STATIC BALANCING OF THE CAL POLY WIND TURBINE ROTOR

A Thesis

presented to

the Faculty of California Polytechnic State University,

San Luis Obispo

In Partial Fulfillment

of the Requirements for the Degree

Master of Science in Mechanical Engineering

by

Derek Simon

August 2012

© 2012
Derek Simon
ALL RIGHTS RESERVED

COMMITTEE MEMBERSHIP

TITLE: Static Balancing of the
Cal Poly Wind Turbine Rotor

AUTHOR: Derek Simon

DATE SUBMITTED: August 2012

COMMITTEE CHAIR: Patrick Lemieux, PhD, PE
Professor, Mechanical Engineering

COMMITTEE MEMBER: Andrew Kean, PhD
Professor, Mechanical Engineering

COMMITTEE MEMBER: Glen E. Thorncroft, PhD
Professor, Mechanical Engineering

ABSTRACT

Static Balancing of the Cal Poly Wind Turbine Rotor

Derek Simon

The balancing of a wind turbine rotor is a crucial step affecting the machine's performance, reliability, and safety, as it directly impacts the dynamic loads on the entire structure.

A rotor can be balanced either statically or dynamically. A method of rotor balancing was developed that achieves both the simplicity of static balancing and the accuracy of dynamic balancing. This method is best suited, but not limited, to hollow composite blades of any size. The method starts by quantifying the mass and center of gravity of each blade. A dynamic calculation is performed to determine the theoretical shaking force on the rotor shaft at the design operating speed. This force is converted to a net counterbalance mass required for each blade. Despite the most careful methodology, there may still be large errors associated with these measurements and calculations. Therefore, this new method includes a physical verification of each blade's individual balance against all other blades on the rotor, with the ability to quantify the discrepancy between blades, and make all balance adjustments in situ.

The balance weights are aluminum plugs of varying lengths inserted into the root of each blade with a threaded steel rod running through the middle. The balance adjustment is thus not visible from outside. The weight of the plug and rod represent the coarse counterbalance of each blade, based on the dynamic calculations. The threaded steel rod acts as a fine adjustment on the blades' mass moment when traveled along the plug. A dedicated blade-balance apparatus, designed and constructed in-house, is used to verify and fine-tune each individual blade and compare it to all other blades on the rotor. The resulting blade assembly is verified on a full rotor static balancing apparatus. The full rotor apparatus measures the steady state tilt of the rotor when balanced on a point. Next, the rotors' tilt is related to its overall level of imbalance with quantifiable error. Most error comes from the fact that the hub, comparable in mass to the blades, creates a false righting moment of the assembly not present in operation. The fully assembled rotor is tested, pre and post balance, in operation on the turbine at a series of predetermined speeds. This is accomplished with a 3-axis accelerometer mounted on the main turbine shaft bearing and a control system which regulates and records turbine speed at 100 Hz.

keywords: wind turbine, rotor balancing, static balancing, reliability, performance

TABLE OF CONTENTS

LIST OF TABLES	vii
LIST OF FIGURES	viii
NOMENCLATURE.....	xii
1. NEED FOR BALANCING.....	1
2. BLADE MEASUREMENT AND SELECTION.....	4
3. MASS MOMENT PROPERTIES (R_i and m_i)	30
3.1. SIMPLE BEAM METHOD FOR DETERMINING R_i and m_i	30
3.2. HANGING METHOD FOR DETERMINING R_i	31
3.3. INITIAL IMBALANCE ANALYSIS.....	35
4. BALANCE WEIGHT DESIGN AND CONSTRUCTION	38
5. BALANCE METHODS	43
5.1. FULL ROTOR	44
5.2. SINGLE BLADE.....	48
6. FULL ROTOR CONSTRUCTION.....	52
7. SINGLE BLADE CONSTRUCTION.....	55
8. FULL ROTOR BALANCING.....	62
9. SINGLE BLADE BALANCING	65
10. FIELD TEST.....	67

11. CONCLUSIONS.....	73
APPENDIX A. DETAILED METHODOLOGY: HANGING METHOD	74
APPENDIX B. DETAILED METHODOLOGY: FULL ROTOR.....	75
APPENDIX C. DETAILED METHODOLOGY: SINGLE BLADE	62
APPENDIX D. DETAILED METHODOLOGY: FIELD TESTING	64
APPENDIX E. FLOWCHART	66
APPENDIX F. COMPLETE BLADE GEOMETRY	67
APPENDIX G. SOLIDWORKS BALANCE WEIGHT DESIGNS	74
APPENDIX H. FULL ROTOR ERROR FUNCTIONS	77
APPENDIX I. MATLAB 3-D COMPARISON OF BLADE LEADING EDGES .	77
APPENDIX J. EES ANALYSIS OF BLADE C.M.....	78
APPENDIX K. EES ANALYSIS OF ROTOR SHAKE FORCE	80
APPENDIX L. EES ANALYSIS OF δ_y.....	81
APPENDIX M. EES ANALYSIS OF BALANCE WEIGHT DESIGN	81
APPENDIX N. EES ANALYSIS OF FULL ROTOR METHOD.....	82
APPENDIX O. SOLIDWORKS DRAWINGS OF SINGLE BLADE METHOD PARTS.....	84
APPENDIX P. FIELD TESTS RESULTS AT ALL ROTOR SPEEDS	87

LIST OF TABLES

Table 2.1. Sample measurements of blade obtained with use of propeller table.....	29
Table 3.2.1: Final blade mass properties	35
Table 9.1: Observed and predicted load cell values for single blade method.....	66

LIST OF FIGURES

Figure 1.1: Dynamic formulation of rotor shake force and balance correction.....	1
Figure 1.2: Failed planetary gearbox and failed roller bearing.....	3
Figure 2.1: Normalized height of leading and trailing edge of turbine blades when measured on a propeller table.....	5
Figure 2.2: Propeller table in Cal Poly laboratory	6
Figure 2.3: Height gauge used to measure as-built blade properties. Blade is mounted on propeller table and viewed from the tip, parallel to span.....	7
Figure 2.4: Blade coordinate system.....	7
Figure 2.5: Leading edge comparison of wind turbine blades. Each line represents the position of the leading edge of a blade relative to all other blades. The coordinate system used is consistent with Figure 2.4. MATLAB script included in APPENDIX I.....	8
Figure 2.6: Blade leading edge height comparison.....	9
Figure 2.7: Blade leading edge lateral comparison.....	9
Figure 2.8: Blade leading edge lateral and height comparison	10
Figure 2.9: Blade chord comparison.....	11
Figure 2.10: Representation of blade-root interface	12
Figure 3.1.1: Simple beam model used to determine center of mass location.....	30
Figure 3.2.1: Hanging method formulation	32
Figure 3.2.2: Hanging blade to determine the location of its center of mass	33

Figure 3.2.3: Distance to C.M. of each blade is measured using a fine scale.....	34
Figure 3.2.4: Example of how center of mass location is determined.....	35
Figure 3.3.1: Dynamic formulation of rotor shake force and balance correction.....	36
Figure 3.3.2: Shake force due to imbalance mass of 1 pound at a distance of 20 inches from the axis of rotation	38
Figure 4.1: Section view of aluminum plug.....	39
Figure 4.2: Model of balance plug showing lip, anchor bolt, and weep holes. The centerline hole will contain the steel threaded rod	40
Figure 4.3: Model view of balance plug showing lip, socket recess, weep, and anchor bolt holes. The inner most hole will contain the steel threaded rod	40
Figure 4.4: Aluminum balance weight with hole for anchor bolt.....	41
Figure 4.5: Balance weight plug with internal threaded rod inside blade root.....	42
Figure 5.1: Dynamic balancing.....	43
Figure 5.2: Static balancing	44
Figure 5.1.1: Sample measurement of d_y	45
Figure 5.1.2: Full rotor method formulation.....	46
Figure 5.1.3: Full rotor method top view	47
Figure 5.1.4: Full rotor method spike position components	48
Figure 5.2.1: Single blade method formulation	49
Figure 5.2.2: Single bade method components.....	50
Figure 5.2.3: Detail view of load cell and counterweight.....	51
Figure 6.1: Detail view of spike used in full rotor method.....	53
Figure 6.2: Sketch of main shaft with conical cavity and index marks	53

Figure 6.3: Hub balancing on spike with bulls eye level sitting on top main shaft with index marks	54
Figure 7.1: Knife edge with slots to bolt onto the steel base	55
Figure 7.2: Knife edge bolted to base. A c-clamp is used to secure the base to a table ...	56
Figure 7.3: Pivot plate with slots to bolt onto the bottom of the counterweight.....	57
Figure 7.4: Bottom view of counterweight with aluminum pivot plate attached	57
Figure 7.5: Counterweight, front view. Blade flange bolts to the front where the grey rings are positioned. Pivot plate is bolted to the bottom, and adjustable weighs travel along threaded rods that extend out the back	58
Figure 7.6: Front view of counterweight with blade flange bolt pattern	59
Figure 7.7: Full assembly of single blade method	61
Figure 8.1: Example of measureable rotor tilt	63
Figure 8.2: Relationship between δ_y and δ_θ	64
Figure 10.1: Accelerometer orientation. The X-axis is in the plane of the rotor, the Y-axis is along the main shaft and the Z-axis is parallel to the tower	67
Figure 10.2: Average main shaft accelerations for the Cal Poly Rotor at a speed range of 201 - 220 RPM using RMS values. Data recorded on 7/20/12	69
Figure 10.3: Average main shaft accelerations for the Cal Poly Rotor at a speed range of 201 - 220 RPM using averages of the absolute values. Data recorded on 7/20/12	70
Figure 10.4: Acceleration in the X-direction as a function of rotor speed. Averages of the absolute values of acceleration were used in this plot. Note that between 0 and 75 RPM the acceleration steady at around 0.0275g for all	

three tests. This is likely the systematic error in the data	71
Figure 10.5: Average acceleration for locked rotor (0 RPM)	72

NOMENCLATURE

RPM	≡	Revolutions Per Minute
MPH	≡	Miles Per Hour
MW	≡	Mega Watt
kWh	≡	kilowatt hour
C.M.	≡	Center of Mass
RMS	≡	Root Mean Square
d_y	≡	distance between top of taper lock and main pivot shaft index (in)
d_{bubble}	≡	distance bubble in bulls eye level has moved during full rotor test (mm)
δ_y	≡	vertical distance between rotor center of mass and pivot point (in)
δ_x	≡	horizontal distance between hub center of mass and pivot point (in)
δ_θ	≡	angle of full rotor tilt (deg)
R_i or R_b	≡	distance to any blade center of mass from its pivot point (in)
R_c	≡	distance to counterweight center of mass from the pivot point (in)
R_{hub}	≡	radius of the hub (in)
L_i	≡	straight line connecting pivot point and any blade center of mass
m_i	≡	mass of any blade (lb_m)
M_{pi}	≡	moment of any blade about its pivot point (in- lb_f)
m_h	≡	mass of the hub (lb_m)
M_{ph}	≡	moment of the hub about its pivot point (in- lb_f)
F_{LC}	≡	force experienced by the load cell (lb_f)
L_{LC}	≡	distance to load cell from the pivot point (in)
m_{cal}	≡	mass of the calibration blade (lb_m)
R_{cal}	≡	distance to the calibration blade from the pivot point (in)

1. NEED FOR BALANCING

The Cal Poly wind turbine is a variable-speed 3 kW machine entirely designed and built in house by students and faculty. The turbine sits on a 70 ft tall tower located on the Cal Poly property of Escuela Ranch and has an approximately 12 ft diameter rotor. The direct drive permanent magnet generator has a nominal operating speed of about 210 RPM, giving the rotor a tip speed ratio of 4. The tip speed ratio for a wind turbine is the ratio of the blade tip velocity to the velocity of the wind.

Each blade is about 69 inches long, not including the radius of the hub, and has a hollow construction of carbon composite [1]. The blades are glued to steel roots with flanges that bolt to the turbine hub. Because the blades are hand-made they vary in length, profile, and mass and are therefore inherently imbalanced until corrected. Each blade is analogous to a mass rotating about an axis as illustrated in Figure 1.1 below. The vector sum of the centripetal forces is ultimately the imbalance that will be corrected.

Figure 1.1: Dynamic formulation of rotor shake force and balance correction

An extensive literature search found wind turbine balancing standards based on practical applications of ISO 1940-1 [2]. The G16 balancing grade is the currently accepted grade used by the wind community. For grade G16 the permissible residual unbalance may be calculated by Equation (1.1) below where W is the mass of the rotor in pounds and N is the rotational speed. Using these units U_{per} will have units of oz-in.

$$U_{per} = 6.015(16)(W/N) \quad (1.1)$$

There are several patented methods for wind turbine rotor balancing. One popular method in the public domain which is not proprietary is that of blade pitch offset [3]. This involves measuring the strain inside the root of each blade and calculating corresponding bending moment on the main shaft. The pitch of each blade can then be varied to aerodynamically offset that bending moment to reduce vibrations. This method, while successful in reducing main shaft vibrations, must necessarily reduce the power capture of the rotor due to not having optimal aerodynamic blade pitch. Other balancing methods include externally adding blade weighs [4] or in-situ balancing [5].

Rotor Balancing is needed to ensure safe, reliable, and smooth operation of the machine. Balancing is important because of excessive noise, safety in populated areas, improved machine reliability and return on investment. An imbalanced rotor could also lead to gearbox or bearing failures such as those in Figure 1.2.

Figure 1.2: Failed planetary gearbox and failed roller bearing

We consulted the IEC 61400 standards for vibration, shake force, and imbalance requirements but no detailed guidelines are given. The standards mention that rotor imbalance should be taken into account for power production calculations and that protection functions are required for excessive vibration [6]. The level of vibration that is excessive, however, is not specified. The international standards also state that the certification body may require vibration tests [7]. Again, no other details are provided.

Applying Equation (1.1) to the Cal Poly wind turbine has a permissible mass imbalance of approximately 0.075 pounds, resulting in a shake force of 1.9 lb_f at design speed. Normalizing this value leads to a shake force of about 3.8% of total rotor weight. If we apply this standard to commercial and larger scale wind machines, the need for balance becomes apparent. A Vestas V90 spinning can experience main shaft shake force of 2500 lb_f due to construction tolerance alone [8]. This is approximately 3% of the rotor weight and is not an insignificant shake force, even for a large turbine. Note that this is just the shake force due to rotor imbalance and does not include the imbalanced due to

blade angle errors, pitch errors, or blade damage. A smaller turbine such as a Bergey Excel 10 can experience a shake force up to 60% of its rotor weight at 400 RPM due to a 1% blade weight tolerance [9]. Given that the Cal Poly rotor is expected to experience speeds upwards of 350 RPM, the need for balancing becomes obvious. At 300 RPM each blade experiences a baseline load due to centripetal force of about 560 lb_f. Any shake force due to imbalance is cumulative with the rotor base load. The forces referred to in these calculations do not include wind gusts and uneven aerodynamic loads. Based on this need for balancing, a method was developed in-house for large scale wind machines and was tested and verified on the Cal Poly turbine.

2. BLADE MEASUREMENT AND SELECTION

The first crucial step in balancing the Cal Poly Turbine rotor was to select the three out of seven manufactured blades to use in turbine operation. Figure 2.1 shows the normalized height, above a propeller table surface, of the leading and trailing edge of each blade.

Figure 2.1: Normalized height of leading and trailing edge of turbine blades when measured on a propeller table

The propeller table used for these measurements is shown in Figure 2.2. We measured the leading and trailing edge of each blade using a height gage as shown in Figure 2.3. The turbine hub was mounted to a propeller table and a blade was bolted to it. The blade extends along the graduated length of the propeller table and measurements of

blade properties at any section can be measured, In the case of Figure 2.1, a height gage was used to measure the distance above the propeller table of both the leading and trailing edges of a blade. These measurements were then normalized to the centerline of the hub for ease of comparison. The leading edges of the blades in Figure 2.1 are the slightly positive sloping lines in the top of the figure and the trailing edges are the lines that curve up from the bottom of the figure. Because each composite blade is glued into a steel root, the leading edge location in 3-D space is inconsistent. A representation of the blade-root assembly can be seen in Figure 2.10. The blade coordinate system is shown in detail in

Figure 2.4. The leading edge inconsistency is most easily seen in Figure 2.6 and Figure 2.7. Figure 2.5 and Figure 2.8 show additional views of the blade Leading edge offset. The MATLAB script used to generate these plots can be found in APPENDIX I.

Figure 2.2: Propeller table in Cal Poly laboratory

Figure 2.3: Height gauge used to measure as-built blade properties. Blade is mounted on propeller table and viewed from the tip, parallel to span

Figure 2.4: Blade coordinate system

Figure 2.5: Leading edge comparison of wind turbine blades. Each line represents the position of the leading edge of a blade relative to all other blades. The coordinate system used is consistent with Figure 2.4. MATLAB script included in APPENDIX I.

Figure 2.6: Blade leading edge height comparison

Figure 2.7: Blade leading edge lateral comparison

Figure 2.8: Blade leading edge lateral and height comparison

Another important blade measurement is the chord length at each section. The chords were measured using a large micrometer and are plotted in Figure 2.9. Note that every blade chord is almost identical, though differing from the design chord by a near constant amount, so chord profiles were assumed to be irrelevant in the blade selection process.

Figure 2.9: Blade chord comparison

The consistency, however, of the chord offset suggests a systematic length misalignment of the blade-root assembly. This is a separate and additional misalignment as mentioned in the previous discussion. The steel roots extend into the blade as can be seen in Figure 2.10, and it appears that the carbon fiber blades were not slid far enough down onto the root when they were glued in place. When calculated, this length is approximately 1.5 inches. Therefore each blade is 1.5 inches longer than the original design specification.

Figure 2.10: Representation of blade-root interface

The blade geometry affects many aspects of wind turbine operation. The aspects, however, of most concern with respect to rotor balancing are R_i and m_i which can be seen in Figure 2.4 and explained in

NOMENCLATURE. Each geometric difference among the blades has a distinct effect on the mass and C.M. Some blades have more glue or resin, and all blades have slightly different lengths and twists. Therefore each blade has a different R_i and m_i , resulting in rotor imbalance.

Based on the measured results, a sample of which can be seen in Table 0.1, blades 3, 4, and 5 were chosen to use in turbine operation because of their similar twists and root offsets. This gives the chosen three blades the most uniform aerodynamic properties. Any set of three blades can be balanced, but if they differ too much aerodynamically, then the balancing will ultimately not be as successful. Complete geometry and measurements for all blades are included in APPENDIX F.

Table 0.1. Sample measurements of blade obtained with use of propeller table

Blade 3						
	H _{LE} Distance from Table Edge = 5 13/16 (in)		Hub Height = 3.350 (in)		Weight = 10.15 (lb _f)	
Section	Length From Hub Axis (in)	H _{LE} (in)	H _{TE} (in)	Normalized H _{LE} (in)	Normalized H _{TE} (in)	Chord Length (in)
1	13 7/8	7.146	1.340	1.796	-4.010	12.864
2	17	7.008	2.354	1.658	-2.996	12.568
3	20	7.008	3.195	1.658	-2.155	12.156
4	23	7.018	3.888	1.668	-1.462	11.595
5	26	7.078	4.529	1.728	-0.821	10.963
6	29	7.101	5.042	1.751	-0.308	10.321
7	32	7.091	5.439	1.741	0.089	9.719
8	35	7.118	5.758	1.768	0.408	9.160
9	38	7.134	6.060	1.784	0.710	8.625
10	41	7.151	6.288	1.801	0.938	8.172
11	44	7.162	6.478	1.812	1.128	7.749
12	47	7.167	6.637	1.817	1.287	7.299
13	50	7.187	6.766	1.837	1.416	6.932
14	53	7.228	6.880	1.878	1.530	6.608
15	56	7.237	6.974	1.887	1.624	6.302
16	59	7.266	7.037	1.916	1.687	6.023
17	62	7.286	7.102	1.936	1.752	5.741
18	65	7.298	7.178	1.948	1.828	5.475
19	68	7.329	7.225	1.979	1.875	5.279
20	71	7.332	7.237	1.982	1.887	5.072
21 (tip)	73 5/32	7.332	7.285	1.982	1.935	4.884

3. MASS MOMENT PROPERTIES (R_i and m_i)

The mass properties of each part are crucial to balancing any type of rotating machinery. The important properties of each blade are its mass and the location of its center of mass. The mass of each blade is simply measured using a scale and is between 10.5 and 11.5 pounds. The location of the C.M. is much more difficult to find, but it varies between 19.5 and 20.5 inches. These properties were found using two different methods; simple beam, and hanging.

3.1. SIMPLE BEAM METHOD FOR DETERMINING R_i and m_i

The simple beam method is advantageous because its results give both the mass and center of mass location. A disadvantage of this method, however, is that it only gives the location of the y -coordinate of the center of mass. The hanging method described in the next section directly gives the x and y -coordinates of the C.M. and the z -coordinate can be interpolated. Each blade is placed on a set of scales in a manner consistent with Figure 3.1.1.

Figure 3.1.1: Simple beam model used to determine center of mass location

By summing moments about each simple support, the distance to the C.M. can be determined. The weight of the entire blade is known and so is the force on each scale. Only two of these three values, however, are needed to determine the location of the center of mass. Therefore the location is calculated using all three combinations of the three variables and the final results are averaged. It should be noted that force values were not recorded if the force measured on each scale did not add up to the known total weight of the blade. This would occur sometimes during experimental resets due to scale tolerance and accuracy. If this was the case, the test was reset and repeated until the scales each read the appropriate values. This analysis is included in APPENDIX J. Ultimately, this was not the chosen method used to determine the location of the C.M., and instead the hanging method shown below is used in the final measurements.

3.2. HANGING METHOD FOR DETERMINING R_i

This method was performed by hanging each blade from a hoist and using a weighted line as a plumb. The formulation of this method can be seen below in Figure 3.2.1. The experimental setup is shown in Figure 3.2.2.

Figure 3.2.1: Hanging method formulation

It is known from physics that if a line is dropped across any shaped object, while suspended from a fixed point, that it will cross the location of the center of mass. In Figure 3.2.1 the blade is hung freely from a point using a main cable. This cable is attached to an eye bolt which is currently replacing one of the anchor bolts in the root of

the blade. The mass of the eye bolt is assumed negligible compared to the blade system for the purpose of this measurement. From the other side of the eye bolt a line is suspended with a weight on the end. At steady state the system will come to rest with the line crossing the location of the blade's center of mass. A weighted line is hung in front of the blade as shown in the figure but also behind the blade as well.

Figure 3.2.2: Hanging blade to determine the location of its center of mass

In experimental practice the intersection of two lines is needed to determine the position of the center of mass. To accomplish this we simply remove the eye bolt and

reattach it to the opposite side of the root. By taping the lines onto the surface to the blade we can then measure the distance from the blade flange to the intersection point which can be seen in Figure 3.2.3 and more closely in Figure 3.2.4. The true location of the center of mass is inside the blade and can be found by linearly interpolating its position from the center of mass points on the front and back of the blade. See APPENDIX A for detailed procedure on this process.

Figure 3.2.3: Distance to C.M. of each blade is measured using a fine scale

Figure 3.2.4: Example of how center of mass location is determined

The final mass properties of the balanced blades are shown in Table 3.2.1 below. Based on this information the rotor will have a total shake force of 0.7% of total rotor weight at a design speed of 210 RPM.

Table 3.2.1: Final blade mass properties

Blade	Weight (lb _f)	Distance to Center of Gravity (in)	Mass Moment (in*lb _f)
3	11.36	19.38	220.16
4	11.03	19.95	220.05
5	10.89	20.17	219.69
Hub	15.81		
Rotor	51.08		

3.3. INITIAL IMBALANCE ANALYSIS

We performed a dynamic analysis, illustrated in Figure 3.3.1, using a simple model of three rotating masses. Each mass is approximately 120° apart and the net shake force on the main shaft is calculated at 300 RPM. The full analysis is included in

APPENDIX K. In this analysis Blade 5 was chosen to be the "dummy" blade in which no mass would be added. All three blades have roughly the same mass moment so the choice to use blade 5 as the dummy was arbitrary. In order to accommodate the anchor bolts in the root of blade 5, however, a non-adjustable aluminum plug was added to the blade which causes the balance masses in blades 3 and 4 to be larger.

Figure 3.3.1: Dynamic formulation of rotor shake force and balance correction

The shake force calculated in APPENDIX K is used to calculate a specific counterbalance mass that goes inside each blade to cancel the shake force. This analysis

can be seen in APPENDIX M. This analysis then calculates a mass of aluminum plug that fits inside the root of each blade in order to cancel out the shake force. The model also includes the steel threaded rod inside each aluminum plug to act as a fine adjustment on the blade's mass moment. The effect of traveling the rod is also a variable input in the analysis to study the range of mass moments that can be achieved with this configuration. The results from APPENDIX M were then used to design the aluminum plugs in SolidWorks and eventually machine them.

The blade shake force can get large very quickly as the rotor's speed increases. This is why the above analyses are conducted at 300 RPM instead of at the design speed of approximately 210 RPM. Designing a rotor to be balanced at a higher speed is a way of building a safety factor into the balance analysis to compensate for any inaccuracies. The effect of speed on an imbalanced rotor is shown in Figure 3.3.2 where the shake force due to a 1 pound imbalance mass on a spinning part is displayed. It should be noted that the shake force in the main shaft increases with shaft speed squared. At 400 RPM the shake force reaches almost 100 lb_f which could be very detrimental because the Cal Poly rotor, at only 50 lb_m, may see speeds of this magnitude in operation.

Figure 3.3.2: Shake force due to imbalance mass of 1 pound at a distance of 20 inches from the axis of rotation

4. BALANCE WEIGHT DESIGN AND CONSTRUCTION

A set of aluminum plugs was designed for each blade and is anchored into the steel roots with 1/4 inch bolts. These anchor bolts' main function, other than holding the plugs in place, is to keep composite blade attached to the steel root in case the glue fails and the blade is thrown off the rotor. The bolt locations are shown Figure 4.1. In operation, the blades will experience a centripetal force of over 550 lb_f. Without the anchor bolts, the glue and resin of the blade-root interface are the only mechanism with which the assembly is held together.

Figure 4.1: Section view of aluminum plug

All three plugs were made from a solid stock of 6061 aluminum and were machined on a CNC lathe to ensure a tight slip fit into the blade root. Figure 4.2 and Figure 4.3 show SolidWorks models of the plugs. Each plug has a 0.030 inch thick lip that presses into the blade flange pilot. The lip needs to be thin enough not to interfere with the blade flange while it bolts to the hub flange, but also strong enough to prevent the plugs from slipping into the hollow blade. If the plugs slid into the blade, they would not be able to be removed because the steel root extends far into the blade. This assembly is represented in Figure 2.10. The assembly is much like a lobster trap, when the lobster may freely enter, but cannot escape the same way it came.

Figure 4.2: Model of balance plug showing lip, anchor bolt, and weep holes. The centerline hole will contain the steel threaded rod

Figure 4.3: Model view of balance plug showing lip, socket recess, weep, and anchor bolt holes. The inner most hole will contain the steel threaded rod

The plugs have a 1/16 inch square weep hole in it to act as a vent for air. Many design requirements were considered for the weep hole. Mainly, it was a question of whether to have a sealed or unsealed blade. The problem with an unsealed blade is that it could fill with water and the rotor would have to be disassembled to drain it. This could be solved by drilling holes in the tip of the blades. We did not choose this option, however, because we were not sure if would actually cause the blades to collect more water. The problem with a sealed blade is that the air inside, while being heated and cooled throughout the turbine's life, would add pressure forces on the composite blade structure that the blades were not designed to experience. No formal analysis was conducted on how the blades would respond to this fluctuating pressure so we did not choose this design either. We chose the weep hole as the compromise design because it should let most air pass freely, while minimizing the amount of water that could trickle into the blade. Recall that blade 5 needed a dummy plug to accommodate the anchor bolts. This caused the balance masses in blades 3 and 4 to be larger, but it was a design compromise in order to seal all three blades from water, while still allowing air to pass. A partially finished aluminum plug can be seen in Figure 4.4 below.

Figure 4.4: Aluminum balance weight with hole for anchor bolt

Inside each plug is a steel threaded rod that can be used as a fine adjustment on the blade's balance. The rods are 1/2-13 coarse thread and have 5/16 inch hex heads machined at one end to allow turning with a socket wrench. An example of the full balance weight assembly is shown in Figure 4.5. Blade 5, however, does not have an adjustable threaded rod and its central threaded hole is sealed with a short bolt. After balanced adjustments are made the rods are locked in place with lock nuts, ensuring they do not move until desired. The entire balance assembly in each blade is fully contained in the steel blade rod and no external weights or balancing mechanisms are visible on the blade exterior.

Figure 4.5: Balance weight plug with internal threaded rod inside blade root

5. BALANCE METHODS

There are two different types of rotor balancing practices; static and dynamic. Dynamic balancing, illustrated seen in Figure 5.1, involves minimizing the shake force or $rM\omega^2$ of a rotating part using a variable speed motor. Dynamic balancing is accurate, but is also more expensive. It is also impractical for large wind turbines because it requires space the size of the rotor swept area. Static balancing, illustrated in Figure 5.2, involves matching the mass moment about a pivot or rM of each rotating part while stationary. Static balancing is safe and relatively economical in terms of space and cost, but not as accurate. Static balancing was chosen for the Cal Poly rotor for its simplicity, economy, and it would be the most applicable to larger machines. Two different static balance methods were used on the Cal Poly rotor; full rotor and single blade.

Figure 5.1: Dynamic balancing

Figure 5.2: Static balancing

5.1. FULL ROTOR

The entire assembled rotor is balanced on a spike or "frictionless" point and the total rotor tilt is measured. The steady state tilt of the rotor can then be related to its level of imbalance. The formulation of this method can be seen in Figure 5.2. For clarification on the variables shown below refer back to the nomenclature section.

The assembly, shown in Figure 5.3, consists of the assembled rotor with a main shaft inserted into the taper lock. This shaft has a conical cavity drilled into one of its ends and is indexed to measure d_y , the distance between top of the taper lock and the zero mark on the main shaft. A measurement of d_y is shown in Figure 5.1 below.

Figure 5.1: Sample measurement of d_y

When the shaft is placed at the zero mark, the rotor is pivoting at its center of mass. From the measured index d_y , the distance between the pivot point and the C.M., or δ_y , is calculated using the equation below. The expression has units of inches and comes from the analysis in APPENDIX L.

$$\delta_y = 0.8525d_y - 0.3471 \quad (5.1.1)$$

Figure 5.2: Full rotor method formulation

Figure 5.3: Full rotor method top view

The conical cavity is what the spike pivots inside of. The spike assembly consists of a shaped hardened steel dowel pin press fit into the shaft of the propeller table. Figure 5.4 shows the spike and its orientation inside the hub. Note that the spike rests deep inside the hub to ensure the system is stable and the rotor will not fall.

The final aspect of the full rotor balancing method is a bulls eye level that sits atop the main shaft. Figure 5.3 shows a top view of the level with the bubble displaced by a few millimeters. The level has a sensitivity of $24 \text{ min}/0.1''$, or 24 minutes of angle per tenth of an inch, and reads the value d_θ defined in Figure 5.2.

Figure 5.4: Full rotor method spike position components

5.2. SINGLE BLADE

In this method a single blade is balanced on a knife edge with a counterweight. The blade and the counterweight each create a moment about the pivot plane of the knife edge. A load cell is attached to the counterweight and it reads a force F_{LC} . F_{LC} , at a distance from the pivot plane of L_{LC} , is the difference in moment of the two masses. The formulation of this method can be seen in Figure 5.2.1. Each blade's mass moment can be adjusted until the load cell reads the proper, thus balancing the rotor. For more detail on these values refer to Section 9 and APPENDIX C.

Figure 5.2.1: Single blade method formulation

Figure 5.2.2 below shows the setup of the single blade method. The blade is bolted on one front face of the counterweight and the base of its flange is at the pivot plane, ensuring the only mass moment on the right side of the knife edge is from the blade. There is a plate bolted to the bottom of the counterweight with a groove machined into it which the knife edge pivots inside.

Figure 5.2.2: Single blade method components

Figure 5.2.3: Detail view of load cell and counterweight

Figure 5.2.3 above shows close up views of the counterweight and load cell system. The counterweight is adjustable by traveling the threaded hex stock along the rods extending from the counterweight. The large counterweight, 45 lb_f, and adjustable mass moment are necessary to ensure the load cell is not overloaded. For this method to

be accurate, the load cell must have a very high precision. Most high precision load cells have small load limits. We chose a $10 \pm 0.01 \text{ lb}_f$ load cell based on its cost and availability. The counterweight can be adjusted so that the load cell only experiences a small force. With additional time and resources the current load cell could be replaced a 1 lb_f load cell with a higher accuracy for better results.

6. FULL ROTOR CONSTRUCTION

Both balance apparatus' were designed and built at Cal Poly in the Building 13-128 machine shop with the assistance of Jim A. Gerhardt, Cal Poly M.E. Department Technician.

The full rotor balance apparatus consists of a coupling nut attached to the $1 \frac{1}{2} - 8$ threaded shaft on the propeller table. Jammed together with the nut is a bolt with its head machined off. A hole was drilled into the bolt and a $1/8$ inch hardened steel dowel pin, ground to a point, was press fit into the hole. A close up of the spike is shown in Figure 6.1 below.

Figure 6.1: Detail view of spike used in full rotor method

The main shaft, at 1.5 inches in diameter, has a conical cavity machined into one face that the spike may pivot inside shown in Figure 6.2; there are also index marks on the shaft to allow measuring of d_y . d_y is zero at the indicated mark and increases to the left. The cavity was first machined using a 45° milling bit and then sharpened to a point with a chanfer tool. Next the index marks were lightly cut into the shaft using a threading tool bit.

Figure 6.2: Sketch of main shaft with conical cavity and index marks

The complete assembly is shown below with the exception of the blades. The hub is balancing on the spike and the indexed shaft can clearly be seen with the bulls eye level on top.

Figure 6.3: Hub balancing on spike with bulls eye level sitting on top main shaft with index marks

7. SINGLE BLADE CONSTRUCTION

The major parts that needed to be constructed for the single blade balance apparatus are the knife edge, pivot plate, and counterweight. The triangular base was already built and recycled from a previous application.

The knife edge, illustrated in Figure 7.1, was machined from a 1/4 inch thick steel plate and the 1/2 inch wide was cut with an end mill. The sharp edge was machined by tilting the mill head to the desired angle and cutting across the plate. The plate was then turned over and the process was repeated on the other side to give a strong, sharp edge. The as-built knife edge attached to its base is shown in Figure 7.2.

Figure 7.1: Knife edge with slots to bolt onto the steel base

Figure 7.2: Knife edge bolted to base. A c-clamp is used to secure the base to a table

The knife edge interacts with the pivot plate illustrated in Figure 7.3. The pivot plate is made from 3/16 inch aluminum in order to reduce its weight. The weight of the pivot plate is important because a small portion of it must necessarily hang over the knife edge on the blade side. Therefore this part was made as light as possible while maintaining strength and function. The 45° slot appears to be triangular in shape in the model below but the as-built plate has a flat bottom and is shaped like a trapezoid. This is because a 45° milling bit has to have a finite width cutting edge. If the bit came to a perfect point, there would be nothing on that point to cut with. Because of trapezoidal shape of the pivot slot, the knife edge must be aligned with one of the trapezoid corners in operation. The as-built pivot plate is shown in Figure 7.4. For more detailed operating procedure see APPENDIX C.

Figure 7.3: Pivot plate with slots to bolt onto the bottom of the counterweight

Figure 7.4: Bottom view of counterweight with aluminum pivot plate attached

Next a large adjustable counterweight, illustrated in Figure 7.5, was machined from five separate blocks of steel. Five blocks were used because a single piece of steel this large is very expensive. The five blocks were individually cut to size, chamfered, and TIG welded together. The important faces of the counterweight were then faced on a mill to ensure flat and parallel surfaces. The counterweight is so large, about 45 lb_f, because its C.M. is close to the pivot plane and it needs to balance a blade induced mass moment of approximately 220 in- lb_f. The blade bolts to the front of the counterweight with the 5-bolt pattern shown below and the placement of the flange and pilot illustrated by the grey circles.

Figure 7.5: Counterweight, front view. Blade flange bolts to the front where the grey rings are positioned. Pivot plate is bolted to the bottom, and adjustable weights travel along threaded rods that extend out the back

Figure 7.6: Front view of counterweight with blade flange bolt pattern

The central slot in the counterweight is so that the threaded rod inside the root may be accessed by a socket wrench. Note the as-built counterweight, pictured in Figure 7.6, only has three bolts to hold the blade. In the back of the counterweight are three holes that will hold 5/8-11 threaded rods. These are most visibly seen in Figure 5.2.3. On these rods there will be steel threaded hex-sock weights that may be traveled to change the counterweight's mass moment. The bottom of the counterweight also has steel ear tabs welded on, post machining, to attach the ball joints of the load cell and safety tie rods.

The safety linkage is one continuous tie rod and the load cell linkage consists of the load cell and a tie rod portion beneath it. This assembly is illustrated in Figure 7.7. The tie rods were machined from aluminum hex stock with 1/4-28 threads. The threads on one side are right hand and the threads on the other are left hand. Mated to these

threads are a right and left hand thread ball joints respectively. This allows the rods, when turned, to extend or contract. This is important for enduring the level, relative to the horizontal, of the apparatus. Each tie rod may also be locked in place with a set of jam nuts.

The full single blade apparatus assembly is represented in Figure 7.7. The load cell and safety rod may freely pivot up and down on their respective ball joints and are held in place with 1/4 inch bolts that act as pins. Refer to APPENDIX C for detailed procedure on using the single blade balance apparatus and APPENDIX O for SolidWorks drawings of the main components.

Figure 7.7: Full assembly of single blade method

8. FULL ROTOR BALANCING

The Cal Poly Rotor was balanced using the two static balancing methods described previously. These methods were developed because little information in the literature exists on wind turbine rotor balancing.

In the full rotor balancing method the rotor is assembled with all three blades (at design pitch) bolted to the hub, as near to the operational state as possible. This position is important because the blade's centers of mass do not lie along the pitch axis of rotation. If one blade is miss-pitched during the experiment, then in operation the rotor will have a different state of balance. An indexed shaft is inserted into the hub's taper lock and the markings on it are ultimately used to calculate δ_y , the distance between the pivot point and the center of mass of the whole system. This parameter controls the sensitivity of the method. Note that the rotor must pivot above its center of mass, i.e. $\delta_y > 0$, in order to be a stable system. The smaller δ_y gets the more sensitive and unstable the system becomes. Recall that the shaft has a conical cavity machined into its bottom face in which a spike pivots inside of. The whole rotor is mounted on the spike and a bulls-eye level is used to measure the angle of tilt of the rotor. An example of the rotor tilt reading is shown in Figure 8.1. From this angle and the known distance δ_y , the level of total imbalance can be determined from the steady state tilt of the rotor. The above mentioned relationships can be seen in Figure 8.2. This figure was generated using the script in APPENDIX N.

Figure 8.1: Example of measureable rotor tilt

Figure 8.2: Relationship between δ_y and δ_θ

In this analysis it is important to note that the hub, at an angle of tilt δ_θ , creates a righting moment of the system that does not occur in turbine operation. This righting moment of the hub is significant because even though the distance its center of mass is displaced is small, its mass is comparable to that of a blade. Further analysis and comparisons show that the level of imbalance concluded from this method can be in error of an order of magnitude. The full rotor method is also not practical for larger rotors as it requires the space of the rotor swept area to perform. Using this method, with a distance

δ_y of approximately 0.73 inches, the balanced rotor has a steady state tilt of about 0.08 degrees (0.5 mm of lateral bubble displacement). This corresponds to a rotor within 0.0084% of total balance. This value, however, corresponds to the generically written expression below, where B is the level of balance.

$$B = \frac{R_2 m_2}{R_1 m_1 + R_{hub} m_{hub}} \quad (8.1)$$

As can be seen in the above equation, the righting moment of the hub in the denominator gives a false level of balance confidence. A more accurate level of balance is estimated to be 0.13%, by zeroing $R_{hub} m_{hub}$. The Shake force based on this value is approximately 0.7% of total rotor weight based on a dynamic analysis using the known blade weights and R_i vectors. Refer to APPENDIX B for detailed procedure on conducting the full rotor balancing exercise.

9. SINGLE BLADE BALANCING

Recall that the single blade balancing method involves a blade bolted to a counterweight that pivots on a knife edge. The counterweight is connected to a 10 lb_f load cell and the cell reads a force at $5\frac{3}{8}$ inches from the pivot point and is the difference in mass moment between the blade and counterweight. Blade 5 or the dummy blade is attached to the apparatus and the load cell is zeroed. Because the pivot point on the single blade apparatus is shorter than the actual rotor operational pivot point, the radius of the hub, 4.557 inches, must be subtracted from the R_i value of the blade. This is illustrated

more clearly in Figure 5.2.1. This means that the load cell will never read zero for all three blades and it will display some offset according to the analysis below.

$$F_{LC}L_{LC} = [(R_i - R_{hub})m_i - (R_{cal} - R_{hub})m_{cal}] \quad (9.1)$$

$$F_{LC}L_{LC} = [R_im_i - R_{hub}m_i - R_{cal}m_{cal} + R_{hub}m_{cal}] \quad (9.2)$$

$$F_{LC}L_{LC} = [\cancel{R_im_i} - R_{hub}m_i - \cancel{R_{cal}m_{cal}} + R_{hub}m_{cal}] \quad (9.3)$$

In order for the rotor to be balanced, R_im_i and $R_{cal}m_{cal}$ must be equal. Because the distance to the pivot point, however, is short by R_{hub} , the remaining mass moment experienced by the load cell can be seen in equation (9.4) below.

$$F_{LC}L_{LC} = R_{hub}[m_i - m_{cal}] \quad (9.4)$$

After the calibration blade is zeroed, each consecutive blade's mass moment is adjusted until the load cell reaches the desired value calculated above. The results of which are seen in Table 9.1. A more accurate load cell would be required to balance the rotor any further. See APPENDIX C for detailed procedure on conducting the single blade balancing exercise.

Table 9.1: Observed and predicted load cell values for single blade method

Blade	Observed F_{LC}	Calculated $ F_{LC} $	Load Cell Error (%)
3	-0.34	0.339	-0.29
4	-0.06	0.059	-1.17
5	0	0	N/A

10. FIELD TEST

An accelerometer is attached to the main shaft bearing, sampling at 100 Hz, and oriented as shown in Figure 10.1. The X -axis is orientated in the plane of the rotor, the Y -axis is along the main shaft and the Z -axis is parallel to the tower.

Figure 10.1: Accelerometer orientation. The X -axis is in the plane of the rotor, the Y -axis is along the main shaft and the Z -axis is parallel to the tower

A gear tooth sensor measures shaft rotations and a control system can vary the electrical load seen by the generator. When the control system is active during turbine operation, a speed set point may be chosen for the rotor. The control system will vary the load seen by the generator to maintain a shaft speed within a certain range, depending on wind speed. For example: A speed set point of 200 RPM is set on the controller. If the wind speed increases and the shaft speed increases above the set point, the generator load is increased and vice versa. The load consists of large heating elements submerged in barrels of water. This, however, is not how the load was controlled during the final balance test due to a faulty solid state relay. Instead, the load was pulsed by hand to try and maintain a rotor speed between 100 and 300 RPM during each test run.

The field test results are displayed in Figure 10.2 below where the error bars represent one standard deviation of each data series presented at the end of APPENDIX P. The balance state of the rotor during the test was the best balanced we achieved using the single blade method. During the 7/20/12 test the rotor was run in its balanced state, then the rotor was disassembled and the balance weights removed and it was run again. Finally, the balance weights were re-installed and the rotor was run for the third time to verify the improvement in balance. There is the least amount of vibration in the Z-direction because vibration is damped due to the mass of the tower. The vibration is slightly larger in the Y-direction but still relatively small. This is expected since the axis is along the main shaft of the turbine. The X-direction experiences the most vibration because the rotor is free to move and yaw, allowing the most shake force. Figure 10.2 was created using the RMS value of the accelerations over 1 second time intervals for

each axis. The RMS values were then binned into speed groupings of 20 RPM and the average values of those bins are shown in the Figure 10.2. Figure 10.3 shows acceleration data analyzed in the same way but instead uses the average of the absolute value of acceleration.

Figure 10.2: Average main shaft accelerations for the Cal Poly Rotor at a speed range of 201 - 220 RPM using RMS values. Data recorded on 7/20/12

Figure 10.3: Average main shaft accelerations for the Cal Poly Rotor at a speed range of 201 - 220 RPM using averages of the absolute values. Data recorded on 7/20/12

The vibration in the *X*-direction is reduced by 19% due to rotor balancing on the day of the test. This is also the largest vibration reduction of the three coordinate directions. Refer to APPENDIX P for full field test results and data. The wind speed data noted in APPENDIX P is an average for each test as the wind speed varied significantly throughout the day. The best experimental balancing recorded during the 7/20/12 test resulted in an overall minimum shake force of 3.5% of rotor weight; reduced from 4.3% during the unbalanced test. Refer to APPENDIX D for detailed procedure on conducting a field test.

Recall Figure 3.3.2 in which the exponential increase in shake force with speed is illustrated. Comparing with Figure 10.4 below we do not see as dramatic an increase in acceleration, even in the unbalanced configuration. Also notice that the acceleration in the re-balanced test remains nearly constant throughout all rotor speeds. This test had the lowest average wind speed and the almost constant suggests a systematic error in the measurement setup which could be addressed with additional testing. Between 0 and 75 RPM the acceleration steady at around 0.0275g for all three tests.

Figure 10.4: Acceleration in the X-direction as a function of rotor speed. Averages of the absolute values of acceleration were used in this plot. Note that between 0 and 75 RPM the acceleration steady at around 0.0275g for all three tests. This is likely the systematic error in the data

The systematic error in the accelerometer alignment can better be seen in Figure 10.5 where the rotor is locked and not spinning for all wind speeds. We still see accelerations in the X, Y, and Z axes though. The acceleration in the X-direction is approximately 0.0275g as we saw in Figure 10.4. This suggests that during the re-balanced rotor test the acceleration at all rotor speeds is nearly identical to the locked rotor values and therefore a very successful balancing.

Figure 10.5: Average acceleration for locked rotor (0 RPM)

11. CONCLUSIONS

The ultimate effect of rotor balancing is to increase annual energy output. Current methods such as pitch offset do reduce main shaft vibrations due to imbalance but at the loss of energy capture. According to Moog, a pitch control company, A 2.5 MW turbine could lose 18000 kWh per month due to blade imbalance [10]. Not only do imbalanced rotors have reduced energy capture but the maintenance and repair cost of replacing bearings and gearboxes over the lifetime of the machine reduces overall return on investment on the turbine.

Using a combination of both the full rotor and single blade balance methods we achieved a reduction in main shaft acceleration of 19% in the *X*-direction and if added to the manufacturing process, could greatly decrease the need for in-situ rotor balancing or pitch-offset. Ultimately we were able to achieve a rotor with a shake force of 3.5% of its weight which is within the G16 standard of 3.8% calculated for our rotor. We believe, however, that with a more accurate load cell and refinement of the single blade method, it has the potential to be successfully applied to larger wind turbines.

APPENDIX A. DETAILED METHODOLOGY: HANGING METHOD

1. Select any blade.
2. Remove one anchor bolt from the root and thread an eye bolt into the hole. Preferably at least 12 inches long but the longer the bolt, the more accurate the results will be.
3. Attach a rope or cable to the end of the eye bolt and the other end of the cable to a hoist or mount. The hoist should be high enough that the blade is allowed to swing and turn freely and not be contacted by any other object or surface.
4. Cut two lengths of fishing line at least 6 feet long and attach a weight to the end of each line.
5. Tie the free end of each line to the cable that is connecting the eye bolt to the hoist or mount. Let one line drop down the top of the blade and the other line drop down the bottom of the blade. Make sure that the lines are not being interfered with by the main cable, each other, or the eye bolt in the middle.
6. Let the system come to rest.
7. Once the system has come to rest, pluck each line several times to ensure it has settled in its natural location (the line through which the C.M is located).
8. Let the system come to rest once again and observe the position of each line on the top and bottom of the blade.
9. Carefully tape each line in place and cut the weights off the line.
10. Cut the top of the fishing line that is tied to the main cable so the only portion of the fishing line left is that which is taped down onto each side of the blade.
11. Remove the main line from the hoist and take blade down and set on floor or table.

12. Remove eye bolt and replace anchor bolt.
13. Repeat steps 2 through 12 with the other anchor bolt location on the root
14. Once steps 1 through 13 have been completed, there should be two intersecting fishing lines taped to each side of the blade. The intersection of those lines should be the location of the C.M. of that blade.
15. Measure the distance from the root to the intersection point on both sides of the blade.
16. Average the two values to linearly interpolate the location of the true C.M. which is somewhere inside the blade.
17. Add the radius of the hub to the value recorded in step 16 the final result is the location of the C.M. of the blade.
18. Repeat for as many blades as desired.

APPENDIX B. DETAILED METHODOLOGY: FULL ROTOR

1. Assemble full rotor at some consistent pitch. Design pitch or zero pitch are good reference points.
2. Insert the indexed shaft that the spike pivots inside into the hub taper lock.
3. Start with the shaft higher up in the taper lock and lock it in place. The farther above the center of mass the rotor pivots, the more stable and less sensitive the system will be. Make sure, however, to have at least 1.5 inches of shaft inside the taper lock to ensure shaft is secure and straight.
4. Measure and record the distance from the top of the taper lock to the main index on the shaft. This index represents the estimated location of the rotor C.M. Therefore, when the shaft is positioned in this spot, the rotor should be pivoting at its center of

mass. Do not attempt this however because this mark is only an approximation and the system would likely be unstable and the rotor would fall. The distance between the taper lock and the main index is the value d_y referred to in

5. **NOMENCLATURE.** From this value δ_y can be calculated using the equation (5.1.1). The value δ_y is the approximation of the actual distance between the C.M and the pivot point.
6. Lift the rotor (2 people required) and gently place the conical cavity of the shaft onto the spike. It is important to make sure both parts are free of burs, divots, and metal fragments.
7. Gently spin the rotor a few degrees back and forth a few degrees to ensure the spike is not only centered, but is not caught on anything or out of position.
8. Place bulls eye level on top of shaft and let rotor come to rest. The more sensitive the analysis it, the smaller d_y , the longer it will take the rotor to settle. When the analysis becomes very sensitive later in the balancing, even wind from the door or a person walking by -will disturb the rotor.
9. Once the rotor has come to rest, measure the distance the bulls eye level has moved from its' center with a fine ruler and record this value, known as d_{bubble} . Using the level's sensitivity rating, this distance can also be converted to an angle.
10. Once δ_y and d_{bubble} have been found, use Figure 8.2 to find the appropriate error curve. If between curves, use a linear interpolation.
11. Once the appropriate error curve has been found refer to APPENDIX H and solve for the value of B for that curve. B is the value of equation (8.1) and is the fraction that

the blade with the largest mass moment is larger than the vector sum of the other two blade's mass moments, plus the mass moment of the hub. The reason B is a linear function of d_θ is because the moment arm of the hub, d_x , is a direct function of the rotor's tilt.

12. Place small weights (washers work well) on the tips of the appropriate blade or blades to level the rotor as accurately as possible.
13. The mass of the washers multiplied by their distance from the pivot point represents the mass moment that needs to be added to that blade(s), or subtracted from the other blade(s) to level the rotor. Also, do not forget to use a vector sum of the mass moments when applying weights to multiple blades.
14. Remove all weights and carefully remove the rotor assembly from the spike (2 people required) and place it back on a table or floor and remove all blades.
15. Use the internal threaded rod to adjust the mass moment of each blade by the appropriate amount calculated above.
16. Reassemble the rotor and repeat steps 5 through 10 to verify the rotor's new state of balance. If the bull's eye level appears to be centered then this iteration of balancing is complete. Continue to step 16 if more accurate balancing is desired.
17. Carefully remove the rotor from the spike (2 people required) and place it back on a table or floor
18. Loosen the taper lock on the hub and move the shaft down by a small amount; 1/8 to 1/4 inch depending on how balanced the rotor was in the previous iteration. Also, move the shaft in smaller increments as balancing near the true C.M. is approached.

19. Lock the shaft in place and repeat steps 4 through 15 until the shafts placement makes the rotor too sensitive to precede with any further balancing. This occurs at a d_y of about $0.7^{+0.05}_{-0.00}$ inches.

APPENDIX C. DETAILED METHODOLOGY: SINGLE BLADE

1. Make sure the load cell is not connected to the counterweight and the safety tie rod is in place
2. Plug in the load cell and let it warm up for 15 minutes
3. Attach the dummy blade, blade 5 for this analysis, to the appropriate side of the counterweight using two bolts and washers. Make sure the blade is indexed to a repeatable amount and that all other blades can be set to this value.
4. Use a rubber mallet to ensure the counterweight has the appropriate horizontal alignment on the knife edge using the two red marks. The pivot plate should be centered between these marks as accurately as possible and should be re-centered after every adjustment or blade change.
5. Tilt up the load cell and connect the ball joint to its appropriate tabs. The load cell and safety rod should both be connected, vertical and straight.
6. Remove the top bolt from the tab connecting the safety rod to the counterweight and tilt it down until it comes to rest on the base plate.
7. Press the pivot plate towards the load cell to ensure the knife edge is not crooked and the pivot plate is in a consistent and appropriate position. This is required because the pivot plate has a trapezoidal groove machined into it so there are two corners in which

the knife edge can rest. It does not matter which corner the knife is resting on, as long as its position and is stable and repeatable for all tests.

8. Repeat step 4.
9. With the dummy blade attached, zero the load cell.
10. Tap on both the counterweight and then the blade and when the system comes to rest, verify that the load cell still reads zero. If not, then repeat steps 7 and 8 until the load cell remains zeroed consistently.
11. Replace the safety rod.
12. Remove the load cell.
13. Remove the dummy blade.
14. Repeat steps 3 through 8 with one of the other blades.
15. Use equation (9.4) to calculate what force the load cell needs to read in order to balance the rotor.
16. Unlock the nut holding the threaded rod in place travel the threaded rod a small amount in the appropriate direction to increase or decrease the mass moment of the blade.
17. Repeat step 4
18. Check the reading on the load cell. If it is not the desired reading then repeat steps 16 and 17 until the desired value is reached. Note that the threaded rod may only be traveled so far into the plug as it needs room for the lock nut to work properly. The lock nut should always have full threads throughout its length. The threaded also may not protrude past the pilot of the blade and stick into the hub. Because of these constraints, it may be necessary to remove the plug-rod assembly and jam nuts

- together on the rod to increase its mass and mass moment. It may also be necessary to remove the threaded rod from the plug and cut the rod down to decrease its mass and mass moment.
19. Once the load cell reads the appropriate value, lock the threaded rod in place. Note that this will travel the rod by at least half a turn and may change the reading on the load cell. If it does change the reading on the load cell, unlock the rod, compensate for this error, and re-lock the rod in place.
 20. Replace the safety rod.
 21. Remove the load cell.
 22. Remove the blade.
 23. Repeat steps 14 through 22 with the remaining blade to complete the rotor balancing

APPENDIX D. DETAILED METHODOLOGY:

FIELD TESTING

1. Tilt down tower and access SD card slot in the bottom of the nacelle. Removal of Nacelle fairing will be required.
2. Insert SD card into slot. Make sure the card has the proper and current configuration file already on board.
3. Connect 9V battery. Care should connect the battery to the proper terminals as a backwards battery may severely damage the system
 - a. Each time the battery is connected the SD card will create a new file with the name DATA_XXX. The file names will start at 000 and increase consecutively each time a battery connection is established. Later a

MATLAB script is needed to convert the raw file data into times and accelerations measured in g's.

4. Make sure the red LED on the circuit board is flashing before proceeding. If the light is not flashing then remove battery and card and repeat steps 2 and 3.
5. Tilt tower up
6. Make sure the load is turned off and disengage turbine brakes
7. Vary the load by hand or using the on-board PWM controller to approximately maintain the desired rotor speed for a given duration of time
8. Once the desired number of test runs are complete engage the turbine brakes and turn off the load
9. Lower the tower
10. Disconnect battery and remove SD card
11. Run MATLAB script to clean the raw data
12. Output file should be an Excel document with columns from left to right in the following order: time, shaft speed in RPM, acceleration in the x -direction, acceleration in the y -direction, acceleration in the z -direction. All accelerations are measured in g

APPENDIX E. FLOWCHART

APPENDIX F. COMPLETE BLADE GEOMETRY

Blade 1						
	H _{LE} Distance from Table Edge (in)	5.75	Hub Height (in)	3.269		
Section	Length From Hub Axis (in)	H _{LE} (in)	H _{TE} (in)	Normalized H _{LE} (in)	Normalized H _{TE} (in)	Chord Length (in)
1	13 15/16	7.095	1.309	1.826	-3.960	13.157
2	16 15/16	7.028	2.375	1.759	-2.894	12.720
3	19 15/16	7.078	3.247	1.809	-2.022	12.280
4	22 15/16	7.065	4.365	1.796	-0.904	11.695
5	25 15/16	7.185	4.633	1.916	-0.636	11.068
6	28 15/16	7.239	5.180	1.970	-0.089	10.359
7	31 15/16	7.243	5.637	1.974	0.368	9.733
8	34 15/16	7.260	5.990	1.991	0.721	9.153
9	37 15/16	7.307	6.250	2.038	0.981	8.657
10	40 15/16	7.329	6.503	2.060	1.234	8.175
11	43 15/16	7.402	6.718	2.133	1.449	7.713
12	46 15/16	7.437	6.909	2.168	1.640	7.285
13	49 15/16	7.489	7.078	2.220	1.809	6.925
14	52 15/16	7.500	7.198	2.231	1.929	6.618
15	55 15/16	7.555	7.319	2.286	2.050	6.300
16	58 15/16	7.603	7.395	2.334	2.126	6.013
17	61 15/16	7.622	7.488	2.353	2.219	5.752
18	64 15/16	7.682	7.569	2.413	2.300	5.509
19	67 15/16	7.704	7.651	2.435	2.382	5.277
20	70 15/16	7.743	7.712	2.474	2.443	5.088
21 (tip)	73 5/8	7.768	7.778	2.499	2.509	4.815

Blade 2						
	H _{LE} Distance from Table Edge (in)	5 29/32	Hub Height (in)	3.342		
Section	Length From Hub Axis (in)	H _{LE} (in)	H _{TE} (in)	Normalized H _{LE} (in)	Normalized H _{TE} (in)	Chord Length (in)
1	13 7/8	7.195	1.429	1.853	-3.913	12.926
2	17	7.076	2.429	1.734	-2.913	12.678
3	20	7.098	3.215	1.756	-2.127	12.270
4	23	7.087	3.999	1.745	-1.343	11.690

5	26	7.076	4.531	1.734	-0.811	11.039
6	29	7.027	5.031	1.685	-0.311	10.365
7	32	7.027	5.423	1.685	0.081	9.740
8	35	7.025	5.734	1.683	0.392	9.204
9	38	7.025	5.967	1.683	0.625	8.676
10	41	7.025	6.183	1.683	0.841	8.189
11	44	7.022	6.364	1.680	1.022	7.747
12	47	7.002	6.499	1.660	1.157	7.327
13	50	7.002	6.608	1.660	1.266	6.947
14	53	7.018	6.691	1.676	1.349	6.622
15	56	7.018	6.762	1.676	1.420	6.314
16	59	7.018	6.819	1.676	1.477	6.018
17	62	7.000	6.890	1.658	1.548	5.723
18	65	7.000	6.924	1.658	1.582	5.472
19	68	7.000	6.920	1.658	1.578	5.278
20	71	7.000	6.949	1.658	1.607	5.100
21 (tip)	73 9/16	6.969	6.978	1.627	1.636	4.885

Blade 3						
	H _{LE} Distance from Table Edge (in)	5 13/16	Hub Height (in)	3.350	Weight (lb _f)	10.15
Section	Length From Hub Axis (in)	H _{LE} (in)	H _{TE} (in)	Normalized H _{LE} (in)	Normalized H _{TE} (in)	Chord Length (in)
1	13 7/8	7.146	1.340	1.796	-4.010	12.864
2	17	7.008	2.354	1.658	-2.996	12.568
3	20	7.008	3.195	1.658	-2.155	12.156
4	23	7.018	3.888	1.668	-1.462	11.595
5	26	7.078	4.529	1.728	-0.821	10.963
6	29	7.101	5.042	1.751	-0.308	10.321
7	32	7.091	5.439	1.741	0.089	9.719
8	35	7.118	5.758	1.768	0.408	9.160
9	38	7.134	6.060	1.784	0.710	8.625
10	41	7.151	6.288	1.801	0.938	8.172
11	44	7.162	6.478	1.812	1.128	7.749
12	47	7.167	6.637	1.817	1.287	7.299
13	50	7.187	6.766	1.837	1.416	6.932
14	53	7.228	6.880	1.878	1.530	6.608
15	56	7.237	6.974	1.887	1.624	6.302
16	59	7.266	7.037	1.916	1.687	6.023
17	62	7.286	7.102	1.936	1.752	5.741

18	65	7.298	7.178	1.948	1.828	5.475
19	68	7.329	7.225	1.979	1.875	5.279
20	71	7.332	7.237	1.982	1.887	5.072
21 (tip)	73.15625	7.332	7.285	1.982	1.935	4.884

Blade 4						
	H _{LE} Distance from Table Edge (in)	5 3/16	Hub Height (in)	3.342	Weight (lb _r)	10.05
Section	Length From Hub Axis (in)	H _{LE} (in)	H _{TE} (in)	Normalized H _{LE} (in)	Normalized H _{TE} (in)	Chord Length (in)
1	13.5	7.133	1.227	1.791	-4.115	12.931
2	17	7.099	2.341	1.757	-3.001	12.708
3	20	7.105	3.185	1.763	-2.157	12.312
4	23	7.105	3.873	1.763	-1.469	11.738
5	26	7.116	4.493	1.774	-0.849	11.103
6	29	7.086	5.042	1.744	-0.300	10.417
7	32	7.086	5.454	1.744	0.112	9.784
8	35	7.086	5.779	1.744	0.437	9.229
9	38	7.096	6.071	1.754	0.729	8.678
10	41	7.107	6.303	1.765	0.961	8.170
11	44	7.116	6.482	1.774	1.140	7.717
12	47	7.125	6.605	1.783	1.263	7.336
13	50	7.140	6.732	1.798	1.390	6.977
14	53	7.162	6.834	1.820	1.492	6.662
15	56	7.162	6.930	1.820	1.588	6.324
16	59	7.162	7.030	1.820	1.688	6.029
17	62	7.162	7.062	1.820	1.720	5.766
18	65	7.178	7.120	1.836	1.778	5.535
19	68	7.197	7.166	1.855	1.824	5.302
20	71	7.205	7.193	1.863	1.851	5.094
21 (tip)	73 23/32	7.205	7.202	1.863	1.860	4.916

Blade 5						
	H _{LE} Distance from Table Edge (in)	5 11/16	Hub Height (in)	3.345	Weight (lb _r)	10.13
Section	Length From Hub Axis (in)	H _{LE} (in)	H _{TE} (in)	Normalized H _{LE} (in)	Normalized H _{TE} (in)	Chord Length (in)
1	13.5	7.131	1.222	1.786	-4.123	13.038
2	17	6.946	2.389	1.601	-2.956	12.706
3	20	7.114	3.262	1.769	-2.083	12.234

4	23	7.131	3.986	1.786	-1.359	11.644
5	26	7.137	4.635	1.792	-0.710	10.999
6	29	7.138	5.139	1.793	-0.206	10.391
7	32	7.150	5.558	1.805	0.213	9.772
8	35	7.160	5.900	1.815	0.555	9.225
9	38	7.161	6.217	1.816	0.872	8.666
10	41	7.207	6.446	1.862	1.101	8.162
11	44	7.227	6.616	1.882	1.271	7.742
12	47	7.236	6.761	1.891	1.416	7.360
13	50	7.265	6.890	1.920	1.545	6.963
14	53	7.273	7.014	1.928	1.669	6.648
15	56	7.315	7.115	1.970	1.770	6.298
16	59	7.330	7.210	1.985	1.865	5.977
17	62	7.346	7.244	2.001	1.899	5.786
18	65	7.357	7.299	2.012	1.954	5.583
19	68	7.370	7.353	2.025	2.008	5.349
20	71	7.382	7.373	2.037	2.028	5.125
21 (tip)	73 11/16	7.399	7.394	2.054	2.049	4.950

Blade 6						
	H _{LE} Distance from Table Edge (in)	5 11/16	Hub Height (in)	3.338		
Section	Length From Hub Axis (in)	H _{LE} (in)	H _{TE} (in)	Normalized H _{LE} (in)	Normalized H _{TE} (in)	Chord Length (in)
1	13.5	6.973	1.309	1.635	-4.029	12.997
2	17	6.929	2.486	1.591	-2.852	12.691
3	20	6.987	3.369	1.649	-1.969	12.209
4	23	6.983	4.092	1.645	-1.246	11.614
5	26	7.053	4.716	1.715	-0.622	11.011
6	29	7.108	5.256	1.770	-0.082	10.368
7	32	7.165	5.691	1.827	0.353	9.724
8	35	7.230	6.030	1.892	0.692	9.197
9	38	7.254	6.340	1.916	1.002	8.699
10	41	7.280	6.563	1.942	1.225	8.191
11	44	7.350	6.799	2.012	1.461	7.720
12	47	7.379	6.966	2.041	1.628	7.330
13	50	7.454	7.129	2.116	1.791	6.918
14	53	7.498	7.260	2.160	1.922	6.602
15	56	7.574	7.378	2.236	2.040	6.329
16	59	7.628	7.493	2.290	2.155	6.031
17	62	7.650	7.579	2.312	2.241	5.751

18	65	7.715	7.660	2.377	2.322	5.513
19	68	7.781	7.737	2.443	2.399	5.328
20	71	7.843	7.834	2.505	2.496	5.161
21 (tip)	73 5/8	7.894	7.897	2.556	2.559	4.904

Blade 7						
	H _{LE} Distance from Table Edge (in)	5 27/32	Hub Height (in)	3.352		
Section	Length From Hub Axis (in)	H _{LE} (in)	H _{TE} (in)	Normalized H _{LE} (in)	Normalized H _{TE} (in)	Chord Length (in)
1	13.5	7.082	1.201	1.730	-4.151	12.991
2	17	6.991	2.369	1.639	-2.983	12.675
3	20	7.047	3.214	1.695	-2.138	12.195
4	23	7.060	3.947	1.708	-1.405	11.606
5	26	7.028	4.590	1.676	-0.762	10.933
6	29	7.148	5.100	1.796	-0.252	10.268
7	32	7.168	5.551	1.816	0.199	9.665
8	35	7.171	5.860	1.819	0.508	9.102
9	38	7.174	6.142	1.822	0.790	8.617
10	41	7.179	6.360	1.827	1.008	8.158
11	44	7.197	6.546	1.845	1.194	7.752
12	47	7.228	6.703	1.876	1.351	7.339
13	50	7.258	6.835	1.906	1.483	6.953
14	53	7.281	6.966	1.929	1.614	6.596
15	56	7.293	7.037	1.941	1.685	6.292
16	59	7.322	7.109	1.970	1.757	6.046
17	62	7.334	7.157	1.982	1.805	5.782
18	65	7.362	7.222	2.010	1.870	5.526
19	68	7.373	7.275	2.021	1.923	5.303
20	71	7.380	7.298	2.028	1.946	5.155
21 (tip)	73 5/8	7.387	7.328	2.035	1.976	4.945

Blades 3,4,5 Average Geometry								
	H _{LE} Distance from Table Edge (in)	5 9/16						
Section	Length From Hub Axis (in)	H _{LE} (in)	H _{TE} (in)	Normalized H _{LE} (in)	Normalized H _{TE} (in)	Chord Length (in)	Twist (deg)	r/R
1	13.625	7.137	1.263	1.791	-4.083	12.944	27.77	0.19
2	17	7.018	2.361	1.672	-2.984	12.661	22.93	0.23

3	20	7.076	3.214	1.730	-2.132	12.234	19.48	0.27
4	23	7.085	3.916	1.739	-1.430	11.659	16.84	0.31
5	26	7.110	4.552	1.765	-0.793	11.022	14.40	0.35
6	29	7.108	5.074	1.763	-0.271	10.376	12.35	0.39
7	32	7.109	5.484	1.763	0.138	9.758	10.69	0.44
8	35	7.121	5.812	1.776	0.467	9.205	9.26	0.48
9	38	7.130	6.116	1.785	0.770	8.656	7.82	0.52
10	41	7.155	6.346	1.809	1.000	8.168	6.66	0.56
11	44	7.168	6.525	1.823	1.180	7.736	5.70	0.60
12	47	7.176	6.668	1.830	1.322	7.332	4.90	0.64
13	50	7.197	6.796	1.852	1.450	6.957	4.10	0.68
14	53	7.221	6.909	1.875	1.564	6.639	3.32	0.72
15	56	7.238	7.006	1.892	1.661	6.308	2.61	0.76
16	59	7.253	7.092	1.907	1.747	6.010	1.92	0.80
17	62	7.265	7.136	1.919	1.790	5.764	1.57	0.84
18	65	7.278	7.199	1.932	1.853	5.531	0.98	0.88
19	68	7.299	7.248	1.953	1.902	5.310	0.49	0.92
20	71	7.306	7.268	1.961	1.922	5.097	0.29	0.97
21 (tip)	73.521	7.312	7.294	1.966	1.948	4.917	0.00	1.00

Horizontal Alignment

Blade	Vertical Location (in)	Horizontal Location (in)	Distance From Hub Centerline (in)	Slope (deg)
1	72	6 1/16	2.93750	0.213
	30	5 29/32	3.09375	
2	72	5 9/16	3.43750	-0.298
	30	5 25/32	3.21875	
3	72	5 27/32	3.15625	0.043
	30	5 13/16	3.18750	
4	72	5 3/4	3.25000	-0.043
	30	5 25/32	3.21875	
5	72	5 19/32	3.40625	-0.085
	30	5 21/32	3.34375	
6	72	6 3/16	2.81250	0.384
	30	5 29/32	3.09375	
7	72	5 9/16	3.43750	-0.256
	30	5 3/4	3.25000	
Hub Centerline (in)	N/A	9		

Vertical Alignment

Blade	Vertical Location (in)	Height (in)	Slope (deg)
1	Tip	2.499	0.722
	32	1.974	
2	71	1.658	-0.040
	32	1.685	
3	Tip	1.982	0.336
	32	1.741	
4	Tip	1.863	0.163
	32	1.744	
5	Tip	2.054	0.342
	32	1.805	
6	Tip	2.556	1.003
	32	1.827	
7	Tip	2.035	0.301
	32	1.816	

APPENDIX G. SOLIDWORKS BALANCE WEIGHT DESIGNS

APPENDIX H. FULL ROTOR ERROR FUNCTIONS

Function	B for First Positive Value of δ_y (in)	Equation
e_1	0.00700	$B = 0.000299*d_\theta + 1.00408$
e_2	0.00600	$B = 0.000298*d_\theta + 1.00350$
e_3	0.00500	$B = 0.000297*d_\theta + 1.00292$
e_4	0.00400	$B = 0.000296*d_\theta + 1.00233$
e_5	0.00300	$B = 0.0002948*d_\theta + 1.001750$
e_6	0.00200	$B = 0.0002950*d_\theta + 1.001167$
e_7	0.00150	$B = 0.0002940*d_\theta + 1.000874$
e_8	0.00100	$B = 0.0002938*d_\theta + 1.000585$
e_9	0.00075	$B = 0.0002941*d_\theta + 1.000438$
e_{10}	0.00050	$B = 0.0002938*d_\theta + 1.000291$
e_{11}	0.00025	$B = 0.0002940*d_\theta + 1.000144$
e_{12}	0.00010	$B = 0.0002959*d_\theta + 1.000061$

APPENDIX I. MATLAB 3-D COMPARISON OF BLADE LEADING EDGES

```
% leading edge of all 7 blades in 3-D space
%
% X1, Y1, Z1 are the coordinates of blade 1 and ect...
% in relation to the prop table, X is the lateral coordinate of the
% leading edge, Y is the distance from the hub along the length of
% the blade, and Z is the vertical coordinate of the Leading edge
%
% all values are taken from the spreadsheet Blade Characteristics.xlsx

X1 = [3.09375, 2.9375];
Y1 = [30, 70];
Z1 = [1.974, 2.499];

X2 = [3.21875, 3.43750];
Y2 = [30, 70];
Z2 = [1.685, 1.658];

X3 = [3.18750, 3.15625];
Y3 = [30, 70];
Z3 = [1.741, 1.982];

X4 = [3.21875, 3.25000];
```

```

Y4 = [30,70];
Z4 = [1.744,1.863];

X5 = [3.34375,3.40625];
Y5 = [30,70];
Z5 = [1.805,2.054];

X6 = [3.09375,2.81250];
Y6 = [30,70];
Z6 = [1.827,2.556];

X7 = [3.25000,3.43750];
Y7 = [30,70];
Z7 = [1.816,2.035];

plot3(X1,Y1,Z1,'-xg', X2,Y2,Z2,'-xb', X3,Y3,Z3,'-xr',... %
      X4,Y4,Z4,'-xk', X5,Y5,Z5,'-xm', X6,Y6,Z6,'-xy',... %3-D plot
command
      X7,Y7,Z7,'-xc', 'LineWidth',2) %

title('Leading Edge Comparison of Wind Turbine Blades',... %
      'fontweight','b','fontsize',16) %
xlabel('X, Lateral Position (in)','fontweight','b') %
ylabel('Y, Distance From Hub (in)','fontweight','b') %graph
formatting
zlabel('Z, Vertical Position (in)','fontweight','b') %
grid on; %
axis square; %
hleg1 = legend('Location','EastOutside','Blade 1','Blade 2','Blade
3',...
      'Blade 4','Blade 5','Blade 6','Blade 7');

```

APPENDIX J. EES ANALYSIS OF BLADE C.M.

"This program calculates the distance R from the root of the blade to the center of mass"

"Blade 3"

$$-R_3(W_3) + L_3F_{23} = 0$$

$$(L_3 - R_3)W_3 - F_{13}L_3 = 0$$

{W_3 = 10.15 "weight of blade based on single scale measurement in lbf"}

L_3 = 68.599 "length of blade from flange face to tip in inches"

F_13 = 7.71 "reaction force from scale 1 for blade 3 in lbf"

F_23 = 2 + (7/16) "reaction force from scale 2 for blade 3 in lbf"

"Blade 4"

$$-R_4(W_4) + L_4F_{24} = 0$$

$$(L_4 - R_4) \cdot W_4 - F_{14} \cdot L_4 = 0$$

$$W_4 = 10.04$$

$$L_4 = 69.162$$

$$F_{14} = 7.58$$

$$F_{24} = 2 + (7.4/16)$$

"Blade 5"

$$-R_5 \cdot (W_5) + L_5 \cdot F_{25} = 0$$

$$(L_5 - R_5) \cdot W_5 - F_{15} \cdot L_5 = 0$$

$$W_5 = 10.13$$

$$L_5 = 69.131$$

$$F_{15} = 7.62$$

$$F_{25} = 2 + (8.1/16)$$

"Blade 2"

$$-R_2 \cdot (W_2) + L_2 \cdot F_{22} = 0$$

$$(L_2 - R_2) \cdot W_2 - F_{12} \cdot L_2 = 0$$

$$W_2 = 10.46$$

$$L_2 = 69.001$$

$$F_{12} = 7.79$$

$$F_{22} = 2 + (10.7/16)$$

"Blade 7"

$$-R_7 \cdot (W_7) + L_7 \cdot F_{27} = 0$$

$$(L_7 - R_7) \cdot W_7 - F_{17} \cdot L_7 = 0$$

$$W_7 = 10.06$$

$$L_7 = 69.068$$

$$F_{17} = 7.62$$

$$F_{27} = 2 + (7/16)$$

APPENDIX K. EES ANALYSIS OF ROTOR SHAKE FORCE

"force components in Y direction"

$$F_y = (m_3 r_3 \omega^2) \cos(.043) - (m_4 r_4 \omega^2) \sin(30.043) - (m_5 r_5 \omega^2) \sin(29.915) - (m_{Al_5} r_{Al_5} \omega^2) \sin(29.915) - (m_{rod} r_{rod} \omega^2) \sin(29.915)$$

"force components in the X direction"

$$F_x = (m_3 r_3 \omega^2) \sin(.043) - (m_4 r_4 \omega^2) \cos(30.043) + (m_5 r_5 \omega^2) \cos(29.915) + (m_{Al_5} r_{Al_5} \omega^2) \cos(29.915) + (m_{rod} r_{rod} \omega^2) \sin(29.915)$$

$$W_3 = 10.15 \text{ "weight of blade 3 in lbf"}$$

$$W_4 = 10.04$$

$$W_5 = 10.13$$

$$m_3 = W_3 / 32.2 \text{ "mass of blade 3 in slugs"}$$

$$m_4 = W_4 / 32.2$$

$$m_5 = W_5 / 32.2$$

$$r_3 = 21.037 / 12 \text{ "distance to center of gravity of blade 3 in feet"}$$

$$r_4 = 21.517 / 12$$

$$r_5 = 21.667 / 12$$

"total imbalance force and angle"

$$F_B = (F_x^2 + F_y^2)^{.5} \text{ "magnitude of total balance force"}$$

$$\theta_B = \arctan(F_y / F_x) \text{ "angle of total balance force, located in the fourth quadrant"}$$

$$\omega = 31.42 \text{ "rotational speed of rotor in rad/s"}$$

"design of dummy aluminum plug is needed to make sure plugs in the other blades are long enough"

$$L_{Al_5} = 2.65 \text{ "length of dummy aluminum plug in inches, just so other blades can have a plug 2.5 inches long"}$$

$$m_{Al_5} = (((\pi/4)(D_{plug}^2) - (\pi/4)(D_{rod}^2))) L_{Al_5} \rho_{aluminum} - ((\pi/4)(D_{socket}^2)) L_{socket} \rho_{aluminum} / 32.2 \text{ "mass of aluminum plug in slugs"}$$

$$r_{Al_5} = ((L_{Al_5} / 2) + 4.557) / 12 \text{ "distance to center of gravity of aluminum plug in feet"}$$

$$W_{Al_5} = m_{Al_5} \cdot 32.2$$

$$r_{rod} = ((L_{rod} / 2) + 4.557 + .5) / 12 \text{ "distance the rod C.M. is from the axis of rotation in feet"}$$

$$m_{rod} = ((\pi/4)(D_{rod}^2) L_{rod} \rho_{steel}) / 32.2 \text{ "mass of rod in slugs"}$$

$$W_{rod} = m_{rod} \cdot 32.2$$

$$D_{socket} = 1.25 \text{ "approximate outside diameter of socket needed to reach lock nut in inches"}$$

$$L_{socket} = .25 \text{ "depth of socket hole in inches"}$$

$$D_{rod} = (1/2) \text{ "diameter of threaded rod in inches"}$$

$$L_{rod} = 1.25 \text{ "length of rod in inches"}$$

$$D_{plug} = 2.185 \text{ "approximate inside diameter of blade pilot in inches"}$$

$$\rho_{aluminum} = 0.09754 \text{ "density of aluminum in lbm/in}^3$$

$\rho_{\text{steel}} = .284$ "density of steel in lbm/in³"

APPENDIX L. EES ANALYSIS OF δ_y

"this program estimates the hub c.m. for the full rotor balance test as a function of shaft position"

$m_{\text{TL}} = 2$ "mass of taper lock on hub in lbm"

$m_{\text{hub}} = 13.7$ "mass of hum in lbm"

$m_{\text{shaft}} = 2.748$ "mass of shaft in lbm"

$m_{\text{hub_balance}} = .182$ "mass of balance bolts lbm added to hub"

$z_{\text{TL}} = 3.74$ "C.M. of taper lock relative to hub centerline in inches"

$z_{\text{hub}} = .84$ "C.M. of hub relative to hub centerline in inches"

$m_{\text{total}} = m_{\text{hub}} + m_{\text{TL}} + m_{\text{shaft}} + m_{\text{hub_balance}}$ "total mass of system in lbm"

$m_{\text{total}} * Z_{\text{sys}} = m_{\text{hub}} * z_{\text{hub}} + m_{\text{TL}} * z_{\text{TL}} + m_{\text{shaft}} * z|1_{\text{shaft}}$ "solves for Z_{sys} , the total C.M. location above the hub c.l."

$z_{\text{shaft}} = 2.6675$ "C.M. of hardened shaft relative to shaft centerline"

$z|1_{\text{shaft}} = z_{\text{offset}} + z_{\text{shaft}}$ "C.M. of shaft relative to hub cal."

$z_{\text{offset}} = 1.25 + d_y$ "distance of bottom of shaft to hub cal."

{ $d_y = 2.875$ "distance shaft pivot point is moved up relative to nominal hub c.m. of 1.25 inches"}

$\delta_y = z_{\text{offset}} - Z_{\text{sys}}$ "actual distance pivot point is above system C.M."

APPENDIX M. EES ANALYSIS OF BALANCE

WEIGHT DESIGN

$F_B = 27.23$ "total force needed to balance the rotor in lbf calculated in APPENDIX K"

$\theta_B = 40.32$ "angle of balance force in degrees, located in the fourth quadrant"

$-F_4 * \cos(30.043) + F_3 * \sin(.043) = -F_B * \cos(\theta_B)$ "trig to reposition the total banalce weight to blades 3 and 4 in the x direction"

$-F_4 \sin(30.043) + F_3 \cos(.043) = F_B \sin(\theta_B)$
 $L_{rod} = 8.75$ "length of rod in inches"
 $R_{rod} = ((L_{rod}/2) + 4.557 + .5)/12$ "distance the rod C.M. is from the axis of rotation in feet"
 $D_{rod} = (1/2)$ "diameter of threaded rod in inches"
 $m_{rod} = (\pi/4) * (D_{rod}^2) * L_{rod} * \rho_{steel}$ "mass of rod in lbm"
 $D_{plug} = 2.185$ "approximate inside diameter of blade pilot in inches"
 $D_{socket} = 1.25$ "approximate outside diameter of socket needed to reach lock nut in inches"
 $L_{socket} = 1.6875$ "depth of socket hole in inches"
 $\rho_{steel} = .284$ "density of steel in lbm/in³"
 $\rho_{aluminum} = 0.09754$ "density of aluminum in lbm/in³"
 $m_{Al_4} = (((\pi/4) * (D_{plug}^2)) - ((\pi/4) * (D_{rod}^2))) * L_{Al_4} * \rho_{aluminum} - ((\pi/4) * (D_{socket}^2)) * L_{socket} * \rho_{aluminum}$ "mass of aluminum plug in lbm for blade 4"
 $m_{Al_3} = (((\pi/4) * (D_{plug}^2)) - ((\pi/4) * (D_{rod}^2))) * L_{Al_3} * \rho_{aluminum} - ((\pi/4) * (D_{socket}^2)) * L_{socket} * \rho_{aluminum}$
 $R_{Al_4} = ((L_{Al_4}/2) + 4.557)/12$ "distance the aluminum plug C.M. is from the axis of rotation in feet, when solved, L_{Al} will be in inches"
 $R_{Al_3} = ((L_{Al_3}/2) + 4.557)/12$
 $\omega = 31.42$ "rotational speed of rotor in rad/s"
 $m_{rod}/32.2 * (R_{rod}) * (\omega^2) + m_{Al_4}/32.2 * (R_{Al_4}) * (\omega^2) = F_4$ "solves for R_{Al} and then uses the result to determine the length of the aluminum plug"
 $m_{rod}/32.2 * (R_{rod}) * (\omega^2) + m_{Al_3}/32.2 * (R_{Al_3}) * (\omega^2) = F_3$
 $m_{Al_4}/32.2 * (R_{Al_4}) * (\omega^2) = F_{Al}$ "force contribution due to aluminum plug in blade 4"
 $m_{rod}/32.2 * (R_{rod} + (x/12)) * (\omega^2) = F_{rod}$ "force contribution due to rod"
 $x = 0$ "representation of moving the rod's center of gravity down the aluminum plug in inches"
 "with $x = 0$, $F_{Al} + F_{rod}$ is equal to F_4 "

APPENDIX N. EES ANALYSIS OF FULL ROTOR METHOD

"Full Rotor Model with C.M. Displaced Below the Pivot Point"

"the model assumes a clockwise angular displacement from a level rotor"

The geometry in this model is based on that shown in **Error! Reference source not found.**

$$d_y_in = d_y * 12$$

$$\Delta y_in = 0.8525 * d_y_in - 0.3471 \text{ "relationship between shaft position and actual distance C.M. is away from pivot point"}$$

$$\Delta x_in = \Delta y_in * \sin(d_theta) \text{ "lateral distance mass of the hub has been displaced due to rotor tilt"}$$

$$d_theta = d_phi$$

$$R_1 = 20.2/12 \text{ "distance of blade C.M. to hub axis in feet"}$$

$$R_2 = 20.201/12 \text{ "distance of blade C.M. to hub axis in feet"}$$

$$m_1 = 11.1 \text{ "mass of blade in lbm"}$$

$$m_2 = 11.1006 \text{ "mass of blade in lbm"}$$

$$m_hub = 18.63 \text{ "mass of hub with piggott shaft and balance bolts"}$$

$$\phi_1 = \arctan(d_y/R_1) \text{ "angle between R and X in deg"}$$

$$\phi_2 = \arctan(d_y/R_2) \text{ "angle between R and X in deg"}$$

$$L_1 = R_1 / \cos(\phi_1) \text{ "distance of blade C.M. to pivot point in feet"}$$

$$L_2 = R_2 / \cos(\phi_2) \text{ "distance of blade C.M. to pivot point in feet"}$$

$$M_P_1 = m_1 * X_1 \text{ "mass moment of blade 1 about pivot point"}$$

$$M_P_2 = m_2 * X_2$$

$$M_P_hub = m_hub * (\Delta x_in / 12)$$

$$(M_P_1 + M_P_hub) = M_P_2 \text{ "moment balance about pivot point"}$$

$$B = (R_2 * m_2) / (R_1 * m_1 + ((\Delta x_in / 12) * m_hub))$$

"Geometric Relationships"

$$X_1 = L_1 * \cos(\phi_1 - d_phi) \text{ "x-coord of blade C.M."}$$

$$X_2 = L_2 * \cos(\phi_2 + d_phi)$$

$$Y_1 = -(d_y) + R_1 * \sin(d_theta) \text{ "y-coord of blade C.M."}$$

$$Y_2 = -(d_y) - R_2 * \sin(d_theta)$$

$$\gamma_1 = \phi_1 - d_phi$$

$$\gamma_2 = \phi_2 + d_phi$$

APPENDIX O. SOLIDWORKS DRAWINGS OF SINGLE BLADE METHOD PARTS

APPENDIX P. FIELD TESTS RESULTS AT ALL ROTOR SPEEDS

Balanced 7-20-12, wind = 14mph

	average of abs			rms		
RPM	AVG X	AVG Y	AVG Z	AVG X	AVG Y	AVG Z
100-120	0.027706	0.015274	0.998511	0.031324	0.018985	0.998617
121-140	0.029714	0.014141	0.998352	0.033297	0.017709	0.998496
141-160	0.029042	0.014612	0.998808	0.033099	0.018434	0.998949
161-180	0.028340	0.012998	0.998906	0.031742	0.016339	0.999058
181-200	0.028805	0.013163	0.998833	0.032544	0.016492	0.999003
201-220	0.030024	0.012874	0.999600	0.035600	0.016400	1.000163
221-240	0.031825	0.014432	0.999662	0.037791	0.018292	1.000290
241-260	0.031825	0.015200	0.999830	0.037993	0.019338	1.000366
261-280	0.032171	0.016777	0.999677	0.038773	0.021422	1.000130
281-300	0.036643	0.022968	0.999686	0.043606	0.028009	1.000378

Unbalanced 7-20-12, wind = 12mph

	average of abs			rms		
RPM	AVG X	AVG Y	AVG Z	AVG X	AVG Y	AVG Z
100-120	0.028537	0.015740	0.996107	0.032306	0.018860	0.996183
121-140	0.029152	0.016182	0.996333	0.033114	0.019546	0.996444
141-160	0.029315	0.015793	0.996190	0.034271	0.019195	0.996320
161-180	0.032680	0.015619	0.996433	0.039497	0.019254	0.996566
181-200	0.032480	0.014394	0.996288	0.038903	0.017973	0.996524
201-220	0.032843	0.014103	0.996617	0.039672	0.017446	0.996951
221-240	0.033962	0.014365	0.997097	0.041630	0.017707	0.997540
241-260	0.037012	0.017793	0.997390	0.045147	0.022242	0.997797
261-280	0.037777	0.021852	0.997187	0.046015	0.026723	0.997525
281-300	0.039646	0.045363	0.998089	0.048244	0.052398	0.998800

Rebalanced 7-20-12, wind = 11mph

	average of abs			rms		
RPM	AVG X	AVG Y	AVG Z	AVG X	AVG Y	AVG Z
100-120	0.027177	0.015371	0.994843	0.028155	0.016718	0.994872
121-140	0.027128	0.014343	0.995215	0.029435	0.016300	0.995283
141-160	0.027478	0.014488	0.994847	0.030014	0.016851	0.994919
161-180	0.027301	0.014015	0.995270	0.030092	0.016646	0.995361
181-200	0.028245	0.013173	0.995509	0.032335	0.015921	0.995813
201-220	0.027847	0.012473	0.996060	0.032002	0.015154	0.996279
221-240	0.027979	0.012768	0.995564	0.032054	0.015187	0.995874
241-260	0.028054	0.012805	0.995501	0.032285	0.015355	0.995798
261-280	0.027489	0.012535	0.996486	0.031505	0.014998	0.996743
281-300	N/A	N/A	N/A	N/A	N/A	N/A

REFERENCES

- [1] B. K. Edwards, "Composite Manufacturing of Small Wind Turbine Blades," Bryan Kyle Edwards, San Luis Obispo, 2009.
- [2] E. Becker and J. Losl, "Reducing Vibration by Balancing Rotor Blades," 2009.
- [3] R. P. S. Kirk Gee Pierce, "Methods and Apparatus for Balancing a Rotor". United States Patent US 20090035136A1, 5 February 2009.
- [4] D. Class, "Method and Apparatus for Balancing Wind Turbine Rotors". United States Patent 5219454, 15 Jun 1993.
- [5] O. W. Larsen, "In Sity Balancing of Wind Turbines". United States Patent 5140856, 25 August 1992.
- [6] International Electrotechnical Commission, "IEC 61400-1," IEC, Geneva, 2005.
- [7] International Electrotechnical Commission, "IEC 61400-22," IEC, Geneva, 2010.
- [8] Vestas, "Wind Turbine Warehouse," 18 October 2006. [Online]. Available: http://www.windturbinewarehouse.com/pdfs/vestas/Vestas_V90_SAC_DSM_3_20_07.pdf. [Accessed May 2012].
- [9] Bergey, "NREL," October 2005. [Online]. Available: <http://www.nrel.gov/docs/fy06osti/38550.pdf>. [Accessed May 2012].
- [10] Moog, "Moog Info," [Online]. Available: <http://www.mooginfo.com/RMS/web/increase-outputs-308T7-922BC.html>. [Accessed 17 July 2012].

- [11] J. M. A. R. J.F. Manwell, Wind Energy Explained, 2nd Edition, West Sussex: John Wiley & Sons Ltd., 2009.
- [12] IRD Balancing, "Balance Quality Requirements of Rigid Rotors," 2009.