PROBABLISTIC FAULT DISPLACEMENT HAZARD ANALYSIS FOR REVERSE
FAULTS AND SURFACE RUPTURE SCALE INVARIANCE

A Thesis
presented to
the Faculty of California Polytechnic State University,
San Luis Obispo

In Partial Fulfillment
of the Requirements for the Degree
Master of Science in Civil and Environmental Engineering

by
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March 2011

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EXECUTIVE SUMMARY

Probabilistic Fault Displacement Hazard Analysis For Reverse Faults and Surface Rupture Scale Invariance

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A methodology is presented for evaluating the potential surface fault displacement on reverse faults in a probabilistic manner. This methodology follows the procedures put forth for Probabilistic Fault Displacement Hazard Analysis (PFDHA). Empirical probability distributions that are central to performing a PFDHA are derived from field investigations of reverse faulting events. Statistical analyses are used to test previously assumed properties of scale invariance with respect to magnitude for normalized displacement. It is found that normalized displacement is statistically invariant with respect to magnitude and focal mechanism, allowing for the combination of a large number of events into a single dataset for regression purposes. An empirical relationship is developed using this single dataset to be used as a fault displacement prediction equation. A PFDHA is conducted on the Los Osos fault zone in central California and a hazard curve for fault displacement is produced. A full sensitivity analysis is done using this fault as a reference, to test for the sources of variability in the PFDHA methodology. The influence of the major primary variables is quantified to provide a future direction for PFDHA.

Keywords: Fault Displacement, PFDHA, surface rupture

ACKNOWLEDGMENTS

I would like to thank Professor Robb Moss for helping me to find a field of study that I am truly passionate about. Without him, my academic path—including this research—would not be heading in the direction it is today. I am thoroughly grateful for having had the privilege to be his student.

I also would like to thank Gary Hughes, Scott Johnston, and Rakesh Goel for taking the time out of their busy schedules to be an integral part of my master's thesis experience. It is wonderful to have had such strong support and assistance from the faculty at Cal Poly, and I feel I am a far better person because of it.

This material is based on work supported by the U.S. Department of Homeland Security under Grant Award Number 2008-ST-061-ND0001. Administration of this grant is conducted through the Department of Homeland Security Center of Excellence for Natural Disasters, Coastal Infrastructure and Emergency Management (DIEM). The views and conclusions contained in this document are those of the authors and should not be interpreted as necessarily representing the official policies, either expressed or implied, of the U.S. Department of Homeland Security.

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Appendix A: Probabilistic Fault Displacement Hazard Analysis for Reverse Faults

Abstract

We present a methodology for evaluating potential surface fault displacement due to reverse faulting events in a probabilistic manner. This methodology, called probabilistic fault displacement hazard analysis, or PFDHA, follows procedures that were originally applied to normal faulting. We present empirical distributions for surface rupture, maximum and average displacement, spatial variability of slip, and other random variables that are central to performing PFDHA for reverse faults. Additionally, a sensitivity analysis is conducted on all independent variables in the PFDHA procedure. The Los Osos fault zone of Central California is used as the test case, and results are presented in the form of a hazard curve. The influence each of the variables has on a hazard curve is quantified to provide direction for future research in PFDHA. It is seen that a distribution for slip spatial variability is the least influential term in the procedure, and a term for the probability of surface rupture has the most influence.

Introduction

Fault displacement hazard is a critical issue for infrastructure in tectonically active regions. Unlike the hazard from strong ground motion, the primary method of mitigating fault displacement hazard is usually avoidance. For some projects, however, crossing a fault is inevitable and providing a reliable estimate of surface fault rupture and its associated displacement is crucial for these projects. For example, the Trans-Alaska Pipeline has sections that were designed with the anticipation of surface rupture from the Denali fault, which most recently occurred in the 2002 event. Currently, standard engineering methods for predicting the levels of potential fault displacement are based on deterministic scenarios and thus are susceptible to overestimation and inefficiency in project design. Over the last several decades, probabilistic seismic hazard analysis (PSHA) has become one of the most valuable tools available for ground motion prediction and seismic code construction. As a result, methods have been proposed (Youngs et al., 2003; Todorovska et al., 2007) to estimate fault displacement in a similarly probabilistic manner. Probabilistic fault displacement hazard analysis (PFDHA), is one such procedure which provides an estimate of expected levels of slip on a fault due to surface rupture.

One of the best known uses for PFDHA was in the Yucca Mountain project (Stepp et al., 2001), where the tectonic environment is primarily extensional producing normal faulting. The distributions developed for the Yucca Mountain project were derived from empirical normal faulting data and therefore are only applicable to this type of faulting. To be used for other types of faulting, it must be reformulated using data from other slip types. We present a complete PFDHA methodology for faults with reverse mechanisms and provide new empirical equations for the necessary probability terms.

Additionally, in recent years the methods used in PSHA have been scrutinized and consensus is building on how best to deal with the various forms of functional and computational uncertainty [e.g. Thomas et al. 2010]. PFDHA, in comparison, is a relatively new field with much of this unaddressed, and therefore quantifying the functional uncertainty underlying the components of a PFDHA will help improve the accuracy of the procedure. We conduct a sensitivity analysis of all independent variables involved to determine how they influence the resulting hazard curve.

Methodology

The PSHA methodology forms the basis for our PFDHA, and thus follows the general procedures outlined by Cornell (1968). PSHA uses empirical probability distributions to model the estimated ground motion levels at a given site. It does so by accounting for all possible events from all locations on every fault in a region. The result is an annual rate of events which exceed a specified level of ground motion and can be computed using the following integral:

$$\nu(z) = \alpha \int_{m_0}^{m_{max}} \int_0^\infty f(m)f(r)P(Z > z|m, r) dr dm \quad (1)$$

Here, $f(m)$ is a distribution for magnitude occurrence that is based on a range of magnitude values that a fault is believed capable of producing. The truncated exponential or characteristic models are the most commonly used. The distribution $f(r|m)$ estimates the source to site distances for all events possible in a region. α is defined as the mean rate of all earthquakes per year from a specific source and is generally given as an annual rate. $\nu(z)$ is therefore the mean annual rate of events exceeding the level of ground motion, z . Lastly, the term $P(Z > z|m, r)$ is often called a ground motion prediction equation and defines how a parameter such as peak ground acceleration varies as a function of distance and magnitude.

In a similar manner, Youngs et al. (2003) developed a methodology which estimates the potential levels of fault displacement due to surface rupture. In this formulation PFDHA follows straight from PSHA but now produces an annual rate of events that exceed a given level of fault displacement:

$$\nu(d) = \alpha \int_{m_0}^{m_{max}} f(m)P(D > d|m, r) dm. \quad (2)$$

For PFDHA, the equations are the same except the ground motion attenuation relationship has been replaced with a distribution to model the probability of exceeding a given level of slip. This distribution is based on empirical fault displacement measurements and incorporates the spatial variability of slip along a fault at the ground surface. As the Youngs et al. PFDHA methodology was created for normal faulting, the probability distributions involved were created using empirical normal faulting data. For PFDHA to work with reverse mechanisms, these distributions must be determined from reverse slip data.

The slip exceedance term, $P(D > d|m, r)$, is what distinguishes the results of a PFDHA from a PSHA. It is an inverted cumulative distribution function that is both location and magnitude dependent, and is chosen to correspond with a specific focal mechanism. The slip exceedance is written as the product of two terms:

$$P(D > d|m, r) = P(Slip|m) * P(D > d|m, r, slip) \quad (3)$$

The first term, $P(Slip|m)$, is an empirical probability distribution of surface rupture, yielding the probability of slip occurrence at the ground surface as a function of magnitude. The second term is a cumulative distribution of displacement and is conditional on the occurrence of surface rupture. A distance term, r , can be included in equation 2 to account for off fault displacement:

$$\nu(d) = \alpha \int_{m_0}^{m_{max}} \int_0^\infty f(m) f(r|m) P(D > d|m, r) dr dm. \quad (4)$$

Qualitatively it has been observed that the hanging wall can have fault related displacements on the order of hundreds of meters back from the primary surface expression of the fault plane (Bonilla, 1970). We have been collecting data for off fault displacement on reverse faults and anticipate presenting these results along with other complimentary analyses in a future study.

Probability of Surface Rupture

The probability of fault rupture propagating to the surface, $P(Slip|m)$, is central to the PFDHA procedure. It is different from a PSHA, where the probability of some level of ground shaking is one within a reasonable distance from the event. As slip can only occur when a fault ruptures the ground surface, there is a probability that an event will not yield any fault displacement at all. Wells and Coppersmith (1993) showed that since the outcome of surface rupture is binary, it can be represented through a logistic regression. They created surface rupture probability terms using several hundred events of all slip types with $M > 4$. Youngs et al. (2003) used the same procedure for purely normal faulting events from Pezzopane and Dawson (1996). We have created a similar distribution for the probability of fault rupture on reverse faults using reverse events from Lettis et al. (1997) and six additional events. The events without surface rupture are found in Table 1, and those reported with surface rupture are found in Table 2. All events from Lettis et al. except those reported with

M_I were used. The events that were reported with M_L or M_S were converted to M_W using Heaton et al. (1986). The catalogue of events was then binned from $5.5 \leq M_W \leq 8$ with bin widths of $\Delta M_W = 0.5$ and separated into categories of slip and no slip. These two data sets were then used in a logistic regression to obtain an equation for the probability of surface rupture occurrence. This probability is represented in the form of the logistic function:

$$P(\text{Slip}|m) = \frac{1}{1 + e^{a+b*m}} \quad (5)$$

$$a = 7.30, b = -1.03$$

We find that the probability of surface rupture for all slip types is significantly greater than that of purely reverse events, as shown in Figure 1. The disparity is greatest when the comparison is between normal faulting and reverse faulting. Sediment and rock can sustain compression much more than tension, and it can be seen that the reverse mechanism does not reach a 100% probability of rupturing the surface even at large magnitudes. The probability of surface rupture for all slip types increases rapidly with $M_W > 5$, while reverse faults alone do not exhibit a similar trend.

Spatial Slip Variability

The second term on the right hand side of Equation 3, $P(D > d|m, \text{slip})$, represents the probability of exceeding a given level of displacement provided that surface rupture has occurred. This is an inverse cumulative distribution function which is also based on empirical data. It is determined from two probability distributions: a distribution for the spatial variability of slip along a fault, and a distribution for the average or maximum displacement. The former can be created from a set of earthquake events where slip has been measured regularly along the fault.

Using data of 9 reverse faulting events from Wesnousky (2008) and Kaneda et al. (2008), a distribution was used to model the spatial variability of slip on reverse faults. Following Youngs et al. (2003) we treat slip along a fault as symmetric about the center, but note that this assumption adds additional uncertainty to the model. To account for varying fault rupture lengths from one fault to another, the position along a fault is normalized by the length of the rupture yielding x/L . The assumption here is that faulting is scale independent, which has been shown to hold in a variety of different situations (e.g. (Savage and Brodsky, 2010)). We place the origin at the ends of the fault such that $x/L = 0$, and $x/L = 0.5$ at the center. These values are then normalized by either the average displacement (AD) or the maximum displacement (MD) along the entire rupture length. The choice of proceeding with either AD or MD is analytically unimportant as it will later be shown to cancel out, but MD tends to be a more statistically robust approach because of the larger empirical database.

All of the reverse slip measurements were normalized for both position x/L and displacement D/AD or D/MD , and are plotted in Figure 2. The data

was then binned with a bin width of $\Delta x/L = 0.05$. The D/AD values in each bin were then subjected to a series of Anderson-Darling goodness of fit tests (D'Agostino and Stephens, 1986) for normal, lognormal, gamma, and Weibull distributions. It was found that only 20% of the bins were rejected as Weibull distributed, and 20% of the bins were rejected as gamma distributed. However, for normal and lognormal distributions, 60% of the bins were rejected in the Anderson-Darling tests. These goodness-of-fit tests used a confidence level of 99% to judge which probability distributions relatively fit the best, with the Weibull and gamma distributions showing superior fit. The D/MD values were tested with Kolmogorov-Smirnov tests for the beta distribution because of the upper and lower bounds. These tests used a 95% confidence level and in this case 20% of the bins were rejected.

Based on these results we proceed only with the gamma and Weibull distributions for modeling D/AD and beta for D/MD . The gamma and Weibull distribution parameters, shape and scale, were then fit to the data in each of the bins, including those which were rejected in the goodness-of-fit tests. The beta distribution shape parameters were fit in the same manner. A 95% confidence level could have been used for the D/AD goodness of fit tests, and while the results would be slightly worse, nothing would change with respect to the estimated distribution parameters. This is because we still use the data from bins that are rejected to estimate parameters, and these remain the same no matter what confidence level used. The results are a pair of distribution parameters for each normalized position bin. These parameters are plotted in Figure 3 and were regressed to obtain an equation for their spatial dependence. The functional form used in the D/AD regression is a 3rd order polynomial in log-space and was chosen solely based on its ability to fit the data. For the D/MD data, a simple linear regression was found to be appropriate. At the ends of the rupture $x/L = 0$, slip is required to be zero which creates an issue when using binned data. Since data at the ends falls into the outermost bins, this results in a non-zero displacement associated with the very ends of a fault. This is treated as additional epistemic uncertainty introduced into the PFDHA as part of the modeling process.

The resulting probability distribution from treating the normalized average displacement D/AD as Weibull distributed is defined as:

$$f(z) = \frac{k}{\lambda} \left(\frac{z}{\lambda}\right)^{k-1} e^{-(z/\lambda)^k} \quad (6)$$

$$k = \exp(-31.8 \left(\frac{x}{L}\right)^3 + 21.5 \left(\frac{x}{L}\right)^2 - 3.32 \frac{x}{L} + 0.431)$$

$$\lambda = \exp(17.2 \left(\frac{x}{L}\right)^3 - 12.8 \left(\frac{x}{L}\right)^2 + 3.99 \frac{x}{L} - 0.38)$$

Here, $z = D/AD$. The Weibull distribution parameters shape, k , and scale, λ , are functions of x/L , as indicated above. These parameters are specific to reverse faulting. If we instead treat D/AD as Gamma distributed, the corresponding probability distribution is:

$$f(z) = z^{k-1} \frac{e^{-z/\theta}}{\theta^k \Gamma(k)} \quad (7)$$

$$k = \exp(-30.4 \left(\frac{x}{L}\right)^3 + 19.9 \left(\frac{x}{L}\right)^2 - 2.29 \frac{x}{L} + 0.574)$$

$$\theta = \exp(50.3 \left(\frac{x}{L}\right)^3 - 34.6 \left(\frac{x}{L}\right)^2 + 6.60 \frac{x}{L} - 1.05)$$

Again, $z = D/AD$ and this probability density function is specific to reverse faulting. k is the gamma shape parameter and θ is the gamma scale parameter. Figure 4 contains a plot of both equations (5) and (6) for the full range of D/AD and fixed values of x/L . It is clear that both the gamma and Weibull distributions fit above are very similar for the same values of x/L . If the normalized maximum displacement D/MD is used, the Beta distribution is defined as:

$$f(z) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} z^{\alpha-1} (1-z)^{\beta-1}$$

$$\alpha = 0.901x + 0.713$$

$$\beta = -1.86x + 1.74$$

where Γ is the gamma function, and α, β are shape parameters.

Average and Maximum Displacement

Wells and Coppersmith (1994) showed that both average and maximum displacement have strong magnitude dependence. These variables, when presented in log-space, vary linearly with magnitude. Wells and Coppersmith provided regression equations for strike slip, normal, reverse, and all slip types. The reverse slip equations however have low values for the coefficient of determination, R^2 , and do not present a strong case for a magnitude dependence on AD and MD . The eight events from Table 3 have been added to the reverse events from Wells and Coppersmith to form new equations. In addition, six reverse events from Wells and Coppersmith were also discussed in Wesnousky (2008) and therefore the average values were used. Two events, Golbaf, Iran and Sirch, Iran, have been removed for reasons discussed later. The regressions are shown in Figure 5 along with the original regression equations from Wells and Coppersmith.

$$\log(AD) = 0.3244 * M - 2.2192 \quad (8)$$

$$\sigma = 0.17, R^2 = 0.62$$

$$\log(MD) = 0.5102 * M - 3.1971 \quad (9)$$

$$\sigma = 0.31, R^2 = 0.53$$

The R^2 values have increased substantially, from 0.01 and 0.13 to 0.62 and 0.53, respectively. Furthermore, the previously reported standard deviations of 0.42 for MD , and 0.38 for AD have decreased to 0.31 and 0.17. If we assume

that the residuals of $\log(AD)$ and $\log(MD)$ are normally distributed, then AD and MD are said to be lognormally distributed. This lognormal distribution is a function of magnitude and valid only in the range of the above regression equations: $5.5 \leq M_W \leq 8$.

Slip Exceedance Distribution

The final inverse CDF in Equation 3 is obtained by combining the distribution for spatial slip variability with the lognormal distribution for average or maximum displacement. As each of these PDFs represent a random variable, namely D/AD and AD , the product of these random variables yields a probability density function for D . A product of random variables can be shown to be equivalent to a logarithmic convolution (Glen et al., 2004), and the resulting distribution is:

$$h(D) = \int_{-\infty}^{\infty} f\left(\frac{D}{AD}\right) g(AD) \frac{1}{AD} dAD \quad (10)$$

Here, $f\left(\frac{D}{AD}\right)$ is either Equation 6 or 7 and $g(AD)$ is the lognormal distribution using 8 or 9. Equation 10 must be integrated to obtain the cumulative distribution, $H(D)$, and finally:

$$P(D > d|m, r, slip) = 1 - H(d) \quad (11)$$

We found that numerically solving Equation 10 was a bit too computationally expensive for practical purposes. Therefore in computing this integral we use Monte Carlo simulations to obtain the cumulative distribution. This is done by generating a random value from both the spatial slip variability distribution and the average (or maximum) displacement distribution, taking the product of the two, and repeating on the order of five thousand times to achieve stability in the solution. Figure 6 shows the inverse cumulative distribution function for several magnitude values at $x/L = 0.25$. The increase of displacement with magnitude is expected, as average/maximum displacement also increases with magnitude.

Example: Los Osos Fault Zone

As a method of illustrating the PFDHA procedure and to provide a basis for a sensitivity analysis, we have chosen to use the Los Osos fault zone in central California. The Los Osos fault is reported to have a median slip rate of 0.5 mm per year (Cao et al., 2003) with a maximum magnitude of $M_W = 7$. The fault area is approximately 44 km by 14 km, and has a b-value of 0.8. A shear modulus of 3.75×10^{11} dynes/cm² was used. For this example, the average displacement method is chosen as a point of reference and paired with the gamma distribution. The gamma distribution has parameters as defined in Equation 7. Additionally, it uses both the empirical probability of surface rupture and lognormal distribution for purely reverse mechanisms. This example

uses $x/L = 0.25$. It is assumed that events with $M_W < 5$ have a negligible probability of rupturing the surface, and thus do not contribute to the rates of exceedance.

The results of the PFDHA are presented in the form of a hazard curve, or a curve indicating the annual probability of exceeding a given level of displacement which is shown in Figure 7. Currently there are no generally accepted risk standards for fault displacement, and the traditional standards for ground shaking may or may not be applicable depending on the problem at hand. In our example there is no value of displacement corresponding to a 10% probability of exceedance in 50 years (0.0021/yr). There is however a 2% probability in 50 years of exceeding 55 cm, and 1% probability in 50 years of exceeding 105 cm. These risk parameters have return periods of 2475 years and 4975 years, respectively. We now treat this hazard curve as a reference curve to be used for comparisons in a sensitivity analysis.

Sensitivity Analysis

As PFDHA is a relatively new tool for estimating fault displacement hazard, the major sources of uncertainty in PFDHA are unexplored. Quantifying the uncertainty of the independent variables, and understanding the effect of this uncertainty on the outcome is necessary for confidence in the slip estimates. There are a number of opportunities in the PFDHA methodology for testing the sensitivity of the procedure and all should be examined to determine the extent of their influence.

An independent variable involved in the spatial slip variability is the choice of using either a gamma or Weibull distribution to describe the random variables D/AD . Both gamma and Weibull distributions for D/AD represent 80% of the bins used in the curve fitting, as only 20% were rejected through the Anderson-Darling tests. Since both distributions passed the same number of tests, we cannot say from the tests alone that one is more correct than the other. More data would be required in a future study to assess whether these results change. The differences between PFDHA's using Weibull or gamma distributions are mostly insignificant. The percent difference between the Weibull based hazard curve and the gamma based hazard curve is plotted as a function of displacement in Figure 8, which also contains the results for the entire sensitivity analysis. The differences are below 1% up to 2 m, and slowly increases up to a maximum of 32% by 10 m. The 2% and 1% in 50 year values are, in this case, both approximately 1% different from the reference hazard curve values.

Additional uncertainty is introduced into the PFDHA procedure through the fitting of curves to distribution parameters, Equations 6 and 7. The functional form of the regression was chosen to best fit the parameters because it was necessary to obtain as close of a predictor equation as possible. We used a logarithmic form for the regressions, however other regression forms, such as non-logarithmic polynomials, could provide nearly the same goodness of fit. Therefore testing the influence of the spatial dependence of the distribution parameters on a hazard curve outcome is important. Figure 8 shows the percent

difference between two hazard curves, using parameter regression form as an independent variable. The curves are identical in both using AD based gamma distributions for slip variation, reverse surface rupture distributions and a purely reverse lognormal distribution. The reference hazard curve however uses a logarithmic form for the regression, and the comparison curve uses a simple third order polynomial. It can be seen that the sensitivity of this PFDHA due to the spatial distribution fitting is low until approximately 0.5 m, where the percent difference then slowly begins to rise. It ultimately peaks at 43% for 10 m. The risk parameters for the comparison hazard curve, 2% and 1% in 50 years, are 71 cm and 113 cm respectively. These values are ultimately just under 10% different from those of the reference curve.

Since both average displacement and maximum displacement methods are available for the PFDHA procedure, it is important that the differences between these two be quantified. Either is eventually canceled out through the product of random variables that is defined by Equation 10, therefore the two methods should ideally be equivalent. However, as a result of natural variability in the faulting process, and our fitting and modeling of this phenomenon, differences are expected. Computing a hazard curve via the maximum displacement method requires that the lognormal distribution for AD be changed to one for MD , in addition to using the beta distribution for D/MD . Hazard curves for Los Osos were computed using gamma based AD and beta based MD methods and they are shown in Figure 9. Both methods have a well-defined plateau region, which exists for $D \leq 0.1$ m. The percent difference as a function of the displacement is shown in Figure 8. The MD method ultimately yields a hazard curve with a different curvature, and the probabilities fall off more quickly than the reference curve. The risk parameters here for the comparison curve are 52 cm for 2% in 50 years, and 85 cm for 1% in 50 years. These are 20% different from those of the reference curve.

Each of the major elements of uncertainty in the spatial variation distribution have been discussed thus far, however there still remains two other components that contribute to a PFDHA. The empirical probability of surface rupture has been defined for reverse mechanisms in Equation 5, and for all slip types by Wells and Coppersmith (1993). While the equation for all slip types has a significantly higher probability of surface rupture, the overall impact of such a difference is previously unknown. Additionally it would be important to understand what controls the probability at which the plateau occurs in a PFDHA hazard curve. In Figure 10 we have constructed hazard curves for two PFDHA scenarios. Both PFDHA's are the same except for the different surface rupture distributions used, $P(Slip|m)$. For the comparison curve, the 2% and 1% in 50 years parameters are 95 cm and 142 cm, respectively. These are in turn 46% and 35% different from the reference curve values. The percent difference between the two hazard curves is plotted as a function of the associated displacement level in Figure 8. Contrary to previous comparisons within the spatial variability distribution, the probability at which the hazard curve plateaus has increased by nearly 45%. This is simply from using all slip types in the surface rupture probability instead of just reverse mechanisms. The percent difference itself

remains roughly constant until about 20 cm, where it then begins to steadily increase and reaches nearly 90% by 6 m.

One final source of potential uncertainty analyzed here is the lognormal distribution for average or maximum displacement. We choose to study the effect of the mean value of AD , μ , for all slip types (Wells and Coppersmith, 1994) on a PFDHA compared to μ for purely reverse faulting mechanisms. Both methods use purely reverse surface rupture distributions and AD based gamma distributions for spatial slip variability. The results can be seen in Figure 11. The displacement for 2% probability of exceedance in 50 years is 24 cm, and 49 cm for 1% in 50 years. These values are 62% and 53% different from those of the reference hazard curve, respectively. The full range of the percent difference with respect to displacement is shown in Figure 8. The percent difference begins to increase until approximately $D = 0.7$ m, where it then begins to decrease. The two curves eventually become equal at approximately 6.5 m, and then start to diverge rapidly.

Discussion

PSHA has had several decades of debate about on how best to treat and model the uncertainty associated with the methodology. As the PFDHA methodology is relatively new, the majority of this uncertainty is unaddressed with respect to both computational methods as well as empirical and procedural choices. This uncertainty should be studied in a manner which is independent of the bounds and range of the problem at hand, to sort out variability that is both naturally inherent to the faulting process as well as variability which is due to lack of knowledge. In PSHA, the conventional method of naming these is aleatory and epistemic uncertainty. Standard PSHA practice involves the use of logic trees, which separate out the various possible methods progressing to the same result. Because of this inclusion of all possible methods, weights must be assigned to the different branches in an effort to establish confidence levels on each branch. Therefore the final PSHA is viewed as a weighted average of all possible hazard curves in an attempt to deal with the epistemic uncertainty associated with the procedure.

A PFDHA has three primary terms within the methodology which act as major sources of uncertainty. These three are specifically those which are not involved in a PSHA. They are the empirical probability of surface rupture, the lognormal distribution for average or maximum displacement, and the spatial variability term for D/AD or D/MD . All of these terms play a distinct role in a PFDHA, and studying the uncertainty contributed by each can explain their unique influence on a PFDHA hazard curve. In addition, understanding how the uncertainty propagates through the PFDHA procedure can help to establish areas where the uncertainty is potentially negligible and where more research and data collection is necessary.

The spatial variability term for normalized displacement D/AD or D/MD has the greatest impact on the uncertainty in a PFDHA, yet has the least overall influence in a hazard curve. From Figure 8 it is clear that the percent difference

curves involving spatial variability distribution have a lower absolute maximum than any of the other independent variables. The most probable explanation for why the spatial variability term has the least influence on the resulting hazard curve is due to its dependence being only on position. Because of this, whether the term uses AD or MD , gamma or Weibull, the result is still normalized displacement. Another possible explanation is due to the finite range of the bounds on normalized displacement, seen in Figure 2. MD especially is affected by this, as it has a maximum possible value of 1.

The lognormal distribution for AD and MD has a substantially greater degree of influence in a PFDHA hazard curve than the spatial variability term, and in some cases has the highest sensitivity of all. This distribution is based around the regression of a logarithmic form of displacement versus magnitude, and therefore the characteristics of this function are central to the behavior of the lognormal distribution itself. From the comparison made in Figure 8, where all slip types and reverse mechanisms are used for the regression equations, the hazard curves have on average a percent difference of 50. This is significantly larger than the spatial variability term, and after $D = 3$ m, the two curves completely diverge from each other. This is because a plateau region does not exist for the all slip types curve, and therefore the annual probabilities begin to decrease at even small displacements. The annual probabilities decrease at a much slower rate with displacement than the reference curve. As such, the two curves finally become equal by approximately 6.5 m, and the divergence ultimately occurs due to the high disparity in the rate that annual probability decreases. The most probable explanation for why these two curves have such different behavior is the large change in the standard deviation, σ , from the reverse fault regression to the all slip type regression. As σ for all slip types is more than twice that of reverse events, it is certainly possible.

In the process of determining our regression equations for reverse faults, we decided to omit the 1981 Golbaf, Iran and Sirch, Iran events in the dataset. These two events occurred within two months of each other, and both took place on the Gowk fault system. These events are notable because they had extremely low levels of fault displacement compared to other events of the same magnitude. Walker and Jackson (2002) have shown that this fault system is extremely complicated with normal faults overlaying the larger, main thrust fault. We feel that due to the nature of this fault system, the energy available is partitioned among all of the different parts and results in substantially lower levels of displacement than expected from the thrust fault.

The empirical probability of surface rupture has the most consistent overall influence on the results of a PFDHA and therefore is the most important term for quantifying uncertainty. As the entire PFDHA procedure, with the exception of magnitude, is conditional on the surface rupturing for slip to occur, it is obvious why this distribution would be the most influential. It was shown in Figure 1 that the probability of surface rupture is significantly lower for reverse faults than for other slip types, and such a difference is expected to have a strong influence in the outcome of a PFDHA. Changing the surface rupture probability type in a PFDHA tends to shift the entire curve, especially the

plateau region. This is important because it is the only one of the three terms which is capable of altering the probability at which the plateau occurs. The plateau portion is a region in which the annual probability of exceedance is approximately the same for a range of displacement values. Therefore it can be said that every value of displacement within that range is equally probable of occurring. Since the surface rupture probability controls the value probability plateaus at, identification of the uncertainty underlying surface rupture is crucial to obtaining the most accurate PFDHA results. From Figure 8 it can be seen that the percent difference between the reference and comparison hazard curves is nearly constant for $D \leq 0.2$ m. This is essentially the full plateau region for both hazard curves. We note that for this results in a shift of the entire plateau region upward by 45%. Beyond the plateau region, the percent difference then rises steadily with approximately constant slope for the remainder of the hazard curve.

Conclusion

We present a reverse fault methodology for estimating the levels of potential surface fault displacement at a given site. The methodology is based on the PFDHA procedure developed by Youngs et al. (2003) for normal faulting in the Yucca Mountain project. In this methodology a PFDHA is analogous to a PSHA for estimating strong ground motion and follows the same general procedure developed by Cornell (1968). We have determined reverse-specific distributions for all of the terms involved in a PFDHA. This includes a spatial variability distribution for normalized slip, a lognormal distribution for average (or maximum) slip, and an empirical equation for estimating the probability of surface rupture. The Los Osos fault zone of central California was used as a test case for a reverse fault PFDHA. Each of the terms in PFDHA are then compared in a sensitivity analysis to determine the full extent of their influence on a hazard curve.

As PFDHA is a relatively new procedure, quantifying uncertainty and the influence it has on a PFDHA has not been previously studied. The sensitivity analysis in this study isolates the influence of the independent variables in a PFDHA and shows how these variables contribute to a hazard curve. Understanding the uncertainty associated with the procedure is critical to achieving relative accuracy and confidence in fault displacement estimates. In turn, this provides direction for the future of uncertainty treatment in PFDHA.

Data and Resources

All data used in this paper came from published sources listed in the references.

Acknowledgements

We thank the reviewers for their constructive comments and suggestions on this paper. This material is based on work supported by the U.S. Department

of Homeland Security under Grant Award Number 2008-ST-061-ND0001. Administration of this grant is conducted through the Department of Homeland Security Center of Excellence for Natural Disasters, Coastal Infrastructure and Emergency Management (DIEM). The views and conclusions contained in this document are those of the authors and should not be interpreted as necessarily representing the official policies, either expressed or implied, of the U.S. Department of Homeland Security.

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Table 1: Database of Reverse Events Without Surface Rupture

Date (mm/dd/yyyy)	Event	M_W	Reference
04/04/1905	Kangra, India	7.8	Lettis et al. (1997)
03/01/1925	Charlevoix, Canada	7.0	Lettis et al. (1997)
05/22/1927	Gansu, China	7.7	Lettis et al. (1997)
11/04/1927	Lompoc, California	6.6	Lettis et al. (1997)
01/15/1934	Bihar, Nepal	8.2	Lettis et al. (1997)
11/01/1935	Temiskaming, Canada	6.4	Lettis et al. (1997)
04/29/1941	Imaichi, Japan	6.3	Lettis et al. (1997)
09/09/1954	Orleansville, Algeria	6.91	Lettis et al. (1997)
01/31/1955	Serra do Tombador, Brazil	6.79	Lettis et al. (1997)
05/07/1961	Hyogo, Japan	5.88	Lettis et al. (1997)
08/19/1961	Kita-Mino, Japan	7.23	Lettis et al. (1997)
04/30/1962	Miyagi Prefecture, Japan	6.5	Lettis et al. (1997)
01/18/1964	Southwest Taiwan	6.55	Lettis et al. (1997)
06/16/1964	Niigata, Japan	7.6	Lettis et al. (1997)
11/13/1965	Urunchi, China	6.5	Lettis et al. (1997)
03/24/1970	Lake Mackay, Australia	6.0	Lettis et al. (1997)
10/16/1970	Akita, Japan	6.1	Lettis et al. (1997)
04/24/1972	Taiwan	7.0	Lettis et al. (1997)
08/28/1972	Simpson Desert, Australia	6.0	Lettis et al. (1997)
09/03/1972	Hamran, Pakistan	6.2	Lettis et al. (1997)
02/21/1973	Point Mugu, California	5.7	Lettis et al. (1997)
08/11/1974	Markansu Valley, Tajikistan	7.1	Lettis et al. (1997)
08/11/1974	Markansu Valley, Tajikistan	5.58	Lettis et al. (1997)
08/11/1974	Markansu Valley, Tajikistan	6.16	Lettis et al. (1997)
03/07/1975	Sarkun, Iran	6.16	Lettis et al. (1997)
04/08/1976	Gazli, Uzbekistan	6.9	Lettis et al. (1997)
05/17/1976	Gazli, Uzbekistan	6.8	Lettis et al. (1997)
05/06/1976	Friuli, Italy	6.5	Lettis et al. (1997)
03/21/1977	Khurgu, Iran	6.7	Lettis et al. (1997)
04/01/1977	Khurgu, Iran	6.0	Lettis et al. (1997)
04/06/1977	Naghan, Iran	6.0	Lettis et al. (1997)
11/23/1977	Caucete, Argentina	7.5	Lettis et al. (1997)
08/13/1978	Santa Barbara, California	5.9	Lettis et al. (1997)
04/15/1979	Montenegro	7.0	Lettis et al. (1997)
01/09/1979	Miramichi, Canada	5.6	Lettis et al. (1997)
12/16/1982	Tadjik,Afghanistan	6.5	Lettis et al. (1997)
05/02/1983	Coalinga, California	6.4	Lettis et al. (1997)
03/19/1984	Gazli, Uzbekistan	7.0	Lettis et al. (1997)
01/26/1985	Mendoza, Argentina	6.0	Lettis et al. (1997)
07/03/1985	New Ireland, Papua New Guinea	7.2	Lettis et al. (1997)
08/04/1985	Kettleman Hills, California	6.1	Lettis et al. (1997)
08/23/1985	Wuquai, China	6.9	Lettis et al. (1997)

Table 1 – Database of Reverse Events Without Surface Rupture

Date (mm/dd/yyyy)	Event	M_W	Reference
10/05/1985	Nahanni, Canada	6.7	Lettis et al. (1997)
12/23/1985	Nahanni, Canada	6.8	Lettis et al. (1997)
03/25/1988	Nahanni, Canada	6.2	Lettis et al. (1997)
05/20/1986	Hualien, Taiwan	6.4	Lettis et al. (1997)
11/14/1986	Hualien, Taiwan	7.3	Lettis et al. (1997)
03/06/1987	Northeast Ecuador	7.1	Lettis et al. (1997)
10/01/1987	Whittier Narrows, California	6.0	Lettis et al. (1997)
11/25/1988	Saguenay, Canada	5.9	Lettis et al. (1997)
01/14/1990	Mangyai-Lenghu, China	6.0	Lettis et al. (1997)
03/04/1990	Kalat, Pakistan	6.0	Lettis et al. (1997)
04/26/1990	Gonghe-Xinghai, China	6.4	Lettis et al. (1997)
11/06/1990	Darab, Iran	6.6	Lettis et al. (1997)
12/13/1990	Hualien, Taiwan	6.6	Lettis et al. (1997)
01/28/1991	Hawks Crag, New Zealand	6.0	Lettis et al. (1997)
02/25/1991	Kalpin, China	6.0	Lettis et al. (1997)
03/08/1991	Eastern Siberia, Russia	6.79	Lettis et al. (1997)
04/04/1991	Rioja, Peru	6.4	Lettis et al. (1997)
04/29/1991	Georgia	7.1	Lettis et al. (1997)
06/15/1991	Dzhava-Tzkinvali, Georgia	6.3	Lettis et al. (1997)
06/28/1991	Sierra Madre, California	5.43	Lettis et al. (1997)
10/19/1991	Northern India	7.0	Lettis et al. (1997)
05/20/1992	Peshawar-Kohat, Pakistan	6.2	Lettis et al. (1997)
10/17/1992	Murindo, Columbia	6.7	Lettis et al. (1997)
10/23/1992	Barisakho-Kazbegi, Georgia	6.5	Lettis et al. (1997)
10/02/1993	Xinjiang, China	6.1	Lettis et al. (1997)
01/17/1994	Northridge, California	6.7	Lettis et al. (1997)
02/23/1994	Sefidabeh, Iran	6.1	Lettis et al. (1997)
02/24/1994	Sefidabeh, Iran	6.3	Lettis et al. (1997)
02/26/1994	Sefidabeh, Iran	6.0	Lettis et al. (1997)
05/01/1994	Hindu Kush, Afghanistan	6.1	Lettis et al. (1997)
05/31/1994	Western Venezuela	6.0	Lettis et al. (1997)
06/18/1994	New Zealand	6.8	Lettis et al. (1997)
01/19/1995	Tauramena, Columbia	6.6	Lettis et al. (1997)
01/26/2001	Bhuj, India	7.9	Rajendran et al. (2001)

Table 2: Database of Reverse Events with Surface Rupture

Date (mm/dd/yyyy)	Event	M_W	Reference
08/31/1896	Rikuu, Japan	7.42	Lettis et al. (1997)
12/23/1906	Manas, China	7.95	Lettis et al. (1997)
01/23/1909	Silakhar, Iran	7.23	Lettis et al. (1997)
01/23/1911	Kirgizia, Russia	7.89	Lettis et al. (1997)
04/18/1911	Raver, Iran	6.29	Lettis et al. (1997)
05/01/1929	Baghan, Iran	7.51	Lettis et al. (1997)
06/17/1929	White Creek, New Zealand	7.89	Lettis et al. (1997)
05/06/1930	Salmas, Iran	7.6	Lettis et al. (1997)
02/02/1931	Hawkes Bay, New Zealand	7.89	Lettis et al. (1997)
12/25/1932	Changma, China	7.82	Lettis et al. (1997)
11/28/1933	Behabad, Iran	6.29	Lettis et al. (1997)
04/21/1935	Taiwan	7.23	Lettis et al. (1997)
01/15/1944	San Juan, Argentina	7.6	Lettis et al. (1997)
01/13/1945	Mikawa-Fukozu, Japan	7.02	Lettis et al. (1997)
03/17/1947	Dari, China	7.89	Lettis et al. (1997)
07/10/1949	Tajikistan	7.75	Lettis et al. (1997)
07/21/1952	Kern County, California	7.4	Lettis et al. (1997)
02/12/1953	Torud, Iran	6.67	Lettis et al. (1997)
12/13/1957	Farsinaj, Iran	6.91	Lettis et al. (1997)
09/01/1962	Ipak, Iran	7.42	Lettis et al. (1997)
05/24/1968	Inangahua, New Zealand	7.1	Lettis et al. (1997)
10/01/1967	Meckering, Australia	6.6	Lettis et al. (1997)
07/24/1969	Pariahuanca, Peru	6.1	Lettis et al. (1997)
10/01/1969	Pariahuanca, Peru	6.6	Lettis et al. (1997)
03/10/1970	Calingiri, Australia	5.97	Lettis et al. (1997)
02/09/1971	San Fernando, California	6.6	Lettis et al. (1997)
04/10/1972	Qir, Iran	6.8	Lettis et al. (1997)
09/06/1975	Lice, Turkey	6.6	Lettis et al. (1997)
08/16/1976	Songpan, China	6.9	Lettis et al. (1997)
08/21/1976	Songpan, China	6.4	Lettis et al. (1997)
08/23/1976	Songpan, China	6.7	Lettis et al. (1997)
01/01/1977	Mangya, China	6.1	Lettis et al. (1997)
09/16/1978	Tabas, Iran	7.4	Lettis et al. (1997)
06/02/1979	Cadoux, Australia	6.1	Lettis et al. (1997)
10/10/1980	El Asnam, Algeria	7.1	Lettis et al. (1997)
06/11/1981	Golbaf, Iran	6.6	Lettis et al. (1997)
07/27/1981	Sirch, Iran	7.1	Lettis et al. (1997)
06/11/1983	Coalinga, California	5.4	Lettis et al. (1997)
03/30/1986	Marryat Creek, Australia	5.8	Lettis et al. (1997)
01/22/1988	Tennant Creek, Australia	6.3	Lettis et al. (1997)
01/22/1988	Tennant Creek, Australia	6.4	Lettis et al. (1997)
01/22/1988	Tennant Creek, Australia	6.6	Lettis et al. (1997)

Table 2 – Database of Reverse Events with Surface Rupture

Date (mm/dd/yyyy)	Event	M_W	Reference
12/07/1988	Armenia	6.8	Lettis et al. (1997)
10/29/1989	Chenoua, Algeria	6.0	Lettis et al. (1997)
12/25/1989	Ungava, Canada	6.0	Lettis et al. (1997)
06/20/1990	Rudbar-Tarom, Iran	7.4	Lettis et al. (1997)
08/19/1992	Kyrgyzstan	7.2	Lettis et al. (1997)
09/29/1993	Killari-Latur, India	6.2	Lettis et al. (1997)
09/29/1999	Chi-Chi, Taiwan	7.6	Lee et al. (2002)
06/22/2002	Avaj, Iran	6.5	Walker et al. (2005)
02/22/2005	Zarand, Iran	6.4	Talebian et al. (2006)
10/08/2005	Kashmir, Pakistan	7.6	Kaneda et al. (2008)
05/12/2008	Sichuan Province, China	7.9	Lin et al. (2009)

Table 3: Events added for AD and MD Regressions

Date (mm/dd/yyyy)	Event	M_W	AD	MD	Reference
01/13/1945	Mikawa, Japan	7.02	1.35	2.05	Wesnousky (2008)
08/19/1992	Kyrgyzstan	7.2	N/A	4.2	Lettis et al. (1997)
09/29/1993	Killari-Latur, India	6.2	N/A	0.5	Lettis et al. (1997)
09/29/1999	Chi-Chi, Taiwan	7.6	3.76	12.8	Wesnousky (2008)
06/22/2002	Avaj, Iran	6.5	N/A	0.65	Walker et al. (2005)
02/22/2005	Zarand, Iran	6.4	N/A	1.05	Talebian et al. (2006)
10/08/2005	Kashmir, Pakistan	7.6	1.7	7.05	Kaneda et al. (2008)
05/12/2008	Sichuan, China	7.9	2.09	6.5	Lin et al. (2009)

Figure 1: Probability of surface rupture for reverse, normal, and all slip types. The normal and all slip types are from Youngs et al. (2003). Empirical distributions are fit using logistic regression for a dichotomous outcome. The probability for reverse events is significantly lower than that of normal and all slip types. These distributions are only valid in the range of $5.5 \leq M_W \leq 8$

Figure 2: Combined dataset for normalized slip measurements from 9 reverse faulting events plotted as a function of x/L . $x/L = 0$ is treated as the beginning (or end) of the fault rupture. The top figure is displacement normalized by the average surface displacement, and the bottom is displacement normalized by the maximum surface displacement measured.

Figure 3: Example distribution parameters for spatial slip variability, $P(D > d|m, slip)$. The regressions shown are valid for a Weibull distribution. Each point plotted here is a distribution parameter fit from data in the corresponding bin. The regressions are 3rd order polynomials in log-space. These parameters are inserted into a Weibull distribution to yield the correct shape necessary to represent D/AD anywhere along the surface rupture.

Figure 4: Comparison of gamma and Weibull distributions for spatial variability at two values of position. These examples are for AD and are independent of magnitude. These distributions are used to ultimately obtain a distribution for displacement.

Figure 5: Regression equations for AD and MD on reverse faults following the procedures by Wells and Coppersmith (1994). Additional events have been added to improve the accuracy of the regressions, which are shown along with previous relationships. The slope has increased for both equations as well as the values for R^2 .

Figure 6: This is the complete inverse CDF for displacement, $P(D > d|m, slip)$. Here it is shown for $x/L = 0.25$ and varying magnitude. The CDF is formed by taking a product of the random variables D/AD (lognormally distributed) and AD (here gamma distributed as in Equation 6).

Figure 7: PFDHA for the Los Osos fault zone near San Luis Obispo, California. This is a Class B fault with a maximum magnitude of $M_W = 7$ (Cao et al., 2003). The PFDHA was computed using the *AD* method and a gamma distribution. The hazard curve indicates the annual probability that a given level of displacement will be exceeded.

Figure 8: The results of the entire sensitivity analysis. Percent difference between the reference hazard curve and a specific comparison curve is plotted as a function of displacement. Each comparison curve is one of the independent variables involved in PFDHA. The only variable that is capable of shifting the plateau region is the probability of surface rupture.

Figure 9: Comparison of computed hazard curve using *MD* option to reference hazard curve using *AD*. Both curves use gamma distributions for spatial variability and lognormal distributions for reverse faults.

Figure 10: Comparison of computed hazard curve using surface rupture distribution for all slip types to reference hazard curve using purely reverse distribution. Both curves use gamma distributions for spatial variability and lognormal distributions for reverse faults.

Figure 11: Comparison of computed hazard curve using regression equation for all slip types to reference curve with purely reverse events. Both curves use gamma distributions for spatial variability and purely reverse surface rupture distributions.

Appendix B: Slip Distributions and Surface Rupture Scale Invariance

Abstract

We investigate the behavior of normalized displacements due to surface rupture. The displacement measurements in a suite of events are normalized by either the maximum displacement or average displacement obtained through field investigations of the surface ruptures. We compare a set of strike-slip measurements with $6.3 \leq M_W < 7.1$ to a set of events with $7.1 \leq M_W \leq 7.9$ to test for magnitude independence. It is shown that when normalized, the distribution for displacement appears to be statistically independent of magnitude, and correspondingly, that the slip distribution appears statistically independent of surface rupture length. This allows for a large number of surface rupture events to be studied in a single combined dataset, which previous studies only assumed was possible. We have determined that the position of maximum displacement occurrence does not correlate with magnitude, but does have a higher probability of being located at the center of a rupture than at the ends. Additionally, we compare a set of reverse events, strike-slip events and normal events to show that the distribution for normalized displacement is statistically independent of focal mechanism. Combined with magnitude invariance, this find allows any mapped surface rupture to be used in a generalized displacement data set, and we present regression equations for normalized displacement using such a database.

Introduction

The characteristics and behavior of surface rupture have often been seen to be quite different from one event to the next. Even when a majority of the independent variables are controlled for, there is often very little in common with

two events having very similar source characteristics when the surface rupture measurements are studied. These variables include seismogenic rupture length, surface rupture length, magnitude, maximum surface displacement, and average surface displacement. Previously, attempts have been made to determine the shape and possible patterns existing in the surface displacements. Wesnousky (2008) compiled a set of slip profiles from surface rupture field investigations and fit a variety of curves to 37 events to study displacement characteristics. He found that it is common for the displacements to be asymmetric about the center of the surface rupture, and that the locations with the largest displacements do not correlate with the epicentral location. Additionally, no single functional form used for displacement profiles seemed to consistently fit a majority of the events well.

Since the variables that are relevant to surface rupture have a random component to them, a large enough set of similar events must be available to fully resolve the variability and distribution of displacement. The only way to obtain a large enough set of events with regularly measured surface displacements is by normalizing the displacement and rupture length. This was originally proposed by Hemphill-Haley and Weldon (1999) for use in a paleoseismic context. The idea is that since maximum and average displacement may be treated as random variables that are a function of magnitude, normalizing the slip measurements by these quantities may render the slip distribution scale independent.

By using slip normalization, it is possible to sample slip measurements from events all over the world to create a database large enough for statistical analysis. Investigations into the behavior of fault displacement for probabilistic hazard calculations [Youngs et al., 2003][Moss and Ross, 2011][Todorovska et al., 2007] have been conducted with normalized slip measurements for reverse, normal, and strike-slip faults. They have normalized slip by both average and maximum surface displacements to obtain distributions for slip that are assumed independent of magnitude. These are modeled using gamma, Weibull, and beta distributions, and can be used to estimate the probability of exceeding a given level of slip at a specific site. For these distributions, and subsequently the slip estimations to be valid, however, the assumption of magnitude scale invariance must hold.

Methodology

We use a set of 34 events from Wesnousky (2008) and one event from Kaneda et al. (2008) as the basis for our statistical analysis. There are 20 strike-slip events, 9 reverse events, and 6 normal faulting events which have been field mapped along the length of the surface rupture. The full set of events is listed in Table 1 along with the corresponding event characteristics. The slip measurements were given with the corresponding distance along strike, generally starting at the beginning (or end) of the surface rupture trace. For use in the statistical analysis, these values must be transformed and normalized.

We treat surface displacement as symmetric about the center of the rupture. The reason for this is due to the variable location of rupture initiation. If all surface ruptures were unilateral, one could denote the beginning of a surface rupture zone easily and there would be no reason for this requirement. However since bilateral rupture does occur, and is present in many of the events used in our database, these rupture zones would have two ends. This makes these events incompatible with the unilateral surface ruptures and thus a way to systematically combine both types into a single database is by forced symmetry. If the dataset is to be used for probabilistic fault displacement hazard analysis [e.g. Youngs et al. 2003], forcing the slip values to be symmetric about the center introduces additional uncertainty into the analysis. This is because we do not know for sure whether surface fault displacement shows any form of symmetry. However, since we are only comparing the data from one faulting scenario to another, this additional uncertainty would be present in both and therefore is negligible in a statistical analysis of the differences between datasets.

Each slip measurement is normalized by the average surface displacement (AD) or the maximum surface displacement (MD). This ultimately yields two different sets of normalized surface displacement measurements, D/AD and D/MD . Only nonzero displacement measurements were used in computing the average surface displacement, because zero displacement implies rupture did not occur at that location. Additionally, as a majority of events were previously digitized, some measurements which were supposed to have no displacement are on the order of a thousandth of a meter and have been removed from the data set. Furthermore, we screened out displacement measurements smaller than 5 cm in this study as unreliable, especially after digitization. In general, the displacement measurements reported by Wesnousky (2008) have

along-strike distance values which start at zero on one end of the surface rupture. Any along-strike distances, however, which had the rupture beginning at a position value other than zero have been re-zeroed such that the final distance value at the end of the trace is the total length of the rupture trace.

To account for the fact that the surface rupture lengths vary substantially from one event to the next, we then normalize the position measurements, x , by the length of the surface rupture, L , yielding x/L . Secondary or conjugate fault traces are treated as independent surface ruptures with the same magnitude. This is because we feel that these ruptures are still part of the complete picture, and their omission from the set would not account for all of the slip during the event. For example, the 1999 Hector Mine event had a main trace and a secondary trace that ruptured, each with different surface rupture lengths. In this case, the position measurements for the main trace were normalized by the surface rupture length of the main trace, and the measurements for the secondary trace were normalized by the surface rupture length of the secondary trace. The 1992 Landers event however is not used in our analysis, and this choice is discussed later in the article. The normalized positions are then treated as symmetric about $x/L = 0.5$, the center of the rupture. The data then range from $x/L = 0$ to $x/L = 0.5$, with the rupture ends at $x/L = 0$. The majority of events only have a single component of slip, however 6 out of the 35 events have both a horizontal and vertical component. As these events do not have the two components measured at the same positions, it is not possible to compute a vector sum. We choose to use only the dominant slip component for these events. The normalized slip measurements for all events are available in a spreadsheet found in the electronic supplement.

Testing Slip Invariance With Magnitude

We now seek to study the effect of earthquake magnitude on the distributions for normalized slip. The 20 strike-slip events in Table 1 with $M_W \geq 6.3$ have been sorted into two magnitude ranges, $6.3 \leq M_W < 7.1$ and $7.1 \leq M_W \leq 7.9$. The reason for choosing these two ranges is to approximately split the total magnitude range in half. These two sets of data are plotted in Figure 1 for D/AD , as a function of normalized position. It can be seen that there is a general trend of the displacements increasing with x/L in both cases. The two data sets are then binned with $\Delta x/L = 0.05$, resulting in a total of 10 bins for each set. We then, for one position at a time, compare the lower

magnitude measurements to the higher magnitude measurements.

We use the two-sample Kolmogorov-Smirnov (K-S) [e.g. D Agostino and Stephens, 1986] test to compare the two data sets. This is a nonparametric test to determine whether two independent samples could have been drawn from the same population. The test is used here for D/AD first, and then later D/MD . We find that for D/AD , there is no statistical difference between the lower magnitude data set and higher magnitude set for 80% of the bins, at the 95% confidence level. Using the same process for D/MD , we find that there is no statistical difference between the two magnitude sets for 80% of the bins at the 95% confidence level. As there is a different K-S test done here at each of the 10 positions along strike, this corresponds to only two of these positions showing a statistically significant difference with respect to magnitude. These two regions along the normalized fault rupture which were rejected by the K-S tests indicate no correlation to each other spatially, and do not occur at the same places when comparing MD data to AD data. If D/AD and D/MD are to be treated as random variables, these results suggest that they are statistically invariant with respect to magnitude.

Testing Slip Dependence on Focal Mechanism

The next independent variable to test is the focal mechanism. For this comparison, we use all 20 strike-slip events with $6.3 \leq M \leq 7.9$ and 9 reverse faulting events with $5.9 \leq M \leq 7.6$. The magnitude ranges involved are approximately the same, however as it has already been shown that normalized displacement appears statistically independent of magnitude, this should have negligible impact. The two reverse slip data sets are plotted as a function of x/L in Figure 2 for both D/AD and D/MD (Again, there is a noticeable trend of increasing displacements with position). The data is then binned with $\Delta x/L = 0.05$, yielding a total of 10 bins each.

We use the two-sample K-S test as before for comparing the strike-slip measurements to the reverse slip measurements. Each position bin for strike-slip is compared to the corresponding bin for reverse measurements, to check whether the bin pairs could have been drawn from the same population. We find that for D/AD , there is no statistical difference between strike-slip and reverse faulting events for 80% of the bin pairs at the 95% confidence level. For D/MD , there is no statistical difference between the two focal mechanisms for 80% of the bin pairs at the 95% confidence level.

In addition to the strike-slip and reverse fault comparisons, normal fault-

ing must also be tested, for completeness. We use 6 normal faulting events and 9 reverse faulting events for the two different datasets in this comparison. The displacement values are normalized by both average and maximum displacement, and binned with $\Delta x/L = 0.05$ for a total of 10 bins per dataset, for each normalization type. Using a two sample K-S test, we find that there is no statistical difference in 100% of the bin-pairs, for D/MD . The tests for D/AD show that there is no statistical difference between normal and reverse slip events in 90% of the bin-pairs. This series of tests is also done at the 95% confidence level. Lastly, we tested the 20 strike-slip events against the 6 normal faulting events for both average and maximum displacement, again with bin size of $\Delta x/L = 0.05$. For D/AD , there was no statistical difference in 100% of the bin-pairs at the 95% confidence level, and for D/MD 90% of the bin-pairs showed no statistically significant difference. We feel that these results provide strong evidence that there is no statistical difference attributable to focal mechanism.

An understanding of the most likely places for the highest displacements to occur is important for fault displacement hazard in a variety of types of infrastructure projects. As shown in both Figures 1 and 2, there is an overall trend for the displacements to increase as one moves toward the center of the surface rupture, $x/L = 0.5$. To see whether maximum displacement also follows this trend, Figure 3 contains a relative frequency histogram of the 20 strike slip events. It can be seen that 55% of the events have the maximum displacement between $0.3 \leq x/L \leq 0.5$, and that there is a definite increasing trend. As each event generally produces only a single position where the displacement is the greatest, there are not enough events for the normal and reverse mechanisms to produce similar histograms. Figure 4 contains a plot of these same maximum displacement locations as a function of magnitude. A t-test indicates there is no statistically significant linear relationship between magnitude and position of maximum displacement occurrence at the 95% confidence level.

Combined Slip Database Characteristics

As a result of the invariance of normalized displacement with focal mechanism, the next step would involve combining all of the normalized slip data into a single large data set to study the general characteristics of fault displacement. Taking the normal, strike-slip, and reverse data used above, the combined set has 1842 points. In doing so, we use only the D/AD set because

the bounds for D/MD at 0 and 1 are arbitrarily restrictive on the dataset. Obtaining an equation for the mean normalized displacement as a function of normalized position, x/L , requires regressing the combined data. Figure 5 contains the combined database for AD based displacement plotted as a function of normalized position, and two general trends are noticeable. The first is that in the same manner as previous normalized displacement data used in this study, the displacements increase with x/L . Second, for D/AD , the dispersion of the data is increasing as one gets closer to the center of a surface rupture. Using a Box-Cox transformation [e.g. Kutner et al. 2004], the maximum likelihood estimate of λ , the power parameter, is 0.46, which is very close to a square-root. By using a square-root transformation on D/AD , the data takes a form that has a constant variance:

$$\sqrt{Y_i} = E(\sqrt{Y_i}) + \epsilon_i \quad (1)$$

where $Y = D/AD$, and Y_i , $E(\sqrt{Y_i})$ are the observed and expected normalized displacements, respectively. ϵ_i is an error term with a mean of zero and variance σ^2 . The transformed data are plotted as a function of normalized position in Figure 6, and it can be observed that the data now have a constant variance and appear to follow a linear trend. Estimating the mean, $\sqrt{\hat{Y}}$, using least squares regression yields the following relationship:

$$\sqrt{\hat{Y}} = \hat{\beta}_0 + \hat{\beta}_1 x. \quad (2)$$

Squaring this equation yields $\hat{Y}$, the median normalized slip:

$$\hat{Y} = (\hat{\beta}_0)^2 + 2\hat{\beta}_0\hat{\beta}_1 x + (\hat{\beta}_1 x)^2. \quad (3)$$

Here, x is normalized position, and $\hat{\beta}_i$ are regression coefficients. The regression coefficients and additional related statistics are located in Table 2. An Anderson-Darling test for normality of the residuals, with all 1842 points, indicates no statistical evidence that the residuals are not normally distributed at the 95% confidence level, and a histogram of these residuals is plotted in Figure 7. Equation 3 is plotted in Figure 5 along with the normalized displacement measurements. Using the new combined database for normalized displacement, we can then plot a new relative frequency histogram of the locations where maximum displacements have occurred. This histogram is shown in Figure 8, and is made from strike-slip, normal, and reverse faulting

events. In comparison with Figure 3, the overall trend is very similar and the frequencies have changed little.

Since the displacement measurements for any given event are correlated, and there is an event-to-event structure within the data, a random effects model [e.g. Abrahamson and Youngs (1992)] was tested as well:

$$\sqrt{Y_{ij}} = E(\sqrt{Y_{ij}}) + \eta_i + \epsilon_{ij} \quad (4)$$

where Y_{ij} , $E(\sqrt{Y_{ij}})$ are the observed and expected normalized displacements corresponding to the i th event and j th measurement. η_i and ϵ_{ij} are independent, random normal variates with mean zero and variances τ^2 and σ^2 , respectively. τ is said to be the inter-event standard deviation and σ is said to be the intra-event standard deviation. The regression results for τ and σ are 0.006 and 0.296. This indicates the intra-event variability represents a majority of the total variability, σ_T , since

$$\sigma_T = \sqrt{\sigma^2 + \tau^2}. \quad (5)$$

These values were obtained first with the Abrahamson and Youngs (1992) algorithm, and confirmed with the LMER random effects package in R [R Development Core Team, 2004]. This suggests that the use of a random effects model is unnecessary, and that a simple fixed effects model, such as Equation 1, is sufficient.

Discussion

Previous studies [Savage and Brodsky, 2010, Youngs et al., 2003, Hemphill-Haley and Weldon, 1999] have suggested that the distributions for normalized displacement may be scale invariant, both for subsurface rupture and surface rupture scenarios. When used in probabilistic fault displacement calculations, it has been assumed *a priori* that only along-strike position has an affect on the shape and scaling of the normalized slip distributions. In our tests, 80% of the position bins show no statistically significant difference from one magnitude range to another. We feel this is a strong indication that normalized displacement does not scale with magnitude, nor alter the distribution shape. Maximum (and average) displacement have been shown empirically to increase with magnitude [Wells and Coppersmith, 1994]. By

normalizing displacement measurements, this magnitude dependence cancels out, rendering the overall normalized slip distributions scale invariant.

With regards to the bins that are rejected as being statistically indistinguishable in the Kolmogorov-Smirnov test, it is highly probable that this is due to epistemic uncertainty in the form of measurement errors. Measurement errors are often present in field investigations, and in the past many teams investigating the same event have produced different results. This could certainly explain any disparities between a slip distribution at one magnitude range compared to another, as well as different types of focal mechanisms. As larger magnitude events have higher amounts of slip, the measurement error, if assumed to be generally the same for all size events, would be a smaller part of the total normalized slip compared to a smaller event. This could be sufficient for causing these rejections. It is also possible that there is epistemic uncertainty due to a variable rupture process that is unevenly distributed between the two groups. This could be due to an ergodic assumption in sampling slip measurements from events all over the world [Anderson and Brune, 1999], and ultimately could cause the respective bins to be rejected in the K-S tests if the assumption does not hold true. However, we expect that any differences in epistemic uncertainty are relatively small, as less than 20% of the bin-pairs in any series of K-S tests were rejected as being statistically indistinguishable. Furthermore, as we use a 95% confidence level in our goodness-of-fit tests, even if the epistemic uncertainty was zero in all of the comparisons, under the null hypothesis we expect the K-S test to yield a Type 1 error 5% of the time. This error corresponds to a false rejection of the samples as coming from the same population.

So far much of the discussion has been about magnitude scale invariance resulting from the K-S tests due to normalization by AD or MD , however normalized position cannot be overlooked. In this study we normalized along-strike position by the length of the surface rupture to account for a high variability among surface rupture lengths. As surface rupture length has been shown to be correlated to magnitude [Wells and Coppersmith, 1994], by sorting the strike-slip events into two different magnitude ranges, this also means that the events have been sorted into primarily two different surface rupture length groups. Specifically, the range for surface rupture lengths among the higher magnitude group is 40-421 km, and 0-60 km in the smaller magnitude group. These two ranges overlap between 40 and 60 km, however there is only one event from the larger magnitude range that falls in this region, the 1999 Duzce, Turkey event. Every other event in the

larger magnitude range has a total surface rupture length exceeding 75 km, and thus these two groups remain approximately separate with respect to surface rupture length. As this indicates a higher rupture length group was equivalently tested against a smaller rupture length group, the same K-S test results are valid in assessing invariance with surface rupture length. Thus normalizing both variables are legitimate methods for combining events into a single data set.

A surprising result of this study is that the slip distributions appear to be statistically independent of focal mechanism. As at most 20% of the bin-pairs tested in the K-S tests were rejected as having a statistically significant difference, we feel that this is strong evidence of invariance. This is especially so because some of the tests showed no bin-pairs being rejected. Given that strike-slip, normal, and reverse faults have all been tested to ensure that the results still hold, we conclude that surface faulting is primarily a spatially dependent variable, with much of this dependence appearing to come from boundary effects created by the finite extent of the rupture plane.

For probabilistic fault displacement hazard analysis (PFDHA) [Youngs et al., 2003], these results provide statistical evidence that the distribution for normalized slip is scale invariant. This allows for the proper estimation of slip on a fault due to surface rupture. Furthermore, as only a small number of events for each focal mechanism and magnitude have fully mapped displacements, these results allow for a more precise modeling of surface faulting phenomenon due to a larger and more complete database because of invariance properties. Without the focal mechanism independence, a distribution for reverse normalized slip would be composed of only 9 events. Now, however, it can be calculated using 35 events and capture more of the variability in surface faulting.

The results of combining all of the data from all focal mechanisms together in a single database are shown in Figure 5 for average displacement. This combined database exhibits the same trends as the data for the individual focal mechanisms, as expected from the results of the Kolmogorov-Smirnov tests. The trend of increasing displacements closer to the center of a surface rupture is clearly an artifact of the faulting process. We believe that it is due to edge effects, and that the center of the rupture is may possibly be free of these effects. The linear regression function obtained is now usable in PFDHA as a spatial variability distribution, for estimation of slip along a fault. Previously determined models for spatial variability distributions [e.g. Youngs et al. (2003), Moss and Ross (2011)] used gamma, Weibull,

and beta distributions fit to the data either through maximum likelihood estimation or by point estimates of distribution parameters regressed over normalized position. These distributions were chosen for flexibility and have variable distribution parameters to accommodate a non-constant dispersion. By transforming the data in this study, we have shown that it is possible to find a relationship between D/AD and x/L in which the transformed residuals are normally distributed, the transformed variance is constant, and is fit by a simple linear regression function.

As stated previously, the 1992 Landers event was not used in the regression analysis. The primary reason is because six of the nine fault traces associated with this event have lengths of less than 10km. To be consistent with our definition of a surface rupture, this would require treating each trace as an individual surface rupture. For these extremely small traces, there are only a small number of slip measurements, and we feel that any uncertainty in position associated with these measurements would have significantly more influence than a larger fault trace would. This is because any error in the slip measurements would have a strong influence on the average slip computed for the particular trace, as most of these traces have only around five points. In comparison, most of the events equivalent in size have traces with over 80 points. Furthermore, with such few points per trace, the points at the end of a rupture which are forcibly lower due to the boundary conditions would have a disproportionate influence on the average displacement. Wesnousky (2008) obtained a cumulative slip curve for the combined fault system which treats the slip measurements as coming from a single trace. Figure 9 shows the normalized slip measurements from this cumulative slip curve plotted against the values predicted by our regression equation. The slip values obtained near the ends of the cumulative slip trace are overpredicted by the regression equation, and the location of cumulative maximum slip, including its immediate vicinity, have slip values that are underpredicted by the regression function. It should be noted that as shown previously, while the higher displacements tend to occur near the center of the fault rupture, this is not always the case, and these results illustrate natural variability with respect to a regression model. Approximately 61% of the slip measurements from this event fall within one standard deviation of the mean value predicted by the regression function, so even for this cumulative slip curve the results hold up fairly well.

Conclusion

We investigate the properties of slip distributions on faults due to surface rupture with a set of 35 events with magnitudes $5.9 \leq M_W \leq 7.9$. We show that it is possible to normalize surface displacement and fault position to account for earthquake events that are dissimilar. By assessing which earthquake characteristics are invariant, it allows for a systematic combination of a large number of events into a single database for statistical analysis. As both aleatory and epistemic uncertainties exist in the faulting process, studying a large number of events helps to properly account for this variability.

We show, using a two-sample Kolmogorov-Smirnov test, that the distribution of normalized slip on a fault appears to be statistically independent of magnitude. This indicates invariance with respect to surface rupture length as well. These tests are conducted with both average displacement and maximum displacement as normalization factors at finely spaced intervals along normalized strike position. As normalized displacement has previously been modeled with the *a priori* assumption that it is scale invariant, this provides statistical evidence that the assumption is correct and ultimately allows for the estimation of fault displacement using probabilistic hazard analysis techniques. In addition to magnitude scale invariance, we find that the distributions of normalized slip are statistically independent of focal mechanism. All focal mechanisms are tested to ensure statistical invariance holds. This allows for data from all mechanisms to be combined, creating a single large data set. An empirical relationship for slip is found that involves a simple linear regression function, offering greater simplicity compared to previously proposed distributions for modeling normalized slip. As the faulting process is highly complex, it is important to look for ways to simplify the relationships between characteristic variables. By looking for similarities in events on large scales we can hope to better understand the faulting phenomenon.

Data and Resources

All data used in this paper came from published sources listed in the references.

Acknowledgements

This material is based on work supported by the U.S. Department of Homeland Security under Grant Award Number 2008-ST-061-ND0001. Administration of this grant is conducted through the Department of Homeland Security Center of Excellence for Natural Disasters, Coastal Infrastructure and Emergency Management (DIEM). The views and conclusions contained in this document are those of the authors and should not be interpreted as necessarily representing the official policies, either expressed or implied, of the U.S. Department of Homeland Security.

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Table 1: Database of surface rupture events used

Date	Event	M_W	Type*	Reference
01/09/1857	San Andreas, CA	7.9	SSR	Wesnousky (2008)
05/03/1887	Sonora, Mexico	7.2	N	Wesnousky (2008)
10/28/1891	Neo-Dani, Japan	7.3	SSL	Wesnousky (2008)
08/31/1896	Rikuu, Japan	7.2	R	Wesnousky (2008)
10/02/1915	Pleasant Valley, NV	7.3	N	Wesnousky (2008)
11/02/1930	Kita-Izu, Japan	6.7	SSL	Wesnousky (2008)
12/25/1939	Erzincan, Turkey	7.7	SSR	Wesnousky (2008)
05/19/1940	Imperial, CA	6.9	SSR	Wesnousky (2008)
12/20/1942	Erbaa-Niksar, Turkey	6.8	SSR	Wesnousky (2008)
11/26/1943	Tosya, Turkey	7.6	SSR	Wesnousky (2008)
09/10/1943	Tottori, Japan	6.3	SSL	Wesnousky (2008)
02/01/1944	Gerede-Bolu, Turkey	7.35	SSR	Wesnousky (2008)
01/31/1945	Mikawa, Japan	6.2	R	Wesnousky (2008)
12/16/1954	Fairview Peak, NV	7.0	NSSR	Wesnousky (2008)
12/16/1954	Dixie Valley, NV	6.8	N	Wesnousky (2008)
08/18/1959	Hebgen Lake, MT	7.0	N	Wesnousky (2008)
07/22/1967	Mudurnu, Turkey	6.7	SSR	Wesnousky (2008)
04/08/1968	Borrego Mtn, CA	6.1	SSR	Wesnousky (2008)
02/09/1971	San Fernando, CA	6.7	R	Wesnousky (2008)
06/02/1979	Cadoux, Australia	6.1	R	Wesnousky (2008)
10/15/1979	Imperial Valley, CA	6.3	SSR	Wesnousky (2008)
10/10/1980	El Asnam, Algeria	6.7	R	Wesnousky (2008)
10/28/1983	Borah Peak, ID	6.9	N	Wesnousky (2008)
03/03/1986	Marryat, Australia	5.9	R	Wesnousky (2008)
11/23/1987	Superstition Hills, CA	6.4	SSR	Wesnousky (2008)
01/22/1988	Tennant Creek, Australia	6.6	R	Wesnousky (2008)
07/16/1990	Luzon, Philippines	7.6	SSL	Wesnousky (2008)
03/14/1998	Fandoqa, Iran	6.6	SSR	Wesnousky (2008)
08/17/1999	Izmit, Turkey	7.1	SSR	Wesnousky (2008)
09/21/1999	Chi-Chi, Taiwan	7.4	R	Wesnousky (2008)
10/16/1999	Hector Mine, CA	6.9	SSR	Wesnousky (2008)
11/12/1999	Duzce, Turkey	7.0	SSR	Wesnousky (2008)
11/14/2001	Kunlun, China	7.8	SSL	Wesnousky (2008)
11/03/2002	Denali, AK	7.7	SSR	Wesnousky (2008)
10/08/2005	Kashmir, Pakistan	7.6	R	Kaneda et al. (2008)

Table 1: *Type of earthquake. N is normal, R is reverse, and SSR and SSL are right lateral and left lateral strike-slip, respectively.

Table 2: Regression coefficients and results for combined slip database

	Value
$\hat{\beta}_0$	0.726
$\hat{\beta}_1$	0.817
σ	0.297
R^2	0.131

Figure 1: Datasets for normalized strike-slip measurements as a function of normalized along-strike position. The above dataset has $6.3 \leq M_W < 7.1$, and the bottom dataset has $7.1 \leq M_W \leq 7.9$. The displacements show an increasing trend with position, with the highest values generally found around the center of the rupture, $x/L = 0.5$. More points exist for the higher magnitude range because the surface ruptures are substantially longer. The dispersion is found to increase with normalized position in both datasets.

Figure 2: Datasets for normalized reverse slip measurements as a function of normalized along-strike position. This database is made from 9 reverse slip events with $5.9 \leq M_W \leq 7.6$. The displacements show the same increasing trend with normalized along-strike position as the strike-slip measurements, along with a similar trend for the dispersion. The above dataset is for D/MD , and the bottom set is for D/AD .

Figure 3: Relative frequency histogram for strike-slip showing the fraction of maximum displacement events at each position. The trend increases in the same manner as the actual displacements themselves, indicating the higher displacements are most likely to occur near the center of a fault rupture.

Figure 4: Plot of the position that each maximum displacement value occurs, as a function of the event magnitude. There is no statistically significant relationship between these two variables at the 95% confidence level.

Figure 5: The combined database for normalized slip as function of position using average displacement. This database is composed of 35 events with reverse, normal, and strike-slip mechanisms based on the invariance results. The black line is the median normalized slip, which is obtained after using a square root transformation on the normalized slip measurements.

Figure 6: The upper figure contains a plot of transformed normalized slip values as a function of position. The transformed values now have a constant variance and obey a simple linear regression model. The line is fit using least-squares to estimate the mean, $\sqrt{\hat{Y}}$, and the curves corresponding to the mean plus and minus one standard deviation are shown. The lower plot contains the same curves and data after transforming back to the original space.

Figure 7: Histogram of residuals from the simple linear regression model after D/AD is transformed to $\sqrt{D/AD}$. The residuals show only a modest departure from a normal distribution and an Anderson-Darling normality test with the 1842 points was not rejected at the 95% confidence level.

Figure 8: Relative frequency histogram for all focal mechanisms showing the fraction of maximum displacement events at each position. The higher probabilities of maximum displacement are found closer to the center of the fault in the same manner as Figure 3, and the probabilities are very similar.

Figure 9: Landers normalized slip measurements plotted against the corresponding values of x/L , compared to the values predicted by the regression function. The model overpredicts D/AD at low values of x/L and underpredicts D/AD in the vicinity of the maximum slip, which occurs at $x/L = 0.28$. As the largest displacements do not always occur at the center, this is expected.