Experimental Study and Modeling of the $g_m$-I Dependence of Long-Channel MOSFETs

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Abstract—this paper describes an experimental study and modeling of the current-transconductance dependence of the ALD1106 and ALD1107 arrays. The study tests the hypothesis that the I-$g_m$ dependence of these 7.8 $\mu$m MOSFETs conforms to the Advanced Compact Model (ACM). Results from performed measurements, however, do not support this expectation. Despite the relatively large length, both ALD1106 and ALD1107 show sufficiently pronounced 'short-channel' effects to render the ACM inadequate. As a byproduct of this effort, we confirmed the modified ACM equation. With an $m$ factor of approximately 0.6, it captures the I-$g_m$ dependence quite well. The paper also introduces several formulas and procedures for I-$g_m$ model extraction and tuning. These are not specific to the ALD transistor family and can be applied to MOSFETs with different physical size and electrical performance.

Index Terms—analog; MOSFET; transconductance; model; ACM

I. INTRODUCTION

In electronic design, transistors find use in multiple circuits. Digital circuits typically use transistors as switches, while analog circuits use them as amplifiers. As an amplifier, the voltage across the gate-source terminals of a MOSFET device changes the current flowing through its source-drain terminals. The transistor is biased at a particular DC operating point to maximize potential small-signal gain. The transconductance, $g_m$, is the slope of the transfer characteristic of a transistor evaluated at the operating point. For a MOSFET, it is captured as follows:

$$g_m = \frac{dI_D}{dV_{GS}} \bigg|_{o.p.}$$

(1)

For a given $g_m$, the voltage gain of a circuit is found by multiplying $g_m$ with the output resistance of the stage.

The Advanced Compact Model (ACM) models the I-$g_m$ relations of a MOSFET and is valid for long-channel devices. It provides a continuous function between sub-threshold and strong inversion operating regions. According to the ACM [1], the I-$g_m$ expression for a MOSFET in saturation is:

$$g_m = \frac{2}{1 + \sqrt{1 + I_D/I_S}^m} \times \frac{I_D}{nU_T}$$

(2)

The normalization current, $I_S$, is defined in (3), where $U_T$ is the thermal voltage $kT/q$. The rest of the quantities have their usual meaning.

$$I_S = \frac{1}{2} n\mu C_{ox} \frac{W}{L} U_T^2$$

(3)

When $I_D$ is much smaller than $I_S$, the device operates in weak inversion, where the gate transconductance takes the form:

$$g_m = \frac{I_D}{nU_T}$$

(4)

When $I_D$ is much larger than $I_S$, the device operates in strong inversion, where the transconductance is proportional to the square-root of the drain current. However, short-channel MOSFETs behave differently. When the channel length of the transistor is reduced, the channel carrier velocity reaches a limit due to high horizontal electric field values [2]. This effect is only seen at higher currents in strong inversion and manifests itself in a weaker than square-root $g_m$-I dependence.

According to [3], the impact of strong electric fields can be modeled by replacing the square root in (2) with a large-value exponent $m$. This modified equation, which has not been previously experimentally verified, is (5).

$$g_m \cong \frac{2}{1 + (1 + I_D/I_S)\bar{m}} \times \frac{I_D}{V_{norm}}$$

(5)

Here $V_{norm}$ equals $nU_T$, and $I_S$ takes the place of $I_S$. The modified ACM is attractive because it is simple yet arguably capable of accounting for various levels of 'high-field' effects.

In this work, we study the behavior of two MOSFETs, ALD1106 and ALD1107, with the goal of demonstrating their I-$g_m$ characteristics conform to (2). Expecting a good agreement with the ACM, we learned that capturing the I-$g_m$ relation needed the modified ACM instead.

Section II discusses data collection and data preprocessing. Section III deals with various aspects of model tuning, and Section IV presents the results.

II. I-V DATA COLLECTION AND PREPROCESSING

As stated in the Introduction, we wish to find whether the I-$g_m$ dependence of ALD1106 and ALD1107 obeys (2). This is a valid supposition because both transistors have a channel length of 7.8 $\mu$m, implying a 'long-channel' device [2], [4]. The first step of the process is obtaining reliable I-V data that covers the operating range of the device. This includes weak, moderate, and strong inversion.

Our experimental data consists of more than one hundred fifty I-V points per transistor studied. As shown in Table I, the current spans approximately seven decades, and for the n-channel devices (ALD1106), it ranges from 1 nA to 19 mA. The range for the p-channel devices (ALD1107) is somewhat smaller: 1 nA to 6.5 mA. Different maximum currents were
employed to ensure the gate-source voltage of the FET under test does not exceed the 10.6 V limit defined by the manufacturer [5], [6].

The I-V points are pseudo-logarithmically spaced in the current domain with ten uniformly distributed points within each decade. An exception is the 1-to-100 μA range, where we took 100 as opposed to 20, to better capture transistor transition from weak to strong inversion.

All data were obtained using a diode-connected transistor configuration, where the Keithley-2400 serves as a current source to bias the transistor under test. Fig. 1a and 1b show the test setup. The Keithley supplies the drain current and simultaneously measures the \( V_{GS} \) of the device, so no additional hardware is needed.

Extracting I-V data from a diode-connected device is attractive because the setup is simple and the connection keeps the MOSFET in saturation. The disadvantage of the ‘diode-connection’ is that the change in \( I_D \) is due to changes in both \( V_{GS} \) and \( V_{DS} \). This means that the derivative of the drain current with respect to \( V_{GS} \) yields the sum of \( g_m \) and \( g_{ds} \). Assigning the change to \( g_m \) alone, results in an error. The error, however, is relatively small because the \( g_m \) is typically much larger than \( g_{ds} \). This is notably true for devices meant for analog applications.

There are at least two phenomena that could compromise the validity of the data gathered: thermal heating and noise. The thermal heating, a concern at high currents, was addressed by an automated data collection scheme. A LabView program controlled the pseudo-logarithmic current sweep and collected gate-source voltage measurements, reducing the time the MOSFET spends in a high-current regime. Minimizing heating is essential because thermal effects manifest in a manner similar to high-field effects: the \( I_D \) and the \( g_m \) are reduced through reduction in carrier mobility.

Noise and electromagnetic interference are concerns at lower drain currents, where the device becomes very resistive having \( 1/g_{ds} \) in the 100s of \( \Omega \). For currents below 1 μA, ten samples were taken for each data point for preprocessing. The preprocessing, implemented in MATLAB, would take either the median or mean of these ten samples. The difference between median and mean preprocessing was small. When data was preprocessed using the median of the ten current points, the overall error between experimental and model data was smaller than when processed with the mean. While white noise is a zero-mean stochastic process, the attempt at reducing its impact through an ensemble average was not very effective. This could be from taking only ten samples. With more samples, the mean preprocessing might work better than the median.

### Table I: Number of I-V Points Per Transistor and Their Distribution

<table>
<thead>
<tr>
<th>Number of Points</th>
<th>ALD1106</th>
<th>ALD1107</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-10 nA</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>10-100 nA</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>0.1-1 μA</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>0.1-1 mA</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>Beyond 1 mA</td>
<td>19</td>
<td>13</td>
</tr>
</tbody>
</table>

This section deals with selecting \( V_{norm} \), \( I_{norm} \), and \( m \) in (5) to optimize the fit with experimental data.

The fact that the model is in \( I_{g_m} \) domain, while the experimental data are in I-V domain, complicates the tuning task. To match model and experiment, we perform a ‘domain conversion’. One could convert experimental data from I-V to \( I_{g_m} \) domain to compare transconductance values. Fig. 2 clarifies this approach. Alternatively, as depicted in Fig. 3, one could convert the model-generated \( I_{g_m} \) data to I-V domain and perform a comparison of voltage values. Both approaches are employed in this paper because we want to show that (5) is robust, where its parameters are not particularly sensitive to the tuning strategy used.

#### A. Tuning by Comparing Transconductance Values

Extracting and verifying an \( I_{g_m} \) model from measured I-V data requires differentiation typically approximated with forward or backward differences.

Because this study relies on the accuracy of the first derivative, a formula based on quadratic spline interpolation of the data was used instead. The quadratic spline interpolation takes three points and assumes a quadratic function passes through the three points. The derivative of this quadratic at the center point is assumed to be the first derivative of the nonlinear function. Following this idea, we derived (6).

\[
g_m \cong \frac{I_D(k) - I_D(k-1)}{V_{GS}(k) - V_{GS}(k-1)} + \frac{I_D(k+1) - I_D(k)}{V_{GS}(k-1) - V_{GS}(k)}
\]

Expression (6) generalizes the simpler two-point differences. It uses all three differences one could form between three points and guarantees zero error for both linear and quadratic dependencies.

**Fig. 2: Fitting Algorithm that Reduces Error in \( I_{g_m} \) Domain**

```plaintext
\[
\text{(I-V)\text{Measured}} \rightarrow (I_{g_m})\text{Measured} \rightarrow \text{Model Fit} \rightarrow \text{Error}
\]
```
Average errors relate to meaningful interpretations. As seen in (9) and (10), the length of an N-dimensional vector in RN space is its norm.

\[
V_{GS(k+1)} \equiv V_{GS(k)} + \frac{I_D(k+1) - I_D(k)}{g_m(k+1) - g_m(k)}
\]  

The tuning strategy used in (7) ensures the error is always a dimensionless quantity.

B. Tuning by Comparing \( V_{GS} \) Values

Equation (6) generates I-gm points from the experimental I-V set. As such, it does not offer a direct comparison to measured data. Also, the derivative operator (6) may enhance errors present in the measured data. To circumvent this issue, we also consider the tuning strategy of Fig. 3 - an approach based upon (7). Expression (7) derives from (6) and allows us to produce an I-V set from an I-gm one. It can be thought an integral operation.

\[
V_{GS(k+1)} \equiv V_{GS(k)} + \frac{I_D(k+1) - I_D(k)}{g_m(k+1) - g_m(k)} \tag{7}
\]

The tuning strategy in Fig. 3 provides direct comparison to measured data and offers more intuition on how accurately the model matches the actual measurements.

C. Selecting a Cost Function to Minimize

We calculate the error by taking the absolute value of the difference between each experimental and model value and dividing it by the corresponding experimental value. Depending on the tuning strategy used, the error calculation involves either \( g_m \) values or \( V_{GS} \) values, but the normalization ensures the error is always a dimensionless quantity.

Our measured and model-produced data sets consist of N points (N > 150). Hence, the error could be seen as an N-dimensional vector \( \mathbf{e} \) and the tuning process as an effort to minimize the ‘length’ of the vector. According to [7], the length of an N-dimensional vector in RN space is its norm.

\[
||\mathbf{e}||_p = \left( \sum_{i=1}^{N} e_i^p \right)^{\frac{1}{p}}, \quad p \geq 1 \tag{8}
\]

The most common norms are the grid norm (p=1), the Euclidean norm (p=2), and the infinity norm (p=\( \infty \)). In this study, we choose to work with \( l_1 \) and \( l_\infty \) not only because they represent the two ends of the spectrum but also because they have meaningful interpretations. As seen in (9) and (10), the average error relates to \( l_1 \) whereas the maximum error equals \( l_\infty \).

\[
e_{\text{ave}} = \frac{e_1 + e_2 + \ldots + e_N}{n} \equiv \frac{||\mathbf{e}||_1}{N} \tag{9}
\]

\[
e_{\text{max}} = \max\{e_1, e_2, \ldots, e_N\} \equiv ||\mathbf{e}||_\infty \tag{10}
\]

Minimizing the Euclidean (\( l_2 \)) norm, not pursued here, should perform between the 1-norm and the infinity-norm.

D. Defining the Search Space

To find the optimal model, we test various combinations of \( V_{norm} \), \( I_{norm} \), and \( m \) against experimental data. Reducing the number of cases tried requires establishing a meaningful range for each parameter. The considerations are provided below.

Because the transconductance in deep sub-threshold is given by \( I_D/V_{norm} \), an approximate value for \( V_{norm} \) is easily obtained by finding the \( g_m \) at low currents and taking the ratio \( I_D/g_m \).

In this study, one hundred possible normalization voltages are taken from a ten percent window around the calculated approximate value of \( V_{norm} \).

The normalization current, \( I_{norm} \), delineates weak inversion from strong inversion. Examination of the experimental data for our devices reveals that this transition occurs at current levels below 10 \( \mu \)A. Thus, we took 100 evenly spaced current values between 0 and 10 \( \mu \)A.

Ideally, the optimum parameter search will return an \( m \) value of 0.5, validating (2), but larger values for \( m \) are also likely. As discussed in [3], \( m \) values in the range of 0.7 to 1 are common for sub-0.5 \( \mu \)m MOSFETs. This justifies a search range of 0.5 to 1. We again take 100 \( m \) values evenly spaced between 0.5 and 1.

With each parameter having 100 possible values, the tuning algorithm examines one million possible solutions. The one yielding the lowest error is kept and passed as the model fit output. For each transistor, we have four ‘best’ models corresponding to four possible combinations of two tuning schemes and two cost functions. The differences are discussed in the next section.

IV. RESULTS AND DISCUSSION

A. Performance of Original and Modified ACM

As seen in Table II and III, the best fit is achieved with \( m \) values of approximately 0.6. This is true for both the n-channel and the p-channel devices. The exponent value changes little depending on the tuning strategy and cost function.

Fig. 4 and 5 show that a model with \( m=0.5 \) fits the data poorly. This finding is surprising, considering the devices have a channel length of 7.8 \( \mu \)m.

B. Impact of Cost Function

Fig. 4 and 5 show that minimizing the mean error, i.e. using an \( l_1 \)-based cost function, provides a better fit for lower current data points. This stems from a higher concentration of data points at lower currents that shifts the model fit to optimize lower currents. The higher current points suffer more error. Conversely, reducing the maximum error (\( l_\infty \)) provides a better fit for higher current data points at the expense of lower current points. In absence of outliers, the discrepancies between the two cost functions should diminish if the density of the points is more uniform.
C. Impact of Tuning Domain

Table II and III show the tuning domain in this particular study has little impact upon optimum models found. General trends are intuitive and summarize as follows: optimizing in one domain sacrifices performance in the other domain. However, the differences are insignificant. We note that irrespective of tuning domain used, the $V_{CG}$ error is small. Maximum error never exceeds 3.6%, and average error stays below 0.2%. The match between experimental $g_m$ and modified ACM model typically has maximum error of 20-25% and an average of sub-10%. This performance is superior to the performance of the original ACM, where the maximum error exceeds 150%.

D. Differences Between N and P-Channel Devices

Tables II and III show differences between NMOS and PMOS devices. The devices have similar normalization voltages and m parameters, but their normalization currents are vastly different. ALD1106 has much larger $I_\text{norm}$. This larger $I_\text{norm}$ does not stem from a wider n-channel transistor, as both devices have 138 $\mu$m widths [4]. Referring to (3), the larger normalization current of ALD1106 can be explained by the higher mobility of electrons and the slightly larger $n$ factor of ALD1106. The larger $n$ is inferred from the larger $V_\text{norm}$.

V. CONCLUSION

This study shows the ‘deviation from square-law’ is not an exclusive attribute to sub-micrometer MOSFETs. As seen with the ALD1106 and ALD1107, a device with length of 7.8 $\mu$m could exhibit ‘high-field’ effects sufficient to invalidate the ACM model. More importantly, the experimental results corroborate the argument presented in [3]. Namely, the $I-g_m$ dependence of practical MOSFETs can be captured by a simple modification to the ACM. The introduced exponent m, a ‘catch-all’ parameter, accounts for many phenomena that shape the $I-g_m$ dependence at high currents. These are strong vertical and lateral fields and finite source-side contact resistance among others. As such, the modified ACM offers analog designers an insight into the overall behavior of the device and facilitates biasing decisions.

### TABLE II: MODEL PARAMETERS AND PERFORMANCE OF MODIFIED ACM EXTRACTED FROM ALD1106 MEASUREMENTS

<table>
<thead>
<tr>
<th>Tuning Scheme</th>
<th>$V_\text{norm}$</th>
<th>$I_\text{norm}$</th>
<th>m</th>
<th>$g_m$ Error, %</th>
<th>$V_{CG}$ Error, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g_m$</td>
<td>$g_{m\infty}$</td>
<td>39.7</td>
<td>0.581</td>
<td>24.8</td>
<td>3.9</td>
</tr>
<tr>
<td>$g_m$</td>
<td>$g_{m1}$</td>
<td>40.6</td>
<td>1.35</td>
<td>26.4</td>
<td>9.7</td>
</tr>
<tr>
<td>$V_\text{norm}$</td>
<td>$V_{\infty}$</td>
<td>42.0</td>
<td>1.85</td>
<td>26.4</td>
<td>9.7</td>
</tr>
<tr>
<td>$V_\text{norm}$</td>
<td>$V_{1}$</td>
<td>40.1</td>
<td>1.47</td>
<td>25.9</td>
<td>5.5</td>
</tr>
</tbody>
</table>

### TABLE III: MODEL PARAMETERS AND PERFORMANCE OF MODIFIED ACM EXTRACTED FROM ALD1107 MEASUREMENTS

<table>
<thead>
<tr>
<th>Tuning Scheme</th>
<th>$V_\text{norm}$</th>
<th>$I_\text{norm}$</th>
<th>m</th>
<th>$g_m$ Error, %</th>
<th>$V_{CG}$ Error, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g_m$</td>
<td>$g_{m\infty}$</td>
<td>33.0</td>
<td>0.307</td>
<td>10.6</td>
<td>7.8</td>
</tr>
<tr>
<td>$g_m$</td>
<td>$g_{m1}$</td>
<td>35.0</td>
<td>0.408</td>
<td>18.1</td>
<td>3.0</td>
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<td>$V_\text{norm}$</td>
<td>$V_{\infty}$</td>
<td>38.1</td>
<td>0.509</td>
<td>22.2</td>
<td>3.7</td>
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<td>$V_{1}$</td>
<td>16.8</td>
<td>0.509</td>
<td>19.4</td>
<td>3.3</td>
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REFERENCES


