sky so that differential first-order extinction can be neglected” [3]. In other words, if two objects such as a satellite and a star are contained within the FOV of a telescope, they are subjected to the same atmospheric effects. Astronomers quantify the brightness of objects with the magnitude scale, a logarithmic scale based upon human vision’s nonlinear perception of increases in brightness (all references to magnitude in this paper pertains to apparent

Figure 2.4: Least squares fitted polynomial for standard deviation of radial, transverse, and cross-track residuals for Stella satellite.

magnitude, not to be confused with the standardized scale of absolute magnitude). On the magnitude scale, brighter objects are classified with more negative values—the full moon is approximately -12.9 magnitude—while dimmer objects receive more positive values—Uranus at its brightest is approximately 5.3 magnitude. The ratio of one object’s apparent brightness to another’s is converted to magnitudes by the following relationship,

$$M_2 - M_1 = -2.5 \log_{10} \left(\frac{F_2}{F_1} \right) \quad (2.1)$$

where M is magnitude and F is intensity flux. Therefore the +18.2 difference in magnitude between the full moon and bright Uranus is equivalent to a $\sim 2 \times 10^7$ reduction in apparent brightness.

Predicting the apparent magnitude for a given satellite is a combination of several factors including but not limited to the following: observatory altitude, observer-to-satellite range, solar phase angle (SPA), elevation angle, design of

Figure 2.5: Telescope limiting magnitude for a staring system.

telescope, atmospheric seeing conditions, satellite characteristic size, and satellite albedo (reflection coefficient)[1]. An estimate for the expected brightness of target satellite is undeniably important for astronomers, as the difference between the satellite magnitude and the telescope’s limiting magnitude dictates the success of the observation. While investigating blind search techniques for micro-satellites in LEO, Ackermann et al. estimated the limiting magnitudes for variable telescope size[1]. These limits are shown in Fig. 2.5—note that this is for a “staring” system (telescope is kept stationary as satellite passes through the FOV), therefore the detection threshold is lower than that for a comparable tracking system. Certain photometry research includes the modeling of debris magnitude, for which a mean albedo value of 0.175 is applied[18]. To estimate the characteristic length, L_C , of a debris object based upon its apparent magnitude, M_{app} , a Lambert sphere is assumed and the following equation is used,

$$L_C = 10^{(M_{Sun}-M_{app})/5} \frac{R}{\sqrt{A}} \sqrt{\frac{6\pi}{\sin(SPA) + (\pi - SPA) \cos(SPA)}} \quad (2.2)$$

where M_{Sun} is the magnitude of the Sun, M_{app} is the apparent magnitude of the debris object, R is the range, A is the albedo, and PA is the solar phase angle.

Table 2.1: Predicted apparent magnitudes of the CP5 cubesat.

	Mean	Standard Dev.	Ackermann Mean
Undeployed Sail	11.49	0.5	~ 10.5
Deployed Sail	6.99	0.5	—

Additional research includes obtaining light curves for tumbling rocket bodies[7]. Such information would be pertinent to “plan and develop proximity and docking for potential future Active Debris Removal operations”, or to analyze the effects of perturbations upon satellite rotational dynamics.

2.2 Initial Plan to Observe CP5 Cubesat

This project began with a specific target in mind: the CP5 cubesat developed by Cal Poly’s PolySat laboratory and launched September 13, 2012[22]. It was thought to be an ideal small, LEO satellite for optical observation, particularly due to its deployable sail payload. Normally a 0.1 by 0.1 m cube (L_C equal to 0.1414 m), the deployed sail raises the cross-sectional area to ~ 1 m (L_C equal to 1.128 m), therefore making it a more viable target. STK and MATLAB were utilized to predict several parameters of CP5 for use in observation preparation, which are shown in Table 2.1 in comparison to predicted values from Ackermann et al[1]. The discrepancy between the values predicted in this report, and those predicted by Ackermann et al., is likely due to differences to some combination of the following factors: observation altitude, albedo, and mean satellite range. Additional predicted values included the mean apparent motion of CP5, 18.6 arcmin/s, and the maximum apparent motion, 52.7 arcmin/s. But in an unfortunate turn of events, the sail failed to deploy once CP5 was inserted into orbit; due to the failure of the sail deployment, focus was turned toward other satellite targets indefinitely.

CHAPTER 3

INSTRUMENTATION AND PROCEDURE

3.1 Instrumentation

3.1.1 Observatory

Observations were performed at the Cal Poly on-campus dome observatory, shown in Fig. 3.1. While it is located in a convenient location, the observatory suffers from high amounts of light pollution (some effort was made to place boxes over nearby light fixtures) and elevation pointing restrictions. Depending upon the azimuthal direction, the elevation is limited to higher than $20\text{-}30^\circ$ due to surrounding buildings and trees. Automated tracking is available for the dome, but a delay in response time and slow rotation speed would inhibit attempts at active satellite tracking. The approximate location of the observatory is 35.30° N, 120.66° W, at an altitude of 105.8 m.

3.1.2 Telescope

The main optics of a Meade LX 200 12" telescope was used for this project and can be see in Fig. 3.2. The telescope is an Advanced Coma Free (or Schmidt-Cassegrain) optical design and and is mounted on a double-tine, fork type mount in the center of the Cal Poly dome observatory. Due to physical constraints on the mount, the telescope is unable to reach within $\pm 20^\circ$ of North. Table 3.1 lists important specifications of the telescope. At a cost of \$4,499, this telescope

Figure 3.1: Cal Poly Dome Observatory

Table 3.1: Meade LX200 12" Telescope Specifications.

Specification	Value
Aperture	305 mm
Optical Design	Advanced Coma Free (ACF)
Focal Length	3,048 mm
Focal Ratio	f/10
Resolving Power	0.38 arcseconds
Eyepiece	26 mm Series 4000 Super Plossi
Pointing Precision	1 arcminute

is considered to be "amateur-sized", at a price thought to be reasonable for a serious amateur astronomer or undergraduate student.

3.1.3 Video Camera

The Lumenera SKYnyx2-0M astrophotography camera was used for recording images produced out of the main optics of the Meade telescope, and is shown in Fig. 3.3. As the camera utilizes a USB interface, it is relatively simple to connect to a computer or laptop for viewing of a live feed or saving of images. Table 3.2 lists the relevant specifications of the camera. This camera is priced at approximately \$1,500.

Figure 3.2: Meade LX200 Telescope.

Figure 3.3: Lumenera SKYnyx2-0M Astrophotography Camera.

Table 3.2: Lumenera SKYnyx2-0M Camera Specifications.

Image Sensor	Monochrome CCD, 5.8 x 4.9 mm array
Specification	Value
Resolution	640 x 480 pixels
Max. Frame Rate	60 fps
Gain Range	1 to 10 x optimizable
Dynamic Range	>60 dB
Integration Time	1/1000 to 16 sec.
Interface	Standard USB 2.0 high-speed

3.1.4 Focal Reducer

To increase the FOV of the telescope, a focal reducer was used along with both size and thread adapter barrels. The focal reducer used was a Celestron f/3.3—it is pictured in Fig. 3.4. A similar f/6.3 focal reducer is priced at approximately \$130. Figure 3.5 shows the focal reducer attached to the camera (along with a barrel extension and thread converter) and attached to the back end of the telescope. The decision to incorporate a focal reducer was made when observation failures were incorrectly thought to be occurring due to cross-track error—see Appendix 7.2. However, there are additional advantages gained with the utilization of a focal reducer: a satellite is observed for a longer period of time (reduction in the standard deviation of the observed magnitude value) and the likelihood of the inclusion of secondary companion stars increases. The possible advantages of secondary companion stars are discussed in Section 6.1.

3.1.5 Software

Several software applications were involved in the observation process—their uses will be further detailed in the following section. Software Bisque’s TheSkyX v10.2 astronomy software was used for computerized control of the telescope in the dome observatory—the Professional edition sells for \$330. Additional hard-

Figure 3.4: Celestron f/3.3 Focal Reducer.

Figure 3.5: Left: full camera setup. Right: camera attached to back end of Meade telescope.

ware/software involved in the observatory functionality was Technical Innovations' Digital Dome Works v5.2 (DDW) observatory automation system. Such a system allows for a continuous, automated tracking of the telescope by the dome. Lucam Recorder v2.7 software was used for control of the Lumenera camera and archiving of recorded images. It is developed by Heiko Wilkens and is available as a free download. For observation preparation and data analysis, MathWorks' MATLAB R2010a and Analytical Graphics Incorporated's STK v10 were used.

3.2 Procedure

3.2.1 Preparation

On the day of a planned night of observations, preparations normally began two to three hours before astronomical twilight. This process was rather involved, and included the propagation of satellite targets, analysis and selection of those targets, and a generation of observation angles and times at which to observe the targets.

Preparation began with visiting the Space Track website, which provides up-to-date TLE data for all documented objects in orbit, and downloading the most recent TLE's for several satellite sets: Globalstar (52 satellites), Visible (214), Weather (57), and Custom (32)[25]. The first three sets are created and maintained by the Space Track website, but the Custom list was compiled for the purpose of this project and is shown in Appendix 7.3. Automated download tools are available, such as Celestrak's Space Track TLE Retriever, but were not used for this project[27].

These TLE's were opened by the MATLAB file **passPredict.m** with the following user-specified inputs: day and time of observation session, duration of

Start		Max altitude			End		Visible passes		
Date, time	Az	Time	Az	EI	Time	Az	Mag	Map	Action
7/11 05:57	S 189°	06:02	SE 128°	15°	06:06	ENE 67°	-0.1	☐	Details
7/13 05:55	SW 215°	06:00	SE 133°	49°	06:06	NE 53°	-1.8	☐	Details
7/14 05:06	SSW 201°	05:11	SE 130°	25°	05:16	ENE 60°	-0.8	☐	Details
7/15 04:18	S 185°	04:22	SE 128°	13°	04:27	ENE 70°	0.1	☐	Details
7/15 05:54	SW 239°	05:59	NW 322°	48°	06:04	NE 43°	-1.8	☐	Details
7/16 05:04	SW 226°	05:10	SE 143°	86°	05:15	NE 49°	-2.4	☐	Details
7/17 04:16	SW 212°	04:21	SE 133°	42°	04:26	NE 55°	-1.6	☐	Details
7/17 05:53	W 262°	05:58	NW 329°	20°	06:03	NE 36°	-0.4	☐	Details
7/18 03:27	SSW 197°	03:32	SE 131°	22°	03:37	ENE 62°	-0.3	☐	Details
7/18 05:03	WSW 249°	05:08	NW 324°	31°	05:14	NE 40°	-1.1	☐	Details

In this table:

☐	Not visible	☐	Marginal	☐	Good	☐	Excellent
--------------------------	-------------	--------------------------	----------	--------------------------	------	--------------------------	-----------

REAL TIME SATELLITE TRACKING

5-DAY PREDICTIONS FOR ISS (ZARYA)

[Main page](#) [Preferences](#)

Your current location:
Cal Poly Telescopes
[Change](#)
Latitude: **35.300502°**
Longitude: **-120.659941°**
Local time zone: GMT -7

Figure 3.6: Sample Output from N2YO website.

the session, maximum age of the TLE, and minimum elevation attainable by the telescope. Each satellite was propagated forward in its orbit at three minute intervals—satellites found to be both visible (line of sight is possible) and illuminated (at least in partial sunlight) were tagged and the following information was outputted from the code: NORAD ID number, observation time, TLE age, and elevation of observation.

Next, the most attractive targets were selected and their NORAD ID numbers put into the search bar in the real time satellite tracking website n2yo.com[19]. Targets are deemed attractive based upon the length of time they are both visible and illuminated, and their maximum elevation angle; the longer and higher the pass, the better. Maintained by a Ham radio enthusiast (n2yo is their callsign), the website provides valuable information about a satellite’s overhead pass—information that is utilized as a means for validation and comparison. Figure 3.6 shows a sample output from the n2yo website. This information is then saved for

further reference¹.

Chosen satellites were then inserted into a preexisting STK scenario—if the satellite was already designated within the scenario, only its TLE must be updated via the Space Track option in the TLE Source box within the satellite properties window². SGP4 was then used to propagate through a designated time period, from which an Azimuth-Elevation-Range (AER) report was generated for a given observatory-bounded sensor. The sensor was modeled as Simple Conic type with a 90° cone half angle. To detect a mid-pass illumination drop-off (a cessation of sunlight partway through the satellite pass), a Lighting Times report was generated³. Because LEO satellites are located at such a low altitude, illuminated passes occur for only a few hours after sunset—it is a common occurrence that some overhead passes occur at the precise moment that a satellite passes into the Earth’s shadow. If such an expiration in illumination was found, the AER report was limited to the times at which the satellite was illuminated. A comma-separated values (csv) version of the AER report was saved at one-second intervals in a Julian date (Jdate) format.

AER reports generated from STK are loaded into MATLAB, at which point the code **STK2VIZIER.m** was used to convert the azimuth and elevation angles to right ascension and declination angles, which utilizes Algorithms 15 (**LSTime.m**), 28 (**AzElToRaDec.m**), 29 (**SUN.m**), **site.m**, **ecef2eci.m**, and **razel2rv.m** from Vallado[29]. It is worth noting that these RA/Dec angles were then converted from their Current Epoch frame to the more standard J2000 frame (see

¹Because this step is primarily used for prediction validation, with little use in data analysis, it can easily be skipped in any future attempts to replicate this procedure.

²Sometime around the release of STK version 10.0.1, the frequency of updates to the AGI server increased from every 2-3 days, to at least once a day. This makes automatic updating of TLE’s a more attractive option than manually downloading from Space Track.

³This step could potentially be removed if the following two conditions are met: 1) SGP4 propagator used in MATLAB is deemed to be at least as accurate as STK, 2) Propagator interval is significantly reduced from three minutes.

Show constraint information

i Result of VizieR Search (within **6arcmin** of a list of targets) ANDed with 1 other constraint:
 (Vmag: "<8.5")
 ordered by increasing _r

Hide the target form

Simple Target / List Of Targets

To query from a list of positions, you have to enter the name of a file which contains the list of positions or central object names, one per line. (VOTable or tab-separated-value format)

☐ Verbose output ☒ single table ☐ add your input as first column

Target dimension:

Choose File: No file chosen J2000: [dropdown] 6 arcmin [dropdown] Submit

☒ Radius ☐ Box size

No object found around J2000 position 22:10:24.4+73:03:36	22 10 24.415 73 3 36.029
No object found around J2000 position 22:09:07.9+73:08:59	22 9 7.9069 73 8 59.059
	•
	•
	•
	21 45 15.004 74 34 5.3268

The 3 columns in color are computed by VizieR, and are **not part of the original data** (note that the **computed coordinates** are computed from the positions)

Note: Errors found in the Hipparcos catalogue are reported in the file [errata.txt](#)

18.I/239/hip_main The Hipparcos and Tycho Catalogues (ESA 1997)
 I annotation(s) - post The Hipparcos Main Catalogue (118218 rows)

Full	_r	RA(J2000)	DE(J2000)	HIP	RAhms	DEdms	Vmag	RA(ICRS)	DE(ICRS)	Pix	pmRA	pmDE	e_Plx	B-V	Notes
	arcmin	"h:m:s"	"d:m:s"				mag	deg	deg		mas/yr	mas/yr	mas	mag	
[2]	3.2125	21 45 48.339	+74 34 44.69	107449	21 45 48.35	+74 34 44.7	8.45	326.45145294	74.57908405	3.10	-4.25	-0.88	0.68	1.895	

21 43 33.771 74 39 13.566

No object found around J2000 position 21:43:33.8+74:39:14

7.2). These angles were saved to a space-delimited text file, and uploaded to the Strasbourg Astronomical Data Center’s (CDS) VizieR service[4]. This service was used to locate stars that qualify as suitable “companion stars” along the projected path of the satellite. At each of the RA/Dec positions, VizieR was assigned to search for stars brighter than 8.5 apparent magnitude (a level visible at the planned exposure time for the camera) and within less than 1/2 the telescope’s smaller FOV dimension. The Hipparcos Main Catalogue of 118,218 stars was selected, and the RA/Dec text file was queried as a “List of Targets” with the aforementioned target dimension ($< 1/2$ FOV). Figure 3.7 displays a few lines of the VizieR output—for a 10 minutes pass, the total number of lines would be 600. Depending upon the intent of the observations, between 1-4 qualifying stars

were chosen, and the pertinent information was archived for future analysis (star catalog number, star-target separation, and V magnitude). If orbit determination is the intent of the researcher, then at least three angle sets with a qualifying companion star are required. Because a staring method was used for observing the satellites, and the telescope must be slewed and realigned between each angle set, at least ~ 2 minutes was required between each selected set. Between this restriction and those enforced by the pointing limits of the telescope, at times it may be difficult to locate suitable companion stars. There are several methods to combat this problem: 1) reduce the step time to less than one-second intervals, 2) increase exposure time to allow for lower-magnitude companion stars, 3) Query additional star catalogs in VizieR (e.g. Tycho catalog), and 4) increase FOV of the telescope.

Once the angle sets had been selected, the index numbers to the left of the query responses (e.g. “18” in Fig. 3.7) were entered into the latter portion of the **STK2VIZIER.m** code. This portion of the code returned the following information: time of the observation, azimuth and elevation degree values, satellite range, solar phase angle, and right ascension and declination sexagesimal values. Solar phase angle was calculated with the following method,

$$\vec{r}_A = r_{SAT} - r_{SITE}, \quad (3.1a)$$

$$\vec{r}_B = r_{SAT} - r_{SUN}, \quad (3.1b)$$

$$SPA = \arccos \left(\frac{\vec{r}_A \cdot \vec{r}_B}{\|\vec{r}_A\| \|\vec{r}_B\|} \right) \quad (3.1c)$$

where r_{SAT} , r_{SITE} , and r_{SUN} are the position vectors of the satellite, observatory, and Sun, respectively. Table 3.3 summarizes the preparation procedure.

Table 3.3: Information Archived from Preparation.

Source	MATLAB	N2YO	STK	VizieR
Info.	RA/Dec SPA Satellite Name/ID Observation Time	Satellite Magnitude Pass start/end times	Illumination Check Az/Elev Range Observation Jdate	Star Name Star Magnitude

3.2.2 Observation

Nights at the Cal Poly Observatory began with activating power to the dome and telescope, and opening DDW and SkyX software. In the attempt to reduce nearby light pollution, boxes were placed over nearby exterior lights and nearby classroom lights were turned off. However this was not always possible, as certain laboratories remained open into the night. Because many observations approached the lower elevation limit of 20° , the lower portion of the dome shutter was raised to the top of the dome—this created an upper elevation limit of $\sim 65^\circ$. While a desktop computer was used for dome and telescope control, a separate laptop was used for camera control and image recording. The next step involved attaching the Lumenera camera to the rear of the telescope, and connecting the camera to the laptop via USB. Once a live feed was established, the telescope was commanded to slew to a nearby, bright star (e.g. Sirius). This action served to confirm the dome was tracking the telescope properly, enable synchronization of the telescope with SkyX, and to focus the telescope with the use of an Optec Temperature Compensating Focuser. Excel sheets containing the information gathered during preparation were opened and observations began shortly.

Several minutes before a planned observation, the telescope was commanded to slew to a bright star (>5 magnitude) near to the companion star. Synchronizing with this bright star allows for targeting with the use of the finder scope, and usually guarantees that a small slew ($<10\text{-}15^\circ$) to the companion star will not arrive without the companion star within the FOV of the telescope. Be-

cause the companion stars were often of a lower magnitude, the finder scope can not be used for synchronization should the telescope lose its pointing accuracy—should this occur, some form of an outward-spiral search pattern must be used with the telescope’s hand controls. Considering the time-crunch inherent to the slew-and-stare method, it is much more preferable to use the sync-to-bright-star-then-slew-to-companion-star technique.

Having established that the telescope was accurately pointed at the companion star, a final slew was made to the RA/dec angles set through which the satellite is predicted to pass. Confident that the telescope was now in the right location, the camera remained on standby until 30-60 seconds before the predicted time at which the satellite will pass through the center of the FOV, at which point the recording began. The Lumenera camera was set to an exposure time of 60 ms, maximum gain, and an integration rate of ~ 17 fps, and recorded until 30-60 seconds after the predicted time of satellite passage—Fig. 3.8 shows the active interface of the Lucam Recorder. Images captured by the camera were automatically stored into folders classified by date and time—they were saved in .jpg format and are accompanied by .txt files detailing the stream information (exposure, gain, etc.) and the timestamps for each image. Timestamp accuracy was only as good as the laptop’s internal clock, therefore the application Atomic Clock Sync v3.5 was used to synchronize the laptop earlier in the night[6].

Review and analysis of these images, and the archival of the resulting data, will be covered in detail in the following chapter. The entirety of the procedural process is shown step-by-step in Fig. 3.9.

Figure 3.8: Active Interface of Lucam Recorder.

Figure 3.9: Step-by-step Observation Procedure. Includes information sent along each step: satellite Two-Line Element Sets (TLE), satellite apparent magnitude prediction (Mag), time and duration of satellite pass (Time), azimuth and elevation angles (Azi/Elev), right ascension and declination angles (RA/Dec), and companion star details (Star).

CHAPTER 4

ANALYSIS

4.1 Review of Images

Images gathered during a night of observation were reviewed by means of a simple, manual “scan” of a set of images. A rather straightforward process, this involved opening a folder of images in Windows Photo Viewer and holding down the right arrow key to view an effective stop-motion animation of the images. At a frame rate of 17 fps, a 60 second duration recording would include 1020 images—using this scan method, all images can be viewed in approximately two minutes. With as many as 26 individual observation attempts in a single night, it was found to be both beneficial and efficient to begin scanning images 10 seconds before the predicted pass time, as opposed to 30 seconds. Given that the vast majority of along-track errors were found to be less than a 2-3 seconds, and clock errors of ~ 5 seconds or less, the satellites (if visible) would likely be seen within ~ 30 seconds. All image review was carried out in a dark room, allowing for easy identification of a moving satellite⁴. If and when the satellite was identified, the images containing the satellite were moved to a separate folder.

⁴A possible alternative method includes using an application such as Windows Live Movie Maker to both increase the brightness of the images (allowing for easier identification of the satellite) and the speed of the animation.

4.2 Astrometry

4.2.1 Data Reduction

Conversion from image to data began with identifying the position of the pertinent information within the images. Microsoft Paint was utilized to determine the center [x,y] pixel location, pix , of three positions: companion star, first satellite position (after it has fully entered the FOV), and the last satellite position (before exiting FOV), although any similar application that provides cursor information could be used. Because of the finite resolution of the pixels, and the haphazard atmospheric effect of scintillation, an uncertainty of ± 1 pixel was given to the star positions, and ± 2 pixels for the satellite positions. The larger uncertainty for satellites is due to the satellites' motion, as a pronounced blurring effect was encountered.

To obtain the necessary data for further analysis, the pixel locations must be converted to right ascension and declination angle sets. This can be accomplished by using the companion star as a reference position, and the direction of satellite's movement as a reference orientation. RA/Dec ephemeris coordinates for cataloged stars can be gathered from CDS's Simbad service—this is an especially helpful tool, as a list of stars can be queried[5]. For establishing the orientation of the FOV, the RA/Dec coordinates saved from the aforementioned **STK2VIZIER.m** code were used to generate a predicted direction of movement, or slope,

$$m_P = \frac{\delta_N - \delta_T}{\alpha_N - \alpha_T} \quad (4.1)$$

where the subscripts indicate T for main target position, and N for the next predicted position 1 second later. Replacing the predicted RA/Dec positions in Eq. 4.1 with the pix values for the two satellite positions, the actual FOV slope

was found,

$$m_A = \frac{pix_{2y} - pix_{1y}}{pix_{2x} - pix_{1x}}. \quad (4.2)$$

To transform from the pixel frame to the RA/Dec frame, the following rotation shown in Fig. 4.1 was made through the correction angle, θ . Any error in the predicted slope—due to an imperfect TLE propagation—was assumed to be negligible at this scale. Although the Meade LX200 telescope utilizes an equatorial mount, which can ensure a constant RA/Dec orientation (given precise camera attachment), the method shown in Fig. 4.1 will work for non-equatorial mounts as well. Now that the pixel frame had been rotated to the correct RA/Dec orientation, the separation vector between the star and satellite position 1, Δ , is added to the star's RA/Dec position,

$$\theta = \arctan(m_A) - \arctan\left(\frac{m_P}{\cos(\delta_{mean})}\right), \quad (4.3a)$$

$$\begin{bmatrix} \delta \\ \alpha \end{bmatrix}_{sat} = \begin{bmatrix} \delta \\ \alpha \end{bmatrix}_{star} + \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix} \begin{bmatrix} \Delta_y \\ \Delta_x / \cos(\delta_{mean}) \end{bmatrix} PS \quad (4.3b)$$

where PS is the plate scale of the image, i.e. the conversion from pixels to arcseconds. The $\cos(\delta_{mean})$ term in Eq. 4.3 was used to scale the right ascension values. Consider the case of longitude and latitude: as latitude increases away from the equator, the angular distance corresponding to a change in longitude decreases, due to the Earth's spherical shape. Therefore, the change in right ascension must be scaled by the mean declination angle at which the observation occurs. The process of determining the RA/Dec positions from the recorded images was handled by the MATLAB code **angledet.m**.

Figure 4.1: Visualization of Pixel-to-RA/Dec Transformation (Pictured top: Satellite movement from position 1 to 2 (red), separation between star and satellite position 1 (green), and frame axes (yellow). Pictured bottom: Predicted direction (blue), actual direction (red), frame rotation (purple), and RA/Dec axes (black)).

4.2.2 Orbit Determination

Single Angle-Set

As mentioned in Section 2.1.1, the acquisition of at least three RA/Dec angle-sets allows for preliminary orbit determination. For all instances in which three angle-sets were obtained, two angles-only methods were used. These included Vallado’s rendition of Gauss’ Method available online in C++, FORTRAN, Pascal, and MATLAB languages[28]. Scripts based on Vallado’s Algorithm 52, **ANGLES-DOUBLE-R**, and Curtis’ rendition of Gauss’ Method were also used as a measure of comparison[29][8]⁵. The input required for these methods included the following: line of site vectors, Jdates, and ground site vectors. Line of set vectors, q , were calculated from right ascension and declination:

$$q = \begin{bmatrix} \cos(\delta) \cos(\alpha) \\ \cos(\delta) \sin(\alpha) \\ \sin(\delta) \end{bmatrix}. \quad (4.4)$$

Jdates were found by converting the saved timestamps for the gathered images—as a consistent clock error was found in most of the timestamp data, the mean error from a night’s observations was removed. Lastly, ground site vectors were found by using **site.m** to find Earth-Centered, Earth-Fixed position vectors for the Cal Poly Observatory, which were then converted to Earth-Centered Inertial vectors through use of **ecef2eci.m**. The MATLAB code **orbit_determination.m** enacted the operations described by this paragraph, and includes a user-specified selection of the desired orbit determination method. Once the position-velocity state vectors were acquired from the output of the orbit determination method, the algorithm **RV2COE2.m** was used to obtain the COE’s for the observed satellite. These parameters provide a comparative measure of success for the

⁵Issues were encountered when Vallado’s actual Double-R MATLAB code was used.

methods.

Multi Angle-Set

If favorable conditions granted the ability to obtain multiple three-angle-sets for a satellite, either from consecutive passes or on back-to-back nights, more advanced orbit determination methods were used. These advanced methods, an extended Kalman filter (EKF) and a least squares differential corrector (LS), were utilized along with Kepler universal variable propagation to iteratively improve the COE's generated by the initial angles-only orbit determination. For a summarized description of the EKF, see Appendix 7.5. The appendix also details the process noise, covariance, and measurement noise associated with the EKF. Implementation of the EKF and LS was done through the use of **orbit_detM.m**, a similar code to **orbit_determination.m**, and **extKALMAN2.m** and **least-squares.m**. A detailed description of nonlinear, least squares differential correction applied to orbit determination can be in Algorithm 66 of Vallado[29].

4.2.3 Temporal TLE Error Growth

Considering the imperfect nature of TLE-based propagation, and the inherent difficulty in providing an accurate model of orbital perturbations, a measure of temporal TLE error growth is invaluable to a satellite operator. To provide the necessary data for this analysis, observations of a single satellite must be acquired across several days. As a general rule for this project, every satellite successfully captured in a given night was checked for visible passes on the following nights. Should such a visible pass exist—one that does not conflict with a more higher priority observation (such as one involved in orbit determination)—the original TLE used for propagation on the previous night was used to further propagate

the satellite.

The script **pixel_analysis.m** also contains the math responsible for this analysis. To determine the distance between the predicted direction of movement, and the actual, observed path, D , the following equations were used,

$$b_A = \delta_1 - m_A \alpha_1 \quad (4.5a)$$

$$D = \frac{|\delta_T - m_A \alpha_T - b_A|}{\sqrt{m_A^2 + 1}} \quad (4.5b)$$

where b is the “y-intercept” for the direction of movement. This distance is then converted into the corresponding angular, cross-track error,

$$CTE = \tan(D)R \quad (4.6)$$

which is the observed error tangential to the satellite’s direction of movement.

For determining the error in the direction of the satellite’s motion, the recorded timestamps were required. The along-track error was found with the following equation:

$$ATE = tE \sqrt{\frac{\mu}{r_E + h}} \quad (4.7)$$

where tE is the time error between the predicted time at which the satellite crosses the FOV and the mean time of satellite position 1 and 2—additional values are the Earth’s radius, r_E , Earth’s Gravitational parameter, μ , and the satellite altitude, h . This equation assumes a circular orbit to determine the orbital velocity, and that the orbital velocity is equal to the transverse velocity seen by the observer.

4.3 Photometry

4.3.1 Data Reduction

The method used for analyzing the observed brightness of a satellite was the simple, but well-known, differential photometry approach. Performing this method requires the act of quantifying the level of light associated with both the satellite and the companion star. The astronomical software Image Reduction and Analysis Facility (IRAF) is commonly used for photometric analysis of Flexible Image Transport System (FITS) images. But because images for this project were saved in JPEG format, MATLAB was used to accomplish the photometry. Upon reading and opening images into MATLAB, the command **ginput** is used to enable the user to save cursor location ([x,y] pixel index) with every click of the mouse. Each pixel within the image carries an intensity “count”, I , an integer proportional to the amount of light received by that pixel.

With four clicks, three locales within the image were analyzed: the center of the companion star (1st click), two opposite corners of a box surrounding the satellite blur (2nd and 3rd), and an empty region of the image (4th). Because the atmosphere and seeing conditions were inconsistent, the background noise in each image differs, thereby requiring analysis of a region of the image not host to stars or the satellite. The mean value of an 81x81 pixel square area surrounding the 4th clicked pixel, was used to estimate the background noise value, BG . A 41x41 pixel square area around the 1st clicked pixel, and the square area constrained by the 2nd and 3rd clicks, provided the sum total of pixel intensity values for the

star, I_{star} , and satellite, I_{sat} , to be used in calculating the magnitude difference:

$$F_{star} = I_{star} - BG \times N_{star} \quad (4.8a)$$

$$F_{sat} = I_{sat} - BG \times N_{sat} \quad (4.8b)$$

$$M_{sat} - M_{star} = -2.5 \log_{10} \left(\frac{F_{sat}}{F_{star}} \right) \quad (4.8c)$$

where N is the number of pixels for the given area (e.g. 1681 for the star area). This process was repeated for each image within a set—depending upon the particular observation, this could number between 1 and >75 images. A mean $M_{sat} - M_{star}$ value was generated from these images, omitting any outliers seen near the edges of the FOV or when the satellite was in close proximity to a star, along with the standard deviation. Photometric analysis was performed by the code **imgPHOT.m** and additional details can be found in Appendix 7.4.

4.3.2 Magnitude Prediction Model

The MATLAB function **appMag.m** was developed for this project; the function was purposed with providing a prediction of apparent magnitude for a given satellite pass. Relevant information saved from observation preparation—range, elevation, and SPA—was passed into the function. Beginning with an initial guess for apparent magnitude, Eq. 2.2 was looped until the correct magnitude is found. Satellite albedo is assumed to be 0.175, whereas the characteristic length of the satellite was found through a more laborious process. While certain sources do provide some amount of satellite dimensions, such as NASA’s National Space Science Data Center’s Master Catalog, it is often the case that the dimensions are incomplete or unlisted[9]. To provide an estimate of satellite size, the stated mass of a satellite was utilized, along with a mass-volume density relationship. This relationship was determined from a statistical regression of just over two

dozen satellites with known mass and dimensions,

$$d = (5.05 + 0.0353ma)^{1/3} \quad (4.9)$$

where d is satellite diameter and ma is mass—it was assumed that the unknown shape of the satellite is a sphere.

Prediction of the satellite magnitude was done for two spacecraft sizes: a small estimate and a large estimate. If multiple dimensions were published for a given satellite, two different initial small and large estimates were provided. For the cases in which either only a single dimension was known, or Eq. 4.9 was used, this single dimension was used for both the small and large estimates. Further separation of the two size estimates was accomplished by the addition of an uncertainty margin—in the interest of modeling satellite drag, researchers have found that a satellite’s surface area can vary by at least one order of magnitude[31]. Converting the uncertainty in area, to uncertainty in diameter (assuming circular cross-section), it was found that diameter can vary by ~ 3.16 . Several test cases were ran, and it was determined that the magnitude predictor was most accurate when 40% of the margin was given to the small estimate ($0.5846 \times d_{small}$), and 60% to the large ($1.6231 \times d_{large}$). Size estimation is located within the **ObservationResults.m** code.

An additional method involving the use of a satellite’s ballistic coefficient, found using the TLE B* term, was found to be unsuccessful at accurately predicting the satellite magnitude: predicted values exceeded the large-estimate magnitude prediction by an average magnitude of ~ 2.9 . It is hypothesized that this inaccuracy is caused by the B* term’s inclusion of additional, time-varying parameters that describe the satellite[31].

Lastly, the effect of atmospheric extinction was applied. As an astronomical object loses elevation in the night sky, its light passes through more atmosphere

en route to an observer[3]. This increase in path impedance reduces the apparent magnitude of the object by an amount, kX , where k is the wavelength-dependent extinction coefficient, and X is air mass and was found by the following expression,

$$X = \sec(1 - el)[1 - 0.0012(\sec^2(1 - el) - 1)] \quad (4.10)$$

For a wavelength of 550 nm (wavelength associated with the V magnitude band), a k value of 0.17 was used. Both the low and high apparent magnitude predictions, along with the N2YO prediction, which utilizes Michael McCants' method for satellite magnitude estimation, were saved for comparative analysis covered in Chapter 5.

4.4 Telescope Optimization

MATLAB's `fmincon` function, an abbreviation for "find minimum of constrained nonlinear multivariable", is a powerful tool used to optimize a system. `Fmincon` requires a working model of the system (dynamics of the system driven by input variable(s)), and uses system constraints, variable limitations, and a user-selected optimization algorithm (such as 'interior-point' or 'sequential quadratic programming') to determine the input variables that achieve the best results defined by the system model. For this report, `fmincon` was used to optimize a telescope purposed for satellite observations. Table 4.1 gives the various elements involved in the optimization scheme, elements detailed in the following paragraph.

In essence, the optimization determines the best telescope size at which to maximize the probability of a successful observation. The optimization process begins with defining the desired qualities of the observation, telescope, and satellite being observed. Once this had been accomplished, the probability of a suc-

Table 4.1: Details of the Fmincon Telescope Optimizer.

Element	Optimizer	Model	Constraints
Purpose	Initialize fmincon, Provide visual results	Define system dynamics, Calculate function value	Enforce limits on Optimizer
Inputs	Satellite/Telescope qualities: L_C, h, i , TLE age, n_T , minimum el , DMD , temporal error growth scheme	Optimized variables: D_T, FR, SFR	Optimized variables
MATLAB	OptScope	foptScope, PGO PGOmag	fconScope

cessful optimization was calculated by determining the probability that the observation will fail due to two causes: A) cross-track error greater than telescope FOV (**PGO.m**), B) satellite apparent magnitude dimmer than telescope's limiting magnitude (**PGOmag.m**).

To determine the failure probability due to cross-track error, cause A, the chosen temporal error growth scheme is of utmost importance—either the growth shown in Fig. 2.4, or the results of Section 5.2 were used. Given the age of the TLE used for orbit propagation, an estimate of CTE was calculated. As for the FOV of the telescope, the following equation was used,

$$FOV = \frac{0.833n_T(\frac{12}{D_T})}{SFR/10} \quad (4.11)$$

where n_T is the number of telescopes should an optical fence be used, D_T is the telescope diameter in inches, and SFR is the telescope's focal ratio. Assuming that the the orbit propagation error follows a normal distribution, the probability of a successful observation was found in the following manner,

$$x = \frac{FOV}{CTE} \quad (4.12a)$$

$$fProbability(y) = \frac{\exp(-\frac{y^2}{2})}{\sqrt{(2\pi)}} \quad (4.12b)$$

$$Probability = \int_{-x}^x fProbability(y)dy \quad (4.12c)$$

where $fProbability$ is the probability density function.

Considering cause B for an observational error, it is first necessary to determine the effective magnitude limit of the telescope used for observations. The telescope limiting magnitude, TLM , is a function of the telescope's optical geometry, the camera exposure time, and the seeing conditions for a given night. For optimization purposes, however, seeing conditions were not considered. Unlike the user-specified exposure time that presides elsewhere in this project, the exposure time used for optimization is itself optimized—exposure time is set to $1/10^{th}$ the time the satellite takes to pass through the FOV. To simplify this calculation, an assumption of a circular orbit was used for calculating satellite range and velocity, v . The probability of a successful observation, in regards to the telescope limiting magnitude, was found in the following manner,

$$AM = \cot\left(\frac{v}{R}\right) \quad (4.13a)$$

$$ET = \frac{FOV}{10AM} \quad (4.13b)$$

$$TLM = 10 + 1.25 \log(ET) - DMD + 2.1715 \log\left(D_T \sqrt{\frac{SFR \times FR}{10}}\right) \quad (4.13c)$$

$$fProbability(y) = \frac{\exp\left(-\frac{1}{2}\left(\frac{y-M_{sat}}{1.0993}\right)^2\right)}{1.0993\sqrt{(2\pi)}} \quad (4.13d)$$

$$Probability = \int_{-20}^{TLM} fProbability(y) dy \quad (4.13e)$$

where AM is the apparent motion of the satellite, ET is exposure time, and DMD is the desired magnitude difference between the theoretical telescope limiting magnitude and the limiting magnitude used for optimization. DMD was used as a means to apply margin to the optimization, and was set to a value of 1 magnitude. A mean magnitude prediction error of 0.95 and an error standard deviation of 0.75, found from the results of the photometry analysis, give a probability density function with a standard deviation of 1.07. Both Eq. 4.12a and 4.13a utilized

Table 4.2: Fmincon Optimizer Constraints.

Constraint	Minimum	Maximum	Initial Value
Telescope Diameter	3 in.	20 in.	12 in.
Focal Reduction	0.2	1	0.6
Telescope Length	14 in.	34.5 in.	26 in.
Focal Ratio	5	20	10

relationships determined analytically with Sky & Telescope’s Scope Calculator utility[2].

The product of these two probability values was the value-of-interest for the optimizer—fmincon attempts to minimize the value with respect to the constraints on the system. These constraints are listed in Table 4.2, as well as the initial guessed values for the optimized variables. The variable limits were chosen based on the common dimensions seen in amateur-sized telescopes priced under ~\$10,000. Various test cases will be shown in the following chapter.

CHAPTER 5

RESULTS AND DISCUSSION

From February to May, observations were made on 29 separate nights, providing a total of 186 individual attempts at imaging a satellite. This does not include at least a dozen planned nights of operation, upon which either weather or a scheduling conflict (observatory was shared between several research projects) prevented any observations. Of the individual attempts conducted, 96 were successful—22 different satellites were captured and are shown through STK in Figure 5.1. Due to their large number and relatively high altitudes, Loral’s Globalstar spacecraft made for ideal and consistent targets; Globalstar spacecraft accounted for 17 of the satellites captured.

Whereas 96 successful captures could be used for photometric analysis, only a fraction were suitable for astrometry. The requisite observational data for orbit determination consists of three consecutive angle sets, hereto known as “trio-captures”. Over the course of this study, 7 distinct, single trio-captures were acquired. To enable supplementary use of EKF or LS, at least two consecutive trio-captures are required—three double trio-captures were obtained. As for analysis of temporal TLE error, observations of satellites obtained on consecutive days are needed—9 satellites were captured multiple times during a 2-6 day period, pertaining to a total of 42 observations. Several image sets can be seen in Appendix 7.1.

Figure 5.1: Successfully Observed Satellites listed within STK Scenario.

5.1 Orbit Determination

To present the results of the various orbit determination methods, direct comparisons are made between the determined COE's and the actual, true COE's. Truth data is gathered from the original TLE's used for orbit propagation, whereupon it is converted from mean to osculating elements by way of Kwok's method (Vallado 9.9.1), handled by the code **mean2osc.m**[29]. Table 5.1 presents the results of three of the single trio-captures—results of the remaining captures can be found in Table 7.4 of Appendix 7.1. The uncertainty values for the double-r method, shown in Table 5.1, arise from the pixel-selection method—to analyze this error, a simple Monte-Carlo method was used, the details of which are given in Appendix 7.6. Uncertainty values were only given for the double-r method, as the differences in results of the various orbit determination methods (differences in both the COE's and COE uncertainties) were found to be insignificant. For this same reason, only the double-r method was used for the testing of the EKF

Table 5.1: Single Pass Orbit Determination Results.

Satellite	Source	Epoch(UT)	A(km)	e	i ($^{\circ}$)	$\Omega(^{\circ})$	$\omega(^{\circ})$
Globalstar M003	TLE	3/25 18:53	8001.4	9.275E-4	52.02	299.16	127.39
	Vallado Gauss		8666.3	6.646E-2	51.69	300.26	147.61
	Curtis Gauss		8618.3	6.548E-2	51.85	300.45	146.89
	double-r		8618.3 ± 49.2	6.547E-2 $\pm 0.492\text{E-}2$	51.85 ± 0.02	300.45 ± 9.5	146.89 ± 0.86
Globalstar M031	TLE	4/14 3:56	7856.9	3.086E-4	52.00	329.54	145.10
	Vallado Gauss		7050.9	1.058E-1	52.29	329.95	324.56
	Curtis Gauss		7044.6	1.044E-1	52.39	330.06	324.67
	double-r		7044.6 ± 4.6	1.044E-1 $\pm 0.007\text{E-}1$	52.39 ± 0.01	330.06 ± 10.55	324.67 ± 0.09
Globalstar M036	TLE	4/20 11:06	8121.7	1.379E-3	52.00	64.18	180.56
	Vallado Gauss		8998.4	8.714E-2	52.51	62.19	112.37
	Curtis Gauss		8924.7	8.414E-2	52.35	62.06	112.95
	double-r		8924.7 ± 10.2	8.414E-2 $\pm 0.097\text{E-}2$	52.35 ± 0.004	62.06 ± 7.8	112.95 ± 0.07

and LS.

In an attempt to gain insight into the source of error responsible for the COE error, the determined and actual state vectors are compared in Table 5.2. Coefficient of determination, R^2 , values were calculated to reveal any correlation between the state error and the uncertainty in the states (proportional to the states)—large correlation could be an indication that the Monte Carlo error analysis, while correct regarding the sources of error, applied insufficient levels of measurement error. The varied R^2 results indicate that it is likely that additional, unknown sources of measurement or process error are responsible for the state errors. Equations used to derived the COE's from the state vectors are shown in Appendix 7.4, in **RV2COE2.m**, and may be helpful in determining the cause of error in the orbit determination results.

Also apparent in Table 5.2, is an imbalance between the errors in velocity vectors and the errors in the radius vectors—velocity vectors routinely contained larger relative error. For nine cases of single-pass orbit determination (including the first three used in the multi-arc orbit determination), the determined COE and state vector errors were tested for correlation with the use of Minitab. The resulting Pearson correlation coefficients, and their respective p-values, are given

Table 5.2: Single Pass Orbit Determination Results cont.

Satellite	Source	r_x (km)	r_y (km)	r_z (km)	v_x (km/s)	v_y (km/s)	v_z (km/s)
Globalstar M003	TLE	-860.85	7049.60	3696.95	-5.1443	1.7407	-4.5052
	double-r	-813.03 ± 1.36	7104.22 ± 4.77	3697.34 ± 1.63	-5.2637 ± 0.0081	1.8140 ± 0.0098	-4.6311 ± 0.0111
	Error (%)	5.56	0.77	0.01	2.32	4.21	2.79
	R^2	0.36					
Globalstar M031	TLE	-4179.73	5643.08	3511.53	-5.3220	-1.0575	-4.6223
	double-r	-4160.57 ± 0.45	5553.84 ± 0.79	3533.78 ± 0.18	-5.0887 ± 0.0017	-1.0255 ± 0.0014	-4.4279 ± 0.0016
	Error (%)	0.46	1.58	0.63	4.39	3.03	4.20
	R^2	0.17					
Globalstar M036	TLE	-5648.03	-1100.79	5716.33	-1.2410	-6.4438	-2.4736
	double-r	-5652.90 ± 0.15	-1103.76 ± 0.74	5764.17 ± 0.82	-1.3061 ± 0.0010	-6.5926 ± 0.0027	-2.5111 ± 0.0010
	Error (%)	0.09	0.27	0.84	5.25	2.31	1.51
	R^2	0.40					

for correlations below a significance level of 0.15. Pearson correlation coefficients are located on a scale from -1 to 1: -1 indicates perfect inverse correlation, 1 indicates directly-proportional correlation, and 0 indicates no correlation. A p-value of 0.1 indicates that there is a 10% chance that the correlation is coincidental. Because the number of data points are few, the results shown in Table 5.3 are not definitive, and should not be considered representative of all angles-only observations. Because it is not immediately clear how state vector errors translate into measurement errors, or explain the shortfalls of the observation process, two observational parameters were also tested for correlation: arc distance, Δt (time duration covered by a trio-capture), and range coverage, ΔR (difference in minimum and maximum range within a trio-capture). As correlation likely exists between the i , a , e , and ω errors, and error is correlated between many of the state vector dimensions, it may be the case that additional correlation between the COE's and states exist, but are not reflected by the data. The only COE-state error correlations at a significance lower than 0.15 are those between r_y and e , and r_y and i . However, if correlations can be treated as syllogistic (if A implies B and B implies C, then A implies C), then it can be hypothesized that increases in arc distance and range coverage effectively decrease the errors in i , a , e , and

Table 5.3: Correlation between Determined COE and State Vector Errors, and Observational Parameters (Obs.).

Type	Element 1	Element 2	Correlation	p-value
COE-COE	i	a	0.91	0.001
	i	ω	0.62	0.072
	i	e	0.62	0.078
	e	a	0.55	0.122
	e	ω	0.53	0.141
State-State	$\mathbf{r}_z$	$\mathbf{v}_x$	0.91	0.001
	$\mathbf{r}_y$	$\mathbf{r}_z$	0.77	0.016
	$\mathbf{v}_x$	$\mathbf{v}_z$	0.75	0.02
	$\mathbf{r}_y$	$\mathbf{v}_x$	0.58	0.101
	$\mathbf{r}_z$	$\mathbf{v}_z$	0.58	0.101
COE-State	e	$\mathbf{r}_y$	0.68	0.043
	i	$\mathbf{r}_y$	0.58	0.1
State-Obs.	$\Delta \mathbf{R}$	$\mathbf{v}_z$	-0.81	0.008
	$\Delta \mathbf{t}$	$\mathbf{r}_z$	-0.58	0.1
	$\Delta \mathbf{R}$	$\mathbf{v}_x$	-0.58	0.105
	$\Delta \mathbf{t}$	$\mathbf{v}_z$	-0.55	0.124
	$\Delta \mathbf{t}$	$\mathbf{v}_x$	-0.54	0.131

ω .

Additional observational data could provide clearer relationships between the observational parameters and determined COE error, and could in turn, provide beneficial modifications to satellite observation techniques. Based on the data presented in this paper, and assuming syllogistic correlations, it is theorized that observing a satellite for as long of an arc as possible, and near zenith and horizon locations, would achieve the lowest levels of orbit determination error.

A comparison of the mean run times for the three orbit determination methods is shown in Table 5.4. The methods were ran 50 times for each method and for each trio-capture (referred to by the date of the observation). Although Vallado's Gauss method code consistently converged the quickest of the three, and Curtis' Gauss converged the slowest, the differences in run times are considered negligible for the purposes of this project. However, if such codes were used for

Table 5.4: Comparison of Performance Times for Orbit Determination Methods.

Method	3/25	3/27	4/12	4/14 3:56	4/14 4:57	4/20	Mean
Vallado Gauss	0.00676	0.00672	0.0065	0.00656	0.00646	0.00674	0.00662
Curtis Gauss	0.0113	0.0113	0.0113	0.0112	0.0112	0.0117	0.0113
double-r	0.00812	0.00832	0.00834	0.00832	0.00868	0.00834	0.00835

a very large batch of observations, for instance 20,000 for a statistical study, the difference would be substantial. Approximately 1 minute of run time would be saved selecting the double-r method over Curtis’ Gauss, even though the two methods converge to the same level of accuracy. It is prudent, however, to recall that the double-r method is capable of handling observations separated by days, whereas Gauss’ method is limited to observations occurring within 5-10 minutes of each other[29].

For the case of the double trio-captures, both the results of the initial single-pass orbit determination and the post-EKF and -LS results are shown in Table 5.5. Of the 13 observed trio sets, 1 failed to converge—accuracy statistics for the successfully-converged sets are shown in Table 5.6. For two of the three multi-arc cases (Globalstar M032 and Globalstar M008), the second angle-sets converged to unrealistic orbits. Because of this, the EKF and LS methods experienced significant error when the state vector results of the first angle-set were used as the initial state epoch. To circumvent this problem, the source TLE’s were propagated forward to the time of the first observation, guaranteeing an accurate initial state. For the remaining multi-arc case (Globalstar M058), the angles-only orbit determination results of the first angle-set were utilized as the initial state epoch. Therefore two scenarios are presented: an a priori orbit determination, and an a posteriori orbit determination built upon a preexisting TLE-derived state.

As seen in the tables of this section, large errors exist primarily in the determi-

Table 5.5: Double Pass Orbit Determination Results.

Satellite	Source	Epoch (UT)	A (km)	e	i ($^\circ$)	Ω ($^\circ$)	ω ($^\circ$)
Globalstar M058	TLE	5/9 3:37	8090.2	1.90E-4	51.996	35.54	27.07
	Double-R	5/11 3:44	7966.0	1.38E-2	52.042	29.97	306.39
	EKF	5/11 5:47	7974.8	1.33E-2	52.042	29.97	322.30
	LS		8090.1	1.66E-3	52.174	29.77	185.72
Globalstar M032	TLE	5/9 19:31	7791.7	1.65E-4	52.002	54.39	47.32
	Double-R	5/11 4:14	8539.2	8.24E-2	52.460	50.08	113.62
	EKF	5/11 6:10	7789.1	8.45E-4	51.994	50.22	62.96
	LS		7790.2	1.10E-2	52.403	49.29	203.74
Globalstar M008	TLE	5/9 4:03	7902.1	2.12E-4	51.996	56.69	155.37
	Double-R	5/11 4:27	7281.8	7.76E-2	51.634	50.80	298.18
	EKF	5/11 6:24	7900.3	9.13E-4	51.991	50.83	77.94
	LS		7900.8	5.74E-3	52.150	50.39	200.80

Table 5.6: Mean Accuracy of the Orbit Determination Methods (This table shows the mean percent error for the single trio-capture, double-r results, as well as the mean increase (positive), or decrease (negative), in the accuracy (% change in error) once the Kalman filter is applied. The error values are 1σ).

Method	A (km)	e	i ($^\circ$)	Ω ($^\circ$)	ω ($^\circ$)
double-r Error	486 $\pm$ 298	0.055 $\pm$ 0.035	0.21 $\pm$ 0.18	3.0 $\pm$ 2.1	73 $\pm$ 55
EKF Improvement	457 $\pm$ 394	0.053 $\pm$ 0.045	0.27 $\pm$ 0.24	0.06 $\pm$ 0.07	44.0 $\pm$ 25.4
LS Improvement	496 $\pm$ 329	0.052 $\pm$ 0.034	0.04 $\pm$ 0.17	-0.47 $\pm$ 0.30	-23.6 $\pm$ 104.9

nation of the satellites' semi-major axis, eccentricity, and argument of perigee. It is hypothesized that these errors are due in part to the relatively short arc of the satellites' orbits that are observed; the angle-sets are separated by 1.5–8.5 minutes and cover 6.6%–9.5% of the satellites' orbital periods. Although large error values accompany the results of the EKF and LS (only three data sets were used), it is apparent that the use of these techniques at least partially improved the accuracy of the orbit determination—3+ trio-captures would certainly continue to improve the COE's, and such an exercise could prove worthwhile at demonstrating the potential for high-fidelity orbit determination. From the limited data, it is apparent that the extended Kalman filter provided marginally better improvement than the least squares method—the EKF also operated faster than the LS by about an order of magnitude. For the cases in which the source TLE was used for generating the initial state, it is apparent that these methods could

Table 5.7: Circularization of Orbit Determination Results with use of Fmincon (Globalstar M031).

Source	A (km)	e	i ($^{\circ}$)	Ω ($^{\circ}$)	ω ($^{\circ}$)	θ ($^{\circ}$)
Original Determined	7041.7	1.05E-1	52.39	330.1	324.67	180.35
Optimized	7779.6	1.27E-5	52.39	330.1	109.29	35.73
TLE	7856.9	3.09E-4	52.00	329.5	145.10	357.93

potentially be used to continuously update the COE’s with reasonable accuracy. Additional improvement could focus on providing better estimates of covariance and process noise matrices, to potentially increase the effectiveness of the Kalman filter.

In a final attempt at improving the results of orbit determination, sans additional EKF iterations, MATLAB’s `fmincon` function was used to effectively “circularize” the generated orbit. By constraining the optimizer to maintain angular momentum to within 10% of its initial value, and optimizing for a minimum eccentricity value, a much more accurate set of COE’s were generated. This process, handled by the code `orbit_det_withOPT.m`, is predicated upon the knowledge that the satellite of interest has a very low eccentricity (0.0001), limiting its potential use for a priori blind-search observations. Results of this optimization exercise are shown in Table 5.7. From this example, it is clear that such a simple optimization of the orbit’s eccentricity can greatly improve certain orbital elements (e.g. semi-major axis, argument of perigee, and true anomaly) while holding the remaining elements constant.

5.2 Temporal TLE Error Growth

Nine satellites were observed multiple times across several days—the calculated, modulated cross-track errors are shown in Fig. 5.2. The level of CTE seen in the acquired observations far exceeds that shown in Fig. 2.4 taken from Ma-

Figure 5.2: Cross-Track Error Results.

son 2009[15]. For a 1000 km altitude satellite passing directly overhead, a 15 km cross-track error corresponds to a $\sim 0.86^\circ$ angular error. To provide a CTE model, a linear regressive fit to each of the satellites' growth was applied, resulting in Eq. 5.1a.

$$\text{Mean} : CTE = 0.054 \times TLEage + 4.44$$

$$\text{Mean} + 1\sigma : CTE = 0.204 \times TLEage + 7.18 \quad (5.1a)$$

Because the uncertainty in calculated CTE—uncertainty due to the pixel selection process (shown as error bars in Fig. 5.2)—is too low to account for the excessive CTE, it is likely that some unknown source of error might be responsible. However, in the event that the results accurately describe the level of CTE that should be expected when using the observation techniques discussed in this

Figure 5.3: Along-Track Error Results.

paper, it remains relevant to apply Eq. 5.1a to the telescope optimization portion of this project.

As for the case of along-track error, the results were much closer to the levels predicted in Fig. 2.4. But due to the unknown levels of error within the timestamps used to calculate ATE, and the assumptions mentioned in Section 4.2.3, the validity of these results are questionable. That being said, these results may warrant a reduction in the recording time margin (± 30 seconds) used in this project. Considering the case of the lowest-altitude satellite observed during this project—IMS-1 at 627 km—an ATE of 40 km present during an overhead pass, would result in a timing error of only 5.3 seconds. For a 1000 km altitude satellite, a 15 km along-track error corresponds to a ~ 2 second timing error. The calculated ATE's are shown in Fig. 5.3.

5.3 Magnitude Prediction Model

To evaluate the accuracy of the magnitude prediction model, it is compared to the value predicted by N2YO. Because the magnitude prediction model produces both a low and high estimate, the average of the two values will be used for the purpose of evaluation. As the N2YO prediction is provided irrespective of the satellite's elevation in the night sky, it is shifted according to the change in observer-satellite range and atmospheric extinction. Additional analysis of the telescope limiting magnitude estimation, provided by Eq. 4.13a, is also included in the results shown in Fig. 5.4. The variation seen in the telescope limiting magnitude is caused by a term left out of Eq. 4.13a, a reduction proportional to the percent of the moon illuminated, found to be a maximum of -2.9 V magnitude reduction for a full moon[21]. But because the effect of the moon is also dependent upon the unknown separation distance between the satellite and the moon, the maximum magnitude reduction is estimated to be -2 for the purpose of this paper. The margin of error given to the photometric results are the 1σ variation seen within the set of images for a single satellite observation. This variation is likely due to uncertainties in the calculation of noise background within each image, as any error in the noise background will disproportionately affect the amount of light attributed to the satellite vs. the companion star.

A zoomed view of the last 15 observations are shown in Fig. 5.5. Overall, the magnitude prediction model outperformed N2YO; on average, the prediction model used by this paper had an error of 0.93 magnitudes ($\sigma=0.75$), while the N2YO predictions were off by 1.14 magnitudes ($\sigma=0.79$). Furthermore, of the 96 observations, this paper's model provided a prediction within the photometric margin of error 21 times, as opposed to 14 times for N2YO. These results demonstrate the validity of developing simple magnitude prediction models, the

Figure 5.4: Magnitude Prediction Model Results.

likes of which can surpass online resources such as N2YO or Heavens Above. It is also hypothesized that the prediction model would have likely performed better had dimension values for the Globalstar satellites (a majority of the successful observations) been known.

An additional takeaway from this section, is the extent to which the theoretical telescope limiting magnitude accurately describes the actual limiting magnitude. But due to the fact that failed observation attempts can't be attributed entirely to insufficient satellite brightness, as a pointing error may be responsible, it is difficult to provide a complete statistical analysis. Of the 96 successful observations, the upper error bars of 16 observations came within 0.5 magnitudes of the predicted limiting magnitude of the telescope. Considering that the pragmatic limiting magnitude is lower than the theoretical limit, it is likely that a reduction

Figure 5.5: Magnitude Prediction Model Results – Last 15 Observations.

factor such as the camera exposure, focal reduction, or moon effect, is excessive. With this in mind, and considering the predictions made in Table 2.1, the magnitude limit seen in Fig. 5.4 suggests that a normal cubesat is likely observable under ideal conditions (e.g. new moon, or low satellite geometrical phase angle).

5.4 Telescope Optimization

Three test cases were presented to the telescope fmincon optimizer: CP5 cubesat, IMS-1 ($L_C=0.8485$ m and $h=627$ km), and Servis-2. For each of these satellites, the optimizer was ran for both levels of temporal error growth shown in Fig. 2.4 (slow) and Eq. 5.1a (fast). The results of these test cases are given in Table 5.8—each of the optimized telescopes used a 0.2 focal reducer. Visualizations of the optimized telescope dimensions for the IMS-1 case, along with the

Figure 5.6: Telescope Optimization — IMS-1, Slow Temporal Growth.

FOV size, are shown in Figures 5.6 and 5.7, in comparison to the Meade LX200 telescope used for observations in this project. Because all test cases involved in this section were simulated satellite passes, all magnitude predictions were calculated for an elevation of 45° , a solar phase angle of 90° , and a TLE age of 5 days.

While it appears that the probabilities of a successful capture for CP5 contradict the findings of the previous section, it is likely that the probability would increase significantly for a higher elevation observation and/or lower SPA. Having considered specific test cases, the next step involved analysis of a large, diverse population of 2400 satellites. Ranging from picosatellite classification (0.1-10 kg) to large satellites (1500-4500 kg), target objects were tested at altitudes of 400-2000 km. Fig. 5.8 and Fig. 5.9 show the results of this analysis, for the

Figure 5.7: Telescope Optimization — IMS-1, Fast Temporal Growth.

Table 5.8: Telescope Optimization for three test cases. S=slow temporal growth rate, F=fast temporal growth rate.

Satellite	L_C (m)	h (km)	D_T (in) S/F	Scope Length (in) S/F	Probability of Capture (%)	
					Optimized S/F	Meade LX200 S/F
CP5	0.1414	625	20/12	25.4/26	0.77/0.027	0.05/0.003
IMS-1	0.8485	627	20/12.6	22.2/14	89.21/24.13	62.43/0.027
Servis 2	2.6926	1207.7	20/12.6	25.4/14	99.89/54.29	98.44/14.28

Figure 5.8: Telescope Optimization — Full Population, Slow Temporal Growth. The colorbar indicates the probability of a successful observation, and the size of each marker is proportional to the mass of the satellite (within its own category)

two temporal growth rates. A linear relationship between the satellite mass and satellite characteristic length was utilized, and was calculated from 11 satellites within these mass classifications,

$$L_C = 0.0052m_a + 0.3128 \quad (5.2)$$

Satellites appear to cluster around specific sizes of telescopes, suggesting an ideal size for observing a wide range of targets. For the slow temporal growth rate, the larger satellites (Mini-Large) are best observed with either a 20 x 25.5 (f/5.74) or 12 x 26 in. (f/9.75) telescope, while a 20 x 22.2 in. (f/5.0) telescope is selected for the smaller satellites (Pico-Mini). When the fast temporal growth rate was used, clusters existed at $\sim 12\text{-}12.5 \times 14$ in. (f/5.04) for the larger satellites (Mini-Large), at 20 x 22.2 in. (f/5.0) for the smallest satellites (Pico-Nano).

Figure 5.9: Telescope Optimization — Full Population, Fast Temporal Growth. The colorbar indicates the probability of a successful observation, and the size of each marker is proportional to the mass of the satellite (within its own category).

However, many ideal dimensions for Nano and Micro-class satellites were found along the $f/5$ line connecting the two clusters. The commercial telescope most closely matching these sizes are listed in Table 5.9. Because the type of telescope often determines the focal length to telescope length ratio, a matching focal ratio does not always mean a matching telescope length. And although these telescopes match the optimized dimensions, their large size (Dobsonian type telescopes) may inhibit their ability to accomplish satellite observations.

It is clear from Table 5.8 that an optimized telescope is capable of providing a better probability of observational success, for both slow and fast rates of temporal TLE error growth. Should a researcher or institution seek a telescope for observing satellites of a certain size, the utilization of a telescope optimizer

Table 5.9: Commercial Telescopes matched to Optimization Results.

Cluster SFR	Commercial Telescope	Telescope Dimensions	Cost (\$)
f/9.75	Meade LX200	12 in. f/10	\$4500
f/5.0	Obsession 20	20 in. f/5	\$7200
f/5.04	Obsession 12.5	12.5 in. f/5	\$3500

can be both advantageous and money-saving.

CHAPTER 6

CONCLUSIONS

Given the open-ended nature of this thesis’ objective—to demonstrate that amateur telescopes can provide observational data relevant to aerospace research—it is difficult to provide a definitive, quantifiable conclusion. As much of this project’s efforts to provide relevant data are considered preliminary endeavors, the quality of data obtained is admittedly mediocre. But considering the quality of equipment used, and the relative simplicity of the observation method, the investigations within this report can be considered successful as many promising results were obtained. Although the orbit elements determined through the use of angles-only observations would not provide the accuracy necessary for guidance, navigation, and control operations, the demonstration of the extended Kalman filter shows promise. If one or more amateur telescopes could provide repeated observations of a satellite, it is likely that an accurate set of COE’s could be calculated. Furthermore, it was demonstrated that accurate COE’s could be determined through the use of TLE-derived initial states—upon which the gathered data was built upon—and through the use of state optimization (example of eccentricity minimization).

As for the temporal error growth, magnitude prediction, and telescope optimization investigations, the results speak to the question concerning the use of amateur telescopes to target low-priority objects. Given the observed temporal growth of CTE, it is evident that satellites with aging TLE’s (at least up to 6 days old) are still observable within a reasonable FOV available to an amateur

astronomer. In addition, the results of the magnitude prediction model suggest that small satellites, such as cubesats, may be viable targets under ideal observing conditions. Unfortunately, of the few attempts to image microsatellites (10-100 kg), only one was successful: the 0.6 x 0.6 m, 83 kg IMS-1 satellite. But if a cubesat were to be observed under the exact same conditions that IMS-1 was found to have an apparent magnitude of 7.5-7.7, it would be expected to be within the range of 9.5-9.7 magnitude—this prediction would be located ~ 1.4 below the telescope’s limiting magnitude.

The IMS-1 satellite is a prime example of the results seen within this paper: although the generated data is limited in quality and quantity, it has provided information that could improve and inform future research efforts. All things considered, this paper has successfully demonstrated that it is indeed feasible to generate applicable astrometric and photometric data, with the use of low-cost, amateur-sized telescopes.

Considering the preliminary nature of these applications, it is useful to act under the assumption that another student or researcher could advance and improve the investigations carried out in this project. The process, results, and analytical tools detailed in this paper act as useful precursors to future research.

6.1 Recommendations for Future Research

This section details several suggestions for future research that builds upon the material covered in this thesis—any of these recommendations might provide challenging research for a Senior Project or Master’s thesis.

Orbit Determination

As mentioned in the orbit determination section of the Results chapter, further analysis of the usefulness of the extended Kalman filter is recommended. Should 3+ consecutive satellite passes be captured, additional iterations of the EKF and LS can be initiated. Should the accuracy of the generated COE's continue to improve with each iteration, a significant step towards validation—validation that orbit determination with amateur telescopes is feasible—will have been taken. It could also be possible to demonstrate that continuous updates to a satellite's COE's, through the use of an EKF, whether or not large perturbations to a satellite's orbit could be absorbed accurately.

It may also prove worthwhile to continue the investigation into the selective optimization of the obtained COE's. Does such an a posteriori circularization provide a consistent improvement to orbit determination? Could other orbital elements apart from eccentricity act as a variable from which to optimize the orbit?

Optical Fence

Given the difficulty in observing such objects as cubesats that push the boundaries of telescopes' limiting magnitudes, a multiple-telescope optical fence may provide a solution. Because telescope magnification power and FOV are inversely related, it is likely that a telescope designed to observe low-magnitude objects may suffer from failed observations caused by CTE. However, if two telescopes could be aligned perpendicular to a satellite's projected direction of movement, with some crossover, an extended FOV can be obtained. In anticipation of such an operation, the MATLAB code **scopefence.m** was developed to, when given a satellite's projected path and N number of telescopes, provide N RA/Dec target

angles for the telescopes. As a 14 in. Celestron telescope is housed next door to the Cal Poly Dome Observatory, an attempt at enacting an optical fence is a challenging but real possibility. Additional operations could include using the optical fence to identify any significant CTE as the satellite passes through the first observed point of its arc—the telescope best suited for accurate slewing could then correct for the CTE and continue subsequent observations at the second and third points of the satellite’s arc. This would permit operators to devote less resources to controlling the secondary telescopes.

Magnitude Prediction Model/Photometry

There are a few possible additions to the photometric analysis and the magnitude prediction model that could improve the fidelity of the associated results. The magnitude prediction model would benefit greatly from a more accurate sizing model for satellites of unknown dimensions. A simple approach could focus on providing a statistical relationship between the size and mass of satellites of known dimensions, for a greater number of satellites than the sample used for this project (26). A more involved approach could involve considerations of satellite shapes, densities, and extended objects, such as antennas or solar panels. Further possibilities include providing better estimates for the satellite geometrical phase angle (potentially based on assumptions of the satellite’s method of control, e.g. three-axis stabilized) and albedo (satellite’s surface materials).

Improvement of the photometric analysis could be accomplished by accounting for companion star color, and the telescopic effect of vignetting: the reduction in brightness near the edges of an image. Because atmospheric extinction affects stars of different colors by varying amounts, a small error arises in the calculation of a satellite’s apparent magnitude. And as a satellite’s brightness is found

from multiple frames within an image set, some in which the satellite is located near the periphery of the frame, any vignetting would negatively affect the mean apparent magnitude. Quantifying the affect of vignetting—if it is found to be an issue—would dictate a change to the image analysis.

Miscellaneous Improvements to Observation Process

Several improvements to the equipment and process used to observe satellites could improve the functionality of the telescope system. As analysis of ATE was limited by a lack of accurate timestamps, the inclusion of a GPS time-inserter could remedy this issue. It may also be beneficial to save the images generated from the camera in FITS format, as opposed to JPEG. This change may allow more traditional image reduction and analysis used by astronomers, with applications such as Image Reduction and Analysis Facility (IRAF) and SAOImage DS9. Switching to FITS format could potentially automate the photometry analysis.

Another potential improvement includes the use of SGP4 propagation within MATLAB, to remove STK entirely from the observation preparation process. This upgrade was previously attempted, but STK-level accuracy was not achieved. If a new attempt were to be successful, the pool of satellite target candidates could be increased (as much preparation time would be saved by bypassing STK). Presenting a larger number of potential targets could improve aspects of observational research.

For the observations in which a focal reducer was used, the probability of secondary companion stars increased significantly. It may prove worthwhile to utilize these stars as additional reference points for data analysis. In the process of RA/Dec angle determination, the secondary star(s) can be used to determine the orientation of the RA/Dec plane, thereby validating the method used in this

project (alignment of the predicted and actual direction of satellite movement). Furthermore, the additional stars can be used to increase the accuracy of the comparative photometry.

CHAPTER 7

APPENDICES

7.1 Selected Image Sets & Raw Data

Table 7.1: Observational Results — Astrometry Data (Observational data including the predicted target (Tar.) and actual determined (Det.) RA/Dec angles).

Satellite	Tar. RA	Tar. Dec	Det. RA	Det. Dec	CTE (km)	ATE (km)
Meteor 3-5	186.174	23.954	186.296	24.044	2.880	6.772
Globalstar M003	88.509	-15.755	88.707	-15.730	6.712	9.169
Fengyun 1B	86.966	-13.443	87.322	-13.385	8.818	0.492
Fengyun 1B	32.959	48.504	33.316	48.707	10.066	7.357
Geosat	73.587	0.430	73.857	0.481	5.548	6.226
Globalstar M051	204.494	66.744	204.463	66.751	1.398	3.189
Globalstar M003	85.298	-15.289	85.383	-15.291	2.441	7.009
Geosat	135.052	-27.453	135.464	-27.492	9.326	13.143
Globalstar M051	206.931	62.883	207.442	62.956	26.574	5.302
Globalstar M003	30.193	23.345	30.381	23.525	12.636	5.028
Globalstar M003	65.146	0.757	65.134	0.774	0.226	2.068
Globalstar M003	99.858	-30.490	99.976	-30.440	5.971	3.228
Fengyun 1B	101.013	-31.106	101.293	-31.148	7.507	12.949
Fengyun 1B	49.237	43.960	49.599	44.124	8.761	3.419
Globalstar M023	54.065	34.313	54.191	34.406	4.688	8.326
Geosat	77.885	-10.983	78.103	-10.953	5.754	3.118
Globalstar M003	72.267	-8.171	72.479	-8.074	9.183	1.047

Continued...

Satellite	Tar. RA	Tar. Dec	Det. RA	Det. Dec	CTE (km)	ATE (km)
Globalstar M023	155.706	-4.020	155.968	-4.049	7.737	4.282
Geosat	123.899	-30.850	124.041	-31.009	0.733	2.990
Geosat	53.546	44.797	53.833	44.798	2.977	0.373
Fengyun 1B	104.893	-29.993	105.079	-30.090	4.068	8.760
Globalstar M023	58.632	31.006	58.886	31.141	7.732	4.102
Globalstar M023	102.754	5.111	102.959	5.202	5.965	0.229
Globalstar M023	141.455	-28.617	141.600	-28.710	1.252	4.585
Globalstar M031	53.513	31.950	53.874	32.082	9.315	4.919
Globalstar M031	96.498	13.178	96.634	13.163	2.185	1.849
Globalstar M031	133.642	-18.759	133.739	-18.770	2.148	2.767
Geosat	70.309	49.900	70.718	49.952	3.646	14.983
Globalstar M043	105.021	54.850	105.141	54.828	1.851	9.945
Geosat	182.775	64.577	183.276	64.420	7.649	6.999
Globalstar M043	86.500	58.704	86.535	58.915	8.648	4.971
Globalstar M043	184.914	38.003	185.010	37.998	2.698	1.480
Globalstar M031	65.695	16.731	65.907	16.886	11.720	2.910

Continued...

Figure 7.1: Observation of IMS-1. Images taken of the Indian Remote Sensing satellite, IMS-1, on April 17th, 2013, at 21:18 PST. At 83 kg, this satellite was the least massive of all objects observed.

Satellite	Tar. RA	Tar. Dec	Det. RA	Det. Dec	CTE (km)	ATE (km)
Globalstar M031	92.903	-3.695	93.223	-3.484	15.463	4.335
Globalstar M031	120.966	-29.104	120.977	-29.167	1.874	5.563
Cosmos 1900	118.142	10.713	118.409	10.606	4.520	8.256
Globalstar M043	72.079	61.455	72.220	61.468	4.075	9.672
Globalstar M043	185.364	47.195	185.572	47.213	7.042	2.785
Globalstar M043	205.087	17.857	205.298	17.787	7.473	2.320
Globalstar M055	114.790	-29.177	114.966	-29.281	9.930	3.140
Globalstar M055	215.698	37.230	215.836	37.152	4.132	2.464
Oscar 36	199.286	50.548	199.588	50.279	5.361	6.240
IMS-1	213.576	44.384	213.897	44.305	5.170	7.124
Globalstar M043	65.440	47.607	65.585	47.611	3.864	9.656
Globalstar M043	102.123	53.008	102.245	53.069	2.892	3.429
Globalstar M043	180.155	12.474	180.350	12.435	5.232	3.890
Globalstar M055	271.488	65.144	271.074	65.209	16.740	4.243
Globalstar M053	128.166	-15.724	128.418	-15.804	12.269	3.268
Globalstar M053	206.844	42.096	206.952	41.944	7.620	3.268
Globalstar M053	206.266	34.691	206.389	34.561	7.805	3.710
Globalstar M036	251.705	54.436	251.600	54.519	6.026	1.680
Globalstar M008	259.143	62.526	259.281	62.687	8.885	0.979
Globalstar M053	270.152	63.556	270.081	63.480	5.473	0.977
Globalstar M036	73.981	58.002	74.402	58.089	21.631	11.370
Globalstar M036	245.715	62.511	245.874	62.570	6.735	5.284
Globalstar M036	252.234	41.760	252.246	41.735	0.294	4.129
Globalstar M050	239.421	54.776	239.485	54.834	2.949	2.819
Globalstar M036	235.805	52.908	236.339	52.705	23.926	13.437
Globalstar M026	150.700	-33.134	151.024	-33.343	17.493	13.718
Globalstar M026	221.879	21.787	222.084	21.599	11.544	3.212
Globalstar M028	137.310	-30.269	137.653	-30.489	17.739	3.170
Globalstar M028	216.525	28.650	216.705	28.434	9.664	1.997
Globalstar M022	270.572	58.563	270.674	58.716	6.635	3.418
Globalstar M026	273.679	56.609	273.545	56.637	3.659	2.080
Globalstar M028	290.026	56.900	290.021	57.019	1.859	0.039
Globalstar M052	90.024	45.209	90.319	45.011	14.607	11.614
Globalstar M052	112.536	50.149	112.726	49.923	9.177	0.210
Globalstar M052	145.301	48.769	145.462	48.877	4.240	2.602
Globalstar M022	293.052	60.789	292.840	60.983	11.762	9.529
Globalstar M022	291.321	44.920	291.661	45.015	18.719	6.457
Globalstar M058	62.246	53.344	62.065	53.371	8.934	0.656
Globalstar M058	190.502	55.665	190.293	55.845	2.039	1.372
Globalstar M058	208.357	33.803	208.218	33.985	1.688	0.258
Globalstar M058	218.230	13.856	218.144	14.024	0.211	1.598
Globalstar M032	60.550	64.399	60.254	64.196	12.593	7.972
Globalstar M032	242.155	63.417	242.513	63.615	12.921	1.035
Globalstar M032	246.076	37.012	246.009	37.172	1.982	5.469
Globalstar M008	66.426	60.305	66.505	60.203	4.895	3.443
Globalstar M008	241.459	43.707	241.237	43.861	6.942	1.587
Globalstar M008	246.796	21.185	246.882	21.415	6.390	1.536
Globalstar M027	124.329	1.133	124.301	0.939	2.494	1.261
Globalstar M027	133.923	27.879	133.814	27.728	1.803	3.673
Globalstar M027	296.090	55.840	296.193	56.095	9.915	3.156
Globalstar M058	112.087	29.343	111.941	29.528	7.672	6.085
Globalstar M058	127.343	24.324	127.243	24.471	4.270	2.144
Globalstar M058	149.739	10.947	149.566	10.996	2.779	3.118
Globalstar M058	174.182	-10.425	173.968	-10.398	5.123	4.220
Globalstar M032	112.089	42.802	111.907	42.662	4.344	1.717
Globalstar M032	142.371	43.193	142.174	43.239	0.301	3.790
Globalstar M032	167.169	36.444	166.998	36.454	1.918	4.168
Globalstar M008	114.298	40.110	114.150	39.919	8.302	2.768
Globalstar M008	140.285	39.586	140.143	39.784	6.122	2.883
Globalstar M008	174.376	27.387	174.285	27.593	3.498	3.188

Figure 7.2: Observation of Globalstar M035. Images taken of Globalstar M035, on April 20th, 2013, at 23:03 PST. At an apparent magnitude of approximately 11.2, this was the dimmest object observed. For the purpose of presentation, every second image from the set was removed, and the brightness and contrast values for the images increased.

Figure 7.3: Observation of Globalstar M002. Images taken of Globalstar M002, on March 12th, 2013, at 21:41 PST. At an apparent magnitude of approximately 4.93, this was the brightest object observed. Every second image from the set was removed.

Figure 7.4: Observation of Globalstar M058. Images taken of Globalstar M058, on May 10th, 2013, at 20:49 PST. This observation had a notably low cross-track error of only 0.21 km—every second image from the set was removed.

Table 7.2: Observational Results — Photometry Data Part 1 (Observational data relevant to photometric analysis, where D1 and D2 are dimensions of each satellite in meters (if listed as 0, dimension was not known), and TLE is the age of the reference TLE).

Satellite	FOV(')	el (°)	R (km)	TLE(hr)	D1	D2	Mass(kg)	SPA(°)
Globalstar M049	4.01	44.30	2692.00	4.79	0	0	450	89.20
Globalstar M003	12.139	33.90	2458.10	8.06	0	0	450	111.10
Globalstar M002	12.139	35.80	2676.60	8.9	0	0	450	83.70
Meteor 3-5	12.139	25.30	2204.20	7.99	12.7	6.5	2150	25.00
Globalstar M003	12.139	32.30	2516.70	8.16	0	0	450	86.60
Fengyun 1B	12.139	32.20	1469.30	33.81	1.8	1.4	881	95.10
Fengyun 1B	12.139	26.90	1636.80	33.89	1.8	1.4	881	125.80
Geosat	12.139	31.50	1310.60	8.08	3	0	635	108.90
Globalstar M051	12.139	38.90	2689.20	8.82	0	0	450	69.60
Globalstar M003	12.139	32.50	2507.90	31.91	0	0	450	97.40
Geosat	12.139	24.50	1527.00	31.35	3	0	635	53.00
Globalstar M051	12.139	31.90	2965.40	31.9	0	0	450	66.70
Globalstar M003	12.139	21.60	3041.33	55.54	0	0	450	146.90
Globalstar M003	12.139	35.36	2401.43	55.61	0	0	450	119.10
Globalstar M003	12.139	23.19	2943.16	55.71	0	0	450	84.20
Fengyun 1B	12.139	21.08	1874.94	81.73	1.8	1.4	881	83.28
Fengyun 1B	12.139	35.28	1384.10	81.83	1.8	1.4	881	122.11
Globalstar M023	12.139	29.46	2341.58	23.6	0	0	450	123.48
Geosat	12.139	20.65	1686.86	56.42	3	0	635	105.70
Globalstar M003	12.139	34.81	2420.25	79.39	0	0	450	112.29
Globalstar M023	12.139	34.79	2139.70	46.5	0	0	450	29.50
Geosat	12.139	23.81	1554.37	79.78	3	0	635	64.40
Geosat	12.139	36.18	1199.28	79.86	3	0	635	119.93
Fengyun 1B	12.139	22.79	1795.75	129.68	1.8	1.4	881	81.07
Globalstar M023	12.139	35.56	2115.32	71.12	0	0	450	123.00
Globalstar M023	12.139	53.11	1694.06	71.18	0	0	450	83.62
Globalstar M023	12.139	24.49	2563.68	71.27	0	0	450	49.72
Globalstar M031	12.139	23.55	2702.65	8.85	0	0	450	141.75
Globalstar M031	12.139	47.45	1867.95	8.93	0	0	450	105.72
Globalstar M031	12.139	35.73	2181.59	9	0	0	450	65.59
Geosat	12.139	26.04	1479.89	9.97	3	0	635	121.97
Globalstar M043	12.139	42.25	2687.51	11.2	0	0	450	100.47
Geosat	12.139	55.50	915.83	33.34	3	0	635	75.04
Globalstar M043	12.139	36.88	2875.51	34.62	0	0	450	110.71
Globalstar M043	12.139	72.88	2113.58	34.73	0	0	450	49.66
Globalstar M031	12.139	22.27	2769.35	57	0	0	450	137.46
Globalstar M031	12.139	30.52	2375.07	57.07	0	0	450	108.71
Globalstar M031	12.139	22.60	2745.32	57.13	0	0	450	78.13
Cosmos 1900	12.139	51.43	920.28	14.36	0	0	1500	86.38
Globalstar M043	12.139	34.13	2985.84	58.05	0	0	450	116.79
Globalstar M043	12.139	65.18	2196.08	58.15	0	0	450	58.70
Globalstar M043	12.139	44.05	2628.25	58.22	0	0	450	27.43
Globalstar M055	12.139	22.05	2894.03	14.97	0	0	450	86.00
Globalstar M055	12.139	30.85	2478.26	15.17	0	0	450	48.90
Oscar 36	12.139	49.94	801.43	14.13	1.1	0.885	325	61.72

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Satellite	FOV(°)	el (°)	R(km)	TLE(hr)	D1	D2	Mass(kg)	SPA(°)
IMS-1	12.139	41.80	906.44	9.91	0.6	0.6	83	55.65
Globalstar M043	12.139	24.38	3468.03	153.87	0	0	450	130.76
Globalstar M043	12.139	47.56	2533.54	153.87	0	0	450	107.19
Globalstar M043	12.139	60.62	2258.87	153.87	0	0	450	34.93
Globalstar M055	12.139	27.64	2621.26	17.22	0	0	450	89.88
Globalstar M053	12.139	34.87	2922.32	16.38	0	0	450	77.39
Globalstar M053	12.139	47.81	2501.90	16.55	0	0	450	53.67
Globalstar M053	12.139	40.17	2729.06	39.93	0	0	450	46.66
Globalstar M036	12.139	21.77	3196.36	17.96	0	0	450	75.46
Globalstar M008	12.139	27.01	2600.46	17.13	0	0	450	83.86
Globalstar M053	12.139	30.58	3115.92	42.22	0	0	450	88.49
Globalstar M036	12.139	22.06	3180.69	19.95	0	0	450	122.89
Globalstar M036	12.139	42.52	2311.93	20.06	0	0	450	79.66
Globalstar M036	12.139	36.12	2516.11	20.11	0	0	450	66.70
Globalstar M050	12.139	27.76	2741.75	19.7	0	0	450	71.84
Globalstar M036	12.139	32.59	2657.15	24.09	0	0	450	69.30
Globalstar M026	12.139	21.27	2722.63	24.58	0	0	450	62.10
Globalstar M026	12.139	24.23	2572.40	24.73	0	0	450	36.64
Globalstar M028	12.139	23.08	2623.85	16.44	0	0	450	73.65
Globalstar M028	12.139	36.88	2064.96	16.59	0	0	450	43.15
Globalstar M022	12.139	21.41	3110.48	17.66	0	0	450	85.20
Globalstar M026	12.139	25.60	2507.94	26.74	0	0	450	85.36
Globalstar M028	12.139	21.26	2733.71	18.63	0	0	450	93.35
Globalstar M052	12.139	21.09	2740.73	26.47	0	0	450	125.82
Globalstar M052	12.139	36.58	2075.33	26.52	0	0	450	110.14
Globalstar M052	12.139	57.32	1623.80	26.55	0	0	450	89.15
Globalstar M022	12.139	26.49	2834.07	19.76	0	0	450	96.16
Globalstar M022	12.139	20.23	3181.58	19.79	0	0	450	89.72
Globalstar M058	20.896	22.10	3132.79	20.46	0	0	450	142.91
Globalstar M058	20.896	63.02	1874.83	20.58	0	0	450	80.16
Globalstar M058	20.896	54.54	2002.75	20.61	0	0	450	55.06
Globalstar M058	20.896	38.21	2408.72	20.64	0	0	450	33.25
Globalstar M032	20.896	23.69	2601.69	7.9	0	0	450	132.76
Globalstar M032	20.896	39.82	1979.48	7.98	0	0	450	82.05
Globalstar M032	20.896	32.12	2228.53	8.03	0	0	450	57.45
Globalstar M008	20.896	22.76	2816.83	21.14	0	0	450	135.63
Globalstar M008	20.896	39.49	2128.62	21.28	0	0	450	62.80
Globalstar M008	20.896	27.09	2592.58	21.33	0	0	450	43.13
Globalstar M027	20.896	21.30	2724.70	16.58	0	0	450	103.59
Globalstar M027	20.896	42.61	1904.05	16.63	0	0	450	101.97
Globalstar M027	20.896	21.44	2722.84	16.78	0	0	450	93.04
Globalstar M058	20.896	22.01	3137.03	25.94	0	0	450	121.12
Globalstar M058	20.896	31.19	2674.81	25.99	0	0	450	106.93
Globalstar M058	20.896	41.59	2303.06	26.02	0	0	450	82.69
Globalstar M058	20.896	39.47	2365.88	26.09	0	0	450	52.81
Globalstar M032	20.896	24.23	2572.93	9.88	0	0	450	121.15
Globalstar M032	20.896	44.80	1852.04	9.93	0	0	450	99.28
Globalstar M032	20.896	62.31	1561.06	9.96	0	0	450	79.24
Globalstar M008	20.896	22.26	2842.02	23.18	0	0	450	119.63
Globalstar M008	20.896	40.01	2113.02	23.23	0	0	450	99.84
Globalstar M008	20.896	62.71	1673.59	23.28	0	0	450	69.16

Table 7.3: Observational Results — Photometry Data Part 2 (Observational data relevant to photometric analysis—listed are the apparent magnitudes from each source, including the companion star).

Satellite	Star	N2YO	Predicted Low	Predicted High	Actual
Globalstar M049	7.82	7.70	7.13	9.64	8.73
Globalstar M003	7.62	8.18	7.77	10.27	9.06
Globalstar M002	5.23	8.20	7.01	9.52	4.93
Meteor 3-5	6.42	7.68	2.56	6.51	6.54
Globalstar M003	7.51	8.04	6.99	9.49	10.07
Fengyun 1B	7.69	6.98	7.15	10.19	7.64
Fengyun 1B	8.16	7.31	8.75	11.80	9.60
Geosat	6.30	6.13	6.30	8.80	7.59
Globalstar M051	8.14	7.65	6.67	9.17	9.82
Globalstar M003	7.31	8.03	7.31	9.81	8.80
Geosat	7.63	6.72	5.23	7.73	6.38
Globalstar M051	8.19	7.95	6.87	9.37	9.46
Globalstar M003	7.07	8.70	10.65	13.15	9.42
Globalstar M003	8.12	7.88	8.08	10.58	8.26
Globalstar M003	6.14	8.58	7.37	9.87	8.88
Fengyun 1B	5.69	7.74	7.48	10.52	7.97
Fengyun 1B	5.77	6.83	8.11	11.15	8.92
Globalstar M023	7.23	7.58	8.30	10.80	9.70
Geosat	8.07	6.72	6.87	9.37	7.51
Globalstar M003	8.25	7.96	7.78	10.29	10.58
Globalstar M023	6.22	7.33	5.60	8.10	7.96
Geosat	6.63	6.82	5.48	7.98	6.54
Geosat	7.77	6.08	6.57	9.07	7.45
Fengyun 1B	7.51	7.63	7.29	10.34	8.65
Globalstar M023	7.01	7.25	8.00	10.50	8.96
Globalstar M023	7.13	6.62	5.94	8.44	8.38
Globalstar M023	6.51	7.84	6.34	8.85	7.92
Globalstar M031	7.10	8.47	9.90	12.40	8.58
Globalstar M031	6.83	7.35	6.89	9.39	8.89
Globalstar M031	7.66	7.80	6.15	8.65	7.97
Geosat	8.07	6.77	7.23	9.73	7.80
Globalstar M043	7.16	8.28	7.50	10.00	7.24
Geosat	7.76	5.36	4.34	6.84	5.07
Globalstar M043	6.84	8.45	8.08	10.58	6.51
Globalstar M043	6.86	7.63	5.69	8.20	7.13
Globalstar M031	6.08	8.46	9.64	12.14	8.73
Globalstar M031	8.14	7.93	7.63	10.13	9.83
Globalstar M031	7.81	8.44	7.08	9.58	9.23
Cosmos 1900	8.21	5.93	4.05	6.55	5.29
Globalstar M043	7.51	8.57	8.45	10.95	8.04
Globalstar M043	6.52	7.71	5.93	8.43	7.16
Globalstar M043	8.43	8.22	5.98	8.48	7.13
Globalstar M055	7.64	8.18	7.41	9.91	6.01
Globalstar M055	7.68	7.69	6.18	8.68	7.76
Oscar 36	8.17	8.58	5.97	8.95	7.72
IMS-1	8.37	9.35	7.48	9.98	7.55
Globalstar M043	8.28	8.96	9.64	12.14	9.90
Globalstar M043	8.06	8.13	7.61	10.11	8.01
Globalstar M043	7.12	7.85	5.67	8.17	7.58
Globalstar M055	7.90	7.78	7.22	9.72	7.82
Globalstar M053	7.73	8.49	7.05	9.55	10.74
Globalstar M053	6.82	8.07	6.17	8.67	7.08
Globalstar M053	7.77	8.33	6.29	8.79	8.23
Globalstar M036	8.79	7.78	7.36	9.86	10.89
Globalstar M008	8.39	8.34	7.04	9.54	8.35
Globalstar M053	7.35	8.73	7.52	10.02	9.88
Globalstar M036	8.43	7.85	9.04	11.54	11.22

Continued...

Satellite	Star	N2YO	Predicted Low	Predicted High	Actual
Globalstar M036	8.51	6.82	6.55	9.05	9.08
Globalstar M036	7.81	7.08	6.48	8.98	5.60
Globalstar M050	5.33	8.24	6.85	9.35	8.66
Globalstar M036	8.02	7.21	6.68	9.18	6.87
Globalstar M026	7.07	7.99	6.74	9.24	10.07
Globalstar M026	8.51	7.78	6.19	8.69	8.19
Globalstar M028	6.03	8.48	6.86	9.36	8.31
Globalstar M028	8.47	7.74	5.66	8.16	6.78
Globalstar M022	7.32	8.27	7.55	10.05	9.95
Globalstar M026	7.05	7.76	7.02	9.52	9.00
Globalstar M028	8.47	8.59	7.52	10.02	9.25
Globalstar M052	8.16	8.03	8.90	11.40	8.59
Globalstar M052	8.06	7.25	7.35	9.85	8.46
Globalstar M052	7.36	6.63	5.99	8.49	7.07
Globalstar M022	8.66	7.81	7.60	10.10	9.39
Globalstar M022	7.18	8.25	7.76	10.26	9.79
Globalstar M058	7.75	8.38	10.35	12.85	10.44
Globalstar M058	8.47	6.90	6.05	8.55	9.19
Globalstar M058	7.20	7.08	5.69	8.19	7.96
Globalstar M058	8.40	7.59	5.87	8.37	8.54
Globalstar M032	8.71	7.88	9.16	11.66	9.33
Globalstar M032	7.13	7.01	6.29	8.79	8.61
Globalstar M032	8.62	7.37	6.07	8.57	8.37
Globalstar M008	9.22	8.56	9.54	12.04	9.59
Globalstar M008	7.79	7.68	6.02	8.52	7.35
Globalstar M008	8.36	8.29	6.24	8.74	7.74
Globalstar M027	8.02	8.57	7.86	10.36	9.87
Globalstar M027	5.49	7.49	6.81	9.31	8.36
Globalstar M027	8.09	8.56	7.49	9.99	9.93
Globalstar M058	8.12	8.36	8.92	11.42	9.93
Globalstar M058	7.38	7.82	7.81	10.31	10.54
Globalstar M058	7.70	7.35	6.63	9.13	8.62
Globalstar M058	8.55	7.44	6.08	8.58	9.62
Globalstar M032	8.29	7.82	8.45	10.95	8.97
Globalstar M032	8.68	6.91	6.64	9.14	7.90
Globalstar M032	8.44	6.47	5.63	8.13	7.27
Globalstar M008	6.79	8.65	8.62	11.12	9.30
Globalstar M008	6.93	7.79	6.97	9.47	7.92
Globalstar M008	8.35	7.17	5.55	8.05	5.27

Table 7.4: Additional Single Pass Orbit Determination Results (Comparison of COE's generated by angles-only orbit determination method, to COE's gathered from propagation-source TLE's).

Satellite	Source	Epoch (UT)	A (km)	e	i (°)	Ω (°)	ω (°)
Globalstar M023	TLE	3/26 19:58	7792.7	1.09E-3	52.01	327.39	356.78
	Vallado Gauss	3/27 4:07	7621.9	2.40E-2	52.01	326.20	323.70
	Curtis Gauss		7622.4	2.12E-2	52.09	326.25	323.66
	Double-R		7622.4	2.12E-2	52.09	326.25	323.66
Globalstar M031	TLE	4/12 18:41	7855.9	1.17E-4	52.00	333.63	97.06
	Vallado Gauss	4/12 3:49	8204.1	3.72E-2	51.92	335.06	140.32
	Curtis Gauss		8194.5	3.88E-2	52.01	335.13	139.15
	Double-R		8194.5	3.88E-2	52.01	335.13	139.15
Globalstar M043	TLE	4/16 3:58	8412.5	8.40E-4	51.94	17.10	102.54
	Vallado Gauss	4/14 4:57	8648.9	2.08E-2	52.02	21.78	123.18
	Curtis Gauss		8585.1	1.83E-2	51.97	21.68	124.99
	Double-R		8585.1	1.83E-2	51.97	21.68	124.99
Globalstar M043		4/17 3:58	Did Not Converge				

7.2 Lessons Learned

Anticipate Bad Weather

Anticipating the disruption of an observation schedule due to poor weather is a common sense issue, but it can not be overstated the extent to which a few cloudy nights can severely reduce the quantity of data acquired. In the case of this project, 4 out of the last 5 planned nights of observations were canceled due to weather. Considering that 23 of the 96 successful observations were recorded on the final observation night, both the quantity and diversity of satellites observed would have increased dramatically had the final nights of observations not been canceled. It is advised that observations should be planned for seasons of the year with the lowest level of cloudy weather, and telescope operators should be flexible concerning the nights on which they observe.

Simple Methods Get Results

Throughout this project, much time and resources were devoted to pursuing observation techniques that were more powerful or efficient than the simple, staring method used for the entirety of the thesis. At different points in time, both active-tracking and a scope fence were considered. Tools were developed to determine the pointing operations of multiple telescopes involved in a scope fence, and active-tracking controllers such as Sidereal Technology's SiTech controller were researched. The staring method was only attempted once the other, more complicated methods were met with operational difficulties. If the staring method had been introduced earlier in the year, the likely outcome would have been an increase in total data acquired.

Current–J2000 Epoch Mistake

In the first weeks of observations, before the installation of the f/3.3 focal reducer, the success rate of capturing satellites was very low—only 2 of 29 observations caught the target satellite. At first, it was thought this low success rate was due to either an inaccuracy in the TLE propagation—leading to cross-track errors larger than the FOV of the telescope—or the satellites were too dim to be observed. Much effort was put into rectifying this supposed problem of an insufficient FOV, including selecting satellite candidates for young TLE ages (<1 day) and low elevation observations (to reduce the angular cross track error). In addition to the clear option of adding a focal reducer to the system, an optical fence (multiple telescopes used in unison) was also considered. Once a focal reducer was acquired and put into place, the success rate improved greatly but remained lower than expected: 75 of 133 observations were successful. Attempts to explain and correct this shortfall continued: recording time was increased from ± 10 s, to ± 30 s, to ensure along-track error was not a problem, and analysis was completed to demonstrate that observation failures were not exclusive to low-magnitude satellites.

It was not until late April (the third month of observations) that the main cause of the issue was uncovered. When RA/Dec angles were generated by MATLAB and STK, they were produced in the Current Epoch frame of reference. As the Earth’s spin axis slowly precesses over a $\sim 26,000$ year cycle, the vernal equinox, pictured in Fig. 2.2, gradually shifts. This shift alters the frame of reference in which the RA/Dec angles are measured. When RA/Dec target angles were being placed into SkyX, they were unknowingly designated as in the J2000 frame, not the Current 2013 frame. Depending on the declination of an angle set, the mismatch in the J2000 and Current frames resulted in errors between ~ 1 -10

arcminutes. These errors can be seen in Appendix 7.1, wherein Figs. 7.1 through 7.3 were obtained before the epoch frame correction, and Fig. 7.4 was obtained after. Once this problem was identified, the correct conversion from Current to J2000 frame,

$$\alpha_{J2000} = \alpha_{Current} - YS(0.01281467 + 0.0055645 \cos(\alpha_{Current}) \tan(\delta_{Current})) \quad (7.1a)$$

$$\delta_{J2000} = \delta_{Current} - 0.0055645 \times YS \cos(\alpha_{Current}) \quad (7.1b)$$

where YS is years-since-2000, was applied to every future observation. After making this correction, 20 out of 20 observations were successful. For data taken before this issue was corrected, all angle-sets were retroactively converted to the J2000 frame in *pixel_analysis.m*.

7.3 Custom List of Satellite Targets

Figure 7.5: Altitude vs. Mass for Satellites of Interest List.

Table 7.5: Satellites of Interest List.

NORAD ID	Satellite	Period (min)	$i(^{\circ})$	Apogee (km)	Perigee (km)
2965	OPS 4947	106.46	89.25	1100	1025
13777	IRAS	102.84	99	908	879
18187	Cosmos 1867	100.63	65.01	803	774
21263	NOAA 12	101.01	98.7	817	796
23088	SL-16 R/B	101.8	71	845	843
23606	Cerise	96.74	98.34	606	599
24954	Faisat 2V	104.36	82.92	993	937
25544	ISS (ZARYA)	92.87	51.65	421	409
25621	Globalstar M023	114.08	52.01	1421	1406
25693	Oscar 36	97.33	64.56	645	617
25736	Mublcom	99.58	97.68	743	734
25886	Globalstar M053	127.7	52	2025	2019
26064	Falconsat	100.3	100.19	800	745
26702	Odin	96.03	97.64	571	565
27608	Unisat 2	97.58	64.56	659	626
27848	Cubesat XI-IV	101.34	98.71	829	815
27939	Mozhayets 4	98.33	97.8	689	669
28372	Aprizesat 1	99.4	98.05	763	696
28493	Nanosat(1)	97.85	98.38	658	653
28650	Hamsat	97.04	97.57	635	599
29678	Corot	102.98	90.01	904	895
29712	Pehuensat 1	96.72	97.65	611	592
32786	IMS-1	97.25	97.75	635	619
36037	Proba 2	99.15	98.3	728	707
36122	Xiwang-1	109.4	100.3	1204	1193
36588	Servis 2	109.44	100.43	1214	1188
36797	Aissat 1	97.14	98.03	631	613
36834	Yaogan 10	97.22	97.85	627	624
38711	MKA-FKI 1	101.14	98.94	822	804
38853	Fitsat 1	87.84	51.63	175	160
39087	AAUSAT3	100.41	98.63	787	770
38763	CP5	97.24	64.67	773	480

7.4 Explanation of Selected MATLAB Codes

This appendix provides additional details for select MATLAB algorithms developed for this project—although only excerpts of each code are presented here, accompanied by commentary, the full files (along with algorithms not listed here) can be made available by contacting the author at bschmalzel@gmail.com.

angledet.m

This code determines the right ascension and declination values from images of a successful observation—the pixel locations for the companion star and two locations of the satellite, are provided as inputs.

```

1 function [obsRD1,obsRD2]=angledet (starPixel,...
2     sc1Pixel,sc2Pixel,RDtarg,RDnSC,RDstar,pscale)
3 %INPUTS:
4 %     starPixel = Approximate Pixel Location of the Companion ...
5 %               Star (can get
6 %               that easily from Microsoft Paint) [x,y]
7 %     sc1Pixel = First Approximate Pixel Location of S/C [x,y]
8 %     sc2Pixel = Second...
9 %     RDtarg,RDnSC,RDstar = same as inputs for pathVisualizer
10 %OUTPUTS:
11 %     obsRD = Observed RA and Dec Angles of S/C Positions
12 %Flip Y-Axis for Pixels:
13 starPixel(1,2)=480-starPixel(1,2);
14 sc1Pixel(1,2)=480-sc1Pixel(1,2);
15 sc2Pixel(1,2)=480-sc2Pixel(1,2);
16 %Pixel Difference between s/c positions (SC) and
17 %s/c position and star (Star):
18 dSC=sc2Pixel-sc1Pixel;
19 dStar1=sc1Pixel-starPixel;
20 %Determine Orientation:
21 thetTN=atan2((RDn(2)-RDt(2)),(RDn(1)-RDt(1)))*...
22 cosd((RDn(2)+RDt(2))/2));
23 thetpixTN=atan2(dSC(2),dSC(1));
24 dthet=thetTN-thetpixTN;
25 dStar1R=[cos(-dthet),-sin(-dthet);sin(-dthet),cos(-dthet)]*...
26 (fliplr(dStar1))';
27 dec=cosd(RDs(2));
28 obsRD1=RDs+(flipud(dStar1R))'*(pscale/3600)./[dec,1];

```

appMag.m

This function is used to estimate the apparent magnitude of a satellite, given a number of variables or assumptions concerning the satellite and its orbit.

```
1 function [amag]=appMag(range,varargin)
2
3 %INPUTS:
4 %   range = Distance from Observer to Spacecraft (km)
5 %   varargin (optional inputs) :
6 %       Lc = Characteristic Length of Satellite (m). Defaults ...
           to 1 m
7 %       elev = Elevation of Satellite (deg). Defaults to 90 ...
           deg (zenith)
8 %       PA = Solar Phase Angle (deg). Defaults to 90 deg
9 %       alb = Albedo. Defaults to 0.175
10
11 %OUTPUT:
12 %   appMag = Apparent Magnitude of S/C
13
14 Msun=-26.75;%<—Magnitude of Sun
15 mag(1,1)=10;diff=100;i=1;
16 %while Loop until correct magnitude is determined
17 while abs(diff)>0.001
18     Lc_calc=10^((Msun-mag(1,i))/5)*range*1000/...
19         sqrt(alb)*sqrt(6*pi/(sin(PA)+(pi-PA)*cos(PA)));
20     diff=Lc-Lc_calc;
21     mag(1,i+1)=mag(1,i)-diff/Lc_calc;
22     i=i+1;
23 end
24 %Atmospheric Extinction Effect:
25 k=0.17;
```



```

26 X=secd(90-elev).*(1-0.0012.*((secd(90-elev)).^2-1));
27
28 amag=mag(1,end)+k.*X;

```

imgPHOT.m

This file is used to quantify the apparent brightness of an observed satellite. It does so through comparative photometry with the use of the companion, reference star within each image. Because the file is written to read-in specifically-named files, it would likely need to be edited for use in other projects.

```

1 while err == 0%<—While Loop runs until no more consecutive ...
    images
2     picID = num2str(firstnum+j-1);
3 %Try/Catch Expression to continue While loop until no more images
4     try
5 [X] = imread(imgstr);
6 imshow(X)
7 [x,y] = ginput(4);
8 %First Click: Star
9 %Second Click: SC Edge 1
10 %Third Click: SC Edge 2
11 %Fourth Click: Empty Area (>40 pixels away from image edges & ...
    SC & Star
12
13 %Find mean background noise by analyzing square area around ...
    4th click:
14 BGnoise(1,j)=mean(mean(X( (y(4)-40:y(4)+40), ...
    (x(4)-40:x(4)+40), 1)));
15 %Determine counts of SC area and subtract BG noise

```

```

16 xl=min([x(2),x(3)]);xh=max([x(2),x(3)]);
17 yl=min([y(2),y(3)]);yh=max([y(2),y(3)]);
18 SCarea=(abs(x(3)-x(2))+1)*(abs(y(3)-y(2))+1);
19 SC(1,j)=sum(sum(X(yl:yh,xl:xh)))-BGnoise(1,j)*SCarea;
20 star(1,j)=sum(sum(X( (y(1)-20:y(1)+20),(x(1)-20:x(1)+20),1)))-...
21     BGnoise(1,j)*1681;
22 if star(1,j)≤0
23     star(1,j)=1;%<—Force dimmer-than-noise star to not have ...
        negative counts
24 end
25 if SC(1,j)≤0
26     SC(1,j)=1;%<—Force dimmer-than-noise s/c to not have ...
        negative counts
27 end
28 %Magnitude Difference between the S/C and the Star:
29 magdiff(1,j)=-2.5*log10(SC(1,j)/star(1,j));
30     catch error_info
31         err=1;
32     end
33 j=j+1;
34 close
35 end
36 %Magnitude Difference between the Star and the Mean Star value
37 stardiff=-2.5.*log10(mean(star)./star);
38 %Plot Magnitude Differences for each image to spot outliers
39 plot(magdiff,'r')
40 hold on
41 plot(stardiff,'p—')
42 ylabel('Magnitude Difference');xlabel('Image Number')
43 legend('Magnitude Difference (SC-Star)','Star Change ...
        (Single-Mean)')
44 %Designate "outlier" images, aka: when s/c passed near a star

```

```

45 donotcount=input(['Ignore outlier images 3, 7, 14, ...
    etc...Enter in form'...
46     '"[3,7,14,...]:" ']);
47     if (isempty(donotcount))
48         donotcount=0;
49     end
50 %Remove outlier images
51 if donotcount(1,1)≠0
52     for i=1:length(donotcount)
53         magdiff(1,donotcount(i))=nan;
54         dDiffdBG(1,donotcount(i))=nan;
55     end
56 end
57 %Report Min, Mean, and Max values, ignoring edge and outlier ...
    images,
58 %along with the mean partial derivative (background noise).
59 fprintf('Greatest Magnitude Difference: %f \n',max(magdiff))
60 fprintf('Lowest Magnitude Difference: %f \n',min(magdiff))
61 L=length(magdiff);
62 if L<3
63     avgmag=nanmean(magdiff);
64     stdmag=nanstd(magdiff);
65 elseif L<7
66     avgmag=nanmean(magdiff(2:L-1));
67     stdmag=nanstd(magdiff(2:L-1));
68 else
69     avgmag=nanmean(magdiff(3:L-2));
70     stdmag=nanstd(magdiff(3:L-2));
71 end
72 diffs=[min(magdiff),avgmag,max(magdiff),stdmag];

```

ObservationResults.m

This code inputs observation data from observations.xls, to be used in both data reduction and analysis, concerning magnitude prediction model validation.

```
1 %% Determine Magnitudes %%
2 %Extinction:
3 k=0.17;X=secd(90-Elev).*(1-0.0012.*((secd(90-Elev)).^2-1));
4 Xo=secd(90-maxElev).*(1-0.0012.*((secd(90-maxElev)).^2-1));
5 zenMag=n2yoMag-k.*Xo;newMag=zenMag+k.*X;
6 %Adjust for Star Extinction as well:
7 newstarmag=starmag+k.*X;
8 %Distance Change Estimate thru extrapolation:
9 diffElev=90-acosd(alt./range)-Elev;
10 maxElevrange=alt./cosd(90-(maxElev+diffElev));
11 magActual=newMag-5*log10(maxElevrange./range);
12 %Calculation with my own Magnitude Code (SPA included):
13 %SIZE DETERMINATION FROM KNOWN DIMENSIONS OR DENSITY ESTIMATION:
14 for i=1:length(range)
15     if size1(i)==0
16         dia2a(i,1)=((5.05+0.0185*mass(i))*6/pi)^(1/3);
17         %dia2a(i,1)=0.0052.*mass(i)+0.3128;
18     else
19         dia2a(i,1)=size1(i);
20     end
21     if size2(i)==0
22         dia2b(i,1)=dia2a(i,1);
23     else
24         dia2b(i,1)=size2(i);
25     end
26 %SELECT MIN & MAX SIZES FROM ABOVE CALCULATIONS + SOME MARGIN:
27 diaS=dia2b(i,1)*0.5547;diaL=dia2a(i,1)*1.7541;
```

```

28 amagH(i,1)=appMag(range(i),diaL,Elev(i),PA(i));
29 amagL(i,1)=appMag(range(i),diaS,Elev(i),PA(i));
30 end
31 %% Magnitude Results Plot (Quantitative Analysis): %%
32 excOBS=[143,164,186];%Excluded observations
33 j=1;pointBrock=0;pointN2Y0=0;
34 for i=1:L
35 if obsDATA(i,5)==1 %and not excluded...
36     starM(j,1)=newstarmag(i,1);
37     magnitudes2(j,:)=magnitudes(i,:);
38     brockmag(j,1)=(magnitudes2(j,1)+magnitudes2(j,3))/2;
39     scM(j,1)=newstarmag(i,1)+meanmagdiff(i,1);
40 %Standard Deviation of Photometry:
41     stdE(j,1)=stdmagdiff(i,1);error(j,1)=stdmagdiff(i,1);
42     N2Y0diff(j,1)=abs(magnitudes2(j,2)-scM(j,1));
43     Brockdiff(j,1)=abs(brockmag(j,1)-scM(j,1));
44     N2Y0diffL(j,1)=abs(magnitudes2(j,2)-scM(j,1)-error(j,1));
45     BrockdiffL(j,1)=abs(brockmag(j,1)-scM(j,1)-error(j,1));
46     N2Y0diffU(j,1)=abs(magnitudes2(j,2)-scM(j,1)+error(j,1));
47     BrockdiffU(j,1)=abs(brockmag(j,1)-scM(j,1)+error(j,1));
48     MagLimit2(j,1)=MagLimit(i,1);
49 if (brockmag(j,1)≤(scM(j,1)+error(j,1)))&&...
50     (brockmag(j,1)≥(scM(j,1)-error(j,1)))
51     pointBrock=1+pointBrock;
52 end
53 if (magnitudes2(j,2)≤(scM(j,1)+error(j,1)))&&...
54     (magnitudes2(j,2)≥(scM(j,1)-error(j,1)))
55     pointN2Y0=1+pointN2Y0;
56 end;j=j+1;end;end

```

orbit_determination.m

This function is used to generate COE's given three observations of a satellite. The user has the ability to comment in/out the desired angles-only orbit determination technique.

```
1 function [a,ecc,incl,omega,argp]=orbit_determination...
2     (JD1,JD2,JD3,i1,i2,i3,obsRD1)
3 %INPUTS:
4 %   JD1,2,3: Julian Dates for Observation 1, 2, and 3
5 %   i1,2,3: Index number for each observation, for the source ...
6 %           excel sheet
7 %   obsRD1: Data read in from pixelanalysis.m
8 %OUTPUTS:
9 %   Common Orbital Elements:
10 %   a = semi major axis (km)
11 %   ecc = eccentricity
12 %   incl = inclination (deg)
13 %   omega = right ascension of the ascending node (deg)
14 %   argp = argument of perigee (deg)
15 lat=35.3005*pi/180;lon=-120.66*pi/180;h=0.105791;%Cal Poly ...
16     Observatory
17 [rs,vs]=site(lat,lon,h);%<—Site Coordinates in ECEF
18 ttt1=((JD1+66.734/(3600*24))-2451545)/36525;
19 [rsite1,~,~]=ecef2eci(rs,vs,[0,0,0]',ttt1,JD1,0.001,0.0424,...
20     0.31,0,0,0);
21 q1=[cosd(obsRD1(i1,2))*cosd(obsRD1(i1,1));cosd(obsRD1(i1,2))*...
22     sind(obsRD1(i1,1));sind(obsRD1(i1,2))];
23 q=[q1,q2,q3];
24 [r2,v2]=angle_rr(q,[JD1,JD2,JD3],[rsite1,rsite2,rsite3],1);
```

```

24  r2=r2';v2=v2';
25  [a,ecc,incl,omega,argp,-]=RV2COE2(r2,v2,4);
26  end

```

passPredict.m

This code predicts which satellites (from a Space Track favorites list) will be visible on a given night.

```

1  function [visibleSAT] = ...
    passPredict(DAYT,dur,sgroup,TLEmax,elevMin)
2  %INPUTS:
3  %   DAYT = PST day/time of observation, in ...
    [mo,day,yr,hr,min,sec] format.
4  %   dur = Desired duration for which passes will be ...
    predicted. (Majority
5  %           of visible passes are within 3 hours of sunset). (hrs)
6  %   sgroup = Which group of satellites to predict (1 = ...
    favorites, 2 =
7  %           globalstar satellites, 3=Navigation, 4 = Special ...
    Interest,
8  %           5 = Weather, 6 = Visible, 7 = Orbcomm, 8 = Iridium)
9  %   TLEmax = Ignore satellites with a TLE age older than ...
    TLEmax (hrs)
10 %   elevMin = Ignore satellites at an elevation below elevMin ...
    (deg)
11 %EXAMPLE: [visibleSAT] = ...
    passPredict([3,16,2013,19,30,0],2,1,40,20);
12 %OUTPUTS:
13 %   visibleSAT = ...
    [satNoradID,Jdate,UTth,m,s,TLEage(hrs),Elevation(deg)]

```

```

14
15 lat=35.3005*pi/180;lon=-120.66*pi/180;h=0.105791;
16 [rs,vs]=site(lat,lon,h);%<—Site Coordinates in ECEF
17 if sgroup==1
18 fid=fopen('SatelliteCandidates\SOI.txt');kL=32;%<—My Favorites
19 elseif sgroup==2
20 %Globalstar Satellites:
21 fid=fopen('SatelliteCandidates\Gstar.txt');kL=52;
22 ...
23 end
24 tempj=1;
25 for k=1:kL
26 TLEinfo=textscan(fid,'%71c%71c',1);
27 L1TLE=TLEinfo{1};L2TLE=TLEinfo{2};
28 [satrec, ~, ~, ~] = twoline2rv(72,L1TLE,L2TLE,'c','d');
29 Jdateobs=JD(DAYT(1),DAYT(2),DAYT(3),DAYT(4),DAYT(5),DAYT(6));
30 %CALCULATE DAY DIFFERENCE BETWEEN TLE AND OBS:
31 Jdiff0=(currentdays-satrec.epochdays(1))*24*60;
32 %~3 minutes Intervals:
33 Jdiff=linspace(Jdiff0,Jdiff0+dur*60,ceil(dur*60/3)+1);
34 Nint=length(Jdiff);
35 r=zeros(Nint,3);v=zeros(Nint,3);cansee=zeros(Nint,1);
36 litup=zeros(Nint,1);elev=zeros(Nint,1);
37 az=zeros(Nint,1);tleAGE=zeros(Nint,1);j=tempj;
38 for i=1:Nint
39 Jdate=Jdateobs+(Jdiff(1,i)-Jdiff0)/(24*60);%<—Julian Date at ...
    each interval
40 ttt=((Jdate+66.734/(3600*24))-2451545)/36525;
41 [rsECI,~,~] = ...
    ecef2eci(rs,vs,[0,0,0]',ttt,Jdate,0.001,0.0424,0.31,0,0,0);
42 [satrec,r(i,:),v(i,:)] = sgp4(satrec,Jdiff(1,i));%<—Propagate
43 los = sight(r(i,:),rsECI','e');%<—Line of Sight Determination

```



```

44 lit = SATlight((r(i,:)),Jdate,'e');%<—Lighting ...
    Determination/6378.1363
45 %Check that Elevation is above 25 degrees:
46 [¬,az(i),elev(i),¬,¬,¬] = rv2raze1...
47     (r(i,:)','v(i,:)','lat,lon,h,ttt,Jdate,0.001,0.0424,0.31,0,0,0);
48 elev(i)=elev(i)*180/pi;
49 tleAGE(i)=Jdiff(i)/60;
50 %If statement to determine if observation is possible:
51 if (strcmp(los,'yes')==1)&&(strcmp(lit,'yes'))&&(elev(i)≥...
52 elevMin)&&( tleAGE(i)≤TLEmax)&&(tleAGE(i)≥0) )
53     litup(i,1)=1;cansee(i,1)=1;
54     timepass=jul2cal(Jdate);
55 visibleSAT(j,:)=[satrec.satnum(1),Jdate,timepass(4:6),...
56 tleAGE(i,1),elev(i,1)];
57     j=j+1;tempj=j;
58     elseif strcmp(los,'yes')==1
59         cansee(i,1)=1;
60     elseif strcmp(lit,'yes')==1
61         litup(i,1)=1;
62     end
63 end;end;end

```

pixel_analysis.m

This code inputs observation data from images_pixelvals.csv, to be used in both data reduction and analysis, concerning orbit determination and temporal error growth.

```

1 IPV=csvread('images_pixelvals.csv',1,1);
2     %Convert RDs/c from Current Epoch to J2000 for older obs:
3     if i≤70

```

```

4     ys2000=13+IPV(i,36)/365.25;%<Years since 2000
5     decTempn=decn;decTempt=dect;
6     n=0.0055645;m=0.01281467;
7     decn=decn-(n*cosd(RAn))*ys2000;
8     RAn=RAn-(m+n*sind(RAn)*tand(decTempn))*ys2000;
9     [RDnSC(1,1:3),RDnSC(1,4:6)]=RDdegtoRD6(RAn,decn);
10    dect=dect-(n*cosd(RAt))*ys2000;
11    RAt=RAt-(m+n*sind(RAt)*tand(decTempt))*ys2000;
12    [RDtarg(1,1:3),RDtarg(1,4:6)]=RDdegtoRD6(RAt,dect);
13    end
14    %Use angledet.m to find RA/dec values of satellite
15    [obsRD1(i,:),obsRD2(i,:)]=angledet([IPV(i,7),...
16        IPV(i,8)],[IPV(i,9),IPV(i,10)],[IPV(i,11),IPV(i,12)],...
17        RDtarg,RDnSC,RDstar,IPV(i,31));
18    slopeA=(obsRD2(i,2)-obsRD1(i,2))/((obsRD2(i,1)-obsRD1(i,1))*...
19        cosd((obsRD1(i,2)+obsRD2(i,2))/2));
20    bA=obsRD1(i,2)-obsRD1(i,1)*slopeA;
21    dist=abs(dect-slopeA*RAt-bA)/sqrt(slopeA^2+1);
22    CTE(i,1)=(tand(dist)*IPV(i,33));%<—Cross Track Error (km)
23    ATE(i,1)=abs(mean([IPV(i,3)+IPV(i,4)/60,IPV(i,5)+IPV(i,6)/60])-...
24        (IPV(i,1)+IPV(i,2)/60));%<—Along Track (min)
25    estDIST(i,1)=ATE(i,1)*60*sqrt(398600.4415/...
26        (6378+IPV(i,35)));%<Along Track (km)
27    %% Orbit Determination
28    % Globalstar M003
29    i1a=10;i1b=11;i1c=12;
30    JD1a=JD(3,25,2013,3,IPV(i1a,3),IPV(i1a,4));
31    JD1b=JD(3,25,2013,3,IPV(i1b,3),IPV(i1b,4));
32    JD1c=JD(3,25,2013,3,IPV(i1c,3),IPV(i1c,4));
33    [a1,ecc1,incl1,omega1,argp1]=orbit_determination...
34        (JD1a,JD1b,JD1c,i1a,i1b,i1c,obsRD1);
35    Idata=[a1,ecc1,incl1,omega1,argp1];

```

```
36 other_info=[JD1a,JD1b,JD1c,ila,ilb,ilc];
```

RV2COE2.m

This function converts Earth-centered inertial state vectors into classical orbital elements.

```
1 %INPUTS:
2 %   r = vector of [x,y,z] radius elements (km)
3 %   v = vector of [x,y,z] velocity elements (km/s)
4 %   body = primary body of influence, use the following ...
        numerical code from
5 %1-11, where 1 is the Sun and 11 is Pluto. The user can also ...
        enter their
6 %own value for the gravitational parameter in km^3/s^2
7 %[Sun,Mercury,Venus,Earth,Moon,Mars,Jupiter,Saturn,Uranus,...
8 %Neptune,Pluto]
9 all_mu=[1.32712428E11,2.2032E4,3.257E5,3.986004415E5,4902.799,...
10 4.305E4,1.268E8,3.794E7,5.794E6,6.809E6,9E2];
11 if body ≤ 11
12 mu=all_mu(1,body);
13 else
14 mu=body;
15 end
16
17 %OUTPUTS:
18 %a= semi major axis (km)
19 %ECC= eccentricity (0-1)
20 %i= inclination (deg)
21 %RA= Right Ascension of the Ascending Node (deg)
22 %AP= Argument of Perigee (deg)
```

```

23 %TA= True Anamoly (deg)
24
25 R=norm(r);V=norm(v);
26 h=cross(r,v);H=norm(h);%specific angular momentum
27 k=[0,0,1];
28 n=cross(k,h);N=norm(n);
29 ecc=( (V^2-mu/R)*r-(dot(r,v))*v)/mu; ECC=norm(ecc);%Eccentricity
30 gamma=V^2/2-mu/R;
31 if ECC ~= 1.0
32     a=-mu/(2*gamma);%Semi-Major Axis
33     p=a*(1-ECC^2);%Semi-parameter
34 else
35     a=inf;
36     p=H^2/mu;
37 end
38 i=acosd(h(1,3)/H);%Inclination
39 if n(1,2) < 0
40     RA=360-acosd(n(1,1)/N);%Right Ascension of the Ascending Node
41 else
42     RA=acosd(n(1,1)/N);
43 end
44 if ecc(1,3) < 0
45     AP=360-acosd(dot(n,ecc)/(N*ECC));%Argument of Periapsis
46 else
47     AP=acosd(dot(n,ecc)/(N*ECC));
48 end
49 if dot(r,v) < 0
50     TA=360-acosd(dot(ecc,r)/(ECC*R));%True Anomaly
51 else
52     TA=acosd(dot(ecc,r)/(ECC*R));
53 end

```

STK2VIZIER.m

This file converts the current Azimuth Elevation angles from STK into Right Ascension and Declination angles. It then writes them into a space-delimited file for uploading to Vizier.

```
1 azelev1=csvread('July17\Yubeleiny.csv',1,0);
2 lat=35.3005;lon=-120.659964;h=347.083;%height in feet
3 [rs,vs]=site(lat*pi/180,lon*pi/180,0.105791);
4 L=length(azelev1);RAdec1=zeros(L,2);dateT1=zeros(L,6);
5 for i=1:L
6     date=jul2cal(azelev1(i,1));
7     locST=LST(date(1),date(2),date(3),date(4),date(5),...
8     date(6),-lon,0,'west');
9     dateT1(i,:)=date;
10    [RAdec1(i,1:3),RAdec1(i,4:6)]=AzElev2RaDec(azelev1(i,2),...
11        azelev1(i,3),lat,locST,2);
12    [RA(i),dec(i)]=RD6toRDdeg(RAdec1(i,1),RAdec1(i,2),...
13    RAdec1(i,3),RAdec1(i,4),RAdec1(i,5),RAdec1(i,6));
14    %Convert RA/dec values from Current Epoch to J2000
15    ys2000=(azelev1(i,1)-2451545)/365.25;%<Years since 2000
16    decTemp=dec(i);
17    n=0.0055645;m=0.01281467;
18    dec(i)=dec(i)-(n*cosd(RA(i)))*ys2000;
19    RA(i)=RA(i)-(m+n*sind(RA(i))*tand(decTemp))*ys2000;
20    [rxS(i),ryS(i),rzS(i),-,]= SUN(azelev1(i,1));
21    ttt=((azelev1(i,1)+66.734/(3600*24))-2451545)/36525;
22    [rsECI(i,:),-,]= ecef2eci(rs,vs,[0,0,0]',...
23    ttt,azelev1(i,1),0.001,0.0424,0.31,0,0,0);
24    [rECI(i,:),-]= razel2rv(azelev1(i,4),azelev1(i,2)*pi/180,...
25    azelev1(i,3)*pi/180,0,0,0,lat*pi/180,lon*pi/180,...
```

```

26     0.105791,ttt,azelev1(i,1),0.001,0.0424,0.31,0,0,0);
27     vecA=rECI(i,:)-rsECI(i,:);
28     vecB=rECI(i,:)-[rxS(i),ryS(i),rzS(i)];
29     PA(i,1)=acosd(dot(vecA,vecB)/(norm(vecA)*norm(vecB)));
30 %Convert RA/dec values into readable values for Vizier
31 [RAdec1(i,1:3),RAdec1(i,4:6)]=RDdegtoRD6(RA(i),dec(i));
32     if RAdec1(i,1)<0
33         RAdec1(i,2)=RAdec1(i,2)*-1;
34         RAdec1(i,3)=RAdec1(i,3)*-1;
35     end
36     if RAdec1(i,4)<0
37         RAdec1(i,5)=RAdec1(i,5)*-1;
38         RAdec1(i,6)=RAdec1(i,6)*-1;
39     end
40 end
41 %CHANGE FILE NAME!!
42 dlmwrite('July17\Yubeleiny.txt',RAdec1,'delimiter',' ')
43 %NOW UPLOAD TO VIZIER:
44 %   http://vizier.u-strasbg.fr/viz-bin/VizieR
45 % 1. Choose Catalog (e.g. "HIP" for Hipparcos)
46 % 2. Choose "List of Targets" tab near top of webpage
47 % 3. Set an upper limit in the Vmag constraint (e.g. "<8")
48 % 4. Set Target Dimension to be less than 1/2 FOV (e.g. 2 ...
    arcmin w/o focal
49 %       reducer, 6 arcmin w/ focal reducer)
50 % 5. Click "Choose File" and upload space-delimited file
51 %After uploading to Vizier, select one of the positions with ...
    a companion
52 %star and enter the index number below:
53 ind=358;
54 importDATA=zeros(3,6);importDATA(1,:)=dateT1(ind,:);
55 importDATA(3,1:3)=azelev1(ind,2:4);importDATA(3,4)=PA(ind,1);

```

```
56 importDATA(2,:)=RAdec1(ind,:);
```

7.5 Extended Kalman Filter

In the field of orbit determination, it is often the case that multiple sets of observation angles are gathered for a single satellite. These angle-sets are acquired from multiple overhead passes of the satellite, can be separated by days, and enable differential correction techniques such as least squares and Kalman filtering. This appendix will present a short summary of the technique used in this report, an extended Kalman filter, covered more thoroughly in Vallado's *Fundamentals of Astrodynamics and Applications*[29].

To enable the use of a Kalman filter, it is first necessary to define initial characteristics of the system: initial covariance, P , initial states, X_0 , second moment of process noise, Q , and the measurement noise, R . Results from Appendix 7.6 are used to estimate the measurement noise,

$$B = \begin{bmatrix} 0.0033^\circ \\ 0.0038^\circ \end{bmatrix}$$

$$R = \begin{bmatrix} \frac{1}{B(1)^2} & 0 \\ 0 & \frac{1}{B(2)^2} \end{bmatrix} \quad (7.2a)$$

where b are the sensor biases involved in angle determination. The initial covariance and process noise matrices were estimated to be composed of the following values,

$$P_0 = \begin{bmatrix} 1km^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1km^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1km^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.0001km^2/s^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.0001km^2/s^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.0001km^2/s^2 \end{bmatrix} \quad (7.3)$$

and process noise errors were determined in the following manner,

$$v = \begin{bmatrix} 1km \\ 1km \\ 1km \\ 0.01km/s \\ 0.01km/s \\ 0.01km/s \end{bmatrix}$$

$$Q = v \times v' \quad (7.4a)$$

where v is the process noise. The accuracy of these estimations is unknown, and future effort could be spent on better quantifying these initial values.

Right ascension and declination observations, z , are inputted into the system, and the Kalman filter begins with the determination of the observation partial derivative, H , after which the error state-transition matrix, Φ , is calculated (see Vallado 10.7.1),

$$X = \begin{bmatrix} r_x \\ r_y \\ r_z \\ v_x \\ v_y \\ v_z \end{bmatrix}$$

$$z = \begin{bmatrix} \alpha \\ \delta \end{bmatrix} \quad (7.5a)$$

$$H = \frac{\partial z}{\partial X}$$

$$\Phi = \frac{\partial X}{\partial X_0}$$

Although these matrices can be approximated either analytically or numerically, this project only used the analytical method of finite, central differencing to determine the partials. An algorithm form of a finite differencing technique can also be found in Vallado section 10.4.1. Kepler's universal variable propagator is used to propagate forward the current state, X , to the predicted state, $\hat{X}$, from which the MATLAB code **topocentric.m** is used to convert the states to topocentric right ascension and declination. The filter continues by predicting the subsequent error covariance,

$$\bar{P} = \Phi P \Phi^T + Q \quad (7.6)$$

From this point on, the filter determines the residuals, b , Kalman gain, K , and state error estimate, δx , and proceeds to update the error covariance and state. These steps directly follow those found in Algorithm 68 of Vallado,

$$b = z - H\bar{X}$$

$$K = \bar{P}H^T[H\bar{P}H^T + R]^{-1}$$

$$\delta x = Kb \quad (7.7a)$$

$$P = \bar{P} - KH\bar{P}$$

$$X = \bar{X} + \delta x \quad (7.7b)$$

Table 7.6: Pixel Selection Errors determined by Monte-Carlo Method.

Target Satellite	RA Error (°)	Dec Error (°)	CTE Error (km)
Globalstar M003 3/25	4.16E-03	1.45E-03	0.078
	2.19E-02	3.95E-02	0.637
	1.64E-03	2.12E-03	0.091
Globalstar M031 4/14	3.08E-03	2.62E-03	0.066
	3.81E-03	3.95E-03	0.052
	1.76E-03	1.41E-03	0.071
Globalstar M036 4/20	2.65E-03	3.61E-03	0.166
	2.62E-03	1.64E-03	0.104
	1.65E-03	1.55E-03	0.071

7.6 Error Analysis

Error propagation was performed for single-pass orbit determination, cross-track error, and apparent magnitude. Analysis of along-track error was omitted due to the unknown uncertainty in the computer clock error. Astrometrical uncertainties included pixel selection errors, ± 1 pixel for star positions and ± 2 pixels for satellite positions, and ± 0.004 arcsec/pix for plate scale. Time stamp errors of ± 1 ms and a guessed computer clock error of ± 1 s were used as well⁵. The effect of these uncertainties were analyzed through the use of a 10,000 iteration Monte-Carlo method. Nine angle-sets were used as a sample (three single-trio observations) and 3σ errors were generated from each—these are reported in Table 7.6. For orbit determination, these errors are reported in Tables 5.1 and 5.2. As for cross-track error, a maximum uncertainty was set to the 3σ value of the calculated errors—this uncertainty was calculated to be 0.558 km and is displayed as error bars in Fig 5.2.

⁵Although a computer clock error was used here, it was found that the uncertainties in along-track error were disproportionately affected when compared to the uncertainties in the cross-track error and orbit determination results. This is due to the fact that whatever clock error existed, was added equally to each observation. Therefore it was decided that while little harm would come from applying a general clock error elsewhere in the report (the magnitude of the resulting uncertainty was lower than that due to the pixel-selection errors), the effect of such an error upon the along-track error would not reflect the true uncertainties.

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