CARBON FIBER LEAF SPRINGS
FOR ADAPTIVE CROSS COUNTRY SKIING

A Thesis
presented to
the Faculty of California Polytechnic State University,
San Luis Obispo

In Partial Fulfillment
of the Requirements for the Degree
Master of Science in Mechanical Engineering

By
William Drew Atkinson
August 2009

©2009
William Drew Atkinson
ALL RIGHTS RESERVED

COMMITTEE MEMBERSHIP

TITLE: DIGRESSIVE RATE CARBON FIBER LEAF SPRINGS FOR
ADAPTIVE CROSS COUNTRY SKIING

AUTHOR: WILLIAM DREW ATKINSON

DATE SUBMITTED: AUGUST 2009

COMMITTEE CHAIR: BRIAN SELF, Ph.D.

COMMITTEE MEMBER: ANDREW DAVOL, Ph.D., P.E.

COMMITTEE MEMBER: JOSEPH MELLO, Ph.D.

Abstract

CARBON FIBER LEAF SPRINGS FOR ADAPTIVE CROSS COUNTRY SKIING William Drew Atkinson

This work describes the development of a custom sit ski for US Ski Team paralympian Greg Mallory from concept through prototype fabrication. The ski consists of a custom seat molded specifically for the athlete, carbon fiber leaf springs, and a custom binding attachment system compatible with NNN style cross country bindings. The sit ski is designed to maximize poling power through the use of an upright rather than reclined seating position, allowing for increased utilization of core muscle strength. The springs were designed based on information gathered by a custom National Instruments data acquisition system, and stiffness analysis was conducted using Castigliano's theorem applied to classical laminate theory.

Table of Contents

LIST OF SYMBOLS	VII
CHAPTER 1. INTRODUCTION.....	1
CHAPTER 2. EXPERIMENTATION.....	6
CHAPTER 3. DATA ACQUISITION	12
CHAPTER 4. ANALYSIS.....	18
CHAPTER 5. FABRICATION	27
CHAPTER 6. TESTING	32
CHAPTER 7. CONCLUSIONS	35
BIBLIOGRAPHY	37
APPENDIX A PART DRAWINGS	1
APPENDIX B LABVIEW CODE	10
APPENDIX C EXPERIMENTAL DATA PROCESSING CODE	13
APPENDIX D EXPERIMENTAL DATA.....	15
APPENDIX E STRESS ANALYSIS CODE	17
APPENDIX F STRESS ANALYSIS PLOT TOOL CODE	26
APPENDIX G INSTRON TEST DATA.....	28
APPENDIX H INSTRON TEST DATA ANALYSIS CODE	35

LIST OF FIGURES

FIGURE 1. GREG MALLORY IN A CONVENTIONAL SIT SKI	1
FIGURE 2. CAPRICORN SIT SKI	2
FIGURE 3. THE SIT SKI DEVELOPED BY THE ADAPT ^X SENIOR PROJECT TEAM	4
FIGURE 4. THE FLEXIBLE FRAME ALLOWED FOR IMPROVED CORNERING.....	5
FIGURE 5. SOLID MODEL OF THE ALUMINUM TEST FRAME.....	6
FIGURE 6. CG LOCATIONS FOR ABLE-BODIED SKIER.....	8
FIGURE 7. SOLID MODEL OF THE BINDING ATTACHMENT.....	10
FIGURE 8. THE ALUMINUM FRAME MOUNTED ON ROLLER SKIS	11
FIGURE 9. NATIONAL INSTRUMENTS COMPACTRIO (NATIONAL INSTRUMENTS)	12
FIGURE 10. WIRING DIAGRAM FOR A WHEATSTONE BRIDGE (NATIONAL INSTRUMENTS).....	13
FIGURE 11. STATIC CALIBRATION FOR REAR LEG	14
FIGURE 12. STATIC CALIBRATION FOR FRONT LEG	15
FIGURE 13. DIAGRAM OF STATIC TEST SETUP.....	15
FIGURE 14. FORCES IN THE SIT SKI DURING A SPRINT	17
FIGURE 15. TOTAL VERTICAL FORCE ON THE SIT SKI AND FORCE ON THE SKI POLE DURING A SPRINT	17
FIGURE 16. FORCE DIAGRAM, FREE BODY DIAGRAM, AND RESULTANT FORCES	19
FIGURE 17. PROFILE OF SPRINGS WITH INCREASING ARC RADIUS	21
FIGURE 18. FABRICATING THE SEAT MOLD	28
FIGURE 19. ASSEMBLED SIT SKI.....	31
FIGURE 20. DIAGRAM OF INSTRON TEST SETUP	32
FIGURE 21. COMPARISON OF THEORETICAL AND EXPERIMENTAL DEFLECTION.....	33
FIGURE 22. COMPARISON OF THEORETICAL AND EXPERIMENTAL SPRING RATES	34

List of Symbols

δ	Displacement of a point due to the application of a force
U	Strain energy
F	Force applied at a point
M	Moment
E	Young's modulus
I	Area moment of inertia
A	Cross sectional area
e	Eccentricity
F_θ	Normal force in a cross section of a curved beam
R_n	Neutral radius
L	Arm length
θ_1	Lower limit of integration in a semi-circular curved beam
θ_2	Upper limit of integration in a semi-circular curved beam
$\overline{EA}$	Equivalent axial stiffness for a composite beam
$\overline{EI}$	Equivalent bending stiffness for a composite beam
ε_x	Strain in x direction
ε_y	Strain in y direction
γ_{xy}	Shear strain
κ_x	Curvature in x direction
κ_y	Curvature in y direction
κ_{xy}	Curvature in xy direction
N_x	Normal line load in x direction
N_y	Normal line load in y direction
N_{xy}	Normal line load in xy direction
M_x	Line moment in x direction
M_y	Line moment in y direction
M_{xy}	Line moment in xy direction

Chapter 1. Introduction

The International Paralympic Committee recently relaxed the rules regarding cross country sit ski configuration, prompting a collaboration between the U.S. Adaptive Ski Team and the Cal Poly Mechanical Engineering Department with the goal of developing an innovative new sit ski design. The cross country team, led by Head Cross Country Coach Jon Kreamelmeyer, requested a new design that is lightweight, with the capacity for suspension-style articulation of the skis as well as adjustable track width. The primary goal of the project, however, is to place the athlete in the optimum position to develop a powerful poling motion.

Figure 1. Greg Mallory in a conventional sit ski

US ski team racers source their sit skis from a number of suppliers. Greg Mallory previously used a sit ski

(shown in Figure 1) made by Michael Byxbe of Sierra Sit Skis operated out of Truckee, CA. This ski places the athlete's legs extended out in front of the seat. The frame is made from welded aluminum tubing and attaches to the ski using a New Nordic Norm binding with an additional rubber stop to keep the frame from rotating forwards around the axle.

Capricorn Sit Skis are made by Chris Klebl in Utah, and use an upright seating position similar to wheelchair racers (see Figure 2).

Figure 2. Capricorn Sit Ski

His most recent ski is characterized by a low seat and raised thighs, and is also made of welded aluminum tubing. One of the more significant features that differentiates Capricorn Sit Skis is the use of a second NNN binding on each ski to constrain the frame in pitch.

This task began as a senior project for a group called AdaptX who produced a carbon composite frame (see Figures 3 and 4) in which the athlete assumes an upright position with the legs tucked underneath the seat of the ski. This position is intended to allow the athlete to maximize efficiency and power output by improving the biomechanics of the poling motion. The optimization of ergonomics and the reduction in weight should provide a significant competitive advantage compared to traditional sit skis. A secondary goal of the new design is to allow the athlete to preserve momentum while cornering. The traditional sit ski design scrubs off a significant amount of speed and also requires a burst of increased exertion at each corner. Higher cornering speed could also provide an important time saving advantage.

Figure 3. The sit ski developed by the AdaptX senior project team

The prototype frame was evaluated at a test session with paralympian Greg Mallory in Truckee, CA, and it became clear that while the concept was promising, there is room for improvement. The primary shortcoming of the current frame is that the seating position is not ideally suited to the athlete's disability. The seat slopes too steeply forward, so Greg couldn't lean back far enough to utilize his abdominal muscles while poling. Secondly, the frame deflects excessively while poling, leading to instability. In spite of these problems, Greg is optimistic that the concept is sound and only needs further refinement.

Figure 4. The flexible frame allowed for improved cornering

The concept for the new sit ski is to incorporate the prototype's improvements in ergonomics and correct its deficiency in stiffness while still preserving its cornering characteristics. In order to accomplish this the frame will be designed with improved leaf springs that strike a compromise between stiffness necessary during poling and compliance desirable during cornering. Greg is known for freestyling around turns on one ski in order to avoid scrubbing off speed, so a frame that flexes to keep both edges engaged with the snow could offer improved cornering traction.

Chapter 2. Experimentation

In order to determine the appropriate spring rate for the carbon frame, the forces applied to the frame during use must be determined. The first step in developing an experiment to measure these forces was the design and fabrication of a rigid frame which could be used as a load cell. This frame could also be used to test the radically altered body position requested by the athlete. The solid model of the experimental frame is presented in Figure 5, and dimensioned drawings are provided in Appendix A.

Figure 5. Solid model of the aluminum test frame

The critical geometric factors for the frame are longitudinal center of gravity location, seat pan height, seat pan angle, and lateral ski spacing. The ski spacing is determined by the FIS (The International Federation of Skiing) standard for cross country ski tracks which is 9 inches center-to-center. Seat pan angle and height were determined experimentally with Greg Mallory using an adjustable sit ski frame, leaving longitudinal CoG location as the last unknown.

The binding attachment was intended to mimic the force distribution on the ski of a stand-up skier as closely as possible, beginning with placing the athlete's center of gravity in the same location as a stand-up skier. According to Chris Hall, director of Nordic racing for Fischer Skis, the binding on a ski is typically mounted on balance, meaning that the axle is directly above the balance point of the ski. The balance point is the transverse location of the center of gravity. Different mounting locations can be used to alter the performance of the ski, for instance the binding can be moved backward to improve performance during glide or forward to improve cornering. Cross country skis are classified as soft, medium, and stiff, depending on how much force is required to flatten them against a steel plate. As shown in Figure

6, the force is applied in two locations, 8cm and 15cm behind the center of gravity of the ski. These dimensions come from the assumption that the binding will be mounted on balance, that the ball of the skier's foot is 8cm from the axle, and that the heel is 15cm from the axle.

Figure 6. CG locations for able-bodied skier

Mr. Hall indicated that the skier tends to put more weight on the ball of the foot during cornering but concentrates weight on the heel while tucked and gliding. Based on these factors we agreed to put the design location for the athlete's center of gravity 10cm behind the binding axle, assuming the location of the binding on the ski can be adjusted as necessary. The location of the skier's center of gravity was approximated using a solid model

generated using Mannequin biomechanics software. The data from Mannequin was imported into Solidworks in order to determine the relative position of the seat and the bindings to meet the design specifications.

Weight, stiffness, and manufacturability were the primary concerns when designing this prototype frame. This led to the selection of 6061 aluminum, which is an appropriate compromise between light weight and manufacturability. Wall thickness for this application is driven by impact resistance; based on previous frames and material availability tube dimensions were selected at 0.5 x 0.049 inches for primary structural members and 0.375 x 0.049 inches for reinforcements.

The method used to attach the skis to the frame of the sit ski is critical to the functionality of the entire system. The binding attachment needs to be lightweight and rigid while allowing for quick ski changes, and ideally remain compatible with the NNN binding system used by most of the athletes on the team. In order to meet these requirements, the binding attachment should closely mimic the boot used by an able-bodied skier but with a feature to prevent the heel end of the binding from lifting off the ski.

Figure 7. Solid model of the binding attachment

1 $\frac{1}{2}$ x 1 $\frac{1}{2}$ x 1/8 inch 6061 aluminum u-channel is readily available and easily machined as well as meeting stiffness, strength, and weight goals for the system. This size of u-channel mimics the rails on a NNN ski boot distributing the weight of the skier along the length of the binding. The front axle is made from a 1 $\frac{3}{4}$ x 1/8 inch 416 stainless steel dowel retained by e-rings in machined grooves. 416 SS was selected due to its combination of high strength and corrosion resistance, which is very important for a component that will spend its working life covered in water. The rear attachment for the binding is a tongue and groove that constrains the ski with minimal play when the front axle is locked to the binding, but easily slides apart when the axle is released.

Figure 8. The aluminum frame mounted on roller skis

Chapter 3. Data Acquisition

The data acquisition system used for this project was the National Instruments (Austin, TX) Compact RIO embedded controller with an analog input module specifically designed for strain gage bridges. This piece of equipment was selected based on its stand-alone data acquisition capabilities as well as its portability and ruggedness.

Figure 9. National Instruments CompactRIO (National Instruments)

A pair of custom virtual instruments (VI) had to be created in LabVIEW in order to read, record, and display data from the strain gages fitted to the sit ski. The primary VI, referred to as the host or real-time VI, logs the data. The purpose of the additional FPGA VI is to activate the module and transfer the data to the real time VI on the controller, where it can be processed and stored.

The data of interest for this frame was the axial force in the primary structural members during race conditions. The lightest, most durable tool for determining these forces is the strain gage. With proper geometric and electrical setup, it is possible to create an axial force load cell insensitive to effects such as bending strain or temperature change. The way to achieve this is through the use of a Wheatstone bridge. The Wheatstone bridge provides an additional benefit in that it is much more sensitive than a single strain gage, a feature that becomes crucial when the strains to be measured are small due to the rigidity of the specimen.

Figure 10. Wiring diagram for a Wheatstone bridge (National Instruments)

The strain gages selected for this test were Vishay CEA-13-250UW-350/P2, which is a 350 Ohm, 0.250 inch gage calibrated for the coefficient of thermal expansion of aluminum.

Once the gages were epoxied and wired into full bridges with a sensitivity of 2.6, they needed to be calibrated against dead weights to determine the null offset and the change in voltage with applied load. This was done by fixturing the frame upside down and hanging weights from the legs, simulating a vertical compressive load similar to what is expected in the field. Due to the high level of electrical noise in the lab, the voltage was averaged over several seconds at each load point. A linear curve fit was then applied to the data points in Excel to determine the true conversion coefficient shown in Figures 11 and 12.

Figure 11. Static calibration for rear leg

Figure 12. Static calibration for front leg

A ski pole was instrumented in the same way to provide a timing signal used to determine the relationship between poling timing and load in the ski frame.

Figure 13. Diagram of static test setup

Physical testing was conducted with the athlete on snow in order to determine the magnitude of the forces exerted by the athlete in the field. This test session was conducted during a training camp hosted by the ski team in Steamboat Springs, Colorado. The site for the testing was a high-elevation mountain pass, which was one of the few

locations with any remaining snow at that time. This testing included sprints, turns, uphill, and downhill. Because the testing occurred late in the season, the snow was soft and slushy. There was a window of only a few hours every morning where the snow was solid enough to ski on; it was usually too soft by midmorning. This testing indicates that the total vertical force on the instrumented legs usually doesn't exceed 100 lbs, and is typically around 70 lbs (which matches expectations because the athlete weighs 140 lbs). Figure 14 presents typical data acquired during the test session. This particular run was a straightline maximum effort sprint down a slightly declined slope. The separate load traces indicate that a large component of the fluctuation in load is generated by the athlete rocking his weight back and forth on the ski, rather than lifting himself off the ground with the poles. However, Figure 14 shows that there is a clear reduction in force applied to the legs of the ski while the athlete is propelling himself forward with the poles.

Figure 14. Forces in the sit ski during a sprint

Figure 15. Total vertical force on the sit ski and force on the ski pole during a sprint

Chapter 4. Analysis

Initial analysis for the sit ski concentrated on determining the effect of geometric changes on the behavior of the spring. Due to the complex shape of the spring the use of a strain energy method is most practical for determining deflection, leading to the application of Castigliano's theorem. This theorem states that *when forces act on elastic systems subject to small displacements, the displacement corresponding to any force, collinear with the force, is equal to the partial derivative of the total strain energy with respect to that force* (Shigley et al., 2004, pg. 201). In mathematical terms, Castigliano's theorem is

$$\delta_i = \frac{\partial U}{\partial F_i}$$

Where δ_i is displacement of the point of application of force F_i and U is the strain energy. Knowing the general shape of the spring, it is possible to identify the modes by which strain energy could be stored and from that deflection can be predicted. For purposes of this analysis the spring was broken up into two portions- the upper arm and the arc. The upper arm is the horizontal portion of the spring which mounts to the seat, while the arc is the

vertically oriented, curved portion of the spring. The junction between the arc and the bottom arm is treated as a fixed joint. The arm stores strain energy in bending, while the arc stores energy through bending, compression, and the moment resulting from that compression (Figure 16). Shear in both the arm and arc was accounted for in early revisions of the analysis but was shown to be negligible and was not included in the later versions.

Figure 16. Force diagram, free body diagram, and resultant forces

The strain energy in the arm due to bending is

$$U_1 = \int_0^L \frac{M^2 dx}{2EI}$$

The energy in the arc due to bending is

$$U_2 = \int_{\theta_1}^{\theta_2} \frac{M^2 d\theta}{2AeE}$$

The energy in the arc due to compression is

$$U_3 = \int_{\theta_1}^{\theta_2} \frac{F_{\theta}^2 R_n d\theta}{2AE}$$

The energy in the arc due to the resultant moment is

$$U_4 = \int_{\theta_1}^{\theta_2} \frac{MF_{\theta} d\theta}{AE}$$

Evaluating the integral for the strain energy in the arm results in the following

$$\delta_1 = \frac{F L^3}{3EI}$$

while pulling out the partial derivative with respect to force in the remaining equations results in

$$\delta_2 = \int_{\theta_1}^{\theta_2} \frac{M}{AeE} \left(\frac{\partial M}{\partial F} \right) d\theta$$

$$\delta_3 = \int_{\theta_1}^{\theta_2} \frac{F_{\theta} R_n}{AE} \left(\frac{\partial F_{\theta}}{\partial F} \right) d\theta$$

$$\delta_4 = \int_{\theta_1}^{\theta_2} \frac{1}{AE} \left(\frac{\partial (MF_{\theta})}{\partial F} \right) d\theta$$

Based on a free body diagram of the structure, the variables in the strain energy equations could be replaced with terms that use the geometry and loads of the spring.

$$M = F[L + R_n(\cos \theta - \cos \theta_1)]$$

$$F_{\theta} = F \sin \theta$$

$$MF_{\theta} = F^2 R_n \sin^2 \theta$$

Taking the partial derivative of these terms

$$\frac{\partial M}{\partial F} = L + R_n(\cos \theta - \cos \theta_1)$$

$$\frac{\partial F_{\theta}}{\partial F} = \sin \theta$$

$$\frac{\partial(MF_{\theta})}{\partial F} = 2FR_n \sin^2 \theta$$

The deflection can now be described in a single equation which is linear in F

$$\delta = \frac{FL^3}{3EI} + \int_{\theta_1}^{\theta_2} \frac{FL + FR_n(\cos \theta - \cos \theta_1)}{AeE} d\theta + \frac{FR_n}{AE} \int_{\theta_1}^{\theta_2} \sin^2 \theta d\theta + \frac{2FR_n}{AE} \int_{\theta_1}^{\theta_2} \sin^2 \theta d\theta$$

This equation can be numerically evaluated in Matlab using a function like *quadl*. There are four variables of interest when evaluating different spring geometries: width, thickness, arm length, and arc radius. A change in any one of these results in a change in the spring rate, so it was necessary to devise a way to compare all possible geometries. This was accomplished through the use of nested *for* loops sweeping across a range of values for each variable. The result is a four dimensional matrix of all possible combinations of spring shapes.

Figure 17. Profile of springs with increasing arc radius

The next step in the analysis is to incorporate laminate properties for stiffness and strength into the code in order to design a layup schedule for the prototype. This means that equivalent EI and EA terms must be evaluated for the laminate then substituted into the existing strain energy equation to determine the spring curve. The compliance matrix takes the form

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{16} & b_{11} & b_{12} & b_{16} \\ a_{12} & a_{22} & a_{26} & b_{12} & b_{22} & b_{26} \\ a_{16} & a_{26} & a_{66} & b_{16} & b_{26} & b_{66} \\ b_{11} & b_{12} & b_{16} & d_{11} & d_{12} & d_{16} \\ b_{12} & b_{22} & b_{26} & d_{12} & d_{22} & d_{26} \\ b_{16} & b_{26} & b_{66} & d_{16} & d_{26} & d_{66} \end{bmatrix} \begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix}$$

Where N and M are respectively line loads and line moments. The terms in the compliance matrix are generated from material properties (Young's modulus and Poisson's ratio) and lamina layup schedule (ply thickness, location, and angle). For a balanced and symmetric layup, a number of a and d as well as all b matrix terms become small enough to be approximated as 0.

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} = \begin{bmatrix} a_{11} & a_{12} & 0 & 0 & 0 & 0 \\ a_{12} & a_{22} & a_{26} & 0 & 0 & 0 \\ 0 & 0 & a_{66} & 0 & 0 & 0 \\ 0 & 0 & 0 & d_{11} & d_{12} & 0 \\ 0 & 0 & 0 & d_{12} & d_{22} & 0 \\ 0 & 0 & 0 & 0 & 0 & d_{66} \end{bmatrix} \begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix}$$

Equivalent EI and EA terms can be pulled out of the compliance matrix as shown. For example, given an assumed stress stain the x-direction only

$$E_x = \frac{\sigma_x}{\epsilon_x}$$

And

$$\sigma_x = \frac{N_x}{t}$$

where t is the thickness of the cross section, we can show that

$$E_x = \frac{N_x}{\epsilon_x t}$$

And from the laminate compliance matrix we can see that

$$\epsilon_x = a_{11} N_x$$

therefore

$$E_x = \frac{1}{a_{11} t}$$

which leads to

$$\overline{EA} = E_x w t$$

thus

$$\overline{EA} = w / a_{11}$$

Similar manipulation leads to

$$\overline{EI} = w D_{11}$$

where

$$D = inv[d]$$

The EI and EA terms in the original strain energy equations can be replaced with $\overline{EA}$ and $\overline{EI}$, allowing deflection to be calculated for a composite structure in the given uniaxial stress state. Once strength of materials methods are replaced with classical laminate theory in the code, it becomes possible to begin designing the spring.

The athlete requested a seat height of 11.75 inches, and the preliminary analysis indicates that the arm length should be as short as possible. Practically it needs to be long enough to keep the structure from rocking back and forth while the athlete is poling. There are also packaging requirements based on the shape of the ischial relief/mounting surface built into the seat. These two factors mean that the shortest possible length from the arc to the athlete's center of gravity is 4 inches. Based on the preliminary analysis, the arc of the spring should have the largest possible radius to maximize stiffness.

Increasing the radius of the arc increases the stiffness of the spring by reducing the bending moment generated by the load F. As the arc radius becomes large relative to the arc length, the chord of the arm becomes so

small that the profile of the arm can be approximated as a straight line during manufacturing.

With those design parameters accounted for, the next step is to iterate layup schedules to achieve sufficient stiffness. Based on material availability the spring will be made from 9.0 oz T700 unidirectional carbon tape 3 inches wide, the properties of which are available in Mil Handbook 17-2F Polymer Matrix Composites Materials Properties. T700 is the strongest and stiffest form of carbon fiber that is commonly available to the public, and uni can be utilized for this application because the stresses are almost entirely oriented in one direction.

The experimental data for the loads in the frame indicates that the springs should be relatively rigid up to 100 lbs. After experimentation with different combinations of layup thickness and width, the best combination of spring rate and manufacturability is a spring 26 plies thick and 2.5 inches wide. The width was selected because the tape is supplied in 3 inch wide rolls and it is necessary to have excess material at the edges of a layup that can be trimmed back after cure, while 26 plies were used in order to build adequate thickness to meet the stiffness goals. Initial stiffness for these springs is predicted to be 506 lb/in. Because this design is

stiffness critical and the axis of the fibers aligns with the direction of the principal stress, the lowest failure index appears on the fibers in compression. When 140 lbs is applied to one leg, this failure index is 0.161.

Chapter 5. Fabrication

The first component to be fabricated for this project was the custom seat. A custom seat has a number of advantages over a generic seat for this application, the most significant of which is superior fit. A seat that fits the athlete properly will improve comfort, reducing the possibility of "hot spots" and bruising; it will also improve the athlete's ability to control the sit ski by eliminating the slack present in a generic seat bucket. Secondly, a custom carbon fiber seat will weigh significantly less than the the state of the art, such as the Sierra Sit Ski polyethylene seat.

Because a controlled flat surface is required on the outside of the seat for mounting the springs, the seat must be formed from a female mold. The process for fabricating the seat begins with taking a mold of the athlete's hindquarters while he sits in the kneeling position. This was accomplished by building a four-sided cardboard box with an open top and front, then taping a double layer of plastic garbage bags down inside. The garbage bags served to contain the foam and also as a mold release film. Two-part expanding urethane foam was then poured under and around the subject. As the foam expanded, it filled the

voids between the athlete and the inside walls of the box. After cure, the athlete was pulled from the mold, leaving behind a perfect impression.

Figure 18. Fabricating the seat mold

The next step in the process was to fabricate a plug of the seat. This was done in a similar fashion to the original mold. The seat mold was placed into a box, then two-part foam was poured into a trashbag and allowed to fill the voids. The resulting plug was trimmed to shape with a saw. An important feature of the seat was the combined relief pocket for the ischial bone structures (protrusions on the bottom of the pelvic bone) and mounting surface for the frame or springs. This feature was formed by casting (in the same urethane foam) a boss based on the relief in a similar sit ski seat, then foam-gluing it to the plug. A flat was then machined on the boss, and a 10

inch by 4 inch piece of 3/8 precision ground aluminum plate was glued to the flat to form the mounting face on the plug. Finally, the foam was coated with polyester body filler and sanded smooth, followed by a coat of Duratec primer.

Before fabricating the race-ready seat, a fit-check prototype was made directly from the plug. This was a simple splash directly on the plug with some scrap carbon weave, which was shipped to the athlete for evaluation. Based on his feedback, the relief was deepened and the height of the sides was increased by $\frac{3}{4}$ inch for the final version. Further, an additional relief was added at the back of the seat for the coccyx.

Once these modifications were made to the plug, a female mold made of fiberglass could be created. The benefits of using composite tooling for this application are threefold. First, it is a very quick way to create a female mold from a male plug. Secondly, if the plug is properly finished the mold will typically only require minimal preparation before it is ready for layups. Finally, fiberglass and carbon fiber have similar coefficients of thermal expansion. This is important if the final part is to be cured in an autoclave because it ensures that the finished part has the same dimensions as

the plug used to create the mold. The mold was made by laying up 6oz tooling fiberglass on the plug with West Systems 105 marine resin and 206 slow hardener. After curing at ambient temperature for two days, the mold was post-cured in the autoclave at 140° for two hours.

The final version of the seat was made up of 12 plies of 3k 4HS carbon fiber weave, with four additional plies reinforcing the mounting flat. The layup schedule was [45/0/45/0/45/0]s, determined based on past experience fabricating parts out of similar material.

Like the seat, the springs must be made in a female mold in order to produce a controlled surface on the outside of the part. Therefore the first step is the manufacture of a plug. In this case ½ inch Medium Density Fiberboard was selected to form the structure of the plug. The MDF was cut to shape by hand and glued together, then coated with several layers of gel coat to seal the surface. After sanding the part surface to a class A finish, the plug was ready to make the mold.

For the spring mold, the layup consisted of 14 plies of 4oz boat and tooling fiberglass sandwiched between four plies of unidirectional carbon tape. This layup was used to create a rigid mold with a coefficient of thermal expansion similar to carbon fiber at a relatively low cost.

The springs themselves were manufactured using conventional wet-layup technique. Strips of carbon tape were wet out with resin, and then excess resin was squeegeed out resulting in plies of what was essentially room-temp cure prepreg. These plies could then be laid up in the mold, vacuum bagged, and cured overnight at ambient temperature followed by a 2 hour post cure in an oven. The fully assembled custom seat and springs are shown in Figure 19.

Figure 19. Assembled sit ski

Chapter 6. Testing

Before conducting subjective testing of the performance of the new sit ski, the stiffness of the springs was measured in a laboratory. The spring was fixtured in the Instron so that the bottom arm remained horizontal throughout its length, while compressive force was applied to the upper arm centered 4 inches from the arc.

Figure 20. Diagram of instron test setup

The Instron was set to compress the spring 0.375 inches at a rate of 0.003 inches per second. After some initial settling in, the spring exhibits a nearly constant spring rate of about 500 lb/in. The settling in period means that the calculated and experimental deflection curves are offset, but the spring rate curves match up almost exactly (refer to Figure 21).

Figure 21. Comparison of theoretical and experimental deflection

The spring rate can be calculated by determining the slope of the force/deflection curve. Due to the high level of noise in the raw data, the instantaneous slope fluctuates too much to represent the spring rate. An averaged slope over a range of data results in the spring rate curve shown in Figure 22. The equation used to determine the spring rate is of the form

$$d_n = \frac{F_{n-50} - F_{n+50}}{\Delta_{n-50} - \Delta_{n+50}}$$

Where d is the slope at a data point n , F is the force, and delta is the deflection.

Figure 22. Comparison of theoretical and experimental spring rates

Chapter 7. Conclusions

This thesis was just one part of a series of projects related to adaptive cross country skiing. There are recreational adjustable sit skis developed in senior projects in the Mechanical Engineering Department, as well as biomechanical analysis being conducted through the Kinesiology Department. The original carbon-sprung racing sit ski was a senior project in 2007-2008. Therefore it is important to look at the results of this project in the context of being a component of a larger effort rather than a one-off racing prototype.

This project resulted in the successful testing of the upright body position for a top-level athlete, as well as the development of a simple and durable binding attachment system. The upright position allows the athlete to use more core muscle strength and should provide a competitive advantage through its more powerful poling motion. An additional product of this project was the development of the software necessary to conduct stand-alone data acquisition with the National Instruments CompactRIO embedded controller. The relevant VI's are applicable to any high-frequency (2-50kS/s) load cell application, and can easily be adapted to accommodate other types of sensors.

Based on the test session with the aluminum frame, it is clear that there is a maximum typical force the athlete applies to the frame while sprinting on a straight. Therefore a frame that is rigid below that force and compliant above that force could still be used to gain some advantage by carving rather than skidding around corners. Furthermore, it appears that a buckling column type leaf spring could be used to generate a digressive spring rate if the deflection occurred primarily in the arc. I would recommend using a non-continuous layup with added reinforcement plies at the junction between the arm and the arc and fewer plies in the flexible portion of the spring. While Castigliano's theorem is a good starting point for analysis of this type of stress analysis, it would be beneficial to go into further depth with an additional method such as finite element analysis.

Bibliography

1282923. REI, Inc. 19 August 2009
<<http://media.rei.com/media/v/1282923.jpg>>.

Agarwal, Bhagwan D., and Broutman, Lawrence J. *Analysis and Performance of Fiber Composites*. New York: Wiley-Interscience, 1990.

The Composite Materials Handbook – MIL-17. Pennsylvania: Technomic, 1997.

NI 923x Continuous DMA.vi. Computer software. Vers. 8.6. Austin, TX: National Instruments, 2008.

Saibene, F., Cortili, G., Roi, G., and Colombini, A. "The Energy Cost of Level Cross-Country Skiing and the Effect of the Friction of the Ski" *European Journal of Applied Physiology* Vol. 58 1989.

Dally, James W., and Riley, William F. *Experimental Stress Analysis*. Knoxville: College House Enterprises, 2005.

NI 9237 Getting Started.vi. Computer software. Vers. 8.6. Austin, TX: National Instruments, 2008.

Lynch, Tennyson. *ManneQuinPro User Manual Competitive Sit Ski Application*, 2008

Measuring Strain With Strain Gages. Austin, TX: National Instruments, 2009.

Shigley, Joseph E., Mischke, Charles R., and Budynas, Richard G. *Mechanical Engineering Design*, 7th edition. New York, McGraw-Hill, 2004.

Griffin, Jr., O. Hayden, Johnson, Eric R., and Sensmeier, Mark D. "Nonlinear Response of Graphite Epoxy Wide Columns subject to Eccentric Load" *Journal of Applied Mechanics* Vol. 60 March 1993.

shuntcaloffset.vi. Computer software. Vers. 8.6. Austin, TX: National Instruments, 2008.

Sollner, George, and Molitor, Mike. "Unique Sequences in Composite Tooling." *Composites Applications*. 1989

Urkzsvwk58831. National Instruments. 9 June 2009
<<http://www.ni.com/cms/images/vzone/tut/urkzsvwk58831.jpg>>.

Appendix A Part Drawings

DWG NAME: PLAN B FRAME 14 NOV 2008
 DRAWN: D. ATKINSON
 ALL DIMENSIONS IN INCHES

NOTE:
 -BRACKET APPEARS FOUR TIMES IN WELDMENT
 -OUTSIDE CORNERS DO NOT NEED TO BE SHARP, WATERJET OK

Appendix B Labview Code

Appendix C Experimental Data Processing Code

```
%CompactRIO data processor
%15 May 2009
%Drew Atkinson

clc

Offset1 = -0.000527916163333295; %mV
Offset2 = 0.000746216632877048; %mV
Offset3 = -1.783400000000000e-04; %mV

Cal1 = 1446312; %lb/mV
Cal2 = 54031; %lb/mV
Cal3 = -500000; %lb/mV
DataRate = 2000; %kS/s

Results = zeros(length(Run3_xc), 4);
n = length(Run3_xc);

for i = 1:n;
    Results(i,1) = i/DataRate;
    Results(i,2) = (Run3_xc(i,1) - Offset1)* Cal1;
    Results(i,3) = (Run3_xc(i,2) - Offset2)* Cal2;
    Results(i,4) = Results(i,2) + Results(i,3);
    Results(i,5) = (Run3_xc(i,3) - Offset3)*Cal3;
end

% %-----2 Channel-----
% plot(Results(:,1), Results(:,2)) %, Results(:,1), Results(:,3))
% legend('Front' %, 'Back');
% title('Calibration')
% xlabel('Load (lbs)')
% ylabel('Time (s)')

% % -----2 Channel with summation-----
% subplot(2,1,1); plot(Results(:,1), Results(:,2), Results(:,1),
Results(:,3))
% legend('Front', 'Back');
% subplot(2,1,2); plot(Results(:,1), Results(:,4))
% legend('Summation');

% -----Working Pole-----
subplot(2,1,1);
plot(Results(:,1), Results(:,2), Results(:,1), Results(:,3),
Results(:,1), Results(:,5))
legend('Front', 'Back', 'Pole');
subplot(2,1,2); plot(Results(:,1), Results(:,4), Results(:,1),
Results(:,5))
legend('Summation', 'Pole');

%-----Only summation and pole-----
% plot(Results(:,1), Results(:,4), Results(:,1), Results(:,5))
```

```
% legend('Summation', 'Pole');
```

Appendix D Experimental Data

Appendix E Stress Analysis Code

[illegible]

```

% size command to get number of plies
n = size(layup,1)

%-----TEMPURATURES-----
Tcure = 140           % cure temperature
T = 30.0             % operating temperature
dT = T-Tcure;        % change in temperature

%-----MATERIAL PROPERTIES-----

      %E1      E2      v12      G12      a11      a22
E = [20.0e6    1.4e6    .30     .93e6    -.5e-6    15.0e-6; % AS4/3501-6
     .241e6    .549e6    .03     .362e6     0      0 ;      % core for
final
     15e6      15e6      .30     .93e6    -.5e-6   -.5e-6;]; % T300 4HS
Fabric

%intitialize the ply distance and ABD matrices

Ntherm = [0;0;0];
Mtherm = [0;0;0];
h = zeros(n+1,1);
A = zeros(3);
B = zeros(3);
D = zeros(3);

% Form R matrix which relates engineering to tensor strain
R = [1  0  0;
     0  1  0;
     0  0  2];

% find the total thickness
total = sum(layup,1);
thick = total(1,2)

% locate the bottom of the first ply
h(1) = -thick/2.;
imax = n + 1;

%loop for rest of the ply distances from midsurf
for i = 2 : imax ;
    h(i) = h(i-1) + layup(i-1,2);
end

%loop over each ply to integrate the ABD matrices
for i = 1:n;

    %ply material ID
    mi=layup(i,3);
    v21 = E(mi,2)*E(mi,3)/E(mi,1);
    d = 1 - E(mi,3)*v21;

```

```

%Q12 matrix
Q = [E(mi,1)/d      v21*E(mi,1)/d      0;
      E(mi,3)*E(mi,2)/d  E(mi,2)/d      0;
      0              0              E(mi,4)];

%ply angle in radians
a1=layup(i,1)*pi/180;

%Form transformation matrices T1 for ply
T1 = [(cos(a1))^2      (sin(a1))^2
2*sin(a1)*cos(a1);
      (sin(a1))^2      (cos(a1))^2      -2*sin(a1)*cos(a1);
      -sin(a1)*cos(a1)  sin(a1)*cos(a1)  (cos(a1))^2-(sin(a1))^2];

T2 = R*inv(T1)*inv(R);

%Form Qxy
Qxy = inv(T1)*Q*R*T1*inv(R);

%load alphas into and array
a=[E(mi,5); E(mi,6); 0.0];

%-----XY Thermal Strains-----
axy = inv(T2)*(a*dT);

% build up the laminate stiffness matrices
A = A + Qxy*(h(i+1)-h(i));
B = B + Qxy*(h(i+1)^2-h(i)^2);
D = D + Qxy*(h(i+1)^3-h(i)^3);

%get thermal loads N(x,y) M(x,y)

Ntherm = Ntherm + Qxy*axy*(h(i+1)-h(i));
Mtherm = Mtherm + .5*Qxy*axy*(h(i+1)^2-h(i)^2);

%end of stiffness loop
end

LaminateAlphas= inv(T2)*a;

%change the display format for compliance matrix
format short e

A = 1.0*A;
B = .5*B;
D = (1/3)*D;

K = [A, B;

```

```

    B, D];

%-----MECHANICAL LOADS-----

MechLOAD = [ 800;      %Nx
             0.0;      %Ny
             0.0;      %Ns
             800*Arm;   %Mx
             0.0;      %My
             0.0;]      %Ms

ThermLOAD = [Ntherm;Mtherm];

LOAD = MechLOAD + ThermLOAD;

% Plate compliance
%
C = [inv(K)];

%-----MATERIAL COORDINATE STRAINS-----
---
e = C*LOAD;

e(5);

%----- reduction factor for ultimate (pseudo A-basis use .80)
RF=.80;

% allowable strains reduced to account for ultimate strength after
impact
% transverse prperties assumed same
% load allowable strains into array
%      ELUT      ELUC      ETUT      ETUC      ELTU
ea = [RF*.01      RF*.011      RF*.02      RF*.022      RF*.0251;    % Carbon
      RF*.02      RF*.018      RF*.0067      RF*.031      RF*.0296;
      RF*.01      RF*.011      RF*.01      RF*.011      RF*.0251 ]; % Hexcel 1168

%zero out results array
ERES = zeros(2*n,6); %strain results
SRES = zeros(2*n,6); %stress results

% loop over each ply and calculate strain
for i=1 : n;
    %loop over top and bottom of each ply
    for j=1 : 2;
        % one is bottom two is top for loc
        ply = i;
        loc = j;

```

```

z = h(i-1+j);

% need angles and transform back to principal directions
el= [ e(1)+z*e(4); e(2)+z*e(5); e(3)+z*e(6) ];

%ply material ID
mi=layup(i,3);
v21 = E(mi,2)*E(mi,3)/E(mi,1);
d = 1 - E(mi,3)*v21;

%---Q12 matrix-----
Q = [E(mi,1)/d      v21*E(mi,1)/d      0;
      E(mi,3)*E(mi,2)/d  E(mi,2)/d      0;
      0                0                E(mi,4)];

a1=layup(i,1)*pi/180;

%Form transformation matrices T1 for ply
T1 = [(cos(a1))^2      (sin(a1))^2
      2*sin(a1)*cos(a1);
      (sin(a1))^2      (cos(a1))^2      -2*sin(a1)*cos(a1);
      -sin(a1)*cos(a1)  sin(a1)*cos(a1)  (cos(a1))^2-(sin(a1))^2];

%-----Thermal Strains in Principle Axis-----
a=[E(mi,5); E(mi,6); 0.0];
aLT = a*dT;

% ply strain in principal coords subtracting the thermal strains
ep = (R*T1*inv(R)*el)-aLT;

% ply stress in principal material coords
sp = Q*ep;

%-----MAX Strain criteria-----
-
%failure index now looks at two different materials

% check fiber direction
if ep(1) > 0.0;
    FI = ep(1)/ea(mi,1);
    FIF=FI;
elseif ep(1) < 0.0;
    FI = abs( ep(1) )/ea(mi,2);
    FIF=FI;
end

%chck transverse direction
if ep(2) > 0.0;
    F1 = ep(2)/ea(mi,3);
elseif ep(2) < 0.0;
    F1 = abs( ep(2) )/ea(mi,4);
end
%
```

```

    if F1 > FI;
        FI = F1;
    end
%
%
% check shear
    F1 = abs( ep(3) )/ea(mi,5);
    if F1 > FI ;
        FIe = F1;
    elseif F1 < FI;
        FIe = FI;
    end

% FIF is failure index on fiber failure
% FIe is the lowest failure index which could be fiber, transverse or
% shear

%load the results array
    % strain
    ERES(2*i+j-2,1)=layup(i); % ply angle
    ERES(2*i+j-2,2)=ep(1); % strain in ply 1 direction
    ERES(2*i+j-2,3)=ep(2); % strain in ply 2 direction
    ERES(2*i+j-2,4)=ep(3); % strain in ply 12 or shear strain
    ERES(2*i+j-2,5)=FIe; % lowest failure index
    ERES(2*i+j-2,6)=FIF; % failure index on fiber

    %stress now, note failure index is based on max strain and just
repeated
    %here now with the stresses
    SRES(2*i+j-2,1)=layup(i);
    SRES(2*i+j-2,2)=sp(1);
    SRES(2*i+j-2,3)=sp(2);
    SRES(2*i+j-2,4)=sp(3);
    SRES(2*i+j-2,5)=FIe;
    SRES(2*i+j-2,6)=FIF;

end
%
end

Inducedloads= K* [1.3897e-002;
                  -1.1452e-002;
                  -4.3797e-018;
                   000;
                  -1.8697e+000;
                  -7.7288e-016;]

ERES=ERES*1
SRES=SRES*1

d= inv(D);

```

```

diary off

%----- 3D PLOT -----

%plate W
L=20;

%Constants of integration
C2 = -(L/2)*e(4);
C3 = -(L/2)*e(5);

% create X and Y over the domain of the function
[X,Y] = meshgrid(0:L,0:L);

% this is the integrated assumed constant curvatures which result in w
% displacemnt in the z direction.
for i = 1:L+1;
    for j = 1:L+1;
        w(i,j) = .5*(e(4)*X(i,j)^2+e(5)*Y(i,j)^2+e(6)*X(i,j)*Y(i,j))...;
        + C2*X(i,j) + C3*Y(i,j);
    end
end

%w
%w(1,L)

mesh(w, 'edgecolor', 'red')

surf(X,Y,w,'FaceColor','red','EdgeColor','none')
camlight left; lighting phong
surf(X,Y,w)
%colormap hsv
%colorbar

xlabel('x (units)'), ylabel('y (units)'), zlabel('z (units)') ;

maxw=max(10);
minw=min(10);

%-----Spring Stuff-----
Force = [5:5:225];
Deflection = zeros(length(Force),1);
Spring_Rate = zeros(length(Force),1);

%Geometry-----

```



```

Ri = Rc - thick/2;
Ro = Rc + thick/2;
Rn = thick/log(Ro/Ri);
ecc = Rc - Rn;
h_axis = Ho - thick;

EA = width*A(1,1)
EI = width/d(1,1)

for i = 1:length(Force);

    %Limits of Integration-----
    theta = 2*asin(h_axis/(2*Rn));
    Angle1 = -theta/2;
    Angle2 = theta/2;

    if ((h_axis/(2*Rn))>1);
        cant_do=(h_axis/(2*Rn))
        disp('impossible geometry');
        break
    end

    bending = @(x) Force(i)*((Arm+Rc*(cos(x)-
cos(Angle1))).^2)/(EA*ecc);
    U1 = quadl(bending,Angle1,Angle2); %Bending in Arc-----
    U2 = (Force(i)*Rn)/(EA)*quadl(@sin_2,Angle1,Angle2); %Normal
Force-----
    U3 = (2*Force(i)*Rn)/(EA)*quadl(@sin_2,Angle1,Angle2);
    %Resultant Moment-----
    U4 = (Force(i)^2*Arm^2)/(6*EI); %Bending in Arm-----

    %Approx_Deflection(i) = (pi*F(i)*Rn^3)/(2*E*I);
    Deflection(i) = U1+U2+U3+U4; %Summation of Castigliano's-----

end

for i = 1
    Spring_Rate(i) = (Force(i)-Force(i+1))/(Deflection(i)-
Deflection(i+1));
end
for i = 2:length(Force)-1
    Spring_Rate(i) = (Force(i-1)-Force(i+1))/(Deflection(i-1)-
Deflection(i+1));
end
for i = length(Force)
    Spring_Rate(i) = (Force(i)-Force(i-1))/(Deflection(i)-Deflection(i-
1));
end

Rawdata = dlmread('Instron9.txt');

n = length(Rawdata);

```

```

Calibrated = zeros(n,3);

Offset = Rawdata(1,2);

for i = 1:n;
    Calibrated(i,1) = -Rawdata(i,2) + Offset;
    Calibrated(i,2) = abs(Rawdata(i,1));
end

for i = 1:50
    Calibrated(i,3) = (Calibrated(i,2)-
Calibrated(i+50,2))/(Calibrated(i,1)-Calibrated(i+50,1));
end
for i = 51:n-51
    Calibrated(i,3) = (Calibrated(i-50,2)-
Calibrated(i+50,2))/(Calibrated(i-50,1)-Calibrated(i+50,1));
end
for i = n-50:n
    Calibrated(i,3) = (Calibrated(i,2)-Calibrated(i-
100,2))/(Calibrated(i,1)-Calibrated(i-100,1));
end

figure(2)
plot(Force, Spring_Rate, 'r') %, Calibrated(:,2), Calibrated(:,3), 'b')
title('Spring Rate')
%legend('Calculated', 'Experimental')
ylabel('Spring Rate (lb/in)')
xlabel('Force (lbs)')

figure(3)
plot(Force, Deflection, 'r') %, Calibrated(:,2), Calibrated(:,1))
title('Deflection')
%legend('Calculated', 'Experimental')
ylabel('Deflection (in)')
xlabel('Force (lbs)')

```

Appendix F Stress Analysis Plot Tool Code

```
%Leaf Spring Design Tool Plotter
%Drew Atkinson
%18 February 2009

clc

%Curve 1
Radius1 = 17;
Length1 = 11;
Thickness1 = 10;
width1 = 17;

%Curve 2
Radius2 = 17;
Length2 = 11;
Thickness2 = 24;
width2 = 8;

%Curve 3
Radius3 = 17;
Length3 = 11;
Thickness3 = 36;
width3 = 4;

Graphs = zeros(4,3);

Graphs = [1, Rc(Radius1), L(Length1), t(Thickness1), w(width1);
          2, Rc(Radius2), L(Length2), t(Thickness2), w(width2);
          3, Rc(Radius3), L(Length3), t(Thickness3), w(width3);
          4, length(Rc), length(L), length(t), length(w)];

format short

disp('          Radius      Length    Thickness  Width')
disp(Graphs)

I1 = (1/12)*w(width1)*((t(Thickness1))^3)
I2 = (1/12)*w(width2)*((t(Thickness2))^3)
I3 = (1/12)*w(width3)*((t(Thickness3))^3)

A1 = w(width1)*t(Thickness1)
A2 = w(width2)*t(Thickness2)
A3 = w(width3)*t(Thickness3)

% subplot(2,1,1),
plot(F, Spring_Rate(:, Radius1, Length1, Thickness1), 'b'), title('Spring
Rate')
% hold on
```

```

% subplot(2,1,2), plot(F,Deflection(:,Radius1,Length1,Thickness1),'b'),
title('Deflection')
% hold on
% subplot(2,1,1), plot(F,Spring_Rate(:,Radius2,Length2,Thickness2),'y')
% subplot(2,1,2), plot(F,Deflection(:,Radius2,Length2,Thickness2),'y')
% subplot(2,1,1), plot(F,Spring_Rate(:,Radius3,Length3,Thickness3),'r')
% subplot(2,1,2), plot(F,Deflection(:,Radius3,Length3,Thickness3),'r')
% hold off

figure (1)
plot(F,Spring_Rate(:, Radius1, Length1, Thickness1,
width1),'b'),title('Spring Rate')
hold on
plot(F,Spring_Rate(:, Radius2, Length2, Thickness2, width2),'g')
plot(F,Spring_Rate(:, Radius3, Length3, Thickness3, width3),'r')
legend('I = 0.0155 in^4' , 'I = 0.0370 in^4', 'I = 0.0633 in^4')
xlabel('Applied Load (lbs)')
ylabel('Spring Rate (lbs/in)')
hold off

figure (2)
plot(F,Deflection(:, Radius1, Length1, Thickness1, width1),'b'),
title('Deflection')
hold on
plot(F,Deflection(:, Radius2, Length2, Thickness2, width2),'g')
plot(F,Deflection(:, Radius3, Length3, Thickness3, width3),'r')
legend('I = 0.0155 in^4' , 'I = 0.0370 in^4', 'I = 0.0633 in^4')
xlabel('Applied Load (lbs)')
ylabel('Deflection (in)')
hold off

```

Appendix G Instron Test Data

3.662	0.575	493.800
3.052	0.575	494.000
3.052	0.575	494.200
3.052	0.575	494.400
3.662	0.575	494.600
2.441	0.575	494.800
3.052	0.575	495.000
3.052	0.575	495.200
2.441	0.575	495.400
3.052	0.575	495.600
2.441	0.574	495.800
3.052	0.572	496.000
3.052	0.571	496.200
3.052	0.570	496.400
3.052	0.569	496.600
-1.221	0.568	496.800
0.000	0.566	497.000
0.610	0.565	497.200
0.000	0.563	497.400
0.000	0.563	497.600
0.000	0.561	497.800
-4.883	0.560	498.000
0.000	0.559	498.200
-3.052	0.558	498.400
-3.052	0.557	498.600
-3.052	0.556	498.800
-3.052	0.555	499.000
-3.052	0.553	499.200
-3.662	0.552	499.400
-3.052	0.551	499.600
-6.104	0.549	499.800
-3.052	0.548	500.000
-6.714	0.547	500.200
-6.104	0.546	500.400
-6.714	0.545	500.600
-6.104	0.544	500.800
-7.935	0.542	501.000
-9.155	0.541	501.200
-7.935	0.540	501.400
-8.545	0.539	501.600
-7.935	0.538	501.800
-8.545	0.536	502.000
-9.155	0.535	502.200
-11.597	0.534	502.400
-10.986	0.533	502.600
-12.207	0.532	502.800
-11.597	0.530	503.000
-11.597	0.529	503.200
-11.597	0.528	503.400
-13.428	0.527	503.600
-13.428	0.525	503.800
-13.428	0.524	504.000
-13.428	0.523	504.200

-14.038	0.522	504.400
-17.090	0.521	504.600
-17.090	0.520	504.800
-17.090	0.518	505.000
-16.479	0.517	505.200
-17.090	0.516	505.400
-17.090	0.515	505.600
-19.531	0.513	505.800
-17.090	0.512	506.000
-20.142	0.511	506.200
-19.531	0.510	506.400
-20.142	0.509	506.600
-18.921	0.507	506.800
-22.583	0.506	507.000
-22.583	0.505	507.200
-21.973	0.504	507.400
-23.193	0.503	507.600
-22.583	0.501	507.800
-25.024	0.500	508.000
-25.635	0.499	508.200
-25.024	0.498	508.400
-25.024	0.497	508.600
-25.024	0.496	508.800
-25.635	0.495	509.000
-24.414	0.493	509.200
-28.076	0.492	509.400
-27.466	0.491	509.600
-28.076	0.489	509.800
-31.128	0.489	510.000
-31.128	0.487	510.200
-30.518	0.486	510.400
-30.518	0.485	510.600
-30.518	0.484	510.800
-31.128	0.482	511.000
-30.518	0.481	511.200
-33.569	0.480	511.400
-33.569	0.479	511.600
-33.569	0.478	511.800
-33.569	0.476	512.000
-33.569	0.475	512.200
-36.621	0.474	512.400
-36.621	0.473	512.600
-36.621	0.472	512.800
-36.011	0.470	513.000
-35.400	0.469	513.200
-39.673	0.468	513.400
-39.673	0.467	513.600
-39.673	0.466	513.800
-39.673	0.464	514.000
-39.062	0.463	514.200
-42.725	0.462	514.400
-42.725	0.461	514.600
-42.725	0.460	514.800
-42.114	0.458	515.000
-42.114	0.457	515.200
-45.776	0.456	515.400
-45.776	0.455	515.600

-45.776	0.454	515.800
-46.387	0.452	516.000
-48.828	0.451	516.200
-48.218	0.450	516.400
-48.828	0.449	516.600
-48.828	0.448	516.800
-48.218	0.446	517.000
-51.270	0.445	517.200
-48.218	0.444	517.400
-51.270	0.443	517.600
-50.659	0.442	517.800
-51.880	0.441	518.000
-50.659	0.439	518.200
-54.321	0.438	518.400
-54.321	0.437	518.600
-53.711	0.436	518.800
-53.711	0.435	519.000
-53.711	0.433	519.200
-56.763	0.432	519.400
-57.373	0.431	519.600
-56.763	0.430	519.800
-57.373	0.429	520.000
-57.373	0.428	520.200
-59.814	0.426	520.400
-59.814	0.425	520.600
-59.204	0.424	520.800
-59.814	0.423	521.000
-63.477	0.421	521.200
-62.866	0.420	521.400
-62.866	0.419	521.600
-62.866	0.418	521.800
-63.477	0.417	522.000
-63.477	0.415	522.200
-65.918	0.414	522.400
-65.918	0.413	522.600
-66.528	0.412	522.800
-65.918	0.410	523.000
-66.528	0.409	523.200
-68.359	0.408	523.400
-68.970	0.407	523.600
-68.970	0.406	523.800
-68.970	0.405	524.000
-71.411	0.404	524.200
-72.021	0.402	524.400
-72.021	0.401	524.600
-72.632	0.400	524.800
-72.021	0.399	525.000
-74.463	0.397	525.200
-74.463	0.396	525.400
-75.073	0.395	525.600
-75.073	0.394	525.800
-74.463	0.393	526.000
-76.904	0.391	526.200
-77.515	0.390	526.400
-78.125	0.389	526.600
-76.904	0.388	526.800
-81.177	0.386	527.000

-81.177	0.385	527.200
-80.566	0.384	527.400
-81.787	0.383	527.600
-80.566	0.382	527.800
-83.618	0.381	528.000
-83.618	0.379	528.200
-83.618	0.378	528.400
-83.618	0.377	528.600
-83.618	0.376	528.800
-87.280	0.375	529.000
-86.670	0.373	529.200
-86.670	0.372	529.400
-87.280	0.371	529.600
-86.670	0.370	529.800
-89.722	0.369	530.000
-90.332	0.367	530.200
-89.722	0.366	530.400
-89.722	0.365	530.600
-89.722	0.364	530.800
-92.773	0.362	531.000
-93.384	0.361	531.200
-93.384	0.360	531.400
-95.825	0.359	531.600
-93.384	0.358	531.800
-95.825	0.357	532.000
-95.215	0.355	532.200
-95.825	0.354	532.400
-95.825	0.353	532.600
-96.436	0.352	532.800
-98.267	0.351	533.000
-99.487	0.349	533.200
-98.267	0.348	533.400
-98.267	0.347	533.600
-101.929	0.346	533.800
-101.318	0.345	534.000
-101.318	0.343	534.200
-101.318	0.342	534.400
-104.370	0.341	534.600
-104.370	0.340	534.800
-104.370	0.339	535.000
-104.370	0.337	535.200
-104.370	0.336	535.400
-106.201	0.335	535.600
-106.812	0.334	535.800
-107.422	0.333	536.000
-107.422	0.331	536.200
-109.863	0.330	536.400
-109.863	0.329	536.600
-109.863	0.328	536.800
-110.474	0.327	537.000
-109.863	0.326	537.200
-112.915	0.324	537.400
-114.136	0.323	537.600
-112.915	0.322	537.800
-112.915	0.321	538.000
-115.967	0.319	538.200
-115.967	0.318	538.400

-115.356	0.317	538.600
-115.967	0.316	538.800
-115.967	0.314	539.000
-119.019	0.313	539.200
-118.408	0.312	539.400
-118.408	0.311	539.600
-118.408	0.310	539.800
-119.019	0.309	540.000
-121.460	0.307	540.200
-120.850	0.306	540.400
-121.460	0.305	540.600
-121.460	0.304	540.800
-120.850	0.303	541.000
-123.901	0.301	541.200
-123.901	0.300	541.400
-123.901	0.299	541.600
-124.512	0.298	541.800
-124.512	0.297	542.000
-126.953	0.296	542.200
-126.953	0.294	542.400
-127.563	0.293	542.600
-130.005	0.292	542.800
-127.563	0.291	543.000
-130.005	0.290	543.200
-130.005	0.288	543.400
-130.005	0.287	543.600
-130.005	0.286	543.800
-133.057	0.285	544.000
-133.667	0.284	544.200
-133.057	0.282	544.400
-133.057	0.281	544.600
-133.057	0.280	544.800
-136.108	0.279	545.000
-136.719	0.278	545.200
-136.108	0.277	545.400
-136.108	0.276	545.600
-136.108	0.274	545.800
-139.771	0.273	546.000
-139.160	0.272	546.200
-138.550	0.270	546.400
-139.160	0.269	546.600
-142.212	0.268	546.800
-142.212	0.267	547.000
-141.602	0.266	547.200
-142.212	0.264	547.400
-145.264	0.263	547.600
-144.043	0.262	547.800
-144.653	0.261	548.000
-145.264	0.259	548.200
-144.653	0.258	548.400
-145.874	0.257	548.600
-147.705	0.256	548.800
-147.705	0.255	549.000
-147.095	0.254	549.200
-150.146	0.252	549.400
-150.757	0.251	549.600
-150.757	0.250	549.800

-150.757	0.249	550.000
-150.757	0.247	550.200
-151.367	0.246	550.400
-153.809	0.245	550.600
-153.198	0.244	550.800
-153.198	0.243	551.000
-153.809	0.242	551.200
-153.809	0.240	551.400
-156.250	0.239	551.600
-156.860	0.238	551.800
-156.250	0.237	552.000
-156.860	0.236	552.200
-159.302	0.234	552.400
-159.302	0.233	552.600
-159.912	0.232	552.800
-159.302	0.231	553.000
-162.964	0.230	553.200
-162.354	0.228	553.400
-162.354	0.227	553.600
-162.354	0.226	553.800
-162.964	0.225	554.000
-165.405	0.223	554.200
-165.405	0.223	554.400
-166.016	0.221	554.600
-166.016	0.220	554.800
-165.405	0.219	555.000
-168.457	0.218	555.200
-168.457	0.216	555.400
-168.457	0.215	555.600
-167.847	0.214	555.800
-167.847	0.213	556.000
-170.898	0.212	556.200
-170.898	0.210	556.400
-171.509	0.209	556.600
-171.509	0.208	556.800
-173.950	0.207	557.000
-173.950	0.206	557.200
-174.561	0.205	557.400
-173.950	0.203	557.600
-173.950	0.202	557.800
-177.612	0.201	558.000
-177.612	0.200	558.200
-177.002	0.200	558.400
-173.950	0.201	558.600
-174.561	0.202	558.800
-177.002	0.201	559.000
-173.950	0.200	559.200
-173.950	0.200	559.400
-173.950	0.201	559.600
-174.561	0.201	559.800
-174.561	0.201	560.000
-174.561	0.201	560.200
-173.950	0.200	560.400
-173.340	0.200	560.600
-174.561	0.200	560.800
-173.950	0.200	561.000
-174.561	0.200	561.200

-174.561	0.200	561.400
-173.950	0.201	561.600
-173.950	0.201	561.800
-173.950	0.201	562.000
-173.950	0.201	562.200
-174.561	0.201	562.400
-175.171	0.201	562.600
-173.950	0.201	562.800
-173.950	0.201	563.000
-173.950	0.200	563.200
-174.561	0.200	563.400
-173.950	0.200	563.600
-174.561	0.200	563.800
-174.561	0.200	564.000

Appendix H Instron Test Data Analysis Code

```
%Instron Plot Tool
%Drew Atkinson
%07 June 2009

clc

n = length(Instron9)
Calibrated = zeros(n,3);

Offset = Instron9(1,2)

for i = 1:n;
    Calibrated(i,1) = -Rawdata(i,2) + Offset;
    Calibrated(i,2) = abs(Rawdata(i,1));
end

for i = 1:50
    Calibrated(i,3) = (Calibrated(i,2)-
    Calibrated(i+50,2))/(Calibrated(i,1)-Calibrated(i+50,1));
end
for i = 51:n-51
    Calibrated(i,3) = (Calibrated(i-50,2)-
    Calibrated(i+50,2))/(Calibrated(i-50,1)-Calibrated(i+50,1));
end
for i = n-50:n
    Calibrated(i,3) = (Calibrated(i,2)-Calibrated(i-
    100,2))/(Calibrated(i,1)-Calibrated(i-100,1));
end

figure(1)
plot(Calibrated(:,1), Calibrated(:,2))
legend('instron')
xlabel('Deflection (in)')
ylabel('Applied Force (lbs)')

figure(2)
plot(Calibrated(:,2),Calibrated(:,3))
legend('instron')
xlabel('Applied Force (lbs)')
ylabel('Spring Rate (lbs/in)')
```