AN ADVANCED CONTROLLER FOR A
SEMI-ACTIVE WHEELCHAIR
SUSPENSION

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ABSTRACT

An Advanced Controller for a Semi-Active Wheelchair Suspension

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An Advanced Controller for a Semi-Active Wheelchair Suspension was designed, built and tested. The suspension consisted of a Goodyear 1S3-011 air spring, IQ Valves high speed proportional solenoid valve, and a custom made accumulator. Several controller designs specific to semi-active suspensions were designed and tested. The controllers investigated were skyhook, acceleration driven damping, and a combined control law employing both a dual and single sensor version. The implementation of skyhook control suffered performance degradation from the idealization due to particular elements of hardware, however acceleration driven damping showed a marked and statistically significant improvement over skyhook control, in hardware, by 14%. The combined control laws exhibited as yet unexplained transient behavior that produced results with low confidence in their veracity. All controllers proposed performed better than a conventional oil damper and spring type suspension.

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Thanks Mom.

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LITERATURE SURVEY

The need for an improved ride comfort in wheelchairs is widespread and explicitly mentioned throughout the literature. The current state of wheelchair rides is undesirable for any medium to long duration use. The majority of research relies on the ISO 2631 document which outlines a method to quantify human health and comfort in a vibration environment. This document allows one to assess the vibration environment based on magnitude, frequencies, and duration of exposure and analyze the data to achieve a single value which can then be used to compare vibration environments. Not only does this document quantify the vibration environment, but states that individuals in a seated position are at risk when subjected to whole body vibration (WBV) environments.

Previously, large scale studies of the population tended to support the idea that chronic vibrations lead to lower back pain as well as spinal injury. A more recent set of studies lessened the credibility of this theory. A recent survey of the state of knowledge in this area noted that current studies lack a thorough exploration of frequency, amplitude, and duration. This oversight may be the primary factor leading the studies to inconclusive or contradictory results. The survey concludes that vibration is still an important consideration and that further controlled studies of the hypothesis are warranted (Hill, Desmoulin, & Hunter, 2009).

For wheelchair riders, low back pain is a common complaint. Vehicle operators that are exposed to whole body vibration also have high rates of low back pain complaints. In certain circumstances, it was found that the vibration levels in wheelchairs are too high for long term exposure and comfort, with regard to the ISO 2631 standard (Weisman & Huston, 1995).

A 2007 study investigated the health risks for users of both manual and powered wheelchairs over a variety of seemingly benign sidewalk surfaces and over a period of three years. The results of the studies showed that both manual and powered wheelchair users may be at risk of secondary injuries from WBV while traveling over common surfaces (Wolf, Cooper, Pearlman, Fitzgerald, & Kelleher, 2007).

A similar study to the previous also investigated the vibration exposure of individuals using wheelchairs on a variety of sidewalk surfaces. The focus of this study was investigating the relative safety of the vibration environment caused by different surfaces' finishes common to routes typically encountered by wheelchair users. The primary focus of this paper was to determine which surfaces provide the safest and most comfortable ride, with regard to ISO 2631. The results of this paper concluded that a particular concrete pour and pattern is optimal for paths traveled by wheelchair users (Wolf, et al., 2005). While this may be suitable for future use in areas where wheelchairs frequent, it would be prohibitively expensive to replace existing infrastructure, lending to the idea of improving the wheelchairs themselves as the practical option.

There have been some recent advances in wheelchair technology with regard to human health and comfort. One study investigated the vibration characteristics of a luxury wheelchair and a prototype wheelchair. This study also confirmed what is documented in the literature, which is that the human body is very sensitive to the frequency range of 0.5-10 Hz. The results of the study concluded that neither wheelchair, both using passive suspensions, performed sufficiently. In fact, the suspension amplified the input signal at a resonant frequency near 3-4.5 Hz (Hostens, Papaioannou, Spaepen, & Ramon, 2003). As the aim of a suspension in this case is rider comfort, amplification of these commonly encountered frequencies is unacceptable.

The paper also concludes that future development must improve the low frequency comfort while maintaining affordable costs.

Some comparison has been done examining the difference in vibration environments when using a wheelchair with and without suspension as well as comparing a front, caster-based suspension and rear, seat-based suspension. The results of this study show that the use of suspension caster forks or a seat based suspension both reduced the peak accelerations as well as shifting the frequency at which it occurred. Though these suspensions did show marginal improvement over a non-suspended system, they are not optimized for use with humans. Both types of suspensions tended to transmit peak accelerations in the resonant frequency range for humans, which is undesirable. Though there were not significant improvements between suspension caster forks and seat-based suspensions the researchers did make one potent observation.

The wheelchairs with rear suspensions performed better, reducing the vibration power by nearly one half, in the octave between 7.81 and 9.84Hz. This octave is within the range of the natural frequency of seated humans. It is interesting to note that there was also a significant difference at the octave between 12.40 and 15.63Hz, with the passive wheelchairs showing a lower vibration power. These results indicate that the vibration transmitted to the wheelchair at the lower octave (7.81–9.84Hz) may be shifted to the higher octave (12.40–15.63Hz) by the rear-suspension system. Shifting vibrations away from the natural frequency range of humans and reducing the amplitude or power are all desirable for reducing the risk of injury due to cumulative trauma. Ideally, the frequency shift would be higher to ensure no overlap with the natural

frequency range of humans. Once suspension systems are shown to reduce vibration within the entire natural frequency range of humans, they could be recommended as a further means of reducing the risk of injury (Cooper, Wolf, Fitzgerald, Bonninger, Ulerich, & Ammer, 2003).

MECHANICAL DESIGN AND SYSTEM MODELING

To familiarize the reader with the design process a short description of the minimum system requirements and functionality is given here. An Invacare Tracer EX-2 manual wheelchair was retrofitted to accept a new suspension. The suspension is comprised of an air bag to act as the spring, an air accumulator, and a valve with an electronically controlled variable orifice size with which to provide damping via air flow across it between the air bag and accumulator.

FRAME DESIGN

The goal of the mechanical design process was to meet all of the basic requirements for the structure and iterate towards a simple, easily buildable design. The wheelchair had to be modified such that the new suspension hardware could be added to create a rigid structure that would withstand the vibration environment employed for testing. The design required that test fixtures be available and readily usable for final testing of the control algorithm.

The wheelchair donated to the project was an Invacare Tracer EX-2 collapsible wheelchair. All extraneous hardware was removed to allow for the retrofit of the new suspension. Because of the folding-type design of this wheelchair, a new frame securing the two halves of the chair was required. Many iterations of this design were considered starting from an original design consisting of a full steel hardback to three c-channel cross members. The requirements of the additions to the wheelchair frame

were for it to hold the original hardware in a rigid orientation, keeping both halves parallel and resist any twisting or bending. Several of the design iterations as well as the final design are shown in Figure 1 and Figure 2.

Figure 1: Design iterations

Figure 2: Final design for prototype

Figure 3: Final hardware for prototype

Figure 4: Top down view inside blue box

Steel C-channel was chosen for its strength and economy. Two C-channels were used across the back of the wheelchair frame and a third across the lower wheelchair frame tubing. By using these three C-channel members the original chair frame could be held in the desired orientation. These cross members also provided a convenient mounting surface for additional hardware.

Securing the C-channel to the wheelchair frame via $\frac{1}{4}$ inch bolts using existing holes in the frame was considered, but that required drilling of additional holes for the lower cross-member. Instead, rubber coated U-bolts were utilized for securing the cross members to the wheelchair frame. The choice of U-bolts was as appropriate option because the tubular sections of the original wheelchair frame provided a circular contact surface to fasten the new structure securely. The rear, lower cross member rests on the lower leg of the wheelchair frame, essentially grounding the cross member and minimizing the loading on the U-bolt. When tightened, the rubber coating allowed the U-bolt to make a secure connection between the cross member and frame.

The dimensions of the C-channel were such that the inner face of the c-channel rested above the mounting nuts on the surface of the shaker. This provided several mounting points to allow for attaching the wheelchair to the shaker table. This design provides the ability to easily place the center of gravity of the system in line with the center of the shaker table, so that the torques were balanced to prevent any rocking or pitching motion.

A third mounting point was added at the rear of the system connecting the lower, rear C-channel to the shaker table surface through use of an L-bracket. This third mounting point was required to provide additional stiffness to resist pitching movement due to any possible mass imbalance.

ADDITIONAL HARDWARE

The air spring by itself has no lateral rigidity with which to constrain its motion in that direction and must be supplemented to restrict the motion to the desired path. In order to account for this, an aluminum plate was fitted with four Thompson pillow blocks with case hardened steel guides. These low friction pillow blocks moving on the guides provide the needed path restriction while introducing minimal friction and resistance to the system. Mounting accessories were used at the ends of the guides to secure the end of guides and have through holes that allow the linear guide to be mounted to the C-channel cross members.

A deadweight was required to simulate the loading produced by a typical passenger. One design considered consisted of a large water tank that could be easily filled and drained. Although, the primary advantage of this dead weight design was the ease of transportation to and from the shaker table, there would be potential leaking as well as sloshing effects during the testing, which are undesirable. A much more suitable

solution was found by using six, 25 lb. weight lifting plates. These plates proved to be easy to transport, simple to safely secure during testing, and more compact in footprint.

A box was constructed to contain the dead weight, valve, and accumulator. The box was directly fixed to the aluminum back plate on which the pillow blocks were mounted and was seated on the air spring. By design of the box, the deadweight was evenly placed and centered over the air spring. Utilizing the weight lifting plates and properly sizing the box, it helped in securing the plates by minimizing any room that would allow them to move. Once secured, the plates were nearly rigid in their position. Using this design, any bending torque created by the forward setting of the weight from linear guide backing would be supported by the air spring, since the center of gravity was located directly over the air spring.

The Goodyear 1S3-011 air spring was chosen as the suspension spring for its small size and desirable properties within the operating range as well as proven use with the technology. The 1S3-011 measures a mere 5.5 inches at the nominal, pressured height. Within this height range, the spring characteristics are highly linear as shown in Figure 5.

Figure 5: Load vs Deflection for a range of constant pressures as given by the 1S3-011 datasheet provided by Good Year

The valve used to obtain the variable damping phenomenon for this system was the isolation balanced standard proportional valve from IQ Valves. This valve was chosen for its fast response time of less than 35 mSec. The valve is pressure balanced and insensitive to inlet and outlet pressures. The valve operates on a supply voltage of 24 volts, 0-0.25 amps, and a command signal voltage of 0-10 volts. An amplifier/controller card, supplied by IQ valves, was used for actuation of the valve. The control signal to the system was generated in real time using MATLAB/Simulink and was output via a DAQ card.

Figure 6: Performance characteristics for IQ Valves isolation balanced high speed proportional solenoid valve

The requirement for a pneumatic accumulator was satisfied by modifying a 1 lb. propane container, since the volume of the 1 lb. propane fell within the bounds of the desired accumulator volume. The original valve stem system for the propane tank was removed and a common propane fitting was modified to go from the propane threads to NPT. The box and weight lifting plates allowed the accumulator to be attached directly to the high-speed valve thereby reducing the number of fittings, adaptors, and tubing/piping necessary for the connection.

Figure 7: Custom adaptor used for NPT to straight thread

A custom adaptor was necessary to go from the straight threads of air spring to the NPT threads of the rest of the pneumatic system. The design of the adaptor was carried out in Solidworks and then machined in a 2"x2" aluminum block. The adaptor served an additional purpose as a T-junction with two ends serving as the straight to NPT adaptation and the third port being used for a resupply in the event the pneumatic system loses air and the mean ride height fall below the nominal design height.

The issue of potential leaks in the pneumatic system and sagging of the seat from the design height was addressed by use of a controlled release height control valve. A typical height control valve reacts immediately to any deviation from the nominal height. The controlled release type design contains a fluid which must drain before the valve reacts and bleeds or resupplies air to the system. In a manner, the controlled release height control valve acts as a high pass filter. The high frequency content of a typical ground profile induced vibration environment can be assumed to be distributed above and below zero. Based on this behavior, the height control valve will

only resupply the system as it leaks over time and has a steady-state offset from the nominal. For the sake of portability, a 5 gallon propane tank, nominally kept at 100 psi, was used as a backup reservoir.

SENSOR SELECTION: STUDY AND ANALYSIS

For this particular system there are a number of states that could be measured. The typical passive suspension is comprised of a sprung or suspended mass, the seat in this case, and the unsprung mass, the wheelchair frame. For this linear translational system, there are six potential states which could be measured, viz, the body position, velocity, or acceleration as well as the respective states of the chassis. It was determined after investigating several suitable control schemes that the measurement of the relative displacement between the seat and frame and the acceleration of the frame would be sufficient data to provide to the controller. By measuring the relative displacement of the seat and frame and taking a numerical time derivative of the relative velocity, $(\dot{x} - \dot{d})$, could be reliably determined. Integration of the frame acceleration would provide $\dot{x}$. Simple subtraction of this data from the relative displacement of the seat and frame then provides the velocity of the seat. Though the mathematical derivation of this process is straightforward, sensor choice is critical for obtaining a good signal to noise ratio for an effective implementation of the control.

STRING POTENTIOMETER

For displacement measurements several varieties of sensors that are readily available can be utilized. Laser, ultrasonic, radar, and magnetic are a few of the numerous varieties. All these prove to be impractical in this application since the laser and radar systems are very expensive and ultrasonic and magnetic devices are noisy and unreliable. A cable extension displacement transducer, also known as a variable resistance string potentiometer (string pot), is a reliable and easy to implement method

for measuring displacement. The typical signal obtained from string pot contains minimal noise, is highly linear, contains no hysteresis, and has little to no potential for outside interference, in addition to being available in a relatively inexpensive package. The Celecso SP1 string pot was chosen for its reputation as well as ease of use. The Celecso SP1 offers a variety of full stroke ranges from which the 0-4.75" was selected. This displacement range would provide distinct signal variation over the stroke of the wheelchair suspension, thus providing an excellent data source. The accuracy of the SP1 is $\pm 1\%$ which, for this application, is excellent. Additionally the SP1 proves to be a robust unit capable of 2.5×10^6 cycles as well as being capable of handling accelerations up to 15 G. Finally, as an analog device the limit of the resolution of the signal is essentially infinite.

Figure 8: Celecso SP1 cable extension transducer

ACCELEROMETER

Measurement of accelerations is most directly accomplished via an accelerometer. Though other signals can be measured and processed, the best method for acceleration signal measurement is direct measurement via an accelerometer. A number of accelerometer designs and technologies exist and are readily available. The

most common accelerometer technology is the piezoelectric accelerometer. This design makes use of the piezoelectric effect of some materials to dynamic changes. The advantage of this design is the relatively low prices compared to other accelerometer technology. Piezoelectric accelerometers are physically robust, but the primary disadvantage to this design is the frequency bandwidth for reliable measurement. All piezoelectric sensors are hindered by the fact that they are incapable of measuring a 0 Hz signal and this theoretical limit is often several Hz higher in practice. There is usually a cut-off frequency below which piezoelectric accelerometers cease to give a reading or give an unreliable reading. This is due to the tendency of the piezoelectric material itself to fade in voltage output, over time, for a constant acceleration, or 0 Hz signal, such as gravity.

The range of frequencies encountered in this project fall into the range where piezoelectric accelerometers provide either poor or no signal. Hence, a second accelerometer technology was considered which does provides excellent performance in this frequency range. The micro electro-mechanical system (MEMS) DC type accelerometer is becoming much more popular for its ability to measure a constant acceleration or 0 Hz signals. The excellent performance of the MEMS accelerometer in the range of frequencies expected to be experienced in this project made it an excellent candidate for selection. The PCB 3741D4HB2G MEMS DC accelerometer gives a 1000 mV/g ($\pm 5\%$) reading up to accelerations of 2 g and over a frequency range of 0 to 70 Hz. The signal conditioning is included in the package as well, reducing the need for elaborate or extensive post-reading filtering.

Figure 9: PCB MEMS accelerometer 3741D4HB2G

STATE MEASUREMENT STUDY

For the previously mentioned method of measuring the absolute velocity of the seat, a simulation using deterministic signals was performed in order to design a basic filter and show a proof of concept for the method. Two deterministic signals were created as superimposed sine waves to represent an arbitrary motion of the wheelchair frame excitation and the ensuing motion of the seat. The arbitrary and created nature of these test signals allowed for easy comparison between the expected values and the results of the simulation. To simulate the noise and illustrate necessity of filtering, white noise centered about zero was then added to the deterministic signal. The simulated measurement, with the added noise, was then filtered and processed, as described previously, to determine the velocity of the seat. One of the most immediate and notable results of the simulation study was the potential drift that was inevitable due to any offset or imbalance of the noise about zero. In a straightforward open loop integration and estimation of these values it could prove to be disastrous if an ever increasing numerical error accrued over time. In Figure 11 and Figure 12 the results of the initial filtering simulation are shown to illustrate the various effects of different filter designs.

$$d(t) = \sum_{i=1}^{11} x_i \sin(\omega_i t) \quad (1)$$

Table 1: Frequencies and magnitudes of superimposed sine wave

freq [Hz]	Mag
0.50	10.00
0.75	8.00
1.00	6.40
1.25	5.12
1.50	4.10
1.75	3.28
2.00	2.62
2.25	2.10
2.50	1.68
2.75	1.34
3.00	1.07

Figure 10: $d(t)$ used for initial filtering simulation

Figure 11: Filtering simulation result

Figure 12: Filtering simulation result

The results of this work helped develop an understanding for the filter designs that would be appropriate to use in the filter with the actual hardware. Initially, for both sensors a type II 8th order Chebyshev low pass filter was used on the incoming readings with a stopband edge frequency of 10 Hz. Such a low cutoff frequency significantly

eliminates the amount of noise with an acceptable level of phase shift. Later it was found that the 100 Hz stopband edge produced better results due to the fact that smoothness of the signal was only required near the zero crossing because of the particular design of the controller.

Figure 13: Filtering results of string pot reading

Figure 14: Time derivative of string pot reading

Figure 15: Filtering results of accelerometer readings

Figure 16: Integrated accelerometer signal using HPF to remove DC offset

SYSTEM MODEL

In order to better facilitate the development and testing of the controllers, a state-space model representing the system was developed. Common and convenient techniques used for analyzing systems are focused on linear, time-invariant models (LTI). The nature of the controller developed in the next section is based on variable system damping, $c(t)$, resulting in a time-varying system model as show in Figure 17 and Figure 19, where $x(t)$ is the displacement of the sprung mass and $d(t)$ is the disturbance input in the form of a displacement. This model was later used to develop results which were used for comparative purposes between the passive and active suspensions.

Figure 17: Representative spring, mass, damper system for modeling wheelchair system with variable damping

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -k/m & -b/m \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} b/m \\ k/m - b^2/m^2 \end{bmatrix} d(t) \quad (2)$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} d(t) \quad (3)$$

where

$$x_1(t) = x(t) \quad (4)$$

$$x_2(t) = \dot{x}(t) - \left(\frac{b}{m}\right)x(t) \quad (5)$$

In order to study the system with variable damping, the above was reconfigured as a mass, spring, and force actuator as shown in Figure 18.

Figure 18: Mass, spring, damper system reconfigured to represent variable damping as force actuator

$$\begin{bmatrix} \dot{x}(t) \\ \ddot{x}(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -k/m & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ k/m & 1/m \end{bmatrix} \begin{bmatrix} d(t) \\ f(t) \end{bmatrix} \quad (6)$$

$$y = [1 \quad 0] \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix} + 0 \quad (7)$$

where

$$f(t) = c(t)(\dot{x}(t) - \dot{d}(t)) \quad (8)$$

CONTROLLER DESIGN

The controller design for this system required special consideration. The suspension in this case is classified as “semi-active.” In a technical differentiation between active and semi-active there are two main points of consideration. The primary point of difference between the two suspensions is that the semi-active suspension cannot add energy into the system. A semi-active suspension may only alter the damping properties of the system. The complimentary advantage that the semi-active suspension holds over a fully active system is the low power requirement. A corollary to the first difference is that the control force of a semi-active suspension is limited in direction to opposing the relative velocity of the sprung and unsprung masses. Several

control laws specific to semi-active suspensions were explored through both simulation and experiment. An explanation of each control law is presented in the following sections.

SKYHOOK

Skyhook control is derived from the consideration of a reconfiguration of a typical suspension as shown in Figure 19. A typical spring, mass, damper system is reconsidered with the damper portion connected to an inertial reference frame. Though this idealization is unrealizable in most practical implementations, it does lead to a powerful control law that can be suitably modified for application. Note that the assumption made in the skyhook configuration is that the damper may provide a reactive force that is proportional to absolute velocity of the body, sprung mass, and not the relative velocity between the sprung and unsprung mass.

Figure 19: Representative schematic for skyhook control derivation

Figure 20: Used with permission (Porumamilla, 2007)

Figure 21: Used with permission (Porumamilla, 2007)

$$\begin{cases} c(t) = c_{max}, & \text{if } \dot{x}(\dot{x} - \dot{d}) \geq 0 \\ c(t) = c_{min}, & \text{if } \dot{x}(\dot{x} - \dot{d}) < 0 \end{cases} \quad (9)$$

This switching condition is a two state approximation of the ideal skyhook control. The advantage of the two-state control law is the simplicity in the hardware required. As opposed to a more expensive piece of hardware with continuously variable damping, a two state damper can be employed. The switching condition can be intuitively explained when reasoned through a few simple example cases. If one considers a situation in which the seat is traveling downward toward the base and the base is traveling upward toward the seat, it would be desirable to have the suspension as stiff as possible to prevent the base and seat from bottoming out as well as keep the seat as close to the nominal height as possible. A converse situation in which the seat has no movement,

but the base is suddenly caused to move away from the nominal position, it would be desirable to have a suspension with no damping for minimum transmissibility.

$$\begin{cases} c(t) = \text{sat}_{[0, c_{max}]} \left[\frac{\alpha c_{max}(\dot{x} - \dot{d}) + (1-\alpha)c_{max}\dot{x}}{(\dot{x} - \dot{d})} \right] & \text{if } \dot{x}(\dot{x} - \dot{d}) \geq 0 \\ c(t) = 0, & \text{if } \dot{x}(\dot{x} - \dot{d}) < 0 \end{cases} \quad (10)$$

A second approximation of the skyhook control law is given above. This is known as the linear version of skyhook control as it implements a continuous control law over the domain of available damping. In this version of skyhook, α is typically 0.5 in order to distribute the damping evenly between sprung and unsprung mass.

ACCELERATION DRIVEN DAMPING (ADD)

ADD is a recent development to the available control laws as applicable to semi-active suspensions. Contrary to the typical two state implementation, or other more complex methods of skyhook control which are meant to approximate the ideal case, ADD can be shown to be an optimal control method. By stating the problem of a quarter car model as an optimal control problem with the rider comfort, as characterized by body acceleration, as the cost function metric, “The Maximum Principle of Pontryagin” can be used to show that the optimal control method for a semi-active suspension, with a road preview, and using rider comfort as the control objective is in fact a two state on-off strategy (Savaresi, Silani, & Bittanti, 2005).

In form, this control law very closely resembles the skyhook control law, except that now the determination of switching condition is based on the acceleration, as opposed to velocity, of the body and the elongation speed of the suspension.

$$\begin{cases} c(t) = c_{max}, & \text{if } \ddot{x}(\dot{x} - \dot{d}) \geq 0 \\ c(t) = c_{min}, & \text{if } \ddot{x}(\dot{x} - \dot{d}) < 0 \end{cases} \quad (11)$$

In order to more intuitively understand the ADD control law, a similar explanation, as given for skyhook control, can be derived by considering a fully active suspension. In the case of a fully active suspension, which can provide force in the same direction as suspension elongation,

For the equation of motion

$$M\ddot{x}(t) = F(t) - Mg \quad (12)$$

the optimal solution is given as

$$F_{in}(t) = \begin{cases} Mg + F_{MAX} & \text{if } (F(t) - Mg) < 0 \\ Mg - F_{MAX} & \text{if } (F(t) - Mg) > 0 \end{cases} \quad (13)$$

subject to the bandwidth of the actuation and the condition

$$|F(t) - Mg| \leq F_{MAX} \quad (14)$$

The solution provided above states that if $F(t)$ is identical to Mg , the acceleration must be zero. Or more realistically, for changing accelerations of the body, a force must be applied to drive the acceleration to zero and the fastest way to do this is to provide maximal restorative force to the point that the acceleration is zero and then instantaneously remove the force. If an actuator could provide infinite frequency response, this control is optimal.

Next, considering the semi-active suspension, $F(t)$ is given by

$$F(t) = c(t)(\dot{x} - \dot{d}) + k(x - d)_{default} \quad (15)$$

Where $c(t)$ is the only controllable portion of $F(t)$. Even though a semi-active suspension lacks the power of a fully active suspension, there is no loss of optimality in

this implementation as the only constraint given is that the force applied be subject to the limits of the maximum available force.

It is worth noting that in a typical application of bang-bang control as suggested here, chattering is a common resulting phenomenon near the switching condition as the state approaches the origin. However, due to the fact that the force generation is proportional to $(\dot{x} - \dot{d})$, for small derivations from the origin and at low speeds, the chattering effect is negligible.

SKYHOOK+ADD MIX

A more recent development involves the combination of skyhook and ADD control laws. The convenience of combining these control laws lies in their model-independent nature. Each control law is a set of switching conditions represented by logical statements. In fact, previous research supports such a combination with the pros and cons of each control law described here.

SH provides a remarkable attenuation benefit around the body resonance frequency ...; around that frequency, ADD provides little or no benefits, if compared with a passive suspension.

ADD provides a remarkable attenuation benefit beyond the first resonance frequency; beyond that frequency, the filtering performances of the SH strategy are similar to those of the standard passive suspension (Savaresi & Spelta, 2009).

The complementary nature of these two control laws lead to the obvious conclusion of combining these two control laws with a switching condition that triggers beyond the first natural frequency. Figure 22 clearly illustrates the performance tradeoff between these two control laws. In each of the control laws respective highest

performance frequency ranges, skyhook essentially performs as a passive suspension with maximum damping and ADD as a passive suspension with minimum damping (Savaresi & Spelta, 2009).

Figure 22: Variance gain of skyhook and ADD control compared with the open loop response for a 2DOF model of a motorcycle (Savaresi & Spelta, 2009).

In order to implement this switch between the two control laws, an additional parameter must be added to the control law which will determine the specific law to be implemented in any given instant. In order to accomplish this control law selection, a time domain based “frequency-range select” and is shown in Equation 16.

$$\omega_f = (\ddot{x}^2 - \alpha^2 \dot{x}^2) \quad (16)$$

where the design parameter α represents the desired crossover frequency between skyhook and ADD control laws. With regard to the interpretation of this equation as a frequency range selector, consider a single-tone periodic signal

$$\dot{x}(t) = A \sin(\omega t) \quad (17)$$

If the function $f(t)$ is defined as

$$f(t) = \ddot{x}(t)^2 - \alpha^2 \dot{x}(t)^2, \alpha \in \mathfrak{R}_+ \quad (18)$$

By plugging $\dot{x}(t) = A \sin(\omega t)$ into $f(t)$ we obtain

$$f(t) = A^2 \omega^2 - A^2 \sin^2(\omega t)(\alpha^2 + \omega^2) \quad (19)$$

Let us consider the problem of studying the positivity of $f(t)$ over one period of this function (the period being $T(\omega) = \pi/\omega$). It is easy to see from equation (19) that

$$f(t) > 0 \Rightarrow \sin^2(\omega t) < \frac{\omega^2}{\omega^2 + \alpha^2} \quad (20)$$

If we call $D_+(\omega) = \{t: f(t) > 0, 0 \leq t \leq T\}$ the domain where $f(t) > 0$, it can be noticed that the measurement of this “positivity-domain,” say $|D_+(\omega)|$, is given by (see Figure 23)

$$|D_+(\omega)| = \frac{2T}{\pi} \sin^{-1} \left(\sqrt{\frac{\omega^2}{\omega^2 + \alpha^2}} \right) \quad (21)$$

Figure 23: Pictorial analysis of the inequality (Savaresi & Spelta, 2006)

And now, according to the preceding, the following holds:

$$\frac{|D_+(\omega)|}{T} \rightarrow 0 \text{ if } \omega \ll \alpha \quad (22)$$

$$\frac{|D_+(\omega)|}{T} \rightarrow 1 \text{ if } \omega \gg \alpha \quad (23)$$

$$\frac{|D_+(\omega)|}{T} \rightarrow \frac{1}{2} \text{ if } \omega = \alpha \quad (24)$$

This means that, over a period T , $f(t) > 0$ for more than $T/2$ if $\omega > \alpha$ and that $f(t) < 0$ for more than $T/2$ if $\omega < \alpha$ and thus $f(t)$ can be considered a simple frequency selector, centered around the frequency $\omega = \alpha$. When $f(t) > 0$ we can assume that $\omega > \alpha$; otherwise $\omega < \alpha$. Since the function $|D_+(\omega)|/T$ rapidly saturates toward 0 or 1 when $\omega \neq \alpha$, a good frequency selection quality is guaranteed (Savaresi & Spelta, 2006).

Figure 24: function $|D_+(\omega)|/T$ (in the normalized frequency) (Savaresi & Spelta, 2006)

Thus, the new combined control law becomes

$$\begin{cases} c(t) = c_{MAX}, & \text{if } [(\ddot{x}^2 - \alpha^2 \dot{x}^2) \leq 0 \wedge \dot{x}(\dot{x} - \dot{d}) > 0] \\ & \text{or } [(\ddot{x}^2 - \alpha^2 \dot{x}^2) > 0 \wedge \ddot{x}(\dot{x} - \dot{d}) > 0] \\ c(t) = c_{MIN}, & \text{if } [(\ddot{x}^2 - \alpha^2 \dot{x}^2) \leq 0 \wedge \dot{x}(\dot{x} - \dot{d}) \leq 0] \\ & \text{or } [(\ddot{x}^2 - \alpha^2 \dot{x}^2) > 0 \wedge \ddot{x}(\dot{x} - \dot{d}) \leq 0] \end{cases} \quad (25)$$

An additional form of the combined control law is available. Based on the development of the time domain frequency range selector and the complementary nature of skyhook and ADD a simple, single sensor version of the combined control has been shown to be effective. The primary advantage of such a control law is the requirement of a single sensor. This control law essentially estimates the dominant frequency content of the acceleration signal, using time domain data only, and selects the appropriate control law. This two state control again coincides with the behavior of

skyhook and ADD as passive suspensions with maximum and minimum damping, respectively.

$$\begin{cases} c(t) = c_{max}, & \text{if } (\ddot{x}^2 - \alpha^2 \dot{x}^2) \geq 0 \\ c(t) = c_{min}, & \text{if } (\ddot{x}^2 - \alpha^2 \dot{x}^2) < 0 \end{cases} \quad (26)$$

ISO 2631: MECHANICAL VIBRATION AND SHOCK – EVALUATION OF HUMAN EXPOSURE TO WHOLE-BODY VIBRATION

As a preface to the following controller testing and analysis, a word must be said about one of the methods used for comparison of the controller performance. In order to evaluate the results of these tests and the controllers a method of evaluation based in human health and comfort as compared to typical engineering quantifications of transmissibility and other performance criteria is needed. The ISO 2631 document outlines a procedure for evaluating a vibration environment with consideration to human health, comfort, and perception.

As for the guidelines from ISO 2631 measurements were made at the principal areas of contact between the human body and source of vibration and the transducer shall be placed between the human and the source. It was convenient that the method described in this document required an accelerometer to be placed such that it can be used both for the control law as well as data collection for evaluation of the vibration environment.

The basic method for evaluating a vibration environment is through the weighted root-mean-square acceleration (RMS acceleration).

$$a_w = \left[\frac{1}{T} \int_0^T a_w^2(t) dt \right]^{\frac{1}{2}} \quad (27)$$

where

$a_w(t)$ is the weighted acceleration as a function of time in m/s^2

T is the duration of the measurement in seconds

The frequency weighted acceleration is a method of taking a weighted average of the acceleration in a manner which will give precedence to those frequencies known to cause negative health effects and perceptions in humans. To achieve this frequency weighting, a transfer function representative of the human body is used as a band limiting filter to frequency weight the recorded time history of the acceleration. The transfer function is a bandpass filter (highpass + lowpass).

The high pass filter is given by

$$\frac{s^2}{s^2 + \sqrt{2}\omega_1 s + \omega_1^2} \quad (28)$$

And the low pass filter by

$$\frac{1}{\frac{s^2}{\omega_2^2} + \frac{\sqrt{2}s}{\omega_2} + 1} \quad (29)$$

where

$$\omega_n = 2\pi f_n; f_1 = 0.4 \text{ Hz}; f_2 = 100 \text{ Hz}$$

And the resulting frequency response being given shown by the bode plot in Figure 25.

Figure 25: Frequency of human body with respect to health, comfort, and perception

It has been established in the relevant literature that the conditions of confinement to a wheelchair are indeed harmful and negative. ISO 2631 confirms this by stating, "...the effects of long-term high-intensity whole-body vibration indicates an increased health risk to the lumbar spine and the connected nervous system of the segments affected." Since then studies have shown that the typical vibration environment of a wheelchair is accepted as harmful, this project does not make an attempt to quantify the specific characteristics of a wheelchair vibration environment. The testing protocol is aimed at obtaining a relative comparison between the performance of the different suspensions. Though ISO 2631 provides additional methods for using the analysis results to determine the severity of a particular environment, this additional analysis does not apply to the testing conditions used here. Shown in Table 3, are the results of the RMS acceleration analysis. The most significant result is the fact that a conventional oil damper and spring type configuration is out

performed by a wheelchair without any suspension. The reason for this profound result is explained in the later sections.

SIMULATION STUDY

The aforementioned model was implemented in Simulink with each of the covered control laws designed and tested. The tests were performed using an arbitrary profile of superimposed sine waves to apply commonly encountered frequencies during typical use of a wheelchair. The benefit of using a deterministic signal for testing is the ability to easily compute derivatives and the repeatability for comparative purposes. The results of these control laws are provided here.

In simulation, the switching of the model parameter introduces some numerical noise, especially near zero-crossings. As the logical condition of skyhook control is based about zero, this exacerbated the situation and required a filter on the results to remove the numerical noise.

In order to characterize the cross over frequency, α , for use in the combined control laws, a variation study was performed where the performance criteria was the frequency weighted average acceleration, as determined by the methods outlined in ISO 2631. It was confirmed via empirical study in simulation, the best performing α was not exactly the same for the combined single and dual sensor control laws, though very close, as shown in Figure 26. The advantage of the two sensor control law is also seen in Figure 22 as the variation in performance with respect to α is much less than that of the single sensor version.

Figure 26: Variation study to characterize best performing α value for mix single and double sensor control laws

The control laws were implemented using Simulink switches. All control laws, save for the two sensor mix law, could be implemented with a single two state switch, an example of the skyhook control law shown in Figure 27 and Figure 28. The control logic for the two sensor mix control law was more complex requiring multiple bit logic operators to achieve the desired result as shown in Figure 29. For a complete documentation of all Simulink models, see Appendix A.

Figure 27: Simulink model of wheelchair with skyhook based semi-active suspension

Figure 28: Implementation of skyhook control law in Simulink

Figure 29: Control logic implementation in Simulink using bit logic operators

The following plots show the typical results of the time domain information. At first glance, the results may seem counterintuitive until the frequency weighted average acceleration is considered. For a head-to-head comparison, the same disturbance signal was used for all plots, in simulation and experiment, and the same time period has been displayed for the simulation results.

Figure 30: Skyhook displacement response to super-imposed sine wave input

Figure 31: ADD response to super-imposed sine wave input

Figure 32: Single sensor Skyhook-ADD response to super-imposed sine wave input

Figure 33: Dual sensor Skyhook-ADD response to super-imposed sine wave input

Figure 34: Comparison of control law displacements

Figure 35: Comparison of control law acceleration

Although inspection of these plots reveals the performance increases with regard to displacement and acceleration attenuation, consideration of the ISO 2631 analysis shown in Table 2 is even more revealing. These results concur with the literature previously cited showing an improvement in ADD over skyhook and a marked improvement of the Mix control law over either. The single sensor Mix law does suffer a performance degradation as previously noted, but still outperforms either individual law.

Table 2: ISO 2631 results for various simulated controllers and peak accelerations seen

Case	A_w [m/s^2]	Peak [m/s^2]	Aw Rel	Peak Rel
Skyhook	1.4849	5.99	77%	79%
ADD	1.477	5.82	77%	83%
Mix 2 sensors	1.204	5.47	100%	90%
Mix 1 sensor	1.4143	4.96	83%	100%

EXPERIMENTAL RESULTS

The experimental portion of the project proceeded in two parts; the first part being the characterization of the system and the second with regard to testing and verifying the controllers.

SYSTEM CHARACTERIZATION

Recalling the control laws previously mentioned, it is necessary to find the settings which produce the minimum and maximum damping. In order to find the orifice settings that produce these values, a frequency response analysis using a sine sweep test with an HP spectrum analyzer was performed in order to determine the system damping for a variety of orifice sizes. Shown in Figure 36 is the system response over the range of openings from full open to nearly closed. It may be observed in this figure that the case of the fully closed valve is not displayed and is not used for the minimum damping setting. Certainly when the valve is closed and no air may flow across the

orifice, there is no damping in the system. There are two reasons why this setting is not used. The first reason is that at the fully closed setting, there is a shift in resonance of the system to an undesirable frequency. The second reason is due to a discontinuity in the damping characteristics as the valve passes from the minimum damping position to the fully closed position. The benefit of these model-independent control laws is illustrated through this process as the orifice setting is the only system characterization that must be performed in order to implement these control laws. Additionally, the only tuning parameter is the crossover frequency α for the combined control law frequency selector, ω_f .

Figure 36: General frequency response of system for valve openings spanning the range from 9.5 v to 5.0 v

Using the information from Figure 36, another series of sine sweeps were run at a much finer resolution in order to better identify the maximum damping point in the

system. These results, shown in Figure 37, identify the 6.9 v signal as the maximum damping point for the system.

Figure 37: Finer resolution sine sweep tests to identify maximum damping

In order to achieve the best results for the combined control law, an empirical method was used to establish the best performing values of the crossover frequency α . Though the theoretical nature to determine this value is thoroughly established, the highly non-linear and complex nature of the actual system gave results which did not directly correlate as expected with the simulation values determined previously. Figure 38 highlights the variation found in determining α for the single sensor case. For convenience, the simulation performance predicted has been repeated in Figure 39.

Figure 38: Frequency weighted average acceleration as a function of the cross over frequency alpha for single sensor combined control law

Figure 39: Comparison of predicted performance and experimental of single sensor mix control law based on α variation

It is clear from these results that that combined control law would likely not be well suited to this hardware implementation. Consider Figure 40 and the error present in this data set. It is clear that one cannot intelligently select an optimal value for α from this data set.

Figure 40: Empirical analysis for choosing optimal α value for Mix 2 sensor controller

CONTROLLER TESTING

The implementation of the measurement and control was done through the MATLAB/Simulink Real-Time target and the National Instruments PCI 6259 high speed card. This multifunction DAQ card allows for 32 inputs, 4 outputs, at a rate of 1 Ms/s for multichannel use. The power and speed of this DAQ card allows for a continuous time control implementation.

Many methods exist for analyzing control designs. Commonly seen methods of analysis include stability, frequency response, performance based criteria, and robustness. However, due to the subjective nature of the primary objective, that is,

improving rider health and comfort, a different approach must be taken in analyzing the results. For all controllers, a frequency domain sine sweep analysis was performed in order to compare with literature and also better understand the controllers' ability to attenuate the frequencies detrimental to human health. Using the methods provided in the ISO-2631 document a time domain analysis was also performed in order to better quantify the relative performance of the controllers and compare them to other configurations.

As a baseline comparison, a "passive" suspension was compared to the results obtained with the semi-active suspension. In this sense, a passive suspension represents a typical oil damper and spring, as is used in at least one commercial wheelchair suspension. The problem in regard to human comfort arises as the frequency input to the system increases. The oil damper begins to act like a high pass filter and essentially becomes a strut at high frequencies. It is this feature of passive systems that highlights one of the benefits of an air-spring based, semi-active suspension (when human comfort is the priority). The air spring acts like a low pass filter. In the case of this particular implementation, any frequencies above 5-6 Hz are attenuated by an order of magnitude.

Due to the fact that a wheelchair fitted with a passive suspension was unavailable for real world testing in this project, a simulation was conducted instead. The same performance analysis that is calculated for the experimental results of the semi-active suspension is also performed on the simulation results of the passive suspension. Though there are some discrepancies between the simulation and experimental results of semi-active suspension, the dynamics of passive suspension are much simpler and easier to model, giving credit to these results. Additionally, the simulation results of the semi-active suspension suffer partially due to the fact that

transfer function fitting was not performed on the system and thus the damping characteristics in the model are approximate.

In Figure 41 and Figure 42, general time domain data is shown to illustrate the physical response of the system in both displacement and acceleration. These figures illustrate the nature of the semi-active system clearly. It is shown in Figure 41 how the ability of the semi-active suspension in responding to a displacement is not a significant improvement over that of a passive spring and oil damper type suspension.

Figure 41: Time domain data of displacements for three controllers and passive suspension

Although the displacement data does not clearly project the differences in performance between the suspension types, the result becomes clear when the frequency content of the acceleration signal is considered, as shown in Figure 42. Here the difference between the passive and semi-active suspensions becomes quite clear.

By simple inspection, the acceleration data seems to intuitively confirm the medical documentation in regard to the discomfort caused by being confined to a wheelchair.

Figure 42: Time domain data of accelerations for three controllers and passive suspension

The frequency response of the system shown in Figure 43 corresponds with the simulated results found in literature. Skyhook performs best below the first resonance and ADD performs best a frequency slightly beyond the first resonance. The combined control law which utilizes the best performance of both the skyhook and ADD schemes outperforms either individual control law.

Figure 43: Frequency response comparison of three controllers

Table 3 shows the comparison of the acceleration data results from testing in both for all the different configurations. The primary points to note before further discussion is how poorly the passive suspension conforms when compared to the best performing, ADD. While the peak acceleration gives an idea of the performance enhancement for the ADD controller, the weighted average acceleration is a better measure for human health and comfort. It is worth noting that the single sensor combined control law does not give any performance increase over ADD in this particular hardware implementation.

Table 3: Frequency weighted average acceleration results from ISO-2631 analysis and peak acceleration data

Case	A_w [m/s^2]	Peak [m/s^2]	Aw Rel	Peak Rel
Passive	2.5841	4.28	-280%	5%
Locked	0.7989	3.37	52%	46%
Spring	1.552	4.5	-88%	-5%
SH	0.6189	2.47	85%	87%
ADD	0.541	2.25	100%	97%
MIX1	0.5456	2.46	99%	88%
MIX2	0.5386	2.19	100%	100%

The validity of the results in Table 3 is highly dependent on the variation of the results. The absolute values of the performance figures are close and the standard deviation is considered. Simple statistical analysis shows the consistence of the results for controllers in Table 4.

Table 4: Statistical comparison of results from several controller tests

run	Skyhook	ADD
1	0.6371	0.5441
2	0.6245	0.5381
3	0.6148	0.5396
4	0.6086	0.5421
5	0.6093	0.5413
Average	0.61886	0.54104
STD - σ	0.012017	0.002306
% Var.	1.941828	0.426231

Though the results of the combined control are somewhat impressive, they need to be corroborated. As noted previously, in attempting to determine the optimal α value, the data did not provide a clear answer. The data used for evaluating the performance is also highly suspect. Consider Figure 44 and the linear trend clearly shown in the data. Based on the simulation results, these data appear to correlate to an α value approaching a performance maxima. However, the data are sorted by order taken,

which happen to correspond to α values of 0,5,10,20,30,25, and 50. Indeed, fitting a linear trend line reveals a correlation of $R^2=0.9347$. Other control laws did not show any of this behavior during testing. Future work is needed to further investigate this phenomena.

Figure 44: 2 sensor combined control law results for a variety of samples characterizing performance, sorted by sample time

CONCLUSIONS

The first and most obvious point to make is with regard to medical literature that clearly documents the need for an improvement to current wheelchair suspensions, which in most cases is no suspension at all. The intention of this project was the study, design, construction, and implementation of an improved wheelchair suspension that utilizes several semi-active suspension control laws for feedback control.

The comparative results of the weighted average acceleration only act to confirm what has already been documented in both subjective and objective terms. Significant improvements can be made to the welfare of those confined to a wheelchair. These results were determined from a repeatable deterministic signal averaged over a five minute period. The truth of the matter is the differences in these suspensions will be much more significant over the lifetime of a user. Figure 45 contains data from ISO 2631 which illustrates how those users confined to a wheelchair for the duration of an entire day can (and do) fall into the caution zone and likely health risk zones with relatively benign looking weighted rms acceleration values due to the length of time they are exposed to these vibration environments.

Figure 45: ISO 2631 data for exposure duration with regard to acceptable levels and health

The particular performance of the controllers for the semi-active suspension is worth further consideration. As with any control system, actuator bandwidth, measurement rate, measurement noise, plant noise, plant/model variation, and the appropriateness of a particular controller are all of concern. The predicted performance results, particularly of the single sensor mix controller, are achieved through this hardware implementation. The underperformance of this particular control law comes down to essentially a single major cause: delay. The mix control law is partially skyhook, due to which the delay that is inherent in pneumatic damping causes significant deviation between the expected and realized performance. The skyhook control used here is an approximation of a quasi-ideal control law. In simulation, the damping change can be affected immediately and the ensuing dynamic changes are also instantaneous in the next time step. Table 2 shows clearly the near identical performance of skyhook and ADD in simulation. Table 3 reveals a significant performance difference. It is not

that in reality ADD over-performs, but rather that skyhook is underperforming. The main sources of delay in hardware, and the subsequent loss of performance, are found in the mechanical actuation of the valve and the compressibility of air itself. Though not predominant, certainly the delay of the solenoid must be accounted for. The reaction speed of the valve at less than 35 mS is significantly fast, but not fast enough to be considered instantaneous with respect to the bandwidth of the controller, 1 mS. Hence, the primary source of delay is from the compressibility of the air itself. Though this factor increases the low pass filter quality of the system, it is due to the fact that air is a compressible medium, subject to highly non-linear fluid and thermodynamic laws. The air must first compress and only then it will move across the valve and introduce damping into the system. For these reasons, the approximation of the semi-ideal skyhook control law fails to perform up to the simulation results.

With regard to seeming robustness of the Acceleration Driven Damping control law to these significant delay issues, one explanation is offered in the derivation of the law itself. Though Skyhook control is somewhat more ad hoc, as was mentioned previously, ADD can be shown to be an optimal controller in its stated form with regard to rider comfort. In fact, it can also be shown that the semi-active implementation of the controller is not even an approximation of the ideal situation, but is itself and true implementation of the ideal ADD control law.

Both combined control laws far underperformed their predicted abilities. There are a few possible explanations. The combined control laws are a combination of ADD and skyhook. As skyhook also underperformed, this combination could be a source of the deviation. The basis of the combined control law is the frequency range selector, however as was shown in Figure 37, the natural frequency of the system changes depending on the damping actuation. The goal of the combined control is to switch just

after the first natural frequency of the system. This floating resonant frequency value may introduce some of the unexplained phenomena. Figure 44 is most peculiar and suggests some transient phenomena not yet identified, but only affecting this multimode controller design. A possible solution that was considered was the extreme of $\alpha = 0$ and $\alpha = \infty$. In these cases the control should have behaved as either skyhook alone or ADD alone. This did not happen and as shown, the choice of α seemed to have no effect on the real world implementation for this case.

FUTURE WORK

The importance of improving the quality of life of fellow man cannot be overstated. There are several aspects that would behoove this project continue work upon. The frame and structure of this prototype are very crude, but do well in serving its purpose. Some work has been started by a tangent group in creating a more friendly version of the hardware. Though this sort of refinement is obvious, it is worth stating.

Though three control laws specific to semi-active suspension controllers were presented here, they are by no means the totality of controllers that could be implemented. Methods are available to correct for the delays that plague the performance of skyhook control. Some modeling was pursued in this project, but it is by no means exhaustive. Further additions to the model of this particular system could be made and developed upon for further refinement of the simulations. Included in this modeling work could be an attempt to explain the unexpected behavior of the combined control laws.

Several implementation improvements could be made for the implementation of the controller itself. Many pathways encountered by a wheelchair rider may be smooth and very benign. In these cases the low pass filter properties of the airbag alone would be sufficient to provide an improved ride. There are many other situations which might

require a more severe arrest of the motion, such as a drop off a steep curb. A multimode control would be useful for these reasons.

Though the DAQ card used was excellent for data collection and real time control, an entire computer is clearly unwieldy to transport with a wheelchair. Once a final controller design had been settled, a logical step would be the development of a microcontroller board for this project. The power and economy of many new microcontrollers are more than sufficient to hand the computation needs of this control system and could even easily handle multimode control and integration of accelerometers into the circuit board for reduced footprint. A final note on sensors is that the filtering and signal processing done in this project was very basic. There are certainly more advanced filtering methods available, such as Kalman filters. Especially because of the not incredibly complex model of this system, even when taking into account some of the higher orders ignored for these controllers, better state estimation and filtering could be reasonably performed both with the lab quality sensors used in this project and, with some effort, the lower end sensors that would be inevitably used in a final product.

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APPENDIX A: SIMULINK BLOCK DIAGRAMS

Figure 46: Skyhook Simulink model

Figure 47: Skyhook logic and control model

Figure 50: MIX 2 sensor Simulink model

Figure 51: MIX 2 sensor control and logic Simulink model

Figure 52: MIX 1 sensor Simulink model

Figure 53: MIX 1 sensor logic and control Simulink model

Figure 54: Passive suspension Simulink model

APPENDIX B: A STUDY OF LQG

As a preliminary study regarding this thesis project, an independent study of linear quadratic Gaussian control was conducted to better understand the application of control and more readily prepare for the problem at hand. Though LQG and Kalman filter was considered for this project, with the exploration of other more suitable controller designs it was decided that LQG was not necessary or beneficial for this project. It was also shown that these controller designs are robust to imperfect measurements, and in fact receive a marginal performance increase by less filtering and subsequently less phase lag. The work done in this independent study resulted in a paper submitted to the American Controls Conference and reproduced here.

A STUDY OF LQG WITH APPLICATION TO LIFE SUPPORT

EXTRACORPOREAL SUPPORT

Extracorporeal support, in general refers to a medical procedure that occurs outside the body, most often applied to circulatory procedures. Examples include hemodialysis, hemofiltration, plasmapheresis, apheresis, extracorporeal membrane oxygenation (ECMO), and cardiopulmonary bypass [2]. This particular study is focused on the use of advanced control methodologies to regulate the arterial partial pressures of O_2 and CO_2 during an ECMO process. ECMO is performed as a method of support for the heart and/or lungs in severely ill patients. In this process, the blood is circulated out of the body and into an ECMO machine where the O_2 and CO_2 gas levels are restored using a membrane oxygenator before being pumped back into the body. This membrane oxygenator performs in a similar function to that of the human lungs by mixing O_2 and CO_2 with the blood. In addition to oxygenating the blood, an ECMO machine maintains the blood temperature at the appropriate level with the help of a heat exchanger such that the body temperature does not drop when the blood is returned, which has serious

implications [2]. Additional components of a typical ECMO system include pressure monitors, pump and various drug administration interfaces as shown in Fig. 55.

Fig. 55: Typical ECMO hardware setup

NEED FOR CONTROL

An in-depth literature survey was conducted to first understand the general protocol involved in a typical extracorporeal membrane oxygenation procedure and further to identify the complications involved in such complex medical techniques. Risks associated with ECMO support itself include bleeding and blood clots. Close monitoring of the patient by a trained professional is required to reduce these risks [3]. In addition, inadequate performance of the machine and/or personnel in maintaining partial pressures of critical blood gases (O₂ and CO₂) can additionally result in gas toxicities. Oxygen toxicity symptoms include disorientation, breathing problems, and vision changes. If an ECMO process needs to be utilized for prolonged time duration, as is often the case, dire consequences such as oxidative damage to cell membranes, retinal detachment, and seizures can occur [4]. Carbon dioxide toxicity on the other hand can lead to equally harmful consequences with effects ranging from increased heart rate and blood pressure to dizziness, confusion, headache, sweating, tremors and loss of consciousness [5]. The importance for automatic sensing and control of an ECMO

procedure is thus summarized in [6]: “Ultimately, the management of each patient involved an empirical search for the combination of volume and flow which would yield normal blood pressure, adequate oxygenation and a manageable fluid requirement. In some patients this was easily achieved, but in others it was never reached. The difficulty in establishing stable perfusion has brought about the consideration of the system at its next level of complexity: the distribution of volumes and flows.”

STATE SPACE MODELING

The main components of a typical ECMO system include a membrane oxygenator, pump, fluids and heparin administration interfaces, heat exchanger, pressure monitor, and blood gas sensors. In this study, only the membrane oxygenator and the blood gas sensors need to be considered for quantifying the system under consideration. As explained earlier, the membrane oxygenator replicates the functionality of the human lungs in that it exposes the blood to regulated amounts of O_2 and CO_2 for assimilation. Firstly, the choice of membrane is critical, since the material should be impermeable to blood, but permeable to O_2 and CO_2 so as to achieve the desired objective [7]. However, the possibility of clots and other obstructions can both impede the functionality of any membrane oxygenator in terms of its mixing capability [8] thereby making closed loop control of the device essential. Fig. 56 shows a typical membrane oxygenator available in the market and the method by which the O_2 and CO_2 gases are introduced into the oxygenator for mixing.

Fig. 56: Membrane blood oxygenator

Owing to new advances in technology, continuous monitoring of blood gases is now possible not only in the form of flow rates of these gases, but also their partial pressures in the blood. This is achieved by the use of a typical blood gas sensor shown in Fig. 57. With this sensor, continuous blood gas monitoring is achieved via a combination of opto-chemical and fiber-optic detectors. These advanced detectors that can measure pH, PCO_2 , PO_2 and temperature can be inserted in-line with the bloodstream in an artery (8). This component is essential for the modeling of a membrane oxygenator from a controls standpoint, since the amount of O_2 and CO_2 assimilated into the blood is best measured through their arterial partial pressures in the blood.

Fig. 57: Arterial blood gas sensor

Hence, together with the dynamics of the flow rates of O_2 and CO_2 gases through the membrane oxygenator, and the dynamics of the blood gas sensor that measure the partial pressures of these gases assimilated by blood, the desired state space model of the system under consideration is obtained. One such mathematical model laid out in [1] is used for analysis and control in this paper and is as given below.

$$\dot{x}(t) = \mathbf{A}x(t) + \mathbf{B}_u u(t) + \mathbf{B}_w w(t) \quad (1)$$

$$m(t) = \mathbf{C}_m x(t) + v(t) \quad (2)$$

$$y(t) = \mathbf{C}_y x(t) = \mathbf{C}_m x(t) \quad (3)$$

$$x(t) = [x_1 \quad x_2 \quad x_3 \quad x_4]^T$$

$$u(t) = [u_1 \quad u_2] \quad w(t) = [w_1 \quad w_2] \quad (4)$$

where,

$$\mathbf{A} = \begin{bmatrix} -10 & 0 & 0 & 0 \\ 0 & -10 & 0 & 0 \\ 6 & -3 & -5 & 0 \\ 0 & 0.5 & 0 & -5 \end{bmatrix}; \mathbf{B}_u = \begin{bmatrix} 10 & 0 \\ 0 & 10 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\mathbf{B}_w = \begin{bmatrix} 10 & 0 \\ 0 & 10 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}; \mathbf{C}_m = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- $x_1(t)$ - flow rate of oxygen
- $x_2(t)$ - flow rate of carbon dioxide
- $x_3(t)$ - arterial partial pressure of oxygen (P_{O_2})
- $x_4(t)$ - arterial partial pressure of carbon dioxide (P_{CO_2})
- $u_1(t)$ - commanded oxygen flow rate
- $u_2(t)$ - commanded carbon dioxide flow rate
- $w_1(t)$ - error in the oxygen valve position
- $w_2(t)$ - error in the carbon dioxide valve position.

CONTROLLER DESIGN

In this study, several H_2 linear optimal control techniques based on the Linear Quadratic Gaussian (LQG) control formulation were designed and simulated. The LQG control system design which is based on the use of a stochastic linear quadratic regulator in cascade with a Kalman filter aims at minimizing a quadratic cost function involving selected states and inputs of the system. The original LQG formulation is such that it functions as a regulator (all states are driven to zero). In order to be able to track a non-zero reference, the LQG control methodology was modified in two ways.

LQG AND LQG/LTR WITH FEEDFORWARD CONTROL

In the basic LQG scheme, the feedback control law is defined as,

$$u(t) = -\mathbf{K}\hat{x}(t) \tag{5}$$

where $\mathbf{K}$ is the Linear Quadratic Regulator gain.

As shown in Fig. 4, in order to incorporate feedforward control, a feedforward control gain (K_r) is added to the above expression. The new control input to the plant is now given by

$$u(t) = -\mathbf{K}\hat{x}(t) + \mathbf{K}_r r(t) \quad (6)$$

This feedforward control gain can be selected to drive the performance outputs to the desired values after first generating the Linear Quadratic Regulator feedback gains ($\mathbf{K}$). A unique expression to obtain this desired K_r value exists and is given by the expression,

$$\mathbf{K}_r = -\{\mathbf{C}_y(\mathbf{A} - \mathbf{B}_u\mathbf{K})^{-1}\mathbf{B}_u\}^{-1} \quad (7)$$

Using the plant dynamics given by (1)-(3) and the LQG with feedforward control law given by (6), the closed loop system equations can be obtained and are as shown below.

$$\begin{bmatrix} \dot{x}(t) \\ \dot{e}(t) \end{bmatrix} = \begin{bmatrix} \mathbf{A} - \mathbf{B}_u\mathbf{K}(t) & \mathbf{0} \\ \mathbf{0} & \mathbf{A} - \mathbf{G}\mathbf{C}_m \end{bmatrix} \begin{bmatrix} x(t) \\ e(t) \end{bmatrix} + \begin{bmatrix} \mathbf{B}_w & \mathbf{0} & \mathbf{B}_u\mathbf{K}_r \\ \mathbf{B}_w & -\mathbf{G} & \mathbf{0} \end{bmatrix} \begin{bmatrix} w(t) \\ v(t) \\ r(t) \end{bmatrix} \quad (8)$$

$$y(t) = [\mathbf{C}_y \quad \mathbf{0}] \begin{bmatrix} x(t) \\ e(t) \end{bmatrix} \quad (9)$$

As is common knowledge, the basic LQG control scheme loses much of the robustness to modeling and other uncertainty that is provided by the LQR control methodology. Using a technique called Loop Transfer Recovery (LTR), it has been noticed that some of this robustness could be recovered. LTR achieves this objective by adding a fictitious noise input to the control input during Kalman Filter design, which has the effect of reducing the filter's reliance on the control input and also making it faster at the same time. This results in the estimated states reaching the actual states quicker,

thereby making the closed loop appear more like that of the LQR. For comparison, an LQG/LTR controller design was also undertaken in this study. The new state equation for the plant dynamics in the case of the LQG/LTR controller becomes

$$\dot{x}(t) = \mathbf{A}x(t) + \mathbf{B}_u u(t) + [\mathbf{B}_w \quad \mathbf{B}_u] \begin{bmatrix} w(t) \\ w_f(t) \end{bmatrix} \quad (10)$$

And the new spectral density matrix is given by,

$$\mathbf{S}_{\begin{bmatrix} w \\ w_f \end{bmatrix}} = \begin{bmatrix} \mathbf{S}_w & \mathbf{0} \\ \mathbf{0} & S_{w_f} \mathbf{I} \end{bmatrix} \quad (11)$$

The closed loop system then becomes

$$\begin{bmatrix} \dot{x}(t) \\ \dot{e}(t) \end{bmatrix} = \begin{bmatrix} \mathbf{A} - \mathbf{B}_u \mathbf{K} & \mathbf{0} \\ \mathbf{0} & \mathbf{A} - \mathbf{G} \mathbf{C}_m \end{bmatrix} \begin{bmatrix} x(t) \\ e(t) \end{bmatrix} + \begin{bmatrix} \mathbf{B}_w & \mathbf{B}_u & \mathbf{0} & \mathbf{B}_u \mathbf{K}_r \\ \mathbf{B}_w & \mathbf{0} & -\mathbf{G} & \mathbf{0} \end{bmatrix} \begin{bmatrix} w(t) \\ w_f(t) \\ v(t) \\ r(t) \end{bmatrix} \quad (12)$$

In this study, the LQG and LQG/LTR, both modified by feedforward control were designed and analyzed from a performance as well as robustness standpoint. A schematic of the closed loop system with these two controller configurations is shown in Fig. 4. The results of the simulation are presented in Sec. IV.

Fig. 58: LQG and LQG/LTR with Feedforward control

LQG AUGMENTED WITH INTEGRAL ACTION

Although the LQG methodology works well for white noise inputs, the controller does not increase the system type. If the plant being controlled intrinsically does not have an integrator, the closed loop system with the basic LQG controller even with the feedforward modification cannot achieve good performance to inputs of a higher type such as a step input. Hence, to achieve good tracking and disturbance rejection to step inputs, the LQG controller has to be augmented with integral action. In this study, one such LQG controller augmented with integral action is designed, and is compared to the above mentioned two LQG variants from a performance and robustness standpoint. A schematic of the LQG control scheme augmented with integral action is shown in Fig. (5).

Fig. 59: LQG augmented with integral control

Unlike the basic LQG methodology, the Kalman filter in the integral control system estimates both the plant state and the disturbance input. In order to accomplish this objective, a second disturbance input $w_0(t)$ is augmented to the white noise disturbance input $w(t)$ in (1). This disturbance input, $w_0(t)$, which is of the step input type, is considered to be defined by (13).

$$\dot{w}_0(t) = w_T(t) \quad (13)$$

where, $w_0(t)$ is the disturbance input now considered to be the augmented system state and $w_T(t)$ is the white noise that drives it. In order to account for the additional integral action, (1) has to be augmented with (13). These new augmented system equations given by (14) and (15) are used for finding the Kalman gains.

$$\begin{bmatrix} \dot{x}(t) \\ \dot{w}_0(t) \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{A} & \mathbf{B}_{w_0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}}_{\mathbf{A}_f} \begin{bmatrix} x(t) \\ w_0(t) \end{bmatrix} + \underbrace{\begin{bmatrix} \mathbf{B}_u \\ \mathbf{0} \end{bmatrix}}_{\mathbf{B}_{uf}} u(t) + \underbrace{\begin{bmatrix} \mathbf{B}_w & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}}_{\mathbf{B}_{wf}} \underbrace{\begin{bmatrix} w(t) \\ w_T(t) \end{bmatrix}}_{\mathbf{w}_d}$$

$$\left(\dot{x}_f(t) = \mathbf{A}_f x_f(t) + \mathbf{B}_{uf} u(t) + \mathbf{B}_{wf} w_d(t) \right) \quad (14)$$

$$m(t) = \underbrace{[\mathbf{C}_m \quad 0]}_{\mathbf{C}_f} \begin{bmatrix} x(t) \\ w_0(t) \end{bmatrix} + v(t)$$

$$(y_f(t) = \mathbf{C}_f x_f(t) + v(t)) \quad (14)$$

Where

$$\mathbf{B}_{w_0} = [\mathbf{I} \quad 0]^T$$

In order to generate the control, the LQR gains need to be computed. This also requires the knowledge of the integration action, since the feedback gains must include a separate gain for the integral controller. Hence, the augmented plant must now include the integral error state equation as well. As can be seen from the schematic, the error state equation is defined as,

$$\hat{e}_I = \int \hat{e} dt \rightarrow \dot{\hat{e}}_I = \int (-\mathbf{C}_y x + r(t)) dt \quad (15)$$

By differentiating the above equation we get the governing error state equation to be,

$$\dot{\hat{e}}_I = -\mathbf{C}_y x + r(t) \quad (16)$$

Hence the new augmented plant equation for computing the LQR gains is given by the equations (18) and (19).

$$\begin{bmatrix} \dot{\hat{x}}(t) \\ \dot{\hat{w}}_0(t) \\ \dot{\hat{e}}_I(t) \end{bmatrix} = \underbrace{\begin{bmatrix} \mathbf{A}_f - \mathbf{B}_{uf}[\mathbf{K}_x & 0] - \mathbf{G}\mathbf{C}_{mf} & -\mathbf{B}_{uf}\mathbf{K}_I \\ -[\mathbf{C}_y & 0] & 0 \end{bmatrix}}_{\mathbf{A}_c} \begin{bmatrix} \hat{x}(t) \\ \hat{w}_0(t) \\ \hat{e}_I(t) \end{bmatrix} + \underbrace{\begin{bmatrix} \mathbf{G} \\ 0 \end{bmatrix}}_{\mathbf{B}_c} m(t) + \underbrace{\begin{bmatrix} 0 \\ 0 \\ \mathbf{I} \end{bmatrix}}_{\mathbf{B}_r} r(t)$$

$$(\dot{x}_c(t) = \mathbf{A}_c x_c(t) + \mathbf{B}_c m(t) + \mathbf{B}_r r(t)) \quad (17)$$

$$u(t) = \underbrace{[\mathbf{K}_x \quad 0 \quad \mathbf{K}_I]}_{\mathbf{C}_c} \begin{bmatrix} \hat{x}(t) \\ \hat{w}_0(t) \\ \hat{e}_I(t) \end{bmatrix}$$

$$(y_c(t) = \mathbf{C}_c x_c(t)) \quad (18)$$

Hence the closed loop system is given by,

$$\begin{bmatrix} \dot{x}(t) \\ \dot{x}_c(t) \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B}_u \mathbf{C}_c \\ \mathbf{B}_c \mathbf{C}_m & \mathbf{A}_c \end{bmatrix} \begin{bmatrix} x(t) \\ x_c(t) \end{bmatrix} + \begin{bmatrix} \mathbf{B}_w & 0 & \mathbf{B}_{w_0} & 0 \\ 0 & \mathbf{B}_c & 0 & \mathbf{B}_r \end{bmatrix} \begin{bmatrix} w(t) \\ v(t) \\ w_0(t) \\ r(t) \end{bmatrix} \quad (5)$$

$$y(t) = \begin{bmatrix} \mathbf{C}_y & 0 \\ 0 & \mathbf{C}_c \end{bmatrix} \begin{bmatrix} x(t) \\ x_c(t) \end{bmatrix} \quad (19)$$

UNCERTAINTY ANALYSIS

Uncertainty analysis is a critical aspect that has to be considered whenever mathematical modeling is performed to capture the dynamics of a real-world system. For the analytical modeling of the extracorporeal membrane oxygenation procedure, the uncertainty is assumed to be structured and of a parametric type affecting the values in the system “A” matrix as shown by (22).

$$\begin{bmatrix} -10 & 0 & 0 & 0 \\ 0 & -10 & 0 & 0 \\ k_1 & -k_2 & -5 & 0 \\ 0 & k_3 & 0 & -5 \end{bmatrix} \quad (20)$$

Where the parametric values k_1 , k_2 , and k_3 are uncertain but bounded in the range as given below.

$$k_1 \in [4,8]; k_2 \in [2,4]; k_3 \in [0.1,0.9] \quad (21)$$

Comparing (1) with (23), it can be noticed that the mean values of the uncertainty range is chosen for the nominal plant dynamics.

The stability of a system to a structured uncertainty is determined by analyzing the feedback system shown in Fig. 60 and Fig. 7. Since the nominal closed-loop system is assumed to be stable, any unstable poles of this system are therefore caused by closing the loop through the perturbation (Δ). It can be shown that the feedback system in Fig. 6 and Fig. 61 are internally stable for all possible perturbations if the magnitude of the structured singular value (SSV) of the transfer matrix seen by the Δ over the entire range of desired frequencies is less than or equal to one. This condition is mathematically stated in (24).

$$\sup_{\omega} \{ \mu_{\bar{\Delta}} [N_{y_d w_d}(j\omega)] \} < 1 \quad \forall \Delta(j\omega) \in \bar{\Delta} \text{ and } \|\Delta(j\omega)\|_{\omega} \leq 1 \quad (22)$$

$N_{y_d w_d}$ (the transfer matrix seen by the Δ) is nothing but the transfer matrix within the dotted box (Fig. 60 and Fig. 61).

$$N_{y_d w_d}(j\omega) = \mathbf{M}[s\mathbf{I} - \mathbf{A} + \mathbf{G}\mathbf{C}_m(s\mathbf{I} - \mathbf{A} + \mathbf{P} - \mathbf{G}\mathbf{C}_m)\mathbf{B}_u\mathbf{K}]^{-1} \quad (24)$$

where,

$$\mathbf{P} = \mathbf{B}_u[s\mathbf{I}[s\mathbf{I} - \mathbf{K}_i\mathbf{K}_x\mathbf{C}_m]^{-1}\mathbf{K}_x]$$

RESULTS

The main objective of control design for an ECMO process was to accurately track the arterial partial pressures of O_2 and CO_2 in the blood by regulating the flow rates of the gases. To ascertain the tracking performance, the closed loop systems were subjected to step reference inputs in the partial pressures of both these gases, (27).

$$\begin{bmatrix} r_1 \\ r_2 \end{bmatrix} = \begin{bmatrix} 50 \\ 5 \end{bmatrix} \begin{matrix} \rightarrow P_{O_2} \\ \rightarrow P_{CO_2} \end{matrix} \quad (25)$$

To further ascertain the tracking performance of the different controllers in the presence of disturbances, step inputs in disturbance with a magnitude of 40% that of the reference input magnitudes after steady state values for reached for the tracking reference input. A step disturbance input, as given in (2), was applied to the system. This step change in in disturbance could be thought of as an error in the flow rate valve position that might be caused due to nonlinear actuator dynamics in the form of sudden jerks. An extreme case of disturbance where the oxygen flow rate was reduced and at the same time, the carbon dioxide flow rate was increased was simulated.

$$\begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} -20 \\ 2 \end{bmatrix} \begin{matrix} \rightarrow P_{O_2} \\ \rightarrow P_{CO_2} \end{matrix} \quad (26)$$

With the above mentioned parametric values for the reference and disturbance inputs, the three different controllers were synthesized for optimal tracking and disturbance rejection performance while providing robust stability to the parametric

uncertainty described in the previous section. The various gains and design parameters resulting for the different controllers are given below.

The design parameters shown in Table 1 were identical for the three different controllers.

Table 5: Relevant parameters for LQG formation

$$Q = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad S_w = \begin{bmatrix} 0.02 & 0 \\ 0 & 0.01 \end{bmatrix}$$

$$S_v = \begin{bmatrix} 1 \times 10^{-4} & 0 \\ 0 & 1 \times 10^{-5} \end{bmatrix}$$

$$R = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix} \quad W_{ei} = \begin{bmatrix} 10 & 0 \\ 0 & 100 \end{bmatrix}$$

Table 2 shows a comparison of the different controller gains for the corresponding controllers.

Table 6: Table of gains for LQR and variations

	LQG		LQG/LTR		LQG+Integral	
K^T	0.70	-0.34	0.70	-0.34	0.96	-0.37
	-0.34	0.19	-0.34	0.19	-0.37	0.42
	1.66	-0.82	1.66	-0.82	2.48	-1.06
	0.08	0.29	0.08	0.29	2.24	5.27
K_r^T	3.08	-1.11	3.08	-1.11	-	-
	5.12	10.43	5.12	10.43	-	-
G	83.82	33.02	783.7	756.8	84.2	33.84
	-19.58	98.67	-325.5	2056	-19.70	100.7
	28.86	-4.36	101.5	-21.50	28.93	-4.39
	-0.44	6.03	-2.10	40.10	-0.44	6.13
K_i	-	-	-	-	-9.24	3.81
	-	-	-	-	-12.06	-29.23

Step response analysis was carried out with the above mentioned parametric values obtained from the synthesis of the controllers. Fig. 62 and Fig. 63 show the step response of the arterial partial pressures of O_2 and CO_2 respectively. It can be clearly seen that while all the controllers provide excellent tracking performance to the reference input, only the LQG controller augmented with integral action is able to totally reject a step input in disturbance given in (28) that was applied to the system at 2.5 s. The ability to reject disturbance is as critical as tracking, since, as already mentioned, variation in blood gas levels over a prolonged time can result in life threatening consequences. Also, it can be noticed that although the LQG augmented with integral control provides slower response than the other two controllers, its well within the accepted limits (≈ 30 sec).

Fig. 62: Step response of O_2 arterial partial pressure

Fig. 63: Step response of CO₂ arterial partial pressure

Fig. 64 and Fig. 65 show the control inputs provided by the different controllers in the form of flow rates of O₂ and CO₂ gases. Using the rise time criteria, a rough idea of the actuator bandwidth can be calculated for the different controllers. Table (3) provides the corresponding parameters.

Table 7: Bandwith of controllers

	LQG + Feedforward	LQG/LTR + Feedforward	LQG + Integral
Bandwidth (Hz) (CO ₂ flow rate)	$\approx 5z$	≈ 5.15	≈ 0.85
Bandwidth (Hz) (O ₂ flow rate)	∞	∞	≈ 1.9

From Table (3) it can be clearly noticed that the LQG and LQG/LTR with feedforward control have much higher bandwidth requirements than the LQG with integral control. This difference is even more pronounced for the O_2 flow rate where only the bandwidth for the LQG with integral control can be physically realized.

Fig. 64: Controller output in O_2 flow rate

Fig. 65: Controller output in CO_2 flow rate

Fig. 12 shows the structured singular values for the parametric uncertainty being considered in (22) and (23). As mentioned earlier, all the controllers were designed so as to ensure that the robust stability condition given by (24) is satisfied. However, it is interesting to notice that the LQG/LTR control slightly reduces the robust stability characteristic of the system compared to the LQG by itself. In Fig. 12, only two out of the four structured singular values have been plotted, since the other two values are close to zero due to the nature of the perturbation. It can be clearly seen from the figure that although addition of integral action reduces the degree of robustness, it still satisfies the robust stability criteria of (24).

Fig. 66: Structured singular value analysis

CONCLUSION

Extracorporeal blood gas support has become a valuable tool in saving lives of patients who would have previously had higher mortality rates. ECMO has become especially useful in neonatal care units for infants with cardiac or pulmonary complications (9). The effective functioning of an ECMO machine depends significantly

on how well the O_2 and CO_2 levels are being maintained by the membrane oxygenator component of the ECMO unit, so as to prevent blood gas toxicity that can result in detrimental long term health problems. Hence, closed loop control is critical for operating ECMO machines and the membrane oxygenator in particular. The paper investigated three different types of LQG based controllers designed for tracking control of the arterial partial pressures of the blood gases (O_2 and CO_2) during a typical ECMO procedure. The system included a membrane oxygenator for introducing the above mentioned gases into the bloodstream and a blood gas sensor for measuring their resulting arterial partial pressures in the blood. The performance of each of the controllers was ascertained both from a tracking as well as disturbance rejection standpoint to step commands in both these inputs. Robustness analysis was also performed on all the closed loop configurations using the structured singular value analysis and the performance of all the controllers was tuned so as to ensure robust stability to the parametric uncertainties considered. While the LQG and LQG/LTR with feed forward control were able to achieve good tracking performance, only the LQG augmented with integral control was able to achieve accurate tracking of the arterial partial pressures of the blood gases in the presence of a step input disturbance in the blood gases flow rates. It was also noticed that the LQG augmented with integral control required lesser bandwidth to achieve the better performance. Extension to this work will involve investigation of H_∞ based control schemes for comparison with the optimal control methodologies designed in this paper.