**PARAMETER ESTIMATION OF FUNDAMENTAL
TECHNICAL AIRCRAFT INFORMATION
APPLIED TO AIRCRAFT PERFORMANCE**

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ABSTRACT

Parameter Estimation of Fundamental Technical Aircraft Information Applied to Aircraft Performance

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Inverse problems can be applied to aircraft in many areas. One of the disciplines within the aerospace industry with the most openly published data is in the area of aircraft performance. Many aircraft manufacturers publish performance claims, flight manuals and Standard Aircraft Characteristics (SAC) charts without any mention of the more fundamental technical information of the drag and engine data. With accurate tools, generalized aircraft models and a few curve-fitting techniques, it is possible to evaluate vehicle performance and estimate the drag, thrust and fuel consumption (TSFC) with some accuracy.

This thesis is intended to research the use of aircraft performance information to deduce these aircraft-specific drag and engine models. The proposed method incorporates models for each performance metric, modeling options for drag, thrust and TSFC, and an inverse method to match the predicted performance to the actual performance. Each of the aircraft models is parametric in nature, allowing for individual parameters to be varied to determine the optimal result.

The method discussed in this work shows both the benefits and pitfalls of using performance data to deduce engine and drag characteristics. The results of this

method, applied to the McDonnell Douglas DC-10 and Northrop F-5, highlight many of these benefits and pitfalls, and show varied levels of success. A groundwork has been laid to show that this concept is viable, and extension of this work to additional aircraft is possible with recommendations on how to improve this technique.

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Nomenclature

a, b	Parametric Variables
$\mathcal{R}$	Aspect Ratio
C_L	Lift Coefficient
ΔC_L	Drag Polar Shifting Coefficient
$C_{L_{max}}$	Maximum Lift Coefficient
C_D	Drag Coefficient
C_{D_0}	Zero-lift Drag Coefficient
D	Drag, lbf
e	Oswald Efficiency Factor
F	Force, lbf
g	Gravitational Constant, ft/sec ²
J	Jacobian
k	Drag due to Lift Factor
L	Lift, lbf
n	Load Factor
M	Mach Number
P	Pressure, psf
p, q	Parametric Variables
q	Dynamic pressure, lbs/ft ²
r	Turn Radius
Re	Reynolds Number
RoC	Rate of Climb
S_{ref}	Wing Reference Area,
SAC	Standard Aircraft Characteristics
T	Thrust, lbf
$Temp$	Temperature, degrees Fahrenheit
$TSFC$	Thrust Specific Fuel Consumption, lb/hr/lb
V	Velocity, ft/sec
V_H	Horizontal velocity component, ft/sec
V_T	Velocity along aircraft's body axis, ft/sec
W	Weight, lbf
$\dot{W}$	Fuel Flow

Greek

γ	Flight Path Angle, Ratio of Specific Heats
δ_0	Non-dimensional Pressure
θ_0	Non-dimensional Temperature
ϕ	Bank Angle
$\dot{\chi}$	Turn Rate

Subscript

i	Variable number
x	Parallel to Freestream Velocity
z	Perpendicular to Freestream Velocity
∞	Free Stream

Chapter 1

Introduction

This research is intended to use aircraft performance data to deduce an engine deck (thrust lapse and TSFC) and a drag polar of the aircraft in question. This process of reverse engineering, known as solving the inverse problem, couples three main areas of work, each of which will be explored independently prior to its application to the real-world problem. The three areas of exploration are the theory of inverse problems, the creation of engine deck and drag polar models, and the aircraft performance equations themselves.

The process utilized to solve this inverse problem will be detailed throughout this paper; an overview of the general process, however, might be useful at this stage to help the reader follow the work in a more complete manner. The process used in this investigation can be seen in Figure 1.1.

The underlying philosophy is as follows: other than an aircraft's weight, wing reference area, aspect ratio and sealevel-static thrust, the only information necessary to predict an aircraft's performance are the drag polar and engine deck. Given the

Figure 1.1: Flowchart of Overall Process

first three parameters mentioned, the derivation of these aero-propulsive functions is possible.³

Gong and Chan³ solved this inverse problem in a more simplified manner than will be attempted here, using a known engine deck to find the parameters in a simplified drag equation. They used only the time-to-climb performance metric and attempted to find the drag parameters for both a Boeing 737 and Learjet 60, and were successful in their attempts.

While this task of solving the inverse problem can be solved perfectly under ideal circumstances (see Chapter 6), it is important to discuss the implications of using real-world data. Data taken from flight tests, as performance data often is, is prone to the conditions of the day and the pilot operating the aircraft, reducing the accuracy of the data. Drag and engine data is often faired and smoothed by hand, adding a human element that is, again, dependent on the person generating the data. For these reasons the “true” drag and engine shapes cannot be perfectly matched (see Chapter 5), which guarantees that the final results will have some error associated with them.

The idea of obtaining full drag and engine models from little amounts of performance data is attractive to many different sectors of the aircraft industry, ranging from premier aircraft companies to academia. Public and private companies could utilize this to understand the effects of changing key aircraft components.

As an example, imagine a company wishes to modify a Cessna Citation X to catch drug runners smuggling drugs into this country. With a maximum speed of Mach 0.92 and a range of about 3,000 nm,⁴ this plane is well suited for the task. However, thousands of pounds of equipment might need to be added to this plane, which could drastically diminish the expected performance. Swapping the Rolls-Royce engines with more powerful ones might deliver the necessary power to keep this plane competitive. Knowing the specific drag information about the original aircraft, which can be obtained with this work, will prove crucial in estimating the new performance characteristics of the heavily modified plane.

This type of retrofitting and upgrading is often done by a company which is not the original manufacturer of the aircraft, making this process of reverse-engineering very necessary. Avionics and systems upgrades occur multiple times throughout an aircraft’s lifetime – which can be as long as 50 to 80 years, in some cases – and require in-depth analysis to ensure that everything will work correctly. Re-engining aircraft can also occur multiple times throughout an aircraft’s lifetime as new, more efficient engines are produced, requiring this same analysis to be conducted.

Design professors nationwide could use this idea to extract historical information from aircraft they otherwise know little about. Practical validation of students’ final design projects for these classes could be completed with the help of this work.

1.1 Motivation

The lack of drag and engine data is a real-world problem in both industry and academia. Often times companies are unwilling or unable to provide this data, making the retrofitting and upgrading discussed in this chapter much more difficult. The data necessary to accurately analyze these aircraft is often classified or proprietary information, and as such, an approximation must be used; this technique of reverse engineering is perfect for this cause. The reverse engineering process, while not able to predict the drag polar and engine deck with 100% accuracy, will provide an “engineering approximation” which will be extremely useful.

This issue can be seen today. The US Air Force has recently ordered new engines

for the E-8 JSTARS, which is a derivative of the Boeing 707-300. The re-engining of these aircraft should allow them to continue flying safely and effectively for another 40-50 years.⁵ The award for new engines, between the GE CFM56 high bypass turbofan and Pratt & Whitney's JT8D-219, went to Pratt & Whitney due solely to the engines having the same weight and center of gravity as the original engines. This choice required less modifications to the aircraft themselves along with significantly less modeling and analysis to compute the new performance.

Having gone through the senior design process, and having watched two subsequent classes since then, it is evident to the author that historical aircraft information is not readily available. For example, many senior design groups from Cal Poly over the past two years used drag data from a DC-10 to validate their designs, which were replacements to the Boeing 737.^{6,7} These aircraft are not of similar weight or size classes, and as such, the usefulness of the validation is limited. The previous class designed an unmanned agricultural sprayer, of which no historical data was available to validate the various drag codes written.

Having drag and thrust functions available for multiple aircraft would help in many aspects of academia. Many classes would benefit from this, including aerodynamics, aircraft performance, and the capstone design sequence. The ability to verify the accuracy of performance codes, for example, against multiple aircraft would greatly enhance the robustness and generality of said codes. The only aircraft of which both engine and drag characteristics are known at Cal Poly are the DC-10 and F-5, and

as such, each project revolves around them.

1.2 History

Parameter estimation and inverse problems have been studied for a very long time. A classic example of the application of inverse problems, prior to the invention of computers, was the discovery of Neptune – the only planet discovered by a mathematical model.⁸ Prior to its discovery, the influence of Neptune on the orbit of Uranus was observed as small perturbations about Newton’s predicted orbital path. These perturbations were originally used as proof that Newton’s law of gravitation was incorrect; two leading scientists, however, found that a large, undiscovered planet orbiting further from the sun could also explain this phenomenon. Urbain Le Verrier and John Couch Adams worked on this problem separately their calculations lead to the prediction of the orbit size and location of Neptune in 1846.

Much closer to home, companies around the world are in constant competition to produce the premier product in their field. A key component to this process is in deducing the characteristics and capabilities of the competition.⁹ GM is said to have bought a brand new Lexus and Toyota Prius with the sole intention of taking it apart to learn each car’s secrets.⁹ This article specifically targets GM’s reverse engineering department – however, GM is not the only company performing these stunts. Maintaining an edge on competitors is a crucial part of industry.

Other commercial uses of reverse engineering involve taking machine made parts

produced without the use of computers and creating a computer model.¹⁰ This is done for many reasons; among the most prevalent are the increased analyses available using computer. In the past, parts were often created individually, with the analysis done by hand. In present day, additional analyses can be conducted, such as Finite Element Analysis and Computational Fluid Dynamics, which can be used to produce a truly optimal part. With current technologies, visual scans of an object may be sufficient to produce an accurate representation of the part at hand, with little tweaking required to adjust it for analysis.¹⁰

The capability to use data readily available to deduce aircraft characteristics has been of interest for years. During the cold war, the U.S. Air Force required immediate analysis of all new Soviet aircraft to discover their capabilities.¹¹ This problem is much more complex than the one posed here but stems from similar ideas. The C.I.A. used pictures – and only pictures – to determine both aerodynamic and engine characteristics, and from there detailed performance calculations. Along the way every branch of the conceptual design process was utilized, including weights and balance, aerodynamics, propulsions, subsystems, and mission performance, to determine the effectiveness of the new aircraft.

More applicable to the work presented here, the theories involved in parameter estimation are most often applied to the determination of stability and control.^{12–16} The analysis of flight test data to determine control derivatives is of prime interest to both NASA and industry; the general model forms are well understood and applicable

in most flight regions, which encourages the use of this technique.¹³ As such, few flight tests are necessary to determine the stability coefficients applicable throughout the majority of the flight regime. These are useful in a safe expansion of the flight envelope for an aircraft, along with validating both wind-tunnel and analytical predictions.

Chapter 2

Setting Up the Problem

The central goal of this research is to investigate the ideas and techniques behind solving the inverse problems of aircraft performance. The process developed should be able to analyze performance data, and from this data, deduce the engine and drag characteristics.

Only a few aircraft parameters are necessary to produce an engine deck and drag polar of decent accuracy. These necessary parameters are weight, T_{SLs} , $\mathcal{R}$ and S_{ref} . Other factors, such as atmospheric properties, are dependant on the data available. Often times the aircraft weight is also dependant on the performance metric; however, this is aircraft specific and depends wholly on how the underlying data is represented.

A quick discussion of the code integration techniques should make the reader more comfortable with the methods explored here, with an emphasis on how the code was actually written. Key code aspects, including a unique flag functionality, will be discussed in enough detail to allow the reader to closely mimic

This problem, already complex in nature, is further complicated by the amount

(and quality) of performance data available. In this chapter the wide range of data available is also explored, with each separate data source providing additional input on the problem at hand.

2.1 Code Integration

As will be discussed further in Chapter 3, a gradient based optimizer has been utilized to match the calculated aircraft performance metrics to the actual metrics. The use of MATLAB's optimizer `lsqnonlin` allows for the ability to write performance codes and include them as subroutines in the general problem. This can be visualized in the same manner Design Structure Matrices (DSM) are used to visualize the flow of information in MDO problems.¹⁷ This can be seen in Figure 2.1. `lsqnonlin` is the optimizer, controlling the flow of variables and information from one subroutine to another. In this sample figure only two routines have been included for the sake of brevity.

Writing the code in this manner allows for each performance subroutine to perform its own unique tasks. If necessary, due to the modality of this writing style, each subroutine can call any other functions as many times as is necessary. Two examples of this are as follows; maximum rate of climb may require an optimizer to find the Mach number at which this occurs, and a takeoff routine would require time integration to determine the takeoff distance. Each of these cases will be discussed in more detail in Chapter 4.

Figure 2.1: DSM Representation of Code Structure

2.2 Flag Functionality

Multiple cases and conditions have been included to allow for different options to be selected while performing this analysis. First and foremost is the option to run only certain performance metrics. This is useful for many reasons, mainly if the aircraft performance data available is lacking in areas. Another interesting capability that this allows is the option to run the same aircraft under multiple scenarios and compare the results, which will be explored further in Chapter 7.

Another option that can be toggled with the use of flags is the Reynolds number effect on drag. In the real world Reynolds number effects can drastically alter an aircraft's performance; however, some older performance routines did not account for changes in Reynolds number, which leads to this being an option.

In optimization problems the scale of the data can be extremely important. Op-

timizing on differences of rate of climb can be order of magnitudes larger than differences in climb gradient, which can have undesired effects on the end result. As such, a flag has been included to allow for normalization of the variables to 1.

In general, the drag and engine characteristics will not be known, as this is the main goal of this work. However, it is entirely plausible that one or two of the drag, thrust or TSFC functions may be known, in which case those values would be used instead of the parametric functions provided. This would allow the program to optimize only on drag, for example, if a complete engine deck is provided. This can be toggled with another flag.

2.3 Data Available: Jane's/Wikipedia

The aircraft data that is most widely available to the general public can be found in *Jane's All the World's Aircraft*¹⁸ and online at www.wikipedia.org.¹⁹ Along with a wealth of background information, these sources often provide limited performance data. The Dassault Falcon 7X, found in *Jane's 2008-2009*, is shown to have the performance characteristics shown in Table 2.1.

In addition to this data, values for S_{ref} , $\mathcal{R}$ and typical operating weight are also available from these sources. These values build a profile of the aircraft's physical information, and represent the only detailed information necessary to perform this analysis.

Table 2.1 shows a few interesting things. First and foremost is that lack of data

Table 2.1: Dassault Falcon 7X Performance Data

Perf. Metric	Source: <i>Jane's</i>		Source: Wikipedia	
	Perf. Value	Units	Perf. Value	Units
Max Operating Mach Number	0.86	[-]	-	-
Max Cruising Speed	481	kts	486	kts
Approach Speed	109	kts	-	-
Time to Climb to FL410	24	min	-	-
Balanced Takeoff Field Length	5,260	ft	-	-
Landing Field Length	2,560	ft	-	-
Range at Mach 0.75	3,090	n.mi.	5,950	n.mi.
Max Certified Altitude	47,000	ft	*51,000	ft

*This is listed as the service ceiling on Wikipedia.

available; eight performance metrics could be found for this aircraft. Due to the restrictions of curve fitting, the aircraft models can have, at a maximum, seven parameters.²⁰ Looking at the data more closely, however, we see that not all of it is of use. For example, the maximum certified altitude is not a function of thrust or drag, but an issue of aircraft certification. Wikipedia does list the service ceiling, but oddly this is higher than the maximum certified altitude reported in *Jane's*. Wikipedia can be edited by anyone, and this fact should be taken into account when using the data on the site. Approach speed is only a function of wing-loading and $C_{L_{max}}$, which is an important parameter, but does not help with the formulation of drag or thrust. Takeoff and landing, while interesting, are not point performance metrics, and are therefore excluded from this study. These areas of flight also use lift augmentation systems, which often change the shape and size of the wing, requiring an entirely different drag polar. Takeoff and landing are discussed in more detail in Chapter 4.

In short, Wikipedia and *Jane's* provide many interesting performance metrics. Many of these metrics, however, while interesting to the general aircraft enthusiast, are of no use to solving the inverse problem under investigation here.

It is also interesting to note the inconsistencies between the two references. While the differences between *Jane's* and Wikipedia are small for two of the comparisons, the difference between the range values is shocking. Noting that the range value reported by Wikipedia is nearly double that of *Jane's*, the initial reaction by the author is to assume that *Jane's* is reporting the radius and Wikipedia is reporting the true range.

Taking a jump forward, a simple drag model will be constructed using only four parameters. For a supersonic aircraft this is the minimum number of parameters to create a viable drag model. The four model parameters included are as follows:

- Subsonic C_{D_0}
- Subsonic k
- Supersonic C_{D_0}
- Supersonic slope for k

These four parameters can create a decent drag model; the results are shown in Figure 2.2. This fit was obtained through the use of `lsqnonlin` and is the best fit available, with this model, for the F-5.

As is seen in Figure 2.2, the model does accurately capture many of the trends of the actual data. While this is promising, of prime concern is the lack of a transonic drag rise. Much of the turns and *RoC* data depends on this region for the analysis,

Figure 2.2: F-5 Drag using Simple Model

and the resulting drag would be largely skewed due to these simplifications. The lack of a transonic drag rise would cause the subsonic C_{D_0} to be larger than it is in reality to offset this effect.

In addition to the transonic approximation, these results use four parameters for drag, while the data available requires that a total of four parameters be used in the analysis. For all of these reasons, it is not recommended that this analysis attempt to be completed with this limited data. Some aircraft have more data reported by either *Jane's All the World's Aircraft* or Wikipedia, but in each case, the number of performance values provided does not allow for complex models to be used.

2.4 Data Available: SAC Chart

Standard Aircraft Characteristics (SAC) charts are intended to present a summary of basic aircraft performance capabilities on the basic mission.²¹ As such, basic data is provided, with abnormal flight conditions and off-design conditions generally ignored. This is seen as an overview of an aircraft's performance and is by no means comprehensive.

While obtaining the Standard Aircraft Characteristics (SAC) chart for an aircraft can be difficult, the immense data found in them makes the SAC chart a great tool for the task at hand. SAC charts not only provide additional performance data, but also dictate the conditions for which the specified values were found. As seen in Table 2.1, the altitude for maximum operating Mach number, maximum cruising speed and range were not specified; this is not the case in SAC charts.

Shown in Table 2.2 at the end of this section is the tabulated data provided in the SAC chart for the Northrop F-5. This data is much more complete than that found in either *Jane's* or Wikipedia. There is still more data contained within the SAC chart, in the form of graphs, shown for the F-5 in Figure 2.3. The performance metrics shown for the F-5 are not indicative of those shown for every aircraft; typical performance metrics provided by a SAC chart include the following:

- Takeoff Distance
- Landing Distance
- Rate of Climb
- Time to Climb

- Maximum Speed
- Mission Radius/Range
- Load Factor
- Radius of Turn
- Acceleration

Figure 2.3: F-5 SAC Chart Data

The inclusion of certain performance metrics is typically a function of the class of aircraft. Although every performance metric is important to the success of an aircraft, a typical SAC chart is only allowed to be six pages in length.²¹ This limits the information that can be portrayed in the document, and as such, it is standard to find different information for different aircraft.

The Northrop F-5, an air-superiority fighter designed for air-to-air combat, focuses on climb, turning abilities and maximum speed, all performance metrics vital to the survivability of the aircraft. The Lockheed S-3A, in contrast, is an aircraft focused primarily on identifying, tracking and destroying enemy submarines; as such, the performance metrics found in its SAC chart focus on its range and endurance capabilities.

Table 2.2: F-5 SAC Chart Data

C O N D I T I O N S			I	II	III	IV	V
			SUBSONIC AREA INTERCEPT	GENERAL PURPOSE & ESCORT	GROUND SUPPORT BOMBER	GROUND SUPPORT BOMBER	FERRY RANGE
TAKEOFF WEIGHT		(lb)	15,777	17,951	20,575	17,917	17,217
Fuel		(lb)	4400	6175 ⑥	6175 ⑥	4400	6175 ⑥
Payload (Ammo)		(lb)	394	394	394	394	—
Payload		(lb)	340 ⑦	340 ⑦	2464 ⑧	2310 ⑨	—
Wing Loading		(psf)	85	97	110	97	93
Stall Speed (Power Off)		(kn)	136	147	156	147	142
Takeoff Ground Roll		(ft)	2030	2950	3800	2950	2450
Takeoff to Clear 50 Ft	①	(ft)	2950	4100	5320	4100	3520
Rate of Climb at SL	②	(fpm)	7930 ⑤	6060	4480	6120	6540
Rate of Climb at SL (OEI)	①	(fpm)	5660 ⑤	4030	2430	4120	4490
Time: Sea Level to 20k Ft	② ③	(min)	3.9 ⑩	4.2	6.2	4.1	3.8
Time: Sea Level to 30k Ft	② ③	(min)	6.3 ⑩	7.6	13.3	7.4	6.8
Service Ceiling	②	(ft)	42,300 ③	37,500	30,800	37,900	38,700
Service Ceiling (OEI)	①	(ft)	36,500	31,700	19,200	32,200	33,700
COMBAT RANGE	④	(n.mi.)	—	—	—	—	950
COMBAT RADIUS	④	(n.mi.)	225	250	220	110	—
Average Cruise Speed		(kn)	502	503	495	457	507
Initial Cruise Altitude		(ft)	37,000	34,700	32,300	25,000	35,600
Final Cruise Altitude		(ft)	40,000	40,400	39,000	35,000	41,100
Total Mission Time		(hr)	0.99	1.34	1.01	0.57	1.88
COMBAT WEIGHT		(lb)	13,275	14,680	15,365	14,070	11,853
Combat Altitude	①	(ft)	50,000 ⑦	35,000	SL	SL	41,1000
Combat Speed	①	(kn)	590	920	645	675	885
Combat Climb	①	(fpm)	1380	11,350	20,800	29,200	9000
Combat Ceiling	①	(ft)	51,700	49,800	47,300	50,600	53,900
Service Ceiling	②	(ft)	44,200	42,200	38,700	43,200	46,600
Service Ceiling (OEI)	①	(ft)	40,200	38,100	33,700	38,300	42,500
Max Rate of Climb at SL	①	(fpm)	34,900	31,400	20,800	29,200	37,250
Max Speed at 36,089 Ft	①	(kn)	940	930	700	860	915
Basic Speed/Altitude	①	(kn/ft)	590/50,000	920/35,000	645/SL	675/SL	385/41,1000
LANDING WEIGHT		(lb)	12,002	12,091	13,175	12,531	11,853
(no chute/with drag chute)							
Ground Roll at SL		(ft)	3580/2630	3600/2650	3860/2320	3700/2730	3430/2550
Total From 50 Ft		(ft)	5130/4180	5150/4200	5480/4440	5270/4300	4950/4070

① Maximum Thrust	⑤ Weight reduced for ground operation and accel to climb speed	⑨ (2) AIM-9J missiles plus MK-84 bomb (centerline)	PERFORMANCE BASIS
② Military Thrust	⑥ With 275 gal centerline tank	⑩ Includes time to T/O & accel to climb speed	(a) Data Source:
③ Allows for weight reduction during ground operation & climb	⑦ (2) AIM-9J missiles		Flight Tests
④ Detailed description of Radius & Range missions found in SAC chart	⑧ (2) AIM-9J missiles plus (4) MK-82 bombs		(b) Performance is based on powers shown in SAC chart

2.5 Data Available: Flight Manual

The flight manual is intended to be all that is necessary for complete safe and efficient operation of an aircraft.²² It is meant to be the book by which the pilots operate the aircraft, and as such, the data in it is extremely thorough. This is necessary because, as stated in the F-5 flight manual, “Unusual operations or configurations are prohibited unless specifically covered herein.”

Flight manuals are available to every pilot of an aircraft. Many flight manuals are for sale online, and simple searches at ebay.com or amazon.com return many results for old manuals. These books contain a wealth of data, not all of which pertains to performance; typical sections include:

- Description and Operation
- Normal Procedures
- Emergency Procedures
- Crew Duties (if applicable)
- Operating Limits
- Flight Characteristics
- Adverse Weather Operation

The appendix, not listed in the above list, contains the performance data. This is intended to be used either as a supplemental to the preceding sections or as a reference manual for the pilots.²² The performance data in the appendix of the flight manual for the F-5 includes, listed in order of appearance in the flight manual:

- Takeoff
- Climb

- Range
- Endurance
- Descent
- Landing
- Combat

Included in each section are detailed breakdowns of a wide variety of conditions at which these parameters may be of interest. As an example, the specific range data covers altitudes from sea level to 40,000 ft, with a range of weights from 11,000 lbs to 24,000 lbs, which cover all of the operating weights allowable for the F-5. Also included is a drag index system, which allows the specific range to be evaluated for any external loading combination. This is much more useful than the values listed in the SAC chart, as those cover only “typical” conditions. As is seen in Table 2.2, the range values are given for one specific loading condition, altitude and velocity; much more data is contained within the flight manual.

As might be expected, much more insight into the drag and engine characteristics is gained with each subsequent source of information. The amount of performance data available dictates not only the complexity of the aircraft models, but also the accuracy of the final solution. Seen in Figures 2.4 and 2.5 is a different representation of the performance data of the F-5. Each star corresponds to a separate data point extracted from both the SAC chart and the flight manual, plotted on the axis of Mach and Altitude for Figure 2.4 and Mach and C_L for Figure 2.5. These data locations show where the resulting drag polar and engine deck from this program will

be the most accurate and point out areas where additional data, if available, should be included.

Figure 2.4: F-5 Data – Altitude vs. Mach

Figure 2.5: F-5 Data – C_L vs. Mach

Chapter 3

Inverse Problems

An inverse problem of using data to deduce model parameters is known as a parameter estimation problem. This is the problem under consideration here, can be thought of simply as reverse-engineering. While this can be a straightforward task, most non-linear parameter estimation problems are ill-posed or ill-conditioned.²³ Returning to the physics and dynamics behind the equations themselves later, a function G may be specified such that m and d are related by Equation 3.1.

$$G(m) = d. \tag{3.1}$$

In practice, m represents some unknown model parameters, and d represents observations in time and space or a set of discrete points. Parameter estimation using discrete data will be the focus of this study as the goal is to analyze aircraft performance data, typically supplied as discrete points.

As is the case in most mathematical proceedings, linear inverse problems are considerably easier to solve than their non-linear counterparts. It can be shown that in the case of the linear inverse problem Equation 3.1 can be written in the form of

a linear system of algebraic equations, as seen in Equation 3.2.²⁴

$$\mathbf{G}\mathbf{m} = \mathbf{d} \quad (3.2)$$

A classic example of this²⁴ involves the fitting of ballistic trajectory data to a quadratic regression model. It is important to note that the linear classification applies only to the parameters, not the model itself; this allows us to cast a quadratic regression model as a linear problem. The mathematic model for a ballistic trajectory is shown in Equation 3.3. It is important in the parameter estimation problem that the parametric function be applicable everywhere, and not only at the points where data exists.

$$y(t, \mathbf{m}) = m_1 + m_2 t - (1/2)m_3 t^2 \quad (3.3)$$

In Equation 3.3 $m_1 - m_3$ are the parameters of interest. This equation is linear in terms of the coefficients, allowing it to be solved in matrix form according to Equation 3.2. The linear problem is shown in Equation 3.4 in matrix form, with data points y_i corresponding to data at time t_i .

$$\begin{bmatrix} 1 & t_1 & -\frac{1}{2}t_1^2 \\ 1 & t_2 & -\frac{1}{2}t_2^2 \\ 1 & t_3 & -\frac{1}{2}t_3^2 \\ \vdots & \vdots & \vdots \\ 1 & t_m & -\frac{1}{2}t_m^2 \end{bmatrix} \begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_m \end{bmatrix} \quad (3.4)$$

Since Equation 3.4 is linear, it can be solved in two ways. First and foremost is by application of the pseudo inverse, seen in Equation 3.5. The plus sign in Equation 3.5

denotes the pseudo inverse.

$$\begin{bmatrix} m_1 \\ m_2 \\ m_3 \end{bmatrix} = \begin{bmatrix} 1 & t_1 & -\frac{1}{2}t_1^2 \\ 1 & t_2 & -\frac{1}{2}t_2^2 \\ 1 & t_3 & -\frac{1}{2}t_3^2 \\ \vdots & \vdots & \vdots \\ 1 & t_m & -\frac{1}{2}t_m^2 \end{bmatrix}^+ \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_m \end{bmatrix} \quad (3.5)$$

This solution technique is useful for many problems. As will be evident in Chapter 5, however, the models selected to represent the drag and engine characteristics are highly non-linear, requiring a different technique for solving these complex problems. The example problem solved above will be examined again in the following section using the non-linear curve fitting techniques for illustration purposes.

3.1 Non-Linear Curve Fitting

Non-linear curve fitting is implemented by using a non-linear optimizer on problems of the form seen in Equation 3.6.²⁵

$$\min \|f(x)\|^2 = \min ((d_1 - G_1(\mathbf{m}))^2 + (d_2 - G_2(\mathbf{m}))^2 + \dots + (d_n - G_n(\mathbf{m}))^2). \quad (3.6)$$

This technique can easily be applied to the example problem in Equation 3.3. Casting the equation in the form of Equation 3.6, a non-linear optimizer is applied to Equation 3.7, with the parameters $m_1 - m_3$ as the variables of interest in the optimization.

$$\min \|f(x)\|^2 = \min ((y_1 - y(t_1, \mathbf{m}))^2 + (y_2 - y(t_2, \mathbf{m}))^2 + \dots + (y_n - y(t_n, \mathbf{m}))^2) \quad (3.7)$$

MATLAB’s function `lsqnonlin` from the optimization toolbox has been used as the non-linear curve fitting tool. Within this optimizer there are two different algorithm options; each one will be explored and compared to find the best option.

3.1.1 Subspace Trust-Region

The subspace trust-region method is based on the interior-reflective Newton method of solving non-linear optimization problems.²⁶ This is the default option for `lsqnonlin` as it is programmed to solve large-scale problems and is the “smartest” option.

The concept behind trust-regions is simple and powerful.²⁶ The idea is to approximate the true function F with a local, simpler function q . The neighborhood where this function is valid is defined as the trust region. This simpler model, found using a truncated Taylor series, is inexpensive to evaluate, and a point of “sufficient” improvement is found. This new point is used to evaluate the true function, and if the actual function value is found to decrease as well, then it is selected as the new point. The trust region is either expanded or contracted depending on the ratio of the actual improvement and the predicted improvement. A good discussion of this method can be found in *Trust-Region Methods* by Conn, Gould and Toint.²⁶ A basic overview of the algorithm, with many of the specifics left out, goes as follows:²⁶

Step 0 : Initialization. An initial point x_0 and an initial trust-region radius Δ_0 are given.

Step 1 : Create Approximation. Construct polynomial approximation $Q(x)$ of $f(x)$ around x_k .

Step 2 : Step Calculation. Search for minimum of $Q(x)$ inside the trust region.

Step 3 : Acceptance of the trial point. If $f(x_{k+1}) < f(x_k)$, accept x_{k+1} , resize trust region radius accordingly and continue. Otherwise, shrink region and try again.

3.1.2 Levenberg-Marquardt

The Levenberg-Marquardt algorithm is a standard in nonlinear optimization.²⁷ Although not an optimal algorithm for speed or error, it works extremely well in practice, and as such has become a standard. The algorithm combines the naïve gradient descent method with a quadratic approximation. While steepest descent works well with steep gradients, it often bogs down in areas of shallow gradients, where the quadratic approximation method shines. As such, these two methods are blended with the use of freely adjusted parameter λ . Seen in Equation 3.8 is Levenberg's equation, without Marquardt's addition; this is to show the original blending of the methods more clearly. It is important to note that in each of these equations the

Hessian ($\mathbf{H}$) is approximated by gradient information.

$$x_{i+1} = x_i - (\mathbf{H} + \lambda \mathbf{I})^{-1} \nabla f \quad (3.8)$$

As λ decreases, Equation 3.8 approaches the quadratic approximation, seen in Equation 3.9, and as λ increases, it approaches the steepest descent method, shown in Equation 3.10.

$$x_{i+1} = x_i - (\mathbf{H})^{-1} \nabla f \quad (3.9)$$

$$x_{i+1} = x_i - \frac{1}{I} \nabla f \quad (3.10)$$

Marquardt improved Equation 3.8 by using the approximated Hessian in the steepest descent portion of the algorithm.²⁷ This extra curvature information helps the steepest descent problem from getting bogged down in “valleys”, and can be seen in Equation 3.11.

$$x_{i+1} = x_i - (\mathbf{H} + \lambda \text{diag}[\mathbf{H}])^{-1} \nabla f \quad (3.11)$$

3.2 Global Optimization

As is often the case in non-linear parameter estimation problems, it was immediately apparent in tackling this problem that often the solution found using the algorithms above was only a local solution. Running the local optimizer 100 times from 100 random starting points on the function developed from this work resulted in 74 unique results; of these, the results that matched were due to the starting points being very similar.

Originally the objective function was thought to be in question. This would have been the case if the performance functions ($G(\mathbf{m})$ in Equation 3.6) were incorrect in any way, or if taking the 2-norm was somehow affecting the results. This disparity in the results led to an investigation into a variety of different penalizing functions, as the performance functions have been validated and the results are trusted. These different penalizing functions, seen in Figure 3.1, weigh outliers in different amounts, which can change the search direction and solution of an optimizer. Although this did not solve the problem of local minima, it was worth investigating.

Figure 3.1: Penalizing Functions

While Figure 3.1 uses values of x to change the penalty, using the objective function value to change the penalty was investigated to determine if the use of an exponential or other functional could reduce the number of local minima. Unfortunately, as can be seen in Figure 3.2, the only method of reducing local minima through the

penalizing function is to have a-priori knowledge of the minimum. This is obviously not possible and requires other methods to be used in solving this problem.

Figure 3.2: Sample Problem with Local Minima

Since there is no simple method available to find the global minimum directly, a global optimizer has been written. There are many options available when selecting a global optimizer, falling into the categories of deterministic, stochastic and heuristic algorithms. Each have their advantages and disadvantages; the choice for this work to use a random tunneling algorithm (stochastic method) took advantage of many of them.

Global optimization is very problem specific. Different problems require special treatment, depending on constraints along with other parameters; for this reason there are very few “cookie-cutter” methods available today. The methods that are available are typically genetic algorithms or other heuristic methods, as these are the

most easily generalized. These methods have many drawbacks, however, and are not well suited to this problem.

The choice of a random tunneling algorithm is twofold; it is based on local optimization results and is (relatively) simple. The fact that this algorithm uses the results of a local optimization allows the use of standard MATLAB functions, such as `fmincon` and `lsqnonlin`, to bear the burden of this procedure. While the details can be found in Kitayama and Yamazaki,¹ the general algorithm can be seen in Figure 3.3.¹

Figure 3.3: Random Tunneling Algorithm¹

In general, this algorithm is broken into three parts. These are:

1. Perform local optimization.
2. Monte-Carlo sampling to find improved starting point.
3. Repeat. Stop when the sequence converges.

This technique, as was mentioned, has both good and bad properties. Both the local optimization and Monte-Carlo sampling allow for constraints, using either constraint functions or simple bounds constraints, which is desired for the problem at hand. In general, this technique is well laid out, and is easy to implement. Unfortunately, due to the random sampling, this algorithm is fairly computationally inefficient. From a variety of test cases run on this problem, a local optimization run can take between 2,000 – 4,000 function evaluations while the global optimization uses anywhere from 20,000 to 200,000 function evaluations. This is due primarily to the large number of function calls necessary to find a new starting point.

Although the efficiency of the code is important, the objective is mainly to investigate this technique, even if it takes too long to be of practical use at this point. MATLAB is a scripting language, and as such, it is much less efficient than any true coding language. If speed is necessary this program can be rewritten in C, C++ or any number of other coding languages with significant improvements in efficiency. The optimizer itself does not use significant computation time; the performance functions and engine/drag models themselves bear the burden of these calculations, and

rewriting these would decrease the computation time significantly.

Another issue with this algorithm is how it terminates. As can be seen upon examination of Figure 3.3, the algorithm terminates only after it fails to find a better point in successive iterations. This means that this algorithm is not guaranteed to find a global minimum; however, if it does find a new minimum, it is guaranteed to be “better” than the original. This property is not ideal, but is a significant improvement from the purely local optimization technique.

For this parameter estimation problem the random tunneling global algorithm was implemented, with MATLAB’s `lsqnonlin` as the local optimization routine. It employs the trust region method to find the local minimum. This technique ensures that as long as the original starting point is close enough to the solution, a minimum will be found.

Chapter 4

Aircraft Performance Models

In order to solve the inverse problems associated with this concept some work must be done to solve the forward problem first. Multiple aircraft performance metrics were looked at and MATLAB was used to implement code to calculate them. The performance metrics investigated here have been chosen to correspond with data typically found in both SAC charts and flight manuals.

Every powered aircraft experiences four main forces in flight – lift, drag, thrust and weight – which can be seen in Figure 4.1. With the exception of the specific

Figure 4.1: Aircraft in Climbing Flight²

range calculation, every performance metric used here can be derived by summing the forces in each direction of the aircraft. First looking at the forces parallel to the freestream velocity, followed by the forces perpendicular, Equations 4.1 and 4.2 are found.

$$F_{\parallel V_{\infty}} = T - D - W \sin \gamma \quad (4.1)$$

$$F_{\perp V_{\infty}} = L - W \cos \gamma \quad (4.2)$$

Using Newton's second law, namely

$$\mathbf{F} = m\mathbf{a},$$

the equations of motion can be seen for an aircraft flying in the plane of the paper. These can be seen in Equations 4.3 and 4.4. It is important to note that turning flight requires a more in-depth analysis – this will be shown in the turns section.

$$m \frac{dV_{\parallel V_{\infty}}}{dt} = T - D - W \sin \gamma \quad (4.3)$$

$$m \frac{dV_{\perp V_{\infty}}}{dt} = L - W \cos \gamma \quad (4.4)$$

These two equations form the basis of almost all performance metrics. The derivation of each specific metric will be shown in its relative section, allowing the reader to use this chapter as either a quick reference guide or a thorough read.

It is important to note a few things before deriving each performance metric. What may have been evident from Figure 4.1 and the equations shown so far, the aircraft is assumed to be a point mass with the weight concentrated at its center

of gravity. Each performance metric investigated here is interested in translational motion only, ignoring any rotational components. This assumption stems from the assumption that the aircraft is trimmed for each mode of flight, which in reality is generally true.

Another important side note is that although data is readily available for both takeoff and landing distances, these performance metrics will not be included in this study. This is due to a variety of reasons; first and foremost is the fact that most aircraft utilize flaps, slats, or other lift-augmentation devices during takeoff and landing. This alters the shape of the wing, and drastically increases drag, leading to the need for a completely separate drag polar for takeoff, landing and normal flight. Each of these polars are relatively unrelated, introducing too many parameters into the problem.

Takeoff and landing are also unique in that they are extremely dependant on the pilot. Each pilot has his own way of piloting his aircraft, leading to tremendous differences in values not seen in any other metric. This variance, along with overall code complexity and the lift-augmentation systems mentioned before, has led to the exclusion of takeoff and landing data from this analysis.

4.1 Steady Flight

The first set of performance metrics are all related to an aircraft in steady flight. The physical relationships have been found by summing the forces in each direction

acting on an aircraft. These are set equal to zero due to the definition of steady flight, that acceleration in any direction is zero.

4.1.1 Maximum Velocity

Maximum velocity is extremely important in fighters, especially during times of war. The ability to outrun an enemy fighter is crucial to minimizing losses. As such, maximum velocity values are often provided for fighters as a function of altitude.

Maximum velocity is the absolute fastest an aircraft can sustain flight at a given altitude. This occurs when thrust is equal to drag, and consequently, when lift is equal to weight. Upon examination of Equation 4.3, for steady, level flight ($\frac{dV}{dt} = 0$ and $\gamma = 0$):

$$T = D. \tag{4.5}$$

Both drag and thrust are functions of velocity, and since we are searching for maximum velocity in this problem, Equation 4.5 must be solved iteratively.

Shown in Figure 4.2 is the maximum velocity data for the F-5. This data varies in an approximately linear fashion above 11,000 ft, and increases until an altitude of approximately 36,000 ft. At this point the maximum Mach number is limited in a different fashion and is not included in this analysis.

This data is extremely important to this analysis. It provides data at the fastest portion of the flight envelope, providing a good basis for the rest of the regions as well. As this research is attempting to determine drag and engine characteristics throughout

Figure 4.2: Maximum Velocity Data for the Northrop F-5

the flight envelope, having data along the entire maximum velocity-limited side of the flight envelope helps in the process.

4.1.2 Climb Gradient

The ability to climb quickly is essential to maximize the efficiency and effectiveness of an aircraft. Typically utilized for obstacle avoidance in low altitude flight, high climb gradients allow flight in mountainous terrain where flight might not otherwise be safe.

Flight manuals for aircraft such as the DC-10 incorporate data spanning various altitude and weight combinations as a reference to the pilots. This is typically for a given velocity, usually some fraction of V_{stall} , and is not indicative of a maximum climb gradient. This is due to FAA regulations which limit both the minimum and maximum velocities below 10,000 ft.²⁸ In the case of the DC-10, the climb gradient data is based on the climbout speed (V_2).²⁹

The climb gradient (CG) is defined to be the ratio of altitude gained to distance

traveled. This can be seen in Figure 4.3 as the tangent of the flight path angle γ . This can also be calculated as the ratio of vertical speed (rate of climb) to ground speed. Accelerated rate of climb (RoC) will be discussed in Section 4.2.1; calculation of the climb gradient requires unaccelerated RoC . RoC may be written as

$$RoC = \frac{dH}{dt} = V_T \sin \gamma \quad (4.6)$$

from Figure 4.3.

Figure 4.3: Aircraft Velocities

Climb gradient is seen from Figure 4.3 as

$$CG = \frac{RoC}{V_H},$$

requiring knowledge of both RoC and V_H .

Derivation of RoC begins by summing forces along the flight path:

$$\begin{aligned}
T - D &= \frac{W}{g} \frac{dV_T}{dt} + W \sin \gamma \\
\frac{T - D}{W} &= 0 + \sin \gamma \\
\frac{T - D}{W} V_T &= V_T \sin \gamma \\
RoC &= \frac{V_T(T - D)}{W} \\
\Rightarrow CG &= \frac{\frac{V_T(T - D)}{W}}{V_H}
\end{aligned}$$

As is seen in Figure 4.3, V_T is the velocity with respect to the aircraft, in the body axis of the aircraft; for this reason, RoC must be divided by V_H instead of V_T . Including this adjustment results in the final equation for CG , shown in Equation 4.7.

$$CG = \frac{RoC}{\sqrt{V_T^2 - RoC^2}} \quad (4.7)$$

4.1.3 Time to Climb

An aircraft's vertical component of velocity is, by definition, the rate of climb. This is simply the time rate of change of altitude. Equation 4.6, repeated here, can be manipulated as follows.

$$\begin{aligned}
RoC &= \frac{dH}{dt} = V_T \sin \gamma \\
dt &= \frac{dh}{RoC}
\end{aligned} \quad (4.6)$$

Solving for total time to climb can be done with an integral from altitude h_1 to h_2 . Since RoC is dependant on altitude it must be included in the integral seen in

Equation 4.8.

$$t = \int_{h_1}^{h_2} \frac{dh}{RoC} \quad (4.8)$$

The integral in Equation 4.8 must be solved numerically. The choice in step size for dh is dependant on the total altitude change from h_1 to h_2 , and must be chosen carefully. The largest change in altitude given in the DC-10 data is only 5,000 ft. Examination of the step size has revealed a 3% error between a single step size and the true integral value, resulting in the author's choice of approximating this integral in one step. In this manner the change in altitude is divided by the average RoC found for the altitude combinations, seen in Equation 4.9.

$$t = \frac{h_2 - h_1}{RoC_{average}} \quad (4.9)$$

4.1.4 Distance Covered During Climb

The ground distance an aircraft covers during a climb segment is entirely dependant on that aircraft's RoC , or more simply, its time to climb. Therefore, the results from Section 4.1.3 will be used in this analysis.

The total ground distance covered is a simple relationship between the ground speed at which the aircraft is flying and the total time spent during the maneuver. This relation is given by Equation 4.10.

$$D = \sqrt{V_T^2 - RoC^2} \times t \quad (4.10)$$

This simple relation quickly and accurately calculates the distance covered during a climb segment.

4.1.5 Fuel Burn During Climb

Fuel burn during a climb segment is calculated similarly to the time to climb and distance covered during climb performance metrics. The flight planning metric involved in this calculation is f_s , namely the fuel consumed specific work. This is defined in Equation 4.11.

$$\begin{aligned} f_s &= \frac{RoC}{\dot{W}_f} \\ f_s &= \frac{RoC}{T \times TSFC} \end{aligned} \tag{4.11}$$

f_s defines the altitude changed per pound of fuel burned. Therefore, to determine the total fuel burned during a climb segment, Equation 4.12 must be used.

$$F.B. = \frac{h_2 - h_1}{f_s} \tag{4.12}$$

It is important to note that climb segments are typically performed at full throttle. Therefore, the value of TSFC used in Equation 4.11 must be for the engine at full throttle. This is in contrast to the value of TSFC used to calculate range, which is for throttled engine performance. This difference in throttle settings requires an otherwise-unnecessary throttling term to appear in the equation for TSFC, detailed in Chapter 5.

The data for each of the three proceeding performance metrics, time to climb, distance to climb and fuel burn during climb, can be seen summarized in Table 4.1. This data was presented only in tabular form.

Table 4.1: DC-10 Climb Data

PRESSURE ALTITUDE (FT)	FUEL (LB) DISTANCE (NM) TIME (MIN)				
	INITIAL CLIMB GROSS WEIGHT (LB)				
	400,000	450,000	500,000	550,000	600,000
41,000	12,700 189 25.3				
37,000	9400 110 15.8	11,800 142 20			
33,000	8300 87 12.8	10,000 105 15.6	12,000 130 18.9	14,900 168 23.8	
29,000	7400 71 11	8700 85 13	10,300 103 15.4	12,200 124 18.2	14,600 152 22
25,000	6300 56 9	7400 66 10.6	8600 79 12.4	10,000 93 14.4	11,800 111 17
20,000	5000 38 6.8	5800 46 8	6700 54 9.1	7700 63 10.4	8800 73 11.9
15,000	3800 26 4.9	4400 30 5.8	5000 36 6.5	5600 41 7.3	6400 48 8.2
10,000	2600 16 3.4	3100 19 4	3400 22 4.4	3800 25 4.8	4200 28 5.2
10,000	2100 12 2.8	2500 14 3.2	2900 18 3.7	3300 20 4.2	3800 24 4.7
5,000	1000 6 1.2	1200 7 1.5	1400 8 1.7	1600 9 1.9	1800 10 2.1

4.1.6 Range

The distance an aircraft can travel while carrying a certain payload is of prime interest in all fields of aviation. As seen in the Breguet range equation,³⁰ range is dictated by three main categories: engine, aerodynamics and weights.

$$R = \left(\frac{1}{TSFC} \right) \left(\frac{VL}{D} \right) \ln \left(\frac{W_1}{W_2} \right) \quad (4.13)$$

Instead of calculating the range, which requires knowledge of the total change in weight, a more typical parameter used in range calculation is the “range factor”. Total range also includes mission assumptions and climb and descent phases, which are additional entirely mission dependant. This value is calculated to produce a normalized value of range, one independent of weight. Range factor, as shown in Equation 4.14, leaves out lift from the Breguet range equation in addition to the weight terms; this is because $L = W$ in cruise, and the range factor is meant to be independent of weight.

$$RF = \frac{V}{\dot{W}} = \frac{V}{TSFC D} \quad (4.14)$$

Range is one of only two functions to incorporate the TSFC function for the DC-10, the other being f_s (Section 4.1.5); for the F-5 it is the only one with TSFC. This is an important fact to remember, as the results are only truly valid in the regions where data is present. The DC-10 range data is only given for 33,000 ft and 35,000 ft, from Mach 0.7 to 0.88, as shown in Figure 4.4, while the F-5 range data spans multiple values of altitude and Mach. The F-5 range data is not presented here as the original

data is in the form of a nomograph.

(a) Altitude of 33,000 ft, Weight of 414,750 lbs (b) Altitude of 35,000 ft, Weight of 424,750 lbs

Figure 4.4: Range Data for the DC-10

The variation of range factor with Mach and altitude is difficult to predict with such limited data. The span of altitudes is not large, limiting the accuracy of this data. While the Mach number ranges from approximately 0.7 to 0.88, this is still not sufficiently large; therefore the models fit for TSFC will only be applicable in this small region of the flight envelope.

4.2 Accelerated Flight

The steady flight performance metrics examined earlier are interesting; however, they do not tell the whole story of aircraft performance. Three more performance metrics have been evaluated, each of which involves acceleration of the airframe.

4.2.1 Rate of Climb

As was true for climb gradient, a quick *RoC* is essential to the success of an aircraft. Cruise performance is typically enhanced at higher altitudes, with decreased

climb time due to a higher RoC resulting in an increase in efficiency. Minimizing intercept time of a fighter taking off is just one example indicating that RoC is an essential part of overall performance.³¹

Steady RoC , as was seen in 4.1.2, is valid for an aircraft flying at a constant true airspeed V_T . The case is different, however, for an aircraft flying at a constant Mach number, equivalent airspeed or calibrated airspeed; due to the changing atmospheric conditions with altitude, the true airspeed is constantly changing. The derivation for the constant Mach climb will be provided here; for constant equivalent airspeed or calibrated airspeed see the McDonnell Douglas performance handbook.²

RoC may be written as

$$RoC = \frac{dH}{dt} = V_T \sin \gamma$$

Summing forces along the flight path gives:

$$\begin{aligned} T - D &= \frac{W}{g} \frac{dV_T}{dt} + W \sin \gamma \\ \frac{T - D}{W} &= \frac{1}{g} \frac{dV_T}{dt} + \sin \gamma \\ \frac{T - D}{W} V_T &= \frac{V_T}{g} \frac{dV_T}{dt} \frac{dH}{dt} + V_T \sin \gamma \\ \frac{T - D}{W} V_T &= \frac{V_T}{g} \frac{dV_T}{dt} RoC + RoC = RoC \left(1 + \frac{V_T}{g} \frac{dV_T}{dh}\right) \\ RoC &= \frac{V_T(T - D)/W}{1 + (V_T/g)(dV_T/dH)} \end{aligned} \tag{4.15}$$

The term $(V_T/g)(dV_T/dH)$ is only necessary when the aircraft is not climbing at a constant V_T .² This ends up being a correction factor, and can be found explicitly

depending on the climb schedule. For a constant Mach climb below 36,089 ft., the factor is

$$(V_T/g)(dV_T/dH) = \frac{MA}{g} \frac{d(MA)}{dH} = \frac{M^2 A}{g} \frac{dA}{dH}$$

Substituting $A_0\sqrt{\theta}$ for A results in

$$M^2 \frac{1}{2g} \frac{d(a^2\theta)}{dH} = \frac{a_0^2}{2g} \frac{d\theta}{dH}$$

where $\theta = T/T_0$. This yields

$$M^2 \frac{a_0^2}{2g} \frac{d}{dH} \left(\frac{T}{T_0} \right) = M^2 \frac{a_0^2}{2gT_0} \frac{dT}{dH}$$

Using the standard lapse rate of $-0.003566^\circ F/ft$ yields

$$(V_T/g)(dV_T/dH) = -0.133M^2$$

The equation for accelerated climb is given by Equation 4.15, where $(V_T/g)(dV_T/dH)$ is an acceleration factor.² Depending on the climb schedule and altitude, this factor varies according to the values found in Table 4.2.

Table 4.2: Acceleration Factor as a Function of Mach Number

Climb Schedule	Constant Mach Number	Constant Equivalent Airspeed	Constant Calibrated Airspeed
Above 36,089 ft.	0	$0.7 M^2$	K
Below 36,089 ft.	$-0.133 M^2$	$0.567 M^2$	$-0.133 M^2 + K$

$$\text{where } K = (1 + 0.2M^2) - (1 + 0.2M^2)^{-2.5}$$

The acceleration terms in Table 4.2 are due to the temperature lapse rates. As such, these were derived using the lapse rates from the 1976 Standard Atmosphere

model.³² This causes the difference in acceleration factors below and above 36,089 ft., as that altitude is the transition between the Troposphere and the Stratosphere. As is seen in Figure 4.6, the acceleration factor for a constant Mach climb increases the actual *RoC*, which can be seen in the data for the Northrop F-5, shown in Figure 4.5. The noticeable kink in the maximum *RoC* curve, which occurs at 36,089 ft., is due to this sudden change in acceleration factor.

Figure 4.5: Maximum *RoC* Data for the Northrop F-5

To demonstrate the significance of this acceleration on the overall *RoC*, the effect of the correction factor is shown in Figure 4.6. As is shown, this acceleration factor can have a large impact on the calculated *RoC*. Shown in Figure 4.6 is the change in climb rate from a steady climb to a constant Mach climb.

In addition to the acceleration factor, high performance aircraft must take into account the angle at which the aircraft is actually flying. Lift is traditionally calculated as being equal to weight, which is not true during climb, and this small angle approximation is negligible for low-performing aircraft. For the Northrop F-5, however, the angle of climb at sea level is approximated 35°, which must be taken into

Figure 4.6: Acceleration Factor for Constant Mach Climb Below 36,089 ft.

account.

Inclusion of this angle in the calculation must be done iteratively, as the climb angle depends on the RoC , which depends on the climb angle. This iterative process has been explored, with the results shown in Figure 4.7. This exploration shows that, although the F-5 is climbing at 35° , the inclusion of the small angle approximation has a negligible effect on the maximum RoC . This is due to the operating point on the drag polar. Since the F-5 is operating at or above Mach 0.9 during this climb, the value of C_L is relatively low, meaning that any variation of C_L will have a small effect on C_D . This fact, and the results seen in Figure 4.7, allow the small angle approximation to be included in the RoC calculation.

Two different approaches have been utilized to calculate the maximum RoC , applying two separate assumptions. A one-dimensional gradient based optimizer, `fminbnd` in MATLAB, applied to the negative of Equation 4.15, finds the true maximum RoC at each altitude. The second method to calculate maximum RoC is to assume the climb is performed at a constant Mach number. This is not far off from

Figure 4.7: Acceleration Factor for Constant Mach Climb Below 36,089 ft.

the true maximum *RoC* climb schedule of the F-5, known to be a linear variation from Mach 0.9 to Mach 0.925; this true schedule is not incorporated as it is assumed that it is not be known.

The option between a truly maximum *RoC* and an approximated maximum *RoC* is given to the user for two reasons; first and foremost is the issue of program speed. Each time `fminbnd` is called, the performance function, drag polar and engine deck are each called approximately 200 times. This greatly decreases the efficiency of the code. The second reason is that climb schedules are typically simplified as much as possible, to decrease the load on the pilot. Requiring a pilot to memorize a complex climb schedule, or interpolate between values based on altitude, is not practical in any real life situation.

4.2.2 Turn Radius

An aircraft is said to be in a steady, level turn if the tangential acceleration is zero, even though there is a radial acceleration component. Defining ϕ as an aircraft's bank angle in a steady, level turn, the equations of motion in each of the aircraft's three directions are seen in Equation 4.16.

$$\begin{aligned}\sum F_{vertical} &= L \cos \phi - W \\ \sum F_{\parallel V} &= T - D \\ \sum F_{radial} &= L \sin \phi\end{aligned}\tag{4.16}$$

Since the only allowable acceleration is radially, applying Newton's second law to Equation 4.16 results in Equation 4.17.

$$L \sin \phi = \frac{W}{g} \frac{V^2}{r}\tag{4.17}$$

Rearranging, it is found that

$$r = \frac{V^2}{g} \frac{W}{L \sin \phi}$$

and since $\frac{L}{W} \equiv n$,

$$r = \frac{V^2}{gn \sin \phi}.$$

Since $n = \frac{1}{\cos \phi}$,

$$r = \frac{V^2}{g \tan \phi}$$

and, since $\tan \phi = \sqrt{n^2 - 1}$,

$$r = \frac{V^2}{g} \frac{1}{\sqrt{n^2 - 1}}.\tag{4.18}$$

It is important to note that Equation 4.18 is dominated by the kinematics of the aircraft, and are universal to all aircraft. This renders Equation 4.18 useless to our problem here as nothing useful is gained through the use of this equation.

The calculation of an aircraft's turn radius can be conducted in another fashion entirely, derived solely by examination of the forces at hand. By definition, an aircraft in a steady, level turn has zero excess power, and therefore zero RoC . This fact can be used to solve for the turn radius in an iterative fashion, by using an optimizer to find the value of the load factor that sets $RoC = 0$. As such, the turn radius for a given Mach number is calculated as follows:

0. Guess initial load factor (n)
1. Calculate C_L using $C_L = \frac{nW}{qS}$
2. Calculate RoC
3. Iterate until $RoC = 0$

This method of calculating the turn radius for a given Mach number incorporates both drag and thrust of the given aircraft, allowing the parameters to be changed, making this a feasible problem. As was true with other performance metrics involving RoC , MATLAB's `fminbnd` was utilized to vary the load factor to find the final value.

Sample turn data can be seen in Figure 4.8. The two different lines in this figure represent two different altitudes of 15,000 ft and 35,000 ft. This data is extremely useful, as the variation of Mach numbers from approximately 0.3 to 1.4 provides a

large range of data. As is seen in Chapter 2, a wide variety of data is required to accurately model thrust and drag for an aircraft. This turn data spans a large range of C_L values, varying from approximately 1.3 at Mach 0.3 to 0.1 at Mach 1.4.

Figure 4.8: Turn Radius Data for the Northrop F-5

Chapter 5

Aircraft Models

There are many options when choosing a parametric equation. In general, any mathematical technique or combination of techniques can be used as a parametric equation, and it is up to the end user to choose the form; some of these options, however, are better suited to the problems at hand. The selection of one equation to model drag throughout the entire flight regime, for example, is rather tricky. This is due to the multiple nonlinearities and must be taken into account when creating a model.

Generalized functions for drag, thrust and TSFC are well understood and have accurate models for them; however, there are intricacies and nonlinearities that are ignored in the development of these models. Drag, for example, can never truly be modeled without solving the full Navier-Stokes equations. A comparison between the models based on the underlying physics to other purely mathematical techniques are explored here. As will be shown, some of these parametric models are better suited to handle these nonlinearities than others, and a few of those options are explored

and compared here.

5.1 Physics Inspired Models

Physics inspired models, defined here as models which are designed to approximate the form of a naturally occurring phenomenon, are an obvious choice when approximating a function. As an example, it is known that subsonic drag, without the effects of compressibility, is reasonably well approximated by Equation 5.1.

$$C_D = C_{D_0} + kC_L^2 \quad (5.1)$$

Equations such as Equation 5.1 go a long way in predicting the behavior of functions such as drag, and are reasonably accurate in most cases.

As will be discussed, there are many underlying physics inspired equations that can be combined in a composite approach. As an example, Equation 5.1 does not take into account transonic drag rise; this drag rise can be added to Equation 5.1 as a separate equation, allowing for many models to be combined into one.

Physics inspired models have both good and bad aspects to them. Since they are based on known phenomenon, interpolation and extrapolation is not as risky as with some of the other methods discussed here, although extrapolation should always be done with caution. Parameter estimation using these models is also simpler, as the parameters can usually be estimated with some reliability and accuracy. As an example, parameters C_{D_0} and k from Equation 5.1 will most likely be approximately 0.02 and 0.2 for a fighter and approximately 0.03 and 0.1 for a transport aircraft.

This user intuition helps to approximate values for some of the parameters, at least within an order of magnitude, and helps the convergence of the overall problem at hand.

5.1.1 Drag Polar

Drag varies with many factors. These include Mach number, lift, Reynolds number, angle of attack, vehicle shape and altitude. Of these variations, only Mach number, lift, Reynolds number and altitude will be explored. Creating a model to span all Mach numbers and all altitudes, spanning a full range of C_L values, creates different issues depending on the type of aircraft in question. Fighter aircraft, for example, operate in subsonic, transonic and supersonic Mach numbers. This creates the need for a continuous function that can include the effects of each flight regime. With the exception of some aircraft (Concord, TU-144), transport aircraft, stay completely subsonic, meaning they do not experience any of the supersonic effects. They do experience transonic drag rise, and while both supersonic and subsonic aircraft experience these effects, the difference in maximum velocity for each aircraft requires a different equation for each.

Subsonic

The creation of a model incorporating the true physical variances seen in real life is challenging for any aircraft. Subsonic aircraft, such as the DC-10, experience a transonic drag rise that varies with C_L . Shown in Figure 5.1 is the actual variation

of drag for the DC-10, showing the effects of both Mach and C_L .

Figure 5.1: Drag of the DC-10

An approximation for subsonic drag^{2,31} is given by Equation 5.2. This equation assumes that both k and C_{D_0} are constant; this is not true, and can be seen in Figure 5.1.

$$C_D = C_{D_0} + k(C_L - \Delta C_L)^2 \quad (5.2)$$

$$k = \frac{1}{\pi e AR}$$

The transonic drag rise seen in Figure 5.1 is both a function of Mach number and C_L . One way to approximate this is to make add a term representing wave drag, $C_{D_{wave}}$.

An equation for $C_{D_{wave}}$ is given by modeling an increase in wave drag as the flow approaches the speed of sound. Equation 5.3, given by Lock,³³ provides an approximation for this.

$$C_{D_{wave}} = A(M - B)^C \quad (5.3)$$

Coefficients A-C are independent parameters. It is important to note that in Lock's original equation, parameter C is given to make Equation 5.3 4th order specific. This

model produces an accurate drag rise for low values of C_L ; however, upon examination of Figure 5.1 it is seen that as C_L increases, the Mach number of drag divergence M_{DD} decreases. M_{DD} is defined differently depending on the source of the information; the two most common definitions are when $\frac{\partial C_D}{\partial M} = 0.1$ and when C_D increases by 10%.

An approximation to the change in $C_{D_{wave}}$ with C_L is given in Equation 5.4. This equation is based solely on the observations of drag trends in Figure 5.1.

$$C_{D_{wave}} = A \left(M - B + C(C_L - D)^E \right)^F \quad (5.4)$$

The transonic drag rise variation with C_L in Figure 5.1 looks to be approximately quadratic in nature. Multiple variations of Equation 5.4 were attempted to find the best equation; after many attempts, it was found that the transonic drag rise variation with C_L is approximately exponential. This realization lead to the form of Equation 5.4, with the additional $C(C_L - D)^E$ term added for this purpose.

At this point there has not been any technique found to capture the variability in k with Mach and C_L . Any paper found to this point uses a numerical method when looking into this complex flow phenomenon. With that said, it is clear with the work done thus far that there is some variation in k with both Mach and C_L , and some of this effect can be seen in Figure 5.1.

Looking at the complete subsonic drag data for the F-5, it is found that in the subsonic region k varies in an approximately linear fashion with C_L . This is handled by a linear term added onto the k term in Equation 5.2, and handles much of the complex variations elegantly. This variation is shown in Figure 5.2.

Figure 5.2: F-5 Drag Data – Variation of k

Supersonic

A drag polar for a supersonic plane must incorporate all aspects of flight. This includes all Mach and altitude ranges, during which C_L changes drastically as well. Development of a succinct model to incorporate each of these elements has been challenging; however, the end result captures most phenomenon seen in actual drag data while retaining the typical parameters seen in simpler models.

The base model for the supersonic drag model is identical to that of the subsonic model, repeated here for convenience.

$$C_D = C_{D_0} + k(C_L - \Delta C_L)^2$$

$$k = \frac{1}{\pi e R} \quad (5.2)$$

This model works well until transonic drag rise is encountered, and it was decided that three models would be explored as options to capture the rise in C_{D_0} typically

seen around and above the speed of sound. Three modeling options were explored to ensure that the best model would be chosen; the options included two models found in research papers,^{34,35} and a composite model created by the author.

The first modeling option is to use a drag coefficient function originally intended to model ballistic weapons.³⁴ According to the Anderson and McCurdy, the form of the function found in Equation 5.5 can accurately model C_{D_0} for a wide range of Mach numbers. It was originally intended to be used with minimal data available, as it was designed to drastically reduce the number of drop tests necessary to accurately predict drag, which corresponds nicely with the ideas presented in Chapter 2.

$$C_{D_0} = A + \frac{\left(1 - e^{-M^B}\right) C}{M^D} \quad (5.5)$$

The coefficients A-D are independent parameters used to adjust the shape of the function. The small number of coefficients in Equation 5.5 make this a prime candidate as a viable function for the zero-lift drag coefficient. As is discussed in Chapter 2, the number of total parameters plays a large part in the efficiency of the code; finding the optimal balance of accuracy and efficiency is difficult, and care must be taken to select an optimal model.

Equation 5.5 was never intended to be used to model drag of a full aircraft. For this reason, multiple test values were run to explore the variety of shapes Equation 5.5 can simulate. The results are shown in Figure 5.3. Each curve shown represents a separate set of input values, intended to show the true flexibility of Equation 5.5. These curves are not intended to represent any particular aircraft's drag polar; instead, they show

the various forms that this equation can represent.

Figure 5.3: Weapons Model Applied to C_{D_0}

The second option to model drag rise is with a hyperbolic tangent function.³⁵ This equation, which is adjusted to allow for either a supersonic drag increase or decrease, is found in Equation 5.6.

$$C_{D_0} = A + \frac{B \tanh[C(M - D)]}{1 + EM^F} \quad (5.6)$$

As was true with Equation 5.5, parameters A-F are independent parameters. Equation 5.6 has one major downfall – it is not able to capture the transonic “bump” seen in the drag data for some aircraft. For this reason this equation will not be used; however, it will still be included in a comparison of all three modeling techniques applied to the F-5.

Equations 5.5 and 5.6 are very similar. Each equation is a different representation of a logistic function,²⁰ and while they both represent the same thing, the slight

difference in formulation leads to different accuracies in modeling.

The third modeling option uses a combination of functions to attempt to fit the drag data. This was initially done due to the literature search not returning any results for functions to describe transonic and supersonic drag effects. Shapes and trends were described, however, leading the author to use his judgement in coming up with the functions described here. The first of these functions is a Gaussian, used to fit the “bump” typically seen at Mach numbers between 0.8 and 1.2. The Gaussian is of the form

$$f(M) = A \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(\frac{(-M - \mu)^2}{2\sigma^2}\right). \quad (5.7)$$

where A , σ and μ are coefficients that can be varied by the overall program to best fit the trends demonstrated in the actual drag polar. μ effects the location of the curve while σ effects the overall width of the curve. A is a simple multiplier to change the overall height of this curve.

This Gaussian approximates a transonic “bump,” and due to its nature, returns to the nominal value. For this reason a separate function is included to handle the drag rise encountered in the supersonic flight regime. The drag increase is approximated by a form of a logistic or sigmoid function, which is essentially a smooth step function.²⁰ This function takes the form of Equation 5.8.

$$f(M) = \frac{B}{C + \exp(-2(M - \mu))} \quad (5.8)$$

where B adjusts the height of the function and μ shifts the location of the curve.

Parameter C adjusts the slope of the function.

The total equation, summed here, results in

$$C_D = C_{D_0} + k(C_L - \Delta C_L)^2 + A \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(\frac{(-M - \mu)^2}{2\sigma^2}\right) + \frac{B}{C + \exp(-2(M - \mu))} \quad (5.9)$$

A generic curve demonstrating the zero-lift components of drag can be seen in Figure 5.4. This employs values for the constants in the full drag equation (Equation 5.9) close to those found for the F-5, adjusted slightly to exaggerate the transonic peak. This shows the Gaussian and logistic functions shown separately, plotted alongside the full function with them included. This is for a general demonstration case only.

Figure 5.4: Generic Drag Rise Incorporating Transonic and Supersonic Corrections

When choosing which function to use for C_{D_0} , it is important to note the number of parameters required by each function. Equation 5.5 uses four parameters, Equation 5.6 uses six parameters, and Equation 5.9 uses eight. Although the number of parameters is not enough to base a choice of functional representation of C_{D_0} on, it is

a very important factor. A side-by-side comparison shows which of the three models is the most accurate. Shown in Figure 5.5 are the three models, given by Equations 5.5, 5.6 and 5.9, matched to the actual C_{D_0} values for the F-5. This comparison shows that all three models do an excellent job of predicting C_{D_0} for the F-5.

Figure 5.5: Comparison of C_{D_0} Modeling Accuracies

Visual inspection of the three fits does not reveal much of a difference between the models; however, the error metrics shown for each figure show a substantial difference between the method outlined for ballistic weapons and the other two methods mentioned. As was mentioned previously, the hyperbolic tangent method does not account for a transonic “bump,” and as such this method will not be used. Although the

ballistic weapons model uses half the number of parameters of the Gaussian model, it also produced the highest error; for this reason the Gaussian modeling technique will be used to model supersonic drag. In this study the accuracy of the model is of more importance than the number of parameters, as the goal is to find the engine and drag characteristics with the least error to the actual data.

The value for k can also vary with both Mach and C_L .³⁶ As no papers have been found describing this effect in a parametric fashion, Equation 5.10 has been created to approximate this variation. Additional terms were added to the value of k in the same fashion as C_{D_0} . A logistic function is used to vary k with C_L in the subsonic regions of flight, taking the form

$$f(M) = \frac{1}{\frac{A}{Cl} + \exp(B(M - C))}. \quad (5.10)$$

This allows for variation of k with C_L until the Mach number reaches the value of C , at which time the variation with C_L is no longer seen.³⁶ A Gaussian models the sharp increase in k directly preceding supersonic flight. The steep rise seen in the generic curve, Figure 5.6, is exaggerated to show the potential effects. This is seen in the F-5 SAC Chart Substantiating Report.³⁶ A logistic function multiplied by the Mach number is used to create the linear increase in k after Mach 1, found in linear supersonic theory.³⁵

A generic curve is portrayed to demonstrate the potential variation of k with Mach number and C_L in Figure 5.6. As was true in the generic drag figure (Figure 5.4), this is meant to be an approximation to what is actually seen in real data, with certain

Figure 5.6: Generic Variation of k Incorporating Transonic and Supersonic Corrections

features exaggerated to show functionality.

5.1.2 Engine Deck

As was true with the drag polar, an engine deck varies with Mach and altitude. It also varies with throttle setting; however, for most of the performance analysis routines investigated here, the thrust is assumed to be at maximum thrust. Whether that includes afterburners or not is dependant on the plane under investigation. The cruise performance is not under maximum thrust, however; this is handled by using drag instead of thrust (see Section 4.1.6).

Few approximations to thrust lapse are found in aircraft design textbooks. The prevailing equations of those found, used to model maximum thrust throughout the entire flight envelope, are provided by Mattingly.³⁷ These equations are provided for multiple types of aircraft engines, ranging from a turboprop to a high bypass ratio

turbofan; each equation is in parametric form, allowing for easy variation. Mattingly does not explicitly state that the coefficients provided in his textbooks can be varied to achieve a best fit, and this must be validated; the validation studies are shown in Section 7.2.3.

Thrust Production

There are multiple engine models to choose from in this analysis. This allows for flexibility in engine type, allowing for switching between models to ensure the best fit. Each is provided by Mattingly,³⁷ and is given for a different engine type. The specific models provided can be seen in Table 5.1.

High Bypass Ratio Turbofan	$\delta_0(1 - a_1 M^{a_2} - \frac{a_3(\theta_0 - a_5)}{a_4 + M})$
Low Bypass Ratio Turbofan, with AB	$a_3 \delta_0(1 - a_1(\theta_0 - a_2)/\theta_0)$
Low Bypass Ratio Turbofan, no AB	$a_3 \delta_0(1 - a_1(\theta_0 - a_2)/\theta_0)$
Turbojet, with AB	$\delta_0(1 - a_3(\theta_0 - 1) - a_1 M^{a_2} - \frac{a_4(\theta_0 - a_5)}{\theta_0})$
Turbojet, no AB	$a_4 \delta_0(1 - a_1 M^{.5} - \frac{a_2(\theta_0 - a_5)}{(a_3 + M)\theta_0})$
Turboprop	$\delta_0(1 - a_1(M - 1)^{a_2} - \frac{a_3(\theta_0 - a_5)}{a_4(M - 0.1)})$

δ_0 and θ_0 are defined as

$$\delta_0 = \frac{P}{P_{sealevel}}(1 + \frac{\gamma-1}{2}M^2)^{\frac{\gamma}{\gamma-1}}$$

$$\theta_0 = \frac{Temp}{Temp_{sealevel}}(1 + \frac{\gamma-1}{2}M^2)$$

Table 5.1: Parameterized Engine Deck Equations

As was mentioned, each parameter a_1 through a_5 was never meant to be varied in the original text. Mattingly provides coefficients in his textbook that create a

“representative” engine deck for each category, meant to be used in the initial aircraft design phase. These approximations can be seen in Figure 5.7, with the true engine thrust lapse values for the CF6 and J85 decks compared to their appropriate counterpart from Table 5.1. In Figure 5.7 the actual thrust lapse is plotted in red and the provided models are shown in blue.

Figure 5.7: Mattingly Provided Models for Thrust Lapse

The graphs shown in Figure 5.7 show that the representative models are not exact. This is expected, as they were designed to represent a “generic” engine of each type. The correct shape of each model shows that the models simply need to be optimized to correctly model the engines at hand. This matching is shown in Chapter 7.

Fuel Consumption

As is true for thrust lapse, Mattingly also provides a parametric equation for TSFC.³⁷ Interestingly, one general equation is enough to describe TSFC for a wide

range of engines. Mattingly's equation, seen in Equation 5.11, varies only in the values of the parameters. This equation, similarly to the thrust equations shown in Section 5.1.2, was not meant to have its coefficients varied. The equation provided for TSFC is shown in Equation 5.11.

$$TSFC = (A + BM)\sqrt{\theta} \quad (5.11)$$

These values are summarized in Table 5.2.

Engine Type	Parameter A	Parameter B
High Bypass Ratio Turbofan	0.45	0.54
Low Bypass Ratio Turbofan, with AB	1.6	0.27
Low Bypass Ratio Turbofan, no AB	0.9	0.30
Turbojet, with AB	1.5	0.23
Turbojet, no AB	1.1	0.30
Turboprop	0.18	0.8

Table 5.2: Parameterized TSFC Coefficients

Unfortunately, this equation is provided for TSFC at maximum thrust only; no equations are provided to account for throttling or cruise conditions. This is not very applicable to the data in this study, as the only performance model that incorporates TSFC is range, which is for throttled engine performance (see Chapter 4). Rather than include some throttling term, it is assumed that the variation of TSFC is reasonably well behaved in the areas of interest. Well behaved, in this instance, means that since an aircraft typically cruises at a reasonably flat region of the thrust hook,

the variation of TSFC with throttle setting will be minimal. This allows the use of Mattingly’s equations to model throttled cruise performance as well.

Shown in Figure 5.8 are Mattingly-provided “representative” models for maximum thrust TSFC. These models are compared to the actual maximum thrust TSFC values of the CF6 and J85 engines. As was the case with thrust lapse, the actual TSFC is shown in red and the generic models are in blue. As was the case with Mattingly’s

(a) Mattingly Turbofan Model Compared to CF6 (b) Mattingly Turbojet Model Compared to J85

Figure 5.8: Mattingly Provided Models for TSFC

generic equations for the thrust, the “representative” TSFC models for both the high bypass ratio turbofan and the turbojet miss the mark in terms of accurate modeling. Another unfortunate realization, upon examination of these Figure 5.8, is that neither model will ever fully capture the true trends they are intended to model. The altitude variation on both models does not show the correct trends leading up to, and above 36,089 feet. According to the Mattingly model, which is a function of temperature

lapse, TSFC does not vary above 36,089 feet, and follows a linear variation with altitude below this break in the atmosphere. This idealization of the atmosphere is not 100% correct, and causes some of the variation in the stratosphere. Figure 5.8(a) shows that as the Mach number decreases the idealization produces worse results.

Figure 5.8(b) shows an even worse trend above 36,089 feet. The inversion of the TSFC trends above this altitude is unexpected, and is cause for concern. Currently nothing is being done to capture this inversion.

Unfortunately, different sources provide fundamentally different models for TSFC. The variation with altitude in Equation 5.11 is controlled solely by $\sqrt{\theta}$, which is a function of the temperature. According to the McDonnell Douglas Performance Short Course, however, TSFC also varies with the δ , which is a function of the pressure.² The equation for TSFC from this performance short course can be found in Equation 5.12, where $\dot{W}(M)$ represents the engine fuel flow.

$$TSFC = \dot{W}(M) T \delta \sqrt{\theta} \quad (5.12)$$

This variation between sources is not a trivial one; the generalized equation is, as discussed in this chapter and in Chapter 5, a crucial piece of this research. Each form of the equation will be examined in Chapter 7, at which point the best fitting form will be used.

5.2 Polynomial Fits

The use of a polynomial to fit a set of data is convenient for many reasons. Polynomials are computationally efficient, are continuous and have continuous derivatives. These facts alone cause them to be the first choice in many curve fitting problems.^{38,39} According to Weierstrass' theorem,⁴⁰ it can be shown that any continuous function can be uniformly approximated by polynomials, in a bounded interval. Polynomials are also convenient as their range is infinite; however, although this is true, extrapolation is not recommended in most circumstances.

Polynomials can model most behavior well, on a bounded interval. The degree choice of the polynomial depends on the size of the interval and the nonlinearity encountered. The larger the degree, the more terms that must be fit, causing this problem to grow quickly. Many models involved in this project are multivariate-nonlinear, and as such, have multiple parameters per equation; increasing the number of terms can drastically change the size of the problem.

5.3 Splines

One way to adjust the accuracy of polynomials is to make the bounded interval as small as possible.⁴¹ This creates a situation where a linear approximation is acceptable, as the property under calculation does not change very quickly inside the element. This fact encourages the progression from a polynomial fit to splines.

Splines are defined as a piecewise polynomial, and are often used for approximating

functions. They are very effective upon examination of one of their original intentions: data smoothing/fitting. Originally used by draftsmen,³⁹ splines were made by anchoring a thin strip of wood at certain points and allowing the wood to follow a shape of minimum strain energy, creating a continuously smooth surface that is both effective and aesthetically pleasing.

This was all done before computers. With the advent of computers, and the faster processors of today, the dependence on splines has grown in many industries.³⁹ They are often used where polynomial approximations would produce Runge’s phenomenon, an issue with fitting polynomials of a higher degree.^{38,42} Runge’s phenomenon occurs when a high degree polynomial used to fit data matches exactly at the data provided but has poor interpolation properties. Of the spline options, cubic splines are the most prevalent, as they ensure continuous first and second derivatives in addition to functional continuity.³⁸

Cubic splines show great functional approximation properties. The true showcase of this is through the Runge function; as seen in Figure 5.9, the polynomial and spline approximations with 6 data points are very similar, and neither does a fantastic job. The graph with 10 data points, however, is completely different. The polynomial approximation has a large “wringing” effect, while the cubic spline closely follows the true shape of the function, shown in Figure 5.9 as the black line. Runge’s phenomenon occurs for all polynomial fits; the actual Runge function shown here is known as a “classic” example to show the drastic effects it can have on the polynomial

Figure 5.9: Runge Function Approximation

approximation of a function.³⁸ The unfortunate effect of increasing the number of data points for the spline is one of computational efficiency. Evaluation of splines is a fairly expensive process from the beginning; increasing the number of data points only magnifies this behavior.⁴³ In solving the inverse problem formed here, each of the drag and thrust functions are evaluated approximately 1.5 million times, and the computational cost of spline interpolation is great on that scale. The true computational cost of splines will be shown at the end of this chapter.

5.4 Padé Approximations

Padé approximations are meant to approximate a function, $f(x)$, at a specific value of x ,³⁸ similar to a Taylor series approximation. A secondary use, according to Dr. Jimenez⁴⁴ and Gershenfeld,⁴⁵ is to approximate functions displaying asymptotic

behavior. The perfect application is transonic drag rise of a subsonic aircraft.

The general form of a one-dimensional Padé approximant is given by Equation 5.13, a rational function composed of a polynomial of degree m divided by a polynomial of degree n .³⁸

$$R(x)_{m,n} = \frac{p_0 + p_1x + p_2x^2 + \cdots + p_mx^m}{1 + q_1x + q_2x^2 + \cdots + q_nx^n} \quad (5.13)$$

Equation 5.13 provides a great approximation to functions that are asymptotic. This is through the creation of poles with the polynomial in the denominator. One large downside is that Padé approximations will, by definition, always model behavior exhibited by poles - even if no pole actually exists. This requires extreme care to be used with the use of the Padé function, as the poles can wreak havoc on the true shape of the function.

Each parametric equation used in this study, including drag, thrust and TSFC, is a two variable function; thus, a two variable form of the equation is necessary. Though many different forms of this equation exist, the form of Equation 5.14 was chosen due to the clear separation of the variables.⁴⁶

$$R(x, y)_{m,n,r,s} = \left(\frac{p_0 + p_1x + p_2x^2 + \cdots + p_mx^m}{1 + q_1x + q_2x^2 + \cdots + q_nx^n} \right) \left(\frac{a_0 + a_1y + a_2y^2 + \cdots + a_ry^r}{1 + b_1y + b_2y^2 + \cdots + b_sy^s} \right) \quad (5.14)$$

The multivariate version of the Padé approximation function is also apt to accurately modeling a function that displays asymptotic behavior. Seen in Figure 5.10 is Equation 5.14 modeling the drag coefficient of a DC-10 , for two values of C_L , in

the transonic region. This region of data was chosen as it displays multiple nonlinearities, and multiple regions of data. As seen in the figure, the values of C_D remain flat until approximately Mach 0.8, at which point they display exponential behavior. The matching of these two complex regions with one function shows great promise.

Figure 5.10: Padé Modeling Transonic Drag Rise

The Padé approximation used in Figure 5.10 was a third order polynomial for each variable. Unfortunately, as the order of each polynomial is of such high degree, extrapolation does not lead to reasonable results. This is the same problem encountered with Sections 5.2 and 5.3, and is an issue caused by two factors. The first is the order of polynomial chosen, as higher order polynomials often display nonphysical shapes.⁴⁵ The second factor is that extrapolation is not recommended under any curve fitting technique.^{23-25,45,47} This is unfortunate, as this particular problem often calls for

extrapolation, and gives weight to the argument that polynomials (or functions of polynomials) might not be the best choice.

5.5 Comparison of Techniques

Four different models have been discussed in this chapter, with each one displaying both good and bad qualities. As such, it is difficult to select just one model as the method to predict the drag and engine characteristics; due to this, each of the four models discussed here will be compared in their accuracy of modeling drag, thrust and TSFC. The results of this comparison will show the highlights and pitfalls of each method, and allow the burden of final model selection to fall with the user of this program.

Figure 5.11 shows a comparison of each of the four modeling methods applied to the DC-10 drag and the resulting approximations to the original functions. In each subplot the different lines denote the different C_L values at which the comparison was run. In each case the red lines denote the actual drag coefficient² and the blue lines represent the best fit of the model.

Figure 5.11 was created using an evenly-spread 40 points from Mach 0.3 to 0.88; clustering the points around the higher gradient areas would not have produced “fair” comparisons. The only C_L values included are those shown in Figure 5.11, and the entire Mach range was used for each C_L value, even though the high C_L /high Mach range is nonexistent.

Figure 5.11: Comparison of Different Approximation Functions of the DC-10 Drag

According to the errors for each model seen in Figure 5.11, the best approximation to the original drag function is the Padé approximation. This is in spite of the fact that the physics inspired model is the only one to accurately capture the changing location of the transonic drag rise with respect to C_L (seen in Equation 5.4). The error value corresponding to each subplot in Figure 5.11 is a sum of squares model. This was done to have a fair comparison of the models.

Each model shown representing drag has its own strengths and weaknesses. The

physics inspired model is the most accurate in terms of the data corresponding to the $C_L = 0.8$ line. Unfortunately, this is in contrast to its representation of transonic drag rise in all other values of C_L . The polynomial model, while more accurate in capturing drag rise for $C_L = 0$, does not show the true asymptotic nature in higher values of C_L . The spline model looks best around $C_L = 0.6$; unfortunately, the natural waviness seen in this line is magnified and also shown throughout the rest of the model. The Padé approximation, while it performs the best in terms of approximating transonic drag rise over multiple C_L values, does not accurately represent it for higher values of C_L .

An obvious additional downside to the spline function are the data points for a C_L value of 0.8 above Mach 0.6. The drag coefficient data does not extend beyond approximately Mach 0.55 at this high value of C_L , and as such, the spline data points do not physically represent any real values. The solution to this problem is to limit the combinations of Mach and C_L only to where data actually exists; unfortunately, this is not possible without prior knowledge of the drag function being modeled. This contradiction limits the applicability of the spline model to this problem, at least in terms of modeling drag: the effectiveness of splines in modeling thrust and TSFC will be explored shortly.

As seen in Figure 5.12, each of the four approximation techniques have also been used to model thrust lapse. As was true with the drag comparisons, the red lines denote the actual DC-10 thrust lapse² and the blue lines represent the best fit model.

The thrust lapse comparison shows the many intricacies of the true function that

Figure 5.12: Comparison of Approximation Functions of the DC-10 Thrust Lapse. The Red Lines Represent Actual Thrust Lapse, Blue Lines Are the Best Fit Model

must be captured before a model can be considered valid. Each model is relatively accurate for most of the flight regime, with the large exception occurring in the high Mach, low altitude portion of the physics inspired graph.

Figure 5.13 shows the comparison of each of the modeling techniques to the actual TSFC for the DC-10. As was true with the comparisons in Figures 5.11 and 5.12, the red lines denote the actual DC-10 TSFC² and the blue lines represent the best fit

model.

Figure 5.13: Comparison of Different Approximation Functions of the DC-10 TSFC. The Red Lines Represent Actual Thrust Lapse, Blue Lines Are the Best Fit Model

After much testing has been done, the author must make a recommendation that the physics inspired model be chosen for this analysis. As is shown in Figures 5.11, 5.12 and 5.13, the physics inspired models do not predict the function they are modeling the best in any of the three cases. In fact, the physics inspired model performs the worst in two of the three cases. This is due, however, solely to the fact that in each matching case, the models are being matched purely to the function they represent,

and data throughout the entire flight profile is known. This will never be the case when applying these models to the inverse problem.

As has been mentioned multiple times thus far, extrapolation with parameter estimation problems is not recommended. In an inverse problem with real life data, often times the performance data is clustered around certain Mach and altitude regions. This can be seen throughout Chapter 8 in the plots indicating the data available; having zero data in a region severely affects the resulting aircraft models. This effect is drastic in the non-physics inspired models, as no underlying effects can be programmed into them. This is in direct contrast to the composite approach of the physics inspired models; even if the supporting performance data is minimal, the form of the equations themselves help guide the shape of the aircraft models. This effect can be seen most directly in the typical drag polars in Chapter 8; for example, Figure 8.26 shows that the low Mach drag polar is fairly accurate through all C_L values, even though the only data point in that region is at C_L of 1.3.

An additional consideration in any comparison is the computational runtime. The time it takes to run each model dictates its usability in both the analysis and results phase; a model that takes a long time to run severely undercuts its value. Table 5.3 shows the time it took to run each model 1000 times. Although this may seem like an excessively high number of evaluations, the work done here can call each of the drag, thrust and TSFC functions upwards of 100,000 times. As is seen in Table 5.3, each of the physics, polynomial and Padé models are relatively quick; this is in direct

contrast to the spline model, which is two orders of magnitude slower. Each of these 1000 function runs was run 20 times and averaged to account for computational anomalies.

Drag		Thrust		TSFC	
Model	Runtime [s]	Model	Runtime [s]	Model	Runtime [s]
Physics	0.0149	Physics	0.0371	Physics	0.0309
Polynomial	0.0144	Polynomial	0.0395	Polynomial	0.0320
Spline	1.8465	Spline	1.8773	Spline	3.7134
Padé	0.0167	Padé	0.0417	Padé	0.0351

Table 5.3: Model Runtimes for 1000 Runs

Chapter 6

Proof of Concept

Validation of this concept can be seen as the first step to proving that the techniques applied in this study are justified. Without some proof that this problem can be solved in an ideal setting, tackling the real world problem with real data is premature. The real data that this technique will be applied to is often a result of flight tests involving inevitable variability in atmospheric conditions, weights, velocities, and many other parameters. Flight test engineers do their best to reduce this data to a standard day, weight and calibrated airspeed; however, error is introduced in each step and makes this problem even more difficult. In addition to this fact, the aircraft models are not perfectly matched by their corresponding functions, adding additional error to the technique. Therefore, as a proof of concept, “ideal” data has been used.

The ideal data is necessary to combat the problems of inaccuracies, both in the performance and aircraft models. In order to remove these inaccuracies, the ideal data must be artificially generated using the tools developed in this paper. It requires the creation of a “representative” engine deck and drag polar, using the models described

in Chapter 5. Ideal performance data can then be generated with the use of the performance equations from Chapter 4. This process uses the equations written for this paper in every step of the creation of this data, ensuring that the inverse problem can be solved exactly. This is not the case for real-world data, as is seen in Section 7.2.

Once the ideal data has been created, the process of solving the inverse problem can be applied. The ideal data was created under the same flight manual conditions as the original data, using the best approximations of the F-5 drag and engine data. This was not done in an attempt to match the F-5, but simply to ensure the creation of viable drag and engine models. 15 data points were used in an even spread for each performance metric. As seen in Figures 6.1, 6.2, 6.3 and 6.4, the performance data has been matched exactly.

The resulting engine and drag models can also be seen in Figures 6.5, 6.6 and 6.7. Initial test runs showed that the solution only converged to the global minimum when the initial parameter values were close to the final values. While this shows that the problem is solvable, it is also slightly discouraging; only starting points with values within about $\pm 15\%$ of the actual values converged. Guessing these parameters to within $\pm 15\%$ limits the usability of this technique in even the ideal case.

Further test cases showed a more promising trend. The convergence depended more on the initial shape of the models than the parameter values themselves. In short, the problem only converged when the initial drag, thrust and TSFC models

Figure 6.1: Ideal *RoC* Data Matched

Figure 6.2: Ideal Maximum Velocity Data Matched

Figure 6.3: Ideal Turn Radius Data Matched

Figure 6.4: Ideal Specific Range Data Matched

still looked like reasonable models. This may seem like an obvious requirement, that the initial models must follow the same trends of the final result. However, selection of these parameters to follow this requirement is not always obvious, due to the non-intuitive parameters involved in each model. Very few parameters, mainly C_{D_0} , ΔC_L and e , have physical meanings; the rest are fairly arbitrary. For this reason it is recommended that the shape that the initial parameters dictate be examined prior to proceeding with the analysis.

Figure 6.5: Ideal Drag Polar Matched

As was shown here, the concept of using performance data to deduce the drag and engine models is well supported, assuming all data is consistent and can be perfectly modeled by the equations at hand. These assumptions are key to the concepts proposed here, and without them this problem is much more difficult.

Figure 6.6: Ideal Engine Model Matched

Figure 6.7: Ideal Engine Model Matched

6.1 The Effects of Data Availability

In this section the different types of data available will be explored to see which performance parameters are more crucial to this analysis than others. This will be done with the artificially generated data from this chapter to ensure that the solution can be found.

All 15 combinations of the data available have been run through this program. Each one used the same initial starting points, the same convergence criteria; everything was the same, with the exception of which data groups were used in the study. In an effort to show only the most interesting and useful data, the performance matches themselves will not be shown; only the resulting drag and engine information will be displayed. A summary of these 15 data combinations can be seen in Table 6.1

Some of the test runs did not yield anything interesting, and these cases will not be shown here. In these cases, the resulting drag and engine data matched the artificially generated data exactly, allowing it to be left out of this study.

In an effort to more effectively show these data combinations, the location of these data points had been included on both Mach/altitude and Mach/ C_L plots. Figures 6.8 show the full range of data available; for each of the above cases in Table 6.1, include the data with the checkmarks only, and this will represent the data included in each study.

Case #	Data Included				Results Worth Discussing?
	<i>RoC</i>	Turns	Max V.	Range	
1	✓	✓	✓	✓	No
2		✓	✓	✓	Yes
3	✓		✓	✓	Yes
4	✓	✓		✓	No
5	✓	✓	✓		No
6			✓	✓	Yes
7		✓		✓	No
8		✓	✓		Yes
9	✓			✓	No
10	✓		✓		No
11	✓	✓			No
12	✓				Yes
13		✓			No
14			✓		Yes
15				✓	No

Table 6.1: Effects of Data Availability

(a) Ideal Data – Altitude vs. Mach

(b) Ideal Data – C_L vs. Mach

Figure 6.8: Ideal Data Locations

6.1.1 Data: Range, Maximum Velocity and RoC

Leaving out the turns data yielded only a slight difference in the models. With this data, reasonable results are still achieved, with the results closely resembling the artificially generated models.

Figure 6.9: Data: Range, Maximum Velocity and RoC – Drag

Figure 6.10: Data: Range, Maximum Velocity and *RoC*- Thrust Lapse

Figure 6.11: Data: Range, Maximum Velocity and *RoC*- TSFC

6.1.2 Data: Turns, Range, and Maximum Velocity

Leaving out the *RoC* data had a much more significant impact than initially anticipated. Since the *RoC* data is limited to one Mach number, the data was (incorrectly) assumed to be less important than some of the other performance metrics.

With that said, the results are still acceptable, and show a great prediction of drag, thrust lapse and TSFC.

Figure 6.12: Data: Turns, Range, and Maximum Velocity – Drag

Figure 6.13: Data: Turns, Range, and Maximum Velocity – Thrust Lapse

Figure 6.14: Data: Turns, Range, and Maximum Velocity – TSFC

6.1.3 Data: Range and Maximum Velocity

Leaving out the *RoC* and turns data had a more significant impact than leaving out only one of either data sources, which is expected after reviewing the previous results. The compounding effect of leaving both of these data sources distorts the results enough that while they are still acceptable, each of the three models does show significant error.

Figure 6.15: Data: Range and Maximum Velocity – Drag

Figure 6.16: Data: Range and Maximum Velocity – Thrust Lapse

Figure 6.17: Data: Range and Maximum Velocity – TSFC

6.1.4 Data: Turns and Maximum Velocity

Exclusion of the range and RoC data seems to only have an effect on drag, although there is an effect on thrust lapse as well; it is just much less than the effect on drag. This is seen due to the data supplied by these two metrics; the only low speed data is for higher values of C_L , which are not shown in Figure 6.18.

Figure 6.18: Data: Turns and Maximum Velocity – Drag

Figure 6.19: Data: Turns and Maximum Velocity – Thrust Lapse

6.1.5 Data: Range

The final three data sources are from only one performance source each. When only looking at one data source, the range of data can be extremely limiting. With that said, however, these next three cases proved to show decent results, even with the limited data.

Only including range data is the least limiting case of these last three. This is due to the widespread data throughout the flight envelope. Even though the C_L data is limited to approximately 0.2 due to the data selection, the drag result is still remarkably accurate. This is most likely due to the artificial nature of this problem and the relatively close starting point to the answer.

Figure 6.20: Data: Range – Drag

Figure 6.21: Data: Range – TSFC

6.1.6 Data: Maximum Velocity

Maximum velocity data is extremely limiting; it only involves data above Mach 1. This requires the subsonic regions to rely completely on the underlying form of the models to predict the characteristics. As is seen in Figures 6.22 and 6.23, this is done with quite a bit of accuracy; this is most likely due solely to the proximity of the starting point to the answer, although each of these cases used identical starting points.

Figure 6.22: Data: Maximum Velocity – Drag

Figure 6.23: Data: Maximum Velocity – Thrust Lapse

6.1.7 Data: RoC

Once again, RoC data is extremely limiting in it's nature due to the lack of variety. In fact, the RoC data is the most limiting of any data source, in that it only provides data for one Mach number. Even so, the results predicted the drag and thrust very well, with the only real error associated with the location of the transonic drag rise.

Figure 6.24: Data: RoC – Drag

Figure 6.25: Data: *RoC*– Thrust Lapse

6.2 Interpretation of Data Requirements

The multiple charts shown in the previous sections of this chapter show the different results of data runs missing different components of the data. This is intended to show the different data combinations required to have a solvable problem.

Although none of the results seem very far off from their “actual” values, this is with one large caveat—the starting values were a mere 4% away from the true solution. This close starting point was chosen to illustrate that some data—the *RoC* data, for example—is crucial to the success of this technique. If the ideal problem cannot be perfectly solved, under ideal conditions and with the starting point only 4% away from the final solution, then it is presumed that the real-world problem would also be unsolvable with the data combination in question. This is an extrapolation that is not provable, but that the author is comfortable in making based on his experiences thus far.

Chapter 7

Validation Studies

Validation studies are imperative to numerical studies and simulations. Without proof that the models and techniques are sufficient at capturing the physical phenomenon there is no basis to trust the results.

7.1 Aircraft Performance Models

In this section the known engine and drag models for both the F-5 and DC-10 are run through the performance equations in an effort to show their accuracy. For the F-5, the values for thrust, drag and TSFC is found in the SAC Chart Substantiating Report.³⁶ For the DC-10, the engine deck has been created with the use of the CF6 installation manual, and the drag is found in the McDonnell Douglas Performance Short Course.²

7.1.1 Maximum Velocity

The maximum velocity validation chart, shown in Figure 7.1, shows that every trend is matched with a high degree of accuracy. The curvature in low altitudes and

the slight curvature at higher altitudes are both picked up by the *RoC* model, showing that this model is indeed performing as intended.

Figure 7.1: Maximum Velocity Model Validation for Northrop F-5

7.1.2 Climb Gradient

Many of the performance metrics found in the F-5 and DC-10 flight manuals are in the form of nomographs. While this form allows many more conditions to be included on the same graph, it makes the graphical representation of this data more difficult as well. For this reason, many of the following performance charts show the actual vs. predicted values for each performance metric. If the data points fall perfectly on the one-to-one line, shown on each of these charts, there is zero error in the models; the further from the line, the more error included.

The climb gradient validation is the least correlated of any of the performance metrics. This is due to the fact that the values of C_L corresponding to this flight

Figure 7.2: Climb Gradient Model Validation for McDonnell Douglas DC-10

are well above the normal flight range, and are between 1.1 and 1.6. These high values of C_L are above the range provided by the high speed polar,² and require an approximate polar to be used in its place. This “ideal” polar, using values provided in the McDonnell Douglas short course,² does not provide an accurate enough representation of the drag. For this reason, in the results section, results for the DC-10 will be shown both with and without the inclusion of the climb gradient data.

7.1.3 Time to Climb

The time to climb data is found in both tabular and graphical forms. The tabulated data has been excluded due to lack of precision, with the graphical data used instead. As is expected, the data points are clustered around the black line, indicating that this model is accurately predicting the function value. While there is inherent error in this prediction, the source of this error is twofold; the aircraft models and

Figure 7.3: Time to Climb Model Validation for McDonnell Douglas DC-10

the performance data each have errors associated with them, and while we must trust these data sources, the error is evident in Figure 7.3.

7.1.4 Distance Covered During Climb

Similarly to the time to climb metric, the distance covered during climb data is presented in a way that does not lend itself well to plotting. For this reason the data is represented in the form of an actual vs. predicted graph. Figure 7.4 shows that the function predicting the distance covered during climb is routinely under-predicting this value. Again, this is most likely due to error in both the given models for thrust, drag and TSFC as well as in the underlying data.

Figure 7.4: Distance Covered During Climb Model Validation for McDonnell Douglas DC-10

7.1.5 Fuel Burn During Climb

The fuel burn during climb is also shown as an actual vs. predicted graph, showing the accuracy of the function predicting fuel burn. As was seen in Figures 7.3 and 7.4, Figure 7.7 shows that this value is only marginally accurate. As was true with the distance covered during climb, this function is routinely under-predicting the correct value.

Unfortunately, the precision of this data is limited. The data for each of the time to climb, distance to climb and fuel to climb functions is limited by the way it is presented. Each of these values was presented in a table format, with a limited number of significant figures. This limits the precision of the data available, making the task of matching for validation purposes more difficult.

Figure 7.5: Fuel Burn During Climb Model Validation for McDonnell Douglas DC-10

7.1.6 Range

The range parameters have been included for both the F-5 and DC-10 and is unique in this aspect. This is the only performance metric applicable to the data available for both aircraft.

Both range parameters show excellent matching abilities, although the DC-10 data is better correlated. This is due most likely to the accuracy of the underlying functions of TSFC.

Figure 7.6: Range Model Validation for Northrop F-5

Figure 7.7: Range Model Validation for McDonnell Douglas DC-10

7.1.7 Rate of Climb

Validation of the *RoC* model was done two ways, using both a constant Mach climb and a best Mach climb. This was done because typical climb schedules are for a given (constant) Mach number to ease the pilot load; however, the performance charts given for the F-5 are indicated as Maximum *RoC*, denoting that Mach number might vary by altitude to provide a line of best *RoC*. With that said, the results for each climb schedule show negligible difference, and the comparison between them is not shown. Figure 7.8 shows that the original data is matched quite well; only the

Figure 7.8: *RoC* Model Validation for Northrop F-5

extreme low and high altitude data points show significant error. The noteworthy component of this chart is the acceleration “kink” around an altitude of 36,000 ft, which is matched extremely well.

7.1.8 Turn Radius

The turn radius data is provided at two altitudes, 15,000 ft and 35,000 ft. The two different SAC chart lines correspond to those separate altitudes. As is seen in

Figure 7.9: Turn Radius Model Validation for Northrop F-5

Figure 7.9, the function is much more accurate at 15,000 ft than it is at 35,000 ft. This is an artifact of the drag polar for the F-5; there is no correction for altitude in the provided data.

7.2 Generalized Aircraft Models

In this section the parametric models for thrust, drag and TSFC discussed in Chapter 5 are matched to their known counterparts. This matching is to show the “best” result possible; this is the closest that the models can come to matching the true behavior.

It is imperative that these models accurately represent the functions that they are trying to represent. The more accurately a quantity can be modeled, the more accurate the final results will be. The models shown here are those created in Chapter 5.

7.2.1 Subsonic Drag Polar

As is expected, the quality of fit for any model is dependant on the range of values included in the study. Included in each of Figures 7.10, 7.11 and 7.12 is the subsonic drag model matched to the DC-10 drag data, with varying ranges of C_L . Figure 7.10 shows the best matching of the three figures, with the transonic drag rise of prime interest.

Figure 7.10 shows the subsonic drag model matched for C_L values of 0, 0.2 and 0.4. Of prime importance in this figure is the matching above Mach 0.7. While not perfect, the subsonic model does capture the drag rise quite well.

As the C_L range is expanded to include 0.6, the quality of the fit diminishes, although only slightly, as seen in Figure 7.11. This is due to the shift in location of the transonic drag rise at higher values of C_L . The capture of this shift affects the fit transonic fit of the lower values of C_L . The noticeable wiggles in the C_L of 0.6 line also decreases the fit quality, as this wiggle is not built in to the subsonic drag model.

Adding the value of C_L of 0.8 once again decreases the overall quality of the fit, although only marginally. Once again, some quality of the transonic drag rise for lower values of C_L is compromised to allow for the Mach shift seen with C_L . Although

Figure 7.10: Drag Validation for McDonnell Douglas DC-10, $C_L = [0, 0.2, 0.4]$

the overall quality is lower, the ability to capture this shifting transonic drag rise is crucial to the success of the model.

As is seen in Figures 7.10, 7.11 and 7.12, the fit of the model is quite good. Although it does not accurately capture some of the finer trends, such as the slight wiggle at C_L of 0.6, the subsonic drag model does match the more crucial aspect of transonic drag rise.

Shown in Figure 7.13 is a traditional representation of a drag polar, plotted as C_L vs. C_D . This representation shows the drag polar for the DC-10 at four different values of Mach number, increasing from left to right. This condensed representation of the drag polars shows the same trends as Figure 7.12 in a more classic format. Each of the first four ticks on the x-axis represents the corresponding Mach number, with each tick signifying an increase in C_D of 0.2.

Figure 7.11: Drag Validation for McDonnell Douglas DC-10, $C_L = [0, 0.2, 0.4, 0.6]$

7.2.2 Supersonic Drag Polar

The drag polar validation shows many things. Showing the general drag fit first, the quality changes drastically depending on the range of C_L values included. Including only C_L values of 0, 0.2 and 0.4 shows a fantastic fit in Figure 7.14. This is expected, as an examination of the F-5 drag polar shows that these values of C_L are well behaved.

Expansion of the C_L range to include $C_L = 0.6$ shows a less-correlated polar in Figure 7.15. This is due to the relative variation of k above $C_{L_{break}}$, as seen in the F-5 SAC Chart Substantiating Report.³⁶

As expected, the fit continues to degrade as additional values of C_L are added. The values for k are reported only to $C_L = 0.8$, and the fit in Figure 7.16 shows the additional error incorporated with including these higher values.³⁶

Figure 7.12: Drag Validation for McDonnell Douglas DC-10, $C_L = [0, 0.2, 0.4, 0.6, 0.8]$

Figure 7.13: Drag Polars Varying Mach Number, Mach Values of $[0.2, 0.4, 0.6, 0.85]$

This fit still incorporates the general trends expected from drag increases due to both C_L and Mach. It is interesting to note, however, that C_D actually decreases through the transonic drag regime prior to a supersonic linear increase. This is due to the drastic and counterintuitive variation of k with Mach number at higher values of C_L and can be seen in the F-5 SAC Chart Substantiating Report. The traditional

Figure 7.14: Drag Validation for Northrop F-5, $C_L = [0, 0.2, 0.4]$

representation of the a drag polar, plotted on axis of C_D and C_L , is shown in Figure 7.17 for the fit with C_L up to 0.8. In order to condense the data, the polar for each Mach number has been plotted on the same graph. Each polar, and therefore each tick on the graph, is offset by 0.1.

Figure 7.15: Drag Validation for Northrop F-5, $C_L = [0, 0.2, 0.4, 0.6]$

Figure 7.16: Drag Validation for Northrop F-5, $C_L = [0, 0.2, 0.4, 0.6, 0.8]$

Figure 7.17: Drag Polars Varying Mach Number, Mach Ranges from 0.2 to 1.6

7.2.3 Engine Deck

Ensuring that the engine model can match the true engine deck is crucial to the success of this program. As is true with the drag model, if there are inaccuracies in the engine model, the accuracy of the results will be greatly decreased.

Thrust Lapse

Figure 7.18 shows the results of matching the thrust lapse model developed in Chapter 5 to the J85, the engine used on the F-5. This matching shows that the parameterized model does an accurate job of capturing the nuances and intricacies of the actual thrust lapse.

Figure 7.18: Thrust Lapse Validation for J85

Although Figure 7.18 is not perfectly correlated throughout the flight envelope, the matched thrust lapse follows each trend with extreme accuracy.

Figure 7.19 shows the thrust lapse validation for the CF6 engine, the engine in use by the DC-10. This deck is not nearly as well correlated as the turbojet model used to match the J85. While that is a problem, the poor correlation is not a deal breaker.

Figure 7.19: Thrust Lapse Validation for CF6

The FAA heavily restricts the low altitude flight of commercial aircraft. This renders the entire bottom right portion of the graph useless, as the aircraft will never actually fly under these conditions.

Fuel Consumption

Examination of the TSFC validation in Figure 7.20 shows that the model does not capture all of the nuances of the true function. This is primarily due to the lower right region of the graph, the combinations of high Mach and low altitude. This

region, although not restricted for military aircraft, is not an optimal cruise region for any aircraft, and as such the engine does not display favorable TSFC results here.

Figure 7.20: TSFC Validation for J85

The important take-away from Figure 7.20 is that the regions of normal cruising flight, seen from approximately Mach 0.6 and lower altitudes and extending to Mach 0.9 at higher altitudes, have well behaving values of TSFC. This region shows an approximately linear relationship which can be well matched in practice.

The CF6 TSFC variation is more well behaved than that of the J85. Seen in Figure 7.21 is the validation of this TSFC model, which matches well for normal cruising regions. This commercial aircraft will typically cruise in a banded region similar to the F-5 as discussed above, with the cruise Mach number increasing as the altitude increases.

Each of the contours seen in Figure 7.21 are approximately linear in nature, which

Figure 7.21: TSFC Validation for CF6

should help the overall results. The region banded approximately by the 0.65 and 0.7 contour lines shows the typical cruise region, which is very well behaved.

This chapter was intended to show the quality of the models created, in preparation for Chapter 8. As was seen in this chapter, the quality of the models relating to the F-5 were of higher quality than those of the DC-10, and the effects of this greatly impact the results in the next chapter. Overall, there are some discrepancies in these models that must be addressed before successful fits are possible.

Overall, each of these models are close to their intended data. This is not close enough, however, and additional work is necessary on some of these models. In particular, the turbofan engine model is lacking in terms of overall fit (see Figure 7.19). The required curvature is much greater in this model than in the turbojet model, which pushes the limits on the abilities of the Mattingly model. In addition,

both TSFC models require additional work as well (see Figures 7.20 and 7.21). The TSFC model used does not capture the trends accurately, and additional terms may be necessary.

This lack of fit shows that regardless of the quality of the performance fits, these models can only ever be as accurate as the models themselves, which in these cases show significant errors. These model errors, coupled with the lack of correlation in each of the four climbing metrics (time to climb, distance to climb, fuel burn during climb and climb gradient), cause some unexpected results in Chapter 8.

Chapter 8

Results

Shown here are the results for multiple runs of this program. The results are for multiple information combinations, with the fits for each aircraft increasing in accuracy as more information is added.

The information for each run is presented in a manner that is consistent with the underlying program. First the performance metrics are shown, with the associated error. This is the overall goal of the program, to minimize this error. After the resulting performance data is shown, the drag and engine information required to produce these performance metrics is shown. It is important to note that the error associated with each performance metric is in terms of % error.

8.1 DC-10

The performance data available for the DC-10 is drastically different than that for the F-5. While this is to be expected to some degree due to the different classes of aircraft, the DC-10 data is surprisingly lacking.

Only two sources of data could be acquired for the DC-10. These were the information from Wikipedia and *Jane's*, and a flight manual purchased from ebay.com. The lack of a SAC chart as an intermediary source of data is most likely due to the differing requirements between military and civilian aircraft.

The difference between military and civilian aircraft poses another problem touched on in the introduction; the reliability of the data is somewhat suspect. As will be evident while reading the flight manual results section, the data doesn't all match. This can be for a number of reasons, all discussed in previous sections, and makes this analysis much more difficult.

8.1.1 Data Available: Flight Manual - All Data Included

The flight manual data is lacking in completeness. Due to the FAA requirements, most of the data included is for OEI and/or takeoff. While OEI does not pose a problem, the takeoff analysis has not been included in this study, severely limiting the usefulness of this data.

Of additional concern for this study is the precision of the data presented. Since the data is for the use of pilots while flying and/or while planning a flight, the pilot does not need to know with exact precision the values presented. Rounding is not an issue due to the inevitable variability in piloting techniques. This poses a problem to this work, however; the lack of precision hurts the results and can be seen in nearly every performance chart.

The flight manual data has been prepared in two parts – first with all of the data,

and second with all of the data with the exception of the climb gradient. This section here has included all of the data.

Figures 8.1 and 8.2 show the locations of the data associated with the DC-10 flight manual. Of prime interest are the black stars, denoting the climb gradient data; this is presented as a flight condition with slats extended and the flaps at 5° . This does not fit well with the rest of the data, presented as slats retracted with zero flaps, and the results of this anomaly are evident throughout these results.

Figure 8.1: DC-10 Flight Manual Data – Altitude vs. Mach

Performance Fits

The program written to solve this problem is matching the performance of the manipulated drag and engine models to the actual performance parameters. This implies that the performance results should be of sufficient quality, as these are the quantities being matched; Figures 8.3–8.6 show that this is not the case for this

Figure 8.2: DC-10 Flight Manual Data – C_L vs. Mach

analysis. Due to the nature of the presentation of the DC-10 performance data, the results are shown as actual vs. predicted plots.

Figures 8.3 and 8.4 are limited by the precision of the data. The tables that presented this data severely limited the precision, and the supporting graphs were not of sufficient quality to digitize to achieve more accurate results. This causes the data to be clustered around the 1-to-1 line of best fit instead of directly on it.

The range data was taken at the 99% best range speed as denoted in the flight manual. This is the typical long-range cruise speed for flights over two hours.²⁹ This data matching is the most accurate of the four parameters included here.

The climb gradient data, due to its abnormal flight condition, is skewed even in the validation. This causes the results to be skewed as well, as there is something not being captured in either the model, performance routine or the data itself.

Figure 8.3: DC-10 Flight Manual Data - Distance to Climb

Figure 8.4: DC-10 Flight Manual Data - Time to Climb

Figure 8.5: DC-10 Flight Manual Data - Range

Figure 8.6: DC-10 Flight Manual Data - Climb Gradient

Drag Polar Result

The resulting drag polar from running this performance analysis is not close to the actual DC-10 drag polar. Figure 8.7 shows that C_{D_0} , e , transonic drag rise, and essentially none of the other drag trends are as they are supposed to be. This is a large problem.

Figure 8.7: DC-10 Flight Manual Data – Drag

Looking at Figure 8.8, the issue becomes clearer. The high C_L data at low Mach numbers is severely skewing the results elsewhere as well. In order to match the higher climb gradients achieved with slats and flaps, the drag is decreased to a point where it affects the entire polar.

Figure 8.8: DC-10 Flight Manual Data – Drag Polars

Engine Deck Result

The low drag found above also caused the required thrust to be low; this effect can be seen in Figure 8.9. Another key feature to note is the matching in the bottom right corner of the graph. Due to FAA regulations, commercial aircraft cannot fly in this region, and therefore the region is completely devoid of data. This causes the extreme lack of correlation in this region. This is not an issue as the aircraft will never fly here.

Figure 8.9: DC-10 Flight Manual Data – Thrust Lapse

The low drag prediction caused the TSFC prediction to be higher than it should be to match the range parameter. This can be seen in Figure 8.10. Also of prime interest is the completely different shape of the TSFC function; the predicted shape is quadratic in nature even though the actual TSFC is linear. This is, once again, an

issue caused by the low drag prediction.

Figure 8.10: DC-10 Flight Manual Data – TSFC

8.1.2 Data Available: Flight Manual - Climb Gradient Excluded

As was mentioned previously, the flight manual data has been prepared in two parts – first with all of the data, and second with all of the data with the exception of the climb gradient. This section has excluded the climb gradient data from the study.

Figures 8.11 and 8.12 show the locations of the data associated with the DC-10 flight manual, with the exception of the climb gradient data. Note the difference between Figures 8.11 and 8.12 and Figures 8.1 and 8.2 from the previous section; the lack of climb gradient data reduces the available high C_L data significantly. However, while the data available is reduced, each of the remaining three performance functions is for no flaps or slats, making the analysis much more uniform, which will be evident in each of the charts presented.

Performance Fits

The program written to solve this problem is matching the performance of the manipulated drag and engine models to the actual performance parameters. The performance results presented in this section are more tightly clustered around the line of best fit, denoting a better match than before.

The same precision issues exist as they did before; in this analysis, however, the results did not need to be skewed nearly as much to achieve the desired performance, allowing the results to match in a more consistent manner.

Figure 8.11: DC-10 Flight Manual Data – Altitude vs. Mach

Figure 8.12: DC-10 Flight Manual Data – C_L vs. Mach

Figure 8.13: DC-10 Flight Manual Data - Distance to Climb

Figure 8.14: DC-10 Flight Manual Data - Time to Climb

Figure 8.15: DC-10 Flight Manual Data - Range

Drag Polar Result

The resulting drag from this analysis is much better when compared to the previous results. Figure 8.16 still shows, however, some areas where improvement is necessary. The transonic drag rise is non-existent in lower C_L values, rendering these results of limited use in a maximum velocity analysis at lower altitudes. As was discussed in the previous section, however, the FAA limits flight at high Mach and low altitude combinations, which causes this lack of correlation.

Another significant problem with the resulting drag in Figure 8.16 is the incorrect variation of drag with C_L . The cause of this issue is still unknown, although the assumed culprit is in the climb data. The precision of this data, along with the high variety of C_L points available from it, indicate that the problem could lie there.

Figure 8.16: DC-10 Flight Manual Data – Drag

One selling point of this result is the variation of transonic drag rise with C_L . This variation, although for the incorrect values of C_L , follows the actual trend very closely. This gives credit to the model in use while simultaneously giving more of an indication that the data is behind the issues seen.

Figure 8.17 shows the same general findings that Figure 8.16 did. The predicted variation of C_D with C_L deviates from the actual C_D variation as C_L increases, as was seen previously. Unfortunately aircraft such as the DC-10 spend the majority of their time flying at a higher C_L than fighters, as is seen in this data; achieving the correct variation of C_D with C_L is crucial.

Figure 8.17: DC-10 Flight Manual Data – Drag Polars

It is interesting to note that the high Mach number variation with C_L is much better than the low Mach number variation. This could indicate a problem with the

model itself in the way this variation is predicted, although the validation seen in Chapter 7 is able to capture this much more accurately than it does in Figure 8.17.

Engine Deck Result

Once again, the low drag results have a tremendous effect on the thrust lapse prediction. While the drag results are more accurate, they are still under-predicting drag in the high C_L regions. This region corresponds directly to the left portion of the graph. Figure 8.18 shows the results for thrust lapse from this run.

Figure 8.18: DC-10 Flight Manual Data – Thrust Lapse

Of prime interest are two things. First and foremost is the lack of fit in the high Mach, low altitude region of Figure 8.18. This has been discussed in the results of the previous run and, in summary, is due to FAA restricts limiting the available data.

The second noteworthy item is that the thrust lapse becomes better correlated as C_L decreases, although it never quite matches exactly. The drag in Figure 8.16 shows

that at lower values of C_L , where the drag results are more accurate, the thrust lapse is also more accurate.

The TSFC prediction, shown in Figure 8.19, is much more accurate than the results from the previous run. Although the low drag still caused the TSFC to be over-predicted, the effect is much less than before. The relative accuracy of the drag allowed for the approximately linear variation be picked up in the model as well.

Figure 8.19: DC-10 Flight Manual Data – TSFC

8.2 F-5

The results for the Northrop F-5 show the true capabilities of the inverse problem solving method. These results show that not only is the problem well posed, but that it is solvable, provided that both the performance and aircraft models have little error associated with them.

In addition, the results for the F-5 are of superior quality to the DC-10 results due to the wider range of data available, and of the superior quality of that data. The higher velocities achieved by the fighter allow calibration of the lower C_L areas of the drag polar, making the drag fit more accurate overall. This more accurate drag fit helps to calibrate both the thrust lapse and TSFC models as well.

8.2.1 Data Available: SAC Chart

The data available in a SAC chart is much more than that of *Jane's* or Wikipedia. It provides detailed performance data, usually with the exception of the specific range parameter. While it does usually include a mission radius, this requires the inclusion of mission-specific parameters, many of which are unknown.

Shown in Figures 8.20 and 8.29 are the data point locations used from the SAC chart data. They correspond to a wide combinations of Mach, C_L and altitude, leading to larger areas of trust in the results. These data points will be included on the resulting drag polar and thrust lapse plots, showing the value of having widespread data.

Figure 8.20: F-5 SAC Chart Data – Altitude vs. Mach

Performance Fits

Figures 8.22, 8.23 and 8.24 show the performance fits for the SAC chart data case. This run of the program included *RoC*, maximum velocity and turn radius performance data that was available from the SAC chart only.

As is expected, Figures 8.22–8.24 each show a great fit. The program written matches performance data to deduce the engine and drag characteristics, inferring that the performance fits should always be matched at least as well as the verification cases shown in Chapter 7. The main reason for this is that the program is manipulating the parameters in the aircraft models in an attempt to match the performance data.

Figure 8.21: F-5 SAC Chart Data – C_L vs. Mach

Figure 8.22: F-5 SAC Chart Data – RoC

Figure 8.23: F-5 SAC Chart Data – Maximum Velocity

Figure 8.24: F-5 SAC Chart Data – Turn Radius

Drag Polar Result

The resulting drag polar from the SAC Chart data is promising. With the exception of the weird “blip” in the transonic region seen in the C_L of 0.4 curve, all of the trends are correct.

Unfortunately, the “blip” is caused by the parameters used to control drag. It means that there is not enough data in corresponding to low Mach and low C_L combinations. As has been mentioned throughout this work, the results can only truly be trusted in regions where data is available; while this may be the case, the nature of the equations used allows for some extrapolation.

Figure 8.25: F-5 SAC Chart Data – Drag

The true polars, shown in Figure 8.26, highlight the problems with extrapolation mentioned above. While the resulting drag from this program matches well in regions

where data is plentiful, with data denoted by the stars, in regions of high Mach and high C_L the results cannot be used at all. The shape of the curves is preserved, which is a result of the equations in use.

Figure 8.26: F-5 SAC Chart Data – Drag Polars

Engine Deck Result

The thrust lapse results are similar to the drag polar results in that they provide a good approximation. While the results are not exact, they would be very useful as an approximation in an aircraft design or performance class.

The SAC chart did not have data on specific range, and therefore no data was available to calculate TSFC.

Figure 8.27: F-5 SAC Chart Data – Thrust Lapse

8.2.2 Data Available: Flight Manual

The flight manual provides the most complete source of data available. In addition to the many performance parameters from the SAC chart, it includes the specific range parameter, allowing for the calculation of TSFC. The specific range, in addition to adding the ability to calculate TSFC, increases the number of data points and adds variety to their locations.

Shown in Figures 8.28 and 8.29 are the data locations corresponding to the data available from the F-5 flight manual. The inclusion of the range parameter shows a wide variety of data in the first chart, spanning multiple altitude and Mach combinations. These points were chosen, however, with varying weight values associated with them, which all correspond to roughly the same points on the drag polar, all of which were approximately at C_L of 0.2.

Figure 8.28: F-5 Flight Manual Data – Altitude vs. Mach

Figure 8.29: F-5 Flight Manual Data – C_L vs. Mach

Performance Fits

As was discussed perviously, the performance fits should have relatively low error associated with them, as the performance matching is the focus of the underlying program. The maximum error is seen in the turn radius data, with the data ranging between $\pm 0.1\%$.

Figure 8.30: F-5 Flight Manual Data – *RoC*

Figure 8.31: F-5 Flight Manual Data – Maximum Velocity

Figure 8.32: F-5 Flight Manual Data – Turn Radius

Figure 8.33: F-5 Flight Manual Data – Specific Range

Drag Polar Result

The drag polar resulting from the F-5 run with full flight manual data is shown in Figure 8.34. In comparison to the results from the SAC chart alone, seen in Figure 8.25, the results shown here are much closer to the actual values.

Figure 8.34: F-5 Flight Manual Data – Drag

The inclusion of the low Mach, low C_L data helped to produce the results seen here. Figure 8.34 shows that the best matching occurs for C_L of 0.2, which can be explained when viewing the data available. The stars in Figure 8.35 represent the data available, and the stars representing range are those that fall around C_L of 0.2. This can be seen easier when referencing Figures 8.28 and 8.29, where the stars are clearly marked to correspond to the data that they represent.

The correlation between the data available and the matching of the drag results

shows the importance of the range data. This data is the only data that does not use the thrust lapse function in its calculation, requiring the calibration of drag and TSFC only; this was discussed in more detail in Chapter 4. The quality of the fit can also be seen in Figure 8.35, represented in the traditional drag polar form; as was true with the previous drag results, the results are only really valid where data is available.

Figure 8.35: F-5 Flight Manual Data – Drag Polars

Engine Deck Result

The thrust lapse results are quite promising, as was similar to the results seen from the last run in Figure 8.27. In fact, the only difference in thrust lapse between the current results, seen in Figure 8.36, and the previous results, are the differences in the results for the drag polars. Since the additional function of range does not utilize the thrust lapse function, the differences must be purely a function of the drag.

Figure 8.36: F-5 Flight Manual Data – Thrust Lapse

The resulting drag polar, seen in Figure 8.35, shows that the results are best correlated in the low C_L regions of flight. This corresponds to, roughly speaking, the right half of the flight envelope seen in Figure 8.36; the results on that portion of the graph are noticeably more accurate.

The TSFC function does not match the original data to a high degree of accuracy.

This is for a number of reasons, with the most prevalent being the data selected for this analysis. The author used the range data points corresponding to various points of “best” range, limiting the results. The blue dots correspond to the range data used in this analysis.

Figure 8.37: F-5 Flight Manual Data – TSFC

While not perfectly correlated, the TSFC results are not a complete waste. Showing with some accuracy the general trend, Figure 8.37 shows that at the data points themselves there is a good degree of correlation. The results are not enough to perform complex analysis with; however, in the cruise areas where this aircraft will typically be flying, the results do come close to telling an accurate story.

Chapter 9

Final Remarks

A technique was developed to investigate the technique of using only aircraft performance data to deduce drag, thrust and TSFC models for a specific aircraft. The entire process of inverse methods was explored, multiple aircraft models were used and performance models were created in order to solve this problem.

Inverse problem methodologies have been explored, with emphasis placed on non-linear curve fitting for this project. A global optimizer has been written to counteract the issue of local minima, multiple optimizer options have been investigated, and the technique has been verified.

A variety of different aircraft models have been explored as options to use with this work. As a stand-alone model many of the strictly numerical techniques work well; in practice, however, these methods have been shown to fall short due to their extrapolation abilities. The selection of aircraft models is of key importance to this work, as the accuracy of the validated models is the best case scenario for the program as a whole.

Multiple aircraft performance functions have been written, tested and validated for use with this program. These represent a variety of “standard” performance metrics that are included in typical SAC Charts and Flight Manuals spanning both the commercial and military industries. Each of these functions has been verified and validated using the known drag and engine functions for the F-5 and DC-10.

The results of this work show many things. First and foremost, the program’s results show that the idea of reverse engineering an aircraft using performance data is possible, albeit with conditions: the underlying aircraft models and performance models must have minimal error, and the data must be free of error. This can be seen throughout the paper, as different aircraft had different levels of error; the artificially created aircraft had zero error, the F-5 had minimal error, and the DC-10 had the most error. This error accumulated from each of the three pillars of this program, and the results are progressively worse as the errors adds up.

The validations of the aircraft models show that it is possible to construct a physically-inspired model that is valid throughout the flight envelope for most cases. Some of the functions are more adept than others at capturing all of the phenomenon; an example of this can be seen in the thrust lapse comparisons. The model for the J85 thrust lapse matches extremely well, while the CF6 model is less correlated throughout.

An original idea of an implementation of this work was to create a universal program to perform this analysis blindly, with any combination of user-provided

input allowable. The ideal setting for this would have been on a website, where a user could enter the data and immediately receive the results. Upon exploration of this topic, however, this is not feasible, for a variety of reasons.

- The runtime on a quad core processor with 3 GB of ram varies from 2 minutes to 10 minutes, depending on the data available and the performance functions utilized. This would be much longer hosted on a website and would render it unusable.
- The endless variety of data available, including unique combinations and functions not explored here, require that the user be allowed to specify custom functions, or at least custom flap/slat conditions, which are not readily built into the models.
- Each aircraft must be custom-tailored in order to have a successful run. While the intention was to build universal models, each one has been built to match the test cases here; deriving a true universal model to be available and useful for every aircraft is not feasible.
- Even with months of work, the resulting drag and engine results from the DC-10 show little use in any practical application. Engineering a truly universal program, able to solve any problem thrown at it, is not practical in any sense. The variability in the true aircraft models and the data available makes this infeasible.

Although many modifications would be necessary, the groundwork has been laid to apply this work to any aircraft, following the steps outlined in Figure 1.1 and detailed throughout this work. Generic drag, thrust and TSFC models have been created, which should provide reasonable results provided that there is enough data and that it is free of error.

9.1 Necessary Information

The many test cases and trial runs examined by the author have provided much insight into the amount, quality and type of performance data necessary to successfully apply the techniques of this work. The test cases seen in Chapter 6 and the results in Chapter 8 attempt to solve this problem of determining the necessary information.

There is not an ideal amount of performance data, in terms of total number of data points, to best run this problem, although the number of data points must be greater than the number of total parameters. In general, it has been found that an equal number of data points for each performance metric provides the best results. In many test cases the end results are biased or skewed due to some performance metrics having more data than others, and it is best to parse the data to achieve a balance.

The data must be free of error. Although this is something that must be assumed up front, it is extremely important that it be true. In addition, each of the performance metrics must be for the same aircraft configuration. If one of the metrics has

5° flaps, all metrics must have 5° flaps, otherwise the process will not work. This can be seen in the DC-10 test cases, where the climb gradient data is for both flaps and slats extended, while the other data is not. In terms of engine performance, it is also crucial that every performance metric be for either with afterburners or without—each thrust lapse and TSFC equation discussed in Chapter 5 is dependant on whether or not afterburners are used, and the equations and parameters are different for each case.

There are many, many performance metrics that may be included in a SAC chart or flight manual. Of these, some are more crucial to the success of this inverse method.

- Turn radius as a function of Mach number provides high C_L data at a condition in the flight envelope where this data would otherwise not be available. Turn data is often the only metric available to provide such high C_L data.
- Maximum velocity is imperative in accurately predicting the high speed portion of the flight envelope. Each additional data point, corresponding to a different altitude, helps to tune the high Mach drag and thrust data.
- Specific range is crucial in establishing an independence between drag and thrust. In most performance functions, if drag is estimated to be too high, thrust lapse is increased to make up for this discrepancy. Range is the only performance function which does not use both drag and thrust; it instead uses drag and TSFC. This different pairing of functions allows drag to be adjusted without respect to thrust, and is unique in this manner.

9.2 Future Work

Of the three pillars of this work, the inverse methods section is the least explored. There are many radically different curve fitting routines available to apply to this inverse problem, only a few of which have been explored here. Bayesian statistics and artificial intelligence techniques would allow for some incorporation of error bounds and uncertainty analysis on the final results, useful in any estimation problem.

Additional global optimization routines should be investigated as well. The choice of a tunneling algorithm was based on ease of programming and the incorporation of gradient based optimizers. Additional methods that incorporate different routines use statistical, heuristic, and deterministic methods, and could increase the efficiency and/or overall quality of the results.

Each of the three main pillars of work have their inaccuracies. Incremental improvements in each individual area will have a tremendous effect on the overall results—this effect can be seen already between the results of the DC-10 and the F-5. The F-5 results are much better than that of the DC-10, due to the improvements in both the performance and aircraft models between the two aircraft. These improvements made all of the difference, and further work will only improve the results.

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