EVALUATION OF SEISMIC FORCE PROVISIONS FOR ANCILLARY SYSTEMS IN PIERS AND WHARVES CONSIDERING EFFECTS OF NONLINEARITY

By

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ABSTRACT

Much of the prior research on understanding seismic response of secondary systems and development of code provisions was conducted either for nuclear power plant facilities or buildings. Most marine structures differ from nuclear power plant facilities or buildings in that the marine structures can be idealized as one-story building type systems. Furthermore, most previous studies have been limited to linear elastic behavior of both primary and secondary systems.

The objectives of this study are to (1) understand effects of nonlinearity, both in primary and secondary systems, on seismic forces in secondary systems, and (2) evaluate seismic force provisions proposed in MOTEMS that were based on linear elastic studies for secondary system considering effects nonlinearity.

Since primary systems of concern in this investigation are marine structures such as piers, wharves, and marine oil terminals, which can be idealized as single-degree-of-freedom (SDOF) systems, this study utilized a simple model with two degrees of freedom, one representing the marine structure and the other representing the ancillary component. This investigation used the SAC ground motion set consisting of 20 ground motions from 10 sites for 10% probability of exceedance in 50 years for a site in Los Angeles, California.

This investigation first studies the effects of damping in the secondary system on forces in the secondary systems. This investigation led to the conclusion that the damping has negligible effects on force in the secondary system for low values of the ratio of the secondary and primary system periods (say less than 0.5) regardless of the primary system period or ratio of the mass of the secondary and primary systems. For larger values of the period ratio, however, force may be 10% to 35% higher in secondary systems with 2% damping compared to systems with 5% damping. Based on this analysis, 2% damping in the secondary system was used in this investigation.

Most previous investigation on nonlinear SDF systems have been focused on seismic displacement demands. It has been found that displacement of systems with non-zero post-yield stiffness is lower than the corresponding elastic-perfectly-plastic SDF systems. However, the effects of non-zero post-yield stiffness is not clear. Therefore, this investigation next examined the effects of post-yield stiffness on system forces. It was found that very-short period system with strength lower than the strength required for it to remain elastic may experience force which may even be higher than force in the corresponding linear-elastic system. Therefore, it recommended that very-short period (or stiff) systems not be designed for strengths much lower than the elastic-level strength. While post-yield stiffness of primary system is often better known, e.g., nonlinear pushover analysis, such may not always be the case for secondary systems. Therefore, careful consideration must be given to force-based design of stiff (or short-period) secondary systems when post-yield stiffness is not known.

The investigation next focused on the effects of nonlinearity in primary and secondary systems on seismic forces in secondary systems. This investigation led to the following conclusions:

- Nonlinearity in the primary system alone leads to: (1) significant reduction of forces in the secondary systems with $T_p/T_s < 1.5$ because nonlinearity in the primary system results in lower acceleration transmitted to the base of the secondary system, which in turn leads to lower force in the secondary system, (2) minimal reduction of forces for systems with
$T_p/T_n > 1.5$ indicating that force in very flexible secondary system is insensitive to nonlinearity in the primary system, and (3) largest reduction for systems for which $T_p/T_n$ is close to one. These trends are essentially independent of period of the primary system.

- Nonlinearity in the secondary system alone leads to: (1) excessive deformation and force, which may be higher than those in corresponding linear elastic systems, for very-short period secondary system with strength lower than the strength required for it to remain elastic, i.e., $R_p$ higher than 1.0, and (2) these trends are most prominent for lower values of $\mu$ and reduce as $\mu$ increases but are essentially independent of period of the primary system.
- It is recommended that very-short period (or stiff) secondary systems not be designed for $R_p$ higher than 1.0.
- Nonlinearity in both primary and secondary systems generally reduces forces in the secondary system. The exception occurs for very stiff secondary systems, i.e., very low $T_p/T_n$ values, where force in nonlinear secondary system exceeds that in its linear-elastic counterpart. The other trends are similar to those noted for systems with nonlinearity in the secondary systems only.

Finally, this investigation evaluated the MOTEMS seismic force provisions for secondary system considering effects nonlinearity. This work led to the following conclusions and recommendations:

- When both primary and secondary systems are expected to remain within the linear elastic range, the MOTEMS simplified and alternate formula may significantly underestimate the force in the secondary system for the period ratios $0.5 < T_p/T_n < 1.5$. Therefore, it is recommended that engineers avoid systems within this period range.
- When both primary and secondary systems are expected to remain within the linear elastic range, the MOTEMS simplified formula leads to significant overestimation which can exceed 100% for period ratios $T_p/T_n < 0.5$ or $T_p/T_n > 1.5$ but the alternate formula reduces this overestimation and generally provides forces that are very close to those from response history analysis.
- When the secondary systems are expected to responds beyond the linear elastic range, both the simplified and alternate MOTEMS formulas may lead to significant underestimation of forces in the secondary system even for period ratios $T_p/T_n < 0.5$. Therefore, it is recommended that either engineers are cautioned against designing secondary systems in this period range for forces much lower than those required to remain linear elastic.
- The higher forces in nonlinear secondary system with period ratio $T_p/T_n < 0.5$ may occur in secondary systems with non-zero post-yield stiffness. Therefore, engineers are also cautioned to design secondary systems with post-yield stiffness to be as close to zero as possible.
- The MOTEMS alternate formula reduces overestimation or provides very good estimates of forces in the secondary systems compared to the simplified formula.
- The MOTEMS alternate formula is preferable when information on period ratio, $T_p/T_n$, is available.
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INTRODUCTION

Seismic response of nonstructural components has been a subject of interest for several decades now (e.g., Singh, 1975, 1988; Kelly and Sackman, 1978, 1979; Sackman and Kelly, 1979, 1980; Der Kiureghian et al., 1983; Gupta and Tembolkar, 1984; Hernreid and Sackman, 1994; Igusa and Der Kiureghian, 1985a, 1985b; Singh and Suarez, 1986; Asfura and Der Kiureghian, 1986; Suarez and Singh, 1987a, 1987b, 1987c; Aziz and Ghobarah, 1988; Chen and Soong, 1988; Gupta, 1997). Scope of this early research includes understanding seismic behavior of nonstructural components and their interaction with the primary structure in nuclear power structures or developing methods to generate floor spectra which provide the input for design of nonstructural components. The later work (e.g., Soong et al., 1993; Bachman and Drake, 1994; Drake and Bachman, 1996; Villaverde, 1996a; Miranda and Taghavi, 2005a, 2005b; Singh et al., 2006a, 2006b; Taghavi and Miranda, 2008; Fathali and Lizundia, 2011) focused on nonstructural components in building applications and eventually led to development of nonstructural component seismic provisions in ASCE 7-16.

Villaverde (1996b) conducted a state-of-the-art review of earthquake resistant design of secondary system and found that much of previous work focused on linear behavior of both the primary and secondary systems. Furthermore, most previous studies did not consider the possibility of much lower damping in secondary systems compared to primary systems. It was noted by Villaverde (1996b) that yielding is expected to occur in primary system during design-level ground shaking and it may significantly reduce input to the secondary system. Therefore, current procedures developed based on linear behavior of primary system may lead to uneconomical and unrealistic design of secondary system. Lin and Mahin (1985) were among the first researchers to consider effects of nonlinearity in the primary system and developed design floor response spectrum from conventional linear floor spectrum accounting for yielding in the primary system. Schroeder and Bachman (1994) examined the response of linear secondary systems supported on nonlinear building models to develop and verify proposed 1994 NEHRP equations for seismic design of secondary systems. It was concluded that NEHRP equations produce more accurate results than previous codes equations and are still very simple to use. Goel (2017b) examined seismic forces in linear secondary systems supported on nonlinear primary systems. It was found that forces in secondary systems supported on linear elastic primary systems are generally larger, and hence conservative, compared to those supported on corresponding nonlinear systems.

A few investigations have considered effects of nonlinearity both in primary and secondary systems. Toro et al. (1989) and Sewell et al. (1989) also examined seismic response of linear and nonlinear structures and equipment in nuclear power plants but were focused on ductility demands rather than forces on secondary systems. Igusa (1990) examined seismic response of 2 degree-of-freedom nonlinear primary-secondary systems using an equivalent linearization and random vibration theory. It was found that small nonlinearity results in significant reduction of the response. However, the results were found to be “accurate” only for low level of nonlinearity.

Much of the prior work has been focused on seismic response of secondary systems in nuclear power plant facilities or buildings. Recently Goel (2017a, 2018) developed and evaluated equations for seismic forces in secondary systems supported marine structures such as piers, wharves, and marine oil terminals. Since many such marine structures can be idealized as single-degree-of-freedom (SDOF) systems, this study used a simple model with two degrees of freedom, one representing the marine structure and the other representing the ancillary component. It was found that acceleration at the base of the secondary system is approximately equal to spectral acceleration at the fundamental period of the primary system. It also proposed an improved
formula to estimate the amplification of acceleration in the secondary system due to its flexibility when mass and period ratios of the secondary and primary system are known. However, this study was limited to linear elastic behavior of both primary and secondary systems.

The objectives of this study are to (1) understand effects of nonlinearity in primary and secondary systems on seismic forces in secondary systems, and (2) evaluate seismic force provisions for secondary system considering effects nonlinearity. The primary systems of concern in this investigation are marine structures such as piers, wharves, and marine oil terminals.
CODE SEISMIC FORCE PROVISIONS FOR SECONDARY SYSTEMS

Seismic forces in ancillary systems, i.e., secondary systems, supported on piers and wharves, i.e., primary systems, are often computed using provisions in the ASCE 7-16 standard (ASCE, 2017). ASCE 7-16 specifies the following formula to compute seismic force in the secondary system that weigh less than 25% of the combined effective weights of the secondary and primary systems:

\[ F_p = \frac{0.4a_p S_{DS} I_p W_p}{R_p} \left( 1 + 2 \frac{z}{h} \right) \]

where \( S_{DS} \) = short period spectral acceleration, \( a_p \) = component amplification factor, \( I_p \) = component importance factor, \( R_p \) = component response modification factor, \( W_p \) = component operating weight, \( z \) = height in structure of point of attachment of component with respect to the base, and \( h \) = average roof height of structure with respect to the base. The values of \( a_p \) and \( R_p \) for different types of nonstructural components are available in ASCE 7-16. The coefficient \( a_p \) is typically set equal to 1 for rigid components and 2.5 for flexible components. ASCE 7-16 permits lower value of \( a_p \) for flexible components if justified by detailed dynamic analysis. The term \( 0.4S_{DS} \) represents the acceleration at the ground level, \( (1 + 2z/h) \) captures amplification of the acceleration from ground to the point of attachment (or base) of the nonstructural component in the building, and \( a_p \) represents further amplification of the acceleration within the component itself. Since ancillary systems on piers and wharves are typically supported at the deck level, \( (1 + 2z/h) = 3 \) and Equation (1) simplifies to:

\[ F_p = \frac{1.2a_p S_{DS} I_p W_p}{R_p} \]

\[ 0.3S_{DS} I_p W_p \leq F_p \leq 1.6S_{DS} I_p W_p \]

ASCE 7-16 also permit an alternative method to compute \( F_p \) :

\[ F_p = \frac{a_p I_p A_x W_p}{R_p} \]

\[ 0.3S_{DS} I_p W_p \leq F_p \leq 1.6S_{DS} I_p W_p \]

in which \( a_i \) = acceleration at the point of attachment of the component from modal (or response spectrum) method, and \( A_x \) = torsional amplification factor computed from
\[ A_x = \left( \frac{\delta_{\text{max}}}{1.2\delta_{\text{avg}}} \right)^2 \]

where \( \delta_{\text{max}} \) is the maximum displacement, and \( \delta_{\text{avg}} \) is the average of the displacements at the extreme points of the structure (see Figure 1). Equation (3) essentially replaces \( 0.4 S_D S_z (1 + 2z/h) \) with \( a_i \) and considers further amplification because of torsion. Finally, ASCE 7-16 permits estimation of \( a_p \) as from Figure 2 if fundamental period of the structure, \( T_n \), and period of the flexible nonstructural component, \( T_p \), are known.

Recent investigations by the author (Goel, 2017a, 2018) recognized that piers and wharves are generally one-level structures and thus can be idealized as SDOF systems. For such SDOF systems, the acceleration, \( a_i \), in Equation (3) is essentially equal to the spectral acceleration at period equal to fundamental vibration period of the primary system, i.e., pier or wharf, and Equation (3) becomes:
in which $S_A$ is the spectral acceleration computed from the design earthquake spectrum at period equal to fundamental vibration period of the primary system, i.e., pier or wharf. Note that the lower and upper limits on $F_p$ specified in Equation (1) also apply to $F_p$ from Equation (2) or Equation (5). Goel (2018) also proposed a formula for $a_p$ as follows:

$$a_p = \begin{cases} 
1.0 & T_p/T_n \leq 0.1 \\
1.0 + 3 \left( \frac{T_p}{T_n} - 0.1 \right) & 0.1 < T_p/T_n < 0.6 \\
2.5 - 2.5 \left( \frac{T_p}{T_n} - 1.4 \right) & 1.4 < T_p/T_n < 2.0 \\
1.0 & T_p/T_n \geq 2
\end{cases}$$

where $\mu$ is the ratio of weights of the secondary and primary systems, $T_p$ is the vibration period of the secondary system, and $T_n$ is the vibration period of the primary system. Use of equations (5) and (6) to compute seismic forces is not permissible for systems with $\mu < 0.2$ and $0.6 \leq T_p/T_n \leq 1.4$ because such systems may exhibit excessive amplification of acceleration in the secondary system.
SYSTEM CONSIDERED

The coupled primary-secondary system considered in this investigation is shown in Figure 3. The parameters that characterize response of a linear elastic system are: (1) ratio of the mass of the secondary and primary system, \( \mu = m_2 / m_1 \); (2) ratio of the vibration periods, \( T_p / T_n \), where \( T_p = 2\pi \sqrt{m_2 / k_2} \) is the vibration period of the secondary system alone and \( T_n = 2\pi \sqrt{m_1 / k_1} \) is the vibration period of the primary system alone; (3) vibration period of the primary system alone, \( T_n = 2\pi / \sqrt{m_1 k_1} \); (4) damping in the primary system, \( c_1 = 2\zeta_p \sqrt{m_1 k_1} \) in which \( \zeta_p \) is the damping ratio in the primary system; and (5) damping in the secondary system, \( c_2 = 2\zeta_s \sqrt{m_2 k_2} \) in which \( \zeta_s \) is the damping ratio in the secondary system. The previous investigation by Goel (2017a, 2018) used Rayleigh damping, selected identical damping ratio in two modes of the coupled system, and hence characterized the damping by a single damping ratio, \( \zeta \). This investigation, however, considered possibility of a lower damping in the secondary system compared to the primary system and therefore explicitly included different damping parameters, \( \zeta_p \) and \( \zeta_s \), for primary and secondary system, respectively.

![Figure 3. Coupled primary-secondary system.](image)

The nonlinearity in the primary system is represented by \( R_p \), which is defined as the ratio of the strength, \( F_p \), required by the primary system to remain linear elastic and its yield strength, \( F_y \). The force-deformation behavior of the primary system (Figure 4a) is represented by Takeda hysteresis relationship (Takeda et al., 1970) found to be appropriate for piers and wharves (CSLC, 2016; Kowalsky et al., 1994). The nonlinearity in the secondary system is represented by \( R_p \), which is defined as the ratio of the strength, \( F_{po} \), required by the secondary system to remain linear elastic and its yield strength, \( F_{py} \). The force-deformation behavior of the secondary system (Figure 4b) is represented by bilinear hysteresis relationship. The post-yield stiffness of the primary system and secondary systems is represented by the coefficients \( \alpha_p \) and \( \alpha_s \), respectively (Figure 4).
Figure 4. (a) Takeda force-deformation relationship (Takeda et al., 1970) used for primary system, and (b) Bilinear force-deformation relationship for secondary system.

Figure 5. Mathematical model of the coupled primary-secondary system.

The nonlinear response history analysis of the coupled primary-secondary system is implemented in the computer program OpenSees (McKenna and Fenves, 2011) using a model shown in Figure 5. For this purpose, zeroLength elements are used for both structural and damping elements. The nonlinearity of the primary system is considered using Hysteretic and that of the secondary system is considered using Steel01 uniaxialMaterial material properties. The damping in the primary and secondary system is modeled using Viscous uniaxialMaterial material properties with appropriate damping coefficients.

A range of system parameters is considered in this investigation. These include six different values of mass ratios: $\mu = 0.01, 0.05, 0.1, 0.15, 0.2, \text{ and } 0.25$; eight different periods of primary system: $T_n = 0.25, 0.5, 0.75, 1, 1.25, 1.5, 1.75, \text{ and } 2$; and twenty-six ratios of periods of secondary and primary systems: $T_p/T_n = 0.01 \text{ to } 3$. Damping ratio in the primary system is selected as $\zeta_p = 5\%$ which is appropriate for most marine structures represented in this investigation. Damping ratio in the secondary system is typically fixed at $\zeta_s = 2\%$ except when effects of damping are investigated where an additional value of $\zeta_s = 5\%$ is considered. Nonlinearity is characterized by $R_y = 2, 4,$
and 8 in the primary system \( R_p = 1.5, 2, 3, \) and 4 in the secondary system. The post-yield stiffness ratio in the primary and secondary systems are selected as \( \alpha_p = 0.05 \) and \( \alpha_s = 0.05 \).
GROUND MOTIONS CONSIDERED

This investigation uses the SAC ground motion set consisting of 20 ground motions from 10 sites for 10% probability of exceedance in 50 years for a site in Los Angeles, California (Somerville et al., 1997). The previous study by the author (Goel, 2017a) also considered a second much larger set consisting of 80 ground motions from 40 sites and found that selection of the ground motion set does not affect the overall conclusions on seismic response behavior of coupled primary-secondary system. Therefore, this study considers only the smaller SAC ground motion set. Figure 6 shows the elastic response spectrum for individual ground motions and median for the ensemble of the selected set.

![Figure 6. Response spectrum for SAC ground motions.](image)

Figure 6. Response spectrum for SAC ground motions.
EFFECTS OF DAMPING IN SECONDARY SYSTEM

As mentioned previously, secondary systems typically possess much lower damping than primary systems (Villaverde, 1996b). The current provisions for estimating seismic force in secondary systems are silent on damping in the secondary system. These provisions generally use 5%-damped spectrum for estimating short period spectral acceleration, \( S_{SS} \), used in Equations (1) and (2) or spectral acceleration, \( S_A \), in Equation (5). Similarly, modal (or response spectrum) method that is used to compute acceleration at the point of attachment of the component, \( a_i \), in Equation (3) typically uses 5% damping for the primary system. Although, component amplification factor, \( a_p \), could have been developed using a different and lower damping in the secondary system, it is tacitly assumed that damping in the secondary system is the same as in the primary system. Therefore, there is a need to explicitly consider different and lower damping in secondary systems compared to primary systems.

The effects of damping in secondary system are examined by comparing ratio of forces in the secondary systems with \( \zeta_s = 2\% \) and 5%. Only linear systems, both primary and secondary, are considered for this part of the investigation because effectiveness of damping in reducing the response are highest for linear systems; this effectiveness is smaller for inelastic systems and decreases as inelasticity increases (Chopra, 2017).

The median results for the selected ground motion suite are presented in Figure 7. These results show that damping has negligible effects on force in the secondary system for low values of \( T_p/T_n \) (say less than 0.5) regardless of the primary system period, \( T_n \), or mass ratio, \( \mu \). For larger values of \( T_p/T_n \), however, force may be 10% to 35% higher in secondary systems with 2% damping compared to systems with 5% damping, and the effects may depend on \( T_p/T_n \), \( T_n \), and \( \mu \). The biggest difference of about 35% occurs for very low mass ratio, i.e., \( \mu = 0.01 \) and \( T_p/T_n \) values close to one; for such cases, the difference is essentially independent of \( T_n \). The difference varies between from 0% to 20% for higher mass ratios, increases with increasing \( T_p/T_n \), and the difference decreases with increasing \( T_n \).
The preceding discussion clearly shows damping may significantly influence force in the secondary system. Therefore, rest of this study considered 2% damping in the secondary system compared to 5% considered by the author in previous studies (Goel, 2017a, 2017b, and 2018).
EFFECTS OF POST-YIELD STIFFNESS ON SYSTEM FORCES

Most previous studies that considered nonlinearity in secondary systems did not explicitly consider effects of post-yield stiffness. It is well known that higher post-yield stiffness results in lower peak displacement. Therefore, elastic-perfectly-plastic idealization is assumed to provide “conservative” estimates of displacement. However, it is not clear that such an idealization will also provide “conservative” estimate of system force. Therefore, this investigation examined effects of post-yield stiffness on both displacement and force in single-degree-of-freedom (SDF) systems. For this purpose, ratio of peak response – displacement and force – of systems with post-yield stiffness ratio, i.e., $\alpha > 0$, and elastic-perfectly-plastic, i.e., $\alpha = 0$, were computed for each ground motion in the selected ground motion suite. A range of system period between 0.05s and 5 sec and three different levels of system nonlinearity, $R = 2, 4, 8$, were considered.

![Graphs showing comparison of displacements and forces for different values of post-yield stiffness ratio.](image)

The median results presented in Figure 8 show, as expected, that displacement of systems reduces with increasing post-yield stiffness. The reduction in displacement is also dependent on the system period with highest reduction occurring for shortest period and the effect reducing with increasing system period. Furthermore, the reduction with increasing post-yield stiffness increases with increasing level of nonlinearity. For longer period systems, the effect of post-yield stiffness on displacement becomes negligibly small and response of elastic-perfectly-plastic provides a sufficiently accurate displacement of systems with other values of post-yield stiffness. As mentioned previously, elastic-perfectly-plastic idealization provides conservative estimate of displacement, even though the level of conservatism may be high for short-period systems.

However, the trends reverse for system force. As apparent from results in Figure 8, the force in the system increases, even though the system displacement reduces, with increasing post-yield stiffness. These effects are more pronounced for short-period systems compared to long-period systems and increase with increasing system nonlinearity. For short-period systems, the force in
system with $\alpha > 0$ can be several times higher than that in elastic-perfectly-plastic system. Therefore, elastic-perfectly-plastic idealization may not be expected to provide conservative estimate of force.

To explain the trend observed in the preceding paragraph for very stiff, nonlinear systems, let us consider the schematic in Figure 9. Let us assume the system deformation and force are $u_e$ and $F_e$ if this system were to remain linear-elastic. Let us next consider a nonlinear system which is characterized by elastic-perfectly-plastic force-deformation behavior. For such a system, yield displacement is $u_y = u_e / R_p$ and associated yield force is $F_y$. When subjected to earthquake ground motion, its peak displacement is $u_p$ and peak force remains that same as its yield force, i.e., $F_y$.

As apparent from Figure 9, the peak force in the short-period, elastic-perfectly-plastic nonlinear secondary system will always be less than that in its corresponding linear counterpart. It is well known that short-period, nonlinear system exhibits very large ductility demand when its yield force, $F_y$, is smaller than the elastic force, $F_e$ (Chopra, 2017). Consequently, the peak displacement can be excessively large for such systems.

For a short-period, system characterized by bilinear force deformation with post-yield stiffness equal to $\alpha$ times linear-elastic stiffness, with $\alpha$ being less than 1.0, the peak displacement, $u_\alpha$, is less than the peak displacement in the corresponding elastic-perfectly-plastic nonlinear secondary system (see Figure 8). However, the peak force, $F_\alpha$, will always be more than the peak force in the corresponding elastic-perfectly-plastic nonlinear secondary system even though the peak displacement in the former system is smaller than that in the later system (see Figure 9). This explains the observation in Figure 8 that the peak force in bilinear system ($\alpha > 0$) is always larger than the peak force in the elastic-perfectly-plastic system ($\alpha = 0$). Depending on values of $\alpha$ and peak displacement $u_\alpha$, it is possible for peak force in short-period, bilinear system to exceed even force in the corresponding linear-elastic system (see Figure 9).

In summary, very-short period system with strength lower than the strength required for it to remain elastic may undergo excessive deformation which may not be readily accommodated in design. Furthermore, very-short period, bilinear system may experience force even higher than force in the corresponding linear-elastic system. Therefore, it is cautioned that very-short period (or stiff) systems not be designed for strengths much lower than the elastic-level strength.

![Figure 9. Schematic description of force-deformation in short-period systems.](image)

The discussion so far also indicates the need to carefully consider post-yield stiffness when estimating forces in short-period systems. While post-yield stiffness of primary system is often better known, e.g., nonlinear pushover analysis, such may not always be the case for secondary
systems. Therefore, careful consideration must be given to force-based design of stiff (or short-period) secondary systems when post-yield stiffness is not known.
EFFECTS OF NONLINEARITY ON SECONDARY SYSTEM FORCES

As noted previously, nonlinearity in the system, either primary or secondary or both, may affect force in the secondary system. This section systematically examines these effects. For this purpose, ratio of the secondary system force in a nonlinear system, $F_{p, NL}$, and in a corresponding linear system, $F_{p, LN}$, are computed in systems defined by selected parameters. These results are generated for each ground motion in the selected SAC suite. The trends are then investigated by examining median results for the entire ground motions suite. Examined first are the effects when secondary system remains linear elastic while primary system yields beyond its linear elastic limit. This is followed by cases when the primary system remains linear elastic and the secondary system yield beyond its linear elastic limit. Finally, effects of nonlinearity in both primary and secondary systems are examined.

Nonlinear Primary System - Linear Secondary System

Figures 10 to 12 present results for three different levels of nonlinearity in the primary system: $R_y = 2, 4, \text{ and } 8$; the secondary system is assumed to remain within its linear elastic range for all these cases. These results show that nonlinearity in the primary system significantly reduces forces in the secondary system with $T_p/T_n < 1.5$. The secondary system experiences lower force with increasing $R_y$. This is expected as increasing nonlinearity in the primary system results in increasingly lower acceleration transmitted to the base of the secondary system, which in turn leads to lower force in the secondary system. The reduction appears to be minimal for systems with $T_p/T_n > 1.5$ indicating that force in very flexible secondary system is insensitive to nonlinearity in the primary system. The largest reduction occurs for systems for which $T_p/T_n$ is close to one.

Nonlinearity in the primary system results in effective elongation of period of the primary system resulting in lowering of the effective $T_p/T_n$ value which in turn leads to lower amplification in secondary system force. A combination of amplification in secondary system force in linear primary-secondary system, and lower amplification in secondary system force due to nonlinearity of the primary system leads to the lowest value of $F_{p, NL}/F_{p, LN}$ when $T_p/T_n$ is close to one. Since the amplification in secondary system force is the highest for systems with lowest value of $\mu$ and decrease with increasing values of $\mu$, the aforementioned trends are also most prominent for lower values of $\mu$ and reduce as $\mu$ increases. The trends are essentially independent of period of the primary system.
Figure 10. Median of ratio $F_{p,NL}/F_{p,LN}$ for systems with $R_y = 2$ and $R_p = 1$.

Figure 11. Median of ratio $F_{p,NL}/F_{p,LN}$ for systems with $R_y = 4$ and $R_p = 1$. 
Figure 12. Median of ratio $F_{p,\text{NL}} / F_{p,\text{LN}}$ for systems with $R_y = 8$ and $R_p = 1$.

Linear Primary System - Nonlinear Secondary System

Figures 13 to 16 present results for four different levels of nonlinearity in the secondary system: $R_p = 1.5, 2, 3, \text{ and } 4$; the primary system is assumed to remain within its linear elastic range for all these cases. These results show that nonlinearity in the secondary system significantly reduces forces in the secondary system with $T_p / T_n > 1.0$ and the reduction is essentially inversely proportional to $R_p$. For example, for cases with $R_p = 2$, $F_{p,\text{NL}} / F_{p,\text{LN}}$ is essentially equal to 0.5 ($= 1/2$). This trend is expected as nonlinearity in the secondary system limits the force that can be generated in the system.

However, $F_{p,\text{NL}} / F_{p,\text{LN}}$ increases with decreasing values of $T_p / T_n$ for systems with $T_p / T_n < 1.0$. In few cases, $F_{p,\text{NL}} / F_{p,\text{LN}}$ exceeds 1.0 indicating that the force in nonlinear secondary system is higher than that in the corresponding linear system. The highest values of $F_{p,\text{NL}} / F_{p,\text{LN}}$ occur for very stiff secondary systems characterized by very low values of $T_p / T_n$.

As demonstrated previously in Figures 8, the peak displacement of a short-period, secondary system characterized by bilinear force deformation is less than that of the corresponding elastic-perfectly-plastic nonlinear secondary system. However, the peak force in bilinear system is always larger than that in the corresponding elastic-perfectly-plastic nonlinear secondary system (Figure 9). Depending on the post-yield stiffness and peak displacement, it is possible for peak force in short-period, bi-linear secondary system may exceed even force in the corresponding linear-elastic system (see Figure 8), which helps explain the trends noted in the preceding paragraph for very-stiff (or very-short period) secondary systems.
In summary, very-short period secondary system with strength lower than the strength required for it to remain elastic, i.e., \( R_p \) higher than 1.0, may undergo excessive deformation which may not be readily accommodated in design. Furthermore, very-short period, bilinear system with \( R_p \) higher than 1.0 may experience force even higher than force in the corresponding linear-elastic system. Therefore, it is cautioned that very-short period (or stiff) secondary systems not be designed for \( R_p \) higher than 1.0.

The trends noted in the previous paragraph are most prominent for lower values of \( \mu \) and reduce as \( \mu \) increases. The trends are also essentially independent of period of the primary system.

![Figure 13. Median of ratio \( F_{p,NL}/F_{p,LN} \) for systems with \( R_y = 1 \) and \( R_p = 1.5 \).](image)
Figure 14. Median of ratio $F_{p,NL}/F_{p,LN}$ for systems with $R_y = 1$ and $R_p = 2$.

Figure 15. Median of ratio $F_{p,NL}/F_{p,LN}$ for systems with $R_y = 1$ and $R_p = 3$. 
Nonlinear Primary System - Nonlinear Secondary System

Figures 17 to 28 present results for three different levels of nonlinearity in the primary system: $R_y = 2$, 4, and 8, and four different levels of nonlinearity in the secondary system: $R_p = 1.5$, 2, 3, and, 4. These results show that nonlinearity in primary and secondary system generally reduces forces in the secondary system. The exception occurs for few combinations of $R_y$, $R_p$, $\mu$ and very stiff secondary systems, i.e., very low $T_p/T_n$ values, where $F_{p,NL}/F_{p,LN}$ may become large and even exceed 1.0 indicating larger force in nonlinear secondary system compared to its linear-elastic counterpart (see Figures 18 to 20). This observation is consistent with the observation in the preceding section that very-short period secondary system with strength lower than the strength required for it to remain elastic, i.e., $R_p$ higher than 1.0, may undergo excessive deformation and may experience force even higher than force in the corresponding linear-elastic system.

For systems with $T_p/T_n > 1.5$, the reduction appears to be dependent only on $R_p$. For example, for cases with $R_p = 2$, $F_{p,NL}/F_{p,LN}$ is essentially equal to 0.5 ($= 1/2$). While $F_{p,NL}/F_{p,LN}$ in many systems with $T_p/T_n < 1.0$ are less than 1.0, indicating lower force in nonlinear system compared to corresponding linear system, $F_{p,NL}/F_{p,LN}$ tends to increase in many cases with decreasing $T_p/T_n$ . Such a trend is observed in systems with larger $R_p$ and is more prominent in systems with lower $\mu$ values. These trends are consistent with previously observed trends for very-stiff secondary systems with strength much lower than that of the corresponding linear-elastic system which are prone to experiencing higher forces.
The largest reduction occurs for systems for which \( T_p/T_n \) is close to one. This trend is consistent with and for the same reasons as mentioned previously for linear secondary system supported on nonlinear primary systems.

The trends noted in previous paragraphs are most prominent for lower values of \( \mu \) and reduce as \( \mu \) increases. The trends are also essentially independent of period of the primary system.

Figure 17. Median of ratio \( F_{p, NL}/F_{p, LN} \) for systems with \( R_y = 2 \) and \( R_p = 1.5 \).
Figure 18. Median of ratio $F_{p,NL}/F_{p, LN}$ for systems with $R_y = 2$ and $R_p = 2$.

Figure 19. Median of ratio $F_{p,NL}/F_{p, LN}$ for systems with $R_y = 2$ and $R_p = 3$. 

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Figure 20. Median of ratio \( \frac{F_{p,NL}}{F_{p,LN}} \) for systems with \( R_y = 2 \) and \( R_p = 4 \).

Figure 21. Median of ratio \( \frac{F_{p,NL}}{F_{p,LN}} \) for systems with \( R_y = 4 \) and \( R_p = 1.5 \).
Figure 22. Median of ratio $F_{p,NL}/F_{p,LN}$ for systems with $R_y = 4$ and $R_p = 2$.

Figure 23. Median of ratio $F_{p,NL}/F_{p,LN}$ for systems with $R_y = 4$ and $R_p = 3$. 
Figure 24. Median of ratio $\frac{F_{p,NL}}{F_{p,LN}}$ for systems with $R_y = 4$ and $R_p = 4$.

Figure 25. Median of ratio $\frac{F_{p,NL}}{F_{p,LN}}$ for systems with $R_y = 8$ and $R_p = 1.5$. 
Figure 26. Median of ratio $\frac{F_{p,NL}}{F_{p, LN}}$ for systems with $R_y = 8$ and $R_p = 2$.

Figure 27. Median of ratio $\frac{F_{p,NL}}{F_{p, LN}}$ for systems with $R_y = 8$ and $R_p = 3$. 
Figure 28. Median of ratio $\frac{F_{p,NL}}{F_{p,LN}}$ for systems with $R_y = 8$ and $R_p = 4$. 
EVALUATION OF SEISMIC PROVISIONS FOR SECONDARY SYSTEMS SUPPORTED ON PIERS, WHARVES, AND MARINE OIL TERMINALS

Proposed Provisions

The horizontal seismic force, $F_p$, in secondary systems supported on piers, wharves, and marine oil terminals are computed from a simplified procedure which does not require information on supporting (or primary) structure:

$$F_p = \frac{1.2 a_p S_{DS} I_p W_p}{R_p}$$

$$0.3S_{DS} I_p W_p \leq F_p \leq 1.6S_{DS} I_p W_p$$

or from an alternate formula which requires information of the supporting (or primary) structure:

$$F_p = \frac{a_p S_{x} I_p A_x W_p}{R_p}$$

$$0.3S_{DS} I_p W_p \leq F_p \leq 1.6S_{DS} I_p W_p$$

where $S_{DS} = \text{short period spectral acceleration in the design-level earthquake}$, $a_p = \text{component amplification factor}$ (see Table 1 or Figure 2), $I_p = \text{component importance factor}$ (see Table 2), $R_p = \text{component response modification factor}$ (see Table 3), $W_p = \text{component operating weight}$, $S_{x}$ is the spectral acceleration computed from the design-level earthquake spectrum at period equal to fundamental vibration period of the primary system, i.e., pier or wharf, and $A_x = \text{torsional amplification factor}$ computed from

$$A_x = \left(\frac{\delta_{\text{max}}}{1.2\delta_{\text{avg}}}\right)^2$$

$$1 \leq A_x \leq 3$$

where $\delta_{\text{max}}$ is the maximum displacement, and $\delta_{\text{avg}}$ is the average of the displacements at the extreme points of the supporting (or primary) structure (see Figure 1).

The horizontal seismic force, $F_p$, in the direction under consideration shall be applied at the center of gravity and distributed relative to the mass distribution of the nonstructural component or nonbuilding structure. The horizontal seismic force, $F_p$, shall be applied independently in at least two orthogonal horizontal directions in combination with service or operating loads associated with the nonstructural component or nonbuilding structure, as appropriate. For vertically cantilevered systems, however, $F_p$ shall be assumed to act in any horizontal direction.

The concurrent vertical seismic force, $F_v$, shall be applied at the center of gravity and distributed relative to the mass distribution of the nonstructural component or nonbuilding structure, as follows:
The proposed procedure may be used to estimate seismic loads on nonstructural components and nonbuilding structures permanently attached to a pier, wharf, or marine oil terminal structure. However, this procedure is not applicable when any of the following apply:

1. Mass of the nonstructural component or nonbuilding structure exceeds 25% of the combined mass of the pier, wharf, or marine oil terminal structure plus nonstructural component or nonbuilding structure;
2. Multiple nonstructural components or nonbuilding structures of similar type (or natural period) when their combined mass exceeds 25% of the total mass of the pier, wharf, or marine oil terminal structure plus nonstructural components or nonbuilding structures;
3. Concrete/Steel pier, wharf, or marine oil terminal structure with irregular configuration and high or medium spill exposure classification;
4. Nonstructural component supported by other nonstructural system permanently attached to pier, wharf, or marine oil terminal structure;
5. Nonstructural component or nonbuilding structure supported by other structure permanently attached to pier, wharf, or marine oil terminal structure;
6. Nonstructural component or nonbuilding structure attached to multiple pier, wharf, or marine oil terminal structures;
7. Nonstructural component or nonbuilding structure attached to structure and ground.

For such cases, seismic response shall be assessed using rational approach that includes consideration of strength, stiffness, ductility, and seismic interaction with all other connected components and with the supporting structures or systems, subject to approval.

Furthermore, the following nonstructural components are exempt from the seismic requirements, i.e., seismic forces need not be considered for such conditions:

1. Temporary or movable equipment unless part of a critical system;
2. Mechanical and electrical components that are attached to the pier, wharf, or marine oil terminal structure and have flexible connections to associated piping and conduit; and either:
   (a) The component weighs 400 lb or less, the center of mass is located 4 ft or less above the pier, wharf, or marine oil terminal deck, and the component Importance Factor, \( I_p \), is equal to 1.0; or
   (b) The component weighs 20 lb or less, or in the case of a distributed system, 5 lb/ft or less.

Table 1. Component amplification factors for nonstructural components and nonbuilding structures.

<table>
<thead>
<tr>
<th>Component or Structure</th>
<th>( a_p^{1,2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rigid components or structures (period less than 0.06 seconds)</td>
<td>1.0</td>
</tr>
<tr>
<td>Rigidly attached components or structures</td>
<td>1.0</td>
</tr>
<tr>
<td>Flexible components or structures (period longer than 0.06 seconds)</td>
<td>2.5</td>
</tr>
<tr>
<td>Flexibly attached components or structures</td>
<td>2.5</td>
</tr>
</tbody>
</table>

1. A lower value shall not be used unless justifies by detailed dynamic analysis and shall in no case be less than 1.0.
2. If the fundamental period of the MOT structure, \( T \), and the period of the flexible nonstructural component or nonbuilding structure, \( T_p \), is known, \( a_p \) may be estimated from Figure 2.
Table 2. Importance factor for nonstructural components and nonbuilding structures.

<table>
<thead>
<tr>
<th>Component or Structure</th>
<th>$I_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Critical</td>
<td>1.5</td>
</tr>
<tr>
<td>Other</td>
<td>1.0</td>
</tr>
</tbody>
</table>

1. A lower value may be utilized, subject to approval.

Table 3. Response modification factors for nonstructural components and nonbuilding structures.

<table>
<thead>
<tr>
<th>Component or Structure</th>
<th>$R_p^1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loading arms</td>
<td>3.0</td>
</tr>
<tr>
<td>Piping/pipelines (welded)</td>
<td>12.0</td>
</tr>
<tr>
<td>Piping/pipelines (threaded or flanged)</td>
<td>6.0</td>
</tr>
<tr>
<td>Pumps</td>
<td>2.5</td>
</tr>
<tr>
<td>Skids</td>
<td>2.5</td>
</tr>
<tr>
<td>Tanks and totes</td>
<td>2.5</td>
</tr>
<tr>
<td>Light fixtures (or luminaires)</td>
<td>1.5</td>
</tr>
<tr>
<td>Electrical conduits and cable trays</td>
<td>6.0</td>
</tr>
<tr>
<td>Mooring hardware</td>
<td>2.5</td>
</tr>
<tr>
<td>Velocity monitoring equipment</td>
<td>2.5</td>
</tr>
<tr>
<td>Instrumentation or storage cabinets</td>
<td>6.0</td>
</tr>
<tr>
<td>Cranes</td>
<td>2.5</td>
</tr>
<tr>
<td>Gangway (column systems)</td>
<td>3.0</td>
</tr>
<tr>
<td>Gangways (truss systems)</td>
<td>Use $R_p$ from frame systems</td>
</tr>
<tr>
<td>Hose towers and racks</td>
<td>Use $R_p$ from frame systems</td>
</tr>
</tbody>
</table>

**Frame systems:**

| Steel special concentrically braced frames       | 6.0     |
| Steel ordinary concentrically braced frames      | 3.5     |
| Steel special moment frames                      | 8.0     |
| Steel intermediate moment frames                 | 4.5     |
| Steel ordinary moment frames                     | 3.5     |
| Light frame wood sheathed with wood structural panels | 6.5 |
| Light frame cold-formed steel sheathed with wood structural panels | 6.5 |
| Light frame walls with shear panels of other materials | 2.0 |
| Other                                            | Subject to Division approval |

1. A higher value may be utilized, subject to approval.

Figures 29 and 30 show variation of the normalized forces from MOTEMS simplified and alternate formulas, respectively, with period of the primary system, $T_n$, and ratio of the period of the secondary and primary systems, $T_p/T_n$, for three values of $R_p = 1, 2, \text{ and } 4$, $A_p = 1$, and $I_p = 1$. Two cases for selection of $a_p$ are considered: (a) $a_p$ from Table 1, and (b) $a_p$ from Figure 2. The lower and upper limits of the normalized force are 0.75 and 4, respectively. When using the
simplified formula of Equation (7), the lower limit controls for larger $R_p$ values and when $a_p = 1$, and upper limit controls for lower values of $R_p$ and when $a_p = 2.5$. When using the alternate formula of Equation (8), the lower limit controls for larger $R_p$ values, lower $a_p$ values, and very flexible primary systems, i.e., longer $T_n$, and upper limit controls for lower values of $R_p$, $a_p = 2.5$, and very stiff primary systems, i.e., short $T_n$.

The results presented in Figure 29 show that the normalized force is independent of the period of the primary system, $T_n$. This is the case because the simplified formula of Equation (7) does not utilize period of the primary system. As expected, simplified formula which utilizes $a_p$ from Table 1, i.e., $a_p$ essentially independent of $T_p/T_n$, provides normalized forces that are essentially independent of $T_p/T_n$ (Figure 29a). The simplifies formula utilizing $a_p$ from Figure 2 leads to lower normalized forces for low and high values of $T_p/T_n$ (Figure 29b). The trends for the alternate formula are similar to those for the simplified formula with the exception that normalized forces also depend on the primary system period $T_n$ and decrease with increasing $T_n$ value (Figure 30). Furthermore, normalized forces from the alternate formula are generally lower than those form the corresponding simplified formula. This is the case because the alternate formula utilized period dependent value of $S_a$ which typically decreases with increasing $T_n$ whereas the simplified formula uses $S_{DS}$ which is independent of $T_n$ and typically corresponds to the highest value of the design spectrum ($S_{DS} = 2.5\ddot{u}_{go}$ in this study).

Figure 29. Normalized force from MOTEMS simplified formula (Equation 7) for systems with $R_p = 1, 2, \text{ and } 4, \ A_x = 1$: (a) $a_p$ from Table 1, and (b) $a_p$ from Figure 2.
Figure 30. Normalized force from MOTEMS alternate formula (Equation 8) for systems with $R_p = 1, 2, \text{ and } 4$, $A_x=1$: (a) $a_p$ from Table 1, and (b) $a_p$ from Figure 2.

**Underlying Assumptions**

There are several underlying assumptions in the proposed seismic provisions for nonstructural components and nonbuilding structures. First, it implicitly assumes adequate performance of nonstructural components and nonbuilding structures for lower-level earthquakes; the nonstructural component or nonbuilding structure is designed only for design-level earthquake. Second, the formula for seismic force in nonstructural components and nonbuilding structures in ASCE 7 and consequently proposed procedure is based primarily on studies that used linear-elastic primary and secondary systems. It is well known that nonlinearity in primary system will reduce acceleration transmitted from its base to top of the deck which will results in lower acceleration input into the secondary system. Therefore, force in the secondary system is expected to reduce because of nonlinearity in the primary system. Similarly, nonlinearity in the secondary system will also limit its force. The proposed formulas in Equations (7) and (8) account for nonlinearity only in the secondary system via use of $R_p$.

**Evaluation MOTEMS Seismic Provisions**

This section presents selected results from comparison of seismic forces in nonstructural components and nonbuilding structures from MOTEMS simplified and improved formulas to those from nonlinear response history analysis. For this purpose, idealized system of Figure 3 is analyzed for a suite of 20 SAC ground motions (Somerville, 1997). In addition to the system parameters defined and used in Goel (2018), the level on nonlinearity in the primary and secondary systems are defined by $R_y$ and $R_p$, respectively: $R_y$ is defined as the ratio of the strength required for the primary system to remain linear elastic and its yield strength, and $R_p$ is defined as the ratio...
of the strength required for the secondary system to remain linear elastic and its yield strength. It is useful to note that the actual strength of a system, either primary or secondary, is generally higher than the design value due to overstrength inherent in the design process. This study does not account for such overstrength in the primary and secondary systems. Furthermore, this study considered different damping in primary and secondary systems: \( \zeta_p = 5\% \) and \( \zeta_s = 2\% \). When needed, the post-yield stiffness ratio in the primary and secondary systems are selected as \( \alpha_p = 0.05 \) and \( \alpha_s = 0.05 \).

The formulas developed for estimating seismic forces in secondary systems supported on piers, wharves, and marine oil terminals are evaluated in this section by examining median trends in ratio of the force computed from nonlinear response history analysis, \( F_{p,NL} \), and from simplified formula in Equation (7), \( F_{p,S} \), or the alternate improved formula in Equation (8), \( F_{p,A} \). For each case, lower and upper limits on forces are imposed. This study used component amplification factor \( \alpha_p \) from Figure 2 because this approach provides lower values of \( F_{p,S} \) or \( F_{p,A} \). The results are generated and examined for several combinations of nonlinearity in the primary system, \( R_y = 1, 2, 4, \) and 8 and in the secondary system, \( R_p = 1, 2, \) and 4. The value of \( S_{05} \) needed in Equation (7) is computed as 2.5 times the peak ground acceleration of the ground motion under consideration. The value of \( S_j \) needed in Equation (8) is selected as the spectral acceleration at period equal to period of the primary structure, \( T_n \), for the selected ground motion.

The median of ratio less than one implies that the MOTEMS seismic formula tends to overestimate seismic force in nonstructural component or nonbuilding structure: the force demand computed from response history analysis (linear or nonlinear) is lower than that provided by the seismic formula. Conversely, median of ratio more than one implies that the seismic formula tends to underestimate seismic force in nonstructural component: the force demand computed from response history analysis (linear or nonlinear) is higher than that provided by the seismic formula.

It is useful to recall that very-short period, bilinear system, i.e., systems with post-yield stiffness larger than zero, experience force much larger than the yield force or force in the corresponding elastic-perfectly-plastic system (Figure 9) and this force may even be higher than force in the corresponding linear-elastic system (see Figure 8). This effect becomes more pronounced with increasingly lower yield strength of the system.

**Simplified Formula**

When both primary and secondary systems remain elastic, i.e., \( R_y = 1 \) and \( R_p = 1 \), the MOTEMS formula underestimate the force in secondary system for the period ratio range \( 0.5 < T_p / T_n < 1.5 \) (Figure 31). The underestimation in this period range is particularly large for low mass ratios, i.e., \( \mu < 0.1 \). The underestimation reduces with increasing period of the primary system, \( T_n \), and increasing mass ratio, \( \mu \). For period ratios \( T_p / T_n < 0.5 \) or \( T_p / T_n > 1.5 \), the MOTEMS formula overestimates the forces in secondary system, the overestimation can exceed by more than 100%, and the overestimation is relatively insensitive to \( \mu \) but increases with \( T_n \).

The aforementioned observations from linear-elastic studies suggest that the secondary system in the period \( 0.5 < T_p / T_n < 1.5 \) are likely to be subjected to forces much higher than those estimated
from the simplified formula and therefore engineers should avoid this period range. However, the
simplified formula provides significant overestimation for secondary systems with period ratios
$T_p / T_n < 0.5$ or $T_p / T_n > 1.5$ and there is opportunity to improve MOTEAMS simplified formula to
reduce the overestimation.

When the system experiences nonlinearity, either in the primary system or the secondary system
of both, as is likely to be the case for the design-level earthquake, the trends begin to deviate from
those observed from linear-elastic studies. For cases when the primary systems remain linear
elastic, i.e., $R_y = 1$, and the nonlinearity is limited to only the secondary system, i.e., $R_p > 1$, the
region where the MOTEAMS simplified formula underestimate the force in the secondary system
shifts towards lower period ratios, i.e., $T_p / T_n < 1$ (Figures 32 and 33). The underestimation
increases with increasing nonlinearity in the secondary system, i.e., increasing $R_p$ value. In
particular, the underestimation is large for period ratios $T_p / T_n < 0.5$. For other period ratios, the
MOTEAMS simplified formula overestimate forces in the secondary systems and trends are similar
as those noted previously based on linear-elastic studies.

To understand why the underestimation region shifts towards the lower period range, it is useful
to examine first how increasing $R_p$ affects the “effective” value of $T_p / T_n$. Recall that short-period
nonlinear systems generally experience larger deformation compared to their linear-elastic
counterparts and thus longer “effective” $T_p$. As a result, the “effective” value of $T_p / T_n$ for short-
period secondary systems, which fell in the period range $T_p / T_n < 0.5$ when linear elastic, shifts
towards the middle period range, i.e., $T_p / T_n$ closer to one where one expects significant
underestimation based on linear-elastic studies.

Second, the significant underestimation occurs because of the post-secondary stiffness in short-
period, nonlinear secondary systems. As mentioned previously, short-period, bilinear system, i.e.,
systems with post-yield stiffness larger than zero, experience force much larger than the yield force
or force in the corresponding elastic-perfectly-plastic system (Figure 9) and this force may even be
higher than force in the corresponding linear-elastic system (see Figure 8). This effect becomes
more pronounced with increasing $R_p$ as apparent from results in Figure 33 compared to Figure 32.

Finally, the underestimation decreases and overestimation increases with increasing nonlinearity
in the primary system, i.e., increasing value of $R_y$ (Figures 32 to 41). It is useful to note that
nonlinearity in the primary system reduces acceleration transmitted to the base of the secondary
system during response history analyses which in-turn leads to lower force in the secondary
system. The force form the MOTEAMS simplified formula, on the other hand, is not affected by the
nonlinearity in the primary system because the formula considers only accelerations transmitted
to the secondary system as if the primary system remains linear elastic, i.e., $R_y = 1$. Therefore, the
nonlinear response history analysis will lead to lower force ratio $F_{p, NL} / F_{p, S}$ with increasing $R_y$.
For large nonlinearity in the primary system, e.g., $R_y = 8$, the MOTEAMS simplified formula leads
to significant overestimation regardless of all system parameters (Figures 40 to 42).
Figure 31. Median of ratio $F_{p,\text{NL}}/F_{p,\text{S}}$ for $R_y = 1$ and $R_p = 1$.

Figure 32. Median of ratio $F_{p,\text{NL}}/F_{p,\text{S}}$ for $R_y = 1$ and $R_p = 2$. 
Figure 33. Median of ratio $F_{pNL}/F_{pS}$ for $R_y = 1$ and $R_p = 4$.

Figure 34. Median of ratio $F_{pNL}/F_{pS}$ for $R_y = 2$ and $R_p = 1$. 
Figure 35. Median of ratio $F_{p,NL}/F_{p,S}$ for $R_y = 2$ and $R_p = 2$.

Figure 36. Median of ratio $F_{p,NL}/F_{p,S}$ for $R_y = 2$ and $R_p = 4$. 

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Figure 37. Median of ratio $\frac{F_{p,NL}}{F_{p,S}}$ for $R_y = 4$ and $R_p = 1$.

Figure 38. Median of ratio $\frac{F_{p,NL}}{F_{p,S}}$ for $R_y = 4$ and $R_p = 2$. 
Figure 39. Median of ratio $F_{p,NL}/F_{p,S}$ for $R_y = 4$ and $R_p = 4$.

Figure 40. Median of ratio $F_{p,NL}/F_{p,S}$ for $R_y = 8$ and $R_p = 1$. 
Figure 41. Median of ratio $F_{p,NL}/F_{p,S}$ for $R_y = 8$ and $R_p = 2$.

Figure 42. Median of ratio $F_{p,NL}/F_{p,S}$ for $R_y = 8$ and $R_p = 4$. 
Alternate Formula

It is useful to recall that the forces in the secondary systems from the response history analysis, \( F_{p, NL} \), are the same as those in the preceding section. Only the force from the alternate formula, \( F_{p,A} \), differs from force from the simplified formula, \( F_{p,S} \). In particular, \( F_{p,A} \) is proportional to primary system period dependent spectral acceleration, \( S_A \), whereas \( F_{p,S} \) is independent of the primary system period and depends on the factor \( 1.2S_{DS} \). Since the cap on spectral acceleration, \( S_A \), is, i.e., \( |S_A|_{\text{max}} = S_{DS} \), \( F_{p,A} \) is always less than \( F_{p,S} \). As a result, the ratio \( F_{p,NL}/F_{p,A} \) presented in this section is always going to be less than the ratio \( F_{p,NL}/F_{p,S} \) presented in the preceding section. In other words, the alternate formula will always lead to higher ratio of forces from response history analysis and the MOTEMS formula.

When both primary and secondary systems remain elastic, i.e., \( R_y = 1 \) and \( R_p = 1 \), the MOTEMS alternate formula underestimate the force in secondary system for the period ratio range \( 0.5 < T_p/T_n < 1.5 \) (Figure 43). The underestimation in this period range is particularly large for low mass ratios, i.e., \( \mu < 0.1 \) but it reduces with increasing mass ratio, \( \mu \). These trends are similar to those noted previously for the simplified MOTEMS formula. However, the underestimation from the alternate formula is larger than that from the simplified formula because the alternate formula gives lower force in the secondary system (see explanation in the preceding paragraph). These effects for the alternate formula are much less dependent on the period of the primary system compared to those for the simplified formula. For the simplified formula, underestimation tends to reduce with increasing \( T_n \) (see Figure 31) but the reduction in underestimation is much with increasing \( T_n \) for the alternate formula (see Figure 43). This is the case because \( F_{p,A} \) reduces with increasing \( T_n \) because reduction in spectral acceleration \( S_A \) with increasing period of the primary system whereas \( F_{p,S} \) is independent of the period \( T_n \).

For period ratios \( T_p/T_n < 0.5 \) or \( T_p/T_n > 1.5 \), the MOTEMS alternate formula either provides very good estimate of the forces in secondary system, as apparent from the ratio \( F_{p,NL}/F_{p,S} \) being close to one or slight overestimation. The overestimation in most cases is lower for the alternate formula compared to the simplified formula, and is relatively insensitive to \( \mu \) and \( T_n \).

When the system experiences nonlinearity, either in the primary system or the secondary system of both, as is likely to be the case for the design-level earthquake, the trends begin to deviate from those observed from linear-elastic studies. For cases when the primary systems remain linear elastic, i.e., \( R_y = 1 \), and the nonlinearity is limited to only the secondary system, i.e., \( R_p > 1 \), the region where the MOTEMS alternate formula underestimate the force in the secondary system shifts towards lower period ratios, i.e., \( T_p/T_n < 1 \) (Figures 44 and 45). The underestimation increases with increasing nonlinearity in the secondary system, i.e., increasing \( R_p \) value. In particular, the underestimation is large for period ratios \( T_p/T_n < 0.5 \). This underestimation for the alternate formula occurs for the same reasons as described previously for the simplified procedure. Furthermore, underestimation is larger for the alternate formula compared to the simplified formula because of lower force \( F_{p,A} \) provided by the alternate formula than the force \( F_{p,S} \) from the simplified formula.
Finally, the underestimation, when it occurs, from the alternate formula decreases with increasing nonlinearity in the primary system, i.e., increasing value of $R_y$ (Figures 46 to 54) for the same reasons as for the simplified formula: nonlinearity in the primary system reduces acceleration transmitted to the base of the secondary system during response history analyses which in-turn leads to lower force in the secondary system, while the force from the MOTEMS alternate formula remains unchanged with $R_y = 1$, which leads to lower force ratio $F_{p,\text{NL}}/F_{p,\text{A}}$ with increasing $R_y$.

For other cases, the MOTEMS alternate formula either provides good estimate, as apparent from the ratio $F_{p,\text{NL}}/F_{p,\text{A}}$ being close to one or slight overestimation. For large nonlinearity in the primary system, e.g., $R_y = 8$, the MOTEMS alternate formula may lead to significant overestimation regardless of all system parameters (Figures 52 to 54).

![Figure 43. Median of ratio $F_{p,\text{NL}}/F_{p,\text{A}}$ for $R_y = 1$ and $R_p = 1$.](image-url)
Figure 44. Median of ratio $F_{p,NL}/F_{p,A}$ for $R_y = 1$ and $R_p = 2$.

Figure 45. Median of ratio $F_{p,NL}/F_{p,A}$ for $R_y = 1$ and $R_p = 4$. 
Figure 46. Median of ratio $F_{p,NL}/F_{p,d}$ for $R_y = 2$ and $R_p = 1$.

Figure 47. Median of ratio $F_{p,NL}/F_{p,d}$ for $R_y = 2$ and $R_p = 2$. 

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Figure 48. Median of ratio $F_{p,NL}/F_{p,d}$ for $R_y = 2$ and $R_p = 4$.

Figure 49. Median of ratio $F_{p,NL}/F_{p,d}$ for $R_y = 4$ and $R_p = 1$. 
Figure 50. Median of ratio $F_{p,NL}/F_{p,d}$ for $R_y = 4$ and $R_p = 2$.

Figure 51. Median of ratio $F_{p,NL}/F_{p,d}$ for $R_y = 4$ and $R_p = 4$. 

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Figure 52. Median of ratio $\frac{F_{p,NL}}{F_{p,A}}$ for $R_y = 8$ and $R_p = 1$.

Figure 53. Median of ratio $\frac{F_{p,NL}}{F_{p,A}}$ for $R_y = 8$ and $R_p = 2$. 
In summary, when both primary and secondary systems are expected to remain within the linear elastic range, the MOTEMS simplified and alternate formula may significantly underestimate the force in the secondary system for the period ratios $0.5 < T_p / T_n < 1.5$. Therefore, engineers should avoid systems within this period range. For period ratios $T_p / T_n < 0.5$ or $T_p / T_n > 1.5$ the MOTEMS simplified formula leads to significant overestimation which can exceed 100% but the alternate formula reduces this overestimation and generally provides forces that are very close to those from response history analysis.

When the secondary systems are expected to respond beyond the linear elastic range, both the simplified and alternate MOTEMS formulas may lead to significant underestimation of forces in the secondary system even for period ratios $T_p / T_n < 0.5$. This occurs because of post-yield stiffness of the secondary systems. Therefore, the engineers are cautioned against designing secondary systems in this period ratio range for forces much lower than those required to remain linear elastic. Since the higher forces are due to post-yield stiffness, the engineers should also be careful to ensure post-yield stiffness of the secondary system to be as close to zero as possible; higher post-yield stiffness may lead to forces that may even exceed those in the linear elastic system.

Finally, the alternate formula reduces overestimation or provides very good estimates of forces in the secondary systems compared to the simplified formula. Therefore, it may be more desirable to use the alternate formula when information on period ratio, $T_p / T_n$, is available.
CONCLUSIONS AND RECOMMENDATIONS

This investigation of the effects of damping in the secondary system on forces in the secondary systems led to the conclusion that the damping has negligible effects on force in the secondary system for low values of $T_p/T_n$ (say less than 0.5) regardless of the primary system period, $T_n$, or mass ratio, $\mu$. For larger values of $T_p/T_n$, however, force may be 10% to 35% higher in secondary systems with 2% damping compared to systems with 5% damping, and the effects may depend on $T_p/T_n$, $T_n$, and $\mu$.

The investigation on effects of post-yield stiffness on system response found that very-short period system with strength lower than the strength required for it to remain elastic may undergo excessive deformation which may not be readily accommodated in design. Furthermore, very-short period, bilinear system may experience force even higher than force in the corresponding linear-elastic system. This occurs due to non-zero post-yield stiffness. Therefore, it recommended that very-short period (or stiff) systems not be designed for strengths much lower than the elastic-level strength. While post-yield stiffness of primary system is often better known, e.g., nonlinear pushover analysis, such may not always be the case for secondary systems. Therefore, careful consideration must be given to force-based design of stiff (or short-period) secondary systems when post-yield stiffness is not known.

The investigation on the effects of nonlinearity in primary and secondary systems on seismic forces in secondary systems led to the following conclusions:

- Nonlinearity in the primary system alone leads to: (1) significant reduction of forces in the secondary systems with $T_p/T_n < 1.5$ because nonlinearity in the primary system results in lower acceleration transmitted to the base of the secondary system, which in turn leads to lower force in the secondary system, (2) minimal reduction of forces for systems with $T_p/T_n > 1.5$ indicating that force in very flexible secondary system is insensitive to nonlinearity in the primary system, and (3) largest reduction for systems for which $T_p/T_n$ is close to one. These trends are essentially independent of period of the primary system.

- Nonlinearity in the secondary system alone leads to: (1) excessive deformation and force, which may be higher than those in corresponding linear elastic systems, for very-short period secondary system with strength lower than the strength required for it to remain elastic, i.e., $R_p$ higher than 1.0, and (2) these trends are most prominent for lower values of $\mu$ and reduce as $\mu$ increases but are essentially independent of period of the primary system.

- It is recommended that very-short period (or stiff) secondary systems not be designed for $R_p$ higher than 1.0.

- Nonlinearity in both primary and secondary systems generally reduces forces in the secondary system. The exception occurs for very stiff secondary systems, i.e., very low $T_p/T_n$ values, where force in nonlinear secondary system exceeds that in its linear-elastic counterpart. The other trends are similar to those noted for systems with nonlinearity in the secondary systems only.

Evaluation of the MOTEMS seismic force provisions for secondary system considering effects nonlinearity led to the following conclusions and recommendations:
• When both primary and secondary systems are expected to remain within the linear elastic range, the MOTEMS simplified and alternate formula may significantly underestimate the force in the secondary system for the period ratios $0.5 < T_p/T_n < 1.5$. Therefore, it is recommended that engineers avoid systems within this period range.

• When both primary and secondary systems are expected to remain within the linear elastic range, the MOTEMS simplified formula leads to significant overestimation which can exceed 100% for period ratios $T_p/T_n < 0.5$ or $T_p/T_n > 1.5$ but the alternate formula reduces this overestimation and generally provides forces that are very close to those from response history analysis.

• When the secondary systems are expected to respond beyond the linear elastic range, both the simplified and alternate MOTEMS formulas may lead to significant underestimation of forces in the secondary system even for period ratios $T_p/T_n < 0.5$. Therefore, it is recommended that either engineers are cautioned against designing secondary systems in this period range for forces much lower than those required to remain linear elastic.

• The higher forces in nonlinear secondary system with period ratio $T_p/T_n < 0.5$ may occur in secondary systems with non-zero post-yield stiffness. Therefore, engineers are also cautioned to design secondary systems with post-yield stiffness to be as close to zero as possible.

• The MOTEMS alternate formula reduces overestimation or provides very good estimates of forces in the secondary systems compared to the simplified formula.

• The MOTEMS alternate formula is preferable when information on period ratio, $T_p/T_n$, is available.
REFERENCES


