EFFECTS OF COLUMN STIFFNESS ON SEISMIC BEHAVIOR OF STEEL PLATE SHEAR
WALLS

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TITLE: EFFECTS OF COLUMN FLEXURAL STIFFNESS ON SEISMIC BEHAVIOR
OF STEEL PLATE SHEAR WALLS

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ABSTRACT

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Steel plate shear walls (SPSWs) are a lateral force resisting system consisting of thin infill steel plates surrounded by boundary frame members. The infill steel plates are allowed to buckle in shear and subsequently form diagonal tension field actions during earthquake events. Hysteretic energy dissipation of this system is primarily achieved through yielding of the infill plates.

Conceptually, in a SPSW system with ideally rigid columns pinned to ground, the infill plates at different stories will yield simultaneously as a result of the lateral loads. However, when the columns become flexible, infill plate yielding may initially occur at one story and progressively spread into the other stories with increasing roof displacement.

This research investigates the effect of column stiffness on infill plate yielding sequence and distribution along the height of steel plate shear walls subjected to earthquake forces. Analytical models are derived and validated for two-story SPSWs. Based on the derived model, probabilistic simulations are conducted to calculate the probability of achieving infill plate yielding in both stories before occurrence of a premature failure caused by excessive inter story drift at the initially yielded story. A total of three simulation methods including the Monte-Carlo method, the Latin Hypercube sampling method, and the Rosenblueth's 2K+1 point estimate method were considered to account for the uncertain infill plate thickness and lateral force distributions in the system.

The investigation is also extended to multi-story SPSWs. Three example six-story SPSWs are evaluated using the Rosenblueth's 2K+1 point estimation method which is identified to be most efficient from the simulation on two-story SPSWs. Moreover, the effectiveness of the column

minimum moment of inertia required in the current code for achieving infill plate yielding at every story of SPSWs is evaluated.

Keywords: Steel Plate Shear Wall, column stiffness, drift concentration, nonlinear finite element.

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Notations

- Δ_i = absolute displacement at the i^{th} story level;
- Δ_y = story yield displacement (absolute yield drift limit);
- δ_i = inter-story (relative) displacement of the i^{th} story;
- k_i = stiffness of the i^{th} story SPSW;
- α = stiffness ratio; Note that, by definition, $\alpha \equiv \frac{EI}{kh_3}$
- γ_i = plate thicknesses ratio;
- λ_l = lateral force ratio;
- Δ_{iy} = displacement at the i^{th} floor when yielding first occurs in the SPSW;
- V_i = shear force at the i^{th} story;
- V_{fy} = shear force at the i^{th} story of the FRAME when yielding occurs in the structure;
- V_c = shear force in the continuou column of the simplified model;
- DCF = drift concentration factor (Note: $DCF \geq 1.0$);
- DCF_{ij} = drift concentration factor for category i , behavior j ;
- DCF_{ic} = critical drift concentration factor for category i , $\forall i = 1 \text{ or } 2$;
- DCF_{cgc} = critical drift concentration factor when $\gamma = 1.0$;
- h_i = the i^{th} story height, $\forall i = 1, 2$;
- H = the total height of the structure; for the 2-story case: $H = h_1 + h_2$;
- μ_i = roof ductility
- $\mu_{ic,j}$ = critical roof ductility for Category j ;
- μ_c = story ductility demand to achieve yielding in both stories in the two-story SPSW;

- μ_{cgc} = story ductility demand to achieve yielding in both stories in the two-story SPSW with $\gamma = 1.0$
- I = moment of inertia of the column
- V = base shear;
- V_y = SPSW base shear when yielding occurs;
- F_y = yield strength of infill plate;
- ε_y = yield strain of infill plate;
- E = Modulus of elasticity;
- t = infill plate thickness;
- L = SPSW bay width;
- β = strip inclination angle;
- M_c = bending moment in the continuous column of the simplified model;
- M_{cc} = critical bending moment in the continuous column associated with the case that yielding occurs in both stories of the simplified model;
- M_{cgc} = critical bending moment in the continuous column associated with the case that yielding occurs in both stories of the simplified model and $\gamma = 1.0$;

Abbreviations

2K+1: Rosenblueth's 2K+1 Point Estimation method

CBF: Centrally braced frame

EBF: Eccentrically braced frame

HBE: Horizontal boundary element (i.e. beam)

L.H.: Latin-Hypercube sampling method

M.C.: Monte-Carlo method

RBS: Reduced Beam Sections

SPSW: Steel Plate Shear Wall

VBE: Vertical boundary element (i.e. column)

*Dedicated to my parents, Cherry Yu, Dr Bing Qu,
and everyone who loves structures.....*

Chapter 1 Introduction

1.1 Scope

The Steel Plate Shear Wall (SPSW) is a lateral force resisting system which includes the thin infill steel panels surrounded by the boundary frame members. Past experimental and analytical studies indicate that this system may behave in a ductile manner and have a high hysteretic energy dissipation capacity compared to the conventional braced frame system and the concrete shear wall system (Timler and Kulal, 1983, Elgaaly, *et al*, 1993, Berman and Bruneau, 2003, e.g.,). In addition, SPSWs can be repaired by replacing the infill panels buckled and yielded in a prior earthquake with new panels and thus can be a more viable option in seismic design community (Qu *et al.*, 2007) .

Current design codes for SPSWs in North America, including the AISC Seismic Design Provisions (AISC 2005) and CSA S-16 Standard (CSA 2005), require that the SPSW columns to be designed using the capacity design procedure. In addition, these codes require a minimum moment of inertia for SPSW columns. While these requirements prevent premature column buckling and excessive column inward deformations caused by the infill plate tension field actions observed in prior experimental research (Lubell *et al.* 2004), no work has been done to evaluate whether these requirements can also ensure plate yielding to occur in all stories of a multi-story SPSW. Conceptually, if the SPSW columns are ideally rigid, the infill panels at different stories will form the tension field actions due to infill plate yielding simultaneously. If the SPSW columns become relatively flexible, infill plate yielding will first occur at a certain story and spread into the other stories upon a larger lateral deformation of the SPSW. The yielding sequence of infill panels in a multi-story SPSW depends on many factors, such as infill plate thickness distribution, lateral force distribution, and SPSW column stiffness.

This thesis investigates the effects of column stiffness on seismic behavior of multi-story SPSWs with consideration of uncertain lateral force and infill plate thickness distributions. Based on the derived model, the minimum SPSW column moment of inertia required by the current design codes will be evaluated.

1.2 Outline of Dissertation

A total of seven chapters are included in this thesis. Key issues in each chapter are addressed as follows:

Chapter 2 reviews prior analytical and experimental research related to this thesis.

Chapter 3 provides detailed derivations for two-story SPSWs based on the simplified strip model. Derivations include SPSW lateral stiffness, story drift concentration, ductility demand to achieve yielding in both stories, and moment in the column at different states for the certain plate thickness and lateral loading distributions.

Chapter 4 verifies the models derived in Chapter 3 based on the nonlinear static analysis results using the finite element (FE) models.

Chapter 5 evaluates the seismic behavior of two-story SPSWs with consideration of uncertain infill plate thickness and lateral force distributions based on the analytical models derived and validated in Chapters 3 and 4 ,respectively. A total of three probabilistic simulation methods, i.e., the Monte-Carlo method, the Latin Hypercube method, and the Rosenblueth's 2K+1 Point Estimation, will be considered and their accuracy and efficiency will be compared.

Chapter 6 extends the discussions and analyses from two-story SPSWs to six-story SPSWs. Using the most efficient simulation method identified in Chapter 5, behavior of three six-story

SPSWs will be evaluated followed by the discussion of the current code requirement on SPSW column stiffness.

Chapter 7 summarizes conclusions and design recommendations obtained from this thesis.

Chapter 2 Literature Review

2.1 General

SPSWs are a lateral force resisting system including thin steel infill panels surrounded by beams and columns which are so-called the horizontal and vertical boundary elements (HBEs and VBEs), respectively. Figure 2-1 shows the schematic of a typical multi-story SPSW system. Past experimental investigations, including monotonic, quasi-static, pseudo-dynamic and shaking table tests, reveal that SPSW systems generally exhibit high initial stiffness, behave in a ductile manner, and are capable of dissipating significant amounts of hysteretic energy (Berman and Bruneau 2003, Sabelli and Bruneau 2007, Qu *et al.*, e.g.). Along with the ease of construction and replaceability of infill panels after severe earthquake, SPSWs have been accepted as a desirable alternative to conventional concentrically braced frame (CBF) and concrete shear walls for seismic design for new buildings as well as seismic retrofit of existing constructions.

Figure 2-1 A typical multi-story SPSW system (Berman and Bruneau 2005)

In this section, some of the major research efforts that have led to a better understanding of the behavior of SPSWs are reviewed. Emphasis is placed on analytical investigations, while some key experimental work is also reviewed. Points of primary interest are modeling technique of infill panels, SPSW ultimate strength, effects of boundary frame moment resisting actions on system performance, design of boundary frame members, determination of system stiffness, and the stiffness requirements for the design of SPSW columns. In addition, prior research on effects of column stiffness on seismic behavior of CBFs, which provides building blocks of the research presented in this thesis, are reviewed.

2.2 Research on SPSWs

2.2.1 Thorburn *et al.* (1983)

Thorburn *et al.* recognized the post-buckling strength of steel panels in SPSWs. The authors developed the strip model shown in Figure 2-2 to simulate the infill panel tension field actions. Based on this simplified model, Thorburn *et al.* found that the stress distribution is not uniform throughout the web, depending on the stiffness of columns, as shown in Figure 2-3. In order to achieve the rigid column, a large value of moment of inertia at the order of 100 times larger than that of an actual column were used.

Figure 2-2 Strip model representation of a typical story (Thorburn *et al.* 1983)

Figure 2-3 Distribution of tensile stresses obtained from the strip model (Thorburn *et al.* 1983)

Figure 2-4 Analytical Model of a buckled SPSW web (Thorburn *et al.* 1983)

Figure 2-3 also shows that the strips oriented in the same direction as the average principal tensile stresses in the panel are capable of transferring the tension field actions. In particular, using the model shown in Figure 2-4, Thorburn *et al.* developed an equation to approximate the infill plate as an equivalent brace as followed:

$$t = \frac{2A \sin \theta \sin 2\theta}{L \sin^2 2\beta} \quad (2.1)$$

where t is the actual thickness of the plate, A is the assigned area of the equivalent strips, L is the bay width, θ is the angle between the vertical axis and an equivalent diagonal

brace; β is the angle of inclination of the principal tensile stresses in the infill plate measured from vertical, which can be found by,

$$\tan^4 \beta = \frac{\left(1 + \frac{Lt}{2A_c} \right)}{\left(1 + \left(\frac{h^3 t}{360A_b} \right) \right)} \quad (2.2)$$

where A_b is the cross-section area of the beam and A_c is the cross-sectional of the column, respectively; h is the story height.

2.2.2 Timler and Kulak (1983)

Timer and Kulak (1983) performed a full scale SPSW testing to verify the strip model developed by Thorburn *et al.* (1983). They found that equation (2.2) failed to include the contribution of column stiffness and revised this equation as followed:

$$\tan^4 \beta = \frac{\left(1 + \frac{Lt}{2A_c} \right)}{\left(1 + ht \left(\frac{1}{A_b} + \frac{h^3}{360LI} \right) \right)} \quad (2.3)$$

where I = moment of inertia of the column.

2.2.3 Berman and Bruneau (2003)

Realizing the behavior of SPSW system is different from plate-girders, Berman and Bruneau developed equations to estimate the plastic strength of SPSW systems using the classic plastic analysis procedure.

Figure 2-5 Strip model for a single-story SPSW with simple beam-column connections (Berman and Bruneau, 2003)

Figure 2-5 shows the strip model of a single story SPSW used for the derivation. Simple beam-to-column connections and truss members in the tension zones are assumed. Based on static equilibrium equations and taking into account the contribution of each zone considered, the plastic strength of the wall can be estimated as

$$V = \frac{1}{2} F_y t L \sin 2\alpha \quad (2.4)$$

where F_y is effective yield stress of steel, α is the angle of inclination of the principal tensile stresses in the infill plate measured from vertical, i.e., equivalent to β as in equation(2.2) and (2.3), as shown in Figure 2-5.

For a multi-story SPSW with all infill plates uniformly yielded along the height of the wall, a general equation for the ultimate strength of the structure can be similarly derived as

$$\sum_{i=1}^{n_s} V_i h_i = \sum_{i=1}^{n_s} \frac{1}{2} F_y (t_i - t_{i+1}) L h_i \sin 2\alpha \quad (2.5)$$

where h_i is the story elevation, n_s is the total number of stories, V_i is the lateral point force at the i^{th} story level, and t_i is the plate thickness at the i^{th} story.

In the case where the beam-to-column connection is rigid, the equation becomes:

$$\sum_{i=1}^{n_s} V_i h_i = 2M_{pcl} + 2M_{pcn} + \sum_{i=1}^{n_s-1} M_{pbi} + \sum_{i=1}^{n_s} \frac{1}{2} F_y (t_i - t_{i+1}) L h_i \sin 2\beta \quad (2.6)$$

where M_{pcl} is the first story column plastic moment, M_{pcn} is the top story column plastic moment, and M_{pbi} is the plastic moment of the i^{th} story beam.

Berman and Bruneau also concluded that a minimum of 10 strips is required to capture the behavior of each infill panel of a conventional SPSW; and the area of each strip can be calculated using the plate thickness and the tributary width of each strip.

2.2.4 Qu *et al.* (2007)

Behaviors of the SPSW beams and replaceability of the SPSW infill panels were experimentally evaluated by Qu *et al.* (2007). Figure 2-5 shows the test setup schematic of this investigation. The beams were designed with the reduced beam section (RBS) connections at all levels. Dimensions of the structure were determined according to the recommendations provided by Berman and Bruneau (2003), where the thicknesses for the infill panel were 3mm and 2mm for the first and second story, respectively. In Phase I test, the specimen was subjected to pseudo-dynamic tests using three ground motions of decreasing intensities. The buckled panels were replaced by new panels in Phase II and the specimen was then subjected to pseudo-dynamic tests and cyclic tests until failure.

Figure 2-6 Specimen schematic and test setup (Qu *et al.* 2007)

It was found that the replaced panel performed desirably as it dissipated significantly amount of hysteretic energy while the boundary frames remained in good condition. The RBS connections between the beams and the columns were found to form plastic hinges during the tests. Figure 2-7 below shows the RBS connection ruptured at the intermediate beam.

Figure 2-7 Rupture at the north end of the intermediate beam (Qu *et al.* 2007)

2.2.5 Qu and Bruneau (2009)

Based on the work from Berman and Bruneau (2003), Qu and Bruneau provided the method to estimate the over-strength in SPSW due to moment resisting action of the boundary frame. In this paper, uniform collapse mechanism was assumed in the derivation. Results indicate that a more economical design and more desirable hysteretic

energy dissipation can be achieved with consideration of the boundary frame moment resisting action.

2.2.6 Topkaya and Atasoy (2009)

Topkaya and Atasoy developed analytical models for evaluation of the lateral stiffness of SPSWs. Through the result comparison between the testing data and analytical predictions, Topkaya and Atasoy found that the simplified strip model seems to underestimate the lateral stiffness (lower limit, flexible solution) while the detailed finite element model using shell elements tends to provide an overestimate (upper limit, stiff solution). Realizing the true stiffness is always between these upper and lower limits and the fact that the total SPSW lateral stiffness includes two portions of stiffness, namely, the elastic stiffness and the post-buckling (inelastic) stiffness, the paper derived a simplified method for calculation of SPSW stiffness. Excluding geometrical nonlinearity and treating the SPSW as a vertical plate girder, in which the columns, beams and infill plates are analogous to the flanges, stiffeners and web of the girder, the strain energy, U , can be approximated as:

$$U = U_b + U_s = \int_0^H \frac{(M(x))^2}{2EI} dx + \int_0^H \beta \frac{(V(x))^2}{2GI^2} dx \quad (2.7)$$

where G = shear modulus of elasticity, U_b = strain energy due to bending, U_s = strain energy due to shear, H = the height of SPSW system, and,

$$\beta = \int_A \left(\frac{Q}{b} \right)^2 dA \quad (2.8)$$

where Q = static moment of the area with respect to neutral axis, b = width of the section, and A = area of SPSW.

Seeing the computational difficulty in equation(2.8) and assuming a linear distribution of the shear stresses along the infill plates as shown in Figure 2-8, equation(2.8) is simplified as:

$$\beta = \beta_1 + \beta_2 \quad (2.9)$$

where β_1 represents the contribution of shear stresses in the column section and β_2 represents the contribution of shear stresses in the infill plate. Note that $\beta_2 \ll \beta_1$.

Figure 2-8 Shear stress variation for a steel plate shear wall system (Topkaya and Atasoy 2009)

According to the Castigliano's second theorem, the displacement at a given location can be found as:

$$\Delta_{total} = \int_0^H \frac{M(x)}{EI} \left(\frac{\partial M(x)}{\partial P} \right) dx + \int_0^H \beta \frac{V(x)}{GI^2} \left(\frac{\partial V(x)}{\partial P} \right) dx \quad (2.10)$$

where P is the force acting at the point of interest.

Therefore, the SPSW stiffness can be found as:

$$k_{SPSW} = \frac{P}{\Delta_{total}} \quad (2.11)$$

Using this approach, Topkaya and Atasoy found that the results from this method agree with those from the finite element method. They also indicated that the post-buckling stiffness of the infill plate is affected by the aspect ratio and the plate slenderness; on the other hand, the post-buckling system stiffness is greatly influenced by the plate slenderness and the stiffness of the boundary elements.

2.2.7 Qu and Bruneau (2010)

In the recognition of out-of-plane buckling and shear yielding of columns from the previous experimental investigation on SPSWs, Qu and Bruneau developed analytical models to capture the column out-of-plane buckling strength and shear strength.

In Qu and Bruneau (2010), the flexibility factor, ω_t , which was derived from prior research on plate girders and was adopted to ensure adequately stiff columns in SPSW design, was reviewed:

$$\omega_t \approx 0.7 h_s^4 \sqrt{\frac{t_{wi}}{2 I_c L}} \quad (2.12)$$

where h_s is the story height and I_c is the column moment of inertia.

Based on Montgomery and Medhekar (2001), the maximum of ω_t should be limited to 2.5, a corresponding I_c can be found as:

$$I_c \geq \frac{0.00307 t_{wi} h_{si}^4}{L} \quad (2.13)$$

This is the equation for minimum column moment of inertia required by the CSA S16 standard and the AISC Seismic Design Provisions for SPSW column design.

Figure 2-9 In-plane free body diagram of the column (Qu and Bruneau, 2010)

Qu and Bruneau also evaluated the shear demand and strength in the SPSW columns.

Based on the free body diagram in Figure 2-9, the following equation for shear demand is derived:

$$V_{u-design} = \frac{2R_y f_y Z_c}{h_{si}} + \frac{\omega_{xci} h_{si}}{2} + \frac{\omega_{yci} d_{ci}}{2} \quad (2.14)$$

where d_{ci} is the column depth; Z_c is the plastic section modulus; ω_{xci} and ω_{yci} are the horizontal and vertical component of infill panel yield forces along the column, respectively; f_y is the yield stress of the boundary frame and R_y is the ratio of expected to nominal yield stress.

Qu and Bruneau claimed equation(2.14) may overestimate the demand for the reasons including the presence of axial force which may reduce the plastic moment at the column ends and the fact that plastic hinges may be formed at the beams instead of at the column, and therefore, the shear demand should be compared to the following shear strength:

$$V_n = 0.6 f_y d_{ci} t_{wci} \quad (2.15)$$

In addition to shear, the undesirable out-of-plane buckling behavior of SPSW columns was also investigated in this paper. Figure 2-10 shows a free-body diagram of the columns under lateral static loadings. Using this diagram, the compression force at the top end of the right-hand-side column, P_{topi} , is derived as followed:

$$P_{topi} = \sum_{j=i}^{n_s} V_{rj} + \sum_{j=j+1}^{n_s} \omega_{ycj} h_{sj} \quad (2.16)$$

where n_s is the total number of stories and V_{rj} is the right end shear of the beam, which can be calculated as:

$$V_{rj} = \frac{(\omega_{ybi} - \omega_{ybi+1})L}{2} + \frac{(\omega_{xbi} - \omega_{xbi+1})d}{2} + \frac{2R_y f_y Z_{ef}}{L_h} \quad (2.17)$$

where d is the depth of the beam, L is distance between the column faces, L_h is the distance between plastic hinge locations and Z_{ef} is the effective plastic section modulus of the beams.

Figure 2-10 VBE free body Diagrams (Qu and Bruneau, 2010)

Figure 2-11 Strong and weak axes for VBEs (Qu and Bruneau, 2010)

Figure 2-11 shows the weak and strong axes orientation for a typical SPSW system, where out-of-plane buckling was generally observed in the plane perpendicular to the weak axis. Based on different combinations of boundary conditions at the top and bottom of the column, the column buckling strength criterion are evaluated and compared to the results from the past experimental research.

2.3 Research on Effects of Column Stiffness on Seismic Behavior of CBFs

2.3.1 MacRae *et al.* (2004)

MacRae *et al.* (2004) investigated the effects of continuous columns on mitigating the story drift concentration in steel concentrically braced frames (CBFs). This paper focuses on a two-story CBF shown in Figure 2-12(a). Pin connections were assumed at the ground levels. In addition, the story height and bracing members are assumed to be identical in both stories. Assuming the lateral forces are linearly distributed, MacRae *et al.* investigated the story drift concentration and moment demand on the continuous columns according to the diagrams shown in Figure 2-12 (d), (e), (f), (g) and (h).

Figure 2-12 Idealization of two-story frame with gravity column deformation and forces (MacRae *et al.*, 2004)

The paper introduced an index called Drift Concentration Factor (DCF) to describe the interstory deformation concentration. For a structure with N stories, DCF is defined as:

$$DCF \equiv \frac{\max \left\{ \frac{\delta_i}{h_i} \right\}}{\frac{\Delta_{roof}}{H}}, \text{ where } i \leq N \quad (2.18)$$

For the two-story case with identical story height,

$$DCF = \frac{\max \left\{ \frac{\delta_2}{h}, \frac{\delta_1}{h} \right\}}{\frac{\Delta_2}{H}}, \quad (2.19)$$

where δ_2 and δ_1 are relative displacements for the second and the first story, respectively,

Δ_2 is the absolute displacement at the second floor, h is the story height and H is the height of the structure.

Note from equation (2.18) and (2.19) that the story drift distribution is uniform when DCF is unity, while a larger DCF value corresponds to a severer drift concentration in the

structure. To relate the column flexural stiffness and the lateral stiffness of the brace members, the paper also introduced the following parameter called stiffness ratio:

$$\alpha \equiv \frac{EI_c}{kh^3} \quad (2.20)$$

where EI_c / h^3 is the flexural stiffness of the column and k is lateral stiffness of the first story brace members. Also note that as $\alpha \rightarrow \infty$, the column becomes ideally rigid; on the other hand, as $\alpha \rightarrow 0$, the column is ideally flexible.

Using the force diagrams shown in Figure 2-12, equations for DCF and M_c are derived in terms of α and the roof ductility, μ_t .

When only the bottom story is yielded,

$$DCF = \frac{10\mu_t + 55\mu_t\alpha - 4 - 10\alpha + 75\alpha^2\mu_t}{5\mu_t(1+5\alpha)(1+3\alpha)} \quad (2.21)$$

$$M_c = \frac{\alpha(15\mu_t\alpha + 5\mu_t - 4 - 10\alpha)}{(2+5\alpha)(1+5\alpha)} V_{fy} h \quad (2.22)$$

When the both stories yield,

$$DCF = \frac{2d_1}{\Delta_2} = 1 + \frac{1}{15\mu_t\alpha} \quad (2.23)$$

$$M_c = V_c h = 0.2V_{fy} h = 0.1V_{fy} H \quad (2.24)$$

where V_{fy} is the yield shear strength of the bottom story.

Using the FE models, MacRae *et al.* evaluated the adequacy of these abovementioned equations as shown in Figure 2-13.

Figure 2-13 Behavior of two-story CBF with various column stiffness and roof ductility (MacRae *et al.* 2004)

2.3.2 Kimura and MacRae (2006)

In comparison with MacRae *et al.* (2004), in which the two-story CBF with continuous columns pinned to ground was investigated, Kimura and MacRae evaluated a high-rise CBF with continuous columns fixed to ground as shown in Figure 2-14. Compared with the pinned column base, the CBF with fully fixed column base results in a higher demand on the foundation while increases the total lateral stiffness of the structure. Column shown in Figure 2-14 can be treated as a cantilever. The braced frames were assumed to be designed such that the brace sizes varied every other story. The brace strength was assumed to match the story shear demand at each story. The frame was under lateral loads with inverse triangular distribution.

$$\begin{bmatrix} \chi_{c1} \\ \chi_{c2} \end{bmatrix} = \frac{10^{-3}}{N} \begin{bmatrix} 2.5 \\ 10\mu - 5 \end{bmatrix} \quad (2.27)$$

where μ is the roof ductility and N is the number of stories.

For small values of α , i.e., flexible columns, DCF can be approximated as:

$$DCF = \min \left\{ \begin{array}{l} \frac{1 + 0.45 \left(1 + \frac{0.4}{\chi_c^{0.1}} \right) (\mu_t - 1)^{0.6} \left(1 - \frac{\beta'}{3} \right) \left(\frac{\beta'^{0.32}}{\alpha^{0.22 + 0.0015\chi_c}} \right) (N^{-0.45} - 1)}{\mu_t^{0.5} \left(1 - \frac{\beta'}{3} \right)} \\ DCF_{\alpha=0}, \text{ where } DCF_{\alpha=0} = \frac{(\mu_t - 1)H}{h_1 \mu_t} \text{ at } \alpha=0 \end{array} \right. \quad (2.28)$$

where,

$$\beta' = \left(1 - \frac{1.3}{N} \right) \beta^{\left(1.25 + \frac{0.5}{N} \right)} \quad (2.29)$$

2.3.3 Ji *et al.* (2009)

Based on the work from MacRae *et al.* (2004), Ji *et al.* investigated the drift concentration of a three-story CBFs with continuous columns. Figure 2-15 shows the plan view and elevation of the structure considered. All gravity columns in Figure 2-15(b) are idealized and condensed to one representative continuous column with a flexural stiffness EI . This idealized column is assumed to remain elastic throughout the analyses.

Figure 2-15 Three-story CBF: (a) plan view; (b) examined braced frame (Ji *et al.* 2009)

The base of the column is assumed to be either fully fixed connection or pinned connection, as shown in Figure 2-16 and Figure 2-17, respectively. The gravity column is simply connected to the braced system via rigid link members at floor levels. All other connections are assumed to be pinned and the story heights are identical along all stories.

Figure 2-16 Simplified elastic model of CBFs for fixed-based case (Ji *et al.* 2009)

Figure 2-17 Simplified elastic model of CBFs for pinned-base case (Ji *et al.* 2009)

DCF, is again adopted to quantify the drift concentration; here, for the three-story case,

$$DCF = \frac{\max \left\{ \frac{\delta_3}{h}, \frac{\delta_2}{h}, \frac{\delta_1}{h} \right\}}{\frac{\Delta_3}{H}} \quad (2.30)$$

In addition, the stiffness ratio, α , as defined in equation(2.20), is also adopted to quantify the relative stiffness of the gravity column. Furthermore, Ji *et al.* evaluated how the change of stiffness ratio would affect DCF due to different degradations in the first story, which is accounted by the γ factor. As shown in Figure 2-18, in general, the authors

found that increase in stiffness ratio will decrease DCF, which physically means the drift distribution is more uniform as the column gets more rigid. .

Figure 2-18 Column stiffness ratio vs drift concentration factor (Ji *et al.* 2009)

Ji *et al.* also found that, for the pinned base case, when $0.1 < \alpha < 2$, the continuous column is most efficient at mitigating drift concentration; whereas for the fixed base case, the column is most efficient at mitigating the drift concentration when $0.01 < \alpha < 0.2$. Note that a smaller γ value leads to a larger DCF value when the column is relatively flexible.

2.4 Summary

This chapter reviews the analytical work related to seismic behavior of SPSWs and CBFs. Papers reviewed in this chapter provide building blocks for deriving the analytical and FE models which will be discussed in detail in the following chapters.

Chapter 3 CLOSED-FORM SOLUTIONS FOR TWO-STORY SPSW SYSTEMS

3.1 Introduction

As described in Chapter 2, prior research indicates that the continuous columns in a multi-story CBF tend to redistribute the drifts along the height of the structure during seismic events. When the continuous columns are ideally rigid, the brace members at different stories yield simultaneously under specified lateral load distributions and a uniform story drift distribution is expected in the building. When the continuous column becomes relatively flexible in a multi-story CBF, yielding in the braced members first occurs at a certain story level, and then progressively spreads into the other stories, resulting in drift concentration and possible premature failures at some stories. While the mathematical model, which quantifies the effects of column for CBFs, exists, such a model for SPSWs remains missing. To fill the critical knowledge gap, this chapter investigates the behavior of a two-story SPSW and formulates the mathematical model to capture the effects of SPSW columns. The following sections describe in detail the assumptions, performance cases and behaviors considered, and step-by-step derivations for the model. Derivations in this chapter will be validated by the finite element model in Chapter 4. In addition, while these equations derived here are only valid for two-story SPSWs, the corresponding results will provide building blocks for the more complicated cases (i.e. multi-story SPSWs with more than three stories) which will be discussed in detail in Chapter 6.

3.2 Derivation assumptions

The schematic of the considered two-story SPSW and its simplified model (i.e. strip model plus continuous column model) is illustrated in Figure 3-1. VBEs and HBEs in the actual SPSW are treated as the boundary frame members and the infill plates are considered by the diagonal strips pinned to the frame members in the simplified model. The SPSW columns are considered by a continuous column member pinned to ground and linked to the boundary frames as shown in Figure 3-1(b). The following sections describe other assumptions made in this model and the corresponding derivations.

Figure 3-1 Actual SPSW and simplified model

3.2.1 Story height assumption

While the story heights may vary from story to story in a real structure, MacRae *et al.* (2004), Kimura and MacRae (2006), and Ji *et al.* (2009) consistently assumed constant story height in prior researches on CBFs. Such an assumption was made based on the fact that the effect of different story heights on the system seismic behavior is negligible in conventional building structures. For simplicity, the two-story SPSW considered in this chapter is also assumed to have identical story heights in both stories. It is recognized that different story heights can be incorporated into the formulation presented below if needed.

3.2.2 Connection assumptions

All the connections are assumed to be pinned in the simplified model, including the member-to-member connections in the boundary frames, the connections between the link beams to the columns, the column-to-ground connections, and infill strips to the boundary frames. It is noted that the above assumptions correspond to the SPSW with simple beam-to-column connections and such assumptions are reasonable due to the fact that the moment resistance of beam-to-column connections can be significantly reduced due to the presence of tension field actions (Qu and Bruneau 2010).

3.2.3 Material property assumptions

All the elements are assigned isotropic steel material properties and self-weight of the elements are neglected. In addition, thermal effects are not considered.

3.2.4 Boundary frame assumptions

Given that the current design codes prevent excessive deflection of boundary frame members caused by the infill tension field actions, the boundary frame members are assumed to be relatively rigid and remain elastic in the analysis. The boundary frame members and the link beams are treated as beam-column elements and truss elements, respectively.

3.2.5 Infill strips assumptions

Past experimental research indicates that the compressional strength of SPSW infill panel is negligible compared with its tensile strength (Thorburn *et al.* 1983); the infill strips are therefore considered by truss elements with tension-only material property in this investigation. The procedures to obtain the infill strips from the given infill panels are presented in detail in Appendix A.

3.2.6 Continuous column assumptions

Buckling and inelastic behavior of the column is out of the scope of this study, i.e., only the elastic column behavior is considered. Such an assumption is made since the capacity design principle requires the columns remain elastic. Moreover, the column is assumed to be pinned to the ground for simplicity. Although the fully fixed column base could be more effective to reduce story drift (Kimura and MacRae, 2006), the corresponding demands on foundation are larger compared to the pinned column base case, which may not be practical in some cases. Additionally, if the continuous column is fixed to ground, the simplified system becomes statically indeterminate, adding complications in derivations of the analytical model.

3.3 Wall stiffness due to infill strips

As shown in a later section, the wall stiffness due to infill strips is required in derivation of the analytical model for quantification of drift concentration. Although a more sophisticated model for determination of SPSW stiffness was developed by Topkaya and Atasoy (2009), their model requires further calculation efforts and may not be practical for the scope of this research. This section focuses on a single story SPSW and derives the wall stiffness due to the infill plate. The equation derived can be used to calculate the story stiffness of the simplified structure shown in Figure 3-1.

According to Thorburn *et al.* (1983), behavior of a single-story SPSW can be approximated by the strip model shown in Figure 3-2. It is noted that the number of strips considered in the model has no impact on the results derived as shown later. Three representative strips, i.e., strips (a), (b) and (c), are selected to represent the strips

anchored to the left column and the top beam, the bottom and top beams, and the bottom beam and the right column, respectively.

Figure 3-2 Strip model for a single-story SPSW

According to Berman and Bruneau (2003), the ultimate strength, V_y , of the considered SPSW can be calculated as:

$$V_y = \frac{1}{2} F_y t L \sin 2\beta \quad (3.1)$$

where β is the inclination angle of infill strips measured from vertical, F_y is the effective yield stress of steel, t is the infill panel thickness and L is the bay width, as shown in Figure 3-2.

When strip (b) reaches its elastic limit, the corresponding lateral deformation at the roof level can be determined as:

$$\Delta_{hb} = \frac{\Delta_{yb}}{\sin \beta} \quad (3.2)$$

where Δ_{hb} is the roof deformation associated with yielding in strip (b), and Δ_{yb} is the elongation of strip (b) associated with its elastic limit.

Figure 3-3 Elongation of the representative strip (b)

From mechanics of material,

$$\epsilon_y = \frac{\Delta_L}{L} = \frac{\Delta_{yb}}{L_b} \quad (3.3)$$

and ,

$$F_y = E\epsilon_y \quad (3.4)$$

where L_b and L both represent the initial length of strip(b), Δ_L is the change of the length of the strip, and ϵ_y is the yield strain of the strip, as shown in Figure 3-3.

Solving for Δ_{yb} from equation(3.3) and substituting it into (3.2) gives:

$$\Delta_{hb} = \frac{L_b \epsilon_y}{\sin \beta} = \left(\frac{h}{\cos \beta} \right) \left(\frac{\epsilon_y}{\sin \beta} \right) = \frac{2h\epsilon_y}{\sin 2\beta} \quad (3.5)$$

Similar to strip (b), strip (a) shown in Figure 3-2 will also undergo elongation. However, because the initial length of strip(a), denoted as L_a , is different from strip (b), the magnitude of the longitudinal elongation requires a further investigation.

Figure 3-4 Elongation of the representative strip (a)

Under the assumption of small deformation, geometry analysis based on Figure 3-4 shows that the initial length for strip (a) can be found as:

$$L_a = \frac{(h - y)}{\cos \beta} \quad (3.6)$$

The elongation of strip (a), ΔL_a , can then be found as:

$$\Delta L_a = h\theta \sin \beta - y\theta \sin \beta = (h - y)\theta \sin \beta \quad (3.7)$$

where θ is the story drift angle, which can be found as:

$$\theta = \frac{\Delta_{ha}}{h} \quad (3.8)$$

where Δ_{ha} is the horizontal component of the elongation of strip (a).

Using equations (3.3), (3.7) and (3.6), the yield strain in strip (a) can be found as:

$$\epsilon_{ya} = \frac{\Delta L_a}{L_a} = \frac{1}{2} \theta \sin 2\beta \quad (3.9)$$

Solving for story drift in equation(3.9) gives,

$$\theta = \frac{2\epsilon_{ya}}{\sin 2\beta} \quad (3.10)$$

Therefore, the corresponding horizontal displacement of strip (a) when it yields, according to equations (3.8) and(3.10), is:

$$\Delta_{ha} = h\theta = \frac{2h\epsilon_{ya}}{\sin 2\beta} \quad (3.11)$$

Note that thermal strain is out of scope of this analysis. In addition, yield strain for any strip is identical, that is,

$$\epsilon_y = \epsilon_{ya} = \epsilon_{yb} \quad (3.12)$$

Therefore, the horizontal elongation component at the yielding state for any strip is also identical according to equations (3.11), (3.5) and(3.12). That is,

$$\Delta_y = \Delta_{ha} = \Delta_{hb} = \Delta_{hc} = \frac{2h\epsilon_y}{\sin 2\beta} \quad (3.13)$$

where Δ_{hc} is the roof deformation component of strip(c).

It is important to realize that equation(3.13) implies all the strips in a SPSW will yield simultaneously. For SPSWs, yielding of infill strip corresponds to the change of slope in the force versus displacement curve. It physically means that the story changes from its elastic state with an elastic stiffness, as shown in the hatched region in Figure 3-5, to its inelastic state, corresponds to the area underneath the flat line in Figure 3-5.

Figure 3-5 Typical Pushover curve

Now, combining equations (3.1) and (3.13) by introducing equation (3.4), the stiffness of SPSW using the strip model can be found as:

$$k_{SPSW} = \frac{V_y}{\Delta_y} = \frac{1}{4} Et \left(\frac{L}{h} \right) \sin^2 2\beta \quad (3.14)$$

where L/h is known as the length-to-height aspect ratio. For a given infill plate thickness and wall width, equation (3.14) indicates that SPSWs become more flexible as their story heights increase. Since past analytical and experimental investigations show that the strip angle, on average, can be approximately to be 45 degree (Berman and Bruneau 2003), i.e., $\beta = 45^\circ$, equation(3.14) can then be further reduced as:

$$k = \frac{1}{4} Et \left(\frac{L}{h} \right) \quad (3.15)$$

3.4 Structural behavior categories for the considered two-story SPSWs

When a two-story SPSW is subjected to the lateral forces, infill plate yielding will first occur in one story and then spread into another story. Whether initial infill plate yielding occurs at the first story or the second story depends on the lateral force distribution and the infill plate thickness distribution. Moreover, whether infill plate yielding can be achieved in both stories depends on the ultimate target displacement at the roof level.

Figure 3-6(a) and Figure 3-6(b) illustrate the two performance categories expected in the considered two-story SPSW, i.e., the cases in which infill plate yielding initially occurs at the first and second stories, denoted as Category I and Category II, respectively. Within each category, the performance of the SPSW can be further differentiated into four behaviors based on the target roof displacement level, namely, Behavior A, in which one story just reaches to its elastic limit and the other story remains elastic; Behavior B, in which the initially yielded story deforms into the inelastic region and the other story remains elastic; Behavior C, which is the critical case corresponding to the minimum deformation demand on the structure to ensure plate yielding in both stories; and Behavior D, in which both stories experience post-yielding behavior. The corresponding infill strip stress versus roof displacement curves associated with these behaviors for the considered categories are shown in Figure 3-6 (c) and (d), respectively.

Figure 3-6 Considered categories and behaviors

In order to determine which performance category controls, define the following two parameters:

$$\lambda = \frac{F_2}{F_1} \quad (3.16)$$

$$\gamma = \frac{t_2}{t_1} \quad (3.17)$$

where F_2 and F_1 are the lateral force at the second and the first story, respectively, and t_2 and t_1 are the infill plate thickness at the second and the first story, respectively.

Essentially, γ and λ describe the infill plate thickness distribution and the lateral force distribution in the considered SPSW.

Given the fact that the shear force at the second story should be smaller than that at the first story, the infill panel at the second story should not be thicker than that at the first story, which requires

$$0 < \gamma \leq 1.0 \quad (3.18)$$

Incidentally, $\gamma = 1.0$ corresponds to the case in which the infill plates at both stories have the same thickness. This may happen when the minimum thickness required at the second story is not available and a thicker plate has to be used. Recalling equations (3.1) and (3.15), the yield strength ratio and stiffness ratio between the two stories are also equal to the plate thickness ratio:

$$\gamma = \frac{t_2}{t_1} = \frac{k_2}{k_1} = \frac{V_{y2}}{V_{y1}} \quad (3.19)$$

where V_{y2} and V_{y1} are the shear strength for the second and first story, respectively.

The lateral force distribution, which is captured by λ , can be determined from the reactive weight distribution. Neglecting the shear force redistribution effect due to the stiffness of the continuous column in the model shown in Figure 3-6, the following

comments may be made: when $\gamma > \frac{\lambda}{\lambda + 1}$, overstrength exists in the second story and

infill plate yielding will initially occur in the first story (i.e., Category I controls); when

$\gamma < \frac{\lambda}{\lambda + 1}$, overstrength exists in the first story and infill plate yielding will initially occur

in the second story (i.e., Category II controls); and when $\gamma = \frac{\lambda}{\lambda + 1}$, both stories will yield

simultaneously; however, this case will be excluded from this thesis since it is unlikely to occur in practice due to the fact that the required plate thickness determined from the ideal lateral force distribution is typically not available in the market and a slightly thicker plate has to be used for design.

As will be shown in the following derivation, DCF of the considered SPSW is the key parameter describing the system performance. By definition, DCF is:

$$DCF = \frac{\max\left\{\frac{\delta_2}{h}, \frac{\delta_1}{h}\right\}}{\frac{\Delta_2}{H}} \geq 1 \quad (3.20)$$

where Δ_2 is the absolute displacement at the roof; h and H respectively represent the story height and the total height of the structure; and δ_1 and δ_2 respectively represent the relative displacements at the first and second story.

Figure 3-7 Schematic of Performance Categories I and II

Per Figure 3-7 the relative displacements in both stories can be calculated as:

$$\delta_1 = \Delta_1 \quad (3.21)$$

$$\delta_2 = \Delta_2 - \Delta_1 \quad (3.22)$$

where Δ_1 and Δ_2 are the absolute displacement at the first and second story, respectively.

When Category I controls, infill plate yielding will first occur at the bottom story,

resulting in $\delta_1 > \delta_2$. Accordingly, equation (3.20) can be simplified as:

$$DCF = DCF_I = \frac{\max\{\frac{\delta_2}{h_2}, \frac{\delta_1}{h_1}\}}{\frac{\Delta_2}{H}} = \frac{\frac{\Delta_1}{h}}{\frac{\Delta_2}{2h}} = 2 \frac{\Delta_1}{\Delta_2} \quad (3.23)$$

Similarly, when performance Category II controls, $\delta_2 > \delta_1$, and the following equation applies:

$$DCF = DCF_{II} = \frac{\max\{\frac{\delta_2}{h_2}, \frac{\delta_1}{h_1}\}}{\frac{\Delta_2}{H}} = \frac{\frac{\Delta_2 - \Delta_1}{h}}{\frac{\Delta_2}{2h}} = 2 \frac{\Delta_2 - \Delta_1}{\Delta_2} \quad (3.24)$$

As required in the following derivations, for both Categories I and II, define Δ_{1y} and Δ_{2y} which respectively represent the lateral displacements at the first and second stories associated with the scenario when infill plate yielding first occurs. Accordingly, the roof ductility can be calculated as:

$$\mu_t = \frac{\Delta_2}{\Delta_{2y}} \quad (3.25)$$

Introducing equations (3.21), (3.22), and (3.25) into (3.23) and (3.24), respectively, one can obtain:

$$DCF_I = 2 \left(\frac{\Delta_2 - \delta_2}{\Delta_2} \right) = 2 \left(1 - \frac{\delta_2}{\Delta_2} \right) = 2 \left(1 - \frac{\delta_2}{\mu_t \Delta_{2y}} \right) \quad (3.26)$$

$$DCF_{II} = \frac{2(\Delta_2 - \Delta_1)}{\Delta_2} = 2 \left(1 - \frac{\Delta_1}{\Delta_2} \right) = 2 \left(1 - \frac{\Delta_1}{\mu_t \Delta_{2y}} \right) \quad (3.27)$$

With the parameters defined above, the following sections derive the mathematical models for Categories I and II, respectively. In each category, the equilibrium equation and deformation compatibility equations are provided followed by evaluation of some factors describing the system performance.

3.5 Derivations for Category I (Qu)

This section presents the derivations for performance Category I. (It is recognized that the majority of the equation derivations here in this section were derived by Qu, however, these equations were included here for completeness.) For illustration purpose, Figure 3-8 presents the displacement of the system together the internal force diagrams of the continuous column in this category. The following sections analyze the behaviors identified in the previous section.

Figure 3-8 Force and moment diagrams for the column under performance Category I

3.5.1 Behavior A

When the infill strips at the first story just reach their elastic limit and the second story infill strips remain elastic, the following equations apply:

$$\delta_{1y} = \Delta_{1y} = \frac{V_{y1}}{k_1} \quad (3.28)$$

$$\delta_{2y} = \Delta_{1y} - \Delta_{2y} < \Delta_{1y} \quad (3.29)$$

where δ_{1y} and δ_{2y} are the relative story displacements when the infill strips yielding just occurs, and V_{y1} is the yield strength of the first story provided by the infill strips.

Per Figure 3-7(a), the story shears are:

$$V_2 = \lambda F_1 = \frac{\lambda}{1 + \lambda} V \quad (3.30)$$

$$V_1 = (1 + \lambda) F_1 = V \quad (3.31)$$

where V is the base shear force, V_1 and V_2 are the shear forces at the first and second story respectively, and F_1 is the lateral force applied at the first story.

As shown in Figure 3-8 (b), the continuous column is considered as a simply supported beam with a point load acting at the midpoint. Accordingly, the free body diagram, shear diagram and bending moment diagram of the continuous column are obtained as shown in (c), (d) and (e), respectively. Since the shear force in the first and second story continuous columns, denoted as V_{c1} and V_{c2} , respectively, are identical, they can be estimated as the following based on the elastic theory:

$$V_{c1} = V_{c2} = V_c = \frac{3EI_c}{h^3} d_c \quad (3.32)$$

where V_c is the column shear force, and d_c can be determined based on Figure 3-9 using the geometric relationship:

$$d_c = \Delta_1 - \frac{1}{2} \Delta_2 \quad (3.33)$$

Figure 3-9 Column displacement for Category I

Substituting equation (3.33) into equation (3.32),

$$V_c = \left(\frac{3EI_c}{h^3} \right) \left(\Delta_1 - \frac{1}{2} \Delta_2 \right) \quad (3.34)$$

Considering the redistribution of story shear forces caused by the continuous column, the redistributed shear forces at the first and second stories when the infill plate yielding just occurs, denoted as V_{f1y} and V_{f2y} , are:

$$V_{f1y} = k_1 \Delta_{1y} = V_1 + V_c = V - 3 \frac{EI_c}{h^3} \left(\Delta_{1y} - \frac{\Delta_{2y}}{2} \right) \quad (3.35)$$

$$V_{f2y} = k_2 (\Delta_{2y} - \Delta_{1y}) = V_2 - V_c = \frac{\lambda}{1 + \lambda} V + 3 \frac{EI_c}{h^3} \left(\Delta_{1y} - \frac{\Delta_{2y}}{2} \right) \quad (3.36)$$

Recall that the stiffness ratio, α , is defined as:

$$\alpha = \frac{EI_c}{k_1 h^3} \quad (3.37)$$

Note that the stiffness ratio only incorporates the first story stiffness. By inspection, it can be seen that smaller α corresponds to a more flexible continuous column and vice versa.

Based on equation (3.19), stiffness of the first and second story satisfies the following relationship:

$$k_2 = \gamma k_1 \quad (3.38)$$

Considering equations (3.37) and (3.38), equations (3.35) and (3.36) can be rearranged as:

$$\Delta_{1y} = \frac{V}{k_1} - 3 \frac{EI_c}{k_1 h^3} \left(\Delta_{1y} - \frac{\Delta_{2y}}{2} \right) = \frac{V}{k_1} - 3\alpha \left(\Delta_{1y} - \frac{\Delta_{2y}}{2} \right) \quad (3.39)$$

$$\Delta_{2y} - \Delta_{1y} = \frac{\lambda}{1+\lambda} \frac{V}{k_2} + 3 \frac{EI_c}{\gamma k_1 h^3} \left(\Delta_{1y} - \frac{\Delta_{2y}}{2} \right) = \frac{\lambda}{1+\lambda} \frac{V}{\gamma k_1} + 3 \frac{\alpha}{\gamma} \left(\Delta_{1y} - \frac{\Delta_{2y}}{2} \right) \quad (3.40)$$

From equations (3.39) and (3.40), Δ_{1y} and Δ_{2y} can be solved as:

$$\Delta_{1y} = \frac{1}{1+3\alpha} \left(\frac{V}{k_1} \right) + \frac{3\alpha}{2(1+3\alpha)} [\chi] \left(\frac{V}{k_1} \right) \quad (3.41)$$

$$\Delta_{2y} = [\chi] \left(\frac{V}{k_1} \right) \quad (3.42)$$

where,

$$[\chi] = \frac{\frac{\lambda}{\gamma(1+\lambda)} + \frac{1+\frac{3\alpha}{\gamma}}{1+3\alpha}}{1 + \frac{3\alpha}{2\gamma} - \frac{3\alpha}{2} \left(\frac{\frac{3\alpha}{\gamma} + 1}{1+3\alpha} \right)} \quad (3.43)$$

As will be shown in the later sections, the following ratio can be used to simplify the derivation:

$$\frac{\Delta_{2y}}{\Delta_{1y}} = \frac{[\chi]}{\frac{1}{1+3\alpha} + \frac{3\alpha}{2(1+3\alpha)} [\chi]} \quad (3.44)$$

When yielding just occurs in the first story, relationships between the inter-story drift and the absolute floor displacement essentially remains identical:

$$\delta_{1y} = \Delta_{1y} \quad (3.45)$$

$$\delta_{2y} = \Delta_{2y} - \delta_{1y} \quad (3.46)$$

Introducing equations (3.41) and (3.42) into equation (3.45) and (3.46), respectively, give:

$$\delta_{1y} = \Delta_{1y} = \frac{1}{1+3\alpha} \left(\frac{V}{k_1} \right) + \frac{3\alpha}{2(1+3\alpha)} [\chi] \left(\frac{V}{k_1} \right) \quad (3.47)$$

$$\delta_{2y} = \Delta_{2y} - \delta_{1y} = \frac{V}{k_1} \left\{ \left(1 - \frac{3\alpha}{2(1+3\alpha)} \right) [\chi] - \frac{1}{1+3\alpha} \right\} \quad (3.48)$$

From equation (3.35), the following relationship is true:

$$\Delta_{1y} = \Delta_{2y} \left(\frac{\Delta_{1y}}{\Delta_{2y}} \right) = \frac{V_{f1y}}{k_1} \quad (3.49)$$

Substituting equation (3.44) into equation (3.49) yields:

$$\frac{V_{f1y}}{k_1} = \frac{\Delta_{2y}}{1+3\alpha} \left\{ \frac{1}{[\chi]} + \frac{3\alpha}{2} \right\} \quad (3.50)$$

Rearranging terms in (3.50), one can find:

$$k_1 \Delta_{2y} = \frac{V_{f1y} (1+3\alpha)}{\left\{ \frac{1}{[\chi]} + \frac{3\alpha}{2} \right\}} \quad (3.51)$$

3.5.2 Behavior B

Behavior B of performance Category I corresponds to the case that the first story infill strips are elongated beyond the yielding point while the second story infill strips remain elastic. With this behavior, the shear forces developed at the first and second stories of the SPSW model (see Figure 3-7(a) and Figure 3-8) are:

$$V_{f1} = V_{f1y} = V - 3 \frac{EI_c}{h^3} (\Delta_1 - \frac{\Delta_2}{2}) \quad (3.52)$$

$$V_{f2} = k_2 (\Delta_2 - \Delta_1) = \frac{\lambda}{1+\lambda} V + 3 \frac{EI_c}{h^3} (\Delta_1 - \frac{\Delta_2}{2}) \quad (3.53)$$

Dividing k_1 on both sides of equation (3.52) and introducing the stiffness ratio defined in equation (3.37) gives,

$$\frac{V_{f1y}}{k_1} = \frac{V}{k_1} - 3\alpha (\Delta_1 - \frac{\Delta_2}{2}) \quad (3.54)$$

Substituting (3.50) into (3.54) and solving for $\frac{V}{k_1}$ give:

$$\frac{V}{k_1} = \frac{\Delta_{2y}}{1+3\alpha} \left\{ \frac{1}{[\chi]} + \frac{3\alpha}{2} \right\} + 3\alpha (\Delta_1 - \frac{\Delta_2}{2}) \quad (3.55)$$

Dividing k_2 on both sides of equation (3.53) and introducing α gives

$$\Delta_2 - \Delta_1 = \frac{\lambda}{1+\lambda} \frac{V}{\gamma k_1} + \frac{3\alpha}{\gamma} (\Delta_1 - \frac{\Delta_2}{2}) \quad (3.56)$$

Substituting (3.55) into (3.56) to solve for Δ_1 :

$$\Delta_1 = \frac{(1 + \frac{3\alpha\lambda}{2\gamma(1+\lambda)} + \frac{3\alpha}{2\gamma})\Delta_2 - \frac{\lambda}{\gamma(1+\lambda)} \left\{ \frac{1}{[\chi]} + \frac{3\alpha}{2} \right\} \Delta_{2y}}{1 + \frac{3\alpha\lambda}{\gamma(1+\lambda)} + \frac{3\alpha}{\gamma}} \quad (3.57)$$

Plugging equation (3.57) into equation (3.22) and also considering equation (3.25), one can obtain the inter-story displacement expression at the second story as followed:

$$\delta_2 = \frac{\left(\frac{3\alpha\lambda}{2\gamma(1+\lambda)} + \frac{3\alpha}{2\gamma}\right)\mu_i\Delta_{2y} + \frac{\lambda}{\gamma(1+\lambda)} \frac{\left\{\frac{1}{[\chi]} + \frac{3\alpha}{2}\right\}}{1+3\alpha} \Delta_{2y}}{1 + \frac{3\alpha\lambda}{\gamma(1+\lambda)} + \frac{3\alpha}{\gamma}} \quad (3.58)$$

Putting equation (3.58) into (3.26) obtains the DCF value for Behavior B of performance Category I as:

$$DCF_{IB} = 2 \left(1 - \frac{\delta_2}{\mu_i\Delta_{2y}} \right) = \frac{2 + \frac{3\alpha\lambda}{\gamma(1+\lambda)} + \frac{3\alpha}{\gamma} - \frac{2\lambda}{\gamma(1+\lambda)} \frac{\left\{\frac{1}{[\chi]} + \frac{3\alpha}{2}\right\}}{1+3\alpha} \frac{1}{\mu_i}}{1 + \frac{3\alpha\lambda}{\gamma(1+\lambda)} + \frac{3\alpha}{\gamma}} \quad (3.59)$$

3.5.3 Behavior C

Beyond Behavior B, continuing loading the structure until the second story infill strips just reach their elastic limit, one can observe Behavior C. When infill strip yielding just occurs in the second story, the inter story drift at the second story satisfies the following relationship:

$$\delta_2 = \Delta_{1y} = \Delta_2 - \Delta_1 \quad (3.60)$$

Substitute equation (3.58) into equation (3.60),

$$\Delta_{1y} = \delta_2 = \frac{\left(\frac{3\alpha\lambda}{2\gamma(1+\lambda)} + \frac{3\alpha}{2\gamma}\right)\mu_i\Delta_{2y} + \frac{\lambda}{\gamma(1+\lambda)} \frac{\left\{\frac{1}{[\chi]} + \frac{3\alpha}{2}\right\}}{1+3\alpha} \Delta_{2y}}{1 + \frac{3\alpha\lambda}{\gamma(1+\lambda)} + \frac{3\alpha}{\gamma}} \quad (3.61)$$

Note that equation(3.61) can be rearranged as followed:

$$\frac{\Delta_{1y}}{\Delta_{2y}} \left(1 + \frac{3\alpha\lambda}{\gamma(1+\lambda)} + \frac{3\alpha}{\gamma} \right) - \frac{\lambda}{\gamma(1+\lambda)} \frac{\left\{ \frac{1}{[\chi]} + \frac{3\alpha}{2} \right\}}{1+3\alpha} = \left(\frac{3\alpha\lambda}{2\gamma(1+\lambda)} + \frac{3\alpha}{2\gamma} \right) \mu_t \quad (3.62)$$

Substituting equation (3.44) into equation (3.62) and solving for μ_t gives,

$$\mu_t = \frac{2+3\alpha[\chi]}{(1+3\alpha)[\chi]} \left[\frac{\gamma + \gamma\lambda - \lambda}{3\alpha(2\lambda+1)} + 1 \right] \quad (3.63)$$

It is noteworthy that the roof ductility value determined from (3.63) corresponds to the minimum displacement requirement to achieve infill strip yielding in both stories when performance Category I controls. This quantity will be important to evaluate the seismic performance for the given two-story SPSWs. As such, define this roof ductility value, μ_{icl} , as followed:

$$\mu_{icl} = \frac{2+3\alpha[\chi]}{(1+3\alpha)[\chi]} \left[\frac{\gamma + \gamma\lambda - \lambda}{3\alpha(2\lambda+1)} + 1 \right] \quad (3.64)$$

Replacing μ_t with μ_{icl} in equation (3.59) gives the DCF expression associated with

Behavior C of Category I:

$$DCF_{IC} = \frac{2 + \frac{3\alpha\lambda}{\gamma(1+\lambda)} + \frac{3\alpha}{\gamma} - \frac{2\lambda}{\gamma(1+\lambda)} \frac{\left\{ \frac{1}{[\chi]} + \frac{3\alpha}{2} \right\}}{1+3\alpha}}{1 + \frac{3\alpha\lambda}{\gamma(1+\lambda)} + \frac{3\alpha}{\gamma}} \frac{1}{\mu_{icl}} \quad (3.65)$$

3.5.4 Behavior D

Figure 3-10 Incremental deflection of the frame when both stories are fully yielded. Beyond Behavior C, continuing increasing the lateral displacement of the system, the infill strips at the first and second stories will be both elongated into the post-yielding range (i.e. Behavior D controls). When the infill strips at both stories are yielded, the system lateral stiffness will be zero and no additional lateral strength will be obtained from the structure if strain hardening is not taken into account. At this stage, the increments of the lateral displacements at the two floor levels, shown in Figure 3-10, satisfy the following relationship:

$$\frac{\Delta_1 - \Delta_{1c}}{\Delta_2 - \Delta_{2c}} = \frac{h}{H} = \frac{1}{2} \quad (3.66)$$

Solving for Δ_1 from (3.66) gives:

$$\Delta_1 = \Delta_{1c} + \frac{1}{2}(\Delta_2 - \Delta_{2c}) \quad (3.67)$$

where Δ_{1c} and Δ_{2c} are first and second floor displacements associated with Behavior C.

Substituting (3.67) and (3.25) into (3.23) gives the DCF value for Behavior D:

$$DCF_{ID} = 2 \frac{\Delta_1}{\Delta_2} = \frac{2 \left[\Delta_{1c} + \frac{1}{2} (\Delta_2 - \Delta_{2c}) \right]}{\mu_t \Delta_{2y}} = 1 + \frac{1}{\mu_t} \frac{\Delta_{2c}}{\Delta_{2y}} \left(\frac{2\Delta_{1c}}{\Delta_{2c}} - 1 \right) \quad (3.68)$$

By definition:

$$\mu_{tcl} = \frac{\Delta_{2c}}{\Delta_{2y}} \quad (3.69)$$

$$DCF_{IC} = \frac{2\Delta_{1c}}{\Delta_{2c}} \quad (3.70)$$

Substituting equations (3.69) and (3.70) into equation (3.68) gives:

$$DCD_{ID} = 1 + \frac{\mu_{tcl}}{\mu_t} (DCF_{IC} - 1.0) \quad (3.71)$$

3.5.5 Other discussions on Category I

3.5.5.1 Bending moment and shear force in continuous column

When performance Category I controls, from Figure 3-8 the maximum bending moment developed in the continuous column can be estimated based on the elastic beam analysis procedure:

$$M_{cl} = \frac{3EI_c}{h^3} \left(\Delta_1 - \frac{\Delta_2}{2} \right) h = 3\alpha h k_1 \left(\Delta_1 - \frac{\Delta_2}{2} \right) \quad (3.72)$$

Considering equations (3.37) and (3.25), equation (3.72) can be rewritten as:

$$M_{cl} = 3\alpha h k_1 \left(\Delta_1 - \frac{\Delta_2}{2} \right) = \frac{3\alpha h \mu_t k_1 \Delta_{1y}}{2} \left(\frac{\Delta_{2y}}{\Delta_{1y}} \right) \left(\frac{2\Delta_1}{\Delta_2} - 1 \right) \quad (3.73)$$

Substituting equation (3.23), (3.44) and (3.35) into equation (3.73) gives

$$M_{cl} = \frac{3}{2} \alpha \mu_t [\chi] h V_{f1y} \frac{DCF_I - 1}{\frac{1}{1+3\alpha} + \frac{3\alpha}{2(1+3\alpha)}} [\chi] \quad (3.74)$$

where V_{f1y} represents the strength of the first story due to the yielded strips at that story

which can be determined from (3.1); and

$$DCF_I = \begin{cases} DCF_{IB} & \text{if } \mu_t < \mu_{icl} \\ DCF_{IC} & \text{if } \mu_t = \mu_{icl} \\ DCF_{ID} & \text{if } \mu_t > \mu_{icl} \end{cases} \quad (3.75)$$

Based on Figure 3-8, the shear force developed in the continuous column can be determined from bending moment diagram as:

$$V_{cl} = \frac{M_{cl}}{h} \quad (3.76)$$

Substituting (3.74) into (3.76) yields

$$V_{cl} = \frac{3}{2} \alpha \mu_t V_{f1y} \left(\frac{\frac{[\chi]}{\frac{1}{1+3\alpha} + \frac{3\alpha}{2(1+3\alpha)}[\chi]}}{1} \right) (DCF_I - 1) \quad (3.77)$$

3.5.5.2 Story Ductility Demand

While the roof ductility, μ_t , is used in the above derivations for describing the target roof deformation expected in the structure, there is no direct information quantifying the ductility on the first yielded story when μ_t is achieved, whereas the ductility on the first yielded story can be more critical to evaluate the system performance. Given that the first story yields first when performance Category I controls, the corresponding story ductility, μ_I , is defined as:

$$\mu_I = \frac{\Delta_I}{\Delta_{Iy}}, \quad (\Delta_I > \Delta_{Iy}) \quad (3.78)$$

It is recognized that the story ductility capacity can be typically obtained from experimental research. If the story ductility capacity of a SPSW is not less than μ_t , the corresponding roof ductility can be achieved; otherwise, the wall will fail at the first yielded story due to excessive deformation.

Considering equation (3.25) and rearranging (3.78) give

$$\mu_t = \frac{\Delta_1}{\Delta_{1y}} = \frac{1}{2} \left(\frac{2\Delta_1}{\Delta_2} \right) \left(\frac{\Delta_2}{\Delta_{1y}} \right) = \frac{1}{2} \left(\frac{2\Delta_1}{\Delta_2} \right) \left(\frac{\mu_t \Delta_{2y}}{\Delta_{1y}} \right) \quad (3.79)$$

Substituting equations (3.23) and (3.44) into equation (3.79) gives

$$\mu_t = \frac{\mu_t}{2} \frac{DCF_t}{\frac{1}{[\chi](1+3\alpha)} + \frac{3\alpha}{2(1+3\alpha)}} \quad (3.80)$$

Within all behaviors considered, the story ductility associated with Behavior C, μ_{IC} is of particular interest since it corresponds to the minimum ductility demand on the first yielded story to achieve infill strips yielding in both stories. Here, μ_{IC} can be obtained by replacing μ_t and DCF_t with μ_{IC} and DCF_{IC} in equation(3.80):

$$\mu_{IC} = \frac{\mu_{IC}}{2} \frac{DCF_{IC}}{\frac{1}{[\chi](1+3\alpha)} + \frac{3\alpha}{2(1+3\alpha)}} \quad (3.81)$$

where μ_{IC} and DCF_{IC} can be obtained from equations (3.64) and (3.65), respectively.

3.6 Derivations for Category II

In practice, the relationship $\gamma < \frac{\lambda}{1+\lambda}$ may apply, indicating performance Category II

may control. The following sections discuss the key behaviors identified in the previous sections.

3.6.1 Behavior A

In this category, the first story includes over-strength and the plate on the second story will yield first under the lateral forces. Figure 3-11 shows the deformation of the system and the force and moment diagrams of the continuous column in Category II. Procedures for analyses in this category are identical to those in Category I and therefore the following equations apply:

$$t_2 = \gamma t_1 \Leftrightarrow k_2 = \gamma k_1 \quad (3.82)$$

$$\alpha = \frac{EI}{k_1 h^3} \quad (3.83)$$

$$\Delta_2 = \mu_t \Delta_{2y} \quad (3.84)$$

Figure 3-11 Shear and moment diagrams for the column under performance Category II
 The continuous column can still be treated as a simply supported beam loaded at the center. The resulting free body diagrams, and shear and moment diagrams are shown in Figure 3-11 (b), (c), (d) and (e), respectively, which are similar to those discussed in

Category I.. According to Figure 3-12, d_c at the first floor can be found using the geometric relationship as:

Figure 3-12 Column displacement for Category II

$$d_c = \frac{\Delta_{2y}}{2} - \Delta_{1y} \quad (3.85)$$

where, Δ_{1y} and Δ_{2y} , in this case, are respectively the absolute displacements at the first and second story when the second story infill strips just yield.

Considering the redistribution effect of story shear caused by the continuous column, the story shears are:

$$V_{f2y} = k_2(\Delta_{2y} - \Delta_{1y}) = \frac{\lambda}{1 + \lambda} V + \frac{3EI}{h^3} \left(\Delta_{1y} - \frac{\Delta_{2y}}{2} \right) \quad (3.86)$$

$$V_{f1y} = k_1 \Delta_{1y} = V + \frac{3EI}{h^3} \left(\frac{\Delta_{2y}}{2} - \Delta_{1y} \right) = V - \frac{3EI}{h^3} \left(\Delta_{1y} - \frac{\Delta_{2y}}{2} \right) \quad (3.87)$$

Introducing equations (3.82) and (3.83) into equations (3.87) and (3.86) respectively gives,

$$\Delta_{1y} = \frac{V}{k_1} - 3\alpha \left(\Delta_{1y} - \frac{\Delta_{2y}}{2} \right) \quad (3.88)$$

$$\Delta_{2y} - \Delta_{1y} = \frac{\lambda}{1+\lambda} \frac{V}{\gamma k_1} + \frac{3\alpha}{\gamma} \left(\Delta_{1y} - \frac{\Delta_{2y}}{2} \right) \quad (3.89)$$

Equation(3.88) shows that Δ_{1y} can be expressed in terms of Δ_{2y} as follows:

$$\Delta_{1y} = \frac{\frac{V}{k_1} + \frac{3\alpha}{2} \Delta_{2y}}{1+3\alpha} \quad (3.90)$$

Substituting (3.90) into (3.89) and solve for Δ_{2y} gives:

$$\Delta_{2y} = [\chi] \frac{V}{k_1} \quad (3.91)$$

where $[\chi]$ is an identical constant as defined in derivations of Category I:

$$[\chi] = \left[\frac{\frac{\lambda}{\gamma(1+\lambda)} + \frac{1+\frac{3\alpha}{\gamma}}{1+3\alpha}}{1 + \frac{3\alpha}{2\gamma} - \frac{3\alpha}{2} \left(\frac{\frac{3\alpha}{\gamma} + 1}{1+3\alpha} \right)} \right] \quad (3.92)$$

Hence, Δ_{1y} becomes:

$$\Delta_{1y} = \left(\frac{1}{1+3\alpha} + \frac{3\alpha}{2(1+3\alpha)} [\chi] \right) \frac{V}{k_1} \quad (3.93)$$

The ratio of Δ_{1y} to Δ_{2y} is also of interest as it will be applied for later substitutions. From

equation (3.91) and(3.93), this ratio can be expressed as:

$$\frac{\Delta_{1y}}{\Delta_{2y}} = \frac{1}{(1+3\alpha)[\chi]} + \frac{3\alpha}{2(1+3\alpha)} \quad (3.94)$$

From the previous discussions, the relative story displacements are:

$$\delta_{1y} = \Delta_{1y} \quad (3.95)$$

$$\delta_{2y} = \Delta_{2y} - \Delta_{1y} \quad (3.96)$$

According to equation (3.86), δ_{2y} becomes,

$$\delta_{2y} = \frac{V_{f2y}}{k_2} = \Delta_{2y} - \Delta_{1y} = \Delta_{2y} \left(1 - \frac{\Delta_{1y}}{\Delta_{2y}} \right) \quad (3.97)$$

Now considering equation (3.94), δ_{2y} can be further simplified as:

$$\delta_{2y} = \left[\left(1 - \frac{1}{1+3\alpha} \right) [\chi] - \frac{1}{1+3\alpha} \right] \frac{V}{k_1} \quad (3.98)$$

Substituting equations(3.94) and (3.97) into equation (3.98), one can get the following expression for δ_{2y} :

$$\delta_{2y} = \frac{\Delta_{2y}}{1+3\alpha} \left[1 + \frac{3\alpha}{2} - \frac{1}{[\chi]} \right] \quad (3.99)$$

3.6.2 Behavior B

Behavior B corresponds to the case where the infill strips at the second story are beyond the yielding limit while the first story strips remains elastic. According to Figure 3-11, story shear forces can be found as:

$$V_{f2} = V_{f2y} = \frac{\lambda}{1+\lambda} V + \frac{3EI}{h^3} \left(\Delta_1 - \frac{\Delta_2}{2} \right) \quad (3.100)$$

$$V_{f1} = k_1 \Delta_1 = V - \frac{3EI}{h^3} \left(\Delta_1 - \frac{\Delta_2}{2} \right) \quad (3.101)$$

Dividing k_2 and k_1 respectively in equation(3.100) and(3.101) and introducing α and γ , equation (3.100) and(3.101) correspondingly become:

$$\frac{V_{f2y}}{k_2} = \frac{\lambda}{1+\lambda} \frac{V}{\gamma k_1} + \frac{3\alpha}{\gamma} \left(\Delta_1 - \frac{\Delta_2}{2} \right) \quad (3.102)$$

$$\frac{V}{k_1} = \Delta_1 + 3\alpha \left(\Delta_1 - \frac{\Delta_2}{2} \right) \quad (3.103)$$

Substituting equations (3.103),(3.97) and (3.99) into (3.102):

$$\frac{\Delta_{2y}}{1+3\alpha} \left[1 + \frac{3\alpha}{2} - \frac{1}{[\chi]} \right] = \left[\frac{\lambda}{1+\lambda} \frac{1}{\gamma} (3\alpha+1) + \frac{3\alpha}{\gamma} \right] \Delta_1 - \left[\frac{\lambda}{1+\lambda} \frac{1}{\gamma} \frac{3\alpha}{2} + \frac{3\alpha}{2\gamma} \right] \Delta_2 \quad (3.104)$$

Dividing Δ_{2y} on both sides of equation(3.104) and solving for Δ_1/Δ_{2y} gives,

$$\frac{\Delta_1}{\Delta_{2y}} = \frac{\frac{1}{3\alpha+1} \left[1 + \frac{3\alpha}{2} - \frac{1}{[\chi]} \right] + \left[\frac{\lambda}{1+\lambda} \frac{1}{\gamma} \frac{3\alpha}{2} + \frac{3\alpha}{2\gamma} \right] \mu_t}{\frac{\lambda}{1+\lambda} \frac{1}{\gamma} (3\alpha+1) + \frac{3\alpha}{\gamma}} \quad (3.105)$$

Substituting equation(3.105) into the DCF_{II} in equation(3.27) gives,

$$DCF_{IIB} = 2 \left(1 - \frac{\Delta_1}{\mu_t \Delta_{2y}} \right) = \frac{\frac{\lambda}{1+\lambda} \frac{1}{\gamma} (3\alpha+2) + \frac{3\alpha}{\gamma} - \frac{1}{3\alpha+1} \left[2 + 3\alpha - \frac{2}{[\chi]} \right] \frac{1}{\mu_t}}{\frac{\lambda}{1+\lambda} \frac{1}{\gamma} (3\alpha+1) + \frac{3\alpha}{\gamma}} \quad (3.106)$$

3.6.3 Behavior C

Similar to the performance in Category I, continuing loading the structure, Behavior C will be observed in which the second story strips are beyond the yielding point and the first story strips just reach yield point. At this state, the absolute displacement at the first floor can be found as followed:

$$\Delta_1 = \Delta_{2y} - \Delta_{1y} \quad (3.107)$$

Dividing Δ_{2y} on both sides of (3.107) gives:

$$\frac{\Delta_1}{\Delta_{2y}} = 1 - \frac{\Delta_{1y}}{\Delta_{2y}} \quad (3.108)$$

Introducing equation (3.94) into(3.108), one can get:

$$\frac{\Delta_1}{\Delta_{2y}} = \frac{(2+3\alpha)[\chi]-2}{2(1+3\alpha)[\chi]} \quad (3.109)$$

Substituting equation(3.109) into(3.105) and solve for μ_i for Category II gives,

$$\mu_{III} = \frac{\left[\frac{\lambda}{1+\lambda} \frac{1}{\gamma} (3\alpha+1) + \frac{3\alpha}{\gamma} \right] \frac{(2+3\alpha)[\chi]-2}{2(1+3\alpha)[\chi]} - \frac{1}{3\alpha+1} \left[1 + \frac{3\alpha}{2} - \frac{1}{[\chi]} \right]}{\frac{3\alpha}{2\gamma} \frac{\lambda}{1+\lambda} + \frac{3\alpha}{2\gamma}} \quad (3.110)$$

The above ductility is critical for understanding the behavior of the system since it corresponds the minimum ductility demand to achieve yielding in both stories. As such, replacing μ_{III} by μ_{icII} gives

$$\mu_{icII} = \frac{\left[\frac{\lambda}{1+\lambda} \frac{1}{\gamma} (3\alpha+1) + \frac{3\alpha}{\gamma} \right] \frac{(2+3\alpha)[\chi]-2}{2(1+3\alpha)[\chi]} - \frac{1}{3\alpha+1} \left[1 + \frac{3\alpha}{2} - \frac{1}{[\chi]} \right]}{\frac{3\alpha}{2\gamma} \frac{\lambda}{1+\lambda} + \frac{3\alpha}{2\gamma}} \quad (3.111)$$

Accordingly:

$$DCF_{nc} = \frac{\frac{\lambda}{1+\lambda} \frac{1}{\gamma} (3\alpha+2) + \frac{3\alpha}{\gamma} - \frac{1}{3\alpha+1} \left[2+3\alpha - \frac{2}{[\chi]} \right] \frac{1}{\mu_i}}{\frac{\lambda}{1+\lambda} \frac{1}{\gamma} (3\alpha+1) + \frac{3\alpha}{\gamma}} \quad (3.112)$$

3.6.4 Behavior D

Because no additional lateral resistance can be provided by the system under Behavior D,

increments of story displacements should be similar to those in Category I, where per

Figure 3-10, the first floor displacement can be found as:

$$\Delta_1 = \Delta_{1c} + \frac{1}{2}(\Delta_2 - \Delta_{2c}) \quad (3.113)$$

Recall from equation (3.27) that DCF in Category II can be found as:

$$DCF_{II} = \frac{2(\Delta_2 - \Delta_1)}{\Delta_2} = 2 \left(1 - \frac{\Delta_1}{\Delta_2} \right) = 2 - \frac{2\Delta_1}{\Delta_2} \quad (3.114)$$

Considering equations (3.84) and (3.113), beyond Behavior C, DCF_{II} in equation(3.114) can be rearranged as followed:

$$DCF_{II} = 2 - 2 \frac{\left[\Delta_{1c} + \frac{1}{2}(\Delta_2 - \Delta_{2c}) \right]}{\mu_t \Delta_{2y}} = 1 - \frac{\mu_{tcII}}{\mu_t} - 2 \frac{\Delta_{2c}}{\Delta_{2y}} \frac{1}{\mu_t} \quad (3.115)$$

Considering equation(3.114), DCF_{II} can be found as:

$$DCF_{II} = \frac{2(\Delta_{2c} - \Delta_{1c})}{\Delta_{2c}} = 2 \left(1 - \frac{\Delta_{1c}}{\mu_{tcII} \Delta_{2y}} \right) = 2 - 2 \frac{\Delta_{1c}}{\Delta_{2y}} \frac{1}{\mu_{tcII}} \quad (3.116)$$

Rearrange (3.116) gives,

$$-2 \frac{\Delta_{1c}}{\Delta_{2y}} = \mu_{tc} (DCF_{2c} - 2) \quad (3.117)$$

Now, substituting equation(3.117) into(3.115), DCF_{II} for Behavior D can be found as:

$$DCF_{ID} = 1 + \frac{\mu_{tcII}}{\mu_t} (DCF_{IIc} - 1) \quad (3.118)$$

where μ_{tcII} and DCF_{IIc} can be calculated according to equation(3.111) and (3.112), respectively.

3.6.5 Other discussions on Category II

3.6.5.1 Bending moment and shear force in continuous column

Following the classic elastic beam analysis procedure and considering Figure 3-11, the column moment can be derived as followed:

$$M_{cII} = \frac{3EI_c}{h^3} \left(\Delta_1 - \frac{\Delta_2}{2} \right) h = 3\alpha h k_1 \left(\Delta_1 - \frac{\Delta_2}{2} \right) = \frac{3\alpha h k_2}{\gamma} \left(\Delta_1 - \frac{\Delta_2}{2} \right) \quad (3.119)$$

Mathematically, equation (3.119) can be rearranged as,

$$M_{cII} = \frac{3\alpha h k_2}{2\gamma} \Delta_2 \left(\frac{2\Delta_1}{\Delta_2} - 1 \right) \quad (3.120)$$

According to equation(3.114), DCF_{II} can be rewritten as:

$$2 \frac{\Delta_1}{\Delta_2} - 1 = 1 - DCF_{II} \quad (3.121)$$

Substituting equations (3.84) and(3.121) into (3.120) gives:

$$M_{cII} = \frac{3\alpha h k_2}{2\gamma} \mu_t \Delta_{2y} (1 - DCF_{II}) = \frac{3\alpha h}{2\gamma} \mu_t (k_2 \Delta_{2y}) (1 - DCF_{II}) \quad (3.122)$$

Considering equation(3.86) and(3.19), the shear strength of the second story can be expressed as:

$$V_{y2} = V_{f2y} = k_2 (\Delta_{2y} - \Delta_{1y}) = V_{y1} \gamma = V_{f1y} \gamma \quad (3.123)$$

Rearranging equation (3.123) gives:

$$k_2 \Delta_{2y} \left(1 - \frac{\Delta_{1y}}{\Delta_{2y}} \right) = V_{f1y} \gamma \quad (3.124)$$

Introducing equation (3.94) into equation (3.124) gives:

$$k_2 \Delta_{2y} = \frac{V_{f1y} \gamma}{\left(1 - \frac{1}{(1+3\alpha)[\chi]} - \frac{3\alpha}{2(1+3\alpha)}\right)} \quad (3.125)$$

Substituting equation(3.125) into(3.122), the column moment can be found as:

$$M_{cl} = \frac{3h}{2} \frac{\alpha \mu_t V_{f1y} (1 - DCF_{II})}{1 - \frac{1}{(1+3\alpha)[\chi]} - \frac{3\alpha}{2(1+3\alpha)}} \quad (3.126)$$

Note that V_{f1y} is the first story shear strength, which can be determined by equation(3.1).

DCF_{II} depends on the capacity level of roof ductility considered, where,

$$DCF_{II} = \begin{cases} DCF_{IIB} & \text{if } \mu_t < \mu_{tcl} \\ DCF_{IIC} & \text{if } \mu_t = \mu_{tcl} \\ DCF_{IID} & \text{if } \mu_t > \mu_{tcl} \end{cases} \quad (3.127)$$

Applying equation(3.76), the shear force in the column can be determined accordingly as followed:

$$V_{cl} = \frac{3\alpha \mu_t V_{f1y}}{2\gamma} \frac{1 - DCF_{II}}{1 - \frac{1}{(1+3\alpha)[\chi]} - \frac{3\alpha}{2(1+3\alpha)}} \quad (3.128)$$

Recall that the minimum value of DCF_{II} is unity, that is, $DCF_{II} \geq 1.0$. Therefore,

equations (3.126) and (3.128) will both yield negative values. To ensure that the moment and shear values are positive, which will be more convenient for comparison, both equations are assigned negative signs.

$$M_{cl} = \frac{3h}{2} \frac{\alpha \mu_t V_y (DCF_{II} - 1)}{1 - \frac{1}{(1+3\alpha)[\chi]} - \frac{3\alpha}{2(1+3\alpha)}} \quad (3.129)$$

$$V_{cII} = \frac{3\alpha V_y \mu_t}{2} \frac{DCF_{II} - 1}{1 - \frac{1}{(1+3\alpha)[\chi]} - \frac{3\alpha}{2(1+3\alpha)}} \quad (3.130)$$

3.6.5.2 Story ductility in Category II

Figure 3-13 Schematic of story displacements in Category II

Figure 3-13 illustrates the schematic of story displacements in Category II. Compared to the discussions in Category I, the story ductility for Category II, denoted as μ_{II} , can be defined as:

$$\mu_{II} = \frac{\Delta_2 - \Delta_1}{\Delta_{2y} - \Delta_{1y}} \quad (3.131)$$

Mathematically, equation (3.131) can be rewritten as

$$\mu_{II} = \frac{\Delta_2 \left(1 - \frac{\Delta_1}{\Delta_2} \right)}{\Delta_{2y} \left(1 - \frac{\Delta_{1y}}{\Delta_{2y}} \right)} \quad (3.132)$$

Note that DCF_{II} in equation (3.114) can be rewritten as followed:

$$\frac{DCF_{II}}{2} = 1 - \frac{\Delta_1}{\Delta_2} \quad (3.133)$$

Substituting equations (3.94), (3.84) and (3.133) into equation (3.132) gives,

$$\mu_{II} = \frac{\mu_{icII}}{2} \frac{DCF_{II}}{1 - \left(\frac{1}{[\chi](1+3\alpha)} + \frac{3\alpha}{2(1+3\alpha)} \right)} \quad (3.134)$$

When the critical case, i.e., Behavior C, applies, the corresponding story ductility, μ_{IC} , is:

$$\mu_{IC} = \frac{\mu_{icII}}{2} \frac{DCF_{IC}}{1 - \left(\frac{1}{[\chi](1+3\alpha)} + \frac{3\alpha}{2(1+3\alpha)} \right)} \quad (3.135)$$

where μ_{icII} and DCF_{IC} can be evaluated using equation(3.111) and (3.112), respectively.

3.7 Brief summary of the derived model

Table 3-1 and Table 3-2 summarize the key equations for estimating M_c , V_c and DCF for the two-story SPSW. Table 3-3 shows the dependent and independent variables.

The followings are supplementary notes for these key equations:

- The parameter, $[\chi]$, can be treated as a constant due to lateral loads and stiffness and is the same for both categories.
- Entrance of category depends on the values of γ and $\frac{\lambda}{\lambda+1}$. Figure 3-14 is the flowchart for determining which category applies..

Table 3-1 Key equations summary (I)

Summary of Key Equations	Category I	Category II
μ_{icj} ($j = I, II$)	$\mu_{icI} = \frac{2+3\alpha[\chi]}{(1+3\alpha)[\chi]} \left[\frac{\gamma+\gamma\lambda-\lambda}{3\alpha(2\lambda+1)} + 1 \right]$	$\mu_{icII} = \frac{\left[\frac{\lambda}{1+\lambda} \frac{1}{\gamma} (3\alpha+1) + \frac{3\alpha}{\gamma} \right] \frac{(2+3\alpha)[\chi]-2}{2(1+3\alpha)[\chi]} - \frac{1}{3\alpha+1} \left[1 + \frac{3\alpha}{2} - \frac{1}{[\chi]} \right]}{\frac{3\alpha}{2\gamma} \frac{\lambda}{1+\lambda} + \frac{3\alpha}{2\gamma}}$
μ_{pC} ($p = I, II$)	$\mu_{IC} = \frac{\mu_{icI}}{2} \frac{DCF_{IC}}{\frac{1}{[\chi](1+3\alpha)} + \frac{3\alpha}{2(1+3\alpha)}}$	$\mu_{IIC} = \frac{\mu_{icII}}{2} \frac{DCF_{IIC}}{1 - \left(\frac{1}{[\chi](1+3\alpha)} + \frac{3\alpha}{2(1+3\alpha)} \right)}$
DCF_{iB} ($i = I \text{ or } II$)	$DCF_{IB} = \frac{2 + \frac{3\alpha\lambda}{\gamma(1+\lambda)} + \frac{3\alpha}{\gamma} - \frac{2\lambda}{\gamma(1+\lambda)} \frac{\left\{ \frac{1}{[\chi]} + \frac{3\alpha}{2} \right\}}{1+3\alpha} \frac{1}{\mu_t}}{1 + \frac{3\alpha\lambda}{\gamma(1+\lambda)} + \frac{3\alpha}{\gamma}}$	$DCF_{IIB} = \frac{\frac{\lambda}{1+\lambda} \frac{1}{\gamma} (3\alpha+2) + \frac{3\alpha}{\gamma} - \frac{1}{3\alpha+1} \left[2+3\alpha - \frac{2}{[\chi]} \right] \frac{1}{\mu_t}}{\frac{\lambda}{1+\lambda} \frac{1}{\gamma} (3\alpha+1) + \frac{3\alpha}{\gamma}}$
DCF_{iC} ($i = I \text{ or } II$)	$DCF_{IC} = \frac{2 + \frac{3\alpha\lambda}{\gamma(1+\lambda)} + \frac{3\alpha}{\gamma} - \frac{2\lambda}{\gamma(1+\lambda)} \frac{\left\{ \frac{1}{[\chi]} + \frac{3\alpha}{2} \right\}}{1+3\alpha} \frac{1}{\mu_{icI}}}{1 + \frac{3\alpha\lambda}{\gamma(1+\lambda)} + \frac{3\alpha}{\gamma}}$	$DCF_{IIC} = \frac{\frac{\lambda}{1+\lambda} \frac{1}{\gamma} (3\alpha+2) + \frac{3\alpha}{\gamma} - \frac{1}{3\alpha+1} \left[2+3\alpha - \frac{2}{[\chi]} \right] \frac{1}{\mu_{icII}}}{\frac{\lambda}{1+\lambda} \frac{1}{\gamma} (3\alpha+1) + \frac{3\alpha}{\gamma}}$
DCF_{iD} ($i = I \text{ or } II$)	$DCF_{ID} = 1 + \frac{\mu_{icI}}{\mu_t} (DCF_{IC} - 1)$	$DCF_{IID} = 1 + \frac{\mu_{icII}}{\mu_t} (DCF_{IIC} - 1)$

Table 3-2 Key equation summary (II)

Summary of Key Equations	Category I	Category II
$[\chi]$	$[\chi] = \frac{M}{N}; \text{ where } M = \frac{\lambda}{\gamma(1+\lambda)} + \frac{1 + \frac{3\alpha}{\gamma}}{1+3\alpha}, \quad N = 1 + \frac{3\alpha}{2\gamma} - \frac{3\alpha}{2} \left(\frac{\frac{3\alpha}{\gamma} + 1}{1+3\alpha} \right)$	
M_{ci} ($i = I \text{ or } II$)	$M_{ci} = \frac{3}{2} \alpha \mu_i h [\chi] V_y \frac{DCF_i - 1}{\frac{1}{1+3\alpha} + \frac{3\alpha}{2(1+3\alpha)}} [\chi]$	$M_{cii} = \frac{3h\alpha\mu_i V_y}{2} \frac{DCF_{ii} - 1}{1 - \frac{1}{(1+3\alpha)[\chi]} - \frac{3\alpha}{2(1+3\alpha)}}$
V_{ci} ($i = I \text{ or } II$)	$V_{ci} = \frac{3}{2} \alpha \mu_i [\chi] V_y \frac{DCF_i - 1}{\frac{1}{1+3\alpha} + \frac{3\alpha}{2(1+3\alpha)}} [\chi]$	$V_{cii} = \frac{3\alpha\mu_i V_y}{2} \frac{DCF_{ii} - 1}{1 - \frac{1}{(1+3\alpha)[\chi]} - \frac{3\alpha}{2(1+3\alpha)}}$
$\frac{M_{ci}}{V_{f1y}h} = \frac{V_{ci}}{V_{f1y}}$ ($i = I \text{ or } II$)	$\frac{M_{ci}}{V_{f1y}h} = \frac{3}{2} \alpha \mu_i [\chi] \frac{DCF_i - 1}{\frac{1}{1+3\alpha} + \frac{3\alpha}{2(1+3\alpha)}} [\chi]$	$\frac{M_{cii}}{V_{f1y}h} = \frac{3\alpha\mu_i}{2} \frac{DCF_{ii} - 1}{1 - \frac{1}{(1+3\alpha)[\chi]} - \frac{3\alpha}{2(1+3\alpha)}}$
DCF_j ($\forall j = I \text{ or } II$)	$DCF_I = \begin{cases} DCF_{IB} & \text{if } \mu_t < \mu_{tcl} \\ DCF_{IC} & \text{if } \mu_t = \mu_{tcl} \\ DCF_{ID} & \text{if } \mu_t > \mu_{tcl} \end{cases}$	$DCF_{II} = \begin{cases} DCF_{IIB} & \text{if } \mu_t < \mu_{tcII} \\ DCF_{IIC} & \text{if } \mu_t = \mu_{tcII} \\ DCF_{IID} & \text{if } \mu_t > \mu_{tcII} \end{cases}$

Table 3-3 Dependent variables for the key equations

Functions of Key Equations	Category I	Category II
M_{ci}	$M_{c1} = f(\alpha, \mu_t, \lambda, \gamma, h, t, F_y, L)$	$M_{c2} = f(\alpha, \mu_t, \lambda, \gamma, h, t, F_y, L)$
V_{ci}	$V_{c1} = f(\alpha, \mu_t, \lambda, \gamma, t, F_y, L)$	$V_{c2} = f(\alpha, \mu_t, \lambda, \gamma, t, F_y, L)$
$\frac{M_{ci}}{V_y h} = \frac{V_{ci}}{V_y}$	$\frac{M_{c1}}{V_y h} = \frac{V_{c1}}{V_y} = f(\alpha, \mu_t, \lambda, \gamma)$	$\frac{M_{c2}}{V_y h} = \frac{V_{c2}}{V_y} = f(\alpha, \mu_t, \lambda, \gamma)$
μ_{icj}	$\mu_{icI} = f(\alpha, \lambda, \gamma)$	$\mu_{icII} = f(\alpha, \lambda, \gamma)$
DCF_{iA}	$DCF_{1A} = f(\alpha, \mu_t, \lambda, \gamma)$	$DCF_{2A} = f(\alpha, \mu_t, \lambda, \gamma)$
DCF_{iB}	$DCF_{1B} = f(\alpha, \mu_t, \lambda, \gamma)$	$DCF_{2B} = f(\alpha, \mu_t, \lambda, \gamma)$
DCF_{ic}	$DCF_{1C} = f(\alpha, \mu_t, \lambda, \gamma)$	$DCF_{2C} = f(\alpha, \mu_t, \lambda, \gamma)$

$$(\gamma, \lambda) \Rightarrow \left\{ \begin{array}{l} \gamma > \frac{\lambda}{1+\lambda} \Rightarrow \text{Category I and} \left\{ \begin{array}{l} DCF_{IB} \text{ if } \mu_t < \mu_{tcl} \\ DCF_{IC} \text{ if } \mu_t = \mu_{tcl} \\ DCF_{ID} \text{ if } \mu_t > \mu_{tcl} \end{array} \right. \\ \gamma < \frac{\lambda}{1+\lambda} \Rightarrow \text{Category II and} \left\{ \begin{array}{l} DCF_{IIB} \text{ if } \mu_t < \mu_{tclII} \\ DCF_{IIC} \text{ if } \mu_t = \mu_{tclII} \\ DCF_{IID} \text{ if } \mu_t > \mu_{tclII} \end{array} \right. \end{array} \right.$$

Figure 3-14 Flowchart for performance determination

Chapter 4 VALIDATION OF THE DERIVED MODELS

4.1 Introduction

In order to confirm the adequacy of the mathematic models derived in Chapter 3, nonlinear static analyses were conducted using the finite element (FE) model developed in the commercially available software package, SAP2000. Consistent with the derivations presented in Chapter 3, the infill panels in the SPSW were simplified as infill strips in the FE model. This chapter describes in detail the selection of a demonstration SPSW, the modeling procedure, determination of analysis parameters, and result comparisons.

Figure 4-1 The two-story SPSW models: (a) strip model for derivation; (b) SAP 2000 FE strip model

4.2 Demonstration structure

A demonstration two-story SPSW was selected in this chapter for validating the analytical models developed in Chapter 3. As a representation in conventional residential building structures, the story height and bay width of the structure were assumed to be 12 ft and 24 ft, respectively. The thickness of the first story infill panel was assigned to be a

constant of 0.07874 inch (i.e., 2 mm), while that of the second story infill panel was varied in the analysis to achieve different values of γ . It is recognized that while the thickness of the second story infill plate determined from the γ values over the range of interest may not be commercially available, it provides useful information for validation purpose. Moreover, the lateral force distribution and the moment of inertia of the continuous column were varied in the analysis to achieve different combinations of λ and α , which allows validation of the analytical model in all considered scenarios.

4.3 FE Modeling

This section presents the development of strip model for the considered two-story SPSW structure using SAP 2000. The strip model, as shown in Figure 4-1(a), is developed based on the procedure proposed by Thorburn *et al.* (1983).

4.3.1 Elements considered

There are essentially four different types of components in the model shown in Figure 4-1(b), namely, boundary frame elements, infill strips, links and continuous column. The beams and columns in the SPSW are assumed to be designed according to the capacity design procedure required by the AISC Seismic Design Provisions (AISC 2005), which ensures that the boundary frame elements and continuous column remain elastic when the infill tension field actions form. Therefore, these members were modeled using elastic beam-column elements in the model.

Table 4-1 Combinations of lateral force and strip area assignments for the FE Model

Category Cases	λ Values	γ Values	Area assignments for the infill strips, [inch ²]		
			First Story	Second Story	
Category I: $\lambda/(1+\lambda) \leq \gamma \leq 1$	1.0	0.667	1.2518	0.8435	
		0.833		1.0427	
		1.0		1.2518	
	2.0	0.778		0.9738	
		0.889		1.1128	
		1.0		1.2518	
	3.0	0.833		1.0427	
		0.9167		1.1475	
		1.0		1.2518	
	Category II: $\gamma < \lambda/(1+\lambda)$	1.0		0.167	0.2091
				0.333	0.4168
		2.0		0.222	0.2779
0.444			0.5558		
3.0		0.25	0.3129		
		0.5	0.6259		

As determined from the capacity design procedure, the infill strips are the only components that will exhibit inelastic behavior in the system during seismic events. In order to capture the inelastic behavior, infill strips were modeled as truss elements with plastic hinges defined at the midpoints. The default hinge length of 0.0012 inch and the symmetric elastic-perfectly-plastic property were assigned to the plastic hinges, as shown in Figure 4-2.

Figure 4-2 Elastic-perfectly-plastic behavior for the plastic hinges in the strips

4.3.2 Boundary conditions and loading condition

Consistent with the derivations of the analytical model, all the member-to-member and member-to- foundation connections were idealized as pinned connections. Concentrated loads were applied at floor levels along the horizontal direction to simulate the seismic forces. The lateral load distribution was determined based on the assumed distribution factor, λ , as defined in Chapter 3. In the nonlinear static analysis, the lateral loads were increased, while their distribution was kept constant, until the roof reaches the expected displacement level (up to 5%) in this investigation.

4.3.3 Parameters and analysis procedures

In the simulation, the lateral force distribution factor, λ , the story stiffness ratio, γ , and the roof ductility, μ_t , are selected such that both the performance categories (i.e. I and II) considered in Chapter 3 will be evaluated. The selected analysis parameters are listed in Table 4-1. Additionally, in order to investigate the effects of column stiffness on seismic behavior of the system, the continuous column moment of inertia, I , is selected as a parameter in the simulation. Solving for I , from equation(3.37) gives:

$$I = \frac{\alpha k h^3}{E} \quad (4.1)$$

where k is the lateral stiffness of the first story infill panel.

Substituting (3.15) into(4.1) gives

$$I = \frac{1}{4} \alpha t L h^2 \quad (4.2)$$

As shown in equation (4.2), the column moment of inertia depends on the thickness of the first story infill plate (2mm), the story height and width of the wall (12 ft and 24 ft, respectively) and the expected stiffness ratio.

For comparison purposes, the stiffness ratio was chosen from 10^{-3} to 10^3 , simulating the continuous column to be extremely flexible to extremely rigid. The selected values of α and the corresponding column moment of inertia determined from equation (4.2) is summarized in Table 4-2.

Table 4-2 Values of column moment of inertia per case of analysis

Analysis Case	Stiffness Ratio α	$I, [inch^4]$
Case 1	0.001	117.558
Case 2	0.01	1175.584
Case 3	0.1	11755.843
Case 4	1.0	117558.452
Case 5	10	1175584.252
Case 6	100	11755842.520
Case 7	1000	117558425.197

Once a set of γ, λ and I values are determined, a simulation was performed to produce a pushover curve, which is a record of the base shear vs. roof displacement. Note that the target roof displacement can be determined using the expected roof ductility, μ_r , and the system yield displacement can be identified from the pushover curve. For validation purpose, the DCF and column moment at the target roof displacement level obtained

from the FE model are compared with those from the derived models. Note that the links are ideally rigid elements that connect the continuous column to the main structure at floor levels. Unlike the boundary frame elements, the links were modeled using truss elements.

According to Berman and Bruneau (2003), a total of twenty strips were assigned for each infill plate. Incidentally, calculations regarding determination of the orientation angle and cross-section properties of these strips are presented in detail in Appendix A. It is recognized that, while the strip model is not able to consider the plate buckling behavior, it provides reasonable estimates of strength and stiffness of this system (Berman and Bruneau 2003, Thorburn *et al.* 1983).

4.3.4 Materials properties

Essentially, steel is the only material used for all the members in the model. For simplicity, self-weight of steel is not included in the analysis. In addition, any effects caused by temperature changes in the system are beyond the scope of this thesis and therefore not considered in the material model. The material constants including modulus of elasticity (E), Poisson's Ratio (ν), nominal yield strength(F_y), and expected yield strength(F_{ye}), respectively equal to 29000 ksi, 0.3, 50 ksi and 55 ksi are assigned to the steel.

4.4 Discussion of simulation results

4.4.1 General observation and model validation

Figure 4-3 shows example pushover curves for two selected analysis cases. Figure 4-3(a) corresponds to performance Category I, where $\gamma=0.9167$ and $\lambda=3.0$; Figure 4-3(b)

corresponds to performance Category II, where $\gamma=0.167$ and $\lambda=1.0$. These two cases, while arbitrarily selected from Table 4-1, provide representative results in general. In each case, seven curves associated with the different column moment of inertia values considered in Table 4-2 are provided.

Generally, if infill strip yielding can be achieved in both stories at the target roof displacement level, the overall system strength will converge to the same value in all cases, although different systems may experience different yielding sequence and processes over the two stories. It is also observed that yielding in both stories can not be achieved at some roof displacement levels in some cases, in particularly, when the column moment of inertia is relatively small, resulting in reduced system strength. This observation is consistent with the fact that overly flexible column in the considered system is less effective to mitigate drift concentration and hence requires excessively large roof displacement to achieve yielding in both stories. Summary of all pushover curves for case studies in Figure B 1 and Figure B 2 can be found in Appendix B.

Figure 4-3 Example pushover curves

As shown in Figure 4-3, the ultimate strength of the two example structures are 711.723 kips and 505.457 kips, respectively. For the walls considered in Figure 4-3(a), following the procedure proposed by Berman and Bruneau (2003), one can obtain the system strength equal to 682.4 kips which is about 4.3% lower than the result from FE analysis, indicating the adequacy of the strip model. It is recognized that a similar study can be conducted on the wall considered in Figure 4-3(b) if needed.

4.4.2 FE model results vs. results from the derived model

As mentioned previously, the moment demand of continuous column and DCF are selected as the performance quantities for comparing the results from the FE model and the derived analytical models. Figure 4-4 and Figure 4-5 present the result comparisons. On each graph, four different values of μ_t (1, 2, 4, and 6) were considered and α values are limited to the range of [0.001, 1000]. As shown, excellent agreements were observed in the result comparison for both performance categories, indicating the adequacy of the derived analytical models.

In addition, results shown in Figure 4-4 through Figure 4-7 allow the following observation::

- In general, as the column stiffness ratio increases, DCF approaches to unity, indicating that more uniform yielding occurs in a system with relatively rigid continuous columns.
- The DCF value reduces rapidly when α increases from 0.01 to 1, indicating it is the most effective range for mitigating drift concentration. It is also found that excessively stiff columns provide limited improvement of system performance.

- For a certain combination of λ and γ , the normalized bending moment expected in the continuous column eventually converge to the same value when yielding occurs in both stories.

Figure 4-4 DCF vs Stiffness Ratio Plots for Category I cases

Figure 4-5 DCF vs Stiffness Ratio Plots for Category II cases

Figure 4-6 Normalized column moment vs Stiffness Ratio Plots for Category I cases

Figure 4-7 Normalized column moment vs Stiffness Ratio Plots for Category II cases

4.4.3 Further discussion on analytical models derived by other researchers

While the analytical models derived in Chapter 3 was based on a two story SPSW, it is recognized that the models can also be used for other systems with a certain type of lateral force resisting action such as CBF. MacRae *et al.*(2004) derived the analytical models for calculating DCF and bending moment in the continuous column in a two-story CBF which has identical braces in both stories and is subjected to linearly distributed lateral forces. Setting $\lambda=2$ and $\gamma=1$ in the analytical models derived in Chapter 3, one can get the following equations:

$$\therefore [\chi] = \frac{\frac{\lambda}{\gamma(1+\lambda)} + \frac{1+\frac{3\alpha}{\gamma}}{1+3\alpha}}{1+\frac{3\alpha}{2\gamma} - \frac{3\alpha}{2} \left(\frac{\frac{3\alpha}{\gamma} + 1}{1+3\alpha} \right)} \stackrel{\text{here}}{=} \frac{5}{3} \quad (4.3)$$

$$\Rightarrow \mu_{icl} = \frac{2+3\alpha[\chi]}{(1+3\alpha)[\chi]} \left[\frac{\gamma + \gamma\lambda - \lambda}{3\alpha(2\lambda+1)} + 1 \right] \stackrel{\text{here}}{=} \frac{(2+5\alpha)(1+15\alpha)}{25\alpha(1+3\alpha)} \quad (4.4)$$

$$\Rightarrow DCF_{IC} = \frac{2 + \frac{3\alpha\lambda}{\gamma(1+\lambda)} + \frac{3\alpha}{\gamma} - \frac{2\lambda}{\gamma(1+\lambda)} \frac{\left\{ \frac{1}{[\chi]} + \frac{3\alpha}{2} \right\}}{1+3\alpha} \frac{1}{\mu_{icl}}}{1 + \frac{3\alpha\lambda}{\gamma(1+\lambda)} + \frac{3\alpha}{\gamma}} \quad (4.5)$$

$$\stackrel{\text{here}}{=} \frac{2+5\alpha - \frac{10\alpha}{1+15\alpha}}{(1+5\alpha)(1+15\alpha)} = \frac{2+25\alpha+75\alpha^2}{1+5\alpha}$$

$$\Rightarrow DCF_{IB} = 1 + \frac{\mu_{icl}}{\mu_t} (DCF_{IC} - 1) \stackrel{\text{here}}{=} 1 + \frac{2+5\alpha}{25\alpha\mu_t(1+3\alpha)} \quad (4.6)$$

$$\therefore DCF_{IB} = 1 + \frac{1}{15\alpha\mu_t} \frac{6+15\alpha}{(5+15\alpha)} \quad (4.7)$$

Figure 4-8 compares the DCF and normalized bending moment in the continuous columns generated from the above equations and those derived by MacRae *et al.* (2004). As shown, excellent agreements were observed, which further confirms the validity of the analytical models derived in this thesis.

Figure 4-8 Result comparison: MacRae *et al.* (2004) vs. derived models

4.5 Summary

In this chapter, the FE model of a demonstration two-story SPSW is developed to validate the analytical models derived in the Chapter 3. In the FE model, the infill panels of SPSWs are modeled using diagonal truss elements with elastic-perfectly-plastic hinges. Nonlinear static analyses were conducted using the developed model. Result comparison indicates that the analytical models derived in Chapter 3 are valid and can be used for further analysis purpose.

Chapter 5 Probabilistic Evaluation of Two-Story SPSWs

5.1 Introduction

The analytical model for two-story SPSWs derived in Chapter 3 has been verified in Chapter 4 using the results from FE analysis. It is noted that the key parameters controlling the system performance are the lateral force distribution and the infill plate thickness distribution along the height of the structure. In practice, uncertainties exist in determination of these two factors due to the following facts: 1) the redistribution of live load and the unexpected plate yielding sequence may change the lateral seismic force distribution (i.e., λ); and 2) the plate thickness determined from the design equations may not be available in the market and slightly thicker plates may have to be used in some cases, leading to a infill plate thickness distribution different from that assumed in design (i.e., changing the factor, γ). Therefore, it is necessary to evaluate the effects of column stiffness on seismic behavior of two-story SPSWs with consideration of the uncertain lateral force and infill plate thickness distributions.

In this chapter, three simulation methods, i.e., the Monte Carlo method, the Latin Hypercube Sampling method, and the Rosenblueth's 2K+1 Point Estimate method, respectively referred to herein as M.C., L.H., and 2K+1, will be used to statistically evaluate the effects of column stiffness on seismic behaviors of two-story SPSWs. The following sections briefly describe these three simulation methods, the response quantity selected for system performance evaluation, and selection of the parameter ranges. In addition, results from these three methods will be compared together with evaluation of the effectiveness of the minimum SPSW column stiffness required by the AISC Seismic Design Provisions.

5.2 Probabilistic Simulation Techniques

In reliability analysis of structural performance, the following equation can be used for quantifying the confidence level of a system.

$$P[Y(X_1, X_2, \dots, X_K) > y] = 1 - P[Y(X_1, X_2, \dots, X_K) \leq y] = 1 - \Phi\left(\frac{y - \theta_Y}{\sigma_Y}\right) \quad (5.1)$$

where Y represents a selected response quantity of the system (i.e. demand) and it is function of a series of random variables describing the system, i.e., X_1 through X_K , y represents the performance limit, i.e., a maximum value of the response quantity allowed in the considered system without causing failures; θ_Y and σ_Y respectively represent the mean and standard deviation of Y , and Φ is the cumulative normal distribution function. It is noted that a goodness-of-fit test is required to confirm that the response quantity of interest follows the normal distribution.

Essentially, equation (5.1) calculates the probability of failure. A larger value of $P(Y > y)$ corresponds to a lower level of confidence on the system. In practice, Y can be selected from the key structural response quantities and the corresponding factor y can be identified from experimental research. Additionally, a certain type of sampling procedure is required for generating the database of Y for evaluation of θ_Y and σ_Y . In this thesis, three different types of sampling procedures, i.e., the M.C., L.H. and 2K+1 methods, will be considered. The following sections respectively introduce these methods.

5.2.1 The M.C. method

The M.C. simulation method is a special numerical technique for consideration of uncertainties existing in a physical problem. The basic idea behind the M.C. simulation is

to numerically simulate some phenomenon and then observe the number of times some event of interest occurs.

Regarding equation (5.1), the M.C. simulation can be performed by following the following steps:

1. generate a large number of sample values for the random variables, X_1 through X_n , and calculate the corresponding Y values which will be used as a database for further analysis;
2. evaluate the mean and standard deviation of Y using the database generated in step 1;
3. select a proper performance limit, y , based on the previous experimental research,;
4. calculate the probability of failure using equation(5.1).

It is recognized that certain probabilistic distributions have to be assumed for X_i when generating their sample values. In addition, equation(5.1) assumes that the response quantity of interest, Y , follows the normal distribution. Such an assumption could be verified through a goodness-of-fit test. The basic concept behind the M.C. simulation is relatively straightforward but the procedure can be computationally intensive as discussed later.

5.2.2 The L.H. method

The M.C. simulation method introduced in the previous section is very useful for performing probabilistic analyses. However, it may be computationally expensive in some instances, in particular when evaluate the problem for a single trial is complex. As a

result, performing a large number of simulations becomes impossible (Nowak and Collins 2000).

The L.H. method is one technique for reducing the number of simulations needed to obtain reasonable results. Essentially, the L.H. method is the same as the M.C. method except that the sampling process is more effective. To be specific, in the L.H. method, the range of possible values of each random variable that determines a response quantity of interest is partitioned into “strata”, a value from each stratum is randomly selected as a representative value. The representative values of each random variable are then randomly combined so that each representative value is considered once and only once in determination of the response quantity of interest (Nowak and Collins 2000).

For the function considered in equation(5.1), Y can be described as follows:

$$Y = Y(X_1, X_2, \dots, X_K) \quad (5.2)$$

Nowak and Collins (2000) recommended the following steps for the L.H. sampling:

1. Partition the range of $X_i (i = 1, 2, 3 \dots K)$ into N intervals such that the probability of a value of occurring in each interval is $1/N$.
2. For each of these N intervals, randomly select a representative value for the interval. In particular, for this analysis, the midpoint value in each of these intervals was chosen.
3. As a result of the previous steps, there are N^K combinations of all representative values. The objective of the L.H. sampling is to select N combinations such that

each representative value will be considered once and only once in the combinations.

4. Calculate Y based on each combination of the representative values, forming a database including N values of Y for determination of θ_Y and σ_Y . Evaluate the system performance using equation (5.1).

Since each interval is only sampled once (i.e., without repeatedly taking sampling values from the same interval), the L.H. method is typically more efficient than the M.C. method. For additional details on the L.H. method and its application in structural reliability analysis, see Han and Wen (1994) and Zhou and Nowak (1988)

5.2.3 Rosenblueth's 2K+1 Point Estimate method

In addition to the L.H. method, complex problems can also be considered by the 2K+1 method proposed by Rosenblueth (1975). Essentially, the 2K+1 method is a simulation technique in which the number of simulations is 2K+1, where K is the number of the input random variables. Basically, the 2K+1 method evaluates a function at 2K+1 critical points and then uses these values to estimate the mean and coefficient of variance of the considered function.

Considering a function described in equation (5.2), Nowak and Collins (2000) summarized the steps of the 2K+1 method as below:

1. For each of $X_i (i = 1, 2, 3 \dots K)$, evaluate its mean value (θ_{X_i}) and standard deviation (σ_{X_i})

2. Evaluate y_0 , which is the value of equation (5.2) when all input variables are set equal to their mean values:

$$y_0 = Y(\theta_{X_1}, \theta_{X_2}, \dots, \theta_{X_k}) \quad (5.3)$$

3. Evaluate equation(5.2) at additional 2K points as follows: for each of the variable X_i , evaluate the function at two values, $\theta_{X_i} \pm \sigma_{X_i}$, while holding all other variables to be equal to their mean values. These two values of the function are denoted as y_i^+ and y_i^- , and can be mathematically expressed as followed:

$$\begin{aligned} y_i^+ &= Y(\theta_{X_1}, \theta_{X_2}, \dots, \theta_{X_i} + \sigma_{X_i}, \dots, \theta_{X_k}) \\ y_i^- &= Y(\theta_{X_1}, \mu_{X_2}, \dots, \theta_{X_i} - \sigma_{X_i}, \dots, \theta_{X_k}) \end{aligned} \quad (5.4)$$

4. For each random variable, evaluate the following two intermediate quantities:

$$\overline{y_i} = \frac{y_i^+ + y_i^-}{2} \quad (5.5)$$

$$V_{y_i} = \frac{y_i^+ - y_i^-}{y_i^+ + y_i^-} \quad (5.6)$$

5. Estimate the mean, $\overline{Y}$, coefficient of variance, V_Y , and standard deviation, σ_Y , as follows:

$$\overline{Y} = y_0 \prod_{i=1}^K \left(\frac{\overline{y_i}}{y_0} \right) \quad (5.7)$$

$$V_Y = \sqrt{\left\{ \prod_{i=1}^K (1 + V_{y_i}^2) \right\} - 1} \quad (5.8)$$

$$\sigma_Y = V_Y \overline{Y} \quad (5.9)$$

With the mean value and the standard variation, the probability of failure defined in equation (5.1) can be evaluated.

5.3 Performance evaluation procedure

5.3.1 Response quantity of interest

Among all the structural response quantities derived in Chapter 3, the story ductility and roof ductility associated with Behavior C, denoted as μ_c and μ_{ic} , are of special interest since they quantify the minimum displacement demand on the system to achieve plate yielding in both stories. Given the fact that limited information about the roof ductility capacity of SPSW can be found in the available literature and detailed information about SPSW story ductility capacity has been documented in prior experimental researches (Berman and Bruneau, 2003, Berman and Bruneau 2005, Chen *et al.* 2006; Vian *et al.* 2009), it was decided to select the story ductility, denoted as μ_c , as the response quantity of interest in this investigation. As derived in Chapter 3, μ_c can be calculated as followed:

$$\mu_c = \begin{cases} \mu_{icI} = \frac{\mu_{icI}}{2} \frac{DCF_{IC}}{\frac{1}{[\chi](1+3\alpha)} + \frac{3\alpha}{2(1+3\alpha)}} & \text{if Category I} \\ \mu_{icII} = \frac{\mu_{icII}}{2} \frac{DCF_{IIC}}{1 - \left(\frac{1}{[\chi](1+3\alpha)} + \frac{3\alpha}{2(1+3\alpha)} \right)} & \text{if Category II} \end{cases} \quad (5.10)$$

where μ_{icI} , μ_{icII} , DCF_{IC} and DCF_{IIC} can be determined from Table 3-1 and Table 3-2.

Accordingly, at each level of α (denoted as α_0), the probability of failure of the system can be quantified by the following conditional probability based on equation (5.1)

$$P[\mu_c(\gamma, \lambda) > \mu | \alpha = \alpha_0] = 1 - P[\mu_c(\gamma, \lambda) \leq \mu | \alpha = \alpha_0] = 1 - \Phi\left(\frac{\mu - \theta_{\mu_c}}{\sigma_{\mu_c}}\right) \Bigg|_{\alpha=\alpha_0} \quad (5.11)$$

where μ_c , γ , λ , and α have been defined in Chapter 3; θ_{μ_c} and σ_{μ_c} respectively represent the mean and standard deviation of μ_c ; and μ represents the story ductility capacity of SPSW which varies from 6 to 12 (Berman and Bruneau, 2003, Berman and Bruneau 2005, Chen *et al.* 2006; Vian *et al.* 2009). Codes written for the analyses can be found in Appendix C, Appendix D and Appendix E.

5.3.2 Selection of parameter ranges

As indicated in equation (5.11), two random variables, γ and λ , are included in the performance evaluation. This section describes the selected ranges of these two random variables and the range of interest for α .

5.3.2.1 Range of α

Theoretically, α varies from zero to infinity. However, this investigation will focus on the range of 0.001 to 1.0 for the following reasons:

1. when α is smaller than 0.001, the SPSW columns may be impractically flexible;
2. when α is greater than 1.0, the difference in system performance caused by the variation of α is negligible as shown in Chapter 4.

For generating the relationship of system probability of failure versus α , a total of 500 equally log spaced α values from 0.001 to 1.0 were considered.

5.3.2.2 Range of λ

As discussed in Chapter 3, λ describes the lateral seismic force distribution in a two-story SPSW. Basically, the lateral seismic forces distributed along a structures depends on the reactive weight distribution and the modal properties of the structure. Per FEMA 450, λ , for the SPSW considered in Chapter 3 can be found as

$$\lambda = \frac{C_{vx2}}{C_{vx1}} \quad (5.12)$$

where,

$$C_{vx1} = \frac{w_1 h^k}{w_1 h^k + w_2 (2h)^k} \quad (5.13)$$

$$C_{vx2} = \frac{w_2 (2h)^k}{w_1 h^k + w_2 (2h)^k} = \frac{w_2 2^k h^k}{w_1 h^k + w_2 (2h)^k} \quad (5.14)$$

where

w_1 and w_2 = reactive weight at each floor level;

h = story height assumed to be the same in both stories;

k = an exponent related to the effective fundamental period of the structure;

C_{vx} = the lateral force distribution vector;

Substituting equations (5.13) and (5.14) into (5.12) gives:

$$\lambda = \frac{C_{vx2}}{C_{vx1}} = \frac{w_2}{w_1} 2^k \quad (5.15)$$

According to FEMA 450, k depends on fundamental period of the structure and can be determined as below:

$$k = \begin{cases} 1.0 & \text{if } T_n \leq 0.5 \text{ sec} \\ 2.0 & \text{if } T_n > 2.5 \text{ sec} \\ 1.0 + \frac{T_n - 0.5}{2} & \text{if } 0.5 \text{ sec} < T_n \leq 2.5 \text{ sec} \end{cases} \quad (5.16)$$

where T_n is the fundamental period of the structure.

Also per FEMA 450, the period of a SPSW system may be approximated as:

$$T_n = C_r h_n^x \quad (5.17)$$

where $C_r = 0.02$ and $x = 0.75$, and h_n is the total height of the structure.

For a two-story SPSW with story height up to 12-feet, the fundamental period determined from equation (5.17) is always less than 0.5 sec. Therefore, according to equation(5.16), $k = 1.0$., which implies

$$\lambda = \frac{2w_2}{w_1} \quad (5.18)$$

Assuming that the reactive weight ratio, w_2/w_1 , varies between 0.5 to 1.5, which physically means that the reactive weight at the second story can be 50% more or less than the weight at the first story, the range of λ is determined to be $[1,3]$.

5.3.2.3 Range of γ and a special analysis case

As discussed in Chapter 3, γ describes the infill plate thickness distribution in a two-story SPSW and it satisfies $0 < \gamma \leq 1.0$ by definition. As derived in Chapter 3, Performance

Category I controls if $\frac{\lambda}{1+\lambda} < \gamma \leq 1$, and Performance Category II controls if $0 < \gamma < \frac{\lambda}{1+\lambda}$.

In order to consider both Performance Categories I and II without being biased, γ was sampled such that the probabilities of γ falling into each category are the same.

Assuming the mean value of λ is 2.0, such a sampling strategy provides a mean value of 0.583 for γ .

In addition to sampling γ as a random variable, another case assuming $\gamma = 1.0$ is included in this thesis. Such a special case corresponds to the SPSWs with identical plates in both stories. This special case is likely to occur in low-rise SPSWs, in which the seismic forces are relatively low and the corresponding required infill plates are not available in the market. It is recognized that performance Category I always controls when $\gamma = 1.0$. Additionally, the story ductility associated with this case is denoted as μ_{cgc} in the following result discussions,

5.4 Result discussions

5.4.1 Kolmogorov-Smirnov goodness-of-fit test

As discussed in Section 5.3, the assumption made in equation (5.11) is that the system probability of failure follows the normal distribution at each level of α . It is necessary to verify the validity of such an assumed probabilistic distribution using the goodness-of-fit test procedure. While quite a few tests for distributions are available, such as the chi-square and the Anderson-Darling methods, the widely used Kolmogorov-Smirnov test, or K-S test, was used in this investigation. The basic premise of this test is to compare the numerical cumulative frequency with the cumulative distribution function of an assumed theoretical distribution. If the maximum discrepancy between the numerical and theoretical frequencies is larger than normally expected for a given size, the theoretical distribution is not acceptable for modeling the underlying population; conversely, if the discrepancy is less than a critical value, the theoretical distribution is acceptable at the prescribed significance level (Ang and Tang, 2007).

To be more specific, based on the database of system performance at each level of α , the K-S test compares the simulation cumulative frequencies, $S_n(y)$ with the cumulative distribution frequencies of the normal distribution, $F_Y(y)$, where Y corresponds to the estimated story ductility value and n is the size of the sample. The maximum difference, D_n , between $S_n(y)$ and $F_Y(y)$ is then compared to the theoretical model, D_n^κ , as followed:

$$D_n = \max |F_Y(y) - S_n(y)| \underset{O.K.}{\overset{if}{<}} D_n^\kappa \quad (5.19)$$

where κ is so-called the significant level.

For $\kappa = 5\%$ and $n > 50$ (which are typically used in practice), D_n^κ can be approximated to be $1.36/\sqrt{n}$ (Ang and Tang, 2007). Therefore, if the condition of $D_n < D_n^\kappa$ is satisfied, that is, if $\max |F_Y(y) - S_n(y)| < \frac{1.36}{\sqrt{n}}$, the sample data set is judged to follow the normal distribution.

Figure 5-1 Normal distribution verification for story ductility data

Based on the K-S procedure described above, distributions of μ_c and μ_{cgc} were evaluated using the results from six tests (three tests for μ_c and μ_{cgc} respectively). Detailed results are presented in Figure 5-1. The size of the sample and the prescribed significance level, i.e., n and κ , were assumed to be 98 and 5%, respectively. Accordingly,

$D_n^\kappa = D_{98}^{0.05} = 0.1374$. As shown in Figure 5-1, while slight discrepancies exist between the results from different tests due to the nature of randomness, all the tests for normal distribution are below the accepted difference level, D_n^κ , indicating the validity of the

assumed normal distribution. It is also recognized that while it seems to be more proper to assume that the system probability of failure follows the log-normal distribution since μ_c and μ_{cgc} are always positive, the result from the K-S test shows that the normal distribution is superior in this specific investigation.

5.4.2 General result comparison

5.4.2.1 Stability of the considered simulation methods

Equation (5.11) calculates the system probability of failure based on the sample values of the random variables, γ and λ . When the sampling number approaches infinity or a sufficiently large value, ideally, the probability of failure from simulation should converge to the theoretical value. However, when the sampling number becomes more practical (which may be relatively small), it is necessary to check the sensitivity of the results to the sampling numbers of the random variables. In this investigation, three simulations were conducted for each numerical method to check their stability. The corresponding results are compared in Figure 5-3. For simplicity, the performance limit, i.e. the story ductility limit was assumed to be 12 ($\mu = 12$). One hundred strata are partitioned respectively for γ and λ in the L.H. method. It is also assumed that γ and λ follow the uniform distribution and normal distribution, respectively, which are the same as those considered for multi-story SPSWs in Chapter 6. It is recognized that the simulations considered were based on different sampling values of the variables and distribution of these sampling values are illustrated in Figure 5-2.

Figure 5-2 Distribution of λ and γ values in the M.C. and L.H. methods

Figure 5-3 Stability comparisons of the considered simulation methods

As illustrated in Figure 5-3, consistent results are observed in all the simulation methods, indicating the stability of these methods. It is also noted that, while slight difference are observed in the 2k+1 method when α is smaller than 0.02, these differences are negligible for practical use.

5.4.2.2 Comparison of different simulation methods

Based on the results generated in Section 5.4.2.1, the results from each method are compared in Figure 5-4. It is observed that all three methods provide almost identical results for the SPSW with identical infill plates in both stories (i.e. the special case in which $\gamma=1$). For the general SPSW cases, while the 2K+1 method underestimates the probability of failure by up to 10% when α becomes relatively small, the α values at

which the probability of failure starts to rise identified from all the three methods are the same. Such an observation indicates that the probability of failure versus α curve may be generated using any of the three considered simulation methods. In addition, as one may notice, the 2K+1 methods requires the least computational efforts since it only considers 2K+1 simulations (significantly less than the other two methods) and therefore this method will be used for evaluating the more complicated case, i.e., multi-story SPSWs in Chapter 6.

Figure 5-4 Result comparisons of the considered simulation methods

5.4.2.3 Effects of number of strata in the L.H. method

The accuracy of the L.H. method depends on the number of strata selected for each random variable. Conceptually, the more strata considered, the more accurate results will be obtained from the L.H. method, at the cost of extra computational efforts. Figure 5-5 compares the results from the L.H. simulations with consideration of different numbers of strata for the random variables. As shown, when the number of strata is greater than 10, the observed differences in the result are negligible.

Figure 5-5 Justification of number of strata required in the L.H Method

5.4.3 Discussion of the code requirement

Based on the definition of the stiffness ratio, α , presented in Chapter 3, one can find that,

$$I = \frac{k\alpha h^3}{E} \quad (5.20)$$

To prevent excessive inward deformation and out-of-plane buckling of SPSW columns caused by the infill plate tension field actions, the Canadian Code, i.e. CAN/CSA S16-01, and the US code, i.e. the AISC seismic design provisions (AISC 2005), requires a minimum value of moment of inertia for the SPSW column:

$$I \geq \frac{0.00307th^4}{L} \quad (5.21)$$

Recall that the continuous column in the derived model and the FE model combines the contributions of both the left and right SPSW columns, therefore, equation (5.21) is actually:

$$I \geq \frac{0.00614th^4}{L} \quad (5.22)$$

Substituting equation (5.20) into equation (5.22) and also considering equation (3.15), one can solve for α as ,

$$\alpha = 0.02456 \left(\frac{L}{h} \right)^{-2} \quad (5.23)$$

where L/h , is known as the SPSW width-to-height aspect ratio.

Per the AISC Seismic Design Provisions (AISC 2005), the aspect ratio for SPSW systems should be limited to the following range:

$$\left(\frac{L}{h} \right) \in [0.8 \quad 2.5] \quad (5.24)$$

Accordingly, the minimum value of α , i.e., $\alpha_{\min}$, required by the code, can be identified for the walls with different aspect ratio as below

$$\alpha_{\min} = \left[\underbrace{0.00393}_{L/h=2.5} \quad \underbrace{0.03838}_{L/h=0.8} \right] \quad (5.25)$$

5.4.3.1 Does the code ensure infill plate yielding in both stories?

With $\alpha_{\min}$ identified from equation (5.25) and the procedures validated in the prior sections, it would be interesting to evaluate whether infill plate yielding in both stores of the considered SPSW can occur if the SPSW columns were designed according to the minimum moment of inertia required by the current design code.

Assuming γ and λ follow the uniform distributions and normal distribution, respectively, over the ranges of interest, (which are identified in Section 5.3.2.2 and 5.3.2.3), Figure 5-6 presents the results of the special SPSW with identical infill plates in both stories and the general SPSWs. The performance limit, μ , are chosen to be 6, 8, 10 and 12 based on the previous experimental researches (Berman and Bruneau, 2005, Berman and Bruneau 2005, Chen *et al.* 2006; Vian *et al.* 2009). The following observations can be consistently obtained for the general SPSWs:

1. As indicated by the probability of failure close to 1.0, infill plate yielding in both stories is unlikely to occur in the walls with the upper bound value of the width-to-height aspect ratios (see the results at $\alpha_{\min} = 0.00393$).
2. For the walls with the lower bound value of the width-to-height aspect ratios, the probability of failure is typically smaller than 0.2, indicating infill plate yielding is more likely to occur in both stories (see the results at $\alpha_{\min} = 0.03838$).

3. Generally, the probability of achieving plate yielding in both stories is higher in the walls with larger $\alpha_{\min}$ values, i.e. in the walls with smaller aspect ratios as indicated in equation(5.23).
4. The ductility capacity of SPSWs also plays an important role. As shown, a larger story ductility provided by the system presents a higher possibility to achieve plate yielding in both stories.

Additionally, results from the special SPSW indicate that infill plate yielding in both stories is less likely to occur in the wall with identical plates in both stories compared to the wall with thinner plate in the upper story.

Figure 5-6 Probability of failure to achieve yielding in both stories of the SPSW

5.4.3.2 Effects of random variable distributions

As required in the simulation methods, proper probabilistic distributions should be assumed for generation of the sampling values of λ and γ . The results presented in the previous sections assume normal and uniform distributions for λ and γ , respectively. These distributions are widely used for probabilistic simulation and judged to be reasonable for the problem considered in this thesis. However, it will be interesting to confirm whether consistent results can be observed if combinations of different types of distributions are considered for λ and γ . This section discusses the results from other

combinations of the probabilistic distributions of λ and γ , listed in Table 5-1. Results of these considered cases are summarized in Figure F1 and Figure F2 (see Appendix F).

Overall, while slight differences exist between different combinations, results consistent with those described in detail in the previous section can be observed.

Table 5-1 Summary of other considered random variable distributions

Combination Number	γ			λ		
	Distribution	θ_γ	σ_γ	Distribution	θ_λ	σ_λ
1	Uniform	0.5833	0.2887	Uniform	2.0	0.5774
2	Uniform	0.5833	0.2887	Normal	2.0	0.2
3	Uniform	0.5833	0.2887	Normal	2.0	0.4
4	Uniform	0.5833	0.2887	Normal	2.0	0.6
5	Normal	0.5833	0.0583	Uniform	2.0	0.5774
6	Normal	0.5833	0.1167	Uniform	2.0	0.5774
7	Normal	0.5833	0.1750	Uniform	2.0	0.5774
8	Normal	0.5833	0.0583	Normal	2.0	0.2
9	Normal	0.5833	0.1167	Normal	2.0	0.2
10	Normal	0.5833	0.1750	Normal	2.0	0.2
11	Normal	0.5833	0.0583	Normal	2.0	0.4
12	Normal	0.5833	0.1167	Normal	2.0	0.4
13	Normal	0.5833	0.1750	Normal	2.0	0.4
14	Normal	0.5833	0.0583	Normal	2.0	0.6
15	Normal	0.5833	0.1167	Normal	2.0	0.6
16	Normal	0.5833	0.1750	Normal	2.0	0.6

As an example, Figure 5-7 shows the results from Combination Number 1. Four capacity levels of story ductility are considered. As shown, the probability of failure is much higher for flexible columns than stiff columns for both the general case and special case. The $\alpha_{\min}$ values from equation(5.25) are also included. Consistent with the results presented in the previous sections, it can be found that the column is less effective to ensure plate yielding in both stories when identical plates are used in both stories. In addition, it is concluded that a slender column is typically inefficient to redistribute the drifts of the two stories and ensure infill plate yielding in both stories when compared to a stocky column. Similar observations can be found in the other results presneted in Appendix F.

Figure 5-7 Uniform Distribution (U.D.) of λ vs U.D. of γ

5.4.4 Additional discussion on bending moment in continuous column

As discussed in Chapter 4, when the infill plates yield in both stories of the SPSW, the bending moment in the continuous column will remain constant at different levels of target lateral displacement. Such an observation indicates the bending moment in the continuous column associated with the case that infill plate yielding occurs in both stories can be used for design of the column. Although the mathematical models derived in Chapter 3 provide a closed form result, the simulation results shown in Figure 5-8 conveniently illustrated the normalized bending moment in the continuous column with consideration of the randomness of λ and γ . As shown in the figure, the normalized bending moment has a similar distribution and reaches the same maximum value at the order of 0.425 at different levels of α . In practice, such an upper bounded value together with the bending moment caused by the infill plate tension field actions may be used for design of the continuous column.

Figure 5-8 Normalized column moment versus α

5.5 Summary

In this chapter, three simulation methods, including the M.C. method, the L.H. method and the 2K+1 method, are used to evaluate the possibility to achieve infill plate yielding in both stories of two-story SPSWs. The stability, accuracy, and respective superiority of these methods are addressed. Results show that all the three methods can provide similar results, however the M.C., L.H. and the 2K+1 methods require decreasing computational efforts. Based on the simulation results, an evaluation was also made to check whether the minimum column stiffness required in the current code ensures infill plate yielding in both stories. It is found that within the walls designed according to the code, the walls with smaller width-to-height aspect ratios are more likely to achieve infill plate yielding in both stories. Other issues including effects of random variable distributions and distribution of bending moment in continuous columns are also discussed.

Chapter 6 Probabilistic Evaluation of Multi-Story SPSWs

6.1 Introduction

Derivations presented in Chapter 3 can be repeated for a multiple-story SPSW, however the derived model will be too tedious for practical use. Inspired by the analysis presented in Chapter 5, this chapter will evaluate the performance of multi-story SPSWs using the probabilistic method. The procedures considered here (including generation of the random lateral force and infill plate distributions along the height of the wall) are the same as those presented in Chapter 5 except that the wall performance will be estimated from the pushover analysis using the strip models in SAP 2000 (version14) due to the absence of the derived models for performance determination. With the advantages identified from Chapter 5, only the 2K+1 method will be used in this chapter. The following sections describe the assumptions of example six-story SPSWs and observations from the result comparisons.

6.2 Model Assumptions and simulation procedure

6.2.1 Basic information of demonstration six-story SPSWs

A proto-type six-story SPSW with bay width and uniform story height respectively equal to 24 feet and 12 feet, as shown in Figure 6-1(a), is selected for analysis purpose. The total height of the structure is 72 feet and the first story infill panel thickness is assumed 5 mm. In addition, the nominal reactive weights are assumed to be uniformly distributed along the height of the structure, as shown in Figure 6-1(b). Note that the assumed thickness of the first story plate will not have any impacts on the results obtained as discussed in detail in the following section.

Figure 6-1 Typical six-story SPSW: (a) Real Model (b) Simulation Model

6.2.2 Determination of analysis parameters

6.2.2.1 Period and nominal lateral force distribution

Per FEMA 450, the period for a SPSW structure may be estimated as followed:

$$T_a = C_r h_n^x \quad (6.1)$$

Note that h_n is the height of the structure in feet., and C_r and x are constant parameters determined to be 0.02 and 0.75 respectively. Accordingly, the period of the structure is determined to be 0.494 sec.

With the period determined from equation (6.1) and the assumed reactive weight distribution, the seismic forces are estimated to be distributed linearly along the height of the structure according to FEMA 450. The force distribution vector, C_{vx} , is tabulated in Table 6-1.

Table 6-1 Calculation Summaries of C_{vx} for each story

Story	$w_x h_x^k$	$\frac{w_x h_x^k}{\sum_{i=1}^6 w_i h_i^k}$	C_{vx}
1	12	0.047619	1
2	24	0.095238	2
3	36	0.142857	3
4	48	0.190476	4
5	60	0.238095	5
6	72	0.285714	6

As a result, the story shear distribution vector, denoted as $\{V_s\}$, can be calculated based on C_{vx} , as followed:

$$\{V_s\} = \begin{Bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{Bmatrix} = \begin{Bmatrix} 21 \\ 20 \\ 18 \\ 15 \\ 11 \\ 6 \end{Bmatrix} F \quad (6.2)$$

6.2.3 Definitions of λ and γ for multi-story SPSWs

In derivation of the models for 2-story SPSW, λ and γ respectively describe the lateral force and infill plate thickness distributions along the height of the structure. Similarly, a vector form of these variables, $\{\lambda_2, \lambda_3, \dots, \lambda_i \dots \lambda_n\}$ and $\{\gamma_2, \gamma_3, \dots, \gamma_i \dots \gamma_n\}$ are defined for multi-story SPSWs:

$$\lambda_i \equiv \frac{F_i}{F_{i-1}}, (i \geq 2) \quad (6.3)$$

and

$$\gamma_i \equiv \frac{t_i}{t_{i-1}}, (i \geq 2) \quad (6.4)$$

Accordingly, the nominal lateral force distribution listed in Table 6-1 gives:

$$\{\lambda\} = \begin{Bmatrix} 1.20 \\ 1.25 \\ 1.33 \\ 1.50 \\ 2.00 \end{Bmatrix} \quad (6.5)$$

In addition, recalling that equation (3.19) shows the plate thickness is proportional to the shear strength, the nominal plate thickness distribution can be determined as:

$$[\gamma] = \begin{bmatrix} 0.9524, \\ 0.90, \\ 0.8333, \\ 0.7333, \\ 0.5755 \end{bmatrix} \quad (6.6)$$

6.2.4 Consideration of uncertainty in infill plate thickness distribution

Equation(6.6) specifies the nominal values of γ_i . It should be noted that these values assume that the plates designed according to the nominal lateral force distributions are available in the market. However, this may not be the case in practice. Typically, a thicker plate may have to be used due to the absence of the required plate. As such, the values specified in equation(6.6) only provides a lower bound of γ_i , denoted as γ_{li} . Additionally, given the fact that the plate at any story is always thicker than or identical to the plate at the story right above, the upper bound of γ_i , denoted as γ_{ui} is identified to 1.0. In practice, γ_i varies between γ_{li} and γ_{ui} as a random variable, depending on the availability of the plates. In this investigation, γ_i is assumed to follow the uniform distribution over the range from γ_{li} to γ_{ui} . The mean value and standard deviation of γ_i , denoted as θ_{γ_i} and σ_{γ_i} , respectively, are summarized in Table 6-2.

Table 6-2 statistical quantities of γ for the general case of six-story SPSW

Story	γ_{li}	γ_{ui}	θ_{γ_i}	σ_{γ_i}
2	0.952381	1.0	0.9762	0.01375
3	0.90	1.0	0.95	0.02887
4	0.833333	1.0	0.9167	0.04811
5	0.733333	1.0	0.8667	0.0767
6	0.545455	1.0	0.7727	0.13122

It is recognized that Figure 6-1 considers a wall with different plates installed at each story. Practically, engineers may select the same plate for neighboring stories for simplicity. In this investigation, a special case, which assumes the infill plates at the 2nd, 4th and 6th stories are the same as those at the 1st, 3rd and 5th stories respectively, is considered. Table 6-2 summarize the corresponding statistical quantities for γ_i . More information about this special case will be provided in Section 6.2.6.

Table 6-3 statistical quantities of γ_i for the special case of 6-story SPSW

Story	θ_{γ_i}	σ_{γ_i}
2	1.0	N/A
3	0.95	0.02887
4	1.0	N/A
5	0.8667	0.0767
6	1.0	N/A

6.2.5 Consideration of uncertainty in lateral force distribution

Equation (6.5) indicates that the nominal lateral forces have a linear distribution, i.e., distributed as an inverse triangle, along the height of the structure. However, the reactive weight redistribution and inelastic behavior of the structure may lead to a different lateral force distribution. Carr (2007) compared the pushover curves of a six-story frame structure under different lateral force distributions. It is observed that the structure may exhibit different performances when it is subjected to a different lateral force distribution,

as shown in Figure 6-2. As such, it would be necessary to consider the uncertainty of lateral force distribution in performance evaluation. Assuming that the value of λ_i determined from the nominal lateral force distribution is the mean of the variable (denoted as θ_{λ_i}) and the random variable λ_i follows normal distribution, Figure 6-3 illustrates the lateral force distributions determined by $\theta_{\lambda_i} \pm \sigma_{\lambda_i}$, where σ_{λ_i} is assumed to be $0.2\theta_{\lambda_i}$. As shown, all the four lateral force distributions investigated by Carr (2007) can be considered under these above assumptions.

Figure 6-2 Loading patterns and Results: (a) Push-over loading pattern for a six-story frame (b) Conventional and Adaptive Pushover Results (Carr, 2007)

Figure 6-3 Possible lateral force distributions

6.2.6 Summary of the three considered SPSWs

To capture and compare the performance of multi-story SPSWs, a total of three models, namely Models 1 through 3, are considered in this chapter. As illustrate in Figure 6-4, Model 1 assumes different infill plates at each story and a constant moment of inertia for the continuous column; Model 2 is the same as Model 1 except that three members are assigned to the continuous column (assuming the same column every two stories); and Model 3 is the same as Model 2 except that three types of plates are assigned to the wall .

In the models with non-constant continuous columns, i.e. Models 2 and 3, the moments of inertia of these columns are reduced using the corresponding factor, γ_i . Table 6-4 summarizes the parameters for each model. The column property I included in the table can be determined for a selected value of α from the following equation:

$$I = k \frac{\alpha h^3}{E} = \left(\frac{1}{4} Et \frac{L}{h} \right) \frac{\alpha h^3}{E} = \frac{1}{4} Lt \alpha h^2 \quad (5.7)$$

Note that detailed values for I , λ and γ for all three models can be found in Appendix G.

Table 6-4 Parameters for the models considered

Model	Story	Plate thickness	Lateral force	Column Moment of Inertia
1	1	t	F	I
	2	$\prod_{i=2}^2 \gamma_i t$	$\prod_{i=2}^2 \lambda_i F$	
	3	$\prod_{i=2}^3 \gamma_i t$	$\prod_{i=2}^3 \lambda_i F$	
	4	$\prod_{i=2}^4 \gamma_i t$	$\prod_{i=2}^4 \lambda_i F$	
	5	$\prod_{i=2}^5 \gamma_i t$	$\prod_{i=2}^5 \lambda_i F$	
	6	$\prod_{i=2}^6 \gamma_i t$	$\prod_{i=2}^6 \lambda_i F$	
2	1	t	F	I
	2	$\prod_{i=2}^2 \gamma_i t$	$\prod_{i=2}^2 \lambda_i F$	
	3	$\prod_{i=2}^3 \gamma_i t$	$\prod_{i=2}^3 \lambda_i F$	
	4	$\prod_{i=2}^4 \gamma_i t$	$\prod_{i=2}^4 \lambda_i F$	$\prod_{i=2}^3 r_i I$
	5	$\prod_{i=2}^5 \gamma_i t$	$\prod_{i=2}^5 \lambda_i F$	
	6	$\prod_{i=2}^6 \gamma_i t$	$\prod_{i=2}^6 \lambda_i F$	
3	1	t	F	I
	2		$\prod_{i=2}^2 \lambda_i F$	
	3	$\prod_{i=2}^3 \gamma_i t$	$\prod_{i=2}^3 \lambda_i F$	$\prod_{i=2}^3 r_i I$
	4		$\prod_{i=2}^4 \lambda_i F$	
	5	$\prod_{i=4}^5 \gamma_i t$	$\prod_{i=2}^5 \lambda_i F$	$\prod_{i=4}^5 r_i I$
	6		$\prod_{i=2}^6 \lambda_i F$

Model <1>

Model <2>

Model <3>

Model <1>

Model <2>

Model <3>

Figure 6-4 Three different models considered in the FE simulations

6.2.7 Response quantity of interest and analysis procedure

The FE models for the six-story SPSWs are similar to the two-story models considered in Chapter 4, in which the strip model is connected to a continuous column member. The member properties and lateral forces determined from Figure 6-4 and Appendix G were assigned to the models and the models were subjected to pushover analysis to a prescribed roof displacement. The maximum story ductility demand associated with the case that infill plate yielding just occurs in all stories, i.e., μ_c , is selected as the response quantity of interest. For the sampled values of γ_i and λ_i , the analysis was repeated, forming a database of μ_c . Then, as described in Chapter 5, the probability of not achieving plate yielding in all stories, called probability of failure, can be assessed using the following equation:

$$P[\mu_c(\gamma_2, \gamma_3, \dots, \gamma_6, \lambda_2, \lambda_3, \dots, \lambda_6) > \mu | \alpha = \alpha_0] = 1 - \Phi \left(\frac{\mu - \theta_{\mu_c}}{\sigma_{\mu_c}} \right) \bigg|_{\alpha = \alpha_0} \quad (5.8)$$

Considering the high computation cost of analyzing six-story SPSWs, the range of α was narrowed down to be [0.001, 0.5]. Another parameter needed in the analysis is the number of α values to be considered, i.e. the resolution of α , for capturing the probability of failure versus α curves. The resolution of α is determined based on the analysis results of the two-story SPSWs as shown in Figure 6-5, which compares the results with different resolutions of α . As shown, when the resolution is 30, the curve from the 2K+1 analysis is relatively smooth and matches well with the more rigorous results from the M.C. simulation. Therefore, over the selected range of α , a total of 30 equally log spaced α values were considered in this investigation. At each selected level of α , the parameters determined from the 2K+1 method were assigned to the FE model

and the probability from equation was evaluated. Connecting the probability values generates the probability of failure versus α curves.

Figure 6-5 α Resolution comparisons

6.3 Result Discussion

Figure 6-6 shows a typical pushover curve found for the six-story SPSW structure, where the horizontal axis is the roof displacement and the vertical axis is the based shear. As shown, the pushover curve corresponds to the progressively developed nonlinear behavior of the wall. The maximum story ductility demand on the SPSW can be identified from the floor displacements associated with the elastic limit and the point where infill plate yielding occurs in all stories. Figure 6-7 shows the progressive development of plate yielding in the FE models of the wall. Example pushover curves can be found in Figure B3 in Appendix B.

Figure 6-6 A typical pushover curve for the six-story SPSW

Figure 6-7 Progressive development of plate yielding in the FE models:

(a) first story yields (b) three stories yield (c) all stories yield

The probability of failure versus α curves for all the three models are presented in

Figure 6-8. While the curves are not complete for some cases over the selected α region,

the observations from these curves are useful for evaluating the effects of column

stiffness on seismic behavior of the wall. As shown, generally, when α increases, the probability of failures will drop, indicating that it is more likely to achieve plate yielding in all stories in a wall with stiffer columns which is consistent with the observation from the two-story SPSWs. In addition, no matter what aspect ratio the multi-story wall has, if the columns are designed only taking into account the minimum moment of inertia specified in the code, it is unlikely to achieve plate yielding in all stories before a certain story reaches its ductility limit as identified by the probability of failure around 1.0. Moreover, results from Models <1> and <2> indicate that the probability of achieving plate yielding in all stories is lower in the wall with decreasing column stiffness along the vertical direction. Furthermore, results from Models <2> and <3> indicate that if plates thicker than required are installed at certain stories, the probability of achieving plate yielding in all stories will be reduced.

Figure 6-8 Probability of failure comparison

6.4 Summary

Probabilistic analyses of three six-story SPSW models were carried out in this chapter.

Developments of all the FE models for the considered 6-story SPSWs are based on those from the two-story SPSWs. Based on the results from nonlinear pushover analysis and the 2K+1 method, it is found that the minimum moment of inertia for SPSW columns specified in the design code may not warrant infill plate yielding in all stories of a multi-story SPSW.

Chapter 7 SUMMARY, CONCLUSIONS, AND FUTURE RESEARCH RECOMMENDATIONS

7.1 Summary and conclusions

This research investigates the effect of column stiffness on seismic behavior of SPSWs. Both two-story and six-story SPSWs were considered. For the two-story SPSW, analytical models for quantifying the effects of column stiffness on structure drift concentration, bending moment in the column caused by the drift redistribution effects, and the ductility demand on SPSWs to achieve yielding in each story were derived. These derived models were verified by the results from nonlinear static analysis using the FE models set up in SAP 2000. Based on the validated models, probabilistic analyses were performed for evaluation of the effect of column stiffness on the system seismic behavior. A total of three probabilistic simulation methods, the M.C. method, the L.H. method and the 2K+1 method, were considered to evaluate their respective accuracies and advantages. Results show that all the three methods provide similar results and are stable, however, the 2K+1 method require the minimum computational efforts. For six-story SPSWs, three different models including different infill plate and column distributions were considered using the 2K+1 procedure.

Results from two-story SPSWs show that

- 1) As the column moment of inertia increases, DCF approaches to unity, indicating that the structure remains straight when displacing laterally and the SPSW columns effectively redistribute the story drifts in the structure, mitigating the drift concentration; however, further increasing the column moment of inertia beyond a certain limit only provides very limited improvement on system

behavior. Accordingly, the drift redistribution effects cause bending moments in the SPSW columns. Such bending moments increase as the column moment of inertia increases but eventually converges to a constant level.

- 2) The three probabilistic evaluation methods provide similar results and all of them are stable, however, the 2K+1 method requires the minimum computational efforts and can be preferably used in evaluation of the more complicated multi-story SPSWs.
- 3) If the columns were designed only taking into account the minimum moment of inertia specified in the AISC Seismic Design Provisions (2005), plate yielding in both stories is more likely to occur in the walls with larger width-to-height aspect ratios.

Results from six-story SPSWs show that

- 1) As the continuous columns become progressively flexible along the height of the structure, they will be less effective to ensure infill plate yielding in all stories.
- 2) When the same plates are installed in neighboring stories, i.e. when plates thicker than required are installed at certain stories, the probability to achieve plate yielding in all stores becomes lower.
- 3) No matter what aspect ratio the wall has, the current code requirement for the minimum column moment of inertia does not necessarily warrant infill plate yielding in all stories.

7.2 Future research recommendations

In this investigation, the beam-to-column and column-to-foundation connections are assumed to be simple connections. It would be interesting to investigate the impacts of moment resistance of these connections in the future.

For the multi-story analysis, this research only considered the nonlinear static analysis to evaluate the behavior of the system. In such an analysis approach, the lateral force distribution does not change as the nonlinear behavior of the structures progressively develops. It would be interesting to further investigate and evaluate the system behavior using the more rigorous nonlinear response history analysis procedure.

Appendix A Infill Strip Development

Both the derived model in Chapter 3 and the FE model in Chapter 4 are based on the strip model for simulation of the SPSWs. This section summarizes the procedures for development of this model. As shown in equation(2.3), developed by Timer and Kulak (1983), the strip inclination angle can be approximated based on the function of the system geometry as followed:

$$\tan^4 \beta = \left\{ \frac{1 + \frac{Lt}{2A_c}}{1 + ht \left(\frac{1}{A_b} + \frac{h^3}{360LI_c} \right)} \right\} \quad (A1)$$

where β is the strip inclination angel with respect to the column, L is the bay width, t is the thickness of the plate, A_c and A_b are the cross-sectional area of the column and beam, respectively, h is the story height and I_c is the column moment of inertia. Rearraning (A1) and solving for β gives:

$$\beta = \text{atan} \left[\left(\frac{1 + \frac{Lt}{2A_c}}{1 + ht \left(\frac{1}{A_b} + \frac{h^3}{360LI_c} \right)} \right)^{\frac{1}{4}} \right] \quad (A2)$$

Assuming that both beams and columns are axially rigid, that is, $A_b \rightarrow \infty$ and $A_c \rightarrow \infty$, equation(A2) reduces as:

$$\beta = \text{Arc tan} \left[\left(\frac{1}{1 + ht \left(\frac{h^3}{360LI_c} \right)} \right)^{\frac{1}{4}} \right] \quad (\text{A3})$$

Recall that, based on the range of α considered, seven values of I_c were correspondingly considered in the FE analysis, which are tabulated in Table A-1. Plug I_c individually into equation(A3), the corresponding β values are tabulated in Table A-1.

Table A 1 Infill strip inclination angles

Analysis Case	$I_c, [inch^4]$	$\beta, [deg]$
Case 1	117.558	35.65
Case 2	1175.584	43.25
Case 3	11755.843	44.80
Case 4	117558.452	44.98
Case 5	1175584.252	45.0
Case 6	11755842.520	45.0
Case 7	117558425.197	45.0

For convenience, β was assumed to be 45 degree for the FE model development. Based on Berman and Bruneau (2003), a total of 20 strips (n=20) were selected in each story of the FE model, which leaves a total of 20 spaces between strips. As shown in the figure below, L_β , which is the length of bay after rotating β degree, can be derived as:

$$L_\beta = h \sin(\beta) + L \cos(\beta) \quad (\text{A4})$$

Figure A 1 Schematic drawing for infill strips

Note that the perpendicular strip spacing, denoted as s_p , is equal to:

$$s_p = \frac{L_\beta}{n} \quad (\text{A5})$$

Therefore, the area of each strip equals to s_p times the plate thickness:

$$A_s = s_p t \quad (\text{A6})$$

Geometry from the figure above shows that the strip spacing in the x direction (i.e., along the HBE), denoted as s_x , and z direction (i.e., along the VBE), denoted as s_z , respectively, can be derived as:

$$\begin{bmatrix} s_x \\ s_z \end{bmatrix} = s_p \begin{bmatrix} (\cos(\beta))^{-1} \\ (\sin(\beta))^{-1} \end{bmatrix} \quad (\text{A7})$$

By inspection, spacing at the corner is half of the regular spacing. that is,

$$\begin{bmatrix} s_{xc} \\ s_{zc} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} s_x \\ s_z \end{bmatrix} \quad (\text{A8})$$

Appendix B Pushover Curves For The FE Model

Figure B1 Pushover Curves For The Two-Story FE Model (Performance Category I)

Figure B2 Pushover Curves For The Two-Story FE Model (Performance Category II)

Figure B3 Example Pushover Curves For The Six-Story FE Model

Appendix C Matlab Codes For The Two-Story Probabilistic Analysis

Sub-Appendix a The "app.m" script

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%M.C.,L.H.& 2K+1 Simulation Matlab Codes for Guo's Thesis
%This is the Main Driver Function
%Written by Guo, XuHua (Gary); Last Date Modified: Mar-07-11
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%THESE FUNCTIONS ARE GOING TO BE TIMED%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% SHOWN IN THE "Command Window" AFTER RUN%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

clear;clc;close all;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
tic;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%Load Inputs (Pre-processing)
disp('Reading data...')
[gamma_prop,cp,alpha_prop,filnam,stat,lamda_prop,fylnam] = inputt;

%The Runn Function
disp('Running the Funciton...')
[alpha,Pfs,vals,rn] = runn(gamma_prop,cp,stat,alpha_prop,lamda_prop);

%Data Validation
disp('Validating Data...')
% [Dn,ln_Dn] = data_val(stat,vals,alpha_prop);

%Output data
disp('Wriring data to the output file...')
[st] =
outputt(filnam,stat,Pfs,alpha_prop,gamma_prop,cp,alpha,rn,vals,lamda_prop,
fylnam);

%Plotting
disp('Plotting Results...')
%plotn(alpha_prop,alpha,Pfs,rn,stat);

if stat(3) == 1
    disp('"Monte Carlo(M.C.)" Method Is Used In This
Analysis..... ')
elseif stat(3) == 2
    disp('"Latin Hypercube(L.H.)" Method Is Used In This
Analysis..... ')
elseif stat(3) == 3
    disp('"2K+1 Method(2K+1)" Is Used In This Analysis..... ')
else
    disp('Error of Input..... ')
    disp('Check "methodd" value in the "inputt.m" file ')
```

end

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%  
toc;  
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

Sub-Appendix b The "inputt.m" function

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%M.C.,L.H.& 2K+1 Simulation Matlab Codes for Guo's Thesis
%This is the INPUT function
%Written by Guo, XuHua (Gary); Last Date Modified: Mar-07-11
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

function [gammaprop,cp,alphaprop,filnam,stat,lamdaprop,fylnam] = inputt
format short g;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%NUMERICAL INPUT VALUES%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%Determine Method(s) For Data Sampling:
    %Monte Carlo Simulation      = 1
    %Latin Hypercube Simulation = 2
    %Rosenblueth's (2K+1)       = 3
    methodd = 3;

%Standard Deviation for xx3

%User Input Percent Error Tolerance==>Unit=[%]:
    tol = 2;

%(For Latin Hypercube Only)Number of Strata In the Domain :

%+++++
%===== This number (numbsegm) has to be
%|=====
%===== an EVEN number as it will crash
%|=====
%===== program otherwise
%|=====
%+++++
%|+++++
    numbsegm = 100;%% <<<=====

%Define The Alpha (Stiffness Ratio) Resolution (Number of Intervals):
    resol = 500; %"resol" has to be an integer!!!

%Define Performance Limits:
    %Roof Ductility Capacity      = mut_cp
    %          DCF Capacity      = DCF_cp
    %1st Floor Ductility Capacity = muo_cp
    %          Moment Capacity   = Mc_cp
    %
    % cp = [mut_cp, DCF_cp, muo_cp, Mc_cp];
    cp = [ 6, 1.5, 8, 0.1];

%Random Number Generation User Defined Standard Deviation for Gamma &
Lamda
    udstdG = 0.3;

```



```

udstdL = 0.2;

%Define The Region For Values of The SPSW Thickness Ratio: gamma
gamma_upper = 1;
gamma_lower = 0;

%Define the Region For Values of The SPSW Structure's Lateral Loads:
lamda_upper = 3;
lamda_lower = 1;

%Define The Limits For The Stiffness Ratio: alpha
AL = 0.001;      %%%%%%%%%Alpha Lower Bound
AU = 1.0;        %%%%%%%%%Alpha Upper Bound

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%CALCULATIONS & DATA OPERATIONS FROM THIS POINT BEYOND%%
%%OUTPUTS ARE STATISTICAL VALUES FROM ALL INPUTS%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

filnam = 'SPSW.txt';

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
fynam = 'Grapher.txt';
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%Generate Gamma Statistical Properties:
gamma_range = gamma_upper - gamma_lower;
gamma_mean  = 7/12;
gamma_cr    = 2/3;
gamma_std   = gamma_range/sqrt(12);%%%%%%%%Assume Uniform
Distribution
gamma_var    = gamma_std/gamma_mean;
gamma_const = 1;%%%%%%%%Special Case Where Plate Thicknesses
Identical

%Generate Lamda Statistical Properties:
lamda_range = lamda_upper-lamda_lower;
lamda_mean  = mean(lamda_lower:lamda_upper);
lamda_std   = 0.3;%%%%%%%%=====>>From Excel
lamda_var    = lamda_std/lamda_mean;

%Generate the Gamma Property Array:
%      1      2      3      4      5
%      6      7      8
% Range      Mean      StandardDev      Upper
Lower      Variance      Critical      Constant
gammaprop = [gamma_range, gamma_mean, gamma_std, gamma_upper,
gamma_lower, gamma_var, gamma_cr, gamma_const];

%Generate the Lamda Property Array:
%      1      2      3      4      5
%      6

```

```

                                % Range      Mean      StandardDev      Upper      Lower
Variance
lamdaprop = [lamda_range, lamda_mean, lamda_std, lamda_upper,
lamda_lower, lamda_var];
% lamdaprop = [lamda_range,      2, lamda_std, lamda_upper,
lamda_lower, lamda_var];

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%disp(gamma(3))
%disp(gammaprop)
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%Actual Number of Intervals In the Domain:
%      actni = numinter - 1;

%Generate An Array for Properties of The Stiffness Ratio: alpha
alphaprop = [AL,AU,resol];

%Total Error: err
value = 1/(tol/100);
AbsErr = gammaprop(2)/value;

%Number of Iterations: N
N = (3*gammaprop(3)/AbsErr)^2;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%disp('N=')
%disp(N)
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%Actual Number of Iterations
if methodd == 1

    actnumbit = N;

elseif methodd == 2

    actnumbit = numbsegm;

else

    actnumbit = alphaprop(3);%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%Revise This for 2K+1

end

%Generate And Array to Collect General Statistical Properties
stat = [actnumbit, tol, methodd, udstdG, udstdL];

end %This function ends here.....

```

Sub-Appendix c The "runn.m" function

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%M.C.,L.H.& 2K+1 Simulation Matlab Codes for Guo's Thesis
%This is the RUN (main) function
%Written by Guo, XuHua (Gary); Last Date Modified: May-31-11
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

function [alpha,Pfs,vals,rn] =
runn(gammaprop,cp,stat,alphaprop,lamdaprop)

format long g;

%Compute array sizes
numbiter = stat(1); %Number of Iteration Recall
gc       = gammaprop(8);%Special Case For Identical Plate Thickness

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%For Random Number Normal Distribution ONLY
    muL    = lamdaprop(2);
    sigmaL = muL*stat(5);%Different than lamdaprop(3)
%=====
    muG    = gammaprop(2);
    sigmaG = muG*stat(4);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

    if (stat(3) == 1)||(stat(3) == 2)

%+++++
%+++++ %Monte Carlo Sampling Method
%+++++
    if stat(3) == 1

        %Random Number Generator
        for randd = 1:numbiter
            %Random Numbers to Enter into The Program (Uniform
Distribution)
            xx1(randd) = random('uniform',0,1);

            %Random Numbers to Generate Gamma Values (Uniform
Distribution)
            %xx2(randd) = random('norm',muG,sigmaG);
            xx2(randd) = random('uniform',0,1);

            %Random Number to Generate Lamda Values Simulate
            %Lateral Forces Applied (Uniform/Normal)
%            xx3(randd) = random('norm',muL,sigmaL);
            xx3(randd) = random('uniform',0,1);

            %%%Generate Gamma Values

```

```

        if xx1(randd) >= 0.5%%%%%Choosing the MidPt value of the
uniform distribution array ranging [0,1]
            gam(randd) = gammaprop(7)+(1-gammaprop(7))*xx2(randd);
        else
            gam(randd) = gammaprop(7)*xx2(randd);
        end

    %%%Lamda values
        lam(randd) = 1+xx3(randd)*(lamdaprop(4)- lamdaprop(5));

    %%%Categorize(cat) the Lamda and Gamma cases as 1 or 2
        if gam(randd) >= lam(randd)/(1+lam(randd))
            cat(randd) = 1;
        else
            cat(randd) = 2;
        end
    end

    %Collecting The Set of These Random Numbers: "rn" Array
    for w = 1:numbiter
        rn(w,1) = lam(w);
        rn(w,2) = gam(w);
        rn(w,3) = cat(w);
    end

    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    +++
        %Latin Hypercube Sampling Method
    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    +++
    elseif stat(3) == 2
        %Compute Number of Segments
        numbsegm = stat(1)/2;

        %Gamma and Lamda Range Section Partitions:
        %Note that gamma=2/3 bisect the range of [0,1]
        gammasec1 = (gammaprop(7)-gammaprop(5))/numbsegm;
        gammasec2 = (gammaprop(4)-gammaprop(7))/numbsegm;
        lamdasec = lamdaprop(1)/stat(1);

        %Generate Arrays for Gamma and Lamda With Equal Section Partitions:
        gammarray1 = gammaprop(5):gammasec1:gammaprop(7);
        gammarray2 = gammaprop(7):gammasec2:gammaprop(4);
        lamdarray = lamdaprop(5):lamdasec:lamdaprop(4);

        %Random Permutation Generation:
        permugamma1 = randperm(length(gammarray1)-1);
        permugamma2 = randperm(length(gammarray2)-1);
        permulamda = randperm(length(lamdarray)-1);

        %Find the Mid-Point Value of Each Sectment;
        for q = 1:numbsegm

```

```

        gammalmids(q) = gammarray1(q)+(gammarray1(q+1)-gammarray1(q))/2;
        gamma2mids(q) = gammarray2(q)+(gammarray2(q+1)-gammarray2(q))/2;
    end

    for q = 1:stat(1)
        lamdamids(q) = lamdarray(q)+(lamdarray(q+1)-lamdarray(q))/2;
    end

    %Random Combination of Gamma and Lamda Values

    %Note that length(gammalmids) = length(gamma2mids)
    for e=1:length(gammalmids)
        gam1(e) = gammalmids(permugamma1(e));
        gam2(e) = gamma2mids(permugamma2(e));
    end

    for e=1:length(lamdamids)
        lam(e) = lamdamids(permulamda(e));
    end

    %Collecting Gamma1 and Gamma2
    %Note that length(gamma1) = length(gamma2)
    lengam = length(gam1);
    gam = zeros(1,2*lengam);

    for mm = 1:lengam
        gam(mm) = gam1(mm);
    end

    for nn = (lengam+1):(2*lengam)
        gam(nn) = gam2(nn-lengam);
    end

    for e = 1:length(lam)
        %%%Categorize(cat) the Lamda and Gamma cases as 1 or 2
        if gam(e) > lam(e)/(1+lam(e))
            cat(e) = 1;
        else
            cat(e) = 2;
        end
    end

    %%%Collecting The Set of These Random Numbers: "rn" Array
    for w = 1:stat(1)
        rn(w,1) = lam(w);
        rn(w,2) = gam(w);
        rn(w,3) = cat(w);
    end

    end

    %+++++
    +++

```

```

%Monte Carlo Simulation & Latin Hypercube Simulation
%+++++
+++

%Calculations of Prob. of Failures for Alphas Based on the "rn" Array
for j = 1:alphaprop(3)

    %Alpha Value Generators: Based on alphaprop(3)=Resolutions
    alpha(j,1)
    =10^(log10(alphaprop(1))+j/alphaprop(3)*((log10(alphaprop(2))-
    log10(alphaprop(1)))));%This is Correct--gives alpha values in Log10

    %Calculations for mu_tc, DCFc, mu_oc & Mc Based on the Equations
    Derived
    for i = 1:numbiter

        %Calculate the Constant [Chi] Value: X & Xgc
        X(i) =
        (lam(i)/(gam(i)*(1+lam(i)))+(1+3*alpha(j)/gam(i))/(1+3*alpha(j)))/(1+3*
        alpha(j)/(2*gam(i))-
        3*alpha(j)/2*((3*alpha(j)/gam(i)+1)/(1+3*alpha(j))));
        %Special Case For Identical Plate Thickness: gam(i) = gc;
        Xgc(i) =
        (lam(i)/(gc*(1+lam(i)))+(1+3*alpha(j)/gc)/(1+3*alpha(j)))/(1+3*alpha(j)
        /(2*gc)-3*alpha(j)/2*((3*alpha(j)/gc+1)/(1+3*alpha(j))));

        %Calculations for Each Category: "mu_tc", "DCFc", "mu_oc", "Mc"
        %All values are checked by the "testing.m" files in the same
        folder
        if cat(i) == 1
            mu_tc(i,1) =
            (2+3*alpha(j)*X(i))/(X(i)*(1+3*alpha(j)))*(1+(gam(i)+gam(i)*lam(i)-
            lam(i))/(3*alpha(j)*(2*lam(i)+1)));
            DCFc(i,1) = 1+(1-
            2*lam(i)/(gam(i)*(1+lam(i)))*(1/X(i)+3*alpha(j)/2)/(1+3*alpha(j))/mu_tc
            (i))/(1+3*alpha(j)*lam(i)/(gam(i)*(1+lam(i)))+3*alpha(j)/gam(i));
            mu_oc(i,1) =
            DCFc(i)/2*mu_tc(i)/(1/(X(i)*(1+3*alpha(j)))+3*alpha(j)/(2*(1+3*alpha(j)
            ))));
            %Normalized Moment, Mc,
            Mcc(i,1) = 3/2*alpha(j)*mu_tc(i)*(DCFc(i)-
            1)/(1/(X(i)*(1+3*alpha(j)))+3*alpha(j)/(2*(1+3*alpha(j))));

        elseif cat(i) == 2
            mu_tc(i,1) =
            (((lam(i)/(1+lam(i))/gam(i)*(3*alpha(j)+1)+3*alpha(j)/gam(i))*((2+3*alp
            ha(j))*X(i)-2)/(2*X(i)*(1+3*alpha(j)))-(1+3*alpha(j)/2-
            1/X(i))/(1+3*alpha(j)))/((lam(i)/(1+lam(i))/gam(i)*3/2*alpha(j)+3*alph
            a(j)/(2*gam(i))));
            DCFc(i,1) =
            ((lam(i)/(gam(i)*(1+lam(i)))*(3*alpha(j)+2))+3*alpha(j)/gam(i)-
            (2+3*alpha(j)-
            2/X(i))/(mu_tc(i)*(1+3*alpha(j)))/((lam(i)/(1+lam(i))/gam(i)*(1+3*alph
            a(j))+3*alpha(j)/gam(i));
            mu_oc(i,1) = DCFc(i)/2*mu_tc(i)/(1-
            (1/(X(i)*(1+3*alpha(j)))+3*alpha(j)/(2*(1+3*alpha(j))));

```

```

        %%%Normalized Moment, Mc
        Mcc(i,1) = 3/2*alpha(j)*mu_tc(i)*(DCFc(i)-1)/(1-
(1/(X(i)*(1+3*alpha(j)))+3*alpha(j)/(2*(1+3*alpha(j)))));

    else
        disp('Attempt For Calculation Is NOT Valid');
        disp('Check The "inputt.m" & "runn.m" Functions...');
        break
    end

        %-----
    ---
        %Special Case For Identical Plate Thickness: gam(i) = gc
    = Constant Gamma;
        mu_tcg(i,1) =
(2+3*alpha(j)*Xgc(i))/(Xgc(i)*(1+3*alpha(j)))*(1+(gc+gc*lam(i)-
lam(i))/(3*alpha(j)*(2*lam(i)+1)));
        DCFcg(i,1) = 1+(1-
2*lam(i)/(gc*(1+lam(i)))*(1/Xgc(i)+3*alpha(j)/2)/(1+3*alpha(j))/mu_tcg
(i))/(1+3*alpha(j)*lam(i)/(gc*(1+lam(i)))+3*alpha(j)/gc);
        mu_ocg(i,1) =
DCFcg(i)/2*mu_tcg(i)/(1/(Xgc(i)*(1+3*alpha(j)))+3*alpha(j)/(2*(1+3*al
pha(j))));
        %%%Normalized Moment, Mc,
        Mccg(i,1) = 3/2*alpha(j)*mu_tcg(i)*(DCFcg(i)-
1)/(1/(Xgc(i)*(1+3*alpha(j)))+3*alpha(j)/(2*(1+3*alpha(j))));
        %-----
    ---

    end%%%%%%%%%%2nd "for: (i)_# of Iteration/alpha" loop ends here

    %Collecting Results from The Calculations for All 4 Items of
    Interest
    valss = zeros(numbiter,size(cp,2)+1);
    for w = 1:numbiter
        valss(w,1) = X(w);
        valss(w,2) = mu_tc(w);
        valss(w,3) = DCFc(w);
        valss(w,4) = mu_oc(w);
        valss(w,5) = Mcc(w);

        %-----
        %Special Case For Identical Plate Thickness:
        valss(w,6) = Xgc(w);
        valss(w,7) = mu_tcg(w);
        valss(w,8) = DCFcg(w);
        valss(w,9) = mu_ocg(w);
        valss(w,10)= Mccg(w);
        %-----
    end
    vals(:,j) = valss;

    %Generate The Array For All 4 Critical Values:
    means1(j) = mean(mu_tc);      stds1(j) = std(mu_tc);

```



```

%=====
%Using Monte-Carlo Method to Generate Lamda and Gamma
%and Take Their Statistical means and standard deviations Below
%=====
for randd = 1:alphaprop(3)
    %%Generate the Lamda(lam)& Gamma(gam)Arrays for total numbers
    %%of Iterations per Alpha

    %Random Numbers to Enter into The Program (Uniform
Distribution)
    xx1(randd) = random('uniform',0,1);

    %Random Numbers to Generate Gamma Values (Uniform
Distribution)
    xx2(randd) = random('uniform',0,1);

    %Random Number to Generate Lamda Values Simulate
    %Lateral Forces Applied (Uniform/Normal)
%    xx3(randd) = random('norm',muL,sigmaL);
    xx3(randd) = random('uniform',0,1);

    %%%Generate Gamma Values
    if xx1(randd) >= 0.5
        gam(randd) = 2/3+(1-2/3)*xx2(randd);
    else
        gam(randd) = 2/3*xx2(randd);
    end

    %%%Lamda values
    lam(randd) = 1+xx3(randd)*(lamdaprop(4)- lamdaprop(5));

    %%%Categorize(cat) the Lamda and Gamma cases as 1 or 2
    if gam(randd) >= lam(randd)/(1+lam(randd))
        cat(randd) = 1;
    else
        cat(randd) = 2;
    end
end
%%Temporately Collecting The Set of These Random Numbers: "rn"
Array
for w = 1:alphaprop(3)
    rn(w,1) = lam(w);
    rn(w,2) = gam(w);
    rn(w,3) = cat(w);
end

%=====
% "2K+1" Point Estimate Codes
%=====

lamda_mean = mean(rn(:,1));    lamda_std = std(rn(:,1));
gamma_mean = mean(rn(:,2));    gamma_std = std(rn(:,2));
catog_mean = mean(rn(:,3));    catog_std = std(rn(:,3));

```

```

%Reorganize the means and std values:
lm = lamda_mean;          lstd = lamda_std;
gm = gamma_mean;          gstd = gamma_std;
cm = catog_mean;          cstd = catog_std;%%For Checking
Purposes Only

%lm = 2;                   lstd = 0.5763;
%gm = 0.5843;              gstd = 0.3;

lmp = lm + lstd;%lamdaplus = lamda_mean + lamda_std
lmm = lm - lstd;%lamdaminus = lamda_mean - lamda_std
gmp = gm + gstd;
gmm = gm - gstd;

%Assembling lmp, lmm, gmp and gmm values
%LG = [lmp, lmm, gmp, gmm];

for j = 1:alphaprop(3)
    %Alpha Value Generators: Based on alphaprop(3)=Resolutions
    alpha(j,1)
=10^(log10(alphaprop(1))+j/alphaprop(3)*((log10(alphaprop(2))-
log10(alphaprop(1)))));%This is Correct--gives alpha values in Log10

%++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
%Generate Chi, [X], Statistical Values
X0(j) =
(lm/(gm*(1+lm)))+(1+3*alpha(j)/gm)/(1+3*alpha(j)))/(1+3*alpha(j)/(2*gm)-
3*alpha(j)/2*((3*alpha(j)/gm+1)/(1+3*alpha(j))));

XLp(j) =
(lmp/(gm*(1+lmp)))+(1+3*alpha(j)/gm)/(1+3*alpha(j)))/(1+3*alpha(j)/(2*gm)
)-3*alpha(j)/2*((3*alpha(j)/gm+1)/(1+3*alpha(j))));
XLm(j) =
(lmm/(gm*(1+lmm)))+(1+3*alpha(j)/gm)/(1+3*alpha(j)))/(1+3*alpha(j)/(2*gm)
)-3*alpha(j)/2*((3*alpha(j)/gm+1)/(1+3*alpha(j))));
XGp(j) =
(lm/(gmp*(1+lm)))+(1+3*alpha(j)/gmp)/(1+3*alpha(j)))/(1+3*alpha(j)/(2*gm
p)-3*alpha(j)/2*((3*alpha(j)/gmp+1)/(1+3*alpha(j))));
XGm(j) =
(lm/(gmm*(1+lm)))+(1+3*alpha(j)/gmm)/(1+3*alpha(j)))/(1+3*alpha(j)/(2*gm
m)-3*alpha(j)/2*((3*alpha(j)/gmm+1)/(1+3*alpha(j))));

XLbar(j) = (1/2)*(XLp(j) + XLm(j));
XGbar(j) = (1/2)*(XGp(j) + XGm(j));

VxL(j) = (XLp(j) - XLm(j))/(XLp(j) + XLm(j));
VxG(j) = (XGp(j) - XGm(j))/(XGp(j) + XGm(j));

```

```

%+++++
%Generate mu_tc Statistical Values

    if gm > lm/(1+lm)
        mu_tc0(j) =
(2+3*alpha(j)*X0(j))/(X0(j)*(1+3*alpha(j)))*(1+(gm+gm*lm-
lm)/(3*alpha(j)*(2*lm+1)));
        %mu_tc0gc(j) =
(2+3*alpha(j)*X0gc(j))/(X0gc(j)*(1+3*alpha(j)))*(1+(gc+gc*lm-
lm)/(3*alpha(j)*(2*lm+1)));%Special Case
    else
        mu_tc0(j) =
(((lm/(1+lm)/gm*(3*alpha(j)+1)+3*alpha(j)/gm)*((2+3*alpha(j))*X0(j)-
2)/(2*X0(j)*(1+3*alpha(j)))-(1+3*alpha(j)/2-
1/X0(j))/(1+3*alpha(j))))/((lm/(1+lm)/gm*3/2*alpha(j)+3*alpha(j)/(2*gm)
));
        %mu_tc0gc(j) =
(((lm/(1+lm)/gc*(3*alpha(j)+1)+3*alpha(j)/gc)*((2+3*alpha(j))*X0gc(j)-
2)/(2*X0gc(j)*(1+3*alpha(j)))-(1+3*alpha(j)/2-
1/X0gc(j))/(1+3*alpha(j))))/((lm/(1+lm)/gc*3/2*alpha(j)+3*alpha(j)/(2*gc)
));%Special Case
    end

    if gm > lmp/(1+lmp)
        mu_tcLp(j) =
(2+3*alpha(j)*XLp(j))/(XLp(j)*(1+3*alpha(j)))*(1+(gm+gm*lmp-
lmp)/(3*alpha(j)*(2*lmp+1)));
        %mu_tcLpgc(j) =
(2+3*alpha(j)*XLpgc(j))/(XLpgc(j)*(1+3*alpha(j)))*(1+(gc+gc*lmp-
lmp)/(3*alpha(j)*(2*lmp+1)));%Special Case
    else
        mu_tcLp(j) =
(((lmp/(1+lmp)/gm*(3*alpha(j)+1)+3*alpha(j)/gm)*((2+3*alpha(j))*XLp(j)-
2)/(2*XLp(j)*(1+3*alpha(j)))-(1+3*alpha(j)/2-
1/XLp(j))/(1+3*alpha(j))))/((lmp/(1+lmp)/gm*3/2*alpha(j)+3*alpha(j)/(2*gm)
));
        %mu_tcLpgc(j) =
(((lmp/(1+lmp)/gc*(3*alpha(j)+1)+3*alpha(j)/gc)*((2+3*alpha(j))*XLpgc(j)-
2)/(2*XLpgc(j)*(1+3*alpha(j)))-(1+3*alpha(j)/2-
1/XLpgc(j))/(1+3*alpha(j))))/((lmp/(1+lmp)/gc*3/2*alpha(j)+3*alpha(j)/(2*gc)
));%Special Case
    end

    if gm > lmm/(1+lmm)
        mu_tcLm(j) =
(2+3*alpha(j)*XLm(j))/(XLm(j)*(1+3*alpha(j)))*(1+(gm+gm*lmm-
lmm)/(3*alpha(j)*(2*lmm+1)));
        %mu_tcLmgc(j) =
(2+3*alpha(j)*XLmgc(j))/(XLmgc(j)*(1+3*alpha(j)))*(1+(gc+gc*lmm-
lmm)/(3*alpha(j)*(2*lmm+1)));%Special Case
    else
        mu_tcLm(j) =
(((lmm/(1+lmm)/gm*(3*alpha(j)+1)+3*alpha(j)/gm)*((2+3*alpha(j))*XLm(j)-
2)/(2*XLm(j)*(1+3*alpha(j)))-(1+3*alpha(j)/2-

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1/XLm(j))/(1+3*alpha(j)))/((lmm/(1+lmm)/gm*3/2*alpha(j)+3*alpha(j)/(2*
gm)));
    %mu_tcLmgc(j) =
    (((lmm/(1+lmm)/gc*(3*alpha(j)+1)+3*alpha(j)/gc)*((2+3*alpha(j))*XLmgc(j)
    -2)/(2*XLmgc(j)*(1+3*alpha(j)))-(1+3*alpha(j)/2-
    1/XLmgc(j))/(1+3*alpha(j)))/((lmm/(1+lmm)/gc*3/2*alpha(j)+3*alpha(j)/(
    2*gc))));%Special Case
    end

    if gmp > lm/(1+lm)
        mu_tcGp(j) =
        (2+3*alpha(j)*XGp(j))/(XGp(j)*(1+3*alpha(j)))*(1+(gmp+gmp*lm-
        lm)/(3*alpha(j)*(2*lm+1)));
        %mu_tcGpgc(j) =
        (2+3*alpha(j)*XGpgc(j))/(XGpgc(j)*(1+3*alpha(j)))*(1+(gc+gc*lm-
        lm)/(3*alpha(j)*(2*lm+1)));%Special Case
    else
        mu_tcGp(j) =
        (((lm/(1+lm)/gmp*(3*alpha(j)+1)+3*alpha(j)/gmp)*((2+3*alpha(j))*XGp(j)-
        2)/(2*XGp(j)*(1+3*alpha(j)))-(1+3*alpha(j)/2-
        1/XGp(j))/(1+3*alpha(j)))/((lm/(1+lm)/gmp*3/2*alpha(j)+3*alpha(j)/(2*g
        mp)));
        %mu_tcGpgc(j) =
        (((lm/(1+lm)/gc*(3*alpha(j)+1)+3*alpha(j)/gc)*((2+3*alpha(j))*XGpgc(j)-
        2)/(2*XGpgc(j)*(1+3*alpha(j)))-(1+3*alpha(j)/2-
        1/XGpgc(j))/(1+3*alpha(j)))/((lm/(1+lm)/gc*3/2*alpha(j)+3*alpha(j)/(2*
        gc))));%Special Case
    end

    if gmm > lm/(1+lm)
        mu_tcGm(j) =
        (2+3*alpha(j)*XGm(j))/(XGm(j)*(1+3*alpha(j)))*(1+(gmm+gmm*lm-
        lm)/(3*alpha(j)*(2*lm+1)));
        %mu_tcGmgc(j) =
        (2+3*alpha(j)*XGmgc(j))/(XGmgc(j)*(1+3*alpha(j)))*(1+(gc+gc*lm-
        lm)/(3*alpha(j)*(2*lm+1)));%Special Case
    else
        mu_tcGm(j) =
        (((lm/(1+lm)/gmm*(3*alpha(j)+1)+3*alpha(j)/gmm)*((2+3*alpha(j))*XGm(j)-
        2)/(2*XGm(j)*(1+3*alpha(j)))-(1+3*alpha(j)/2-
        1/XGm(j))/(1+3*alpha(j)))/((lm/(1+lm)/gmm*3/2*alpha(j)+3*alpha(j)/(2*g
        mm)));
        %mu_tcGmgc(j) =
        (((lm/(1+lm)/gc*(3*alpha(j)+1)+3*alpha(j)/gc)*((2+3*alpha(j))*XGmgc(j)-
        2)/(2*XGmgc(j)*(1+3*alpha(j)))-(1+3*alpha(j)/2-
        1/XGmgc(j))/(1+3*alpha(j)))/((lm/(1+lm)/gc*3/2*alpha(j)+3*alpha(j)/(2*
        gc))));%Special Case
    end

    mu_tcLbar(j) = (1/2)*(mu_tcLp(j) + mu_tcLm(j));
    mu_tcGbar(j) = (1/2)*(mu_tcGp(j) + mu_tcGm(j));

    Vmu_tcL(j) = (mu_tcLp(j) - mu_tcLm(j))/(mu_tcLp(j) +
    mu_tcLm(j));

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Vmu_tcG(j) = (mu_tcGp(j) - mu_tcGm(j))/(mu_tcGp(j) +
mu_tcGm(j));

Ymu_tc_mean(j) =
mu_tc0(j)*((mu_tc_Lbar(j)/mu_tc0(j))*(mu_tc_Gbar(j)/mu_tc0(j)));

%By Definision: Coeffienct of Variation = Std/mean
Vmu_tc(j) = sqrt((1+(Vmu_tcL(j))^2)*(1+(Vmu_tcG(j))^2)-1);
STDmu_tc(j) = Vmu_tc(j)*Ymu_tc_mean(j);

%-----
%Special Case For Identical Plate Thickness: gam(i) = gc
%mu_tc_Lbargc(j) = (1/2)*(mu_tcLpgc(j) + mu_tcLmgc(j));
%mu_tc_Gbargc(j) = (1/2)*(mu_tcGpgc(j) + mu_tcGmgc(j));

%Vmu_tcLgc(j) = (mu_tcLpgc(j) - mu_tcLmgc(j))/(mu_tcLpgc(j) +
mu_tcLmgc(j));
%Vmu_tcGgc(j) = (mu_tcGpgc(j) - mu_tcGmgc(j))/(mu_tcGpgc(j) +
mu_tcGmgc(j));

%Ymu_tc_meangc(j) =
mu_tc0gc(j)*((mu_tc_Lbargc(j)/mu_tc0gc(j))*(mu_tc_Gbargc(j)/mu_tc0gc(j)
));
%-----

%+++++
++
%Generate DCFc Statistical Values
if gm > lm/(1+lm)
    DCFc0(j) = 1+(1-
2*lm/(gm*(1+lm))*(1/X0(j)+3*alpha(j)/2)/(1+3*alpha(j))/mu_tc0(j))/(1+3*
alpha(j)*lm/(gm*(1+lm))+3*alpha(j)/gm);
    %DCFc1(j) = DCFc11(j)=
(2+3*alpha(j)*lm/(gm*(1+lm))+3*alpha(j)/gm-
2*lm/(gm*(1+lm))*(1/X0(j)+3*alpha(j)/2)/(1+3*alpha(j))*(1/(mu_tc0(j))))
/(1+3*lm*alpha(j)/(gm*(1+lm))+3*alpha(j)/gm);
    %DCFcgc0(j) = 1+(1-
2*lm/(gc*(1+lm))*(1/X0gc(j)+3*alpha(j)/2)/(1+3*alpha(j))/mu_tc0(j))/(1+
3*alpha(j)*lm/(gm*(1+lm))+3*alpha(j)/gm);%Special Case
else
    DCFc0(j) = ((lm/(gm*(1+lm))*(3*alpha(j)+2))+3*alpha(j)/gm -
(2+3*alpha(j)-
2/X0(j))/(mu_tc0(j)*(1+3*alpha(j))))/((lm/(1+lm)/gm*(1+3*alpha(j)))+3*a
lpha(j)/gm);
end

if gm > lmp/(1+lmp)
    DCFcLp(j) = 1+(1-
2*lmp/(gm*(1+lmp))*(1/XLp(j)+3*alpha(j)/2)/(1+3*alpha(j))/mu_tcLp(j))/(
1+3*alpha(j)*lmp/(gm*(1+lmp))+3*alpha(j)/gm);
else
    DCFcLp(j) =
((lmp/(gm*(1+lmp))*(3*alpha(j)+2))+3*alpha(j)/gm - (2+3*alpha(j)-
2/XLp(j))/(mu_tcLp(j)*(1+3*alpha(j))))/((lmp/(1+lmp)/gm*(1+3*alpha(j))
)+3*alpha(j)/gm);

```

```

end

if gm > lmm/(1+lmm)
    DCFcLm(j) = 1+(1-
2*lmm/(gm*(1+lmm))*(1/XLm(j)+3*alpha(j)/2)/(1+3*alpha(j))/mu_tcLm(j))/(
1+3*alpha(j)*lmm/(gm*(1+lmm))+3*alpha(j)/gm);
    %DCFcLm(j) = (2+3*alpha(j)*lmm/(gm*(1+lmm))+3*alpha(j)/gm-
2*lmm/(gm*(1+lmm))*(1/XLm(j)+3*alpha(j)/2)/(1+3*alpha(j))*(1/mu_tcLm(j)
))/ (1+3*alpha(j)*lmm/(gm*(1+lmm))+3*alpha(j)/gm); %Same Equation as
above
else
    DCFcLm(j) =
((lmm/(gm*(1+lmm))*(3*alpha(j)+2))+3*alpha(j)/gm - (2+3*alpha(j)-
2/XLm(j)))/(mu_tcLm(j)*(1+3*alpha(j))))/( (lmm/(1+lmm)/gm*(1+3*alpha(j)))
+3*alpha(j)/gm);
    %DCFcLm(j) =
(lmm/(1+lmm)*(1/gm)*(3*alpha(j)+2)+3*alpha(j)/gm-
1/(3*alpha(j)+1)*(2+3*alpha(j)-
2/XLm(j)))/(1/mu_tcLm(j))/(lmm/(1+lmm)*(1/gm)*(3*alpha(j)+1)+3*alpha(j)/
gm));
end

if gmp > lm/(1+lm)
    DCFcGp(j) = 1+(1-
2*lm/(gmp*(1+lm))*(1/XGp(j)+3*alpha(j)/2)/(1+3*alpha(j))/mu_tcGp(j))/(1
+3*alpha(j)*lm/(gmp*(1+lm))+3*alpha(j)/gmp);
else
    DCFcGp(j) =
((lm/(gmp*(1+lm))*(3*alpha(j)+2))+3*alpha(j)/gmp - (2+3*alpha(j)-
2/XGm(j)))/(mu_tcGp(j)*(1+3*alpha(j))))/( (lm/(1+lm)/gmp*(1+3*alpha(j)))
+3*alpha(j)/gmp);
end

if gmm > lm/(1+lm)
    DCFcGm(j) = 1+(1-
2*lm/(gmm*(1+lm))*(1/XGm(j)+3*alpha(j)/2)/(1+3*alpha(j))/mu_tcGm(j))/(1
+3*alpha(j)*lm/(gmm*(1+lm))+3*alpha(j)/gmm);
else
    DCFcGm(j) =
((lm/(gmm*(1+lm))*(3*alpha(j)+2))+3*alpha(j)/gmm - (2+3*alpha(j)-
2/XGm(j)))/(mu_tcGm(j)*(1+3*alpha(j))))/( (lm/(1+lm)/gmm*(1+3*alpha(j)))
+3*alpha(j)/gmm);
end

DCFc_Lbar(j) = (1/2)*(DCFcLp(j) + DCFcLm(j));
DCFc_Gbar(j) = (1/2)*(DCFcGp(j) + DCFcGm(j));

VDCFcL(j) = (DCFcLp(j) - DCFcLm(j))/(DCFcLp(j) + DCFcLm(j));
VDCFcG(j) = (DCFcGp(j) - DCFcGm(j))/(DCFcGp(j) + DCFcGm(j));

YDCFc_mean(j) =
DCFc0(j)*((DCFc_Lbar(j)/DCFc0(j))*(DCFc_Gbar(j)/DCFc0(j)));

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%By Definision: Coeffienct of Variation = Std/mean
VDCFc(j) = sqrt((1+(VDCFcL(j))^2)*(1+(VDCFcG(j))^2)-1);
STDDCFc(j) = VDCFc(j)*YDCFc_mean(j);

%+++++
++
%Generate mo_tc Statistical Values
    if gm > lm/(1+lm)
        mu_oc0(j) =
DCFc0(j)/2*mu_tc0(j)/(1/(X0(j)*(1+3*alpha(j)))+3*alpha(j)/(2*(1+3*alpha
(j))));
    else
        mu_oc0(j) = DCFc0(j)/2*mu_tc0(j)/(1-
(1/(X0(j)*(1+3*alpha(j)))+3*alpha(j)/(2*(1+3*alpha(j)))));
    end

    if gm > lmp/(1+lmp)
        mu_ocLp(j) =
DCFcLp(j)/2*mu_tcLp(j)/(1/(XLp(j)*(1+3*alpha(j)))+3*alpha(j)/(2*(1+3*al
pha(j))));
    else
        mu_ocLp(j) = DCFcLp(j)/2*mu_tcLp(j)/(1-
(1/(XLp(j)*(1+3*alpha(j)))+3*alpha(j)/(2*(1+3*alpha(j)))));
    end

    if gm > lmm/(1+lmm)
        mu_ocLm(j) =
DCFcLm(j)/2*mu_tcLm(j)/(1/(XLm(j)*(1+3*alpha(j)))+3*alpha(j)/(2*(1+3*al
pha(j))));
    else
        mu_ocLm(j) = DCFcLm(j)/2*mu_tcLm(j)/(1-
(1/(XLm(j)*(1+3*alpha(j)))+3*alpha(j)/(2*(1+3*alpha(j)))));
    end

    if gmp > lm/(1+lm)
        mu_ocGp(j) =
DCFcGp(j)/2*mu_tcGp(j)/(1/(XGp(j)*(1+3*alpha(j)))+3*alpha(j)/(2*(1+3*al
pha(j))));
    else
        mu_ocGp(j) = DCFcGp(j)/2*mu_tcGp(j)/(1-
(1/(XGp(j)*(1+3*alpha(j)))+3*alpha(j)/(2*(1+3*alpha(j)))));
    end

    if gmm > lm/(1+lm)
        mu_ocGm(j) =
DCFcGm(j)/2*mu_tcGm(j)/(1/(XGm(j)*(1+3*alpha(j)))+3*alpha(j)/(2*(1+3*al
pha(j))));
    else
        mu_ocGm(j) = DCFcGm(j)/2*mu_tcGm(j)/(1-
(1/(XGm(j)*(1+3*alpha(j)))+3*alpha(j)/(2*(1+3*alpha(j)))));
    end

mu_oc_Lbar(j) = (1/2)*(mu_ocLp(j) + mu_ocLm(j));

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mu_oc_Gbar(j) = (1/2)*(mu_ocGp(j) + mu_ocGm(j));

Vmu_ocL(j) = (mu_ocLp(j) - mu_ocLm(j))/(mu_ocLp(j) +
mu_ocLm(j));
Vmu_ocG(j) = (mu_ocGp(j) - mu_ocGm(j))/(mu_ocGp(j) +
mu_ocGm(j));

Ymu_oc_mean(j) =
mu_oc0(j)*((mu_oc_Lbar(j)/mu_oc0(j))*(mu_oc_Gbar(j)/mu_oc0(j)));

%By Definision: Coeffienct of Variation = Std/mean
Vmu_oc(j) = sqrt((1+(Vmu_ocL(j))^2)*(1+(Vmu_ocG(j))^2)-1);
STDmu_oc(j) = Vmu_oc(j)*Ymu_oc_mean(j);

%+++++
++
%Generate Normalized Moment, Mc's, Statistical Values
if gm > lm/(1+lm)
    Mcc0(j) = 3/2*alpha(j)*mu_tc0(j)*(DCFc0(j)-
1)/(1/(X0(j)*(1+3*alpha(j)))+3*alpha(j)/(2*(1+3*alpha(j)))));
else
    Mcc0(j) = 3/2*alpha(j)*mu_tc0(j)*(DCFc0(j)-1)/(1-
(1/(X0(j)*(1+3*alpha(j)))+3*alpha(j)/(2*(1+3*alpha(j)))));
end

if gm > lmp/(1+lmp)
    MccLp(j) = 3/2*alpha(j)*mu_tcLp(j)*(DCFcLp(j)-
1)/(1/(XLp(j)*(1+3*alpha(j)))+3*alpha(j)/(2*(1+3*alpha(j)))));
else
    MccLp(j) = 3/2*alpha(j)*mu_tcLp(j)*(DCFcLp(j)-1)/(1-
(1/(XLp(j)*(1+3*alpha(j)))+3*alpha(j)/(2*(1+3*alpha(j)))));
end

if gm > lmm/(1+lmm)
    MccLm(j) = 3/2*alpha(j)*mu_tcLm(j)*(DCFcLm(j)-
1)/(1/(XLm(j)*(1+3*alpha(j)))+3*alpha(j)/(2*(1+3*alpha(j)))));
else
    MccLm(j) = 3/2*alpha(j)*mu_tcLm(j)*(DCFcLm(j)-1)/(1-
(1/(XLm(j)*(1+3*alpha(j)))+3*alpha(j)/(2*(1+3*alpha(j)))));
end

if gmp > lm/(1+lm)
    MccGp(j) = 3/2*alpha(j)*mu_tcGp(j)*(DCFcGp(j)-
1)/(1/(XGp(j)*(1+3*alpha(j)))+3*alpha(j)/(2*(1+3*alpha(j)))));
else
    MccGp(j) = 3/2*alpha(j)*mu_tcGp(j)*(DCFcGp(j)-1)/(1-
(1/(XGp(j)*(1+3*alpha(j)))+3*alpha(j)/(2*(1+3*alpha(j)))));
end

if gmm > lm/(1+lm)
    MccGm(j) = 3/2*alpha(j)*mu_tcGm(j)*(DCFcGm(j)-
1)/(1/(XGm(j)*(1+3*alpha(j)))+3*alpha(j)/(2*(1+3*alpha(j)))));
else

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        MccGm(j) = 3/2*alpha(j)*mu_tccGm(j)*(DCFcGm(j)-1)/(1-
(1/(XGm(j)*(1+3*alpha(j)))+3*alpha(j)/(2*(1+3*alpha(j)))));
    end

    Mcc_Lbar(j) = (1/2)*(MccLp(j) + MccLm(j));
    Mcc_Gbar(j) = (1/2)*(MccGp(j) + MccGm(j));

    VMccL(j) = (MccLp(j) - MccLm(j))/(MccLp(j) + MccLm(j));
    VMccG(j) = (MccGp(j) - MccGm(j))/(MccGp(j) + MccGm(j));

    YMcc_mean(j) =
Mcc0(j)*((Mcc_Lbar(j)/Mcc0(j))*(Mcc_Gbar(j)/Mcc0(j)));

    %By Definision: Coeffienct of Variation = Std/mean
    VMcc(j) = sqrt((1+(VMccL(j))^2)*(1+(VMccG(j))^2)-1);
    STDMcc(j) = VMcc(j)*YMcc_mean(j);

%-----
-
%+++++
+
    %Special Case For Identical Plate Thickness:
%+++++
+
%-----
-

    %%%%%%%%%Substitute gm, gmm and gmp for "gc"

    X0gc(j) =
(lm/(gc*(1+lm))+(1+3*alpha(j)/gc)/(1+3*alpha(j)))/(1+3*alpha(j)/(2*gc)-
3*alpha(j)/2*((3*alpha(j)/gc+1)/(1+3*alpha(j))));
    XLpgc(j) =
(lmp/(gc*(1+lmp))+(1+3*alpha(j)/gc)/(1+3*alpha(j)))/(1+3*alpha(j)/(2*gc
)-3*alpha(j)/2*((3*alpha(j)/gc+1)/(1+3*alpha(j))));
    XLmgc(j) =
(lmm/(gc*(1+lmm))+(1+3*alpha(j)/gc)/(1+3*alpha(j)))/(1+3*alpha(j)/(2*gc
)-3*alpha(j)/2*((3*alpha(j)/gc+1)/(1+3*alpha(j))));
    XLbargc(j) = (1/2)*(XLpgc(j) + XLmgc(j));

    VxLgc(j) = (XLpgc(j) - XLmgc(j))/(XLpgc(j) + XLmgc(j));

%+++++
+
    %Generate DCFc Statistical Values
    if gc > lm/(1+lm)
        mu_tcc0gc(j) =
(2+3*alpha(j)*X0gc(j))/(X0gc(j)*(1+3*alpha(j)))*(1+(gc+gc*lm-
lm)/(3*alpha(j)*(2*lm+1)));

    else
        mu_tcc0gc(j) =
(((lm/(1+lm)/gc*(3*alpha(j)+1)+3*alpha(j)/gc)*((2+3*alpha(j))*X0gc(j)-
2)/(2*X0gc(j)*(1+3*alpha(j)))-(1+3*alpha(j)/2-

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```

1/X0gc(j))/(1+3*alpha(j)))/((lm/(1+lm)/gc*3/2*alpha(j)+3*alpha(j)/(2*gc)));
    end

    if gc > lmp/(1+lmp)
        mu_tcLpgc(j) =
(2+3*alpha(j)*XLpgc(j))/(XLpgc(j)*(1+3*alpha(j)))*(1+(gc+gc*lmp-
lmp)/(3*alpha(j)*(2*lmp+1)));
    else
        mu_tcLpgc(j) =
(((lmp/(1+lmp)/gc*(3*alpha(j)+1)+3*alpha(j)/gc)*((2+3*alpha(j))*XLpgc(j)
)-2)/(2*XLpgc(j)*(1+3*alpha(j)))-(1+3*alpha(j)/2-
1/XLpgc(j))/(1+3*alpha(j)))/((lmp/(1+lmp)/gc*3/2*alpha(j)+3*alpha(j)/(
2*gc)));
    end

    if gc > lmm/(1+lmm)
        mu_tCLmgc(j) =
(2+3*alpha(j)*XLmgc(j))/(XLmgc(j)*(1+3*alpha(j)))*(1+(gc+gc*lmm-
lmm)/(3*alpha(j)*(2*lmm+1)));
    else
        mu_tCLmgc(j) =
(((lmm/(1+lmm)/gc*(3*alpha(j)+1)+3*alpha(j)/gc)*((2+3*alpha(j))*XLmgc(j)
)-2)/(2*XLmgc(j)*(1+3*alpha(j)))-(1+3*alpha(j)/2-
1/XLmgc(j))/(1+3*alpha(j)))/((lmm/(1+lmm)/gc*3/2*alpha(j)+3*alpha(j)/(
2*gc)));
    end

    mu_tc_Lbargc(j) = (1/2)*(mu_tcLpgc(j) + mu_tCLmgc(j));

    Vmu_tCLgc(j) = (mu_tcLpgc(j) - mu_tCLmgc(j))/(mu_tcLpgc(j) +
mu_tCLmgc(j));

    Ymu_tc_meangc(j) = mu_tc0gc(j)*((mu_tc_Lbargc(j)/mu_tc0gc(j)));

    %By Definision: Coeffienct of Variation = Std/mean
    Vmu_tcgc(j) = sqrt((1+(Vmu_tCL(j))^2)-1);
    STDmu_tcgc(j) = Vmu_tcgc(j)*Ymu_tc_meangc(j);

%+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
+
    %Generate DCFc Statistical Values
    if gc > lm/(1+lm)
        DCFc0gc(j) = 1+(1-
2*lm/(gc*(1+lm))*(1/X0gc(j)+3*alpha(j)/2)/(1+3*alpha(j))/mu_tc0gc(j))/(
1+3*alpha(j)*lm/(gc*(1+lm))+3*alpha(j)/gc);
    else
        DCFc0gc(j) = ((lm/(gc*(1+lm))*(3*alpha(j)+2))+3*alpha(j)/gc
-(2+3*alpha(j)-
2/X0gc(j))/(mu_tc0gc(j)*(1+3*alpha(j)))/((lm/(1+lm)/gc*(1+3*alpha(j))
+3*alpha(j)/gc);
    end

```

```

        if gc > lmp/(1+lmp)
            DCFcLpgc(j) = 1+(1-
2*lmp/(gc*(1+lmp)))*(1/XLpgc(j)+3*alpha(j)/2)/(1+3*alpha(j))/mu_tcLpgc(j)
)/ (1+3*alpha(j)*lmp/(gc*(1+lmp))+3*alpha(j)/gc);
        else
            DCFcLpgc(j) =
((lmp/(gc*(1+lmp)))*(3*alpha(j)+2))+3*alpha(j)/gc -(2+3*alpha(j)-
2/XLpgc(j))/(mu_tcLpgc(j)*(1+3*alpha(j)))/((lmp/(1+lmp)/gc*(1+3*alpha(
j)))+3*alpha(j)/gc);
        end

        if gc > lmm/(1+lmm)
            DCFcLmgc(j) = 1+(1-
2*lmm/(gc*(1+lmm)))*(1/XLmgc(j)+3*alpha(j)/2)/(1+3*alpha(j))/mu_tcLmgc(j)
)/ (1+3*alpha(j)*lmm/(gc*(1+lmm))+3*alpha(j)/gc);
        else
            DCFcLmgc(j) =
((lmm/(gc*(1+lmm)))*(3*alpha(j)+2))+3*alpha(j)/gc -(2+3*alpha(j)-
2/XLmgc(j))/(mu_tcLmgc(j)*(1+3*alpha(j)))/((lmm/(1+lmm)/gc*(1+3*alpha(
j)))+3*alpha(j)/gc);
        end

        DCFc_Lbargc(j) = (1/2)*(DCFcLpgc(j) + DCFcLmgc(j));

        VDCFcLgc(j) = (DCFcLpgc(j) - DCFcLmgc(j))/(DCFcLpgc(j) +
DCFcLmgc(j));

        YDCFc_meangc(j) = DCFc0gc(j)*((DCFc_Lbargc(j)/DCFc0gc(j)));

        %By Definision: Coeffienct of Variation = Std/mean
        VDCFcgc(j) = sqrt((1+(VDCFcLgc(j))^2)-1);
        STDDCFcgc(j) = VDCFcgc(j)*YDCFc_meangc(j);

%+++++
+
        %Generate mo_tc Statistical Values
        if gc > lm/(1+lm)
            mu_oc0gc(j) =
DCFc0gc(j)/2*mu_tc0gc(j)/(1/(X0gc(j)*(1+3*alpha(j)))+3*alpha(j)/(2*(1+3
*alpha(j))));
        else
            mu_oc0gc(j) = DCFc0(j)/2*mu_tc0gc(j)/(1-
(1/(X0gc(j)*(1+3*alpha(j)))+3*alpha(j)/(2*(1+3*alpha(j)))));
        end

        if gc > lmp/(1+lmp)
            mu_ocLpgc(j) =
DCFcLpgc(j)/2*mu_tcLpgc(j)/(1/(XLpgc(j)*(1+3*alpha(j)))+3*alpha(j)/(2*(
1+3*alpha(j))));
        else
            mu_ocLpgc(j) = DCFcLpgc(j)/2*mu_tcLpgc(j)/(1-
(1/(XLpgc(j)*(1+3*alpha(j)))+3*alpha(j)/(2*(1+3*alpha(j)))));

```

```

end

if gc > lmm/(1+lmm)
    mu_ocLmgc(j) =
    DCFcLmgc(j)/2*mu_tcLmgc(j)/(1/(XLmgc(j)*(1+3*alpha(j)))+3*alpha(j)/(2*(
    1+3*alpha(j))));
else
    mu_ocLmgc(j) = DCFcLmgc(j)/2*mu_tcLmgc(j)/(1-
    (1/(XLmgc(j)*(1+3*alpha(j)))+3*alpha(j)/(2*(1+3*alpha(j)))));
end

mu_oc_Lbargc(j) = (1/2)*(mu_ocLpgc(j) + mu_ocLmgc(j));

Vmu_ocLgc(j) = (mu_ocLpgc(j) - mu_ocLmgc(j))/(mu_ocLpgc(j) +
mu_ocLmgc(j));

Ymu_oc_meangc(j) = mu_oc0gc(j)*((mu_oc_Lbargc(j)/mu_oc0gc(j)));

%By Definision: Coeffienct of Variation = Std/mean
Vmu_ocgc(j) = sqrt((1+(Vmu_ocLgc(j))^2)-1);
STDmu_ocgc(j) = Vmu_ocgc(j)*Ymu_oc_meangc(j);

%+++++
+
%Generate Normalized Moment, Mc's, Statistical Values
if gc > lm/(1+lm)
    Mcc0gc(j) = 3/2*alpha(j)*mu_tc0gc(j)*(DCFc0gc(j)-
    1)/(1/(X0gc(j)*(1+3*alpha(j)))+3*alpha(j)/(2*(1+3*alpha(j))));
else
    Mcc0gc(j) = 3/2*alpha(j)*mu_tc0gc(j)*(DCFc0gc(j)-1)/(1-
    (1/(X0gc(j)*(1+3*alpha(j)))+3*alpha(j)/(2*(1+3*alpha(j)))));
end

if gc > lmp/(1+lmp)
    MccLpgc(j) = 3/2*alpha(j)*mu_tcLpgc(j)*(DCFcLpgc(j)-
    1)/(1/(XLpgc(j)*(1+3*alpha(j)))+3*alpha(j)/(2*(1+3*alpha(j))));
else
    MccLpgc(j) = 3/2*alpha(j)*mu_tcLpgc(j)*(DCFcLpgc(j)-1)/(1-
    (1/(XLpgc(j)*(1+3*alpha(j)))+3*alpha(j)/(2*(1+3*alpha(j)))));
end

if gc > lmm/(1+lmm)
    MccLmgc(j) = 3/2*alpha(j)*mu_tcLmgc(j)*(DCFcLmgc(j)-
    1)/(1/(XLmgc(j)*(1+3*alpha(j)))+3*alpha(j)/(2*(1+3*alpha(j))));
else
    MccLmgc(j) = 3/2*alpha(j)*mu_tcLmgc(j)*(DCFcLmgc(j)-1)/(1-
    (1/(XLmgc(j)*(1+3*alpha(j)))+3*alpha(j)/(2*(1+3*alpha(j)))));
end

```

```

Mcc_Lbargc(j) = (1/2)*(MccLpgc(j) + MccLmgc(j));
VMccLgc(j) = (MccLp(j) - MccLmgc(j))/(MccLpgc(j) + MccLmgc(j));
YMcc_meangc(j) = Mcc0gc(j)*((Mcc_Lbargc(j)/Mcc0gc(j)));

%By Definision: Coeffienct of Variation = Std/mean
VMccgc(j) = sqrt((1+(VMccLgc(j))^2)-1);
STDMccgc(j) = VMccgc(j)*YMcc_meangc(j);

%+++++
+

%+++++
+

    %Probability of Failure
    Pf_mu_tc_PerAlpha(j) = 1-
normcdf(cp(1),Ymu_tc_mean(j),STDmu_tc(j));
    Pf_DCFc_PerAlpha(j) = 1-
normcdf(cp(2),YDCFc_mean(j),STDDCFc(j));
    Pf_mu_oc_PerAlpha(j) = 1-
normcdf(cp(3),Ymu_oc_mean(j),STDmu_oc(j));
    Pf_Mcc_PerAlpha(j) = 1-
normcdf(cp(4),YMcc_mean(j),STDMcc(j));

    %Special Case For Identical Plate Thickness:
    Pf_mu_tcgc_PerAlpha(j) = 1-
normcdf(cp(1),Ymu_tc_meangc(j),STDmu_tcgc(j));
    Pf_DCFcgc_PerAlpha(j) = 1-
normcdf(cp(2),YDCFc_meangc(j),STDDCFcgc(j));
    Pf_mu_ocgc_PerAlpha(j) = 1-
normcdf(cp(3),Ymu_oc_meangc(j),STDmu_ocgc(j));
    Pf_Mccgc_PerAlpha(j) = 1-
normcdf(cp(4),YMcc_meangc(j),STDMccgc(j));

end

    %Assembling Of Probabilities Of Failure Sets
    Pfs = zeros(alphaprop(3),size(cp,2)*2);
    for k = 1:alphaprop(3)
        Pfs(k,1) = Pf_mu_tc_PerAlpha(k);%%%Pf_mu_tc
semilogx()
        Pfs(k,2) = Pf_DCFc_PerAlpha(k);
        Pfs(k,3) = Pf_mu_oc_PerAlpha(k);
        Pfs(k,4) = Pf_Mcc_PerAlpha(k);
        %Special Case For Identical Plate Thickness:
        Pfs(k,5) = Pf_mu_tcgc_PerAlpha(k);
        Pfs(k,6) = Pf_DCFcgc_PerAlpha(k);
        Pfs(k,7) = Pf_mu_ocgc_PerAlpha(k);
        Pfs(k,8) = Pf_Mccgc_PerAlpha(k);
    end
end

```

```

%Collecting Results from The Calculations
valss = zeros(alphaprop(3),size(cp,2)+5);
for w = 1:alphaprop(3)
    valss(w,1) = X0(w);
    valss(w,2) = mu_tc0(w);
    valss(w,3) = DCFc0(w);
    valss(w,4) = mu_oc0(w);
    valss(w,5) = Mcc0(w);
    valss(w,6) = lmp;
    valss(w,7) = lmm;
    valss(w,8) = gmp;
    valss(w,9) = gmm;
end
vals(:, :, j) = valss;

%%=====

%++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
%If Number is Other than 1,2,3 for "methodd" in "inputt.m"
file

%++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
else

disp('=====')
disp('Please Verify Your Input Values for "Methodd" in
"inputt.m"...')
disp('-----')
')
end

end%%This Function Ends Here.....

```

Sub-Appendix d The "ouputt.m" function

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%M.C.,L.H.& 2K+1 Simulation Matlab Codes for Guo's Thesis
%This is the OUTPUT Function
%Written by Guo, XuHua (Gary); Last Date Modified: Mar-07-11
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

function [st] =
ouputt(filnam,stat,Pfs,alphaprop,gammaprop,cp,alpha,rn,vals,lamdaprop,
fylnam)

%Compute array sizes
numbiter = stat(1);
q=1;
qq=0;
qqq=0;

%Open file
fid = fopen(filnam,'w');

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
fprintf(fid,'-----
----- \n');
fprintf(fid,'This File Contains Matlab Data Outputs from Monte Carlo
Simulations for Guo"s Thesis \n');
fprintf(fid,'-----
----- \n');
fprintf(fid,' \n');
fprintf(fid,' \n');

fprintf(fid,'+++++
+++++ \n');
    if stat(3) == 1
        fprintf(fid,'.....USING Monte Carlo (M.C.)
Simulation Method Here..... \n');
    elseif stat(3) == 2
        fprintf(fid,'.....USING Latin Hypercube (L.H.)
Simulation Method Here..... \n');
    elseif stat(3) == 3
        fprintf(fid,'.....USING Rosenblues (2K+1)
Simulation Method Here..... \n');
    else
        fprintf(fid,'.....Something is Wrong in the "inpuutt.m" code,
check input value for "methodd".... \n');
        pause
    end
fprintf(fid,'+++++
+++++ \n');
fprintf(fid,' \n');
fprintf(fid,' \n');
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```

if (stat(3) == 1)||(stat(3) == 3)
    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    fprintf(fid,'I n p u t / O u t p u t   P a r a m e t e r s \n\n');
    fprintf(fid,'          Error          Alpha          Mean          Mean
St.Dev      ST.Dev          Mean          \n');
    fprintf(fid,'          Tolerance  Resolution          Lamda          Gamma
Lamda      Gamma          Category          \n');
    fprintf(fid,'-----
----- \n');
    fprintf(fid,'          %2.2f          %2.2f          %2.2f          %2.2f
%2.2f          %2.2f          %2.2f
\n',stat(2),alphaprop(3),mean(rn(:,1)),mean(rn(:,2)),std(rn(:,1)),std(r
n(:,2)),mean(rn(:,3)));
    fprintf(fid,'          \n');
    fprintf(fid,'          \n');
    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
else
    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    fprintf(fid,'I n p u t / O u t p u t   P a r a m e t e r s \n\n');
    fprintf(fid,'          Error[%]          Alpha          Mean          Mean
St.Dev      ST.Dev          Mean          Numb.Of          \n');
    fprintf(fid,'          Tolerance  Resolution          Lamda          Gamma
Lamda      Gamma          Category          Stranta          \n');
    fprintf(fid,'-----
----- \n');
    fprintf(fid,'          %2.2f          %2.2f          %2.2f          %2.2f
%2.2f          %2.2f          %5.2f          %8i
\n',stat(2),alphaprop(3),mean(rn(:,1)),mean(rn(:,2)),std(rn(:,1)),std(r
n(:,2)),mean(rn(:,3)),stat(1));
    fprintf(fid,'          \n');
    fprintf(fid,'          \n');
    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
fprintf(fid,'L a m d a ( L a t e r a l   F o r c e   R a t i o )   P r o p e
r t i e s\n\n');
fprintf(fid,' Lamda          Lamda          Lamda          Lamda
Lamda          \n');
fprintf(fid,' Range          Average          Stand.Dev          UpperBound
LowerBound          \n');
fprintf(fid,'-----
----- \n');
fprintf(fid,' %6.2f          %6.2f          %6.2f          %6.2f
%6.2f
\n',lamdaprop(1),lamdaprop(2),lamdaprop(3),lamdaprop(4),lamdaprop(5));
fprintf(fid,'          \n');
fprintf(fid,'          \n');
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
fprintf(fid,'G a m m a ( T h i c k n e s s   R a t i o )   P r o p e r t i e
s\n\n');
fprintf(fid,' Gamma          Gamma          Gamma          Gamma
Gamma          \n');

```



```

fprintf(fid, ' Range           Average           Stand.Dev           UpperBound
LowerBound   \n');
fprintf(fid, '-----\n');
fprintf(fid, ' %6.2f           %6.2f           %6.2f           %6.2f
%6.2f
\n', gammaprop(1), gammaprop(2), gammaprop(3), gammaprop(4), gammaprop(5));
fprintf(fid, ' \n');
fprintf(fid, ' \n');
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
fprintf(fid, 'P e r f o r m a n c e   L i m i t s (PL)\n\n');
fprintf(fid, ' Roof Ductility           D C F           1stFloor Ductility
Column Moment           \n');
fprintf(fid, '           PL           PL           PL
PL           \n');
fprintf(fid, '-----\n');
fprintf(fid, '----- \n');
fprintf(fid, ' %1.2f           %1.2f           %1.2f
%4.2f \n', cp(1), cp(2), cp(3), cp(4));
fprintf(fid, ' \n');
fprintf(fid, ' \n');
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

if (stat(3) == 1) || (stat(3) == 2)

    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    %Write Variable Values for Probability of Failure
    fprintf(fid, 'P r o b a b i l i t y   o f   F a i l u r e \n\n');
    fprintf(fid, ' Resolution           AlphaValues           Pf(mu_tc)
Pf(DCFc)           Pf(mu_oc)           Pf(Mcc)           Pf(mu_tcg)
Pf(DCFgc)           Pf(mu_ocg)           Pf(Mccg) \n');
    fprintf(fid, '-----\n');
    -----\n');

    for e = 1:alphaprop(3)
        fprintf(fid, ' %6i           %6.4f           %6.4f
%6.4f           %6.4f           %2.4f           %6.4f
%6.4f           %6.4f           %2.4f \n', e, alpha(e), Pfs(e,1),
Pfs(e,2), Pfs(e,3), Pfs(e,4), Pfs(e,5), Pfs(e,6), Pfs(e,7), Pfs(e,8));
    end
    fprintf(fid, ' \r\n');
    fprintf(fid, ' \r\n');
    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

if q==1;

    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    %Write All Random Variables with Their Corresponding Category

```

```

        fprintf(fid, '      #Iteration      Lamda      Gamma
Category      \n');
        fprintf(fid, '-----
-----\n');
        if stat(3) == 1
            for w = 1:numbiter
                fprintf(fid, '      %6i      %6.4f      %6.4f
%6i\n', w, rn(w,1), rn(w,2), rn(w,3));
            end
                fprintf(fid, '      \r\n');
                fprintf(fid, '      \r\n');
            else
                for w = 1:numbiter
                    fprintf(fid, '      %6i      %6.4f      %6.4f
%6i\n', w, rn(w,1), rn(w,2), rn(w,3));
                end
                    fprintf(fid, '      \r\n');
                    fprintf(fid, '      \r\n');
            end
                %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
end

if qq==1
    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    %Write Variable Values X, mu_tc, DCFc, mu_oc, Mc,
    for j = 1:alphaprop(3)
        fprintf(fid, '      \r\n');
        fprintf(fid, '      # Iteration      Chi(X)      mu_tc
DCFc      mu_oc      Mcc
\n');
        fprintf(fid, '-----
-----\n');
        fprintf(fid, 'Outputs of the %2i th Number of Resolution Are
Found Here\n', j);
        fprintf(fid, '-----
-----\n');
        for i = 1:numbiter
            fprintf(fid, '      %6i      %6.4f      %6.4f
%6.4f      %6.4f      %6.4f\n', i, vals(i,1,j),
vals(i,2,j), vals(i,3,j), vals(i,4,j), vals(i,5,j));
        end
            fprintf(fid, '      \r\n');
        end
            fprintf(fid, '      \r\n');
            fprintf(fid, '      \r\n');
            %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

end %for if qq=1

```

```

%%Outputting Mcc only
if qq==1
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Write Variable Values X, mu_tc,DCFc,mu_oc,Mc,
for j = 1:alphaprop(3)
    fprintf(fid,' \r\n');
    fprintf(fid,' # Iteration Mcc
\n');
    fprintf(fid,'-----
-----\n');
    fprintf(fid,'Outputs of the %2i th Number of Resolution Are
Found Here\n',j);
    fprintf(fid,'-----
-----\n');
    for i = 1:numbiter
        fprintf(fid,' %6i %6.4f
\n',i,vals(i,5,j));
    end
    fprintf(fid,' \r\n');
end
fprintf(fid,' \r\n');
fprintf(fid,' \r\n');
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

end %%for if qq=1

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%=====
=====
elseif stat(3) == 3

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Write Variable Values for Probability of Failure
fprintf(fid,'P r o b a b i l i t y o f F a i l u r e \n\n');
fprintf(fid,' Resolution AlphaValues Pf(mu_tc)
Pf(DCFc) Pf(mu_oc) Pf(Mcc) Pf(mu_tcg)
Pf(DCFcg) Pf(mu_ocg) Pf(Mccg) \n');
fprintf(fid,'-----
-----\n');

for e = 1:alphaprop(3)
    fprintf(fid,' %6i %6.4f %6.4f
%6.4f %6.4f %2.4f %6.4f
%6.4f %6.4f %1.4f
\n',e,alpha(e),Pfs(e,1), Pfs(e,2), Pfs(e,3), Pfs(e,4), Pfs(e,5),
Pfs(e,6), Pfs(e,7), Pfs(e,8));
end

fprintf(fid,' \r\n');
fprintf(fid,' \r\n');

```

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Write All Random Variables with Their Corresponding Category
fprintf(fid, 'Values Below Comes From Monte Carlo Simulation for
Just for The Random Number Generation Purposes...\n\n');
fprintf(fid, '      # Iteration      Lamda      Gamma
Category      \n');
fprintf(fid, '-----
-----\n');

for w = 1:alphaprop(3)
    fprintf(fid, '      %6i      %6.4f      %6.4f
%6i\n', w, rn(w,1), rn(w,2), rn(w,3));
end
    fprintf(fid, '      \r\n');
    fprintf(fid, '      \r\n');
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Close File and Check For Write Error
fclose(fid);
[st] = fclose('all');

if st ~= 0
    disp('Error Data Could NOT Be Written To Output "SPSW.txt"
File...');
    disp('Check "runn.m" & "inputt.m" For Error(s)...')
    pause;
end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%~~~~~
%
%===== The Code Below is developed to be
=====
%===== read for "Grapher.txt" and should
=====
%===== be deleted when post in the disseration
=====
%
%~~~~~
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Open file
fid1 = fopen(fylnam, 'w');

fprintf(fid1, '+++++
+++++ \n');
if stat(3) == 1

```

```

        fprintf(fid1,'.....USING Monte Carlo (M.C.)
Simulation Method Here..... \n');
    elseif stat(3) == 2
        fprintf(fid1,'.....USING Latin Hypercube (L.H.)
Simulation Method Here..... \n');
    elseif stat(3) == 3
        fprintf(fid1,'.....USING Rosenblues (2K+1)
Simulation Method Here..... \n');
    else
        fprintf(fid1,'.....Something is Wrong in the "inputt.m" code,
check input value for "methodd".... \n');
        pause
    end
    fprintf(fid1,'+++++ \n');
    fprintf(fid1,' \n');
    fprintf(fid1,' \n');
    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
    if (stat(3) == 1)||(stat(3) == 2)

        %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
        %Write Variable Values for Probability of Failure
        fprintf(fid1,'P r o b a b i l i t y   o f   F a i l u r e \n\n');
        fprintf(fid1,' Resolution           AlphaValues           Pf(mu_tc)
Pf(DCFc)           Pf(mu_oc)           Pf(Mcc)           Pf(mu_tcg)
Pf(DCFcg)           Pf(mu_ocg)           Pf(Mccg) \n');
        fprintf(fid1,'-----\n');
        -----\n');

        for e = 1:alphaprop(3)
            fprintf(fid1,' %6i %6.4f %6.4f
%6.4f %6.4f %2.4f %6.4f
%6.4f %6.4f %2.4f \n',e,alpha(e),Pfs(e,1),
Pfs(e,2), Pfs(e,3), Pfs(e,4),Pfs(e,5),Pfs(e,6),Pfs(e,7),Pfs(e,8));
        end
        fprintf(fid1,' \r\n');
        fprintf(fid1,' \r\n');
        %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

    elseif stat(3) == 3

        %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
        %Write Variable Values for Probability of Failure
        fprintf(fid1,'P r o b a b i l i t y   o f   F a i l u r e \n\n');
        fprintf(fid1,' Resolution           AlphaValues           Pf(mu_tc)
Pf(DCFc)           Pf(mu_oc)           Pf(Mcc)           Pf(mu_tcg)
Pf(DCFcg)           Pf(mu_ocg)           Pf(Mccg) \n');
        fprintf(fid1,'-----\n');
        -----\n');

        for e = 1:alphaprop(3)

```

```

        fprintf(fid1, '    %6i                %6.4f                %6.4f
%6.4f                %6.4f                %2.4f                %6.4f
%6.4f                %6.4f                %1.4f
\n',e,alpha(e),Pfs(e,1), Pfs(e,2), Pfs(e,3), Pfs(e,4), Pfs(e,5),
Pfs(e,6), Pfs(e,7), Pfs(e,8));
    end

    fprintf(fid1, '                                \r\n');
    fprintf(fid1, '                                \r\n');
    %%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

end

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%Close File and Check For Write Error
fclose(fid1);
[st] = fclose('all');

if st ~= 0
    disp('Error Data Could NOT Be Written To Output "SPSW.txt"
File...');
    disp('Check "runn.m" & "inputt.m" For Error(s)...')
    pause;
end

end%%This Function Ends Here.....

```

Sub-Appendix e The "plotn.m" function

```

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%M.C.,L.H.& 2K+1 Simulation Matlab Codes for Guo's Thesis
%This is the PLOT Function
%Written by Guo, XuHua (Gary); Last Date Modified: Mar-07-11
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

function [] = plotn(alphaprop,alpha,Pfs,rn,stat)
q=0;
if q==1
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% This Function Plots The Pf per Alpha for All 4 Targets
figure
clf
box on
hold on
axis([0.001 1000 0 1])

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

for e = 1:alphaprop(3)
    plot(semilogx(alpha(e)),Pfs(e,1),'-mo');
    %semilogx(alpha(e)),Pfs(e,5),'-bx');

    %semilogx(alpha(e)),Pfs(e,2),'--
    .b',semilogx(alpha(e)),Pfs(e,3),'--gs',semilogx(alpha(e)),Pfs(e,4),'--
    rx')

    %legend(4,'mu_tc','DCFc','mu_oc','Mc');
    xlabel('Stiffness Ratio','fontsize',8,'fontweight','b')
    ylabel('Probability of Failure','fontsize',8,'fontweight','b')
    title('Prob. of Failure','fontsize',10,'fontweight','b')
end
end%%end if q=1

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
if (stat(3) == 1)||(stat(3) == 2)
%This Function Plots The Histograms of Gamma and Lamda
figure
clf
box on
hold on
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
subplot 121;hist(rn(:,1),stat(1));
title('Histogram of lamda
Values','fontsize',10,'fontweight','b')
xlabel('Possible Values of
Lamda','fontsize',8,'fontweight','b')
ylabel('Times of Occurrance','fontsize',8,'fontweight','b')

subplot 122;hist(rn(:,2),stat(1));
title('Histogram of Gamma
Values','fontsize',10,'fontweight','b')

```

```

        xlabel('Possible Values of
Gamma','fontsize',8,'fontweight','b')
        ylabel('Times of Occurance','fontsize',8,'fontweight','b')
    end

end%%This Function Ends Here.....

```



```
function [Dn,ln_Dn] = dataval(stat,vals,alphaPROP)

%Compute array sizes
numbiter = stat(1);

if (stat(3) == 1 || stat(3) == 2)

%+_++_++_++_++_++_++_++_++_++_++_++_++_++_++_++_++_++_+
+_+
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%% Normal Probability
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%+_++_++_++_++_++_++_++_++_++_++_++_++_++_++_++_++_++_+
+_+

%Alpha Resolution = j
for j = 1:alphaPROP(3)

%Sort Accending Values
mu_oc = sort(vals(:,4,j));%mu_oc
ave_mu_oc(j,1) = mean(mu_oc);
stdp_mu_oc(j,1) = std(mu_oc);


mu_ocgc = sort(vals(:,9,j));%mu_ocgc
ave_mu_ocgc(j,1) = mean(mu_ocgc);
stdp_mu_ocgc(j,1) = std(mu_ocgc);


for i = 1:numbiter
%Inner loop for Number of Iterations
Sn(i) = i/numbiter;

%Calculate the Probability of Each Function for Each Iteration
P_mu_oc(i) =
normcdf(mu_oc(i),ave_mu_oc(j,1),stdp_mu_oc(j,1));
P_mu_ocgc(i) =
normcdf(mu_ocgc(i),ave_mu_ocgc(j,1),stdp_mu_ocgc(j,1));

%Absolute Difference Btw (Probability of Each Value i
DiffCDFs_mu_oc(i) = abs(P_mu_oc(i)-Sn(i));
DiffCDFs_mu_ocgc(i) = abs(P_mu_ocgc(i)-Sn(i));

end

%Maximum Values of Absolute Differences in All Inner
Iterations:Dn
Dn_mu_oc(j) = max(DiffCDFs_mu_oc);
Dn_mu_ocgc(j) = max(DiffCDFs_mu_ocgc);
```



```
disp(j)

end

elseif stat(3) == 3
    disp('No need of data validtion for the 2K+1 Method...')
    Dn=0;
    ln_Dn=Dn;

else
    disp('Wrong Input...')
    Dn=0;
    ln_Dn=Dn;

end

end
```

Appendix D Primary Matlab Functions Flowcharts

Figure D1 The "inputt" function structure

Figure D2 The "run" function structure for the M.C. method

Figure D3 The "runn" fuction structure for the L.H. method

Figure D4 The "runn" fuction structure for the 2K+1 method

Appendix E Definitions of Important Variables In The Matlab Codes

gammaprop:	γ properties;
cp:	performance limits (performance capacities);
alphaprop:	α properties;
filename:	the output txt data file name;
stat:	statistical parameters;
lamdaprop:	λ properties;
fylnam:	the output txt data file name for graphing;
alpha:	α values based on the specified resolution;
Pfs:	probability of failure (failure rate);
vals:	data base matrix stored for each α value;
rn:	random number generations for γ, λ and their corresponding categories
Dn:	the maximum difference between the simulation cumulative frequencies and the cumulative distribution frequencies of the normal distribution (See Section 5.4.1);
ln_Dn:	the maximum difference between the logarithm simulation cumulative frequencies and the logarithm cumulative distribution frequencies of the lognormal distribution.
st:	error check for the output function;

Appendix F Probability Curves

Figure F 1 Normal and uniform distributions for λ and γ (Combination Number 2 to 7)

Figure F 2 Normal distribution of λ versus normal distribution of γ (Combination Number 8 to 16)

Appendix G Six-story FE Analysis Parameter Tables

Column	1	2	3	4	5	6	7	8	9	10
Row	$\lambda_{\mu 2}(1+\lambda_{\sigma 2})$	$\lambda_{\mu 3}(1+\lambda_{\sigma 3})$	$\lambda_{\mu 4}(1+\lambda_{\sigma 4})$	$\lambda_{\mu 5}(1+\lambda_{\sigma 5})$	$\lambda_{\mu 6}(1+\lambda_{\sigma 6})$	$\gamma_{\mu 2}(1+\gamma_{\sigma 2})$	$\gamma_{\mu 3}(1+\gamma_{\sigma 3})$	$\gamma_{\mu 4}(1+\gamma_{\sigma 4})$	$\gamma_{\mu 5}(1+\gamma_{\sigma 5})$	$\gamma_{\mu 6}(1+\gamma_{\sigma 6})$
1	2.4 kips	2.0 kips	2.0 kips	2.0 kips	2.0 kips	2.0 kips	2.0 kips	2.0 kips	2.0 kips	2.0 kips
2	3.6 kips	3.6 kips	3.0 kips	3.0 kips	3.0 kips	3.0 kips	3.0 kips	3.0 kips	3.0 kips	3.0 kips
3	4.8 kips	4.8 kips	4.8 kips	4.0 kips	4.0 kips	4.0 kips	4.0 kips	4.0 kips	4.0 kips	4.0 kips
4	6.0 kips	6.0 kips	6.0 kips	6.0 kips	5.0 kips	5.0 kips	5.0 kips	5.0 kips	5.0 kips	5.0 kips
5	7.2 kips	7.2 kips	7.2 kips	7.2 kips	7.2 kips	6.0 kips	6.0 kips	6.0 kips	6.0 kips	6.0 kips
6	Strip_2S	3.0854	3.0854	3.0854	3.0854	3.0854	3.1279	3.0854	3.0854	3.0854
7	Strip_3S	2.9312	2.9312	2.9312	2.9312	2.9312	2.9715	3.0158	2.9312	2.9312
8	Strip_4S	2.6870	2.6870	2.6870	2.6870	2.6870	2.7239	2.7646	2.8163	2.6870
9	Strip_5S	2.3288	2.3288	2.3288	2.3288	2.3288	2.3608	2.3961	2.4409	2.5074
10	Strip_6S	1.7995	1.7995	1.7995	1.7995	1.7995	1.8242	1.8514	1.8861	1.9375
11	Continuous Column	One α Resolution Corresponds to One Constant I_c value for These 21 cases here (See)								

Column	11	12	13	14	15	16	17	18	19	20	21
Row	$\lambda_{\mu 2}(1-\lambda_{\sigma 2})$	$\lambda_{\mu 3}(1-\lambda_{\sigma 3})$	$\lambda_{\mu 4}(1-\lambda_{\sigma 4})$	$\lambda_{\mu 5}(1-\lambda_{\sigma 5})$	$\lambda_{\mu 6}(1-\lambda_{\sigma 6})$	$\gamma_{\mu 2}(1-\gamma_{\sigma 2})$	$\gamma_{\mu 3}(1-\gamma_{\sigma 3})$	$\gamma_{\mu 4}(1-\gamma_{\sigma 4})$	$\gamma_{\mu 5}(1-\gamma_{\sigma 5})$	$\gamma_{\mu 6}(1-\gamma_{\sigma 6})$	$(\lambda_{\mu}, \gamma_{\mu})$
1	1.6 kips	2.0 kips	2.0 kips	2.0 kips	2.0 kips	2.0 kips	2.0 kips	2.0 kips	2.0 kips	2.0 kips	2.0 kips
2	2.4 kips	2.4 kips	3.0 kips	3.0 kips	3.0 kips	3.0 kips	3.0 kips	3.0 kips	3.0 kips	3.0 kips	3.0 kips
3	3.2 kips	3.2 kips	3.2 kips	4.0 kips	4.0 kips	4.0 kips	4.0 kips	4.0 kips	4.0 kips	4.0 kips	4.0 kips
4	4.0 kips	4.0 kips	4.0 kips	4.0 kips	5.0 kips	5.0 kips	5.0 kips	5.0 kips	5.0 kips	5.0 kips	5.0 kips
5	4.8 kips	4.8 kips	4.8 kips	4.8 kips	4.8 kips	6.0 kips	6.0 kips	6.0 kips	6.0 kips	6.0 kips	6.0 kips
6	Strip_2S	3.0854	3.0854	3.0854	3.0854	3.0854	3.0430	3.0854	3.0854	3.0854	3.0854
7	Strip_3S	2.9312	2.9312	2.9312	2.9312	2.9312	2.8909	2.8465	2.9312	2.9312	2.9312
8	Strip_4S	2.6870	2.6870	2.6870	2.6870	2.6870	2.6501	2.6094	2.5577	2.6870	2.6870
9	Strip_5S	2.3288	2.3288	2.3288	2.3288	2.3288	2.2968	2.2616	2.2168	2.1502	2.3288
10	Strip_6S	1.7995	1.7995	1.7995	1.7995	1.7995	1.7747	1.7475	1.7129	1.6615	1.5634
11	Continuous Column	One α Resolution Corresponds to One Constant I_c value for These 21 cases here (See)									

Table G 1 Lateral Force and Strip Area Assignments for Model <1>

	Column	1	2	3	4	5	6	7	8	9	10
Row		$\lambda_{\mu 2}(1+\lambda_{\sigma 2})$	$\lambda_{\mu 3}(1+\lambda_{\sigma 3})$	$\lambda_{\mu 4}(1+\lambda_{\sigma 4})$	$\lambda_{\mu 5}(1+\lambda_{\sigma 5})$	$\lambda_{\mu 6}(1+\lambda_{\sigma 6})$	$\gamma_{\mu 2}(1+\gamma_{\sigma 2})$	$\gamma_{\mu 3}(1+\gamma_{\sigma 3})$	$\gamma_{\mu 4}(1+\gamma_{\sigma 4})$	$\gamma_{\mu 5}(1+\gamma_{\sigma 5})$	$\gamma_{\mu 6}(1+\gamma_{\sigma 6})$
1	F_2	2.4 kips	2.0 kips	2.0 kips	2.0 kips	2.0 kips	2.0 kips	2.0 kips	2.0 kips	2.0 kips	2.0 kips
2	F_3	3.6 kips	3.6 kips	3.0 kips	3.0 kips	3.0 kips	3.0 kips	3.0 kips	3.0 kips	3.0 kips	3.0 kips
3	F_4	4.8 kips	4.8 kips	4.8 kips	4.0 kips	4.0 kips	4.0 kips	4.0 kips	4.0 kips	4.0 kips	4.0 kips
4	F_5	6.0 kips	6.0 kips	6.0 kips	6.0 kips	5.0 kips	5.0 kips	5.0 kips	5.0 kips	5.0 kips	5.0 kips
5	F_6	7.2 kips	7.2 kips	7.2 kips	7.2 kips	7.2 kips	6.0 kips	6.0 kips	6.0 kips	6.0 kips	6.0 kips
6	Strip_2S	3.0854	3.0854	3.0854	3.0854	3.0854	3.1279	3.0854	3.0854	3.0854	3.0854
7	Strip_3S	2.9312	2.9312	2.9312	2.9312	2.9312	2.9715	3.0158	2.9312	2.9312	2.9312
8	Strip_4S	2.6870	2.6870	2.6870	2.6870	2.6870	2.7239	2.7646	2.8163	2.6870	2.6870
9	Strip_5S	2.3288	2.3288	2.3288	2.3288	2.3288	2.3608	2.3961	2.4409	2.5074	2.3288
10	Strip_6S	1.7995	1.7995	1.7995	1.7995	1.7995	1.8242	1.8514	1.8861	1.9375	2.0356
11	ContConlumn 1	Depends On The α Resolution (Use Values in the column named "Column 21" of Column moment of inertia for Model <2>)					Depends On The α Resolution (See "Column 6 to "Column 10" of Column moment of inertia for Model <2>)				
12	ContConlumn 2										
13	ContConlumn 3										

	Column	11	12	13	14	15	16	17	18	19	20	21
Row		$\lambda_{\mu 2}(1-\lambda_{\sigma 2})$	$\lambda_{\mu 3}(1-\lambda_{\sigma 3})$	$\lambda_{\mu 4}(1-\lambda_{\sigma 4})$	$\lambda_{\mu 5}(1-\lambda_{\sigma 5})$	$\lambda_{\mu 6}(1-\lambda_{\sigma 6})$	$\gamma_{\mu 2}(1-\gamma_{\sigma 2})$	$\gamma_{\mu 3}(1-\gamma_{\sigma 3})$	$\gamma_{\mu 4}(1-\gamma_{\sigma 4})$	$\gamma_{\mu 5}(1-\gamma_{\sigma 5})$	$\gamma_{\mu 6}(1-\gamma_{\sigma 6})$	$(\lambda_{\mu}, \gamma_{\mu})$
1	F_2	1.6 kips	2.0 kips	2.0 kips	2.0 kips	2.0 kips	2.0 kips	2.0 kips	2.0 kips	2.0 kips	2.0 kips	2.0 kips
2	F_3	2.4 kips	2.4 kips	3.0 kips	3.0 kips	3.0 kips	3.0 kips	3.0 kips	3.0 kips	3.0 kips	3.0 kips	3.0 kips
3	F_4	3.2 kips	3.2 kips	3.2 kips	4.0 kips	4.0 kips	4.0 kips	4.0 kips	4.0 kips	4.0 kips	4.0 kips	4.0 kips
4	F_5	4.0 kips	4.0 kips	4.0 kips	4.0 kips	5.0 kips	5.0 kips	5.0 kips	5.0 kips	5.0 kips	5.0 kips	5.0 kips
5	F_6	4.8 kips	4.8 kips	4.8 kips	4.8 kips	4.8 kips	6.0 kips	6.0 kips	6.0 kips	6.0 kips	6.0 kips	6.0 kips
6	Strip_2S	3.0854	3.0854	3.0854	3.0854	3.0854	3.0430	3.0854	3.0854	3.0854	3.0854	3.0854
7	Strip_3S	2.9312	2.9312	2.9312	2.9312	2.9312	2.8909	2.8465	2.9312	2.9312	2.9312	2.9312
8	Strip_4S	2.6870	2.6870	2.6870	2.6870	2.6870	2.6501	2.6094	2.5577	2.6870	2.6870	2.6870
9	Strip_5S	2.3288	2.3288	2.3288	2.3288	2.3288	2.2968	2.2616	2.2168	2.1502	2.3288	2.3288
10	Strip_6S	1.7995	1.7995	1.7995	1.7995	1.7995	1.7747	1.7475	1.7129	1.6615	1.5634	1.7995
11	ContConlumn 1	Depends On The α Resolution (Use Values in the column named "Column 21" of Column moment of inertia for Model <2>)					Depends On The α Resolution (See and "Column 16" to "Column 20" of Column moment of inertia for Model <2>)					Follow "Column 21"
12	ContConlumn 2											
13	ContConlumn 3											

Table G 2 Lateral Force and Strip Area Assignments for Model <2>

	Column	1	2	3	4	5	6	7
Row		$\lambda_{\mu 2}(1+\lambda_{\sigma 2})$	$\lambda_{\mu 3}(1+\lambda_{\sigma 3})$	$\lambda_{\mu 4}(1+\lambda_{\sigma 4})$	$\lambda_{\mu 5}(1+\lambda_{\sigma 5})$	$\lambda_{\mu 6}(1+\lambda_{\sigma 6})$	$\gamma_{\mu 3}(1+\gamma_{\sigma 3})$	$\gamma_{\mu 5}(1+\gamma_{\sigma 5})$
1	F_2	2.4 kips	2.0 kips	2.0 kips	2.0 kips	2.0 kips	2.0 kips	2.0 kips
2	F_3	3.6 kips	3.6 kips	3.0 kips	3.0 kips	3.0 kips	3.0 kips	3.0 kips
3	F_4	4.8 kips	4.8 kips	4.8 kips	4.0 kips	4.0 kips	4.0 kips	4.0 kips
4	F_5	6.0 kips	6.0 kips	6.0 kips	6.0 kips	5.0 kips	5.0 kips	5.0 kips
5	F_6	7.2 kips	7.2 kips	7.2 kips	7.2 kips	7.2 kips	6.0 kips	6.0 kips
6	Strip_3S	3.0026	3.0026	3.0026	3.0026	3.0026	3.0893	3.0026
7	Strip_5S	2.4723	2.4723	2.4723	2.4723	2.4723	2.4723	2.8178
8	ContConlumn_1	Depends On The α Resolution (Use Values in the column named "Column 15" of Column moment of inertia for Model <3>)					Use I_c values from "Column 6" & "Column 7" of Column moment of inertia for Model <3>	
9	ContConlumn_2							
10	ContConlumn_3							

	Column	8	9	10	11	12	13	14	15
Row		$\lambda_{\mu 2}(1-\lambda_{\sigma 2})$	$\lambda_{\mu 3}(1-\lambda_{\sigma 3})$	$\lambda_{\mu 4}(1-\lambda_{\sigma 4})$	$\lambda_{\mu 5}(1-\lambda_{\sigma 5})$	$\lambda_{\mu 6}(1-\lambda_{\sigma 6})$	$\gamma_{\mu 3}(1-\gamma_{\sigma 3})$	$\gamma_{\mu 5}(1-\gamma_{\sigma 5})$	$(\lambda_{\mu}, \gamma_{\mu})$
1	F_2	1.6 kips	2.0 kips	2.0 kips	2.0 kips	2.0 kips	2.0 kips	2.0 kips	2.0 kips
2	F_3	2.4 kips	2.4 kips	3.0 kips	3.0 kips	3.0 kips	3.0 kips	3.0 kips	3.0 kips
3	F_4	3.2 kips	3.2 kips	3.2 kips	4.0 kips	4.0 kips	4.0 kips	4.0 kips	4.0 kips
4	F_5	4.0 kips	4.0 kips	4.0 kips	4.0 kips	5.0 kips	5.0 kips	5.0 kips	5.0 kips
5	F_6	4.8 kips	4.8 kips	4.8 kips	4.8 kips	4.8 kips	6.0 kips	6.0 kips	6.0 kips
6	Strip_3S	3.0026	3.0026	3.0026	3.0026	3.0026	2.9159	3.0026	3.0026
7	Strip_5S	2.4723	2.4723	2.4723	2.4723	2.4723	2.4723	2.1527	2.4723
8	ContConlumn_1	Depends On The α Resolution (Use Values in the column named "Column 15" of Column moment of inertia for Model <3>)					Use I_c values from "Column 13" & "Column 14" of Column moment of inertia for Model <3>		
9	ContConlumn_2								
10	ContConlumn_3								

Table G 3 Lateral Force and Strip Area Assignments for Model <3>

$\alpha_i,$ $\forall i=[1, 30] \in Z^+$	CASE 1	$\lambda_{\mu 2}(1+\lambda_{\sigma 2})$	Resolution #	α_i	I [in ⁴]
			1	0.00123017	361.54
	CASE 2	$\lambda_{\sigma 2}(1+\lambda_{\sigma 2})$	2	0.00151332	444.76
			3	0.00186165	547.13
	CASE 3	$\lambda_{\mu 4}(1+\lambda_{\sigma 4})$	4	0.00229014	673.06
			5	0.00281727	827.98
	CASE 4	$\lambda_{\mu 5}(1+\lambda_{\sigma 5})$	6	0.00346572	1018.56
			7	0.00426344	1253.01
	CASE 5	$\lambda_{\sigma 6}(1+\lambda_{\sigma 6})$	8	0.00524476	1541.41
			9	0.00645195	1896.20
	CASE 6	$\gamma_{\sigma 2}(1+\gamma_{\sigma 2})$	10	0.00793701	2332.65
			11	0.00976388	2869.57
	CASE 7	$\gamma_{\mu 3}(1+\gamma_{\sigma 3})$	12	0.01201124	3530.06
			13	0.01477589	4342.58
	CASE 8	$\gamma_{\sigma 4}(1+\gamma_{\sigma 4})$	14	0.01817688	5342.11
			15	0.02236068	6571.72
	CASE 9	$\gamma_{\sigma 5}(1+\gamma_{\sigma 5})$	16	0.02750747	8084.34
			17	0.03383891	9945.12
	CASE 10	$\gamma_{\sigma 6}(1+\gamma_{\sigma 6})$	18	0.04162766	12234.21
			19	0.05120916	15050.17
	CASE 11	$\lambda_{\sigma 2}(1-\lambda_{\sigma 2})$	20	0.06299605	18514.29
			21	0.07749595	22775.75
	CASE 12	$\lambda_{\mu 3}(1-\lambda_{\sigma 3})$	22	0.09533331	28018.08
			23	0.11727632	34467.05
	CASE 13	$\lambda_{\mu 4}(1-\lambda_{\sigma 4})$	24	0.14426999	42400.38
			25	0.17747683	52159.74
	CASE 14	$\lambda_{\mu 5}(1-\lambda_{\sigma 5})$	26	0.21832694	64165.43
			27	0.26857959	78934.48
	CASE 15	$\lambda_{\mu 6}(1-\lambda_{\sigma 6})$	28	0.33039896	97102.95
			29	0.40644739	119453.29
	CASE 16	$\gamma_{\mu 2}(1-\gamma_{\sigma 2})$	30	0.5	146948.03
	CASE 17	$\gamma_{\mu 3}(1-\gamma_{\sigma 3})$			
	CASE 18	$\gamma_{\mu 4}(1-\gamma_{\sigma 4})$			
	CASE 19	$\gamma_{\mu 5}(1-\gamma_{\sigma 5})$			
	CASE 20	$\gamma_{\mu 6}(1-\gamma_{\sigma 6})$			
	CASE 21	$(\lambda_{\mu}, \gamma_{\mu})$			

Table G 4 Column Moment of Inertia for Model <1>

Resolution #	alpha	Model#2 Column	Column 21 (in Table 2)	Column 6	Column 7	Column 8	Column 9	Column 10	Column 16	Column 17	Column 18	Column 19	Column 20
1	0.00123	ContColmn_1	361.54	361.54	361.54	361.54	361.54	361.54	361.54	361.54	361.54	361.54	361.54
		ContColmn_2	335.29	339.90	344.97	335.29	335.29	335.29	330.68	325.61	335.29	335.29	335.29
		ContColmn_3	266.39	270.05	274.08	279.21	286.82	266.39	262.73	258.70	253.57	245.96	266.39
2	0.001513	ContColmn_1	444.76	444.76	444.76	444.76	444.76	444.76	444.76	444.76	444.76	444.76	444.76
		ContColmn_2	412.47	418.14	424.37	412.47	412.47	412.47	406.79	400.56	412.47	412.47	412.47
		ContColmn_3	327.71	332.21	337.17	343.47	352.84	327.71	323.20	318.24	311.94	302.57	327.71
3	0.001862	ContColmn_1	547.13	547.13	547.13	547.13	547.13	547.13	547.13	547.13	547.13	547.13	547.13
		ContColmn_2	507.40	514.38	522.05	507.40	507.40	507.40	500.43	492.75	507.40	507.40	507.40
		ContColmn_3	403.13	408.68	414.77	422.53	434.05	403.13	397.59	391.50	383.74	372.21	403.13
4	0.00229	ContColmn_1	673.06	673.06	673.06	673.06	673.06	673.06	673.06	673.06	673.06	673.06	673.06
		ContColmn_2	624.19	632.78	642.21	624.19	624.19	624.19	615.61	606.17	624.19	624.19	624.19
		ContColmn_3	495.92	502.74	510.24	519.78	533.96	495.92	489.10	481.61	472.06	457.89	495.92
5	0.002817	ContColmn_1	827.98	827.98	827.98	827.98	827.98	827.98	827.98	827.98	827.98	827.98	827.98
		ContColmn_2	767.86	778.42	790.03	767.86	767.86	767.86	757.31	745.70	767.86	767.86	767.86
		ContColmn_3	610.07	618.46	627.68	639.42	656.86	610.07	601.68	592.46	580.72	563.28	610.07
6	0.003466	ContColmn_1	1018.56	1018.56	1018.56	1018.56	1018.56	1018.56	1018.56	1018.56	1018.56	1018.56	1018.56
		ContColmn_2	944.60	957.59	971.88	944.60	944.60	944.60	931.62	917.33	944.60	944.60	944.60
		ContColmn_3	750.49	760.81	772.16	786.60	808.05	750.49	740.17	728.83	714.39	692.93	750.49
7	0.004263	ContColmn_1	1253.01	1253.01	1253.01	1253.01	1253.01	1253.01	1253.01	1253.01	1253.01	1253.01	1253.01
		ContColmn_2	1162.03	1178.00	1195.57	1162.03	1162.03	1162.03	1146.05	1128.48	1162.03	1162.03	1162.03
		ContColmn_3	923.23	935.93	949.89	967.65	994.05	923.23	910.54	896.58	878.82	852.42	923.23
8	0.005245	ContColmn_1	1541.41	1541.41	1541.41	1541.41	1541.41	1541.41	1541.41	1541.41	1541.41	1541.41	1541.41
		ContColmn_2	1429.49	1449.15	1470.76	1429.49	1429.49	1429.49	1409.84	1388.22	1429.49	1429.49	1429.49
		ContColmn_3	1135.74	1151.35	1168.53	1190.38	1222.85	1135.74	1120.12	1102.95	1081.10	1048.63	1135.74
9	0.006452	ContColmn_1	1896.20	1896.20	1896.20	1896.20	1896.20	1896.20	1896.20	1896.20	1896.20	1896.20	1896.20
		ContColmn_2	1896.20	1896.20	1896.20	1896.20	1896.20	1896.20	1896.20	1896.20	1896.20	1896.20	1896.20
		ContColmn_3	1397.15	1416.36	1437.49	1464.37	1504.31	1397.15	1377.94	1356.81	1329.93	1289.99	1397.15
10	0.007937	ContColmn_1	2332.65	2332.65	2332.65	2332.65	2332.65	2332.65	2332.65	2332.65	2332.65	2332.65	2332.65
		ContColmn_2	2163.28	2193.03	2225.73	2163.28	2163.28	2163.28	2133.54	2100.83	2163.28	2163.28	2163.28
		ContColmn_3	1718.73	1742.37	1768.35	1801.42	1850.56	1718.73	1695.10	1669.11	1636.05	1586.91	1718.73
11	0.009764	ContColmn_1	2869.57	2869.57	2869.57	2869.57	2869.57	2869.57	2869.57	2869.57	2869.57	2869.57	2869.57
		ContColmn_2	2661.21	2697.80	2738.04	2661.21	2661.21	2661.21	2624.61	2584.38	2661.21	2661.21	2661.21
		ContColmn_3	2114.34	2143.41	2175.38	2216.06	2276.51	2114.34	2085.27	2053.30	2012.62	1952.17	2114.34
12	0.012011	ContColmn_1	3530.06	3530.06	3530.06	3530.06	3530.06	3530.06	3530.06	3530.06	3530.06	3530.06	3530.06
		ContColmn_2	3273.74	3318.75	3368.25	3273.74	3273.74	3273.74	3228.73	3179.23	3273.74	3273.74	3273.74
		ContColmn_3	2601.00	2636.76	2676.09	2726.13	2800.50	2601.00	2565.24	2525.91	2475.87	2401.50	2601.00
13	0.014776	ContColmn_1	4342.58	4342.58	4342.58	4342.58	4342.58	4342.58	4342.58	4342.58	4342.58	4342.58	4342.58
		ContColmn_2	4027.26	4082.64	4143.53	4027.26	4027.26	4027.26	3971.89	3910.99	4027.26	4027.26	4027.26
		ContColmn_3	3199.68	3243.67	3292.05	3353.61	3445.09	3199.68	3155.68	3107.30	3045.74	2954.26	3199.68
14	0.018177	ContColmn_1	5342.11	5342.11	5342.11	5342.11	5342.11	5342.11	5342.11	5342.11	5342.11	5342.11	5342.11
		ContColmn_2	4954.22	5022.34	5097.25	4954.22	4954.22	4954.22	4886.10	4811.19	4954.22	4954.22	4954.22
		ContColmn_3	3936.15	3990.27	4049.79	4125.52	4238.05	3936.15	3882.03	3822.51	3746.78	3634.25	3936.15
15	0.022361	ContColmn_1	6571.72	6571.72	6571.72	6571.72	6571.72	6571.72	6571.72	6571.72	6571.72	6571.72	6571.72
		ContColmn_2	6094.54	6178.34	6270.49	6094.54	6094.54	6094.54	6010.74	5918.59	6094.54	6094.54	6094.54
		ContColmn_3	4842.14	4908.72	4981.93	5075.09	5213.53	4842.14	4775.56	4702.35	4609.18	4470.75	4842.14

Table G 5 Column moment of inertia for Model <2>: α Resolution # 1 to 15

Resolution #	alpha	Model#2 Column	Column 21 (in Table 2)	Column 6	Column 7	Column 8	Column 9	Column 10	Column 16	Column 17	Column 18	Column 19	Column 20
16	0.027507	ContColumn_1	8084.34	8084.34	8084.34	8084.34	8084.34	8084.34	8084.34	8084.34	8084.34	8084.34	8084.34
		ContColumn_2	7497.33	7600.42	7713.78	7497.33	7497.33	7497.33	7394.25	7280.89	7497.33	7497.33	7497.33
		ContColumn_3	5956.66	6038.56	6128.63	6243.24	6413.54	5956.66	5874.76	5784.69	5670.09	5499.78	5956.66
17	0.033839	ContColumn_1	9945.12	9945.12	9945.12	9945.12	9945.12	9945.12	9945.12	9945.12	9945.12	9945.12	9945.12
		ContColumn_2	9223.01	9349.82	9489.27	9223.01	9223.01	9223.01	9096.19	8956.74	9223.01	9223.01	9223.01
		ContColumn_3	7327.71	7428.47	7539.27	7680.25	7889.75	7327.71	7226.96	7116.16	6975.18	6765.68	7327.71
18	0.041628	ContColumn_1	12234.21	12234.21	12234.21	12234.21	12234.21	12234.21	12234.21	12234.21	12234.21	12234.21	12234.21
		ContColumn_2	11345.88	11501.89	11673.44	11345.88	11345.88	11345.88	11189.87	11018.32	11345.88	11345.88	11345.88
		ContColumn_3	9014.35	9138.29	9274.59	9448.03	9705.75	9014.35	8890.40	8754.10	8580.67	8322.95	9014.35
19	0.051209	ContColumn_1	15050.17	15050.17	15050.17	15050.17	15050.17	15050.17	15050.17	15050.17	15050.17	15050.17	15050.17
		ContColumn_2	13957.38	14149.29	14360.33	13957.38	13957.38	13957.38	13765.46	13554.43	13957.38	13957.38	13957.38
		ContColumn_3	11089.19	11241.67	11409.34	11622.69	11939.73	11089.19	10936.71	10769.05	10555.69	10238.65	11089.19
20	0.062996	ContColumn_1	18514.29	18514.29	18514.29	18514.29	18514.29	18514.29	18514.29	18514.29	18514.29	18514.29	18514.29
		ContColumn_2	17169.97	17406.06	17665.67	17169.97	17169.97	17169.97	16933.88	16674.27	17169.97	17169.97	17169.97
		ContColumn_3	13641.61	13829.18	14035.44	14297.90	14687.92	13641.61	13454.04	13247.77	12985.31	12595.30	13641.61
21	0.077496	ContColumn_1	22775.75	22775.75	22775.75	22775.75	22775.75	22775.75	22775.75	22775.75	22775.75	22775.75	22775.75
		ContColumn_2	21122.01	21412.43	21731.80	21122.01	21122.01	21122.01	20831.58	20512.21	21122.01	21122.01	21122.01
		ContColumn_3	16781.52	17012.26	17266.00	17588.88	18068.66	16781.52	16550.77	16297.03	15974.16	15494.37	16781.52
22	0.095333	ContColumn_1	28018.08	28018.08	28018.08	28018.08	28018.08	28018.08	28018.08	28018.08	28018.08	28018.08	28018.08
		ContColumn_2	25983.69	26340.97	26733.84	25983.69	25983.69	25983.69	25626.42	25233.54	25983.69	25983.69	25983.69
		ContColumn_3	20644.14	20928.00	21240.14	21637.33	22227.55	20644.14	20360.29	20048.15	19650.95	19060.74	20644.14
23	0.117276	ContColumn_1	34467.05	34467.05	34467.05	34467.05	34467.05	34467.05	34467.05	34467.05	34467.05	34467.05	34467.05
		ContColumn_2	31964.40	32403.91	32887.21	31964.40	31964.40	31964.40	31524.89	31041.58	31964.40	31964.40	31964.40
		ContColumn_3	25395.84	25745.03	26129.02	26617.63	27343.70	25395.84	25046.64	24662.66	24174.04	23447.98	25395.84
24	0.14427	ContColumn_1	42400.38	42400.38	42400.38	42400.38	42400.38	42400.38	42400.38	42400.38	42400.38	42400.38	42400.38
		ContColumn_2	39321.69	39862.36	40456.91	39321.69	39321.69	39321.69	38781.02	38186.47	39321.69	39321.69	39321.69
		ContColumn_3	31241.24	31670.80	32143.17	32744.25	33637.44	31241.24	30811.67	30339.30	29738.22	28845.03	31241.24
25	0.177477	ContColumn_1	52159.74	52159.74	52159.74	52159.74	52159.74	52159.74	52159.74	52159.74	52159.74	52159.74	52159.74
		ContColumn_2	48372.42	49037.54	49768.94	48372.42	48372.42	48372.42	47707.30	46975.91	48372.42	48372.42	48372.42
		ContColumn_3	38432.08	38960.52	39541.61	40281.05	41379.82	38432.08	37903.64	37322.54	36583.11	35484.34	38432.08
26	0.218327	ContColumn_1	64165.43	64165.43	64165.43	64165.43	64165.43	64165.43	64165.43	64165.43	64165.43	64165.43	64165.43
		ContColumn_2	59506.38	60324.59	61224.33	59506.38	59506.38	59506.38	58688.16	57788.43	59506.38	59506.38	59506.38
		ContColumn_3	47278.05	47928.12	48642.97	49552.60	50904.27	47278.05	46627.98	45913.13	45003.50	43651.82	47278.05
27	0.26858	ContColumn_1	78934.48	78934.48	78934.48	78934.48	78934.48	78934.48	78934.48	78934.48	78934.48	78934.48	78934.48
		ContColumn_2	73203.05	74209.59	75316.42	73203.05	73203.05	73203.05	72196.51	71089.68	73203.05	73203.05	73203.05
		ContColumn_3	58160.11	58959.81	59839.19	60958.19	62620.99	58160.11	57360.41	56481.03	55362.03	53699.23	58160.11
28	0.330399	ContColumn_1	97102.95	97102.95	97102.95	97102.95	97102.95	97102.95	97102.95	97102.95	97102.95	97102.95	97102.95
		ContColumn_2	90052.31	91290.53	92652.12	90052.31	90052.31	90052.31	88814.09	87452.50	90052.31	90052.31	90052.31
		ContColumn_3	71546.91	72530.68	73612.47	74989.03	77034.56	71546.91	70563.14	69481.35	68104.79	66059.26	71546.91
29	0.406447	ContColumn_1	119453.29	119453.29	119453.29	119453.29	119453.29	119453.29	119453.29	119453.29	119453.29	119453.29	119453.29
		ContColumn_2	110779.78	112303.01	113978.00	110779.78	110779.78	110779.78	109256.56	107581.57	110779.78	110779.78	110779.78
		ContColumn_3	88014.97	89225.18	90555.96	92249.37	94765.72	88014.97	86804.76	85473.98	83780.57	81264.22	88014.97
30	0.5	ContColumn_1	146948.03	146948.03	146948.03	146948.03	146948.03	146948.03	146948.03	146948.03	146948.03	146948.03	146948.03
		ContColumn_2	136278.13	138151.96	140212.48	136278.13	136278.13	136278.13	134404.31	132343.79	136278.13	136278.13	136278.13
		ContColumn_3	108273.51	109762.27	111399.36	113482.55	116578.09	108273.51	106784.75	105147.65	103064.47	99968.93	108273.51

Table G 6 Column moment of inertia for Model <2>: α Resolution # 16 to 30

Resolution #	alpha	Model#2 Column	Column 15 (in Table 3)	Column 6	Column 7	Column 13	Column 14
1	0.00123	ContConlumn_1	361.54	361.54	361.54	361.54	361.54
		ContConlumn_2	335.29	344.97	335.29	325.61	335.29
		ContConlumn_3	266.39	274.08	286.82	258.70	245.96
2	0.001513	ContConlumn_1	444.76	444.76	444.76	444.76	444.76
		ContConlumn_2	412.47	424.37	412.47	400.56	412.47
		ContConlumn_3	327.71	337.17	352.84	318.24	302.57
3	0.001862	ContConlumn_1	547.13	547.13	547.13	547.13	547.13
		ContConlumn_2	507.40	522.05	507.40	492.75	507.40
		ContConlumn_3	403.13	414.77	434.05	391.50	372.21
4	0.00229	ContConlumn_1	673.06	673.06	673.06	673.06	673.06
		ContConlumn_2	624.19	642.21	624.19	606.17	624.19
		ContConlumn_3	495.92	510.24	533.96	481.61	457.89
5	0.002817	ContConlumn_1	827.98	827.98	827.98	827.98	827.98
		ContConlumn_2	767.86	790.03	767.86	745.70	767.86
		ContConlumn_3	610.07	627.68	656.86	592.46	563.28
6	0.003466	ContConlumn_1	1018.56	1018.56	1018.56	1018.56	1018.56
		ContConlumn_2	944.60	971.88	944.60	917.33	944.60
		ContConlumn_3	750.49	772.16	808.05	728.83	692.93
7	0.004263	ContConlumn_1	1253.01	1253.01	1253.01	1253.01	1253.01
		ContConlumn_2	1162.03	1195.57	1162.03	1128.48	1162.03
		ContConlumn_3	923.23	949.89	994.05	896.58	852.42
8	0.005245	ContConlumn_1	1541.41	1541.41	1541.41	1541.41	1541.41
		ContConlumn_2	1429.49	1470.76	1429.49	1388.22	1429.49
		ContConlumn_3	1135.74	1168.53	1222.85	1102.95	1048.63
9	0.006452	ContConlumn_1	1896.20	1896.20	1896.20	1896.20	1896.20
		ContConlumn_2	1896.20	1896.20	1896.20	1896.20	1896.20
		ContConlumn_3	1397.15	1437.49	1504.31	1356.81	1289.99
10	0.007937	ContConlumn_1	2332.65	2332.65	2332.65	2332.65	2332.65
		ContConlumn_2	2163.28	2225.73	2163.28	2100.83	2163.28
		ContConlumn_3	1718.73	1768.35	1850.56	1669.11	1586.91
11	0.009764	ContConlumn_1	2869.57	2869.57	2869.57	2869.57	2869.57
		ContConlumn_2	2661.21	2738.04	2661.21	2584.38	2661.21
		ContConlumn_3	2114.34	2175.38	2276.51	2053.30	1952.17
12	0.012011	ContConlumn_1	3530.06	3530.06	3530.06	3530.06	3530.06
		ContConlumn_2	3273.74	3368.25	3273.74	3179.23	3273.74
		ContConlumn_3	2601.00	2676.09	2800.50	2525.91	2401.50
13	0.014776	ContConlumn_1	4342.58	4342.58	4342.58	4342.58	4342.58
		ContConlumn_2	4027.26	4143.53	4027.26	3910.99	4027.26
		ContConlumn_3	3199.68	3292.05	3445.09	3107.30	2954.26
14	0.018177	ContConlumn_1	5342.11	5342.11	5342.11	5342.11	5342.11
		ContConlumn_2	4954.22	5097.25	4954.22	4811.19	4954.22
		ContConlumn_3	3936.15	4049.79	4238.05	3822.51	3634.25
15	0.022361	ContConlumn_1	6571.72	6571.72	6571.72	6571.72	6571.72
		ContConlumn_2	6094.54	6270.49	6094.54	5918.59	6094.54
		ContConlumn_3	4842.14	4981.93	5213.53	4702.35	4470.75

Table G 7 Column moment of inertia for Model <3>: α Resolution # 1 to 15

Resolution #	alpha	Model#3 Column	Column 15 (in Table 3)	Column 6	Column 7	Column 13	Column 14
16	0.027507	ContConlumn_1	8084.34	8084.34	8084.34	8084.34	8084.34
		ContConlumn_2	7497.33	7713.78	7497.33	7280.89	7497.33
		ContConlumn_3	5956.66	6128.63	6413.54	5784.69	5499.78
17	0.033839	ContConlumn_1	9945.12	9945.12	9945.12	9945.12	9945.12
		ContConlumn_2	9223.01	9489.27	9223.01	8956.74	9223.01
		ContConlumn_3	7327.71	7539.27	7889.75	7116.16	6765.68
18	0.041628	ContConlumn_1	12234.21	12234.21	12234.21	12234.21	12234.21
		ContConlumn_2	11345.88	11673.44	11345.88	11018.32	11345.88
		ContConlumn_3	9014.35	9274.59	9705.75	8754.10	8322.95
19	0.051209	ContConlumn_1	15050.17	15050.17	15050.17	15050.17	15050.17
		ContConlumn_2	13957.38	14360.33	13957.38	13554.43	13957.38
		ContConlumn_3	11089.19	11409.34	11939.73	10769.05	10238.65
20	0.062996	ContConlumn_1	18514.29	18514.29	18514.29	18514.29	18514.29
		ContConlumn_2	17169.97	17665.67	17169.97	16674.27	17169.97
		ContConlumn_3	13641.61	14035.44	14687.92	13247.77	12595.30
21	0.077496	ContConlumn_1	22775.75	22775.75	22775.75	22775.75	22775.75
		ContConlumn_2	21122.01	21731.80	21122.01	20512.21	21122.01
		ContConlumn_3	16781.52	17266.00	18068.66	16297.03	15494.37
22	0.095333	ContConlumn_1	28018.08	28018.08	28018.08	28018.08	28018.08
		ContConlumn_2	25983.69	26733.84	25983.69	25233.54	25983.69
		ContConlumn_3	20644.14	21240.14	22227.55	20048.15	19060.74
23	0.117276	ContConlumn_1	34467.05	34467.05	34467.05	34467.05	34467.05
		ContConlumn_2	31964.40	32887.21	31964.40	31041.58	31964.40
		ContConlumn_3	25395.84	26129.02	27343.70	24662.66	23447.98
24	0.14427	ContConlumn_1	42400.38	42400.38	42400.38	42400.38	42400.38
		ContConlumn_2	39321.69	40456.91	39321.69	38186.47	39321.69
		ContConlumn_3	31241.24	32143.17	33637.44	30339.30	28845.03
25	0.177477	ContConlumn_1	52159.74	52159.74	52159.74	52159.74	52159.74
		ContConlumn_2	48372.42	49768.94	48372.42	46975.91	48372.42
		ContConlumn_3	38432.08	39541.61	41379.82	37322.54	35484.34
26	0.218327	ContConlumn_1	64165.43	64165.43	64165.43	64165.43	64165.43
		ContConlumn_2	59506.38	61224.33	59506.38	57788.43	59506.38
		ContConlumn_3	47278.05	48642.97	50904.27	45913.13	43651.82
27	0.26858	ContConlumn_1	78934.48	78934.48	78934.48	78934.48	78934.48
		ContConlumn_2	73203.05	75316.42	73203.05	71089.68	73203.05
		ContConlumn_3	58160.11	59839.19	62620.99	56481.03	53699.23
28	0.330399	ContConlumn_1	97102.95	97102.95	97102.95	97102.95	97102.95
		ContConlumn_2	90052.31	92652.12	90052.31	87452.50	90052.31
		ContConlumn_3	71546.91	73612.47	77034.56	69481.35	66059.26
29	0.406447	ContConlumn_1	119453.29	119453.29	119453.29	119453.29	119453.29
		ContConlumn_2	110779.78	113978.00	110779.78	107581.57	110779.78
		ContConlumn_3	88014.97	90555.96	94765.72	85473.98	81264.22
30	0.5	ContConlumn_1	146948.03	146948.03	146948.03	146948.03	146948.03
		ContConlumn_2	136278.13	140212.48	136278.13	132343.79	136278.13
		ContConlumn_3	108273.51	111399.36	116578.09	105147.65	99968.93

Table G 8 Column moment of inertia for Model <3>: α Resolution # 16 to 30

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