DEVELOPING DENDRIFORM FAÇADES USING FLOW NETS AS A DESIGN
AID

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ABSTRACT

Developing Dendriform Façades using Flow Nets as a Design Aid

This thesis highlights a method of arriving at form that minimizes the need for high end technology and complex mathematical models, yet has structural principles of load flow at the highlighted methods core. Similar to how graphical statics assisted earlier architects and engineers to arrive at form by relating form and forces, this thesis suggests a method of form finding that relates the flow of stresses within solid masses to possible load-bearing façades. Looking to nature, where an abundance of efficient structural solutions can be found, this thesis focuses on a tree-like structural form called the dendriform. In doing so, this thesis explores the idea that through an understanding of typical load flow patterns and the removal of minimally stressed material of the solid body, dendriforms can be revealed that qualitatively exemplify load flow yet maintain an architectural aesthetic.

Keywords: Dendriform, Flownet, Graphic Statics, Maxwell's Theorem.

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LIST OF NOMENCLATURE

Φ	Airy's stress function
c	dimensionless constant within Airy's stress function
θ	angle measured from the line of action of the force
P	applied force in the positive downward direction
r	distance from applied force to the point of desired stress
σ_r	closed form wedge solution of a fan like stress distribution
α	angle defining the limits of a half wedge solution
P_r	required axial compressive strength or demand
P_c	available axial compressive strength or capacity
M_r	required flexural strength or demand
M_c	available flexural strength or capacity
F_i	internal force of member i
L_i	length of member i
$\vec{P}_i$	external load vector
$\vec{r}_i$	location vector of external load
$\sigma_{\text{prescribe}}$	prescribed axial stress capacity
A_i	cross sectional area of member i
V_i	volume of member i

1.0 SERVING AS ARCHITECT AND ENGINEER

Engineering designs must balance regulatory requirements, structural integrity, and aesthetics, but the days during which one person served as both architect and engineer are far removed from when pioneers such as Antonio Gaudi and Pier Luigi Nervi practiced. People like these were capable of creating forms that even by today's standards are not only architecturally inspiring, but structurally rational. With current technologies, the realization of a unique form can completely take structural logistics out of the process. Technologically driven form does not incorporate guiding principles that structurally validate the means of resisting against wind, earthquake, and gravity loading efficiently. At the opposite end of technologically driven forms are forms in which their derivation is a highly evolutionary process involving cumbersome mathematical models and computer automated form-finding techniques. One flaw to such an evolutionary process includes the need for an initial structural geometry. A second flaw is that the tools necessary to pursue such an involved process, including theoretical background and high end technology, are far removed from a physical understanding of the flow of forces.

This thesis highlights a method of arriving at form that minimizes the need for high end technology and complex mathematical models, yet has structural principles of load flow at the highlighted methods core. Similar to how graphical statics assisted earlier architects and engineers to arrive at form by relating form and forces, this thesis suggests a method of form finding that relates the flow of stresses within solid masses to possible load-bearing façades. Looking to nature, where an abundance of efficient structural

solutions can be found, this thesis focuses on a tree-like structural form called the dendriform. In doing so, this thesis explores the idea that through an understanding of typical load flow patterns and the removal of minimally stressed material of the solid body, dendriforms can be revealed that qualitatively exemplify load flow yet maintain an architectural aesthetic.

1.1 Structural Rationalism

The purpose of this thesis is to explore the possibility of deriving structurally rational form through the assistance of stress flow nets. By exploring solid bodies subjected to a myriad of boundary conditions, loading patterns, and geometries, a goal of this thesis is to validate the use of a dendriform structure as a load bearing façade. A second goal of this thesis is to highlight a method of deriving structural form that is structurally sound and accessible.

1.2 Issues Pertaining to Current Form-Finding Processes

The relation between architecture and structures has undergone a transformation in the recent past. This transformation can be attributed to the frantic race for originality, surprise, or media impact that private and political powers demand when aspiring for iconic landmarks with aims of receiving acclamation or publicity (Calzon and Cruz 2010). The desire for such iconic structures has created numerous problems in arriving at structural form.

The first problem is the advancement of technology created to combat such high demands for creative forms. Existing technologies have nearly eliminated the need for user intelligence. In doing so many people, no matter their background, are capable of

bringing form to fruition without an understanding of structural behavior. An example of such technology is the program Rhinoceros, a computer aided drafting program that literally has a button entitled “form find.” All a person needs to create a structural form is the ability to open the program, draw a preliminary geometry, and finally click “form find.” Guiding principles that should be the driving force behind the form are replaced by competence in navigating a computer interface and motivation to create something aesthetically appealing. In compilation of conference proceedings, J. Martinez-Calzon, explains “The computer has promoted an unrestrained genesis of unprecedented canonical forms that are far from being structurally sound” (Calzon and Cruz 2010). Form is being created solely for the sake of form without any inclination of relating form to structure.

The second problem with current form-finding techniques is two-fold and lies on the opposite spectrum of having the ability to create form unknowingly. ETH’s Phillippe Block uses elaborate form-finding methods: Block uses traditional methods such a reciprocal geometry melded with advances in technology to create forms that have proven structural integrity. However, such elaborate form-finding techniques consist of highly developed automated processes and intricate mathematical models that can be overwhelming for even a person who has the necessary background to understand structural behavior. Although such theoretical techniques adhere to using structural principles as a molding process in the evolution of form, the technique is not reasonable for a deadline-driven industry where knowledge saves time and time is money. Seemingly, the capability of form finding requires one of two things: the competence to

navigate a computer interface while creating unfounded canonical geometries or the theoretical and programming background that would allow for specialized mathematical techniques that can be costly and time consuming.

1.3 Some Common Ground for Form Finding

What is needed is a readily available form-finding technique that takes advantage of the ease that computer-aided methods have to offer in addition to the guiding structural principles of the theoretical approach. Ideally, such a method could be used as a preliminary design tool that would minimize the initial dependency on a computer. This method would also contain an evident relationship between form in which and structure in which the flow of stresses is the underlying principle and load flow can be visualized. Load flow is an understanding of how forces in a structure are resolved between two or more known points. The flow of stresses is one interpretation of load flow that allows the designer to better understand how forces are being resolved within a structure.

1.4 Use of Flow of Stresses as a Design Aid Today

Using load flow in order to arrive at form is not an entirely novel concept. Historically, people such as Pierre Luigi Nervi used flow of forces to design structures such as thin shells and domes. An example of Nervi's work exemplifying the use of load flow in deriving structural form can be seen in Figure A, below. Before Nervi, Antonio Gaudi used the flow of forces to assist in arriving at a form that was structurally stable as can be seen throughout his final project La Sagrada Familia.

Figure A: Pierre Luigi triangular vault

Source: Iori 2009

More recently, load flow has been used to stimulate design of building façades in a very practical manner. Skidmore, Owings and Merrill's (SOM) Transbay Transit Competition Tower is a contemporary example of how load flow, and more specifically the flow of stresses, was used to determine structural form (Figure B, below). In order to arrive at the structural form of the tower, the offices of SOM first looked to nature for logarithmic patterns that respond to strength and stresses. Their research led them to what is known as the Michell truss. In 1904 a mechanical engineer named Anthony George Michell found the optimal structural geometry for cantilever. Idealizing the Tower as a cantilever, SOM arrived at the structural form of the Tower by overlaying the solution to the Michell truss on top the Tower's existing form (Figure C, below). In doing so, a rational structural braced frame system was found which gives architectural appeal to the overall structure. The method of arriving at structural form briefly described in this section is a practical example of using the flow of stresses or flow nets to arrive at structural form. This

method of arriving at preliminary geometry is an underlying concept in developing a unique type of structural form, the Dendriform.

Figure B: Transbay Terminal by Skidmore Owings and Merrill

Source: Courtesy of Skidmore, Owings, and Merrill

Figure C: Michell Truss (Left) and Overlay of Michell Truss

Source: Courtesy of Skidmore, Owings, and Merrill

2.0 A PARTICULAR TYPE OF STRUCTURAL FORM

This section further develops the definition of dendriform first by explaining its origin, second by describing some mechanical principles of trees that will help to explain the structural behavior of the dendriform, and finally by looking at some physical building examples of the dendriform. The link between the dendriform and flow nets is achieved by explaining the dendriform through graphics statics and relating the flow of forces to the flow of stresses by giving some mechanical background on flow nets.

2.1 Defining the Dendriform

The Merriam-Webster definition of dendriform is “treelike in form.” The root “Dendri” originates from the Latin word for tree or “Dendron.” The term “Dendriform” within the context of this thesis will refer to geometry that resembles a tree in that it is

comprised of multiple branch-like elements. A dendriform structure is a type of bio-mimicry, wherein designers look to nature for informal ideas regarding form and efficiency. Comprised of appendages that can be related to the branches of a tree, the dendriform transfers load through mostly axial tension or axial compression with minimal bending involved. Relating the load flow within trees to dendriform flownets, design ideas can be proposed for load-bearing façades that can be classified as dendriform structures. Two key aspects of the mechanical behavior of trees are directly applicable when creating dendriform structures: the principle of minimum lever arm and the axiom of constant stress. Both principles have been investigated extensively by Mattheck and Kubler and are discussed in the following section.

2.2 The Mechanical Design of Trees

The principle of minimum lever arm is best explained in Figure D, below, which shows a man holding a bucket of water with his arm outstretched who, after growing fatigued, decides to carry the bucket on his head. In doing so, the man reduces the moment arm associated with the bucket, l , by aligning the force of the bucket over his center of mass. Similar behavior can be seen in trees, which can also be illustrated by Figure D. After the loss of a tree's leading shoot or branch, which can be related to man's outstretched arm, the lateral succeeding branch straightens to take the place of its predecessor. The need for a leading shoot to always come forth is known as Apical Dominance. Negative geotropism or negative gravitropism describes the self-correcting, countergravity growth phenomenon (Mattheck 1991). The concept of a branch aligning

itself with the center of mass will become useful for the derivation of a dendriform structure using graphic statics in Section 2.2.1 in this thesis.

Figure D: Principle of minimum lever arm

Source: Mattheck 1991

Another mechanical concept of trees that can be implemented in the creation of a dendriform structure was found by Metzger. He found that the tapering of trees will ensure a uniform distribution of the surface bending stress if the stem diameter has a cubic relation to the distance from the effective point of wind load, as seen in Figure E below. The height of application of wind loading can be related to effective height of a structure's seismic mass while undergoing ground motions. The sudden change in stem diameters in the area of branch joints illustrates load distribution, also seen in Figure E. Above and below the branch joints, stresses of equal intensity occurs (Mattheck and Kubler 1997). Studies conducted in Section 5.1 of this thesis implement this idea of incrementally changing an elements cross section to achieve nearly constant stress throughout the dendriform structure.

Figure E: Tapering of tree to achieve constant stress distribution

Source: Mattheck and Kubler 1997

The axiom of uniform stress also helps explain the root structure of a tree. According to Mohr-Coloumb's law, the shear strength in the ground increases proportionally to the degree of surface compression (Mattheck and Kubler 1997). This concept helps describe the behavior of a tree's root system due to wind loading. An applied wind load causes the ground to be compressed on the leeward side while the ground is lifted on the windward side. Considering the weight of the ground to be negligible compared to the wind force, the ground must undergo equally intense stresses. The less shear-resistant windward side must be reinforced by longer roots for equal stresses to occur. Longer roots, meaning more surface area, increase the stresses by increasing the shear friction between the soil and root system (Figure F, below). It should also be noted that roots subjected to tensile forces such as those on the windward side tend to take on a buttress-like geometry. While trees resolve stresses mainly through axial tension or compression, the root system is an area where bending is likely to occur.

Behavior of root systems can be helpful in defining the geometry of dendriform structures near boundary restraints that must resist uplift and compressive forces.

Figure F: Equal shear friction in soil due to root adaptation

Source: Mattheck and Kubler 1997

2.3 Early Dendriform Structures

Early structural dendriforms can be seen in the work of Antonio Gaudi, who was a pioneer in his ability to fuse architectural form with structural rationality. This fusion is a common theme throughout all of his life's work but no more so than in his final masterpiece, La Sagrada Familia in Barcelona. Begun in 1883, La Sagrada Familia contains visually arresting dendriform structures, as seen in Figure G below.

Figure G: Dendriform Columns within La Sagrada Familia

Source: Huerta 2006

The main idea behind Gaudi's use of the dendriform will be referred to as the centroid of force method. To attain equilibrium between various sections of a roof mass, the total weight and centroid of force for each roof section are calculated. Afterward, a base column with an already fixed position diverges into multiple branches aimed at the center of gravity of each roof section. With a particular branch assigned to each roof section, the loads induced by the weight of the roof sections are transferred axially all the way to the ground. An example of the center of force method can be found in Section 2.5, Graphic Statics.

More current examples of dendriform structures include Stuttgart Airport terminal in Germany, shown in Figure H below. Note that the dendriform columns contained in La Sagrada Familia and Stuttgart Airport are both three-dimensional structures in that their geometries reside in an x, y, and z coordinate system. This thesis will concentrate on two-dimensional planar dendriform structures initially and will conclude with a two studies highlighting the potential for three-dimensional dendriforms. The reason for the initial

focus on two-dimensional dendriforms is two-fold. The first reason is thoroughness: The intent of this thesis is to rigorously explore the link between form and forces in the 2D world with hopes that the general concepts and ideas presented can be extrapolated into 3D. The second reason is clarity: The development of the first dendriform structure in this thesis will be done through graphic statics. While graphic statics has potential for analysis and design in three dimensions, such a process is not well defined and therefore can become convoluted, detracting from this thesis's intent of presenting a clear link between form and force.

Figure H: Dendriform Structure in Stuttgart Airport Terminal

Source: Courtesy of Schlaich Bergermann und Partners

2.4 Design Ideas for the Dendriform Façade

With a definition of dendriform in mind and some understanding of the mechanical nature of trees, design ideas can now be suggested that attempt to derive dendriform façades. To reiterate, any dendriform proposed should resemble a tree or at least have tree-like qualities. The first of these qualities is that the dendriform be comprised of various appendages or branches that spawn from a single limb or trunk.

Also, the dendriform must appear to blossom from its stalk or a common base point. In doing so, the representation of canopy is created. As this blossoming occurs, the principles of minimum lever arm and constant stress distribution should be followed as much as possible in order to minimize bending and respond to varying load requirements. Adherence to these concepts necessitates a decrease in branch cross section over the tree height. As will be seen later, applying the center of force method in which all appendages align with the applied center of loading will be more difficult to follow when dealing with three-dimensional structures. This concept is best suited for determinate systems (systems containing equal or fewer number of unknowns in comparison to the number of equations available to solve for the unknowns) in which a dendriform is comprised of two-force axially loaded members. The author intends to explore dendriform façades that are capable of resisting both gravity and lateral loading, implying the introduction of bending in the dendriform. Therefore this thesis will also investigate indeterminate dendriform structures to not limit the possibilities for dendriform façades.

2.5 Graphic Statics

In 1865 Karl Culmann presented his now widely known work, “Die Graphische Statik.” In this work Culmann defined his theory of graphostatics, in which he highlighted the interdependencies of projective geometry and internal forces. More commonly known today as graphic statics, this unique method has the benefit of the designer being able to visually experience the relation between form and forces. The theory developed by Culmann is a hallmark uniting design, calculation, and construction (Gerhardt, Kurrer, & Pichler 2003). Based on Culmann’s mathematical proof of the

projective relationship between a funicular polygon and a force polygon (an idea originally introduced by Pierre Varignon), the qualitative aspect of load flow can be experienced optically (Figure I, below).

Figure I: Funicular Polygon (bottom) and Force Polygon (above)
Source: Varignon 1725

Used by pioneers such as Antonio Gaudi, graphic statics was an initial tool of arriving at an architecturally interesting yet structurally efficient form. Gaudi's last project, La Sagrada Familia, exemplifies the qualitative benefits of using graphic statics. Graphic statics deemed a form viable by proving that a structure in question was stable. The process not only rationalized the design work of the structural engineer, but at the same time created structures with extraordinary gracefulness. All the while, force and construction drawings appear in the dual shape of both the sensory consciousness and the sensory needs (Gerhardt, Kurrer, Pichler 2003).

2.5.1 Graphic Statics and the Dendriform

This section gives an example of developing a dendriform by following the centroid of force method. The dendriform is then analyzed using graphic statics. To begin, a loading situation must be created and design decisions must be made. The loading situation consists of supporting a flat horizontal roof that spans 20 feet and lies 15 feet above the ground. The dimensions chosen are arbitrary. Lateral loading is not included here for simplicity but will be considered later. The weight of the roof can be represented by a uniformly distributed load of $0.2P$ kip per linear foot. The value of $0.2P$ is chosen so that the uniformly distributed load can be broken up into an equivalent loading of four concentrated masses weighing $1P$ each. Notice that the equivalent concentrated load of the four masses is applied at the masses respective centers of gravity. The first design decision is the number of tiers or levels of branches. The author decided that three tiers of branches would adequately represent a tree. Next, the author decided that each tier would double in the number of branches in comparison to the tier below. To do this, each branch sprouts into two branches during the transition between tiers. Figure J, below, shows the loading situation and includes grid lines that will assist in the layout of the individual branches of the dendriform.

Figure J: Equivalent Roof loading with Grids

Source: Author, using REVIT

Beginning from the ground, a vertical element representative of a tree trunk is inserted into the grid system so that its base is directly below the point of the roof's mid-span. The element is then oriented pointing at the center of gravity of the entire roof system. Only the portion of the element that is contained within the bottom tier is used. Next, the roof loading is broken up into a right half and a left half. As mentioned, the second tier will have twice as many branches as the first. Therefore, the trunk element diverges into a left branch pointing directly at the center of gravity of the left half of roof loading and right branch pointing towards the center of gravity for the right half of roof loading. Finally, the top tier spouts into four branches, each pointed directly at the center of gravity of an assigned roof mass. Figure K below displays the evolution of the dendriform.

Figure K: Dendriform evolution using center of gravity method

Source: Author, using REVIT

Now that the geometry of the dendriform structure has been created, an analysis using graphic statics can take place. The first step in a graphical analysis of a dendriform is to define the spaces between loading and elements using Bow's nomenclature, in which each space between loads is labeled with a capital letter while each closed area between elements is labeled with a number, most commonly left to right or counterclockwise. Bow's nomenclature for the dendriform created can be seen below in Figure L below.

Figure L: Bow's nomenclature

Source: Author, using REVIT

After implementing Bow's nomenclature, the force polygon can be constructed. The final force polygon for the considered dendriform is shown in Figure M below. The first step in constructing the force polygon is to choose a drawing scale relating the magnitudes of the internal and external forces to the length of the lines that make up the force polygon. Once a scale is chosen, a series of lines is drawn to represent the applied loading and external reactions. Each line in the force polygon is a measure of force for the external and internal forces that make up the dendriform. All lines are drawn parallel to their respective external load or member. Lines between two letters signify external loads, while lines between a letter and a number signify internal axial forces of dendriform members. For example, force "ab" represents the far left 1P vertical external load while force "a1" is the internal axial force in member "A1". Note that forces are defined with lowercase letters while elements or members are defined with uppercase letters. A closed force polygon indicates that a stable structure exists that resists loading

in pure axial tension and compression. Solid lines of the force polygon denote axially compressive members and forces, while the dashed lines represent axially tensile members and forces.

Figure M: Form and Force Polygon side by side

Source: Author, using REVIT

2.6 Load Flow Interpretation

A convenient tool to better understand the internal behavior of structural elements that will allow for the development of rational structural façades is load flow interpretation. Load flow interpretation consists of envisioning a load path between an applied load and the point of load resolution. The path begins with either an applied external force or internal force such as self-weight. Such forces cause the structural element to undergo internal stresses. The distribution or flow of internal stresses is referred to as flow nets and can be related to the laminar flow of water. The link between the flow of stress and the flow of force is that the stress distribution can be replaced with

force resultants just as done in strut-and-tie modeling and thrust line analysis of arches.

The following section better describes flow nets, the relation between the flow nets and the flow of water, and the link between force and stress as it pertains to strut and tie modeling and thrust line analysis.

2.6.1 Flow Nets Also (Stress Trajectories)

Directly related to load path interpretation, flow nets, also known as stress trajectories, are useful when trying to capture the behavior within a structural body at any location within the body. At any point within the structural body, the peak stresses can be found by investigating a series of vectors and contour lines that are aligned with the principal stresses (Kelly and Tosh 1999). The vectors and contour lines make up a stress trajectory.

To fully understand the concept of stress trajectories, a look into principal stresses is needed. Principal stresses can be defined as the maximum and minimum stresses found from a plane stress element. By discretizing a larger structural body into a finite number of plane stress blocks, the principal stress can be arrived at by transforming the stresses given in a rectangular coordinate system to polar coordinate stresses. The transformation usually consists of using stress transformation equations or is done graphically through the aid of Mohr's circle. In both cases the plane stress block is simply rotated to an angle θ_p in which no shear stress exists (Figure N below). In right-hand image of Figure N, σ_1 and σ_2 are the maximum and minimum stresses. The two stresses are always oriented 90

degrees to one another. It should also be noted that the maximum shearing stresses occur at an angle of 45° from the principal axis.

Figure N: Rotation of plane stress block to achieve principal stresses

Source: Principal Stress 2D

By discretizing a structural body and identifying the pair of maximum and minimum stresses in addition to their principle angle for each body, flow nets can be drawn by connecting the individual vectors. In doing so, stress trajectories are created. Current technologies such as Finite Element Modeling (FEM) programs have the ability to automate the process of calculating principal stresses and angles which can then be display on the structural body. An example of a flow net using an FEM program, SAP 2000, is shown in Figure O below. The figure displays stress trajectories for a simply supported solid wall under self-weight loading.

Figure O: Stress trajectories using SAP 2000

Source: Author, using SAP 2000

2.6.2 Flow of Stresses like Flow of Water

It is helpful for a designer to hand-draw a logical sketch representing the flow of stresses. In order to do so load flow must be simplified. Again looking to nature for a simplistic solution, flow of stresses can be related to the laminar flow water. Laminar flow is defined as a non-turbulent streamline flow in which a fluid behaves as a system of orderly layers, with no eddies or irregular fluctuations. Although most forces in a structure are static, meaning that forces do not literally flow through a body of material, the contour lines that make up flow nets or stress trajectories mimic the patterns witnessed when watching the dynamic motion of a liquid such as water. By understanding some typical flow patterns of water and relating them to structural situations consisting of varying geometry, boundary conditions, and loading patterns, a

hand representation of the flow of forces can be drawn. For example, when water flows through a narrow pathway such as a canal, the currents all flow parallel to one another. Shown in Figure P below, this type of flow is defined as Parallel Flow and can be related to the flow of stresses or forces at a distance away from applied loading and boundary restraints. This distance can be found using Saint-Venant's principle. Saint-Venant's principle states that localized effects caused by any load acting on a body will dissipate or smooth out within regions that are sufficiently away from the location of the load. These regions are typically located at a distance equivalent to the largest dimension over which the loads are acting, such as the width of a wall or depth of a beam. When a narrow pathway of water such as a canal becomes an estuary and opens up to a larger body of water such as the ocean, the pattern looks like a handheld fan. This flow is defined as Half-Plane Flow and can be related to stresses near a concentrated load or a mid-span boundary restraint. If the estuary opens up to the ocean at a 90-degree angle, the flow of water mimics a quarter of the handheld fan and is defined as Quarter-Plane Flow. The Quarter-Plane Flow is similar to stresses near a corner loaded element or boundary restraints near a corner. And finally, if two narrow waterways are connected by a larger body of water, the water flow pattern is similar to a handheld fan fully stretched with ends touching and is defined as Full-Plane Flow. Full-Plane Flow is similar to a short and stout solid body that has dimensions that never allow the flow stresses to transition into the Parallel Flow pattern. Each flow pattern can be seen Figure P below.

Figure P: Typical Water & Force Flow Patterns

Source: Author, using REVIT

2.6.3 Strut and Tie Modeling and Thrust Line Analysis

Devised as a method to envision efficient placement of reinforcing bars in reinforced concrete, strut and tie modeling is a method of load path interpretation. Once the distribution of stresses has been found, which can be done by relating the flow of stresses to the flow of water as described in the former section, a series of equivalent force resultants can take the place of the stress distribution. The force resultant itself can be found through an accepted codified method, such as in Appendix A of ACI 318, or more accurately found by solving closed form elasticity problems. One such solution, known as the wedge solution, is discussed in detail in Section 3.0, Solid Body Studies. Tension ties and compression struts are placed where the force resultants lie within the structural body. The struts and ties make up a truss model that allows for the designer to see the load path between an applied force and its point of resolution. The truss model can then be solved through conventional truss analyses or through graphics statics. A sequence of images demonstrating the strut and tie modeling process is seen in Figure Q

below. Note that the flow pattern is made up of the individual flow patterns mentioned in Section 2.6.2, Flow of Stresses Like Flow of Water. The flow pattern begins with Half-Plane flow directly beneath the point load, transitions into Parallel Flow for the mid region of the wall, and ends with Quarter-Plane Flow at the bottom reactions. In Strut and Tie Modeling, the sections of various flow patterns are broken into two regions, the T-Region representing transitional flow, and the D-Region representing undisturbed flow. Half-Plane and Quarter-Plane Flow occur within the T-Regions, while Parallel flow occurs within the D-Region.

Figure Q: Strut and Tie Modelling Process

Source: Allen and Zalewski 2010

Another form of load flow interpretation is Thrust Line Analysis (TLA). TLA is primarily used to study the stability of arches. Equilibrium of an arch can be visualized through a single line, the line of thrust. After dividing the arch into discrete sections, the line of thrust is created by connecting the compressive resultants of each discrete section.

Graphic statics assists in finding the magnitudes of the compressive resultants. For the arch to be purely in compression, the line of thrust must lie entirely within the cross section. Not only can TLA be used to assess existing arches, but TLA also has the capability to determine the shape of an arch when a distinct line of thrust is desired. Figure R below shows how load flow through an arch can be visualize through single line.

Figure R: Example of Thrust Line Analysis

Source: Allen and Zalewski 2010

3.0 SOLID BODY STUDY

To better understand load flow through a structural body and to be able to depict load flow in a graphical manner, it is useful to look at load flow in concentrated areas through classical elasticity theory. This section investigates the flow of stresses throughout a solid body of material in various regions of the solid body. The solid body studied is a simply supported wall of unit thickness with pinned boundary constraints at the wall's bottom corners. The loading condition investigated is a concentrated load located mid-span at the top of the wall. This structural situation can be seen in Figure S below. First stresses in the structural wall near the concentrated load are explored and later the stresses near the bottom boundary constraints are explored. The reason for choosing a simply supported wall as the structural situation for investigation is that the geometry is similar to that of a tall multi-story building, for which this thesis proposes load bearing façades. By investigating similar geometry and loading at a small scale, a robust understanding of the flow of stresses can hopefully be extrapolated to a larger scale.

3.1 Centrally Located Concentrated Load

An applied load is often the beginning of a load path. Therefore, to fully understand the entire load path or flow of stresses through a structural body it is necessary to understand the flow of stresses near the applied load. The applied load investigated occurs mid span at the top of the wall as shown below in Figure S.

Figure S: Loading Condition for Solid Body Study

Source: Author, using REVIT

As stated in Section 2.6.2, Flow of Stresses Like Flow of Water, the flow of stresses resemble the laminar flow of water. In the case of the applied point load centrally located, the flow pattern is referred to as Half-Plane Flow (Figure P on page 25). The length of the fan can be estimated from Saint-Venant's Principle. Recall from Section 2.6.3 that Half-Plane Flow occurs within a Transitional Region (T-Region) and is directly followed by Parallel flow occurring within in an Undisturbed Region (D-Region). Saint-Venant's Principle states that the distance between the T-Region and the D-Region can be no larger than the distance between the left and right extremes of the parallel flow stress distribution. This distance is the same as the width of the wall. Therefore the maximum length of the fan is equal to the width of the wall (Figure T below).

Figure T: Length of Fan Stress Distribution

Source: Author, using REVIT

With the length of the fan known, the stress distribution of the fan can be found using a semi-infinite compression wedge solution in conjunction with the Boussinesq-Flamant Equation. The Compressive Wedge Solution can be seen below in Figure U.

Figure U: Semi Infinite Compressive Wedge Solution

Source: Urugal and Fenster 1990

By first assuming the Airy's stress function represented by

$$\Phi = cPr \theta \sin \theta , \quad \text{Eq. 1}$$

where c is a constant (dimensionless),

θ is the angle measured from the line of action of the forces (radians), and

P is the positively downward applied force (kips), and

r is the distance from the applied load to the point of desired stress (in).

Known polar coordinate formulas can be substituted into Equation 1 to arrive at the fan-like stress distribution (σ_r),

$$\sigma_r = \frac{2Pc * \cos \theta}{r} . \quad \text{Eq. 2}$$

The expression in Equation 2 is known as the Boussinesq-Flamant Equation and describes the stress at any location within the fan. The expression states that the stresses in the fan pattern have greatest magnitude along the line of action of the external load, and they diminish with increasing angles from that line until reaching a magnitude of zero at 90° to the line of action. As the stress distribution fans out, the magnitude also decreases in proportion to their distance from the point application of the external applied load. While the Boussinesq-Flamant Equation works well for loads applied at any angle and loads that pull rather than push, it should be noted the applied load must occur centrally (mid-point of the wall) to ensure symmetry across the vertical axis.

Because the externally applied load and the force resultant of the fan-like stress distribution must balance one another, the force resultant must equal the applied load. Summing the forces in the vertical axis, the equilibrium condition becomes,

$$2 \int_0^{\alpha} (\sigma_r \cos \theta) d\theta = -P, \quad \text{Eq. 3}$$

where all variables have been previously defined in Equation 1 except for α , which represents the boundary angle defining each half of the wedge (radians). Substituting Equation 2 for σ_r in Equation 3, the integrand becomes,

$$2 \int_0^{\alpha} \left(\frac{2Pc \cos \theta}{r} \cos \theta \right) d\theta = -P. \quad \text{Eq. 4}$$

After integrating, the expression becomes,

$$Pc(2\alpha + \sin 2\alpha) = -P, \quad \text{Eq. 5}$$

and the constant c is found to be,

$$c = \frac{1}{2\alpha + \sin 2\alpha}. \quad \text{Eq. 6}$$

With the constant c known, the stress distribution of the wedge is,

$$\sigma_r = \frac{2P \left(-\frac{1}{2\alpha + \sin 2\alpha} \right) \cos \theta}{r}, \quad \text{Eq. 7}$$

where the only unknowns are the variables α and θ . Equation 7 can be used to find the magnitude of stress at any location within the fan distribution. By substituting the width of the wall (the largest length of the fan as mentioned earlier) for the radius r , the stress distribution towards the end of the fan pattern can be found. If assuming a unit thickness t , and a unit radius r , the stress distribution would be as seen in Figure V below.

Figure V: Stress Distribution of a Centrally Located point Load
Source: Author, using REVIT

3. 2 Quarter Infinite Problem

Another location of loading where the flow stresses explored is near a corner-loaded condition. Replacing the boundary restraint with a point load, this loading condition is analogous to a corner-loaded solid body as seen in Figure W, below. For the purposes of this investigation, this corner loading situation will be referred to as the Quarter Infinite Problem (QIP).

Figure W: Corner loading condition for Quarter Infinite Problem

Source: Author, using REVIT

The QIP is unique because, unlike stresses near the centrally located vertical load explored earlier, the stresses near a loaded corner are no longer symmetric nor are they solely in compression. As mentioned in Section 3.1, the Boussinesq-Flamant equation is only applicable for a condition in which the load is located directly at mid span of the solid body. Although the load is still vertical for the QIP, the location of the load at an extreme edge or corner causes the upper edge of the body to be in tension. Adding tension calls for the adoption of a new method to arrive at the internal stresses.

Now that tension is added to the problem, a second wedge in bending (a cantilever wedge) must be used in conjunction with the previously used compression wedge. The superposition of the bending wedge and compression wedge will allow for a

full encapsulation of the stresses located at the concentrated corner load. The cantilever wedge and compression wedge can be seen in Figure X below.

Figure X: Cantilever Wedge Solution

Source: Urugal and Fenster 1990

The nomenclature used in Figure X above is the same as the nomenclature used when the compression wedge was first presented in Figure U. Because the solution for the compression wedge has already been shown, the focus will now shift to solving the cantilever wedge problem. Now employing the assumed Airy's stress function represented by

$$\Phi = cPr\theta_1 \sin\theta_1, \quad \text{Eq. 8}$$

where c is a constant (dimensionless),

θ_1 is the angle measured from the line of action of the forces (radians), and

P is the applied force (kips) positive downward, and

r is the distance from the applied load to the point of desired stress (in).

The equilibrium condition is

$$\int_{\pi/2-\alpha}^{\pi/2+\alpha} (\sigma_r \cos \theta_1) r d\theta_1 = 2cP \int_{\pi/2-\alpha}^{\pi/2+\alpha} \cos^2 r \theta_1 d\theta_1 = -P,$$

Eq. 9

where σ_r is the radial stress at a point within the wedge (ksi), and

α is the boundary angle defining each half of the wedge (radians).

After integration, c is found to be equal to $-1/(2\alpha - \sin 2\alpha)$. Replacing θ_1 with $(90^\circ - \theta)$ the radial stress (σ_r) becomes

$$-\frac{2P \cos \theta_1}{r(\alpha - \frac{1}{2} \sin 2\alpha)} . \quad \text{Eq. 10}$$

If θ_1 is larger than $\pi/2$, the radial stress is positive signifying that tension exists (Urugall and Fenster 1990).

Now that the solution for the wedge in bending has been derived, the idea of superimposing the compression wedge and cantilever wedge will be developed.

Considering the corner geometry, the boundary angles are 0 degrees being directly under the applied load and located at the vertical side edge of the solid body, and 90 degrees being perpendicular to the applied load and located at the horizontal top edge of the solid body. A graphical representation of the superposition can be seen in Figure Y below.

Figure Y: Graphical representation of superposition

Source: Author, using REVIT

At any point between the boundaries 0 and 90 degrees, the angle from the vertical load can be described as beta (β). A key component in defining the theta for the wedge and cantilever problem is the understanding that theta radiates from the centerline of the corner wedge. With this said, the boundaries of theta can be defined and are listed below.

For the Compression Wedge Solution

If $0^\circ < \beta < 45^\circ$ then $\theta = 45^\circ - \beta$

If $45^\circ < \beta < 90^\circ$ then $\theta = \beta - 45^\circ$

For the Cantilever Wedge Solution

For all β , $\theta = 45^\circ + \beta$

Breaking up the vertical unit load, P , into a component perpendicular to the centerline for the cantilever wedge and a component parallel to the centerline for the compression wedge, the two wedge problems can now be solved individually. Once each wedge

problem has been solved for over the corner boundary, the two solutions can be added together to arrive at the stress distribution shown in Figure Z below. The stress distribution shown assumes a unit thickness, t , and unit radius, r . The angle of 32.5° is called the Direction of Zero Stresses (Allen 2010) and represents the change from compressive stresses to tensile stresses. Above this line, tensile stresses exist while below the line, compressive stresses exist.

Figure Z: Distribution of stresses after superposition

Source: Author, using REVIT

In order to verify superposition results, the QIP was modeled using SAP 2000. To capture the corner section of the wall, thin shell area sections were used and were meshed into 4-noded elements. The membrane had a unit thickness while the meshes were 1" by 1". Where the corner section was cut from the global system, fixed boundary restraints were placed to represent the hindrance of the internal body to rotate or translate.

Self-weight of the membrane was neglected and a 1 kip point load was applied in the negative z direction. The SAP output resulted in a tensile stress of 1.36ksi and compressive stress of 2.12ksi located at a unit radius of to the left and below the point load respectively. The SAP analysis verifies that superposition of a compression and bending wedge is a valid method for determining internal stresses for the QIP. An image of the SAP model can be seen in Figure AA below. Additionally, Figures BB and CC display the location and magnitude of the maximum tensile and compressive stresses, respectively.

Figure AA: SAP model created to mimic corner loaded condition

Source: Author, using SAP 2000

Figure BB: Location and Magnitude of Maximum Tensile Stress
Source: Author, using SAP 2000

Figure CC: Location and Magnitude of Maximum Compressive Stress
Source: Author, using SAP 2000

4.0 FORM FINDING WITH A SIMPLE DENDRIFORM

With a bit more insight into the flow of stresses achieved through classic elasticity theory, the focus of thesis now shifts back to more graphical methods. This section explores the idea of creating a new dendriform by manipulating the force polygon of an existing dendriform. Taking the dendriform that was created by following the center of force method in Section 2.2.1, the possibility of arriving at a new dendriform by changing the geometry of the initial force polygon will be explored.

4.1 Defining a New Force Polygon

The force polygon can be defined as the projected geometry of the form polygon and vice versa. Because of this interdependency between the form and force polygons an entirely new structural form can be achieved by altering the associated force polygon.

To begin the form finding process, the geometry of the force polygon defined in Section 2.5.1 is modified. The original dendriform and respective force polygon is shown again for reference in Figure DD, below.

Figure DD: Original form and force polygons

Source: Author, using REVIT

The modified geometry of the force polygon can be seen in Figure EE. Figure EE shows that nodes 1 and 2 were adjusted. Joint 1 was pulled closer to line “ae” while joint 3 was pushed away from line “ae”.

Figure EE: Altered geometry of force polygon

Source: Author, using REVIT

4.2 Backing-Out the New Dendriform

With a new force polygon at hand, a new geometry for the dendriform structure must be defined in order to keep the relationship between form and forces. The modification of joints 1 and 3 only affect forces “a1”, “a2”, “23”, and “3e”. In turn, the only changes in geometry are with respect to members “A1”, “A2”, “23”, and “3E”. The geometry of the bottom two tiers of the dendriform is unaffected. The new geometry of the dendriform can be found by projecting the changes of the force polygon onto the form polygon. First a line parallel to line “a1” of the force polygon is drawn and is then placed in its respective location within the form polygon. In the case of member “A1”, the respective location would be at the top of member “A2”. This process is repeated for the remaining members that underwent altering in the force polygon. The resulting dendriform can be seen in Figure FF, below. In this figure, the dashed lines within the dendriform denote the previous location of the branches associated with the original dendriform.

Figure FF: New Dendriform Geometry due to changes in Force Polygon
Source: Author, using REVIT

4.3 Limitation to Form Finding Process using Graphic Statics

After attaining the new dendriform, an observation can be made that does not follow the theory behind a purely axially loaded structure and therefore does not lend itself to a graphical analysis. This observation is that the external vertical loads no longer lie directly above the branches of the top tier which implies that some type of bending must be resisted by the roof and in turn by the dendriform branches. The introduction of bending is a problem for two reasons. The first is that the dendriform is a structural system comprised of pin ended elements that is meant to resist forces only through axial compression and tension. By definition, the two force members that make up the dendriform are incapable of resisting bending. Second, a system that undergoes bending cannot be analyzed graphically. A graphical analysis requires a statically determinate structure made up of two force members. The inclusion of bending implies moment-

resisting-connections meaning indeterminacy. A determinate structure is required for a graphical analysis because the force polygon only represents axial forces.

In order to allow for a graphical analysis and the preservation of the link between form and forces, the external loading must be placed above the top tier branches (Figure GG). The vertical component of each of the third tier branches is still $1P$ as clearly seen by the force polygon. But now the original center of force method described earlier is no longer needed. Verification that the loads on the roof of Figure DD are statically equivalent to the loads on the roof of Figure GG can be checked by summing their effective moments about an arbitrary point. The two loading patterns give identical moments. The concept of equivalent moments about an arbitrary point remains true no matter the geometry of dendriform so long as the loading magnitude and pattern remain constant. This concept relates Section 4.4, Insight from Maxwell. An interesting phenomenon is that although the loads change in location, their magnitude remains the same. The magnitudes remain the same because all points along line “ae” of the force polygon are fixed due to the constant horizontal projection of the roof. To move any point along line “ae” up or down would imply adding some sort of inclination to the roof. The fact that the external loading remains constant in magnitude can be verified by inspection of the force polygon. Another way to think of the constant external loading is that the vertical component of the axial forces in the top tier of branches will always equal $1P$. Yet another insight is that axial loads in third tier elements in Figure DD are different from the axial loads of those elements in Figure GG, so the axial tension in the roof elements must respond accordingly.

Figure GG: Moved External Loading with Internal atop Top Tier Branches
Source: Author, using REVIT

Having to redefine the loading condition can be construed as a limitation in the form finding process. Ideally, the loading condition could be held constant while various forms that adequately resist the loading are created. By keeping loading a constant, a variable is removed, simplifying any parametric studies conducted between various forms.

4.4 Insight from Maxwell

Further insight into how form and force are related can be achieved through Maxwell's Theorem as it pertains to the design and analysis of a purely axial dendriform structure. Maxwell's Theorem is at the core of graphical statics in that it is the basis of reciprocal geometry. Reciprocal geometry is the link between the form and force polygon. The insight revealed through the use of described Maxwell's Theorem is as

follows. The theorem equates the internal and external energy in a truss as seen below in equations 11 and 12,

$$\text{Internal Energy} = \text{External Energy} \quad \text{Eq. 11}$$

$$\sum F_i L_i = \sum \vec{P}_i \bullet \vec{r}_i, \quad \text{Eq. 12}$$

where F_i is the internal axial force of member i ,

L_i is the length of member i ,

$\vec{P}_i$ is the force vector with respect to a global coordinate system, and

$\vec{r}_i$ the vector defining the location of that force with respect to the origin of the coordinate system.

Note that the right hand side of the equation is a dot product causing the right hand side to be a scalar. The internal work can be broken into two parts consisting of internal energy due to tension members and internal energy due to compression members as seen in equation 13 below.

$$[\sum F_i L_i]_{\text{tension}} - [\sum F_i L_i]_{\text{compression}} = \sum \vec{P}_i \bullet \vec{r}_i, \quad \text{Eq. 13}$$

Assume that a design criterion is to have each member of the structure experience that same prescribed stress regardless of the force it feels, then using the axial stress equation,

$$F_i = \sigma_{\text{prescribed}} * A_i, \quad \text{Eq. 14}$$

where $\sigma_{\text{prescribed}}$ is the prescribed axial stress capacity (tension and compression),

A_i is the cross sectional area of member i , and

F_i is the internal force of member i as defined previously,

equation 14 can be substituted into equation 13 as so,

$$\left[\sum \sigma_{prescribed} * A_i L_i \right]_{tension} - \left[\sum \sigma_{prescribed} * A_i L_i \right]_{compression} = \sum \vec{P}_i \bullet \vec{r}_i. \quad \text{Eq. 15}$$

Inspection of equation 15 shows that an embedded volume term exists in the form of the product of A_i and L_i . Replacing that product with the variable V_i representing volume of member i , equation 15 becomes

$$\left[\sum \sigma_{prescribed} * V_i \right]_{tension} - \left[\sum \sigma_{prescribed} * V_i \right]_{compression} = \sum \vec{P}_i \bullet \vec{r}_i. \quad \text{Eq. 16}$$

The implication of the above equation is that no matter the change in dendriform geometry, conservation of energy exists. Also if the loading pattern and magnitude remains constant, that is if the right hand side of equation 16 does not change, then changes in the internal arrangement of the tension and compression elements in the truss (the left hand side of equation 16) always results in a constant number, regardless of the changes. Conservation of energy is preserved through a balancing of tension and compression components that comprise the internal energy. It will be shown that the individual tension component (or compression component) carry implications of a dendriforms efficiency.

As mentioned in Section 4.3, insights can be made by linking the summation of moments of a dendriform about an arbitrary point and the use of Maxwell's Theorem. The right hand side of equation 16 is expressed in terms of force times a length, which is analogous to the units of a moment. Therefore Maxwell's can be considered a summation of moments, yet the right hand side of equation 16 is a scalar not a vector. This is why the term "analogous" was used since a moment is a vector. However, as previously mentioned, the left hand side of Maxwell's Theorem is representative of the volume of

the dendriform. The summation of moments is likened to the right hand side of Maxwell's Theorem in the form of the dot product between the external load and location vectors. If the loading remains constant while internal configurations change, the summation of moments will always equate to zero. With respect to Maxwell's Theorem, no matter which arbitrary point is used to the coordinate system and consequents the location vector ($\vec{r}_i$), the dot product will always be constant. Figure HH below shows the various components that make up Maxwell's Theorem with respect to the original dendriform as seen in Figure BB.

One design insight gained from the application of Maxwell's Theorem is the quest for convergence upon an optimum structure. Here optimum is defined as the creating of a structure from a minimum amount of material with all elements equally stressed to some prescribed value. Such a least volume structure would be the optimum structure for a given set of loads. Maxwell's Theorem includes such a volume term and therefore allows for the tracking of a structures efficiency based on volume of equally stressed elements. The efficiency term will be considered as the summation of the absolute values of each of the bracketed terms on the left hand side of equation 16 that make up the internal energy portion of Maxwell's Theorem. The minimum sum would be the most efficient structure. Absolute values of the tension and compression terms on the left hand side of equation 16 are needed to identify progress towards optimization otherwise a progression is not clearly apparent because the sum of the positive and negative terms on the left hand side of equation 16 will always be a constant, as previously described.

Figure HH: Components of Maxwell's Theorem on Center of Force Dendriform

Source: Author using REVIT

To investigate the design implications of Maxwell's Theorem, a study was conducted by the author. The study consisted of exploring the results using absolute value terms of the left hand side of equation 16 of Maxwell's Theorem for four various dendriforms of contrasting geometry. Two of the four dendriform geometries have

already been presented. The first is the original dendriform created through use of the center of force methodology in Section 2.5.1 and is referred to as the “Center of Force Dendriform.” The fourth dendriform is the unsymmetrical dendriform arrived at by manipulation of the force polygon exhibited earlier in Section 4.0 and is called the “Unsymmetrical Dendriform.” The remaining two dendriform geometries were arrived in the same fashion as the “Unsymmetrical Dendriform” in that the force polygon was rearranged imposing a new and unique form polygon but in a more exaggerated manner. The second dendriform involves a “pinching” of the force polygon while the third dendriform involved a “stretching” of the force polygon. All four of the dendriforms and their respective force polygons can be seen in Figure II through LL on the following pages. Each of the figures consists of all information needed for Maxwell’s Theorem including length of members, force magnitudes, and locations to applied loading. The form polygon gives the lengths and orientations of all members and the force polygon has all internal forces needed. Where lengths are not given, symmetry about the vertical axis can be used.

Figure II: Center of Force Dendriform used for Maxwell Study.
Source: Author using REVIT

Figure JJ: “Stretched” Dendriform used for Maxwell Study
Source: Author using REVIT

Figure KK: “Pinched” Dendriform used for Maxwell Study
Source: Author using REVIT

Figure LL: Unsymmetrical Dendriform used for Maxwell Study
Source: Author using REVIT

As an example calculation, Maxwell's Theorem for the Center of Force Dendriform will be presented with reference to Figure II. The value of P was set to equal one pound. Using the internal energy of equation 13 as a starting point, the summation of the force length product of the tension members is as follows;

$$\Sigma[F_i L_i]_{tension} = (FL)_{B1} + (FL)_{C2} + (FL)_{D3} = (1 * 2 + 1 * 2 + 1 * 2) = 6lbft.$$

The next component of equation 13 is the summation of the force length product of the compression members. Because the dendriform is symmetrical about the vertical axis above the dendriform trunk, one side of the dendriform can be summed and multiplied by two and added to the force length product of the trunk. The summation is

$$\begin{aligned}\Sigma[F_i L_i]_{compression} &= 2 * [(FL)_{A1} + (FL)_{12} + (FL)_{A2}] + (FL)_{AE} \\ &= 2[(1.42 * 2.83) + (1 * 2) + (2.23 * 2.24)] + (4 * 2) = -30lbft.\end{aligned}$$

Note that compression is considered to be negative, and therefore the summation of the compression members is negative. Now the proposed efficiency term described earlier in this section can be found by adding the absolute values of the tension and compression force product summations. The efficiency term is represented by the equation

$$[\Sigma F_i L_i]_{tension} - [\Sigma F_i L_i]_{compression} = 6lbft + |-30lbft| = 36lbft.$$

The above term will be useful when comparing the efficiency of the various dendriform geometries. Without the absolute value the above efficiency term becomes the summation of the internal energy and is equal to -24lbft. This number will also be useful later. The final component of Maxwell's Theorem is the external energy and is calculated below.

$$\begin{aligned}
& \Sigma \vec{P}_i \bullet \vec{r}_i \\
&= \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \bullet \begin{Bmatrix} -3 \\ 6 \end{Bmatrix} + \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \bullet \begin{Bmatrix} -1 \\ 6 \end{Bmatrix} + \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \bullet \begin{Bmatrix} 1 \\ 6 \end{Bmatrix} + \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \bullet \begin{Bmatrix} 3 \\ 6 \end{Bmatrix} \\
&= 24lbft.
\end{aligned}$$

It can be seen that the summation of internal and external energy is equal to zero, or that the right hand side of Maxwell's Theorem is equal to the left hand side. Conservation of energy applies to the dendriform no matter its geometry so long as the loading pattern and magnitude are constant between dendriforms. The term *36lbft* calculated using absolute values on the left hand side of equation 13 is also important because it will be used to monitor progress toward optimization. Recall that if the loading magnitude and pattern do not change as internal configurations are made with the hopes of finding an optimal configuration, the result of the dot product will always equal a constant. For this particular loading, the constant will always equal *24lbft*. Closer inspection of the dot product reveals that the dot product of the horizontal components will always equal zero because the loading is strictly in the vertical axis which remains constant. As the top tier of joints move due to changes in geometry of the tension and compression members of the dendriform, the external loads must move with the joint in order to have a closed force polygon signifying a stable structure. Therefore, with all dendriform geometries presented in figures II through LL, the only variables are the horizontal components of the load vector but this is of no significance because the value is multiplied by a horizontal component in the load vector that is of zero magnitude. Therefore, the external

energy or the result of the dot product for this loading condition will always be a particular constant. If horizontal loading were present this would result in a different right hand side. As mentioned previously a constant dot product arises from taking the moments about an arbitrary origin describes the coordinate system of the dendriform. The only variable when taking this moment is the moment arm or perpendicular distance between the applied external load and the base point in which the moment is taken about.

In addition to a piecewise investigation of Maxwell's Theorem, the significant values of average stiffness of the top tier joints and total weight of dendriform were found. Recall that cross sections of members were chosen to ensure that each member experiences a prescribed constant stress. The average stiffness of the dendriform's top tier joints was found by first modeling the dendriform in SAP. The stiffness of each joint was found by dividing the applied joint load by its respective vertical displacement. The average stiffness was then achieved by summing all the vertical stiffness's of the top tier joints and dividing by the number of joints. Hollow structural shapes were used. The length of each element was multiplied by the pound per linear foot for its respective cross section. The summation of all such member weights gives the total dendriform weight. The average stiffness of the top tier joints and total dendriform weight based on identical prescribed member stress was found for all four dendriforms. The process demonstrated for Maxwell's Theorem for the first dendriform, was repeated for the remaining three dendriforms. A Table 1 below summarizes the results.

Table 1

<i>Dendriform</i>	<i>Stretched</i>	<i>Unsymmetric</i>	<i>Center of Force</i>	<i>Pinched</i>
$\Sigma(F_i L_i)_{tension} [lb\ ft]$	12	7	6	4
$\Sigma(F_i L_i)_{compression} [lb\ ft]$	-36	-31	-30	-28
<i>Efficiency Term [lb ft]</i>	48	38	36	32
$\Sigma Internal\ Energy [lb\ ft]$	-24	-24	-24	-24
$\Sigma External\ Energy [lb\ ft]$	24	24	24	24
<i>Average Stiffness [lb/in]</i>	236	304	296	339
<i>Dendriform Weight [lbs]</i>	180	139	135	114

Analysis of the results reveals first that as the dendriform evolves from the stretched geometry to the pinched geometry, the efficiency term decreases meaning an optimum dendriform is being converged upon. Another finding is a stiffening of the structure (with the exception of the unsymmetrical dendriform) and a corresponding decrease in weight of the structure over the same path towards optimization. The unsymmetrical dendriform that does not follow the stiffness pattern is an artifact of the method used to arrive at average stiffness. The findings make structural sense when considering the structural form the dendriform is morphing into. As the dendriform becomes more pinched, the structure begins to resemble a vertical column. A vertically loaded column is a straight line and is the shortest load path between applied loading and the point of load resolution. In turn, less material is required, creating a lighter and more efficient structure. Because a column has no overhang branches, the average stiffness no longer consists of deflecting appendages and all deformation takes place in line with the center of the structure, creating a stiffer structure. A final finding is that when comparing the absolute value term of the left hand side of equation 13, it is seen that the designer

gets penalized twice when diverging from the optimum structure; one penalty due to the tension members and another penalty for the compression members. As the geometry diverges from the “pinched” geometry each of the tension and compression members force length product increase signifying an inefficient structure.

The investigation of Maxwell’s Theorem can be adapted to various structures and can give insight into structural efficiency. Use of Maxwell’s has great potential and the author suggest that seeing how Maxwell’s can be integrated into design practice is an area if interest. When paired with other target structural goals such stiffness and minimal weight, Maxwell’s Theorem can help develop parametric studies with the goal of arriving at a structurally sound solution.

5.0 SOME TYPICAL FLOW NET PATTERNS

As stated earlier, flow net patterns can be arrived at by hand with some understanding that the flow of stress is similar to the flow of water. The aim of this section is to reinforce this concept by exploring the flow nets resulting from a variety of loading, boundary, and geometrical situations of a single wall panel. While not every flow pattern lends itself to the flow of water analogy or to a hand derivation of flow nets, a better understanding of the flow stresses is possible with the assistance of a finite element modeling program. However, it should be reiterated that dependence upon such programs is not the intention of this thesis and no matter the complexity of the structural situation the designer must have an understanding of the flow of stresses. Another purpose of this section is to implement the link between dendriform flow nets and structural geometry. After inspection of the flow nets and acknowledging the common

characteristics, possible dendriform geometries are suggested that adhere to the dendriform design ideas from Section 2.4.

Using SAP 2000, the flow nets of a single cantilevered wall panel composed of thin shell concrete membrane elements (12" thick) were studied. The wall had an aspect ratio of 1:1 (15' high and 15' wide) and was meshed into unit squares. The general process after creating the typical base model was to either change the boundary conditions, the loading pattern, the geometry, or a mixture there of and then, using the “show stresses” option in SAP 2000, plot the maximum principal stresses. Throughout this section, images of the final plotting of principal shell stresses (flow nets) will be shown. The stresses are shown by arrows representing relative magnitudes and orientations of the principal stresses at a particular location. Also, the contour range was adjusted to make the stress arrows more distinguishable. An image of the settings used in SAP 2000 is shown below in Figure MM. Finally, the “model alive” option was used to expedite the iterative process. The “model alive” option allows for an automatic update of the model each time changes are made without having to re-run the analysis.

Figure MM: SAP 2000 Flow Net display settings

Source: Author using SAP 2000

5.1 Cantilevered Wall Subjected to Gravity Loading

The first of the typical flow net patterns investigated consisted of pinned boundary restraints at the bottom corner edges of the wall and gravity loading. The gravity loading was applied by subjecting the wall panel to the self-weight of concrete (150 pounds per cubic foot). The resulting flow net can be seen below in Figure NN.

Figure NN: Flow Net due to Gravity Loading

Source: Author using SAP 2000

The flow net shown above is considered typical because self-weight will always be present and therefore the flow pattern will always be present. Note that the loading due to self-weight is a varying load distributed over the height of the structural body as opposed to point loads shown previously. Relating this flow pattern back to the flow of water, Quarter-Plane flow is clearly apparent near the pinned boundary constraints. Quarter-Plane flow is comparable to water from an estuary transitioning into an open body of water at a 90 degree angle (see Figure P). If reversing the flow of water so that the water transitions from the open body into the estuary at the same 90 degree angle, the flow of stresses near the pinned restraints of the wall panel is simulated. At the location of the pinned restraints the compressive stresses are funneled into a concentrated area. Funneling of stresses into a concentrated area can be explained through the secondary

compressive stresses. Not visible in Figure NN due to their small magnitude in comparison to the primary compressive stresses are the secondary stresses that lie 90 degrees to the primary stresses. The secondary stresses pinch the primary stresses while approaching the pinned boundary restraints. The pinching of the primary stresses is what gives the funicular shape to the flow net at the bottom corner conditions of the wall.

Also noteworthy is that the funneling of stresses between the two pinned restraints creates arching action which envelopes an area of low stressed material towards the bottom portion of the wall. The concept of stresses being funneled to a single point will hold true for any adjacently located boundary restraints. One of the earliest documentations of the funneling of forces and arching action within a load path due to gravity loading can be traced back to the sketches of Fazlur Khan, seen in Figure OO below. Notice that Khan's sketch includes three boundary constraints, all of which attract stress in a funneling manner.

Another trait of the gravity-induced flow net shown in Figure NN that should be noted is that the magnitudes of the arrows representing of the principal stresses become less prominent toward the top of the wall. Similar to the area beneath the arching action, smaller, less prominent stress arrows imply low stress demand because of the accumulation of weight increasing while moving down the wall until the weight is distributed between the two pinned restraints.

Figure OO: Khan's sketches of load path due to gravity loading

Source: Khan 1981.

With a better understanding of the flow of stresses due to pure gravity loading, it is easier to conceptualize the preliminary geometry of one possible dendriform structure. Observed characteristics of the gravity flow nets included arching action, funneling of stresses into the boundary restraints, low stress demand beneath the arch, and decreasing stress demand while ascending the original cantilevered wall. With these traits in mind, a dendriform geometry was created and a perspective view can be seen in Figure PP below. The geometry clearly illustrates arching action as well the funneling of stresses into the boundary restraints. The low stress demand beneath the bottom arch is accounted by completely removing any material in the area. Because low demand exists at this location, no material is needed to resist any internal stresses and therefore the material can be removed. The decrease in stress demand towards the top of the wall is depicted in

two ways. First, the spacing between arches increases while ascending. The larger spacing of the arches implies that each of the arches has a larger tributary area. Because the stress demand decreases with increasing height, each arch is capable of resisting a larger tributary area. Additionally, the cross section of the arches decreases with height as well. The decrease in cross section is reasonable due to the decreasing stress demand while ascending. Ideally, an optimal cross section for each individual element could be found to achieve a nearly constant stress distribution throughout the dendriform. Doing so would capture the most efficient structure based upon volume and stress demand.

Figure PP: Perspective of Possible Dendriform for Gravity Flow Net
Source: Author using SAP 2000

5.1.1 Cantilevered Wall with Window Opening Subjected to Gravity Loading

A common occurrence in structural solid bodies is the placement of openings such as windows. When openings are placed within a solid body the flow stresses is altered.

However, this alteration in flow is not completely foreign to the flow net recently shown in which no openings exist. Figure QQ below displays the flow nets of a single panel wall with a centrally located window opening. It can be seen that arching action is still very apparent, especially below the window opening. A little less obvious is the arching action that finds its way around and over the window opening.

Figure QQ: Flow Nets of Wall with Opening

Source: Author using SAP 2000

Because of the manipulation in geometry, the load path must re-orient itself in order to create a continuous path. Any interruption in this path, such as a window opening, causes a redirection in flow. In fact, the more abrupt the change in geometry, the more abrupt the change in the flow of stresses must be to accommodate the new geometry. This concept can be seen when comparing the two images seen in Figures RR and Figure SS below. Figures RR and SS take the same centrally located window opening (left images), turns it 45 degrees and changes the corner angles of the opening from 90 degrees to 45 degrees (right images). Figure RR shows the flow nets by plotting

the stress vectors as shown thus far. It is seen that the redirection of load path in the wall with the 45 degree opening is less abrupt around the corner of the opening. A less abrupt redirection of load path translates to less accumulation of stresses and in turn lower stress demands. In Figure SS the flow of stress is depicted through stress contours rather than stress vectors. The darker contours signify regions of high concentrated stress which is consistent with the interpretation of Figure RR. The areas that quickly transition from dark to light contours signify high stress gradients or abrupt changes in stress flow. It is clear that the opening with the softer or less abrupt geometry exemplifies lower stresses as well as lower stress gradients. The concept of softer geometries resulting in lower concentrated stresses and in turn a more a smooth flow in stresses can also be explained through the flow of water. Imagining both openings as rocks in a stream of water, the rock with sharp corners would result in a significant redirection in the flow of water. The rock with the softer or rounded corners would allow for a more effortless transition of water around the rock (Allen and Zalewski 2010).

Figure RR: Comparison of Flow Nets with Various Opening Geometries

Source: Author using SAP 2000

Figure SS: Comparison of Stress Contours with Varied Openings

Source: Author using SAP 2000

5.3 Pin-Pin Cantilevered Wall Subjected to Lateral Point Loading

Just as gravity loading, lateral loading is also considered to induce a typical flow pattern. Because structures must be capable of resisting gravity and lateral loading, understanding the flow of stresses from both loading patterns is useful when deriving a load resisting façade. Figure TT shows the flow nets due to a horizontal point load of 1 kip located at a top corner of the cantilevered wall.

Figure TT: Flow Net due to Horizontal Point Load

Source: Author using SAP 2000

Immediately it is apparent that arching action is once again present and that the area the arch encompasses is minimally stressed. A subtle difference in the arching action due to the horizontal point load and the pure gravity loading is that the arch resulting from the point load is concave up, while the arch from gravity loading is concave down. The difference in concavity is due to the principal stresses that outline the arching action

differing in orientation or principal angle. When inspecting the arching action due to the point loaded wall, the principal angle nears 45 degrees while approaching the centerline of the wall. The 45 degree angle is perfectly logical when considering the parabolic shear distribution of a rectangular cross section and Mohr's criterion for maximum shear stress. The highest point of the parabolic shear distribution of a rectangular shape is at the Neutral Axis (N.A.) of the cross section, which corresponds to the maximum shear stress as can be seen in Figure UU below. Also, according to Mohr's criterion, the maximum shearing stress is located at a principal angle of 45 degrees. At 45 degrees the normal stresses are minimal and only shear stresses exist. Therefore the change in concavity between the gravity and horizontal loading signifies that shear and bending are present within the wall.

Figure UU: Shear Distribution of Cantilevered Wall

Source: Author using REVIT

Also observed is that the compressive and tensile flow nets intersect above the arch. The intersection creates diagrid action or an inherent triangulation. In the recent years, the Diagrid has become a prominent façade for bracing the exterior of a building.

The reason for the diagrid's rise in popularity is its efficiency in resisting lateral and gravity loading. Essentially an optimum lattice pattern just like the Michell truss, the diagrid minimizes weight while maximizing resistance to load.

The flow nets due to the laterally loaded wall can be related to the flow of water. Imagining the solid wall as a volume of water, the point load as a water hose, and the pinned boundary constraints as drains, the water current between the hose and the drains is similar to the flow of stress at any point with the wall. Just as water spouts from the hose into the volume of water in a fan like pattern, the point load distributes stress in the same manner. Also, just as the drains funnel the water, the boundary restraints funnel the stresses.

5.3.1 Other Types of Lateral Loading Patterns

When considering lateral loading due to seismic or wind, a point load at the top of the structure is not entirely representative of the loading pattern. Seismic force is the product of mass and acceleration, and therefore the loading pattern for an earthquake may be best input as a imposed horizontal acceleration. A horizontal triangular distributed load is another possible pattern that could be representative of equivalent forces due to seismicity. As for wind loading, a uniform distributed loading pattern could be used to represent wind pressure. Also, a unit shear atop the wall may more accurately capture the force exerted from an attached diaphragm. The pinned restrained wall panel was subjected to all four of these loading patterns to determine how and whether the resulting flow nets differed.

6.0 DEVELOPING A TUBULAR DENDRIFORM

With some understanding of typical flow net patterns and possible façades that are capable of providing an efficient means of resistance to the loading situation, a more complete structure can be developed. In this section a four sided shell structure was subjected to chosen ground accelerations and gravity (self-weight) loading to achieve a sequence of flow nets over time. A dendriform was then derived based upon particular qualities that the flow nets encompassed. Finally, the derived dendriform was subjected to the same ground motions and gravity loading in order to assess the dendriform for structural integrity based upon code compliance, strength, and ductility.

6.1 Interpretation of Flow Nets

Thus far all dendriform studies have been restricted to a two dimensional plane. In order to allow for better visualization of the dramatic appearance the dendriform can achieve and better understand the link between flow nets and the dendriform, it was desirable to delve into three dimensions. A four-story structure was and a nonlinear procedure was undertaken (nonlinear modal history using SAP 2000). Each story measured 15' high and with a footprint of 50' by 50' square. Thin-shell concrete area sections with a unit thickness were used to create the four sides of the cubical structure. The thin shells were meshed to allow for better interpretation of resulting flow patterns. Each corner of the structure was restrained by pinned boundary conditions that resisted vertical and horizontal translations but allowed for rotational freedom. In addition, rigid diaphragms were assigned to the top of all four stories to allow for compatible interaction

between the four walls of the structure. An image of the structure created using SAP 2000 can be seen in Figure VV below. A

Figure VV: 3 Dimensional Four Story Model

Source: Author using SAP 2000

To better mimic actual loading conditions, dynamic loading was chosen for this study as opposed to the static loading used during the previous studies. A nonlinear modal history using the ground motion file from the Northridge earthquake was implemented. The choice to use Northridge ground motions was arbitrary. The file chosen consisted of only accelerations in the x direction. Because only the resulting flow nets were of interest, a simple modal type time history was chosen rather than a direct integration type because of the expedited run time of the modal type analysis. The ground motions were scaled by 0.3937 in order to convert the accelerations from cm/sec^2 to in/sec^2 . Other than the imparted ground accelerations, the only other loading was due to gravity or self-weight of the concrete walls. After having run the load case consisting of

the Northridge ground accelerations and gravity loading, the resulting flow nets of the in plane walls were investigated. The flow nets encountered closely resembled flow nets that would be expected for a static load case for gravity and lateral loading that could be hand drawn using knowledge of typical flow patterns. However, it was discovered that a subtle repeating flow pattern existed due to the cyclic nature of the ground motion. By stepping through the ground accelerations at 0.1-second increments and observing the flow of stresses at each time step, behavior such as arching action became clearly apparent. This repetitive pattern was comprised of five to seven 0.1 second time steps at which time the pattern was reversed and became a mirror image of the initial pattern. In other words, each half of an in-plane façade cycled between being in compression and tension as would be expected for cyclic ground accelerations.

Also observed was that the magnitude of arching action varied throughout the flow pattern. During the onset and conclusion of the pattern, the arching action appeared to be more prominent meaning the void towards the bottom of the wall and between the pinned restraints became larger. Figure WW below shows the flow pattern as seen in SAP 2000 at 0.1-second increments. In this figure the black areas signify compressive stresses, the gray areas signify tensile stresses, and the white areas signify low stresses. Note that at the eighth image the pattern is reversed and at the sixteenth the pattern begins again. By super-imposing the first seven time steps with the remaining six time steps a full arching pattern would result.

Figure WW: In Plane Flow Net Pattern
Source: Author using SAP 2000

Also of interest were the flow nets for the out of plane façades. Arching action towards the base and between the pinned restraints was once again discernible. Unique to the out of plane flow nets was the tendency for centralized compressive and tensile stress to accumulate at the locations where diaphragms tied into the façade. Load flow or load path readily explains this occurrence. The imposed ground acceleration causes the thin shell concrete wall to undulate under the structures natural frequency. The back and forth motion is then transferred to the diaphragm which ties all walls together. With the diaphragm moving back and forth, a push-pull effect is inflicted upon the out of plane walls which explains the concentrated tensile and compressive forces at the locations where the diaphragm and out of plane walls attach. Figure XX below shows the out-of-plane flow net sequence.

Figure XX: Out of Plane Flow Net Pattern

Source: Author using SAP 2000

6.2 From Flow Nets to Dendriform

With an idea of how the shell structure responded to the Northridge ground motions and gravity loading, a dendriform structure can be created by removing minimally stressed material and replacing stress trajectories with tree-like branches.

While an exact tracing of the stress trajectories would achieve the most efficient structure, some creative license is involved when creating a dendriform in order to ensure that the structural geometry is perceived as organic and tree-like. The main criterion for the geometry of the dendriform is that an obvious visual link exists between the response flow nets and the realized structure. In order to do so, certain flow net traits must be evident in the dendriform structure: a void towards the bottom implying arching action; triangulation at the top implying the crossing of principal stresses; and fewer branches or less congestion or light cross sections towards the top implying low stress demand. In addition, traits such as the bent tree root system mentioned in Section 2.2 were instilled to combat uplift. The dendriform that was realized was of modular geometry composed of tree-like limbs truncating into the ground and forming an embedded arch. The module was placed on all four sides of the initial cubical structure. At the corner conditions where the two modules interact, a tree resembling structure is well-defined. An image of the repeating module can be seen in Figure YY and a perspective of the three-dimensional dendriform can be seen in Figure ZZ, both below.

To give an architectural aesthetic to the dendriform, rectangular Hollow Structural Steel (HSS) sections were chosen, particularly HSS 10x8x5/8. This cross section was chosen as the maximum size to be used because all rectangular HSS's above this section are considered non-compact per AISC standards. The overall dimensions of the dendriform are identical to those of the cubical structure, 50' by 50' footprint and four 15' stories. In addition, rigid diaphragms were assigned at every story. Note that each corner of the dendriform façade consists of multiple boundary restraints as opposed to the

singular pin constraint seen in the shell model. The reason for multiple restraints is to reduce the amount of bending that a single cross section must endure. By adding more members and in turn more boundary constraints the stresses and forces to be resisted are distributed more evenly due to added redundancy.

Figure YY: Elevation of Dendriform Module
Source: Author, using SAP 2000

Figure ZZ: Perspective of Dendriform Structure
Source: Author, using SAP 2000

6.3 Loading the Dendriform Structure

A main focus of this thesis is using flow nets within a solid body of material to arrive at a new lighter, more efficient structure that is capable of resisting the initial loading condition of the original solid body. To best compare the initial solid body such as the most recent cubic shell structure to the new lighter dendriform structure, the dendriform must be subjected to a similar loading condition. In doing so an equivalent but lighter structures of similar load carrying capacity is created. The manner in which the Northridge ground accelerations were applied to the shell structure is consistent for the dendriform. However, because the shell structure was constructed of thin shell concrete walls while the dendriform structure was constructed of hollow steel branches, the gravity loading has changed, attributable to not only the difference in density between concrete and steel but also to the dendriform's use of less material. In order to apply a consistent gravity load between the two structures, the density of the new dendriform structure must take on the same density as the original concrete cubic structure. Enforcing a similar density can be done by proportioning the density of steel by the ratio of the summation of the vertical base reactions for the concrete structure to the summation of vertical base reactions for the steel dendriform. In doing so, the new dendriform structure is subjected to a gravity loading similar to that used to arrive at the dendriform -inspiring flow nets.

An example of the process is as follows. The summation of the vertical reactions for the original cubic structure was equal to about 800 kips. The summation of the vertical reactions for the dendriform structure was about 300 kips. This is logical because

the dendriform, made up of branches, is constructed with a less dense material and accounts for less volume than the original concrete solid body. Therefore the dendriform should be lighter. By dividing the 800 kip reaction by the 300 kip reaction a multiplier of 2.67 was found. By multiplying the density of steel by 2.67, the new summation of vertical reactions for the dendriform structure is equal to that of the original concrete structure (800 kips), meaning that a similar gravity loading condition exists.

6.4 Assessing the Dendriform

After subjecting the dendriform structure to Northridge ground accelerations and the proportioned gravity loading, an assessment of the dendriform structure was conducted. To begin the assessment of the dendriform structure the “Steel Frame Design” option in SAP 2000 was used. The preferences for the steel frame design are shown in Figure AAA below.

	Item	Value
1	Design Code	AISC-LRFD99
2	Multi-Response Case Design	Envelopes
3	Framing Type	SCBF
4	Seismic Design Category	D
5	Phi (Bending)	0.9
6	Phi (Compression)	0.85
7	Phi (Tension-Yielding)	0.9
8	Phi (Tension-Fracture)	0.75
9	Phi (Shear)	0.9
10	Phi (Shear-Torsion)	0.75
11	Phi (Compression, Angle)	0.9
12	Ignore Seismic Code?	No
13	Ignore Special Seismic Load?	No
14	Is Doubler Plate Plug-Welded?	Yes
15	Consider Deflection?	Yes
16	DL Limit, L /	120.
17	Super DL+LL Limit, L /	120.
18	Live Load Limit, L /	360.
19	Total Limit, L /	240.
20	Total-Camber Limit, L /	240.
21	Pattern Live Load Factor	0.75
22	Demand/Capacity Ratio Limit	0.95

Figure AAA: SAP 2000 Steel Frame Design Preferences

Source: Author, using SAP 2000

Figure AAA shows that the framing type chosen was a Special Concentric Braced Frame (SCBF). The SCBF was chosen because the nature of the dendriform is to resist loading mostly in axial compression or tension. However, toward the bottom of the structure where the inclination of branches decreases, bending stresses will be induced causing behavior to more resemble that of a moment resisting frame. Therefore, in actuality the true nature of this dendriform would be somewhere between a braced frame and a moment frame. For the purpose of analyzing the code compliance of each individual element, the dendriform elements resemble elements of a braced frame more so than elements of a moment frame.

Because the dendriform created must resist loads in both bending and axial, the dendriform frame elements must be considered to be beam-columns. For this reason,

moment axial interaction must be considered. Moment axial interaction is automatically considered when using the steel design option in SAP 2000. Per the AISC 3rd edition of the Steel Construction Manual, SAP 2000 checked combined bending and axial determining if the interaction equation,

$$\frac{P_r}{P_c} + \frac{M_r}{M_c} \leq 1.0, \quad \text{Eq. 10}$$

is met, where

P_r = required axial compressive strength or demand (kips),

P_c = available axial compressive strength or capacity (kips),

M_r = required flexural strength or demand (kips), and

M_c = available flexural strength or capacity (kip-in).

Equation 10 varies slightly depending on whether the ratio P_r/P_c is less than or greater than 20 percent. So long as the combined effects of axial and bending remain less than or equal to unity, the element is sufficient to resist the loads the element is subjected to.

Figure BBB below shows an elevation displaying the interaction values for a single façade in plane of the earthquake loading. Most values in Figure BBB fall below the unity requirement, meaning that the combined axial and bending equation is adequately met. However, toward the base of the dendriform certain interaction values are above the unity requirement. Two diagonal elements at the base, one on the left and one on the right, display interaction values of 1.341 and 1.32, respectively. Recalling that these two elements were implemented into the dendriform to mimic the root system of a tree and assist in resisting uplift, the elements failing to meet the interaction equation is

logical. Section 2.2 explained the root system assisted in resisting uplift through friction between the soil and the branches. In the created dendriform the root-like elements don't have the advantage of using friction to resist uplift and instead must resist uplift through bending. Figure CCC shows the axial and flexural components that make up the total interaction value of 1.341 for base diagonal element on the left of the façade shown in Figure BBB below. Figure CCC displays an axial component of 0.421 and a flexural component of 0.918, more than twice that of the axial component. The element is clearly acting mostly in bending which means that the orientation of the element does not adequately mimic the flow of stresses.

The only other element that does not satisfy the interaction equation is a vertical member toward bottom right most side of the façade, shown in Figure BBB below. The member displays a value of 1.052. Again looking at the axial and flexural components of the interaction value in Figure DDD on the following page, the element is seen to act mostly in axial having an axial component of almost twice that of the flexural component.

Figure BBB: Moment Axial Interaction
Source: Author, using SAP 2000

Steel Stress Check Information (AISC-LRFD99)

Frame ID: 416 Analysis Section: HSS10x8x5/8
 Design Code: AISC-LRFD99 Design Section: HSS10x8x5/8

COMBO ID	STATION LOC	MOMENT RATIO	INTERACTION CHECK =	AXL + B-MAJ + B-MIN	MAJ-SHR MIN-SHR RATIO	MAJ-SHR MIN-SHR RATIO
DSTL2	0.00	0.422 (C)	=	0.422 + 0.000 + 0.000	0.119	0.000
DSTL2	3.18	0.878 (C)	=	0.421 + 0.455 + 0.001	0.121	0.000
DSTL2	6.35	1.341 (C)	=	0.421 + 0.918 + 0.002	0.123	0.000
DSTL3	0.00	0.243 (C)	=	0.243 + 0.000 + 0.000	0.067	0.000
DSTL3	3.18	0.500 (C)	=	0.242 + 0.257 + 0.000	0.069	0.000
DSTL3	6.35	0.764 (C)	=	0.241 + 0.522 + 0.000	0.071	0.000

Modify/Show Overwrites: Overwrites
 Display Details for Selected Item: Details
 Display Complete Details: Tabular Data

Strength Deflection OK Cancel
 Stylesheet: Default Table Format File

Figure CCC: Axial and Bending Components of total Interaction Value
 Source: Author, using SAP 2000

Steel Stress Check Information (AISC-LRFD99)

Frame ID: 400 Analysis Section: HSS10x8x5/8
 Design Code: AISC-LRFD99 Design Section: HSS10x8x5/8

COMBO ID	STATION LOC	MOMENT RATIO	INTERACTION CHECK =	AXL + B-MAJ + B-MIN	MAJ-SHR MIN-SHR RATIO	MAJ-SHR MIN-SHR RATIO
DSTL1	0.00	0.953 (C)	=	0.533 + 0.282 + 0.138	0.024	0.015
DSTL1	6.00	0.700 (C)	=	0.532 + 0.131 + 0.037	0.019	0.015
DSTL1	12.00	0.611 (C)	=	0.531 + 0.016 + 0.064	0.013	0.015
DSTL2	0.00	1.052 (C)	=	0.583 + 0.316 + 0.153	0.026	0.017
DSTL2	6.00	0.773 (C)	=	0.583 + 0.148 + 0.042	0.021	0.017
DSTL2	12.00	0.662 (C)	=	0.582 + 0.011 + 0.069	0.017	0.017

Modify/Show Overwrites: Overwrites
 Display Details for Selected Item: Details
 Display Complete Details: Tabular Data

Strength Deflection OK Cancel
 Stylesheet: Default Table Format File

Figure DDD: Axial and Bending Components of total Interaction Value
 Source: Author, using SAP 2000

In order to achieve compliance with the interaction equation for the base diagonal members, iterations which slightly changed their orientation could be run until the axial component is sufficiently greater than the flexural component. At such a point, the orientation of the elements amply follows the flow of stresses. Because the vertical

member that does not satisfy the interaction equation has a sufficiently larger axial component when compared to the flexural component, the orientation of the element adequately follows the flow of stresses. To achieve satisfaction of the interaction equation, the member must be replaced with a cross section with a larger interaction capacity. The process of undertaking iterations of changes in geometry until all elements sufficiently pass the stress checks using SAP 2000 is similar to the process shown in Section 4.1, in which the force polygon of the dendriform was manipulated in order to change the geometry of the dendriform structure in section 4.1. Both processes are useful preliminary design and analysis tools for creating initial geometry for load bearing façades that help ensure particular constraints are met. In addition, both processes show the interdependencies between geometry and load flow.

7.0 A NON-RECTILINEAR FORM

As a closing study, a starting shell structure of non-rectilinear geometry was chosen to show that using the flow of stresses to determine structurally rational dendriform façades does not apply to regular geometries only. The starting shell structure chosen was a spherical dome and can be seen in Figure EEE below. The dome has a diameter of approximately 100', a height of 50', and an approach angle of 60 degrees. The dome is constrained by four pin boundary conditions placed in a radial fashion at 90 degree increments. The footprint of the dome is substantially larger than that of the square shell structure studied in section 6.0. The reason for the increase in footprint is to demonstrate that the basic principles and general process of going from flow nets to dendriform remain essentially constant no matter the scale of structure. In addition,

conducting a study having larger scale will create an opportunity to highlight any hidden pitfalls that may lie in the process when dealing with larger scales. Loading for this study remained static and involved no lateral loading. Loading consisted of the self-weight of the thin shell concrete elements and a single point load of 100 kips centrally placed at the top of the dome.

Figure EEE: Starting Shell Structure
Source: Author using SAP 2000

7.1 Interpretation of Flow Nets

Before running the SAP shell model to achieve the flow nets, it is useful to create a hand sketch of the anticipated flow net. The information presented in Section 2.6.2 reveals that the load path will behave as if water is poured on top of the dome and funnels into the four pinned boundary constraints that act like drains. Therefore the load path begins at the point load at the top of the dome and is resolved at the four pinned boundary constraints. The flow of stresses due to the self-weight will similarly be resolved at the

boundary constraints but the initial loading has a distributed origin because the self-weight of the shell is distributed over the shell's surface. Therefore the load path due to self-weight can be considered to magnify the load path that is due to the applied point load. Using the flow of water analogy allows a hand representation of the flow net to be drawn, which can be seen in Figure FFF below. The solid lines represent the primary compressive stresses while the dashed lines represent the smaller tensile stresses. The dashed lines lie at 90 degree angles to the solid lines because they are principal stress. It should be apparent that although the tensile stresses are small, they cannot be ignored.

Figure FFF: Hand Representation of Expected Flow Net
Source: Author using REVIT

With a hand representation of the anticipated flow nets, the SAP shell model can be run. If the flow net resulting from SAP greatly varies from the hand representation, it can be assumed that either an error lies within the SAP model or that the initial the loading situation and the load path was misinterpreted. Figure GGG below displays a

plan view of the SAP flow nets. The similarities between the hand representation of the flow net and the flow net resulting from SAP are clearly apparent. Arching action is visible between locations of boundary constraints in both flow nets as would be expected. Additionally, the circular ring of secondary tensile stresses is apparent in both the hand and SAP flow nets. The secondary stresses in the SAP flow net are subtle due the disparity of magnitudes when comparing the larger primary stresses. Because consistency of flow nets between the hand representation and the SAP model exists, an understanding of how the structural shell is behaving under the loading situation is evident. The next step is to create a dendriform that adequately captures the flow net.

Figure GGG: Flow net resulting from SAP

Source: Author, using SAP 2000

7.2 Flow Net to Dendriform

Creative liberty is allowable when deriving a dendriform that is meant to mimic the flow net. However, too much liberty can lead to a dendriform façade that is not structurally rational and is inefficient in the manner in which the façade resolves the flow of stresses. When an inefficient dendriform is developed, the flow net to dendriform process becomes iterative until a satisfactory dendriform is constructed. An initial attempt at a dendriform structure can be seen in Figures HHH and III below. Distinct connections between the dendriform and the flow nets are noticeable, including the arching action between supports signifying minimally stressed material within the same location on the initial shell structure. Another connection is the tapering of the dendriform toward the boundary supports signifying a funneling of stresses. Mimicking the funneling of stresses leads to an organic dendriform geometry that resembles a flower with four distinct pedals. This imagery is best seen in the plan view of the dendriform. The sizes chosen were arbitrarily large wide flange sections in anticipation of the long un-braced spans of the members. At this stage, geometry is the main concern and member sizes can be revised later in the process.

Figure HHH: Perspective of Dendriform

Source: Author, using SAP 2000

Figure III: Plan View of Dendriform

Source: Author, using SAP 2000

7.3 Loading and the Dendriform

With the dendriform constructed, it can be loaded similarly to how the dome shell structure was loaded. Just as in Section 6.0, the summation of vertical base reactions for the concrete shell structure will be divided by the summation of the vertical base reactions of steel dendriform structure. In doing so, a ratio or multiplier is achieved that will make the density of concrete and steel equivalent and in turn induces a similar self-weight loading condition for structures of two different materials and material volume.

7.4 Assessing the Dendriform

The SAP steel design check for axial and flexural interaction was conducted prior to including the additional point load atop the dendriform. If the dendriform cannot meet the code compliance embedded in the SAP steel designer under self-weight alone, it will surely fail if subjected to additional loading. A plan view of the results of the steel design check for a single petal of the dendriform structure can be seen in Figure JJJ below.

Recalling that the requirement of the steel design check is that the interaction between axial and bending be less than or equal to unity, it can be seen in Figure JJJ that numerous members fail. Failed members appear in black while members that pass appear in gray. The members that make up the perimeter and the lower spine of the petal all have values greater than one. Taking a closer look reveals that numerous members attaching to the common node at the top center of the dendriform fail as well. After individual inspection of the failed members, many of the failed members exhibit an interaction equation with large flexural components. In addition, the large flexural component is due to bending in the weak axis direction. Interaction results for a single dendriform element

can be seen in Figure KKK following Figure JJJ. Note that the bulk of the equation is due to B-MIN, or bending about the minor axis, yet even the major axis bending portion of the equation exceeds the axial portion.

Figure JJJ: Plan View of Steel Design Check Results

Source: Author, using SAP 2000

Steel Stress Check Information (AISC-LRFD93)

Frame ID	74	Analysis Section	W44X335
Design Code	AISC-LRFD93	Design Section	W44X335

COMBO ID	STATION LOC	---MOMENT RATIO	INTERACTION CHECK	---MAJ-SHR---MIN-SHR---/ RATIO RATIO
DSTL1	0.00	2.964 (C)	= 0.038 + 0.239 + 2.687	0.070 0.041
DSTL1	8.49	2.341 (C)	= 0.037 + 0.131 + 2.173	0.057 0.041
DSTL1	16.97	1.740 (C)	= 0.035 + 0.046 + 1.659	0.043 0.041

☒ Strength
 ☐ Deflection

Stylesheet: Default

Figure KKK: Interaction equation results of dendriform element
 Source: Author using SAP 2000

Two conclusions can be made from an assessment of the dendriform using the results from the steel design check. The first is that because many of the failed members are dominated by flexural behavior, the branch-like elements of the dendriform do not accurately follow the flow of stresses. A re-orientation of such elements may be required in a second iteration of the dendriform. The second conclusion is that because most bending occurs in the weak axis direction, tensile stresses perpendicular to the failed members must be pulling in the weak axis direction. This can be confirmed by looking at the original flow net. In both the hand representation of the flow net and the flow net resulting from SAP, secondary tensile forces, known as hoop stresses, perpendicular to the primary compressive forces exist. Such stresses are the result of the diameter wanting to spread out laterally over the height of the dome due to gravity loading pressing down. A good analogy is pressing down on a marshmallow. As the height of the marshmallow

shrinks, its diameter increases. To combat the lateral deformation, a ring of stresses forms that help to confine the diameter of the dome. In the creation of the dendriform, no members were included to mimic these tensile hoop stresses. In a second iteration of the dendriform, members that follow a ring pattern over the height of the dendriform may prove useful in resisting the weak axis bending.

7.5 A Second Iteration on the Dendriform

Because the previous dendriform focused solely on compression flow nets, a second iteration was done. The goals of the second iteration were to more accurately follow the flow of stresses and include dendriform elements that follow the secondary hoop stresses. However, the organic aesthetic of the dendriform should remain intact. The resulting dendriform can be seen in Figures LLL and MMM below. It should be noted that the same arbitrarily large wide flange section used in the first iteration was also used in the second iteration.

Figure LLL: Perspective of second dendriform iteration
Source: Author using SAP 2000

Figure MMM: Plan View of second dendriform iteration

Source: Author using SAP 2000

Some obvious differences exist when comparing the second iteration of the dendriform to the first iteration of the dendriform. One difference is the inclusion of elements in a ring pattern over the height of the dome. As mentioned earlier, the ring of elements are introduced to follow the tensile hoop stresses seen within the shell structure flow net. A second difference is the re-orientation of the members that outline the arching action of the flow net. In the initial iteration of the dendriform, the members outlining the arching action were concave towards the top of the dome or concave up. In the plan view of the first iteration shown in Figure HHH, the upwards concavity can be seen by the rounded edges of the four petals which oppose the outlining shape of the original flow net of the shell structure. In the second iteration of the dendriform, the members outlining the arching action have a more subtle concavity and tend to follow the funneling of stresses more accurately by reducing the concavity of the petal elements. A final difference

between the two iterations of the dendriform is the location at which the elements of the petals perimeter meet at common points. Within the first iteration, the common point is directly above the pinned boundary restraint which then transitions into a singular trunk-like element. This layout of elements causes an abrupt ending in load path in which all stress must be resolved by the trunk-like element. Just as in the discussion of the stresses near an opening in Section 5.1.1, abrupt changes in geometry induce large localized stresses. In the second iteration of the dendriform, the load path is more fluid and the resolution of stresses is distributed between multiple trunk-like elements.

The second iteration of the dendriform was loaded similarly to that of the domed shell structure. First the vertical base reactions of the shell structure and the dendriform were proportioned to make the density of concrete and steel and in turn the self-weight of the shell and the dendriform, equivalent. Additionally a 100kip point load was applied at the top of the dendriform. Both the self-weight and point load were analyzed together as a dead loading condition. The “steel frame design” option was then used to investigate the beam column interaction. The results of a single petal of the dendriform can be seen below in Figure NNN.

Figure NNN: Steel Frame Design results

Source: Author using SAP 2000

The “steel frame design” results show that all members pass because no interaction value is unity, implying not only adequacy of the existing cross sections but the potential to use smaller cross sections. Also note that the interaction value decreases over the height of the dendriform, finally approaching zero toward the top. The decrease in the interaction value signifies that the cross sections used in the dendriform can decrease in size proportionally to the interaction value. The concept of decreasing cross sections over the height of the dendriform is similar to a tree’s trunk tapering over its

height or the trees branches decreasing in cross section. This idea was first introduced in Section 2.2. Both the tapering and decrease in cross section can be attributed to smaller stresses accumulating at higher locations of the tree.

8.0 CONCLUSION

This thesis has proposed techniques of arriving at structurally rational preliminary geometry in the design process all the while highlighting a unique structural form, the dendriform. This closing section aspires to reiterate and give more insight into the form-finding process presented and expand upon the potential of both the process and use of the dendriform as a structural system.

8.1 Expanding Upon the Form-Finding Process

A main goal of this thesis is to exemplify that hand techniques rooted in first principles of structural mechanics result in a structurally sound and efficient form that can launch the design process. First principles such as a plotting of principal stresses allow the designer to visualize load path in the form of flow nets. The flow nets can be used to dictate where structural elements should be placed in order to accomplish mainly axial behavior within the elements. By coupling the tracing of flow nets with graphical statics, the designer has control of load path and is capable of manipulating both form and force simultaneously. Briefly mentioned earlier in this document, the method of graphical statics has the potential of being exhibited in three dimensions. An adaptation of the presented form-finding process using 3D graphical statics is a logical expansion of the form-finding process. Undertaking such a procedure would involve presenting enough

auxiliary views to encompass the three dimensional structure. In doing so, a three dimensional form and force polygon can be achieved. A side by side graphical representation of a three dimensional form and force polygon can be seen below in Figure OOO. The capability of a 3D visualization of the form and force relation has the potential of becoming a muse for undiscovered patterns and geometries. Printed representations of the form and force polygon are somewhat difficult to read, but can be readily interpreted with computer visualization programs, as was done to create Figure OOO.

Figure OOO: 3D Form and Force Polygon

Source: Author using REVIT

Another expansion of the form-finding process is the consideration of other structural problems. While this thesis has attempted to investigate numerous structural situations of varying loading and geometries, by no means have all scenarios been

exhausted. Potential scenarios include cantilevered geometry, varying boundary conditions, and time duration loading such as thermal loading. With a survey of more structural situations, it is likely that new flow patterns and design insights will arise. Also, the potential for modular structural geometries exists due to common characteristics of flow patterns repeating. Looking past the design phase, modular geometry is beneficial for ease of analysis and construction. An example of a structure of modular geometry is Anne Tyng's City Tower, shown in Figure PPP below.

Figure PPP: Model of Ann Tyng's City Tower
Source: Leslie 2005

Modularity can be seen in both the tubular and spherical dendriforms that were generated in this thesis, with the tubular dendriform composed of four identical facades and the

spherical dendriform as having four similar quadrants. Modularity is possible because of re-occurring flow net traits such as arching action between boundary restraints.

8.2 Validation of the Dendriform as a Structural System

All dendriforms presented proved to be successful in their intention of resisting equivalent loading of a similar solid geometry. An obvious benefit of the dendriform is a lighter structure, the result of removing minimally stressed material leaving behind structural elements that trace the principal stresses or flow nets. When this process is coupled with use of Maxwell's Theorem, an even lighter structure is possible. The potential of applying Maxwell's Theorem to a dendriform structure has been presented here in a new and innovative design process. Because Maxwell's Theorem allows for convergence upon an optimal structure based on volume, an already light structure can become even lighter through iterations of the original dendriform. In a seismic region, a lighter structure can be considered advantageous because it would correspond to a reduced seismic weight. A reduced seismic weight allows for a reduction in demand that the dendriform elements will have to resist when subjected to ground motions and a reduced design demand leads to savings in construction costs and time. Another economic benefit of the proposed designs is the dendriform's ability to resolve stresses through mainly axial behavior, which is far more structurally efficient than flexural behavior. When a structural design minimizes flexural action, connections can be designed as pinned elements and therefore savings in detailing is obtained.

Other factors that validate the use of the dendriform are the potential of having a structure that undergoes constant stress and the dendriform's inherent redundancy.

Redundancy has the obvious benefit of alternate load paths. Constant stress implies a very sophisticated and efficient design. By achieving constant stress a structures safety of factor against failure is well known because no extra material is used and each element is pushed to the brink of its capacity. Additionally, constant stress is the basis of optimization as seen through Maxwell's Theorem. It should be noted that the idea of a constant stress structure is somewhat fanciful in that such an objective is not practical in a typical design process. Achieving a constant stress structure is beneficial as an academic exercise for a given set of loads giving insight into how geometry affects efficiency.

Finally, use of the dendriform can also be validated by looking at architectural demands. The dendriform has the ability to keep much of the load bearing structure to the perimeter of a building. In addition, if intermediate framing is required inside of the structure, the dendriform has the benefit of funneling to a single point. Both of these qualities maximize available floor space. Additionally, the dendriform contains voids between branch-like elements and therefore minimizes obstructed exterior views while allowing space for the placement of electrical and HVAC inside the building.

8.3 Further Possible Advances through Automation

The form-finding process undertaken throughout this thesis can be considered as the very first steps of optimization. In optimization, material is iteratively re-distributed in the design space to optimize for a structural parameter such as maximizing structural stiffness or achieving constant stress throughout the structure. Because the optimization process is iterative, the process is often automated. This thesis has presented other structural parameters having automated potential that could create optimization process

with yet more insight. Such structural parameters include automation of Maxwell's Theorem and the automation of the beam-column interaction. The automation of three dimensional graphical statics could also be advantageous to the form-finding process.

Automation of Maxwell's Theorem would allow for the comparison of dendriforms with varying geometries in an expedited manner seeking an optimum solution based on a predetermined criterion. Maxwell's Theorem could be implemented in an automated process in two ways, as a penalty function, or as a limiting constraint. A penalty function is a global objective function that is paired with any constraints. The purpose of a penalty function is to minimize the objective function while satisfying all constraints (Joghataie, and Takaloozadeh, 2009) . In relation to Maxwell's Theorem, the objective function would be based upon a minimal structural volume, and constraints to be satisfied could be any number of parameters such as stiffness or achieving constant stress. Beam-column interaction would be an ideal constraint to implement when optimizing a dendriform. Typically constraints are included as inequalities or equalities. Beam-column interaction would most likely be included as an inequality constraint in which the summation of the flexural and axial ratio of equation 10 would be less than unity. If not implemented as a penalty function, Maxwell's Theorem has the potential of being a constraint by deciding upon a maximum target volume and keeping all potential dendriform designs below this target volume.

The reciprocal geometry that graphical statics is based upon also lends itself to being automated. Automation of the reciprocal geometry would most likely be in the form of parametric modeling in which form and force are continuously updated as

geometry and or loading changes. Parametric modeling would allow for immediate insight into the relation between form and force. A two dimensional example of such automation can be found on the website entitled “Active Statics” (Greenwold, Allen, and Zalewski 1998). A three dimensional process can be found in the studies of thrust line analysis by Phillipe Block who used it to find compression only forms for 3D vaults. The potential for the automation of graphical statics for other structural geometries including the dendriform is apparent. Another beneficial quality of automating the graphical form-finding process, other than an optical experience of the link between form and force, is that no single unique answer exists. Depending on a multitude of variables such as architectural constraints and loading, numerous structural geometries can be achieved for any one situation. If particular constraints exist such as limited capacity in a particular structural element, the geometry can be arranged in manner that meets such a limitation. Having the advantage of real time updates as the design evolves allows for an interactive thought process in which possible complications due to complex relations are addressed before they arise.

Automation of a process implies use of a computer. It should be reiterated that the aim of this thesis is not to direct the designer to be dependent upon a computer. The control of the form-finding process must always lie in the hands of the designer and must not be overshadowed by computer methods that are not fully understood and have no structural rationalism. Origins of structural patterns and geometries should not be hidden within computer coding nor should any changes be based solely upon aesthetics. The tubular and spherical dendriforms created toward the end of this thesis may be thought to

have been manifested by a computer program but in actuality the computer was used to check the behavior of geometries that were originated by looking to nature for design ideas.

Engineers have a rich tradition of creating forms based on mathematical truths. Early pioneers such as Gaudi and Nervi were capable of fusing these mathematical truths with architectural aesthetic. Such fusion resulted in structures that exhibited and understanding of load flow all while staying true to architectural constraints. This thesis learns from the early tradition and extends it into a new and innovative direction.

9.0 REFERENCES

- (Allen and Zalewski 2010) Allen, Edward, and Wacław Zalewski. *Form and Forces: Designing Efficient, Expressive Structures*. Hoboken, NJ: John Wiley & Sons, 2010. Print.
- (Beres and Rabbat 2007) Beres, Attila B., and Basile G. Rabbat. "Strut-and-Tie Model for Structural Concrete Design." *Structural Engineer*. Portland Cement Association, Oct. 2007. Web. Aug. 2010. <<http://gostructural.com>>.
- (Calzon and Cruz 2010) Martínez-Calzón, J., and Paulo J. S. Cruz. "Treatment of the Form in Structural Engineering." *Proceedings of the First International Conference on Structures and Architecture*. Treatment of the Form in Structural Engineering, Portugal, Guimaraes. London: CRC, 2010. 7-10. Print.
- (Efunda 2010) *Principal Stress 2D*. Digital image. *EFunda: The Ultimate Online Reference for Engineers*. EFunda Inc. Web. Feb. 2010. <http://www.efunda.com/formulae/solid_mechanics/mat_mechanics/plane_stress_principal.cfm>.
- (Gerhardt, Kurrer, and Pichler 2003) Gerhardt, Rolf, Karl-Eugen Kurrer, and Gerhard Pichler. "The Methods of Graphical Statics and Their Relation to Structural Form." *Proceedings of the First Congress on Construction History, Madrid, 20th-24th*, January 2003. Print.
- (Greenwold, Allem, and Zalewski 2003) Greenwold, Simon, Ed Allen, and Wacław Zalewski. "Active Statics." *Active Statics*. Massachusetts Institute of Technology, 2003. Web. Aug. 2011.
- (Huerta 2006) Huerta, Santiago. *Structural Design in the Work of Gaudi*. Invited Paper. Madrid: Department of Structural Design, Escuela Técnica Superior de Arquitectura, Universidad Politécnica de Madrid, 2006. *Google Scholar*. Web. Jan. 2010. <http://oa.upm.es/703/1/Huerta_Art_002.pdf>.
- (Iori 2009) Iori, Tullia. "Projects." *Pier Luigi Nervi*. Milan: ORE Motta Cultura, 2009. 83. Print.
- (Joghataie, and Takalloozadeh, 2009) Joghataie, A., and M. Takalloozadeh. "Improving Penalty Functions for Structural Optimization." *Scientific Iranica* 16.4 (2009): 308+. *Google Scholar*. Web. Nov. 2011.

- (Khan 1981) Khan, Fazlur. "The Chicago Architectural Journal." *Structural Theoreis and Their Architectural Expression-A Review of Possibilities* 1981st ser. 1 (1981): 32+. Print.
- (Kelly and Tosh 2000) Kelly, D.W., and M.W. Tosh. *Interpreting Load Paths and Stress Trajectories in Elasticity*. Tech. 2nd ed. Vol. 17. Sydney: MCB UP, 2000. *Google Scholar*. Web. Feb. 2011. <<http://www.emeraldinsight.com/journals.htm?Articleid=1455437>>.
- (Kurrer 2008) Kurrer, Karl E. "The Discliine-Formation Period of Theory of Structures." *The History of the Theory of Structures*. Berlin: Ernst and Son, 2008. 319. Print.
- (Leslie 2005) Leslie, Thomas. "Yale University Art Gallery New Haven, Conneticut." *Louis I. Khan: Building Art, Builiding Science*. New York: George Braziller, 2005. 85. Print.
- (Mattheck and Kubler 1997) Mattheck, Gerhard Claus., and Hans Kubler. *Wood-The Internal Optimization of Trees*. Berlin: Springer, 1997. Print.
- (Mattheck 1991) Mattheck, C. *Trees: the Mechanical Design*. Berlin: Springer-Verlag, 1991. Print.
- (Urugal and Fenster 2003)Urugal, Ansel, and Saul K. Fenster. "Stresses Due to Concentrated Loads." *Advanced Strength and Applied Elasticity*. 4th Edition ed. New Jersey: Prentice Hall, 2003. 117+. Print.