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Linking cyclic stress and cyclic strain based methods for assessment of cyclic liquefaction triggering in sands

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Contradictory conclusions may arise when assessing liquefaction resistance of sands based on penetration tests and shear wave velocity. To provide a more unified analysis framework, this letter couples cyclic stress and cyclic strain based analysis of liquefaction triggering using site-specific correlations between penetration resistance and small strain shear modulus from shear wave velocity using the seismic cone tests. Cyclic strain theory provides a robust lower limit to liquefaction resistance, and analyses indicate that relatively high ratios of small strain stiffness to penetration resistance lead to high liquefaction resistance at relatively low cyclic stress ratios. The increased resistance to liquefaction from relatively high stiffness values is suspected to break down at higher cyclic stress ratios, where liquefaction resistance is controlled by the potential for soil to dilate, which correlates well with effective stress normalised cone tip resistance. The analysis framework is in general agreement with laboratory and field data that are predominantly Holocene in origin; however further validation by a comprehensive lab testing programme is warranted.

KEYWORDS: in situ testing; liquefaction; sands

INTRODUCTION

Analysis of soil liquefaction resistance based on in situ tests can result in contradictory conclusions when different measurements are used, particularly high-strain measurements from penetration tests and low-strain measurements from shear wave velocity. Liquefaction resistance is controlled by two distinctly different mechanisms – where high soil stiffness initially resists the development of shear strains and where high dilation angles minimise the consequences of liquefaction at higher strain levels. This letter sets out a framework in which these two aspects of liquefaction resistance are addressed separately and correlations between penetration resistance and shear modulus are used to link the two mechanisms within conventional design charts.

Cyclic strain theory (Dobry *et al.*, 1982) provides a robust lower limit for liquefaction resistance: if a threshold shear strain is not exceeded, no shear-induced pore pressures will develop and there will be no reduction in stiffness or strength due to cyclic loading. This letter explores the following hypotheses.

(*a*) Cyclic strain theory, with resistance based on G_0 (small-strain shear modulus) and a threshold shear strain, provides a lower limit for liquefaction resistance.
(*b*) The influence of dilation at higher strain levels limits the consequences of liquefaction and better correlates to penetration resistance, specifically cone penetration test (CPT) cone tip resistance (q_c) for soils of the same high-stress compressibility.
(*c*) Correlations between G_0 and q_c exist that can link cyclic stress and cyclic strain based approaches, but these correlations are not unique and should be updated with results from seismic cone tests.
(*d*) Coupled cyclic stress and cyclic strain based assessment of liquefaction resistance indicates that soils with high relative ratios of G_0/q_c (i.e. K_G (see the next section)) have a liquefaction resistance at low levels of cyclic loading that is greater than typical liquefaction resistance correlations based on the q_c of Holocene sands.

By incorporating relationships between G_0 and q_c, increases in stiffness due to factors such as soil ageing or cementation become an implicit component of a liquefaction analysis.

CORRELATIONS BETWEEN CONE TIP RESISTANCE AND SHEAR WAVE VELOCITY

Correlations between G_0 and q_c exist, but they are not unique (e.g. Rix & Stokoe, 1991). G_0 is a small-strain property controlled by the relative number of particle contacts, the characteristics of those particle contacts and the effective stress state (e.g. Santamarina *et al.*, 2001), whereas q_c is a large-strain property that is controlled by a larger strain shear stiffness, high-stress crushability of the soil, dilation angle and the horizontal effective stress state (e.g. Salgado *et al.*, 1997). Therefore, measurement of in situ G_0 and q_c through the use of a seismic cone test can be used to reduce the uncertainty in correlations between strength and stiffness, which is particularly useful for aged and cemented sands. Since the ratio of G_0/q_c is influenced by relative density and effective stress as well as age and cementation (e.g. Eslaamizaad & Robertson, 1996; Fahey *et al.*, 2003; Schnaid *et al.*, 2004), G_0/q_c cannot be used alone to reduce uncertainty in liquefaction-triggering correlations. The empirical parameter K_G, as defined in equation (1) (after Rix & Stokoe (1991)), will be used to account for sand relative density and effective stress level on the correlation between G_0 and q_c:

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$$K_G = \frac{G_0/q_c}{\left[(q_c/p_{\text{ref}})/(\sigma'_{v0}/p_{\text{ref}})^{0 \cdot 5}\right]^{-m}} = \frac{G_0/q_c}{q_{c1N}{}^{-m}} \tag{1}$$

where σ'_{v0} is the initial in situ vertical effective stress, p_{ref} is a reference stress equal to 100 kPa, m is an empirical exponent typically taken as 0·75 (e.g. Rix & Stokoe, 1991; Schneider *et al.*, 2004) and q_{c1N} is the stress normalised cone tip resistance using a median sand overburden stress exponent of 0·5 (Moss *et al.*, 2006b).

Seismic cone data from eight studies are summarised in Table 1 and Fig. 1. Aged, cemented and calcareous sands tend to have values of K_G between 330 and 1100, while Holocene sands tend to have a K_G of 110–330, with a median value of 215. These ranges are in good agreement with previous recommendations of Rix & Stokoe (1991) and Schnaid *et al.* (2004). The following section develops a lower limit of liquefaction resistance in sandy soils based on cyclic strain theory.

CYCLIC STRAIN BASED ASSESSMENT OF LIQUEFACTION TRIGGERING

The lower limits of liquefaction resistance can be defined where induced cyclic strains fall below the elastic threshold shear strain. An elastic threshold strain would be independent of the number of cycles of loading, therefore the number of loading cycles is not explicitly accounted for in this study. Data reported by Dobry *et al.* (1982) and Stokoe *et al.* (2008) support an elastic threshold strain of 1×10^{-4} that is independent of the number of cycles of loading typical of earthquakes (<30 cycles). The small-strain shear modulus (G_0) normalised by the initial in situ vertical effective stress (σ'_{v0}) can be used to calculate a threshold shear strain (γ_{th}) based on the cyclic stress ratio (CSR = τ/σ'_{v0}):

$$\gamma_{th} = \frac{\tau_{th}}{G_0} = \frac{\tau_{th}/\sigma'_{v0}}{G_0/\sigma'_{v0}} = \frac{\text{CSR}}{G_0/\sigma'_{v0}} \tag{2}$$

Figure 2(a) compares field case histories (Andrus and Stokoe, 2000) with shear wave velocity data where

Table 1. Summary of data used in evaluation of correlations between CPT cone tip resistance and small-strain shear modulus

Sand	Median K_G*	K_G CoV	Mineralogy	Single site	Aged/cemented	Lab/field	Fig.	Reference
Italian	212	0·14	Siliceous	No	No	Field	1(a)	Baldi *et al.* (1989)
Washed mortar	361	0·13	Siliceous	Yes	No	Lab	1(a)	Rix & Stokoe (1991)
Heber Road	181	0·17	Siliceous	Yes	No	Lab	1(a)	Rix & Stokoe (1991)
Mississippi River	221	0·24	Siliceous	No	No	Field	1(a)	Schneider *et al.* (2004)
Shenton Park	513	0·20	Siliceous	Yes	Yes	Field	1(b)	Schneider *et al.* (2008)
Ledge Point	491	0·22	Calcareous	Yes	No	Field	1(b)	Schneider & Lehane (2010)
Quiou	303	0·18	Calcareous	Yes	No	Lab	1(b)	Fioravante *et al.* (1998)
Perth aged	518	0·49	Siliceous / Calcareous	No	Yes	Field	1(c)	Fahey *et al.* (2003)
Holocene liquefaction	181	0·49	Siliceous	No	No	Field	1(d)	Roy (2008)
Holocene no liquefaction	184	0·56	Siliceous	No	No	Field	1(d)	Roy (2008)
Pleistocene liquefaction	213	0·43	Siliceous	No	Yes	Field	1(e)	Roy (2008)
Pleistocene no liquefaction	368	0·67	Siliceous	No	Yes	Field	1(e)	Roy (2008)
Laboratory Holocene	210	0·20	Siliceous	No	Yes/No	Lab	1(f)	Roy (2008)
Laboratory Pleistocene	408	0·25	Siliceous	No	Yes/No	Lab	1(f)	Roy (2008)
All siliceous Holocene sands in this table	214	0·39	Siliceous	No	No	Both	—	—

*For exponent m = 0·75

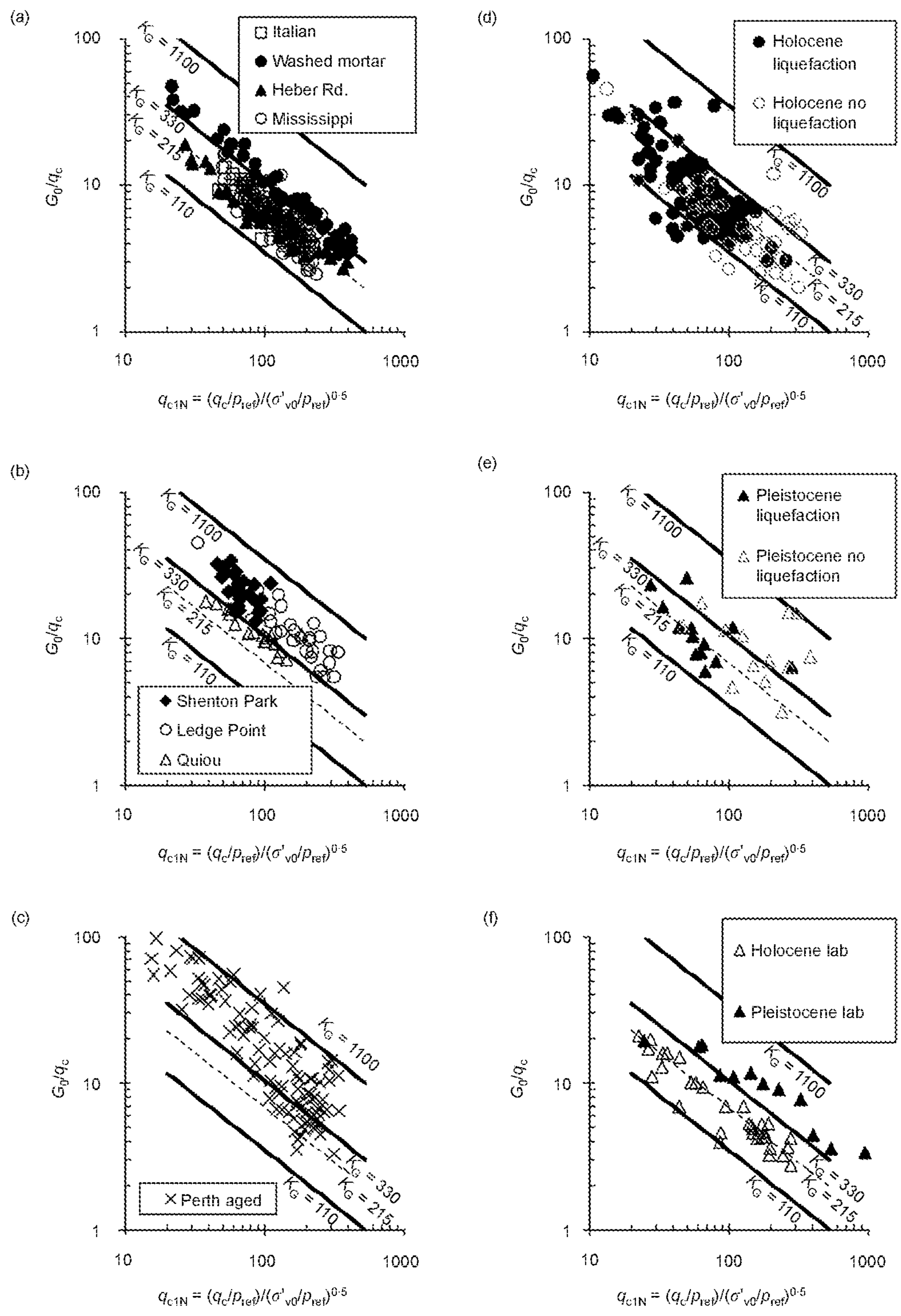

Fig. 1. Correlations between normalised cone tip resistance and small-strain shear modulus measured in seismic cone tests for various sand deposits (see Table 1 for details)

liquefaction was observed during earthquakes to a threshold shear strain of 1×10^{-4}. Figure 2(b) presents case histories where liquefaction was not observed, compared with a higher equivalent threshold shear strain of 4×10^{-4}. Normalised average cyclic loading from field case histories (CSR = τ_{avg}/σ'_{v0}) tends to increase with G_0/σ'_{v0}. For case histories where soils have liquefied, the CSR is the maximum cyclic resistance ratio (CRR = τ_{avg}/σ'_{v0}) that the soil could have experienced during the earthquake. For case histories where soils have not liquefied, the CSR is the minimum CRR that the soil could have experienced during the earthquake. A threshold shear strain of 1×10^{-4} provides a minimum level of liquefaction resistance for the database of case histories with a typically initial vertical effective stress level of 50 kPa. Layers that were subjected to higher cyclic stresses but did not result in observations of liquefaction likely had the consequences of liquefaction minimised due to dilation at higher shear strains.

CPT case histories where G_0 was not explicitly measured can be evaluated within the same framework if a correlation between G_0 and q_c is assumed. The database from Moss *et al.*

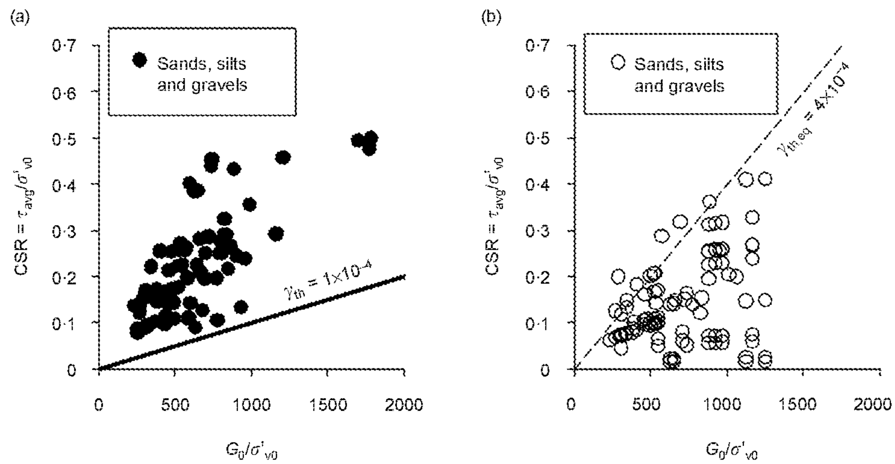

Fig. 2. Field database of liquefaction case histories in sandy soils (Andrus & Stokoe, 2000) with shear wave velocity ($G_0 = \rho V_s^2$) compared with threshold strain levels: (a) liquefaction cases; (b) non-liquefaction cases

(2006a) is analysed in Fig. 3 using equation (1) with $m = 0{\cdot}75$ and $K_G = 215$ (median of Holocene siliceous subset database). A similar minimum liquefaction resistance related to a threshold shear strain of 1×10^{-4} is observed for the CPT database when the median correlation between G_0 and q_c is used.

COMBINING CYCLIC STRAIN WITH CYCLIC STRESS BASED ASSESSMENT OF LIQUEFACTION TRIGGERING

The minimum value of liquefaction resistance, represented by the threshold strain, can be equated to the CRR using the ratio of G_0/σ'_{v0}, as illustrated in equation (3a) (cf. equation (2)). Using the correlation between G_0 and q_c (equation (1)), the minimum liquefaction resistance can be expressed as a function of cone tip resistance and initial vertical effective stress (equation (3b)).

$$\mathrm{CRR} \geq \frac{\tau_{th}}{\sigma'_{v0}} \geq \frac{G_0\gamma_{th}}{\sigma'_{v0}} \tag{3a}$$

$$\mathrm{CRR} \geq \frac{\tau_{th}}{\sigma'_{v0}} \geq \frac{K_G\, q_{c1N}{}^{-0{\cdot}75}\, q_c\, \gamma_{th}}{\sigma'_{v0}} \tag{3b}$$

Figure 4 compares the cyclic resistance from laboratory tests on Holocene and Pleistocene samples collected using freezing techniques (Roy, 2008) with cyclic resistance calculated using equation (3b). A threshold strain level of 1×10^{-4} and effective vertical stress of 50 kPa is used in equation (3b) for consistency with Figs 2 and 3 and databases of liquefaction case histories. The effects of stress level on threshold strain have been discussed in the literature (e.g. Dobry *et al.*, 1982; Santamarina & Aloufi, 1995; Santamarina *et al.*, 2001); the influence of stress level on equation (3b) requires further study. Use of the Roy (2008) dataset in Fig. 4 allows exploration of the following two issues.

(*a*) Since the data are from laboratory tests, there is less ambiguity as to the stress ratio that causes liquefaction when compared to field case histories.

(*b*) Pleistocene soils are not typically included in liquefaction case history databases and are expected to have higher values of liquefaction resistance (e.g. Arango & Migues, 1996; Hayati & Andrus, 2009; Leon *et al.*, 2006; Moss *et al.*, 2008; Pyke, 2003; Roy, 2008; Seed, 1979).

Incorporation of soil age into design method formulations generally does not significantly reduce uncertainty in the analysis for two reasons. First, the actual age of a soil deposit is rarely measured in a geotechnical site investigation. Second, in areas that are subject to high seismic activity, liquefiable soils tend to re-liquefy in historical seismic events (Youd 1984). In relation to the second point, recurring historic liquefaction events may destroy the

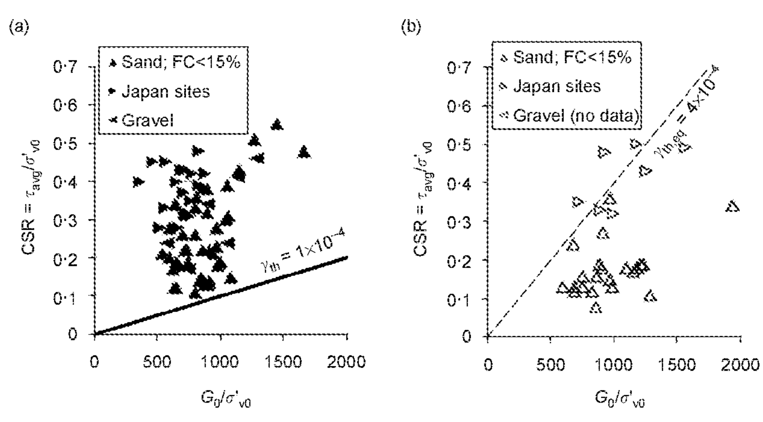

Fig. 3. Field database of liquefaction case histories in sandy soils (Moss *et al.*, 2006a) with shear modulus estimated from cone tip resistance ($G_0/q_c = 215 q_{c1N}{}^{-0{\cdot}75}$) compared with threshold strain levels: (a) liquefaction cases in sands and gravels; (b) non-liquefaction cases in sands and gravels

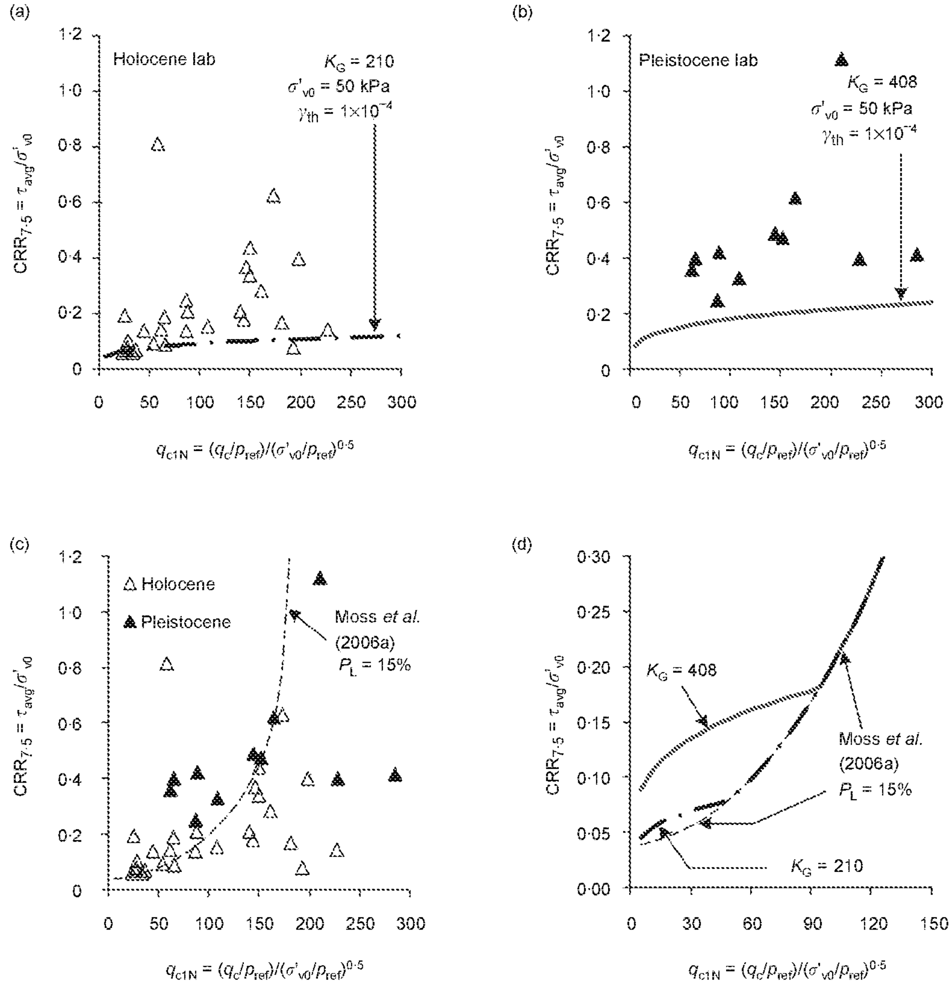

Fig. 4. Combination of cyclic stress and cyclic strain based analysis of liquefaction triggering for $\gamma_{th} = 1 \times 10^{-4}$ and $\sigma'_{v0} = 50$ kPa. (a) Cyclic strain assessment compared with Holocene lab data with effective confining stress $\sigma'_{c0} < 250$ kPa from Roy (2008). (b) Cyclic strain assessment compared with Pleistocene lab data with $\sigma'_{c0} < 250$ kPa from Roy (2008). (c) Holocene and Pleistocene lab data with $\sigma'_{c0} < 250$ kPa from Roy (2008) compared with cyclic stress boundary for Holocene sands (Moss *et al.*, 2006a). (d) Conceptual combined analysis using cyclic strain theory in Fig. 4(a) and 4(b) with cyclic stress theory from Fig. 4(c)

structure and/or bonding effects often induced by ageing that increase liquefaction resistance. An aged soil that had previously liquefied could then have similar engineering properties (strength and stiffness) as a young soil, and therefore have similar resistance to liquefaction during a seismic event.

Analysis of Roy's (2008) data uses a sample median K_G value of 210 for Holocene sands in Fig. 4(a) and 408 for the Pleistocene sands in Fig. 4(b) (Table 1). The Pleistocene sands tend to show higher liquefaction resistance than Holocene sands for q_{c1N} less than about 150, where dilation is considered to have less influence on liquefaction resistance. For a number of cases with q_{c1N} greater than about 150, the liquefaction resistance of Holocene sands approaches that of Pleistocene sands with the same cone tip resistance. These data tend to fall along the cyclic stress based ($P_L = 15\%$) liquefaction resistance curve of Moss *et al.* (2006a). For data falling along this curve, the consequences of liquefaction are limited by dilation at larger strain levels and any increase in resistance is minimised when ageing effects have been broken down at larger strains.

Figure 4(d) is a conceptual diagram that combines cyclic strain based liquefaction resistance with cyclic stress based curves. At low values of normalised cone tip resistance, stiffness controls liquefaction resistance and Pleistocene soils have a higher liquefaction resistance for the same cone tip resistance, as indicated by K_G. For these curves, the increase in liquefaction resistance is approximately a factor of 2, in agreement with the previous studies of Arango and Migues (1996) and Moss *et al.* (2008). At higher values of normalised cone tip resistance, the consequences of liquefaction are limited by dilation, and correlations to q_{c1N} for Holocene soils may be applicable to both Holocene and Pleistocene deposits. When combining these two methods in one plot, the strain based approach generally controls the behaviour of aged soils below q_{c1N} of about 100 and the stress based approach above $q_{c1N} = 100$. There are some Pleistocene points that follow the trends of the cyclic strain lower limit in Fig. 4b; these need further investigation through controlled laboratory studies.

CONCLUSIONS

Cyclic strain theory produces a lower limit for liquefaction resistance through the threshold strain. Application of correlations between G_0 and q_c link cyclic stress and cyclic strain based assessment of liquefaction analyses within

traditional CRR versus q_{c1N} charts. For soils of low normalised cone tip resistance in which dilation does not significantly increase liquefaction resistance, cyclic strain based assessment of liquefaction resistance matches well with cyclic stress based liquefaction limits in Holocene soils. For Pleistocene soils with higher ratios of stiffness to strength (i.e. high K_G), the lower limits of liquefaction resistance based on cyclic strain theory result in CRR values that are approximately twice those of Holocene soils for a q_{c1N} less than about 100. These higher levels of liquefaction resistance are consistent with previous observations and provide a conceptual framework for understanding this behaviour. A comprehensive programme of laboratory testing of uncemented, cemented and aged soils, with measurements of liquefaction strength and small-strain shear modulus, would help to extend the analysis discussed here.

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