NONLINEAR MODELING OF A
SUSTAINABLE MATERIAL

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by
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ABSTRACT
Nonlinear Modeling of a Sustainable Material
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This study developed a nonlinear constitutive model for a sustainable orthotropic material. Existing methods for constitutive models of wood were improved upon to include the nonlinear stress-strain response not only in the two orthogonal axes but at any orientation to the strong axis of this material. This method also simplifies the nonlinear stress-strain relationships into bilinear stress-strain curves which can be valuable in hand calculations as well as finite-element analyses. The effectiveness of the proposed constitutive model is demonstrated by comparing bilinear stress-strain predictions to experimental data.

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LIST OF NOMENCLATURE

A	Cross-sectional area
E	Young's modulus
E_{linear}	Young's modulus in the linear range
E_{nonlin}	Young's modulus in the nonlinear range
E_s	Apparent Young's modulus
E_x	Young's modulus in the 0° axis
G	Shear modulus
I	Moment of inertia of the cross-section
l	Length of the specimen
MOR	Modulus of rupture
P	Applied load
α	Constant used in calculating bilinear point strain
γ	Shear strain
δ	Maximum vertical deflection
δ_b	Vertical deflection due to bending
δ_s	Vertical deflection due to shear
ε	Strain
σ	Stress
τ	Shear stress
ν	Poisson's ratio
h/l	Depth/Span ratio

1.0 INTRODUCTION

This thesis is part of a larger research project called *The Emergency Shelter Project*. The Emergency Shelter Project is being developed by the Department of Architectural Engineering, the Department of Architecture, and the Department of Materials Engineering at California Polytechnic State University, San Luis Obispo. The overall goal of the larger project is to design a temporary shelter that is durable, economical, rapidly assembled, and made out of a sustainable material. The purpose of this thesis was to find the material's stress-strain properties for use in nonlinear analyses. These properties were found experimentally and then further explored via finite element analyses. The gathered data was used for the development of a new semi-empirical constitutive model.

1.1 The Material

The sustainable material that was used is a 65% plastic and 35% fiber composite. In this study, the plastic was a polypropylene (PP) homopolymer, but a high-density polyethylene (HDPE) has also been used. Polypropylene can be recycled from used plastic products such as grocery bags and milk jugs. The fibers are ≈ 1 mm long kenaf fibers and are used as reinforcement within the recycled plastic matrix. Kenaf is a plant that is not normally used in current wood/plastic products. The renewable resources for the fibers and the plastic matrix make this material sustainable. The larger *Emergency Shelter Project* team investigated the use of this material to create rapidly assembled shelters.

The plastic is mixed with the organic fibers and is extruded out of a machine as shown in Figure A.

Figure A: PP/kenaf Extrusion Process

Source: Saliklis 2009

Usually, the PP/kenaf material is extruded into planks, but it can also be injection molded into virtually any shape or size. Both processes align most of the fibers along the long or longitudinal axes, creating an **orthotropic**¹ material similar to wood since the other axes have fewer aligned fibers. All test specimens in this study came from extruded planks

¹ Key terms in bold are defined in the glossary

with the dimensions of 5.125" wide, 1.125" thick, and 48" long. Throughout this study, the 0° axis will refer to this longitudinal extrusion axis.

1.2 The Experimental Testing

Experiments were conducted on this material to find the shear modulus G and Young's modulus E at varying angles to the strong axis in both the **linear-elastic range** and **nonlinear range**. Fifty-six tension tests were performed to solve for Young's modulus, and nine four-point bending tests were performed to solve for the shear modulus. These experimentally obtained values formed the basis of the body of data used to create a new constitutive model.

1.3 The Mathematical Model

The proposed constitutive model developed for this material is an extension of two previously derived constitutive models for orthotropic wood based materials. The first wood-based model, derived by Saliklis and Falk (2000), predicts the shear modulus and Young's modulus at varying angles to the strong axis in an orthotropic material. The second model, derived by Saliklis, Urbanik, and Tokyay (2003), predicts the shear modulus and Young's modulus in the nonlinear range in the strong axis and at ninety degrees to the strong axis. A weakness of the first model is that it applies only to the linear region. A weakness of the second model is that it does not capture any response other than 0° and 90°. The newly proposed constitutive model is a combination of the two wood-based models as it predicts both a linear and nonlinear Young's modulus at any orientation to the strong axis. The advantage of this model is the application of a simple bilinear stress-strain relationship for a clearly nonlinear material. A bilinear

stress-strain curve simplifies hand analyses and may be necessary when inputting material properties into many structural analysis programs. This model's ease of implementation will be demonstrated by a nonlinear finite element analysis (FEA) of a lateral load resisting frame. A weakness of the new model is that it does not mechanistically link the shear response to the axial response as did the previous two models. Further experimental testing would be required to create such a mechanistic linkage.

2.0 LITERATURE REVIEW

A growing focus of the building industry is to create “greener” buildings. In addition to designers creating better energy-saving methods, greater attention is being given to the use of recyclable materials such as steel and plastics. The high-density polyethylene (HDPE) or polypropylene (PP) composites are attractive because they can be recycled from some of the enormous waste stream currently being generated. However, further research of the design properties is necessary to give designers the confidence they need to begin using the product in structural systems.

Compared to wood, the industrial scale fabrication of the HDPE ensures a more consistent material makeup: fewer knots, less grain inclination, etc. According to Saliklis’ and Arens’ paper “Rapidly assembled emergency shelters made from ‘green’ materials” (2009), the more consistent makeup results in a lower standard deviation and comparable design values. Saliklis’ and Arens’ paper gives experimental **modulus of rupture MOR** data for an HDPE material similar to that used in this study and is shown in Table 1 below.

Material	Modulus of Rupture (MOR)	MOR coeff of variation	MOR std. deviation	Design strength *
Loblolly Pine	88.2 MPa (12,800 psi)	25%	22.0 MPa (3,200 psi)	24.7 MPa (3589 psi)
ag. fiber/plastic composite	55.2 Mpa (8,000 psi)	4%	2.2 MPa (320 psi)	24.5 MPa (3559 psi)

* for wood design strength = (MOR – MOR Std. Dev.*1.645)/2.1

Table 1: Flexural Strength (Modulus of Rupture) for Wood and Composite
Source: Saliklis 2009

Their data shows that pine has a modulus of rupture of 12,800 psi, with a standard deviation of 3,200 psi and design strength of 3589 psi. In comparison, the modulus of

rupture data for their fiber reinforced HDPE is 8,000 psi, with a standard deviation of 320 psi and design strength of 3559 psi. The comparable design strengths of wood and HDPE make it possible to use HDPE as a structural material.

However, before HDPE can be considered in structural uses, allowable design values and material properties need to be established such as the elastic moduli and yield stresses.

Young's modulus can be experimentally derived through a number of tests including bending, compression, and tension tests. American Society for Testing and Materials (ASTM) standards prescribes known procedures to solve for Young's modulus. However, finding the shear modulus is not as straightforward. Difficulties arise in test setups and trying to obtain data in the nonlinear range. The shear modulus can be derived many different ways including bending theory, torsion tests, and pure shear tests, as stated in "A New Test Method for Measuring the Longitudinal and Shear Moduli of Fiber Reinforced Composites" by S. Javad Jalali and Farid Taheri (1999). Possible shear tests as suggested by the literature review include the following:

- ASTM 3044
- Torsion test
- Iosipescu shear test (Yoshihara 1999)
- Three-point bending test (Yoshihara 1998)
- Four-point bending test (Yoshihara 2002)

It is not certain whether the ASTM shear test could be used for this plastic-fiber composite because it is intended for wood-based panels. In addition, the ASTM test for shear cannot produce data past the linear-elastic range because the shear properties found are derived from elastic theory. Both the torsion test and Iosipescu shear test would be

ideal since they load the member in pure shear with no bending deflection. However, the torsion test assumes the shear modulus is the same for all planes parallel to the strong axis, which is correct to assume for an **isotropic** material but incorrect to assume for this orthotropic material. The Iosipescu shear test consists of a complicated test setup requiring a special test fixture.

Yoshihara's article "Measurement of the shear modulus of wood by asymmetric four-point bending tests" gives a comparison among the four-point bending test, three-point bending test, and a pure shear test. The results of Yoshihara's study show that a four-point bending test can properly solve for the shear modulus when using a shear factor of 1.5 and a specimen length at least 20 times greater than its depth (Yoshihara 2002). Using shallow beams in a shear test seems counter-intuitive; however, Yoshihara's study demonstrated that smaller depth/span ratios reduce the extra deflection caused by stress concentrations. This study also compared the use of 1.2 and 1.5 for the shear factor, and showed that a value of 1.2 overestimated the shear modulus and a value of 1.5 was more appropriate. Other research has also investigated using a shear factor of 1.5 (Bindzi and Samson 1995). Yoshihara's paper provides a relationship between the shear modulus and Young's modulus, such that both can be obtained through a least squares error minimization.

Each of Yoshihara's bending tests applies Timoshenko Beam Theory, which assumes that the deflection of a beam is composed of bending deflection and shear deflection. Because these tests rely on elemental bending theory, they are not used for obtaining data past the linear-elastic range. The inherent difficulty of obtaining sound

nonlinear experimental shear data explains why this thesis did not attempt to mechanistically link nonlinear shear response to nonlinear axial response.

The model proposed here is an extension of the bilinear model proposed by Saliklis (2003). A key idea in this model is a distinct yield point on each of the three orthogonal axes. Shih and Lee (1978) also used this approach which allowed for different but linked yield points on the orthogonal axes. It is this criterion that ANSYS has adopted for nonlinear orthotropic materials and this was the reason for the extension of the previous constitutive models. The bilinear model works extremely well for materials that do not exhibit large nonlinearity.

3.0 BACKGROUND

Part of the basis of this project is testing that has been done on earlier generations of the HDPE/PP composite material. Figure B below shows elastic modulus results from a study that used UV light to simulate aging (Carpenter 2009).

Figure B: Elastic Modulus with Respect to UV Exposure Hours

Source: Saliklis 2009

The graph above shows experimentally-found elastic moduli in relation to hours of exposure to UV light. Another study that investigated the bending and compressive strength of the HDPE/PP material has shown the material to be orthotropic and very nonlinear (Lee 2007). On the next page is a sample of a compression specimen after testing, Figure C.

Figure C: Sample Compression Specimen

Source: Lee 2007

Figure D, on the following page, shows sample stress-strain curves for a similar HDPE material subject to compression.

Figure D: HDPE/Fiber Compression Tests
Source: Lee 2007

The stress-strain relationship shows that the material is very non-linear. Yet even for such markedly nonlinear curves, a bilinear curve fit can still determine the initial elastic and nonlinear moduli. A sample stress strain curve with a bilinear approximation is shown in Figure E on the following page.

Figure E: Similar Material Stress-Strain Curve with Bilinear Approximation

Source: Saliklis 2009

In the figure above, the slopes of the two curve-fitted lines represent the modulus, one in the linear and one in the nonlinear range. Literature often refers to the nonlinear range as the post-yield range for many materials, defining the point at which the stress-strain curve changes slope as the yield point. However, testing was not done to determine whether this PP/kenaf material actually yields at this apparent yield point. Therefore, this study defines the point at which the slope changes as the **bilinear point** and the subsequent modulus as the **nonlinear modulus**.

4.0 EXPERIMENTAL TESTING

This section will discuss the experimental test methods used to solve for the material properties of the PP/kenaf composite material.

4.1 Tension Test

Pure tension tests were performed to determine Young's modulus in both the linear-elastic and nonlinear range. Young's modulus E can be found using the experimental stress σ and strain ε data along with the relation known as Hooke's law, shown below.

$$\sigma = E\varepsilon \quad \text{Eq. 1}$$

When the initial stress-strain data is fit with straight lines, the slopes of the lines can be recorded as the resulting moduli.

Thin test specimens were cut from planks in the 1-2 plane, see Figure F, at angles θ of 0° , 33° , 45° , 68° , and 90° from the 1 axis.

Figure F: Labeled Axes of a Board

The 12 plane was chosen because it could yield samples that were at least 5" long, or long enough to perform the tension test. Test specimens had relatively similar rectangular cross-sectional areas ranging from 0.251 to 0.396 in². The dimensions were measured using a vernier scale caliper. Samples of the specimens are shown in Figures G and H on the following pages.

Figure G: Tension Test Specimens

Figure H: Measuring Test Specimen Dimensions

The length and width of the original planks were 48" and 5.125" respectively. The 0° specimens could be cut up to 48" long. However, the length of the specimens cut at 90° could only be 5.125" long, making it difficult to adequately clamp the specimen with the attached extensometer. Figure I on the following page shows the test setup.

Figure I: Tension Test Setup

The Epsilon model 3542 axial extensometer was plugged into the Tinius Olsen High Force Electromechanical Tester, which is connected to the computer. The specimen was then clamped into the machine and the extensometer was connected to the specimen between the clamps. The Tinius Olsen software, along with the extensometer peripheral, recorded the tensile force and corresponding strain at each load in tabular form.

The longer specimens were easier to test because the resulting data was more consistent. Many of the shorter specimens didn't have enough material in the clamps and slipped or broke in the clamp leading to unreliable data near failure. The desirable failure mechanism is shown in Figure J on page 18, and the undesirable failure mechanism is shown in Figure K on page 18.

Figure J: Desirable Failure Mechanism

Figure K: Undesirable Failure Mechanism

Stress concentrations near the clamps led to an unreliable ultimate stress, and slippage often caused jumps in the strain data.

Another factor that affected ultimate stress was the cross-sectional area. Specimens with smaller cross-sections failed at lower stresses than specimens with larger cross sections. A reason for this behavior may be that during production of the material, some fibers settle too close together in the matrix creating weak spots called fiber pockets.

As a result, larger cross-sections are more uniform than smaller ones. Studies have been done on similar fiber-reinforced composites showing the relationship between fiber dispersion and tensile strength (Raj and Kotka 1991). Fiber dispersion can be clearly seen in Figures L and M below.

Figure L: Tension Specimen after Failure

Figure M: Fiber Dispersion

Although some specimens failed prematurely due to fiber pockets, the stress-strain data was still usable when calculating Young's modulus. However, the same data could not be used to calculate ultimate stress.

The force data was divided by the specimen area to change the forces to nominal stresses. The end result of each tension test was a stress-strain curve which was then fitted with a bilinear curve. A sample stress-strain curve and line fitting is shown in Figure N below.

Figure N: Sample Tension Test Stress-Strain Curve

The slopes of the best fit lines are the values of Young's modulus in both the linear-elastic and nonlinear range. All of the modulus values were collected and averaged for each angle. Table 2 on the next page shows the results for the experimental linear modulus E_{linear} , the nonlinear modulus E_{nonlin} , and the bilinear point stress σ_b as a function of orientation to the strong axis. Graphical summaries of E_{linear} and E_{nonlin} are shown in Figures O and P.

Angle (degrees)	Average E_{linear} (psi)	Average E_{nonlin} (psi)	Average σ_b (psi)
0	1064000	588000	4650
33	735000	511000	3180
45	634000	415000	2690
68	517000	389000	2310
90	495000	360000	2220

Table 2: Tension Test Summary

Figure O: Initial Linear Young's Modulus Summary

Figure P: Nonlinear Young's Modulus Summary

The most important values to note are the linear-elastic Young's moduli in the strong 0° and weak 90° directions, which are 1,064,000 psi and 495,000 psi respectively.

Fifty-six total specimens were tested: 12 at 0° , 13 at 33° , 11 at 45° , 10 at 68° , and 10 at 90° . For each angle, the sample size was increased until the standard deviation was less than 20% of the value of Young's modulus. As expected, the samples cut closer to 0° had higher values for Young's modulus and the samples cut closer to 90° had lower values for Young's modulus. The PP/kenaf planks have a clear strong axis running the direction of the long axis of the planks. Significantly, the values of Young's modulus found for PP homopolymer are almost at the same level as those found for structural timber.

4.2 Four-Point Bending Test

Four-point bending tests were performed to determine the linear-elastic shear modulus G of the material. The advantage of this test is that the shear modulus G can be

found even when Young's modulus E_x is unknown. The test setup is shown in Figure Q below.

Figure Q: Four-Point Bending Test Setup

Source: Yoshihara 2002

The deflection for the four-point bending test is derived from elastic bending theory and can be found by means of virtual work. The real and virtual shear and moment diagrams are shown in Figure R on the following page.

Figure R: Shear and Moment Diagrams for 4-Point Bending Test

The deflection due to bending δ_b and the deflection due to shear δ_s at the points of loading are calculated through integration of the real and virtual shear and moment diagrams as follows:

$$\delta_b = \frac{1}{EI} \int m(x)M(x)dx = \frac{1}{EI} \left[2 \frac{1}{3} \left(\frac{Pl}{12} \right) \left(\frac{l}{12} \right) \left(\frac{l}{3} \right) + 2 \frac{1}{3} \left(\frac{Pl}{12} \right) \left(\frac{l}{12} \right) \left(\frac{l}{6} \right) \right] = \frac{Pl^3}{432EI},$$

$$\delta_s = \frac{s}{GA} \int v(x)V(x)dx = \frac{s}{GA} \left[\left(\frac{P}{4} \right) \left(\frac{1}{4} \right) \left(\frac{l}{3} \right) + \left(-\frac{P}{2} \right) \left(-\frac{1}{2} \right) \left(\frac{l}{3} \right) + \left(\frac{P}{4} \right) \left(\frac{1}{4} \right) \left(\frac{l}{3} \right) \right] = \frac{sPl}{8GA},$$

Eq. 2

and the total vertical deflection δ at the points of loading, as shown in Eq. 3, is

(Yoshihara 2002)

$$\delta = \delta_b + \delta_s = \frac{Pl^3}{432E_x I} + \frac{sPl}{8GA}, \quad \text{Eq. 3}$$

Where P is the total load applied,
 l is the overall length of the specimen,
 I is the moment of inertia of the cross section,
 A is the cross-sectional area,
 s is Timoshenko's shear factor (1.5 for this experiment),
 E_x is Young's modulus in the longitudinal direction, and
 G is the shear modulus.

If the shear deformation is ignored, the displacement is defined in terms of the apparent Young's modulus E_s in Eq. 4 as follows (Yoshihara 2002):

$$\delta = \frac{Pl^3}{432E_s I}. \quad \text{Eq. 4}$$

The following relation (Eq. 5) is then obtained from Eq.'s 3 and 4 (Yoshihara 2002):

$$\frac{1}{E_s} = \frac{1}{E_x} + 4.5 \frac{s}{G} \left(\frac{h}{l} \right)^2. \quad \text{Eq. 5}$$

The shear modulus G can be found even if Young's modulus E_x is unknown. If E_x is known, then G can be solved using simple algebra. However, if E_x is not known then E_x and G can be solved for simultaneously using varying height-to-length ratios and the method of least squares. This experiment solved for E_x and G simultaneously and then compared E_x to the E_{linear} from the tension tests to assess the accuracy of the results.

Two series of tests were performed using the four-point bending setup described by Yoshihara (2002). The first series of tests consisted of four unplanned boards and four planed boards tested at their full length. For this series, only two different h/l (depth/span) ratios were explored and more testing was needed. Series two of testing consisted of eight more specimens with varying h/l ratios.

The boards were tested using the universal testing machine (UTM). The UTM is controlled manually and displays force and vertical displacement values. The test setup is shown in Figure S below.

Figure S: Four-Point Bending Test Setup

In this experiment, the specimens were tested in shear in the 1-3 plane; see Figure F on page 14. This orientation was chosen because it required the least specimen preparation, since the span was suggested to be 20 times larger than the depth (Yoshihara 2002).

Therefore this experiment would yield a shear modulus for the 1 axis in the 3 direction

G_{13} .

The supports and loads were alternated and placed at third-points of the beam. Total beam lengths that were used were 18", 25.5" and 42". The internal loading point takes $\frac{3}{4}$ of the total load and the cantilevered loading point takes $\frac{1}{4}$ of the total load. In theory, the two loading points displace the exact same amount. The two load point

displacements were checked to be the same using dial gauges as shown in Figure T, below.

Figure T: Displacement Check

For this check, both gauges were visually read and recorded at a given load. The difference between the two displacements was found to be less than 5%. Therefore it was assumed that both points displace the same amount. Also, the displacement given digitally by the universal testing machine was shown to match the displacement given by the dial gauges.

The load P was the applied load from the UTM. Both the load and the vertical head displacement were recorded manually by reading the load gauge and head deflection display. The UTM output devices are shown in Figure U on page 28, following.

Figure U: Universal Testing Machine Readouts

For each test, deflection vs. load data was collected at increments of 25 and 50 lbs. Since the four-point bending method works best when the material is in the linear-elastic range, data was collected at smaller load increments in the initial stages of loading.

In the first series of tests, four planks were tested without modification, and four of the planks were planed to make the section rectangular. The width of each board was 5.125". The moment of inertia of an unplanned board was found by hand to be 0.479 in^4 . Subsequently, the thickness of the planed boards was reduced to 0.828", and the moment of inertia to 0.242 in^4 . Figure V on the following page shows an unmodified cross-section.

Figure V: Cross-Section of Unplaned Board

As seen in the figure above, the exact height, or thickness, of the unplaned board was difficult to measure because of the uneven top surface. Therefore, exact h/l ratios could not be determined and the unplaned board tests were ultimately not used in determining G . Specimen 23 was the only specimen taken from the first series of testing because that series of testing only included one useful h/l ratio. In the second series of testing, the widths of the boards were 4.453" or 4.328", and the heights of the boards varied among 0.664", 0.781", 0.836", and 0.844". The final range of h/l ratios used was 0.0197 to 0.0469.

E_s , the apparent Young's modulus assuming only bending deformation, needed to be calculated for each test specimen using the load-deflection data. Eq. 4 on page 25 was rearranged to the slope-intercept form of a line as follows:

$$\frac{Pl^3}{432I} = E_s \delta. \quad \text{Eq. 6}$$

In this form, the left hand side represents the dependent variable, δ represents the independent variable, and E_s is the slope of the line. For each test, the left hand side was plotted as a function of δ , and the slope was recorded as E_s . A sample plot is shown in Figure W on the following page.

Figure W: Sample Plot of δ vs. $PL^3/432I$, Eq. 6, for a Specimen

A best fit line was used to determine E_s , the slope, for each plot. Table 3 below gives a summary of the specimens, their respective h/l ratios and E_s values.

Specimen Label	width (in)	height (in)	length (in)	h/l (in/in)	E_s (psi)
23	5.125	0.828	42.0	0.0197	1006054
2A1	4.453	0.844	18.0	0.0469	906425
2B1	4.453	0.664	18.0	0.0369	1006502
2C1	4.453	0.836	18.0	0.0464	929482
2D1	4.328	0.781	18.0	0.0434	943069
2A2	4.453	0.844	25.5	0.0331	975654
2B2	4.453	0.664	25.5	0.0260	1050863
2C2	4.453	0.836	25.5	0.0328	1081665
2D2	4.453	0.781	25.5	0.0306	1018660

Table 3: 4pt Bending Experiment Specimen Summary

Then, Eq. 5 from page 25 was used, with the varying E_s and h/l ratios, to solve simultaneously for E_x and G . Rearranging Eq. 5 gives

$$G = 4.5 \frac{s}{\frac{1}{E_s} - \frac{1}{E_x}} \left(\frac{h}{l} \right)^2. \quad \text{Eq. 7}$$

Substituting the values of h/l and the corresponding E_s for all nine specimens gave a set of nine equations with two unknowns, G and E_x . All nine equations were then simultaneously plotted on one graph of G as a function of E_x , as shown in Figure X below.

Figure X: Plot Used in Solving for E_x and G_{13}

Theoretically, all nine functions would cross at the same point and the values of E_x and G could be read straight off the graph. However, as experimental results aren't perfect, statistics were needed to determine the desired values. The highlighted area in the graph above shows where many of the functions cross and where the values of E_x and G will fall. The method of least squares was used to find values of E_x and G that most closely

satisfied all nine functions at the same time and these values were found to be $E_x = 1,087,000$ psi and $G_{13} = 97,000$ psi.

In conclusion, the values found from this test were reliable. There is less than a 3% difference between Young's modulus found from this test ($E_x = 1,087,000$) and Young's modulus found from the tension tests ($E_{linear} = 1,064,000$). This finding helps to validate this experiment and verify the value found for the shear modulus G_{13} .

4.3 Simulated Four-Point Bending Test

Four-point bending tests were simulated using a finite element analysis (FEA) in order to assess the accuracy of the four-point bending test and verify the results of the physical experiment.

The experimental values found for Young's modulus and the shear modulus, from the four-point bending experiment, were input as material properties in the finite element model to show that the simulated experiment predicts these same values. The beam was modeled in the finite element computer program ANSYS, using beam elements. The material properties used in these simulations were defined as E_x for Young's modulus in the longitudinal direction, G_{xy} for the shear modulus, ν_{xy} for Poisson's ratio, and s for the shear factor; the values of these constants were as follows:

$$\begin{aligned} E_x &= 1,064,000 \text{ psi,} \\ G_{xy} &= 97,000 \text{ psi,} \\ \nu_{xy} &\text{ is estimated as 0.1, and} \\ s &= 1.2. \end{aligned}$$

Because there was no experimental data for Poisson's ratio for this material, the value was estimated based on known values for wood materials. The deflection of the four-

point bending test does not depend on Poisson's ratio and therefore has no effect in this experiment; however, ANSYS required that it be defined to run the analysis.

The initial set of tests was run with the more commonly known shear factor of 1.2 instead of 1.5 as suggested by Yoshihara. The test setup and meshed beam elements are shown in Figure Y below.

Figure Y: Simulated Experiment Test Setup and Beam Mesh

Three fourths of the total load is applied to the inner loading point B while one fourth of the total load is applied to the outer loading point D. Deflection can be measured at either loading point because they both displace the same amount. For each specimen, a total load of 20 pounds was applied and the resulting deflection was recorded. Then, E_s for each specimen was solved for using Eq. 6. A summary of the specimens used and their corresponding h/l ratios and E_s values is shown in Table 4 on the following page.

Specimen Label	width (in)	height (in)	length (in)	h/l (in/in)	E_s (psi)
1	4.000	0.800	16	0.0500	897795
2	4.453	0.844	18	0.0469	915032
3	4.453	0.664	18	0.0369	966612
4	4.453	0.836	18	0.0464	917482
5	4.453	0.781	18	0.0434	933818
6	4.453	0.844	24	0.0342	974745
7	4.453	0.664	24	0.0277	1006923
8	4.453	0.836	24	0.0348	976292
9	4.453	0.781	24	0.0325	986644
10	4.000	0.800	36	0.0222	1026460
11	4.000	1.000	36	0.0278	1006492
12	4.000	1.200	36	0.0333	983117
13	4.000	1.800	36	0.0500	897795

Table 4: 4pt Bending Simulated Experiment Specimen Summary

Then Eq. 7 was used to plot G as a function of E_x for each specimen, shown in Figure Z below.

Figure Z: Plot Used in Solving for E_x and G for Simulated 4pb Experiment ($s = 1.2$)

The resultant $E_x = 1,063,984$ psi and $G = 121,278$ psi are easily read off the plot.

However, the value for G is off by a factor of 1.25 which is also the ratio of the two shear factors, 1.5 and 1.2. Another set of tests were run with the shear factor equal to 1.5 and the resultant E_x vs. G plot is shown in Figure AA.

Figure AA: Plot Used in Solving for E_x and G for Simulated 4pb Experiment ($s = 1.5$)

With a shear factor of 1.5, the moduli are predicted as $E_x = 1,064,000$ psi and $G = 97,000$ psi. These results show that, theoretically, the four-point bending test can accurately predict Young's modulus E_x and the shear modulus G when using a shear factor of 1.5. Therefore, my experimental values for the longitudinal Young's modulus and shear in the 1-3 plane are reliable.

4.4 Experiment Results

It was initially thought that G_{I2} would not differ greatly from G_{I3} . For wood, the properties in the two weak axes are very similar to each other. However, evidence shows that G_{I2} is much larger than G_{I3} for this material. Since a four-point bending experiment was not performed in the 1-2 plane, the shear modulus G_{I2} was extracted from the classic orthotropic elasticity equation, Eq. 8 (Jones 1975).

$$\frac{1}{E_{\theta}} = \frac{\cos^4 \theta}{E_1} + \frac{\sin^4 \theta}{E_2} - \frac{2\nu_{I2} \cos^2 \theta \sin^2 \theta}{E_1} + \frac{\cos^2 \theta \sin^2 \theta}{G_{I2}}, \quad \text{Eq. 8}$$

where E_1 is known,
 E_2 is known,
 ν_{I2} is estimated as 0.1,
 E_{θ} is known for certain values of θ , and
 G_{I2} is unknown.

In this equation, the value of Poisson's ratio was based on known wood values and is inversely related to the shear modulus. Therefore, if ν_{I2} was estimated to be larger than 0.1, the resulting shear modulus G_{I2} would be smaller.

E_{θ} was plotted as a function of θ , and a value of G_{I2} was found that would minimize the error between Eq. 8 and the tension experiment data. A plot of this equation, with two different values of G_{I2} , along with the tension data is shown in Figure BB.

Figure BB: Fitting Eq. 8 to Experimental Data

G_{12} is found to equal 284,000 psi when using Eq. 8. Assuming that $G_{12} = G_{13}$ would show that the classic orthotropic elasticity equation predicts a shear modulus with an error of 193%, too large to be entirely assigned to the inapplicability of the equation to our material. Therefore, the plastic-fiber composite material must have different properties in the 2 and 3 axes. Further testing in both axes is needed to support this claim. For the purpose of this thesis, it is assumed that $G_{12} = 284,000$ psi and $G_{13} = 97,000$ psi.

Reasons for differing behavior in the 2 and 3 axes may be partly attributed to the length of the fibers. In wood, the length of the cells ensures that all of them lie in the same general direction, giving it the properties of one strong axis and two similarly weaker orthogonal axes. The fibers in this material are much smaller, $\approx 1\text{mm}$, and may not all align perfectly in one direction. When this 1.125" x 5.125" board is extruded, the

fibers may align most towards the 1 axis, second most towards the 2 axis, and the least towards the 3 axis.

5.0 CONSTITUTIVE MODEL

The stress-strain constitutive model for this plastic-fiber composite was developed from two known wood-based constitutive models. The proposed constitutive model predicts the bilinear stress-strain relationship at any orientation to the strong axis, given a nonlinear or bilinear stress-strain relationship in the strong axis.

5.1 First Wood-Based Constitutive Model

Research was done to find a constitutive model for this material that could predict the linear-elastic Young's modulus at varying orientations to the strong axis. Three models were considered, and one was chosen that best matched the experimental tension test data for the 1-2 plane.

The first model considered was the classic orthotropic elasticity equation, Eq. 8, which was shown to fit the experimental data when the shear modulus was 284,000 psi. This model was ultimately not chosen because of the dependence on the shear modulus G , which was not solved for experimentally in the 1-2 plane.

The second model considered for this material is the empirical relationship for plywood-type panels proposed by Saliklis (2000). This relationship, shown in Eq. 9 below, looks similar to the orthotropic elasticity equation except without the Poisson's ratio term.

$$\frac{1}{E_{\theta}} = \frac{\cos^4 \theta}{E_1} + \frac{\sin^4 \theta}{E_2} + \frac{\cos^2 \theta \sin^2 \theta}{(A^{2A})G_{12}}, \quad \text{Eq. 9}$$

where $A = E_2/E_1$, and
 $G_{12} = 284,000$ psi.

A plot of Eq. 9 against the experimental data is shown in figure CC on the next page.

Figure CC: Fitting Eq. 9 to Experimental Data

This model underestimates Young's modulus between the strong and weak axes because it was calibrated to work for plywood, which is strongest at 0° and 90°, but weaker in between.

The final model tested for this material was the empirically derived Hankinson's formula shown below.

$$E_{\theta} = \frac{E_1 E_2}{E_1 \sin^n \theta + E_2 \sin^n \theta}. \quad \text{Eq. 10}$$

This model has been shown to work well for many different species of wood, with the exponent value n varying between 1.5 and 2. A minimization of error approach was used to find the value $n = 1.8$ for this plastic-fiber composite. Figure DD on the next page shows Eq. 10, with $n = 1.8$ and $n = 2$, against the experimental data.

Figure DD: Fitting Eq. 10 to Experimental Data

Hankinson's formula provides the best match to the tension experiment data, whether $n = 2$ or 1.8 , and was chosen as the best model that predicts the linear-elastic Young's modulus at varying angles for this material. The only drawback with using Hankinson's formula is that it cannot be used to solve for the shear modulus.

5.2 Second Wood-Based Constitutive Model

Further research was done to find a constitutive model that could predict bilinear stress-strain curves in the 1 and 2 axes. The model that was considered was the one proposed by Saliklis (2003). This model can take a bilinear stress-strain curve in the 1 axis and predict the bilinear stress-strain curve for the 2 axis or vice versa. It can also be used to predict the bilinear stress-strain relationship for shear in the 1-2 plane. The bilinear stress-strain relationship is shown in figure EE.

Figure EE: Bilinear Constitutive Model

In this model, A, B and C describe the two straight lines of the stress versus strain curve.

First, the bilinear stress-strain relationship for the 2 axis is defined as follows:

$$\begin{aligned}\sigma_2(\varepsilon_2) &= A_2 \varepsilon_2 & \text{for } \varepsilon_2 \leq \frac{C_2}{A_2 - B_2}, \\ \sigma_2(\varepsilon_2) &= B_2 \varepsilon_2 + C_2 & \text{for } \varepsilon_2 \geq \frac{C_2}{A_2 - B_2}.\end{aligned}\quad \text{Eq. 11}$$

Next, the bilinear stress-strain prediction for the 1 axis is as follows:

$$\begin{aligned}\sigma_1(\varepsilon_1) &= A_1 \varepsilon_1 & \text{for } \varepsilon_1 \leq \frac{C_2}{A_2 - B_2} \sqrt{\frac{\nu_2}{\nu_1}}, \\ \sigma_1(\varepsilon_1) &= B_1 \varepsilon_1 + C_1 & \text{for } \varepsilon_1 \geq \frac{C_2}{A_2 - B_2} \sqrt{\frac{\nu_2}{\nu_1}},\end{aligned}\quad \text{Eq. 12}$$

where

$$A_1 = A_2 \frac{\nu_1}{\nu_2},$$

$$B_1 = B_2 \frac{\nu_1}{\nu_2},$$

$$C_1 = C_2 \sqrt{\frac{\nu_1}{\nu_2}}. \quad \text{Eq. 13}$$

For simplicity, the Poisson's ratio terms ν_{12} and ν_{21} were shortened to ν_1 and ν_2 respectively. If the Poisson's ratios are unknown, the ratio ν_1/ν_2 can be found using the initial moduli in the 1 and 2 axes.

$$\frac{\nu_1}{\nu_2} = \frac{A_1}{A_2}. \quad \text{Eq. 14}$$

Third, for shear stress τ and shear strain γ in the 1-2 plane, the bilinear stress-strain prediction is as follows:

$$\begin{aligned} \tau_{12}(\gamma_{12}) &= A_{12} \gamma_{12} & \text{for } \gamma_1 &\leq \frac{C_2}{2(A_2 - B_2)} \sqrt{\frac{A_2}{G_{12}}}, \\ \tau_{12}(\gamma_{12}) &= B_{12} \gamma_{12} + C_{12} & \text{for } \gamma_1 &\geq \frac{C_2}{2(A_2 - B_2)} \sqrt{\frac{A_2}{G_{12}}}, \end{aligned} \quad \text{Eq. 15}$$

where

$$\begin{aligned} A_{12} &= G_{12}, \\ B_{12} &= G_{12} \frac{B_2}{A_2}, \\ C_{12} &= \frac{C_2}{2} \sqrt{\frac{G_{12}}{A_2}}. \end{aligned} \quad \text{Eq. 16}$$

If the initial shear modulus G_{12} is unknown it can be estimated by means of the empirical relationship:

$$G_{12} = \frac{A_2 \sqrt{\nu_1 / \nu_2}}{2(1 + \sqrt{\nu_1 \nu_2})}. \quad \text{Eq. 17}$$

Poisson's ratio was not known for this material, nor was it solved for experimentally, therefore the previous value of $G_{12} = 284,000$ psi was used.

Representative stress-strain curves for the 1 and 2 axes were collected from the tension test data. The 1 axis curve was then fit with a bilinear relationship, shown below,

Figure FF: Fitting a Bilinear Curve to the 1-Axis

Then, the initial modulus of the 2 axis, $A_2 = 495,000$ psi, was used to predict the bilinear relationship for the 2 axis. Similarly, the initial shear modulus, $G_{12} = 284,000$ psi, was used to predict a bilinear stress-strain relationship for the 1-2 plane. The results are shown on the next page in Figure GG.

Figure GG: Bilinear Stress-Strain Predictions

This model seems to work well for this material. The nonlinear behavior on the 2 axis is fairly accurately captured by the bilinear prediction. However, the accuracy of the bilinear prediction for shear in the 1-2 plane cannot be assessed due to the lack of shear data past the linear-elastic region. If, however, future data can be found for nonlinear shear response, then this model can be readily applied to such data.

5.3 New Constitutive Model

The proposed constitutive model is a combination of both wood-based models as it predicts the bilinear stress-strain relationship at any orientation to the strong axis, given

a nonlinear stress-strain relationship in the strong axis and the Poisson's ratios or the linear-elastic modulus at 90° to the strong axis.

A series of simulated off-axis tension tests, under the bilinear stress-strain model, were performed in ANSYS to create a continuous set of data for the linear-elastic modulus A , nonlinear modulus B , and bilinear point strain ε_b . This set of data was compared to the physical experimental data. New equations for A , B and ε_b were developed to include angle to the strong axis θ . These equations were based on a modified version of Hankinson's formula shown in Eq. 18 below.

$$E_\theta = \frac{E_1 E_2}{E_1 \sin^4 \theta + \alpha(E_1 + E_2) \sin^2 \theta \cos^2 \theta + E_2 \cos^4 \theta}. \quad \text{Eq. 18}$$

This new formula was created to include the variable α , which would make it possible to better fit the ε_b data. It is also important to note that when $\alpha = 1$, Eq. 18 reduces to Hankinson's formula. Eq. 18 can be further modified by relating E_2 to E_1 or ε_{b2} to ε_{b1} using the ratio ν_2/ν_1 , see Eq. 19 below.

$$E_\theta = \frac{E_1 \left(\frac{\nu_2}{\nu_1} \right)}{\sin^4 \theta + \alpha \left(1 + \frac{\nu_2}{\nu_1} \right) \sin^2 \theta \cos^2 \theta + \left(\frac{\nu_2}{\nu_1} \right) \cos^4 \theta},$$

and

$$\varepsilon_{b\theta} = \frac{\varepsilon_{b1} \sqrt{\frac{\nu_1}{\nu_2}}}{\sin^4 \theta + \alpha \left(1 + \sqrt{\frac{\nu_1}{\nu_2}} \right) \sin^2 \theta \cos^2 \theta + \sqrt{\frac{\nu_1}{\nu_2}} \cos^4 \theta}. \quad \text{Eq. 19}$$

Because the bilinear point strain is smaller at 0° and larger at 90° , see Figure JJ, but still linked through the Poisson's ratios, it uses ν_1/ν_2 instead of ν_2/ν_1 . Using this form of the equation, both the A and B data could be matched well when $\alpha \approx 1$ and the ε_b data could be matched well when $\alpha \approx 2$. The A , B , and ε_b data along with their corresponding forms of Eq. 19 are shown in Figures HH through JJ.

Figure HH: Matching A Data

Figure II: Matching B Data

Figure JJ: Matching ε_b Data

Through Eq. 19, the bilinear model can consider any orientation between the 0° and 90° axes. A summary of the new constitutive relationship is given in Eq.'s 20 and 21 following.

$$\begin{aligned}\sigma_\theta(\varepsilon_\theta) &= A_\theta \varepsilon_\theta & \text{for } \varepsilon_\theta \leq \varepsilon_b, \\ \sigma_\theta(\varepsilon_\theta) &= B_\theta \varepsilon_\theta + C_\theta & \text{for } \varepsilon_\theta > \varepsilon_b,\end{aligned}\tag{Eq. 20}$$

where

$$\begin{aligned}A_\theta &= \frac{A_1 \left(\frac{\nu_2}{\nu_1} \right)}{\sin^4 \theta + \left(1 + \frac{\nu_2}{\nu_1} \right) \sin^2 \theta \cos^2 \theta + \left(\frac{\nu_2}{\nu_1} \right) \cos^4 \theta}, \\ B_\theta &= \frac{B_1 \left(\frac{\nu_2}{\nu_1} \right)}{\sin^4 \theta + \left(1 + \frac{\nu_2}{\nu_1} \right) \sin^2 \theta \cos^2 \theta + \left(\frac{\nu_2}{\nu_1} \right) \cos^4 \theta}, \\ \varepsilon_{b\theta} &= \frac{\varepsilon_{b1} \sqrt{\frac{\nu_1}{\nu_2}}}{\sin^4 \theta + 2 \left(1 + \sqrt{\frac{\nu_1}{\nu_2}} \right) \sin^2 \theta \cos^2 \theta + \sqrt{\frac{\nu_1}{\nu_2}} \cos^4 \theta}, \\ C_\theta &= \varepsilon_{b\theta} (A_\theta - B_\theta),\end{aligned}\tag{Eq. 21}$$

and

$$\frac{\nu_2}{\nu_1} = \frac{A_2}{A_1}.$$

To demonstrate the effectiveness of this model, bilinear stress-strain curves were predicted for orientations of 33° , 45° and 68° to the strong axis. These bilinear

predictions were then compared to the experimental stress-strain data and can be seen in Figures KK through MM following.

Figure KK: Bilinear Prediction for 33° Data

Figure LL: Bilinear Prediction for 45° Data

Figure MM: Bilinear Prediction for 68° Data

The previous figures show that this proposed constitutive model accurately captures the stress-strain relationships for any orientation to the strong axis. A summary plot of the bilinear constitutive model with the experimental data is shown in Figure NN.

Figure NN: Summary of Bilinear Stress-Strain Predictions

The plot above shows a comparison of bilinear stress-strain predictions for 0°, 33°, 45° and 90°. The 68° orientation is not included in the summary plot for visual clarity. As expected, the bilinear predictions for angles between 0° and 90° are enveloped by the bilinear curves of each.

6.0 APPLICATION IN FINITE ELEMENT ANALYSIS

The bilinear constitutive model was used to perform a nonlinear finite element analysis (FEA) using the computer program ANSYS. A one-story lateral load resisting frame with a distributed load at the roof level was analyzed.

6.1 Computer Modeling

Plane elements with a thickness of 0.5 inches were used to model all parts of the frame, resulting in a braced frame with moment connections. The frame elevation with dimensions is shown in figure OO below.

Figure OO: Analyzed Frame

The linear-elastic material properties used in this simulation were defined as E_x and E_y for Young's modulus in the longitudinal and transverse directions, G_{xy} for the shear modulus, and ν_{xy} for Poisson's ratio; the values of these constants were as follows:

$$\begin{aligned} E_x &= 1,064,000 \text{ psi,} \\ E_y &= 495,000 \text{ psi} \\ G_{xy} &= 284,000 \text{ psi, and} \\ \nu_{xy} &\text{ is estimated as 0.1.} \end{aligned}$$

Generalized Anisotropic Hill Potential was used to simulate nonlinearity by defining yield stresses and post-yield moduli. Although this PP/kenaf material was not proven to yield, this nonlinear model still applied because plasticity was not an issue. The required constants included σ_{xyield} and σ_{yyield} for the yield stresses in the x and y directions, $\tau_{xyyield}$ for the yield shear stress, and $E_{xplastic}$, $E_{yplastic}$ and $G_{xyplastic}$ for the corresponding post-yield moduli; the values of these constants were as follows:

$$\begin{aligned} \sigma_{xyield} &= 2727 \text{ psi,} \\ \sigma_{yyield} &= 1857 \text{ psi,} \\ \tau_{xyyield} &= 703 \text{ psi,} \\ E_{xplastic} &= 578,000 \text{ psi,} \\ E_{yplastic} &= 268,000 \text{ psi, and} \\ G_{xyplastic} &= 154,000 \text{ psi.} \end{aligned}$$

The frame mesh is shown in figure PP on page 55 following.

Figure PP: Frame Mesh

The element coordinates were adjusted to align the strong axes along the lengths of the members, simulating fiber alignment from extrusion or injection molding. Therefore, the strong axes of the braces are in the same orientation as the braces themselves. It was modeled this way because injection molding is one possible construction solution using this material. Another construction solution would be to take one large sheet of extruded material and cut out pieces leaving one continuous frame. The strong axis would then be aligned at all points on the frame including the braces. This would have resulted in the

braces having a lower Young's modulus and failure load. A pushover analysis would have shown decreased frame stiffness and a decreased ultimate load and deflection. The simulated fiber alignment can be seen in figure QQ below.

Figure QQ: Close-up of Element Local Axes

Lateral load was placed at the roof level and horizontal deflections were monitored at point A. The frame was pushed until the stress in the braces reached 4650 psi, the average failure stress at 0° from the tension tests; see Table 2 on page 21. A force displacement relationship was recorded.

6.2 Analysis Results

A simple hand calculation using the bilinear-stress strain prediction shows the effectiveness of this model in both hand and finite element analyses. The force-displacement relationship for both is shown below in figure RR.

Figure RR: Force-Displacement Relationship for the Analyzed Frame

Both models predict very similar pushover curves and demonstrate the effectiveness of the proposed constitutive model in nonlinear analyses of this plastic-fiber composite.

7.0 CONCLUSIONS

Experimental work was done for Young's modulus at varying orientations E_θ and for the shear modulus G . Physical experimental results were verified through simulated experiments using ANSYS. The experimental testing done for the shear modulus gives evidence that this PP/kenaf material has noticeably different properties in the two weak axes which is contrary to the initial assumptions. However, there was difficulty encountered when trying to relate the shear stress τ to the shear strain γ past yield. New experimental data is required to complete the linkage between E_θ and G . Therefore this study focused on a nonlinear constitutive model of E .

A semi-empirical approach was used to create a new constitutive stress-strain model that has been demonstrated to predict the nonlinear stress-strain response of this material at varying orientations to the strong axis. An important strength of this model is the application of simple bilinear curves to distinctly nonlinear stress-strain data. This simplification is demonstrated through hand analyses of load resisting systems.

The second strength of this model is that it relates the apparent yield strains in all orientations to the strong axis. Bilinear approximations also assure distinct yield points, which is why ANSYS uses the bilinear model for orthotropic materials. Nonlinear finite element analyses of the larger emergency shelter were performed to demonstrate the effectiveness of this model when implemented into ANSYS.

The third strength of this model is that it captures the stress-strain response in any possible orientation, whereas previous methods only linked the three orthogonal axes.

Weaknesses stem from the lack of shear data available, which is why this model is based on the empirical Hankinson's formula. Further testing may be done to see if a theoretical relationship, such as the classic orthotropic elasticity equation, can be used to derive a model similar to the one proposed here. This constitutive model has been shown to work well for this PP/kenaf material; however, further research may also be done to investigate the effectiveness of this model with other orthotropic materials such as wood.

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9.0 APPENDIX

Four-Point Bending Test Displacement Check

Check displacements at a load of 300 lbs

Machine Reading = 0.210"

Dial 1 Reading = 0.214"

Dial 2 Reading = 0.225"

% error between dials = $(0.225 - 0.215) / 0.225 \times 100 = 4.89\%$ error ←

Predicted displacement from dials (through geometry) = 0.217"

% error between predicted and machine =

$(0.217 - 0.210) / 0.217 \times 100 = 3.11\%$ error ←

Check displacement at a load of 900 lbs

Machine Reading = 0.663"

Dial 1 Reading = 0.681"

Dial 2 Reading = 0.715"

% error between dials = $(0.715 - 0.681) / 0.715 \times 100 = 4.76\%$ error ←

Predicted displacement from dials (through geometry) = 0.690"

% error between predicted and machine =

$(0.690 - 0.663) / 0.690 \times 100 = 3.84\%$ error ←

Frame Pushover Hand Calculation:

Given:

$$\begin{aligned}
 A_{\text{brace}} &= 0.5'' \times 4'' = 2 \text{ in}^2 \\
 L_{\text{brace}} &= \sqrt{(47^2 + 90^2)} = 102'' \\
 E_{\text{elastic}} &= 1064000 \text{ psi} \\
 E_{\text{plastic}} &= 578000 \text{ psi} \\
 \sigma_{\text{yield}} &= 2720 \text{ psi} \\
 \sigma_{\text{fail}} &= 4650 \text{ psi}
 \end{aligned}$$

Solution:

$$K_{\text{before yield}} = (2)(2 \text{ in}^2)(1064000 \text{ psi}) \cos^2(62)/(102'') = 9196 \text{ lb/in}$$

$$F_{\text{brace yield}} = (2720 \text{ psi})(2 \text{ in}^2) = 5440 \text{ lbs}$$

$$P_{\text{yield}} = (2)(5440 \text{ lbs}) \cos(62) = 5108 \text{ lbs} \quad \leftarrow$$

$$\Delta_{\text{yield}} = (5108 \text{ lb})/(9196 \text{ lb/in}) = 0.555'' \quad \leftarrow$$

$$K_{\text{after yield}} = (2)(2 \text{ in}^2)(578000 \text{ psi}) \cos^2(62)/(102'') = 4996 \text{ lb/in}$$

$$F_{\text{brace fail}} = (4650 \text{ psi})(2 \text{ in}^2) = 9300 \text{ lbs}$$

$$P_{\text{fail}} = (2)(9300 \text{ lbs}) \cos(62) = 8732 \text{ lbs} \quad \leftarrow$$

$$\Delta_{\text{fail}} = 0.555'' + (8732 - 5108 \text{ lbs})/(4947 \text{ lb/in}) = 1.288'' \quad \leftarrow$$

10.0 GLOSSARY

bilinear point – the point at which the stress-strain curve changes slope.

isotropic – material properties are the same in every direction.

linear-elastic range – the range in which the maximum strain is less than the bilinear point strain for any portion of a specimen.

nonlinear modulus – the slope of the stress-strain curve past the bilinear point.

nonlinear range – the range in which the minimum strain is more than the bilinear point strain for any portion of a specimen.

modulus of rupture – the normal stress on the tension side of a specimen loaded in bending, when the material starts to yield.

orthotropic – material properties depend on the direction in which they are measured.

11.0 ACRONYMS

PP	polypropylene
HDPE	high-density polyethylene
FEA	finite element analysis
ASTM	American Society for Testing and Materials
NDS	National Design Specification
UTM	universal testing machine