Equilibrium Without Statics
The Modern Muller Breslau Method

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Modern Muller Breslau Method

This senior project is an investigation of the Modern Muller Breslau (MMB) Method. This method can be used to analyze both determinate and indeterminate 2D and 3D structures. The scope of this project is to analyze determinate beams, trusses, and 3-hinged arches. Next, the project demonstrates the MMB Method on indeterminate beams and frames. This project concludes by demonstrating the power of this method in the form of plastic analysis.

In addition to the analyses described, this project dives into the proof of concept of a newly developed board game which makes finding internal forces of a truss very simple.

The Modern Muller Breslau method is a new method which has improved upon the Classic Muller Breslau Method. The Classic Method was developed by Muller Breslau and is also known as the influence line method. Even though the Classic Method uses similar principles to the Modern Method, the power of the Classic Method is limited to gravity loads. The Modern Muller Breslau Method was developed by Edmond Saliklis, PhD, PE. It can be used for advanced structural analysis and will be the focus of this report.

This method has the ability to solve a plethora of different problem types, many which will be described in this report, by using only one equation. The equation is the fundamental work equation.

$$Unknown * \Delta + \Sigma(Force_i * Loft_i) = 0 \qquad \text{(Equation 1)}$$

To use this equation, the principle revolves around a perturbation of the unknown being sought. This unknown could be an external equilibrating force, an external equilibrating moment, an internal equilibrating force, or an internal equilibrating moment. When the unknown is perturbed an amount, Δ, all the forces are lofted. For example, to find the vertical reaction at the roller support of a simply supported beam, this reaction would be called the unknown. Then, a vertical perturbation at the location of the roller would need to be created. Since the roller reaction is an external reaction, the member itself should not change length and therefore should simply rotate. The vertical perturbation will be called Δ in Equation 1. This equation is a work equation which means that the work done by the external forces requires that we seek the loft only in the direction of the forces. Next, in the case of a vertical gravity load, the loft is the distance that all the forces have moved along the direction of the force. In this case, the loft is a vertical distance. This loft represents the amount of work done by the force and therefore is sensitive to positive and negative. If the Loft is in the same direction at the force, it will be considered positive work, and if it is in the opposite direction, that represents negative work. This positive and negative sign convention will apply the same to the Loft term in Equation 1.

Determinate Beams

The following example is a traditional simply supported beam with a point load applied. This problem can be solved very simply by applying the Modern Muller Breslau method. When trying to find the vertical reaction of the roller, a vertical perturbation, Δ will be applied. When applying the perturbation, it is important to keep the length of the beam constant. This will require a circle with radius equal to the length of the beam. By placing a new point on the circle, a perturbation of any vertical distance can be applied to the structure without changing the length of the beam. Next, the force must stay at the same location on the beam. To do this, another circle with radius equal to the distance between the applied force and the pin will be used. The force will follow along this radius, allowing a vertical loft to be developed without changing the location of the force on the beam. Since this beam has only a gravity force being applied, the answer will be accurate, no matter how large the perturbation. This means that the Modern Muller Breslau Method will exactly match the Classic Muller Breslau Method no matter how large the perturbation. Now, the equations shown above can be applied to solve for the vertical reaction at the roller.

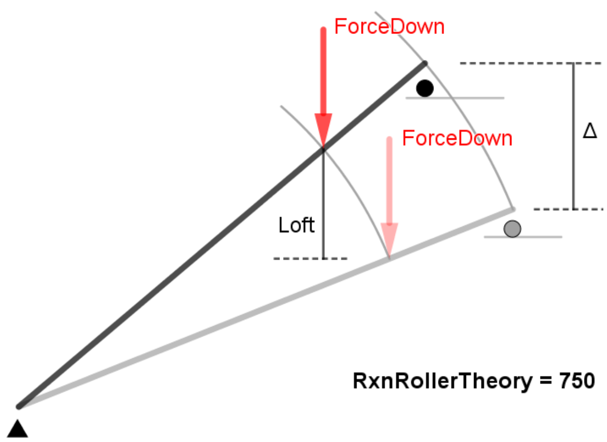
To apply Equation 1 to this example, the unknown is the vertical reaction of the roller. In addition, it needs to be determined whether Loft is positive or negative. In this case, since the direction of Loft is opposing the direction of ForceDown, work is negative.

$$RxnRoller * \Delta - (ForceDown * Loft) = 0 \qquad \text{(Equation 1.1.1)}$$

The following steps including the values of each of the variables are shown below in Figure 1.

Figure 1. Support Reaction of Determinate Beam

In addition to horizontal determinate beams, this method can also be applied to slanted determinate beams, which is proved by the following example. Since the only force acting on the beam is downward, the value of the roller reaction will be exact at both large and small perturbations. The perturbation occurs around a point in order to maintain its length as shown by the arc in Figure 2. In addition, the force moves along a similar circular arc. This allows the force to remain on the same location of the beam at any perturbation. As shown in previous examples, the reaction at the roller is then found by multiplying the vertical loft by the force, and dividing that quantity by the perturbation, Δ.



RxnRollerMMB = (Force * Loft) / △

RxnRollerMMB = (1000 * 2.22) / 2.96 = 750

Figure 2. Support Reaction of Slanted Determinate Beam

The following example is another slanted simply supported beam, but with a horizontal load. It can be seen here that the size of the perturbation does indeed make a significant difference in the accuracy of the Modern Muller Breslau method when analyzing situations with lateral loads. Figure 3 shows this beam with a large perturbation and Figure 4 is under the exact same condition but has a very small perturbation. While the necessity for a small perturbation may appear to complicate things, the remainder of the steps to complete the problem is no different than the previous examples. In the example in Figure 3 and Figure 4, the vertical reaction of the right roller is being sought. Solving for this reaction on the right requires inducing a vertical perturbation to the roller on the right. It can be seen in Figure 3 that there will be some horizontal translation of the roller due to maintaining the length of the beam. It is important that the horizontal translation is occurring at the location of the roller rather than the pin. This will ensure that the boundary conditions maintain satisfied.

This example has only one load applied, ForceRight. Since this is a horizontal force, the loft will be measured as a horizontal distance rather than vertical, unlike the previous examples.

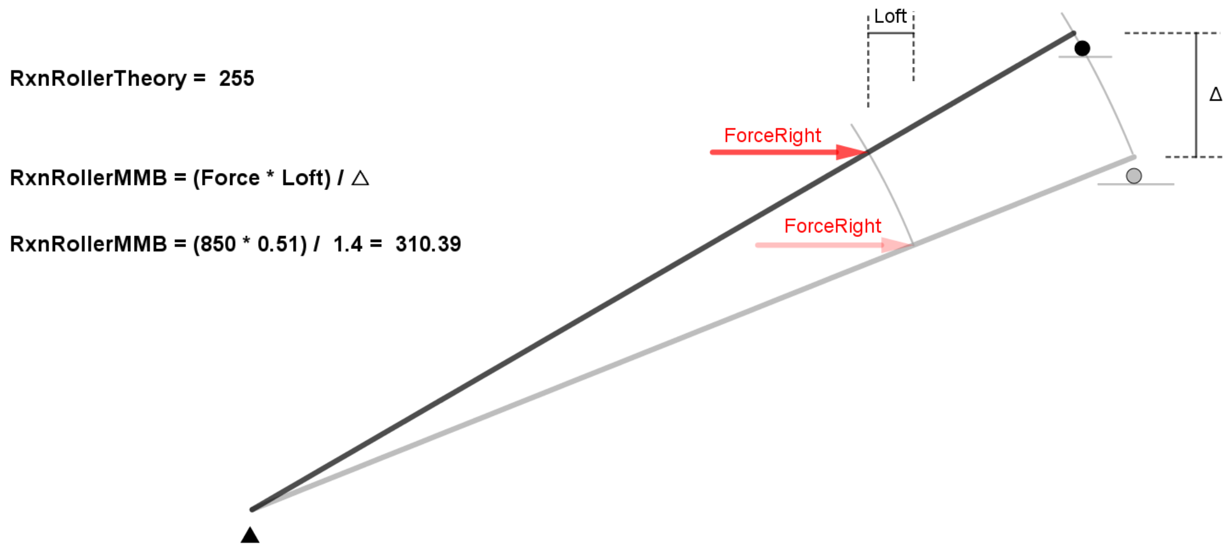


Figure 3. Support Reaction of Slanted Determinate Beam with Horizontal Force

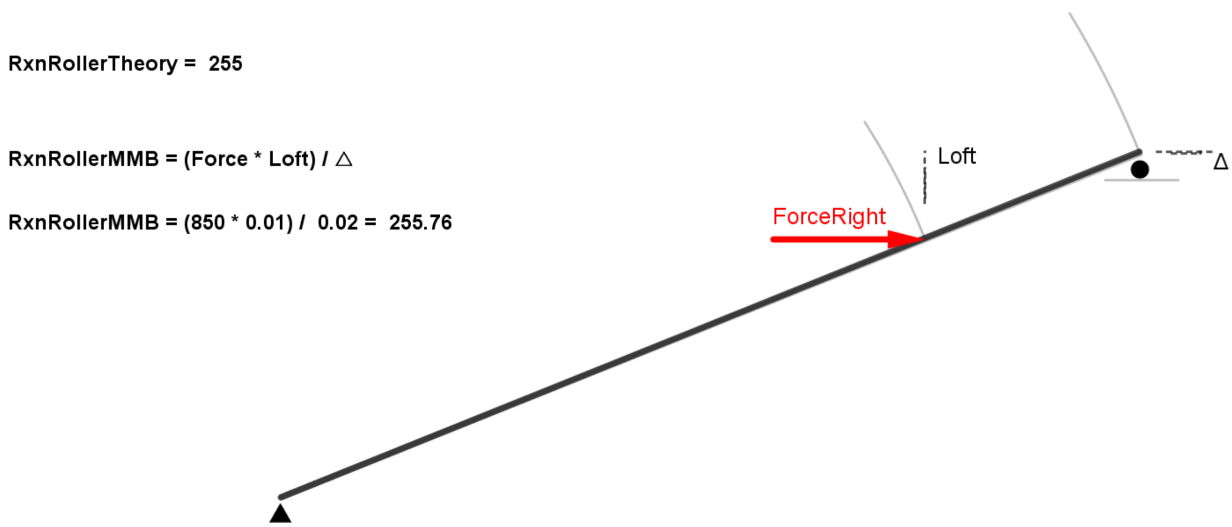


Figure 4. Support Reaction of Slanted Determinate Beam with Horizontal Force

Figure 5 and Figure 6 show a beam with a bend in it. This bend is not a kink, but rather a fixed angle at a bend in a beam. It is important to note that the angle of the bend stays constant at 30° through the perturbation. Creating this unchanging angle is done in Geogebra by linking the two members which is shown in Figure 7 and Figure 8. This example also demonstrates the power of a small perturbation. It can be seen that with a large perturbation, ForceRight1 has an impact on the solution. As the perturbation gets smaller, the horizontal loft of ForceRight1 gets very small. With a small perturbation, it is clear that ForceRight1 is actually a horizontal force on the horizontal portion of the beam, therefore having no effect on the right vertical reaction. In the MMB method, this is clear because the loft reaches zero at an infinitely small perturbation. This can be seen as LoftRight1 in Figure 5 and Figure 6.

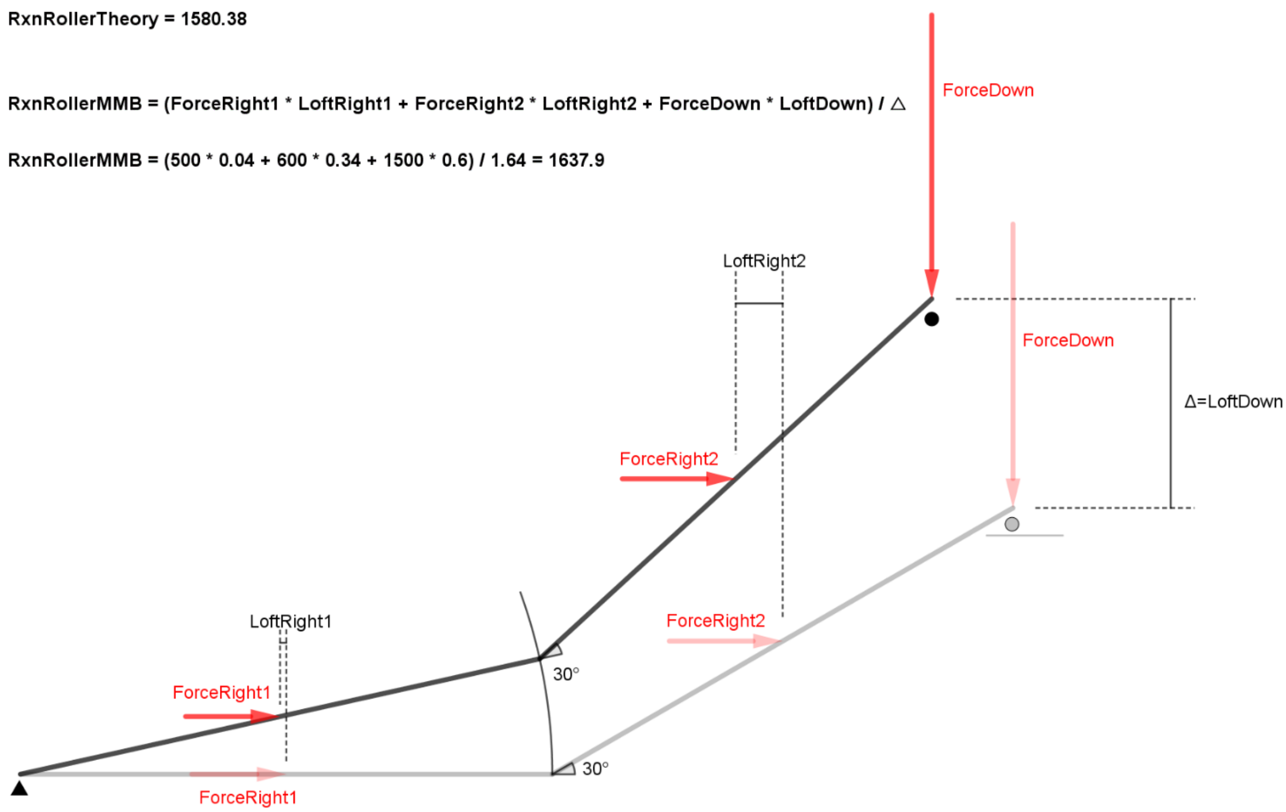


Figure 5. Roller Support Reaction of Bent Beam

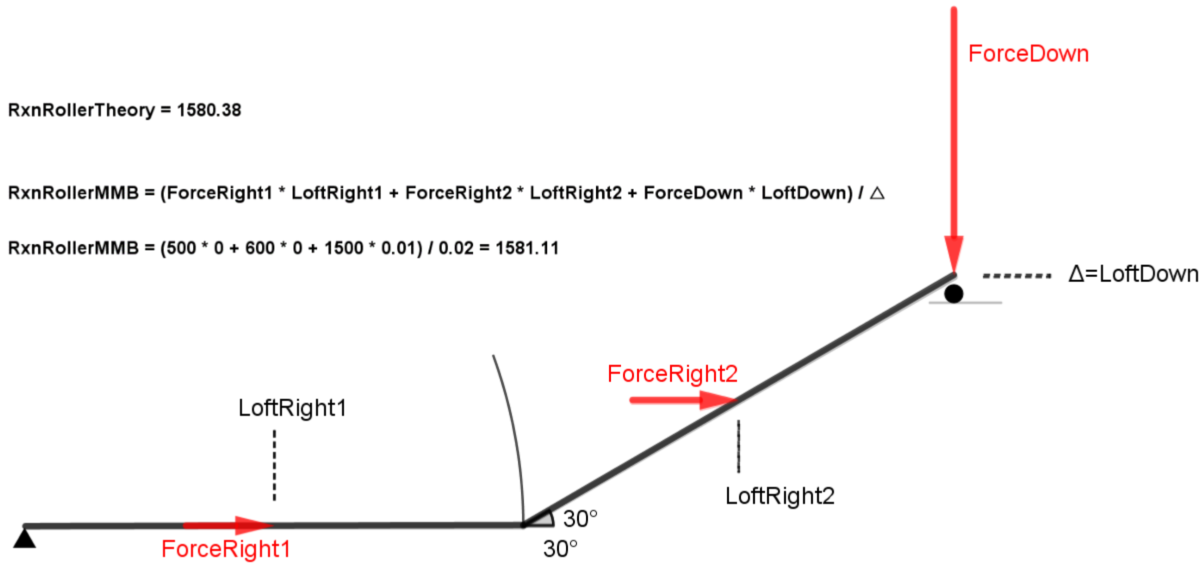


Figure 6. Roller Support Reaction of Bent Beam

To create the rigidly linked beams, the original shape should be created, and the angle to be preserved is measured. Next, the left portion of the segment is created by forming a circle with center at the pin and radius equal to the length of the original left portion. Using the circle will preserve the exact length of the left portion. The new left portion should be drawn with any arbitrary angle. As shown below, the point where the horizontal and slanted portion meet will be the point to move and create any perturbation.

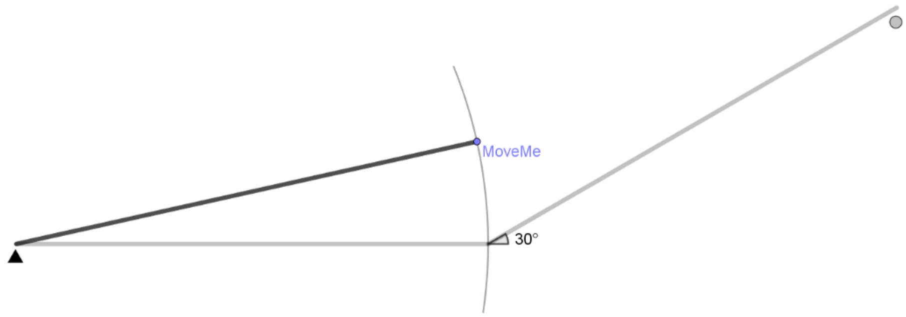


Figure 7. GeoGebra- Linked beams

To create the sloped segment of the beam, the "Angle with Given Size" function will be used. In this case, the given angle is 30°. This function will drop a point at a location which will be along the 30° angle relative to the left segment. Once this point is located, a line should be drawn using the points MoveMe and the new point found by the angle. Next, a circle should be drawn as shown in Figure 10. This circle is drawn with center at MoveMe and radius length equal to the length of the sloped span of the original shape.

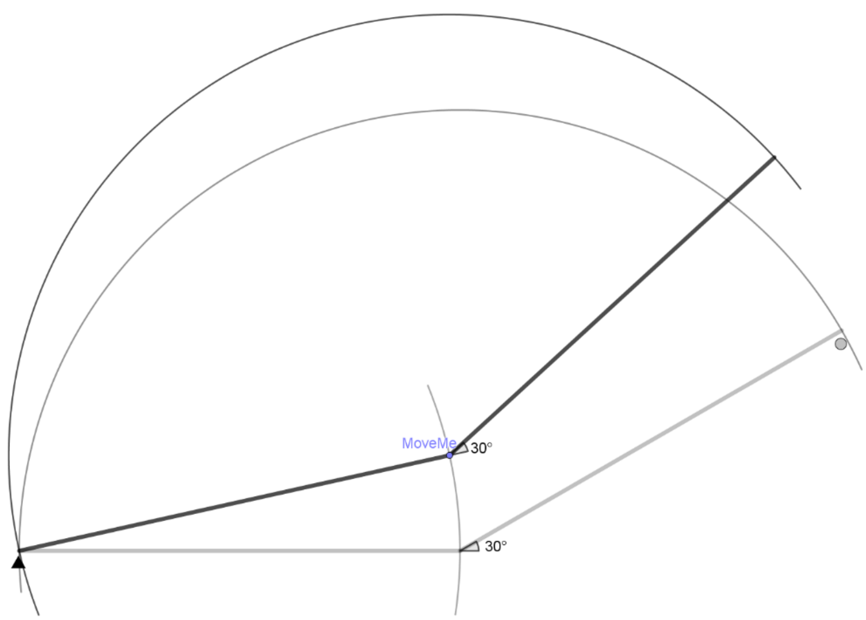


Figure 8. GeoGebra- Linked Beams

The Modern Muller Breslau Method can also be used to find the shear force in a member. Figure 9 demonstrates this method. Similar to finding the moment, a crack in the member is created and the total displacement created from the displacements above and below will be equal to Δ. One side of the beam will perturb upward and the other side downward. This perturbation is done by stretching the member, but only the member being analyzed. In Figure 9 the desired shear force is right at the center of the beam, therefore the perturbation above and below the original shape are both equal to Δ/2.

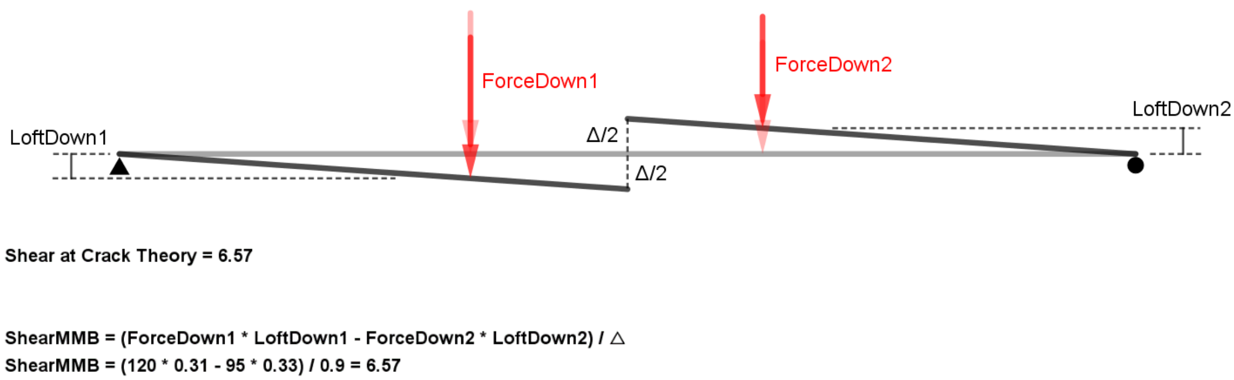


Figure 9. Shear Force in Determinate Beam

Finding the shear force in the center of the beam is very simple because the displacement on either side is equal, as shown above in Figure 9. When the shear force is desired in another location of the beam, the height of the displacements above and below the beam must be found. Figure 10, shown below demonstrates the way to calculate these values given the location of the crack.

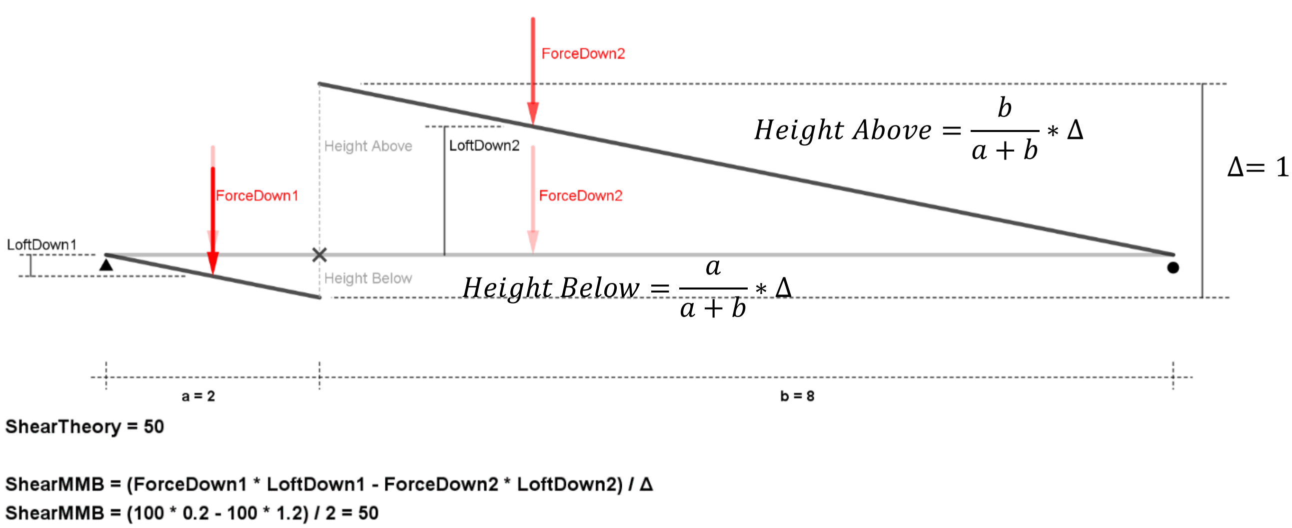


Figure 10. Shear Force in Determinate Beam

In addition to shear, the Modern Muller Breslau Method can be used to find the moment in a member. In Figure 11, the moment is found in the center of a simply supported beam. The equation used is still Equation 1. A crack is applied to the beam where the moment is to be found and a displacement perpendicular to the beam is formed. Since this example is one member, the portion of the beam on either side of the crack will be stretched to meet at a point with the height of the displacement. Since the unknown here is an internal moment, the perturbation occurs by stretching the member and therefore, the roller will not have any displacement. To apply Equation 1 to this example, the perturbation is the relative angle of between the two members formed by the displacement. Before any perturbation is applied, the perturbation, in this case, the angle, is equal to zero. In general, the larger the displacement, the larger the relative angle formed. Since the member is stretching, the force will move only vertically, and the loft is the vertical distance from the original location to the perturbed location. When finding the moment within a member, it is important to use a small perturbation. This can be seen by the difference in solutions between Figure 11 and Figure 12.

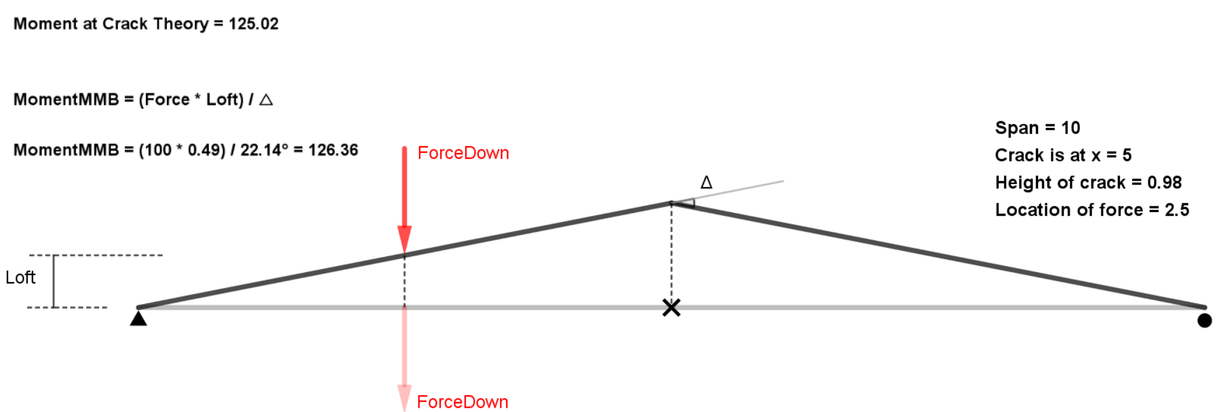


Figure 11. Moment in Determinate Beam

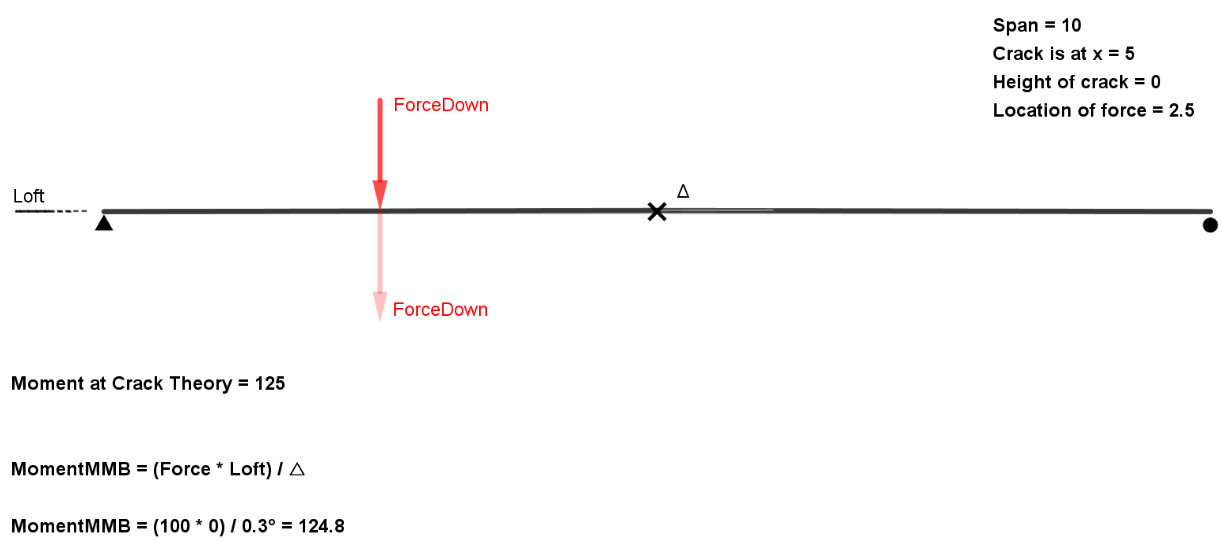


Figure 12. Moment in Determinate Beam (moment in flat beam)

Figure 13 is an example very similar to Figure 11 and Figure 12, but this time with two loads applied. This example proves that the method remains the same even with another load applied.

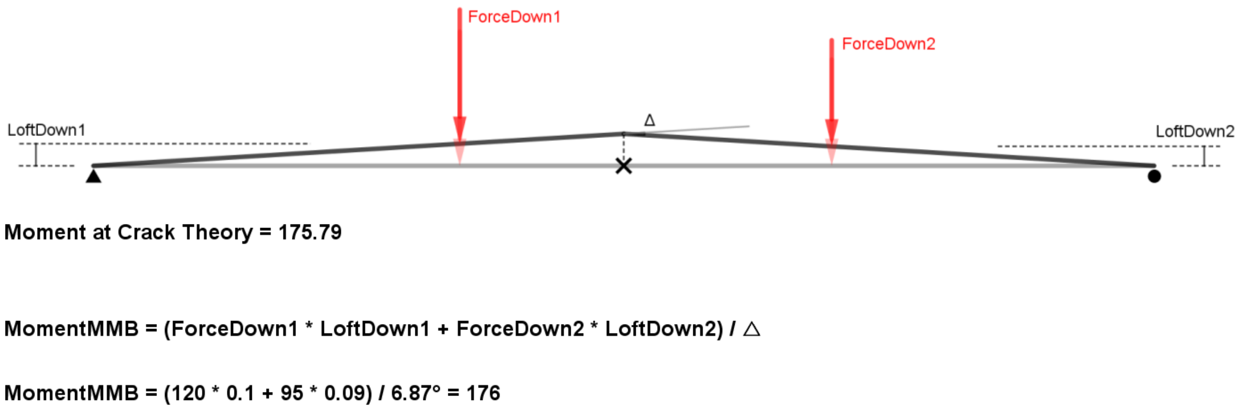


Figure 13. Moment in Flat Determinate Beam

The method of finding a moment in the member does not change when the beam is slanted. A slanted, simply supported beam is shown in Figure 14. To create the perturbation and find Δ, the same process as above will be followed. With the slanted beam, it is especially important for the displacement of the crack to be perpendicular to the beam as shown in Figure 14. Similar to the above examples, a small perturbation is required when finding the moment even in a slanted beam. This is shown in Figure 14 and Figure 15.

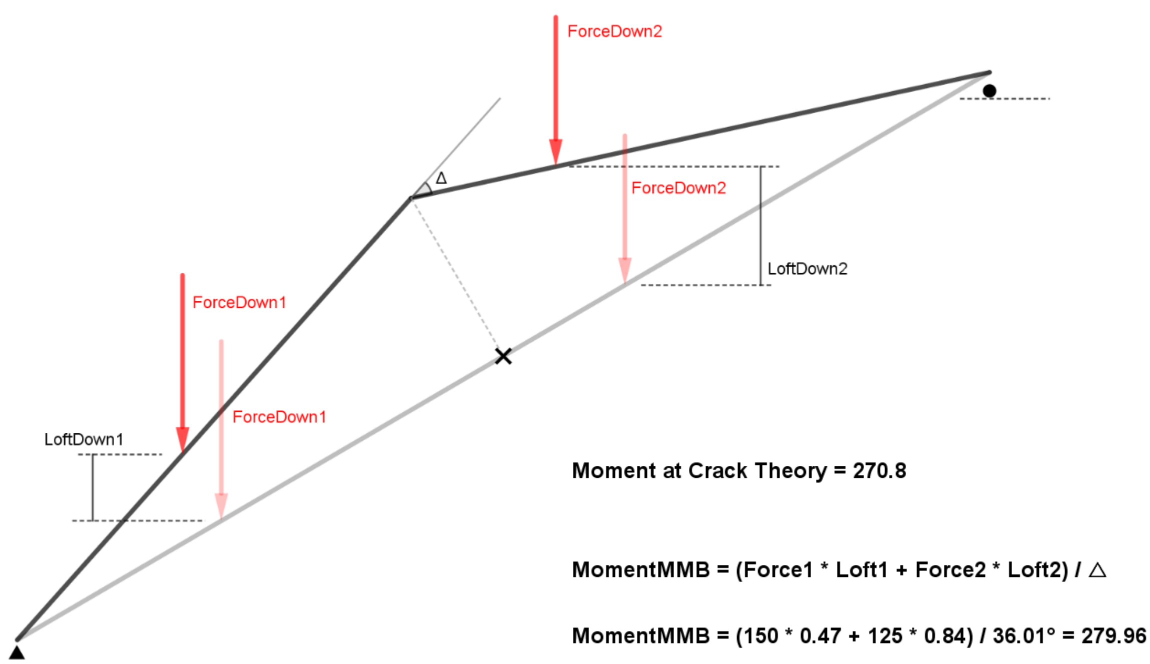


Figure 14. Moment in Slanted Determinate Beam

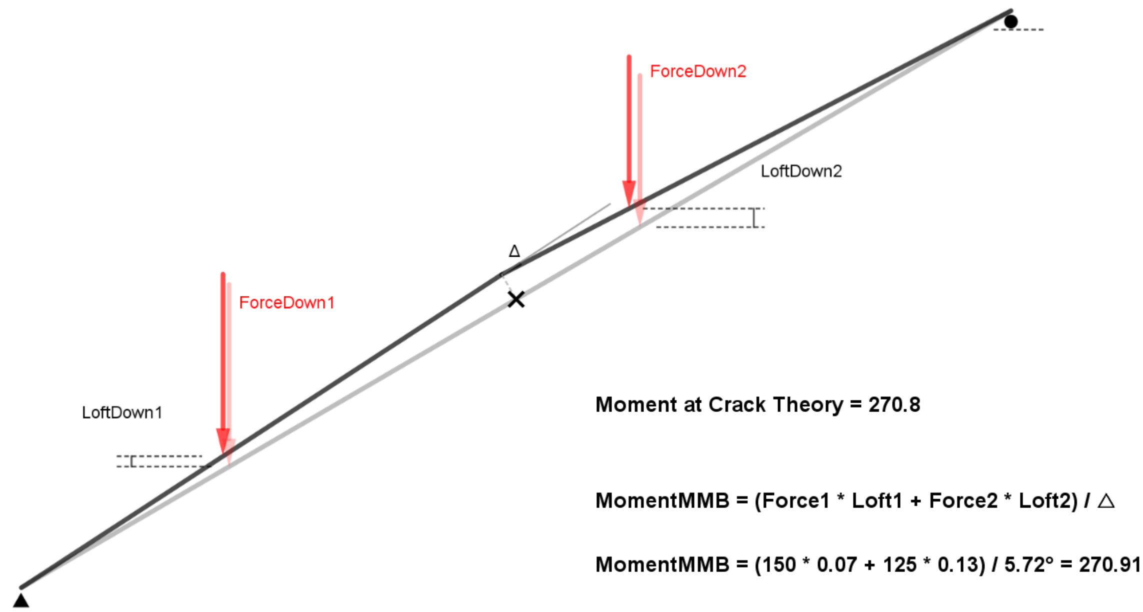


Figure 15. Moment in Slanted Determinate Beam

Three-Hinged Arches

As mentioned above, Equation 1 can be used to solve many different types of problems. Here, Equation 1 will be used to solve three-hinged arches. This type of problem can get a bit more involved when building in GeoGebra, but solving for reactions as well as internal moments of the members, the following examples follow the same basic idea. In this section, boundary errors are introduced. A boundary error is when one of the boundary conditions is not satisfied. For example, a typical roller has a vertical reaction. When creating the perturbed shape, assuming the perturbation is not meant to be imposed upon that roller, if any vertical translation is introduced at that roller, that vertical translation would be the boundary error. In the previous sections, the building in GeoGebra was much simpler so it was possible to satisfy the boundary errors in all conditions. With three-hinged arches, it is important to try to keep the boundary error as close to zero as possible. The errors caused by boundary conditions will be shown throughout the following sections.

The following example is a simple three-hinged arch. Since the unknown is the vertical reaction of the left pin, it is clear that the left pin must be perturbed vertically. In Figure 16 and Figure 17, the importance of keeping a small perturbation with the horizontal load is especially clear.

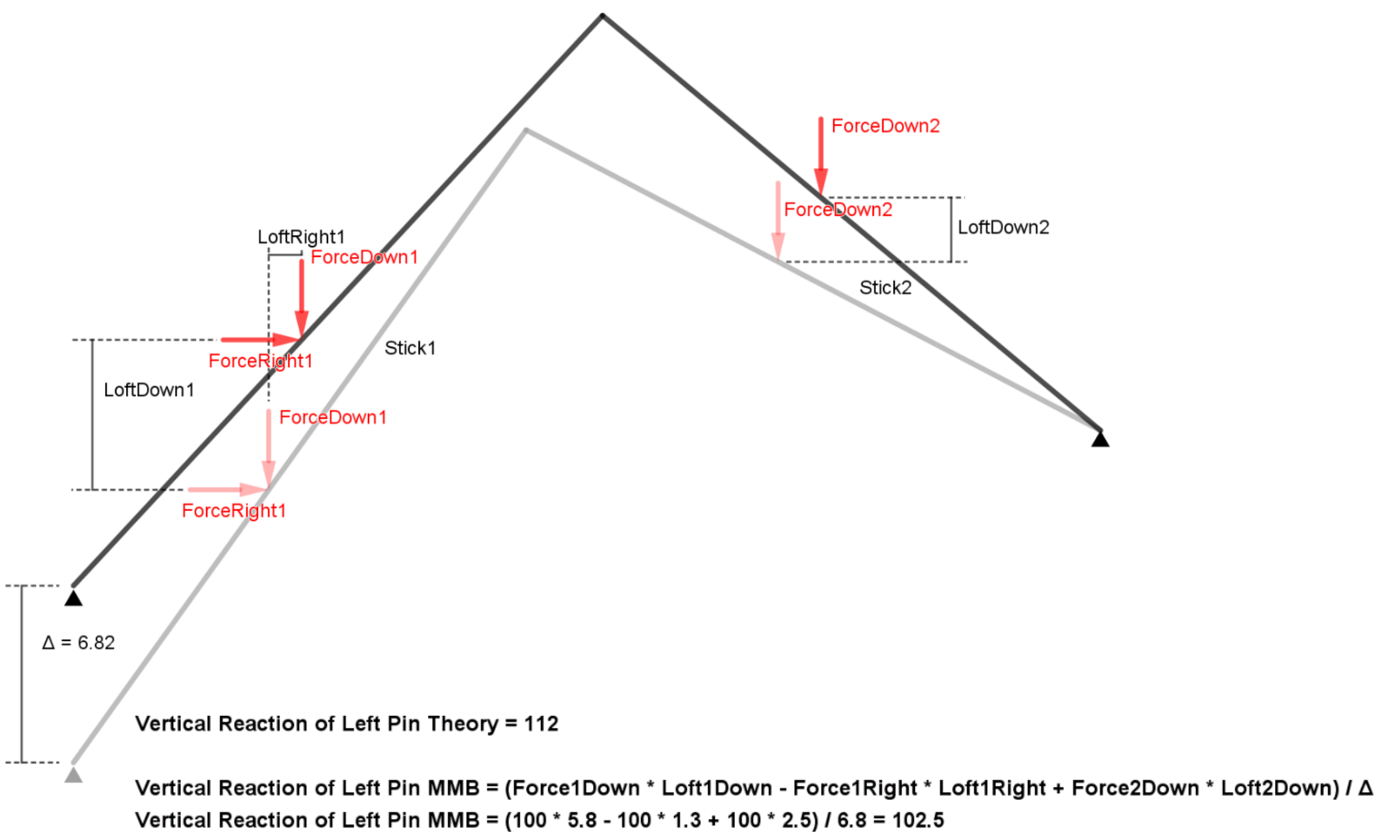


Figure 16. Vertical Support Reaction of Three-Hinged Arch

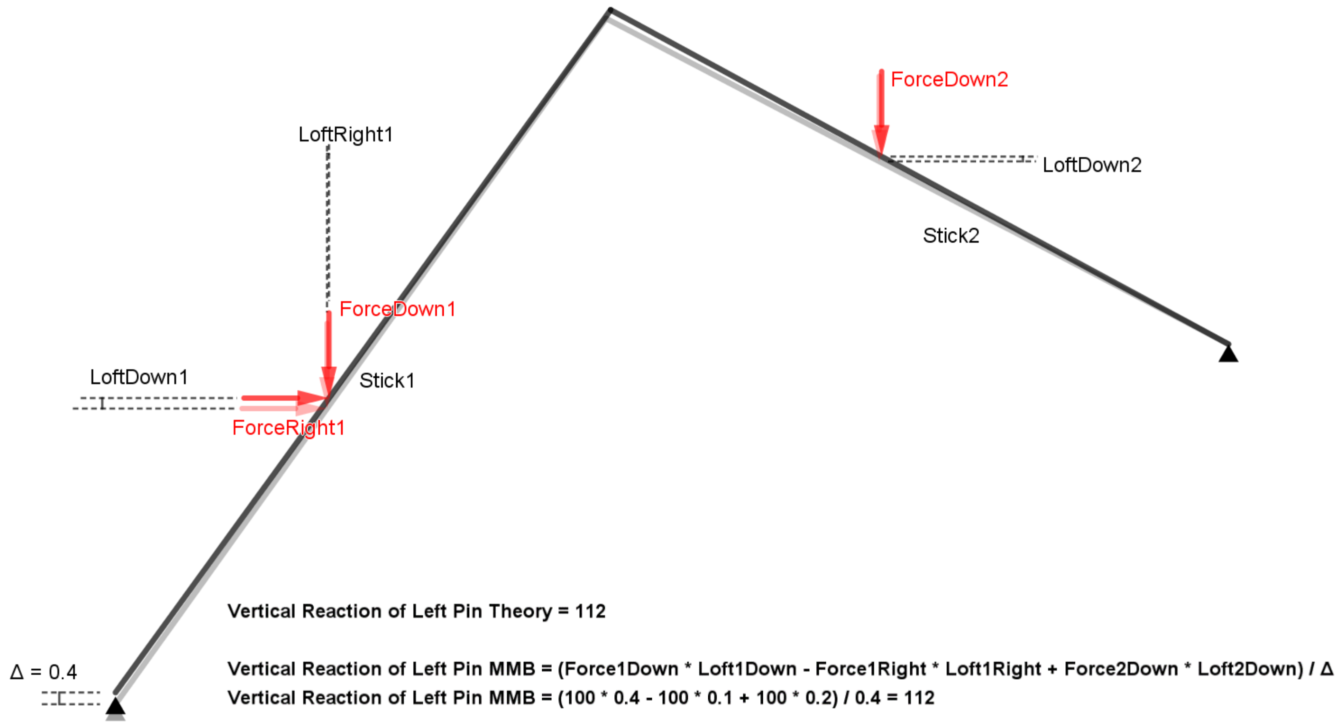


Figure 17. Vertical Support Reaction of Three-Hinged Arch

In Figure 18 and Figure 19, a bend is added to the simple three-hinged arch shown in Figure 16 and Figure 17. This added complexity doesn't have any effect on the way Equation 1 is used to solve this problem. The following example is solving for the vertical reaction of the pin on the right instead of the left. The added complexity of the geometry does influence the way it was built in GeoGebra and there is now a boundary error. Figure 18 has a very large boundary error and therefore has a solution very far from the theoretical solution. This boundary error was brought very close to zero in Figure 19 which gave a solution very close to the theoretical value.

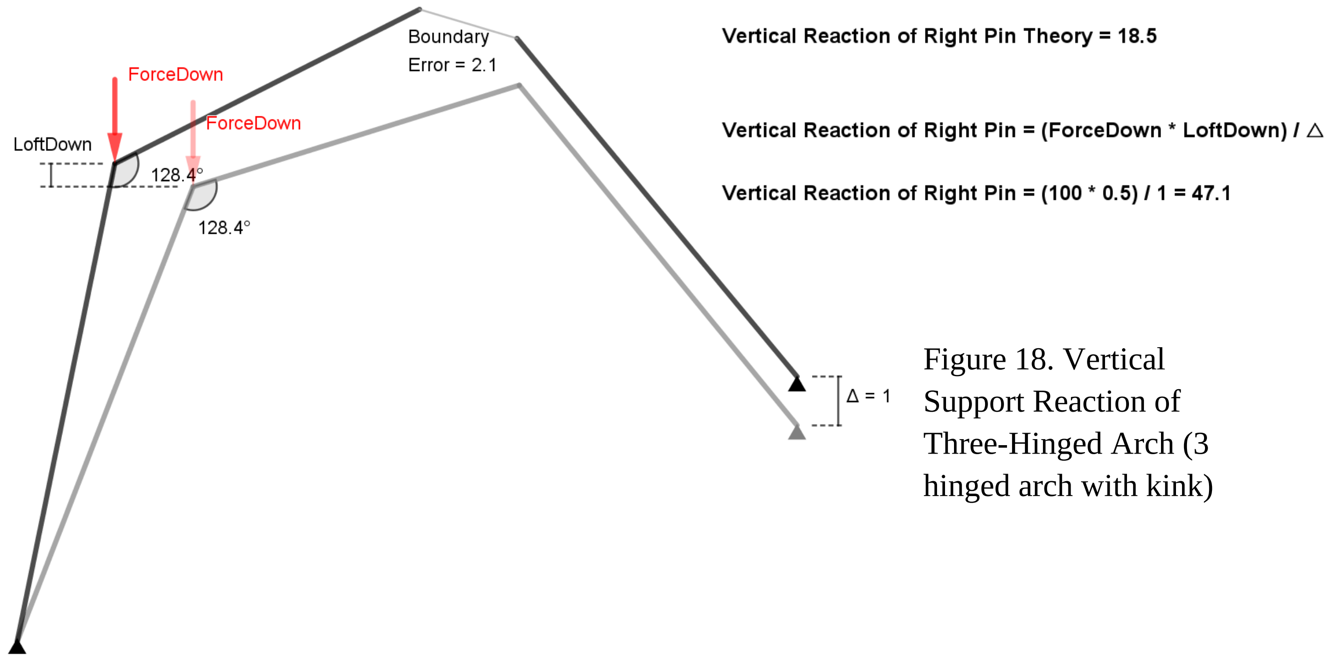


Figure 18. Vertical Support Reaction of Three-Hinged Arch (3 hinged arch with kink)

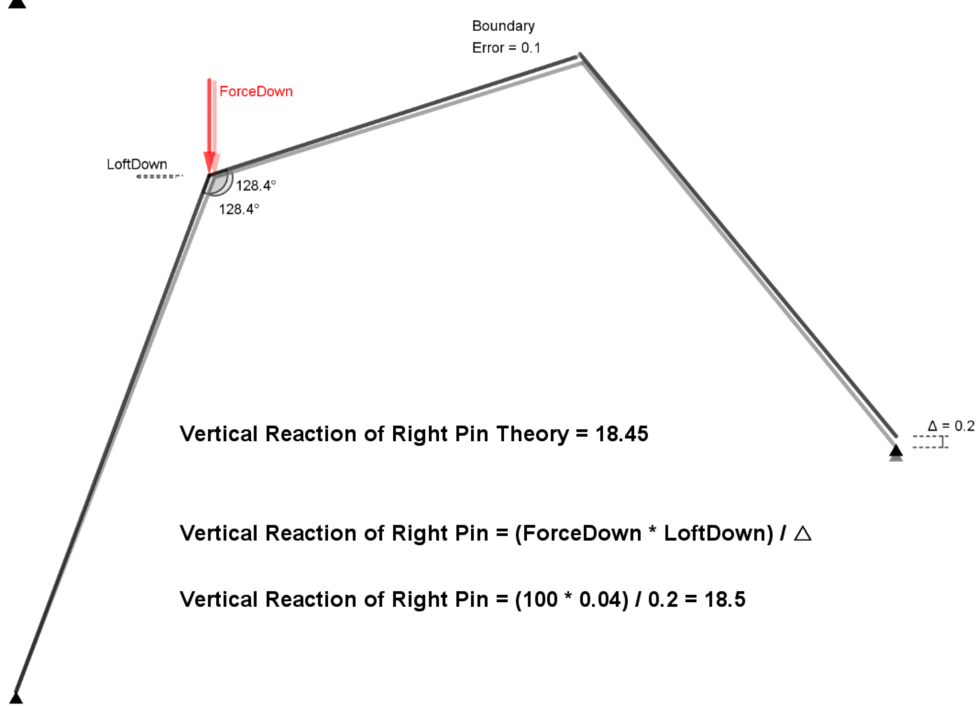


Figure 19. Vertical Support Reaction of Three-Hinged Arch

As shown above, a bend can be added to the three-hinged arch without any change to the process of applying the Modern Muller Breslau Method. Next, a second bend is added to the three-hinges arch. With this geometry, it is again very important to bring the boundary error as close to zero as possible. In addition, there are now two angles that must remain constant through the perturbation. The process of creating this linkage is shown in Figure 7 and Figure 8. Figure 20 and Figure 21 demonstrate that the horizontal reaction of the pin can be found by applying a horizontal perturbation to the pin.

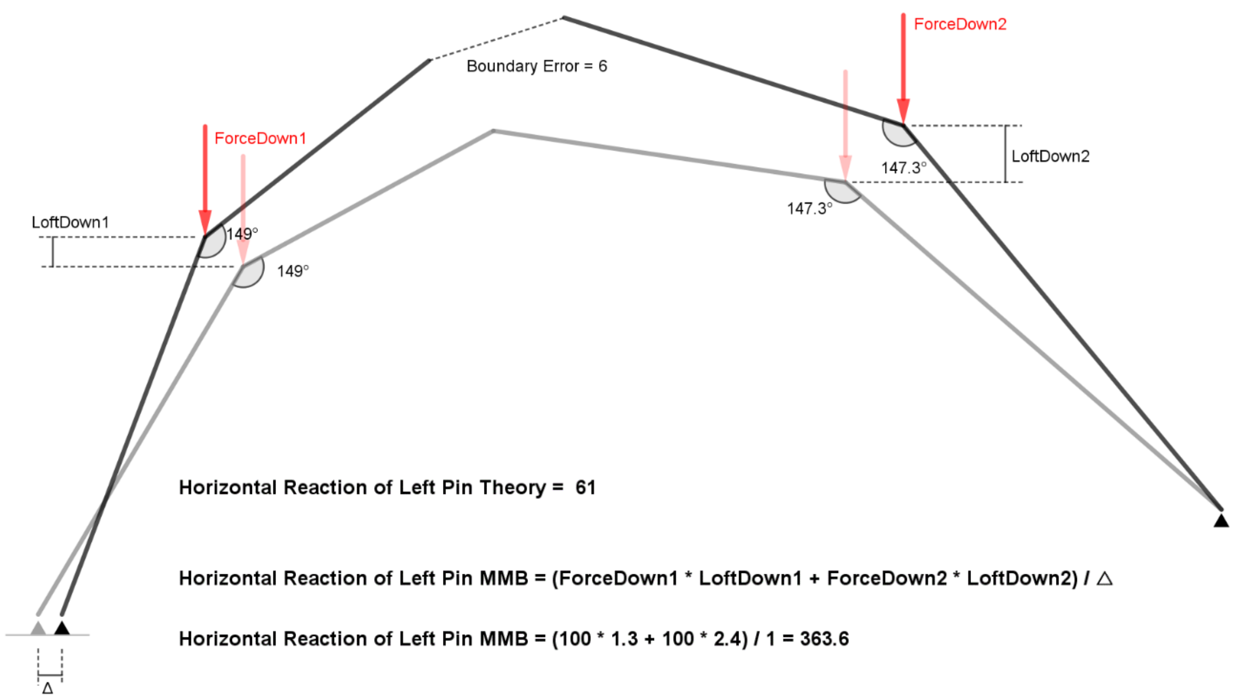


Figure 20. Vertical Support Reaction of Three-Hinged Arch

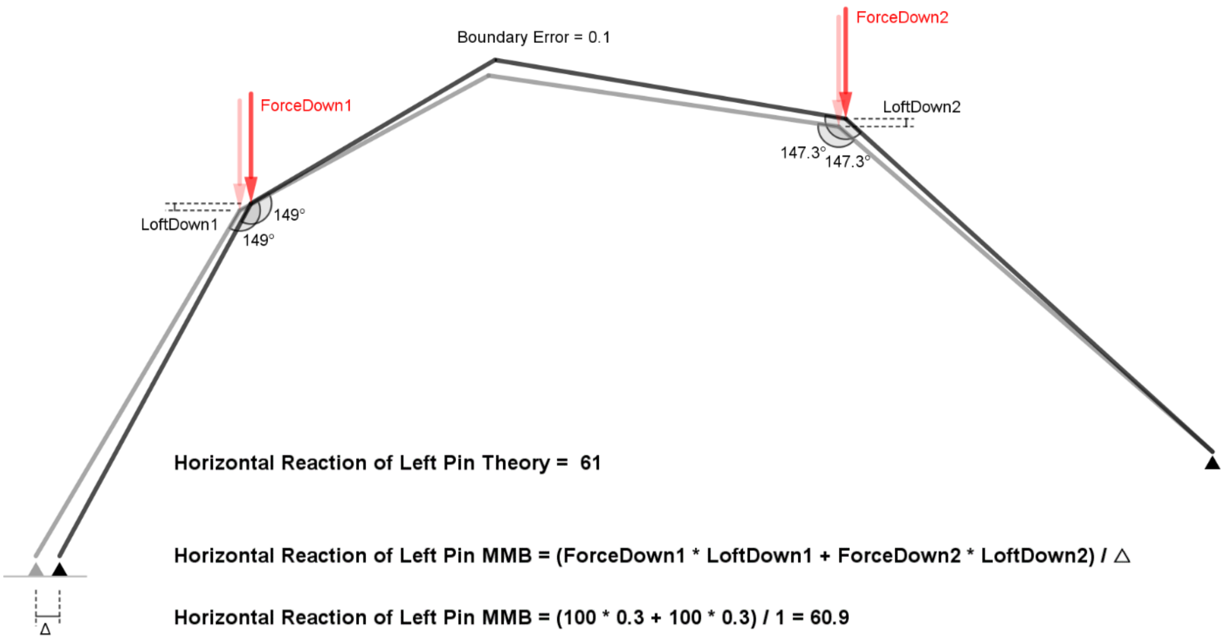


Figure 21. Horizontal Support Reaction of Three-Hinged Arch

Figure 22 and Figure 23 show the same three-hinged arch as Figure 20 and Figure 21, however, here the vertical support reaction is found. The difference in this solution is that the perturbation is now vertical rather than horizontal.

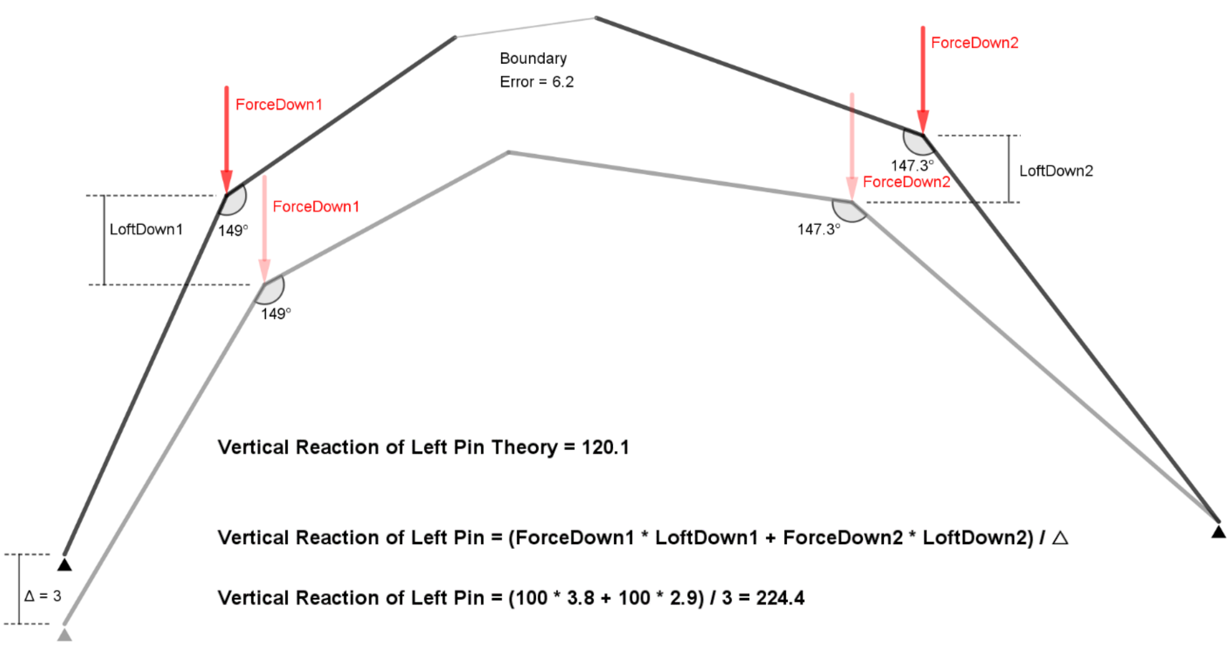


Figure 22. Vertical Reaction of Three-Hinged Arch Support

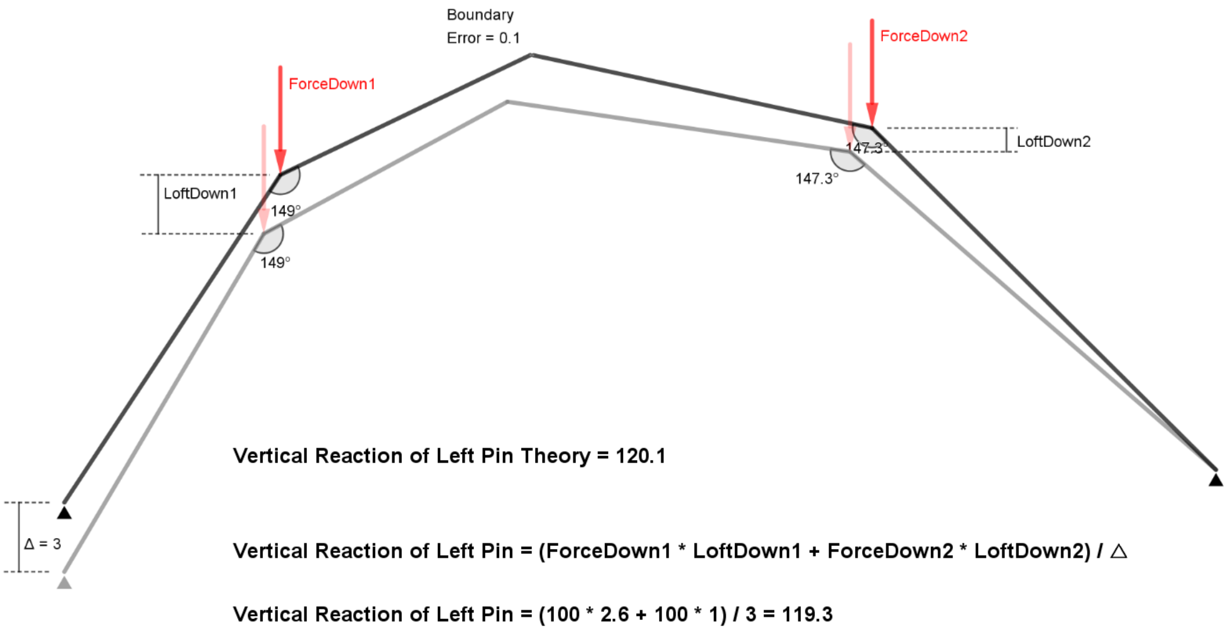


Figure 23. Vertical Reaction of Three-Hinged Arch Support

As mentioned above, in addition to the reactions, the Modern Muller Breslau Method can also be used to find the moment within the members of the three-hinged arch. Finding the moment in one of these members follows the same process as finding the moment in a slanted beam, as shown above. One important point to note is that only the member under investigation will have any perturbation. This is shown in Figure 24 and Figure 25. It is clear that Stick2 remains unaffected by the perturbation.

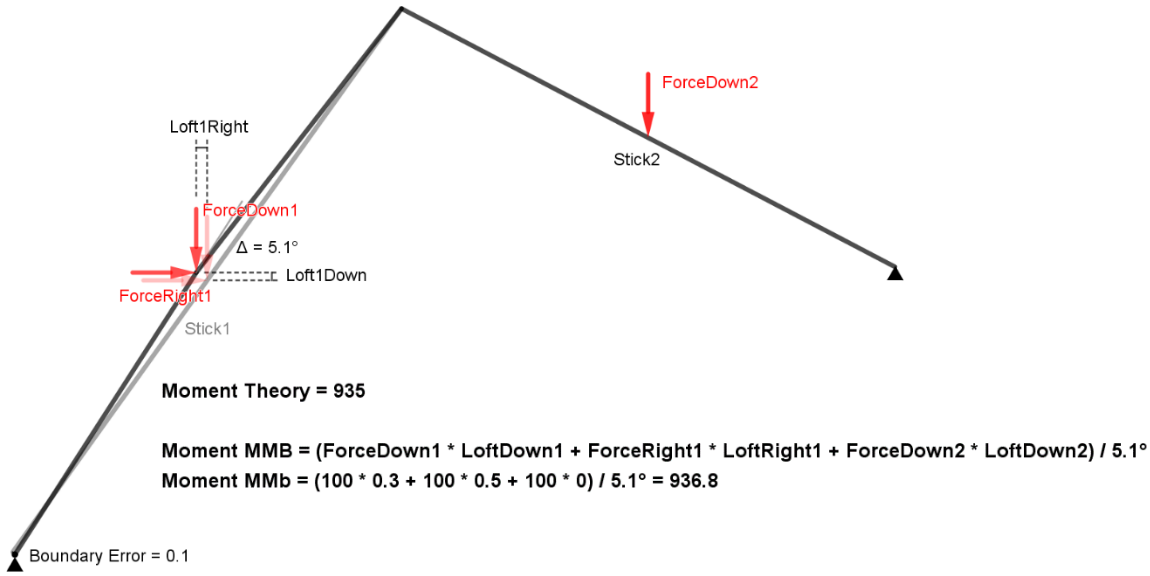


Figure 24. Moment in Three-Hinged Arch

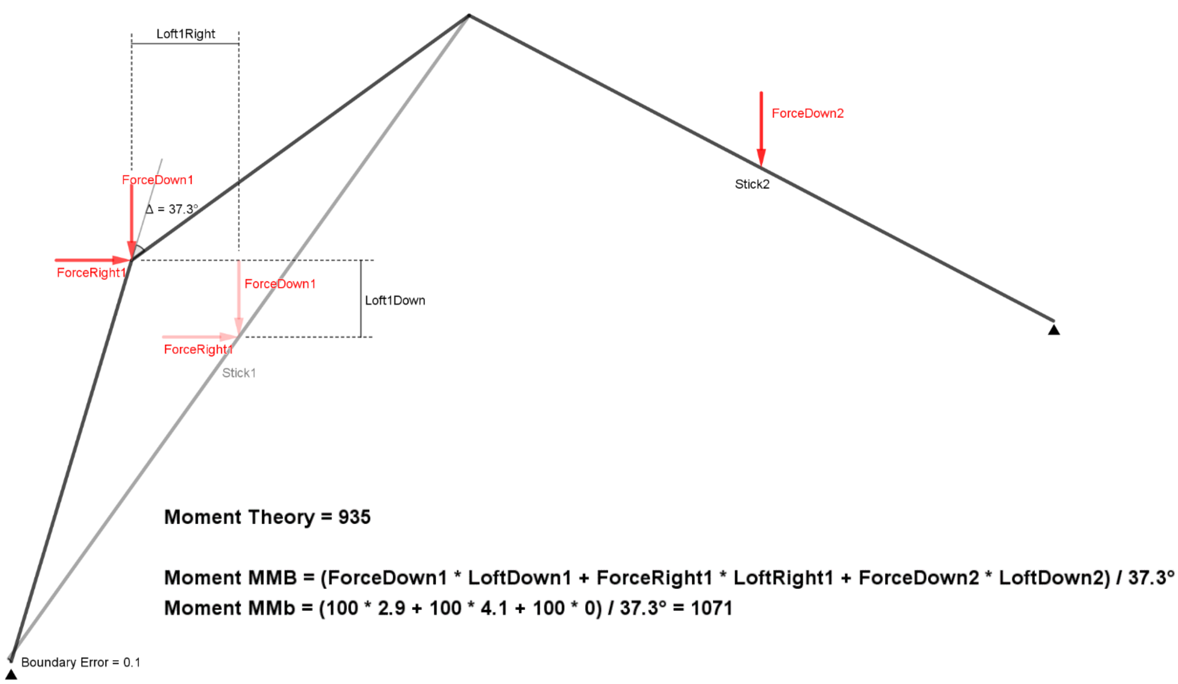


Figure 25. Moment in Three-Hinged Arch

Trusses

In this report, three member trusses and five member trusses are analyzed. To find the support reactions, Equation 1 will be used, and the process will be the same as above. In addition to the reactions of the supports of the trusses, this report will also include how to find the axial force within the truss members. Since this is an internal reaction, Equation 1 will be used in a slightly different form. The internal forces will resist the external forces. Whether the perturbation is an elongation or a compression, the internal forces will oppose the external forces. Figure 26 and Figure 27 demonstrate the directions of the internal and external forces for tension and compression forces.

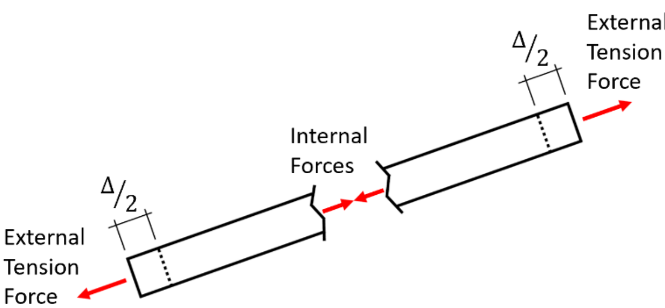


Figure 26. External and Internal Forces

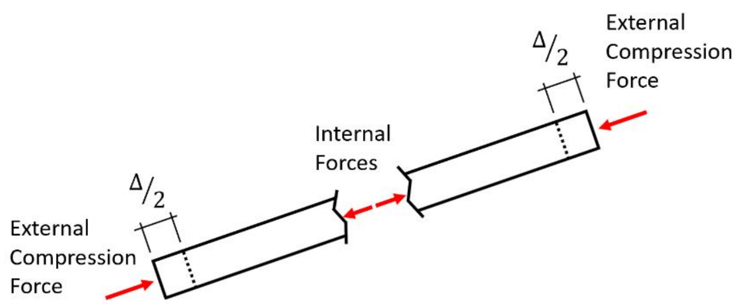


Figure 27. External and Internal Forces

To find the internal axial forces of the members of a truss, a new form of Equation 1 will be used. This equation incorporates how the internal forces oppose the external forces. To represent this, the terms representing the internal and external forces of the system will have opposite signs, as shown below.

$$-(Unknown * \Delta) + \Sigma(Force_i * Loft_i) = 0 \qquad \text{(Equation 1A)}$$

In Figure 28 and Figure 29, the force in Stick 2 is being sought. In order to find this force within the member, the member itself needs to be perturbed. In this case, since it is an axial force to be found in the member, the perturbation is equal to the change in length of the member. These examples are very similar and show that no matter the lengths of the members of the truss, the process does not change.

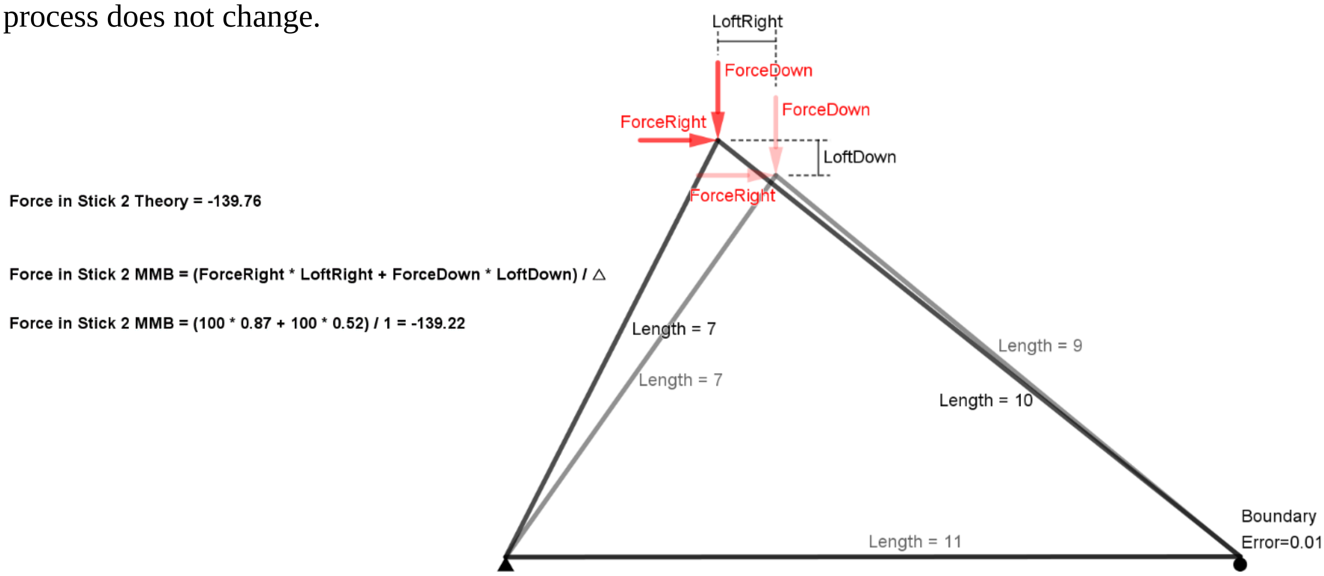


Figure 28. Three Member Truss Axial Force

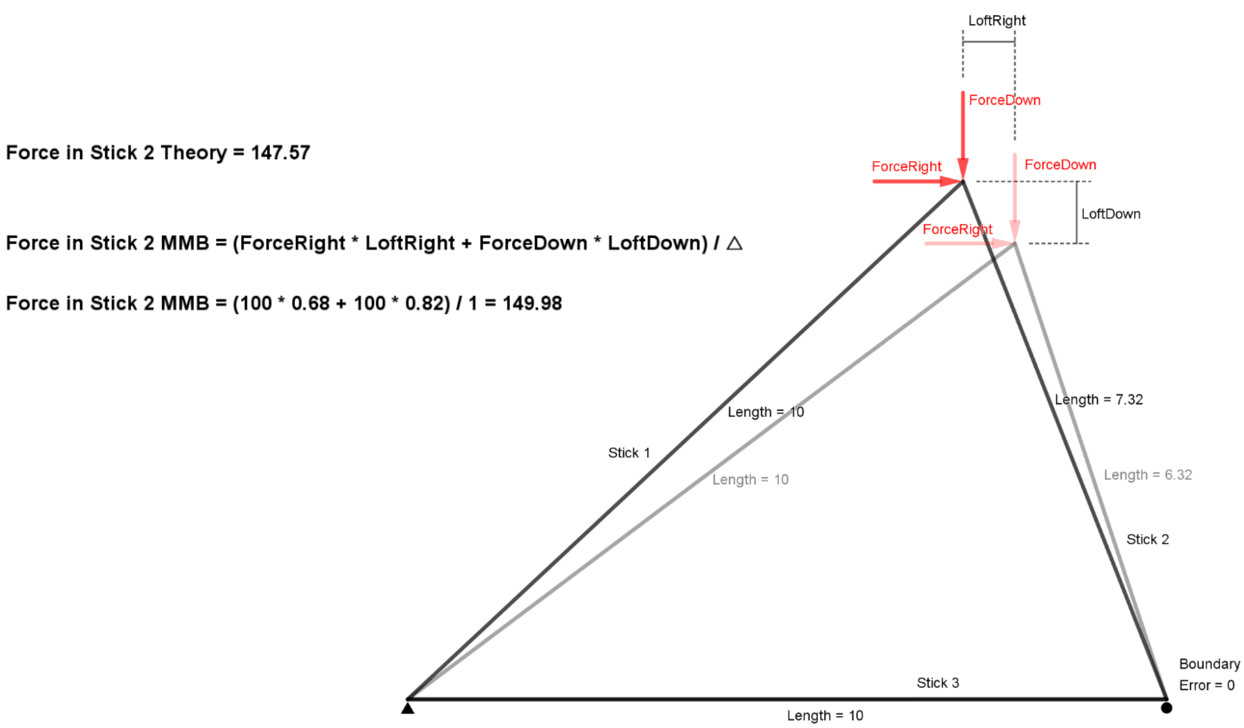


Figure 29. Three Member Truss Axial Force

In the following example, the rection at the roller supporting the truss is being sought. In order to find this reaction, the vertical perturbation is shown in Figure 30. It Is clear by the following figures that the size of the perturbation influences the result. When the perturbation is reduced to 0.01, the value found by the Modern Muller Breslau method is converging to the theoretical value.

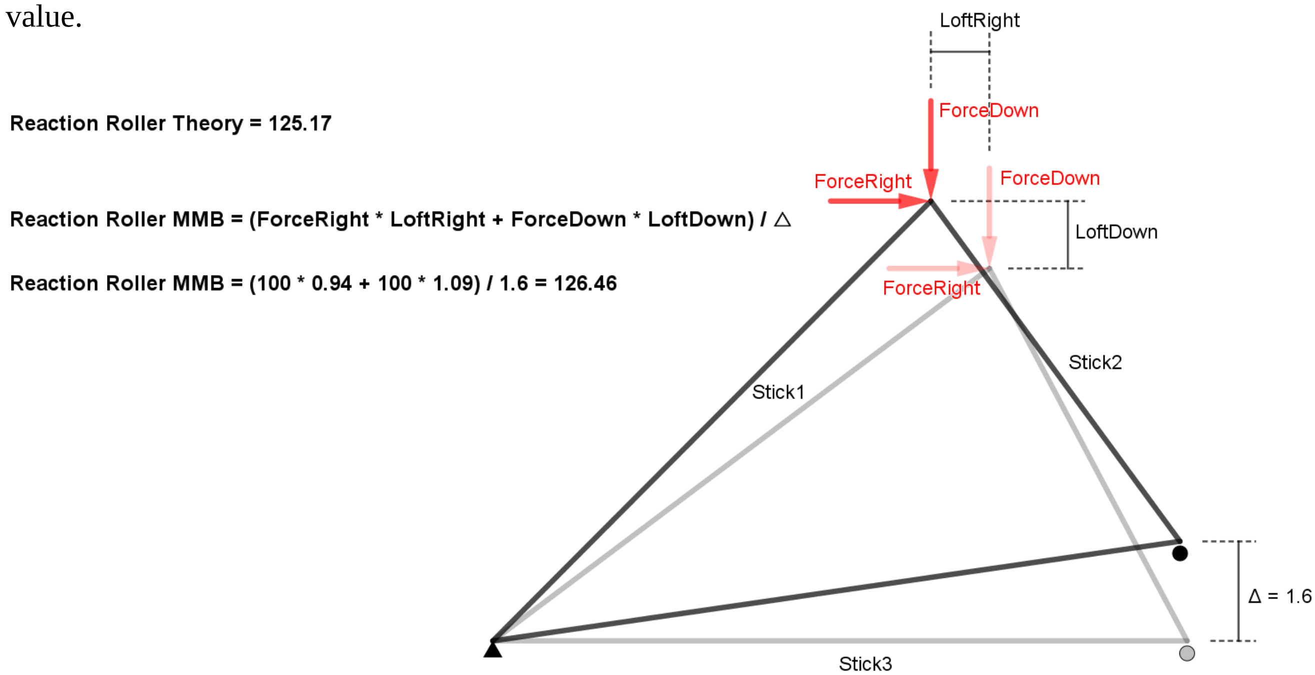


Figure 30. Three Member Truss Axial Force

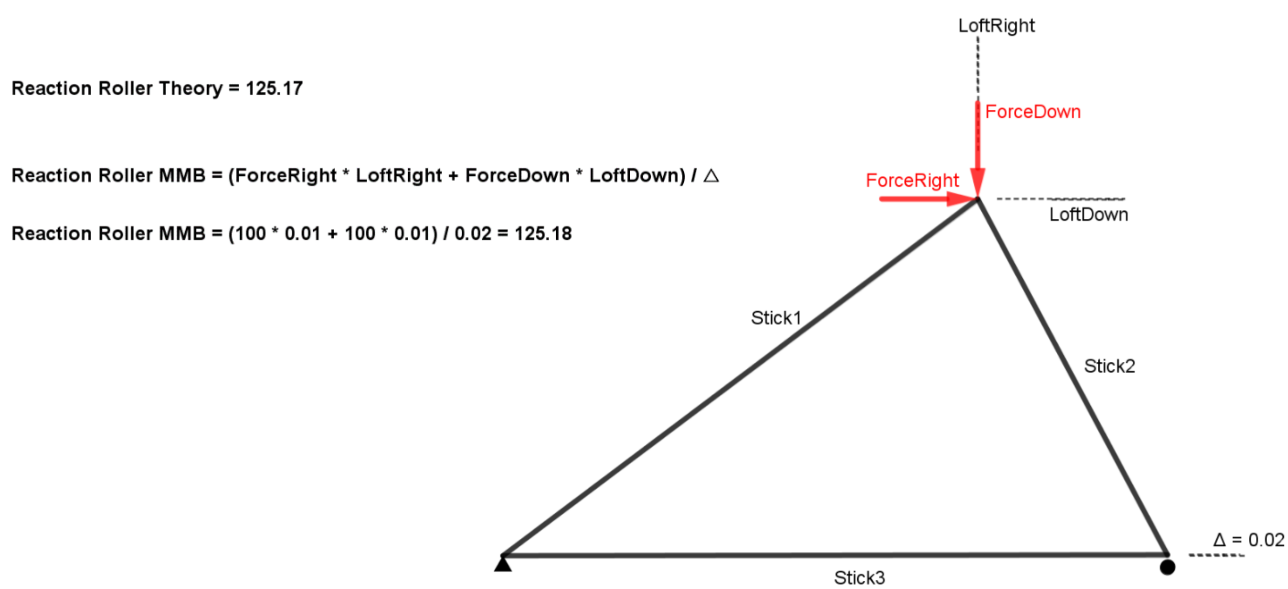


Figure 31. Three Member Truss Support Reaction

The MMB Method has also been proven in the case of a tilted three member truss. This is shown below in Figure and Figure 32. It can be seen that again that the boundary error will make a large difference in the result, therefore it needs to be kept as close to zero as possible.

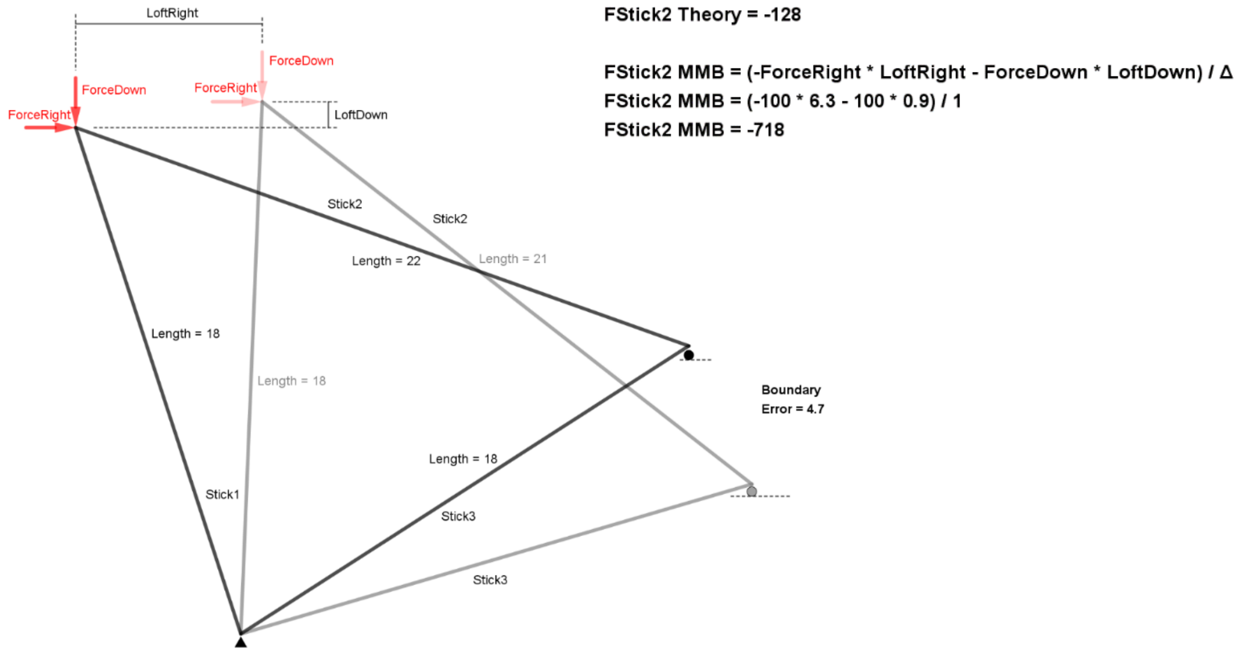


Figure 32. Three Member Truss Axial Force

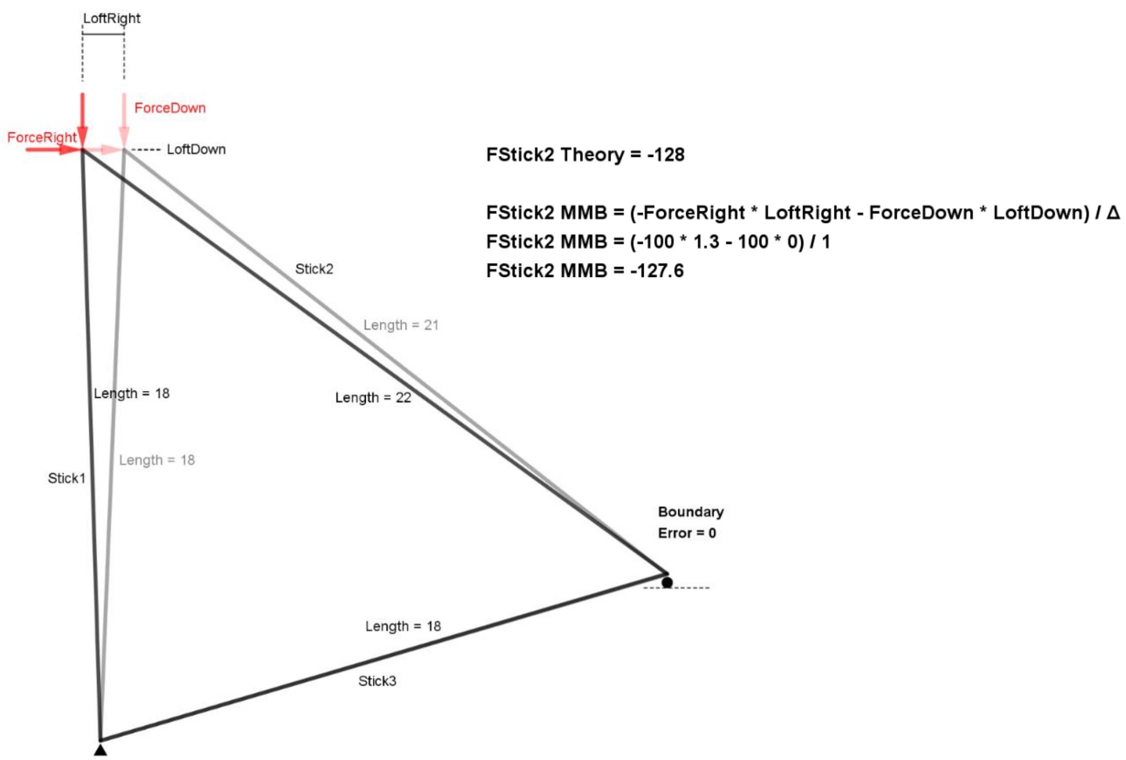


Figure 33. Three Member Truss Axial Force

Through this project, the theory of a game was proved using a combination of GeoGebra examples along with physical models. The idea of this game is to prove how simple it is to solve for the force in members of trusses. To solve for the force, one would start with three members which will be referred to as Stick1, Stick2, and Stick3 moving forward. When solving for the force in one of the sticks, the player would increase the length of that stick by 1 unit. This means that delta, Δ will be equal to 1 to keep the following calculations as simple as possible. The player would then need to rebuild the new truss using the new stick length. After satisfying all boundary conditions, the player should measure the lofts of each force. Lastly, the player will apply equation 1 using the information found by the geometry of the truss to find the force in the stick.

Below, in Figure 34 and Figure 35 an example of the physical model proof of concept of the game along with the corresponding GeoGebra solution is shown. In this example, the player is looking for the force in Stick2, therefore, the original stick with length of 13 was replaced with a stick with a length of 14. Both the Geogebra example and the physical model produce solutions very close to the theoretical solution for the force in Stick2.

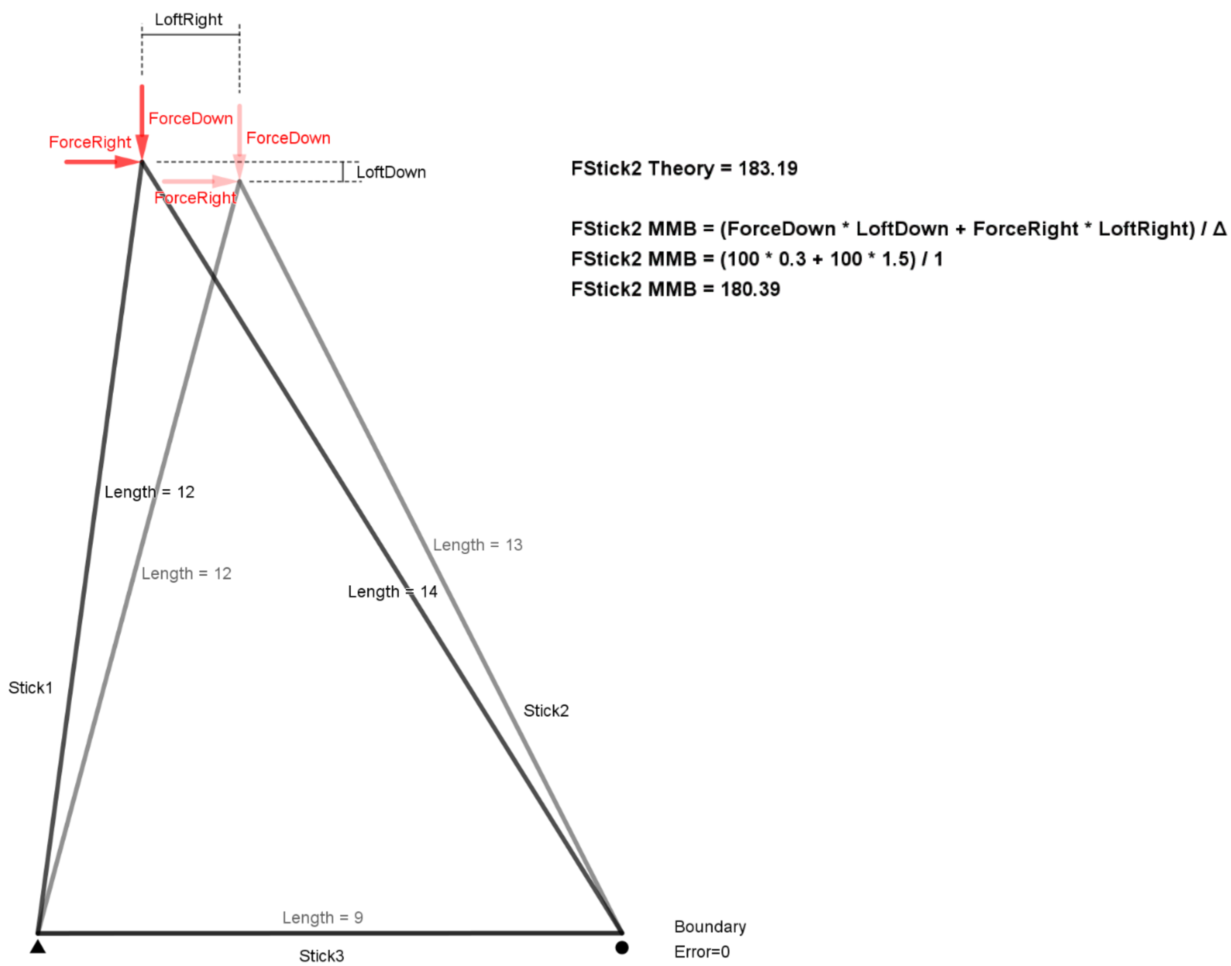


Figure 34. Three Member Truss Axial Force

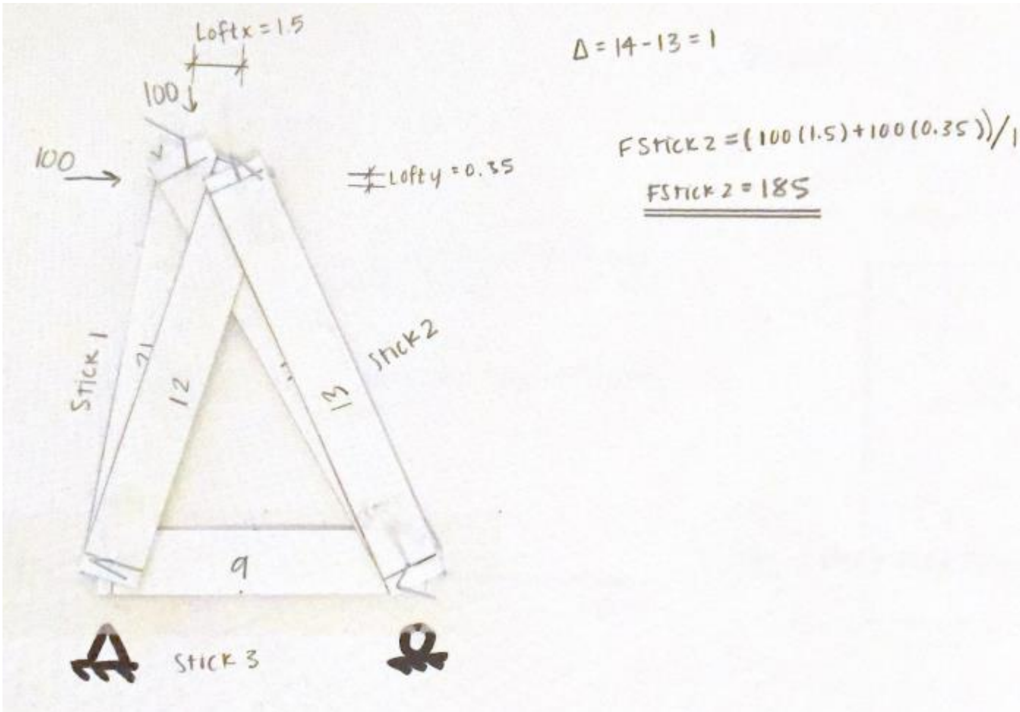


Figure 35. Three Member Truss Axial Force – Physical Proof of Concept

A proof of concept was also proved to find the force in Stick1, shown in Figure 36 and Figure 37. This further shows the validity of this concept. Similar to the first example with the physical model solution, this example also had great success in obtaining a solution very close to the theoretical value with both the GeoGebra solution and the physical proof of concept.

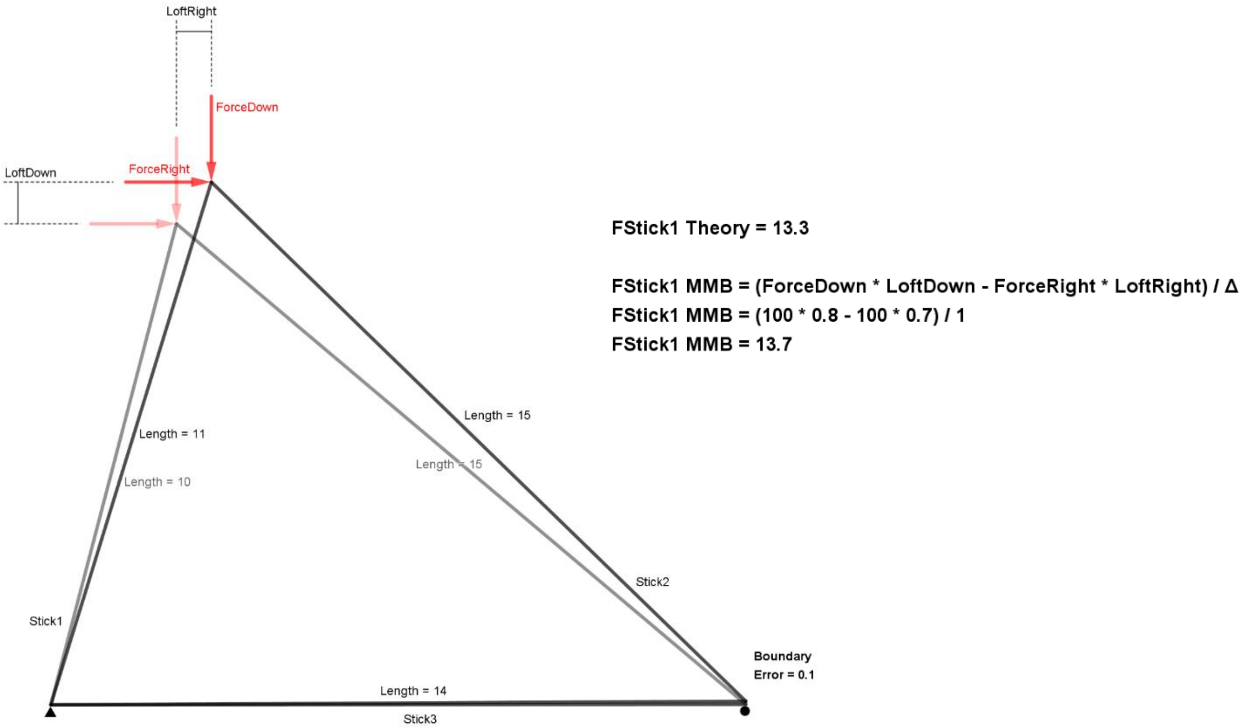


Figure 36. Three Member Truss Axial Force

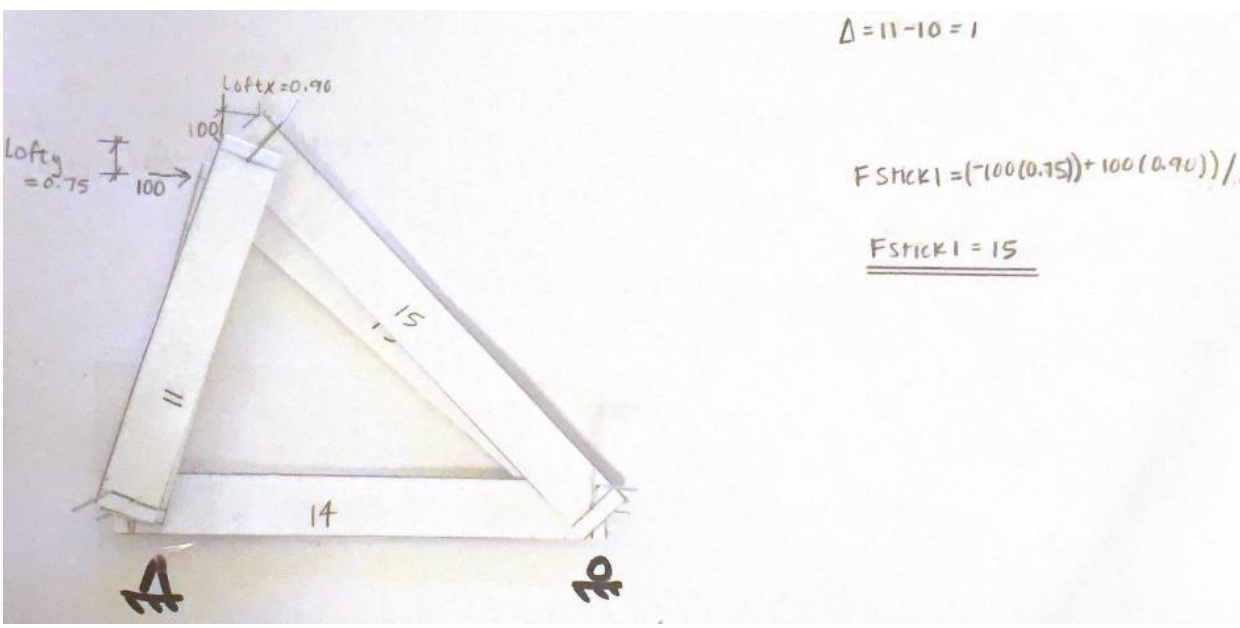


Figure 37. Three Member Truss Axial Force – Physical Proof of Concept

In addition to the three member trusses shown above, five member trusses were also investigated. The example is shown below in Figure 38 and Figure 39. In this example, the force in Stick3 is the unknown. The length of this member increased from 243 to 244 to find this axial force.

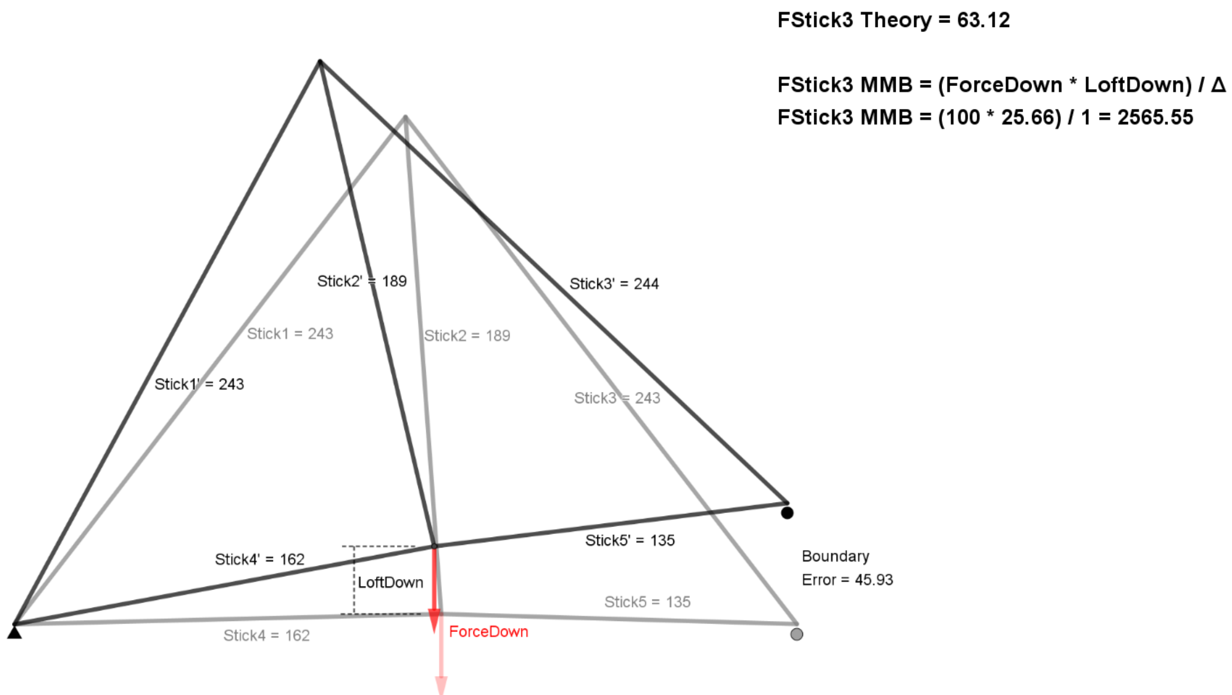


Figure 38. Five Member Truss Axial Force

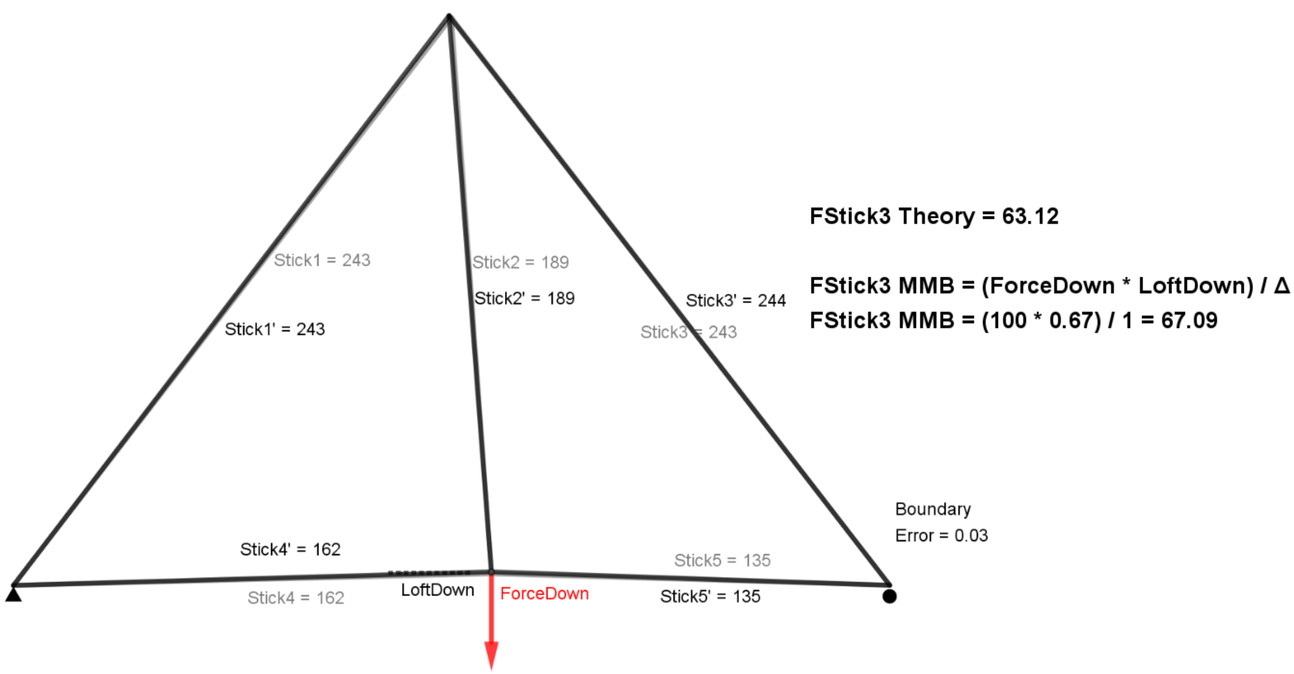


Figure 39. Five Member Truss Axial Force

Determinate Frames

When it comes to analyzing frames, finding the force in specific members can become quite complex. With the Modern muller Breslau method, this process is quite simple and can be done using only equation 1. The following examples show how to find the moments in particular members which is done very similar to above when moments were found in beams. The frames prove to be a bit more of a challenge as the angles in the connection from beam to column cannot change when the shape is perturbed. In GeoGebra, it is important to ensure that the beam and column joist becomes linked, as described below, to keep this condition satisfied. In addition to the angles, the boundary condition at the roller must also be satisfied. Since it is a roller, it can translate along only one direction, so it is important in the case of the frames that there is not any substantial vertical displacement.

The following examples show the same frame and solve for the moment in the beam and one of the columns. To create this in GeoGebra, the original shape of the frame needs to be constructed. This can be done by defining the height of the columns as well as the length of the beam. Next, one should create a circle with radius equal to the height of the columns and center around the pin on the left. This circle will ensure that the length of the left column does not change. Once that is created, simply put a point on the circle as shown in Figure 40. Next, the Angle with Given Size GeoGebra function is used. Refer to the process described above for linked beams. The beam on the left side of the crack is dependent on the location of the crack to ensure that the location of the crack is preserved in the perturbed shape. Next, another circle is formed to create the perturbed right half of the beam. To create the right column, the angle of the column beam connection must be 90° and the length of the column must stay the same, as shown in Figure 41. With this construction, the vertical displacement of the crack can be changed which will affect the relative angle of the beam. In addition, the right column should be moved to ensure a small boundary error.

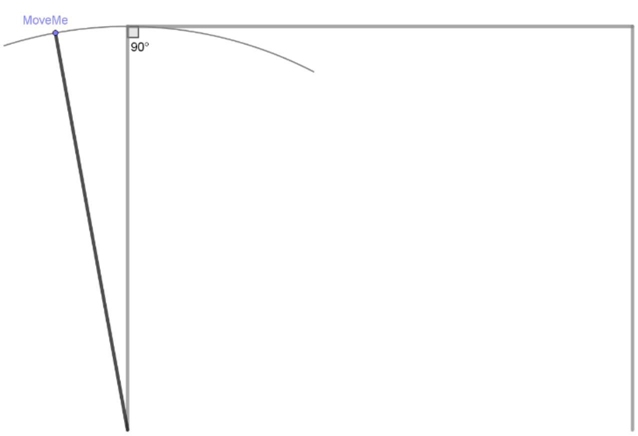


Figure 40. Left Column Construction

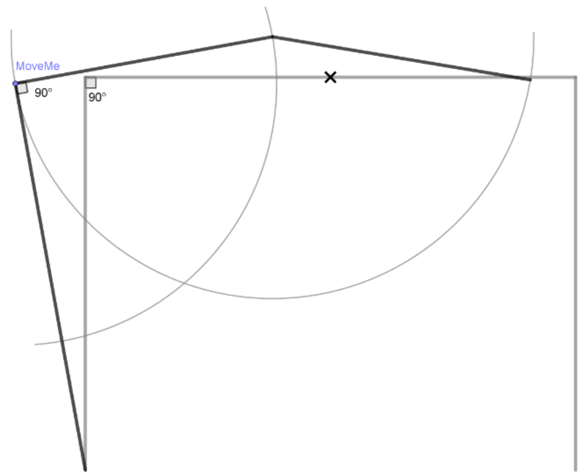


Figure 41. Cracked Beam Construction

As mentioned above, the MMB Method can be used to find the moment within the members of a determinate frame. Similar to some previous sections, the boundary errors need to be as close to zero as possible. In Figure 42 and Figure 43, the moment in the beam of the determinate frame is found. Figure 42 demonstrates the solution when there is a large boundary error and a large perturbation. In Figure 43, that boundary error and perturbation were both reduced.

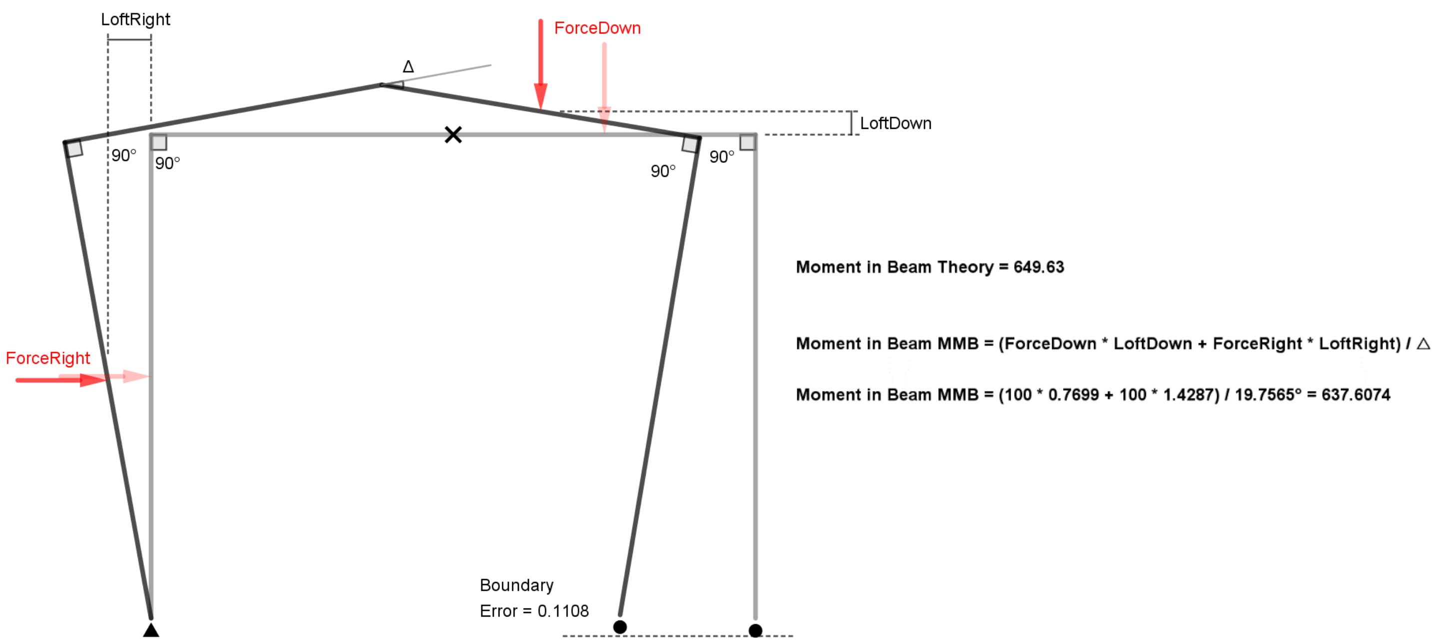


Figure 42. Moment in Beam of Frame

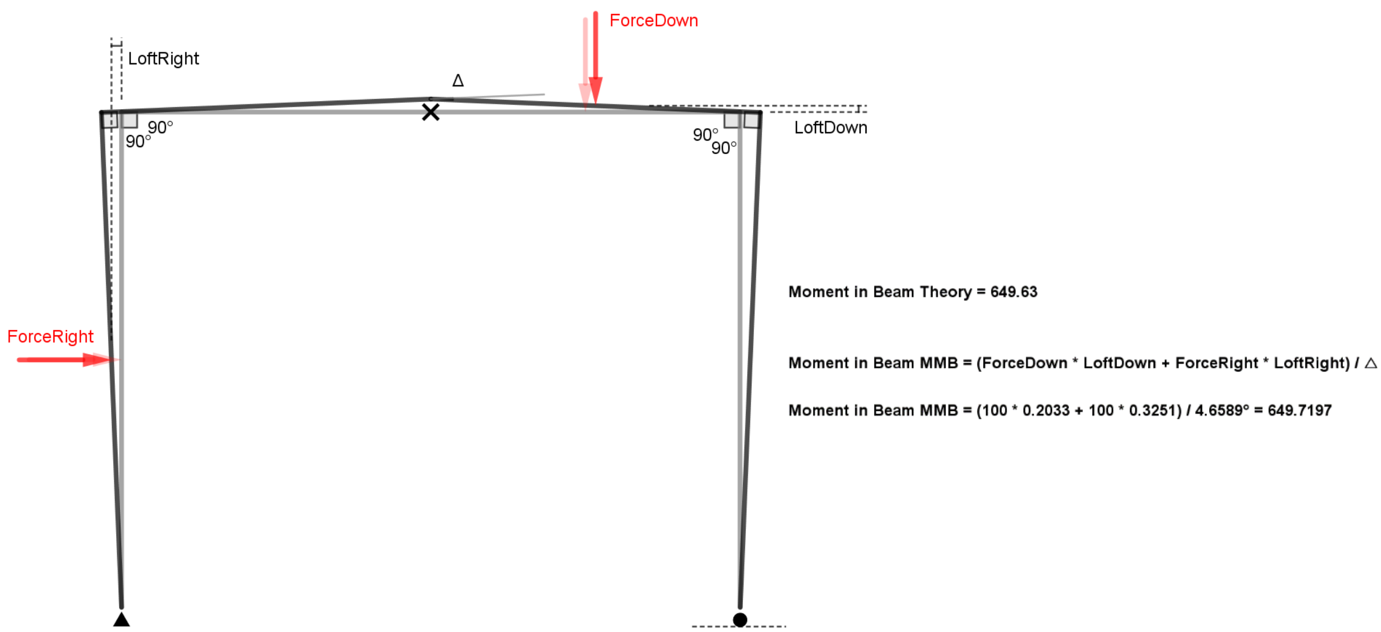


Figure 43. Moment in Beam of Frame

In addition to finding the moment in the beam of the frame, the moment in the columns can also be found very simply. Figure 44 shows the method to find the moment in the left column of the same frame shown in Figure 42 and Figure 43.

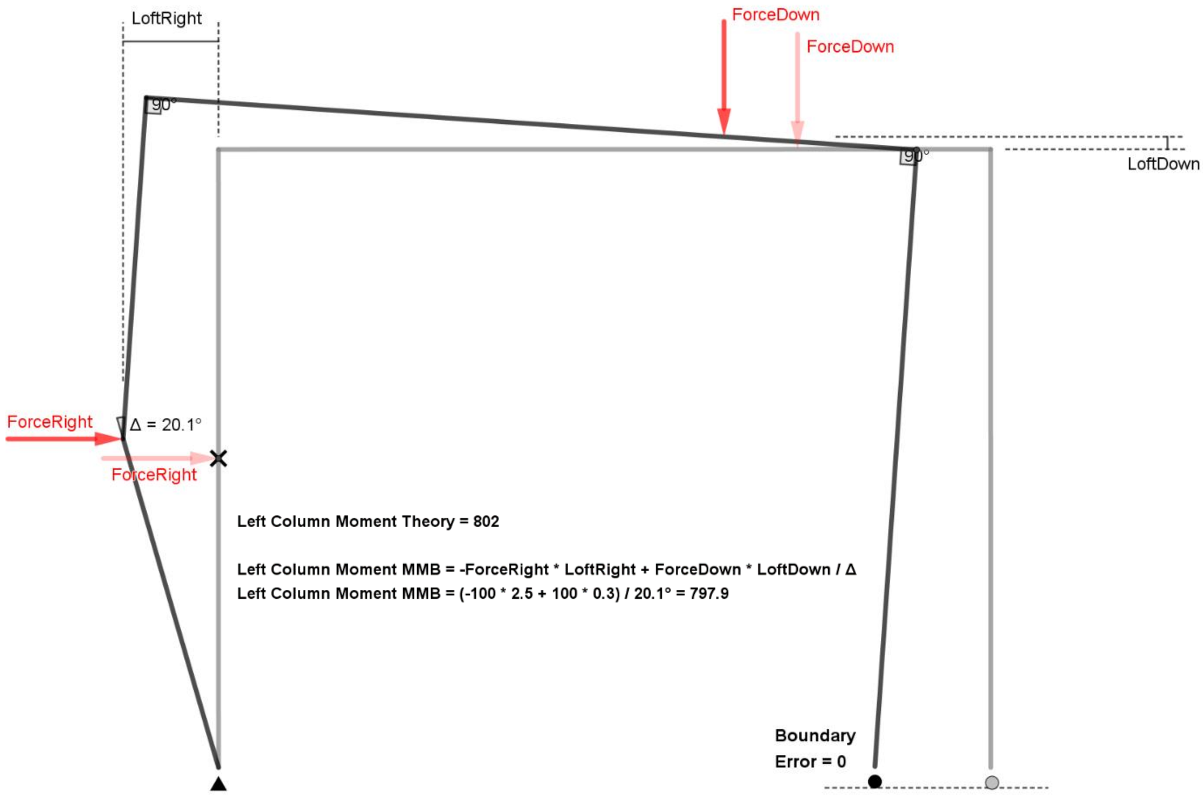


Figure 44. Moment in Left Column of Frame

Plastic Analysis

The Modern Muller Breslau Method can also be used for plastic analysis. The application of this method in regard to plastic analysis is finding the minimum force to create a plastic hinge as well as the location where the plastic hinge will occur. To find the location, the example was created on GeoGebra using a crack that has the ability to slide along the beam. This gives the ability to slide the crack along the length of the beam while finding the corresponding force to create a plastic hinge at every location along the beam. Without knowing the location of the hinge, this makes it very simple to see where the minimum force would be required. In solving these problems, an altered form of Equation 1 will be used, similar to the adjustment made to find the axial forces within the truss members. In this application, Figure 45 and Figure 46 show the way that the internal and external components oppose each other. For a positive rotation as shown in Figure 45, the internal moment is reacting to the positive rotation with a negative moment. The opposite is happening in Figure 46.

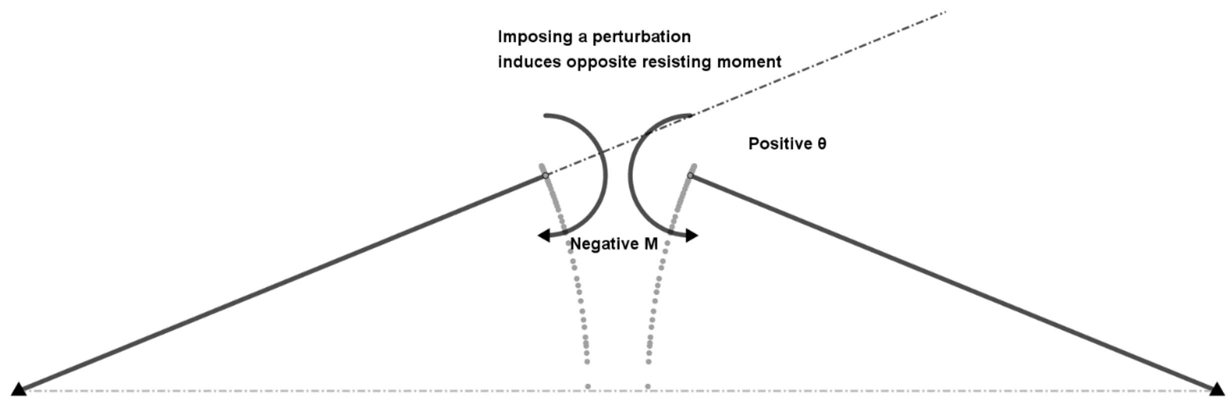


Figure 45. Work of Internal Moment

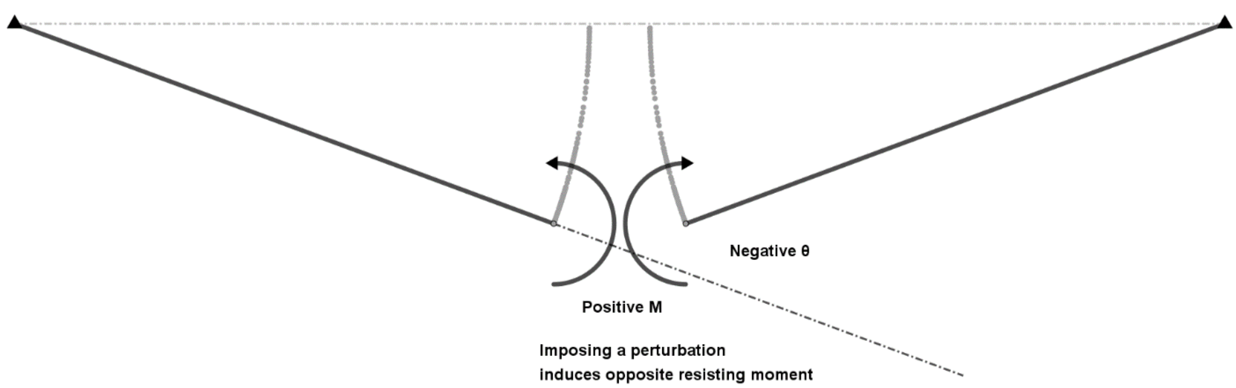


Figure 46. Work of Internal Moment

As shown in the figures above, the sign of the moment is always opposite the sign of the theta created. This is represented in the following equation by imposing a negative sign on the term with Moment and θ, as shown below.

$$\sum -(Moment_i \cdot \theta_i) + \sum Force_i \cdot loft_i = 0 \qquad \text{(Equation 1B)}$$

In order to have the ability to slide the crack along the length of the member, a new GeoGebra technique was introduced. This is the minimum function which helps to find the accurate value of the loft depending on where the member is being cracked. To start the construction of Figure 50, the original beam with a fixed beam should be created. Then the perturbed shape with the crack. This should be done by placing the crack as a point on the original beam that can slide along the beam. The perturbed shape should be dependent on the crack and therefore should adjust to where the crack is located. This idea is clear in Figure 45 and Figure 46. Whether the location of the crack is on the left or the right of the load determines where the loft should be measured. To set this up, start by drawing two segments of where the loft would be with the location of the crack on either side of the load. These segments are called L1 and L2 and are shown in Figure 45 and Figure 46.

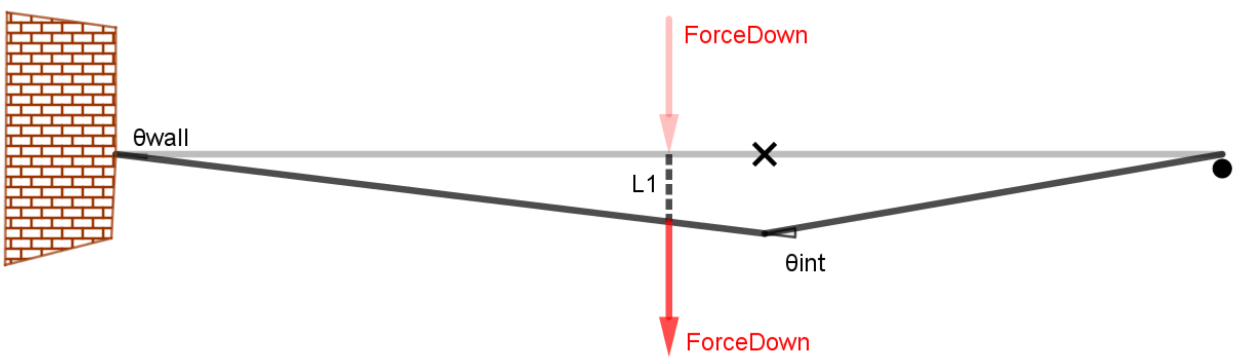


Figure 47. Propped Cantilever Plastic Analysis Construction

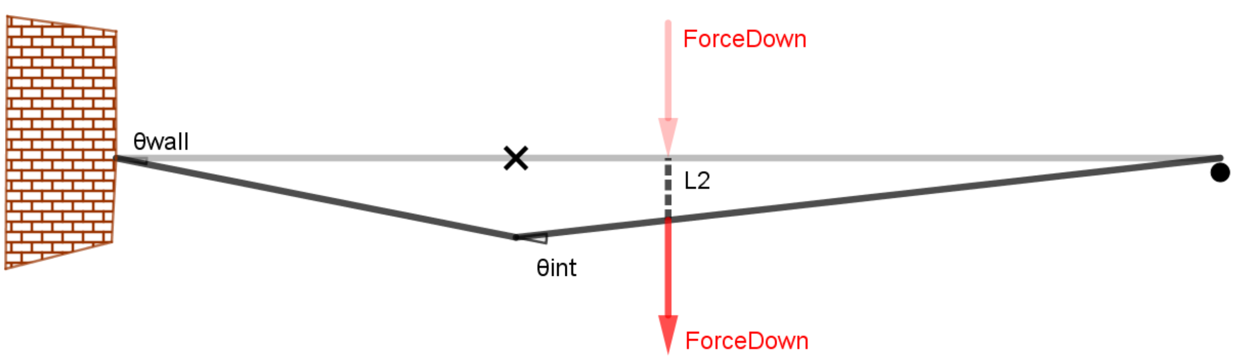


Figure 48. Propped Cantilever Plastic Analysis Construction

Once the segments are created, the length of the loft can be defined. This is where the minimum function is used. The GeoGebra input for this length is shown in Figure 47.

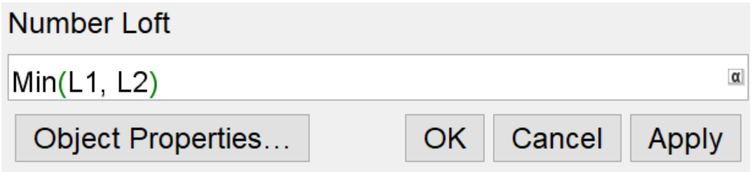


Figure 49. GeoGebra Input - Minimum Function

To complete the construction, a circle with center at the point of the original force with radius equal to the length of Loft needs to be created. Next, a point at the intersection of that circle and a vertical line needs to be found. Lastly, the loft can be drawn using the point of the load and the intersection point which represents the length of the loft.

Using the GeoGebra techniques described above, the propped cantilever in Figure 50 and Figure 51 can be analyzed below. To find F1, Equation 1B will be used and shown below.

$$\sum -(Moment_i \cdot \theta_i) + \sum Force_i \cdot loft_i = 0 \qquad \text{(Equation 1B)}$$

$$\sum Force_i \cdot loft_i = \sum (Moment_i \cdot \theta_i)$$

$$F1\ MMB = (\theta_{int} + \theta_{wall})/Loft$$

In Figure 50, the minimum force to create a plastic hinge at 4 units from the left is 0.79 * Mpr, while in Figure 51, the minimum force to create a plastic hinge at 5 units from the left is 0.6 * Mpr. As mentioned above, the location of the plastic hinge can be moved to any location on the beam. The following figures demonstrate how one can converge to the correct solution of the location of the plastic hinge with this configuration.

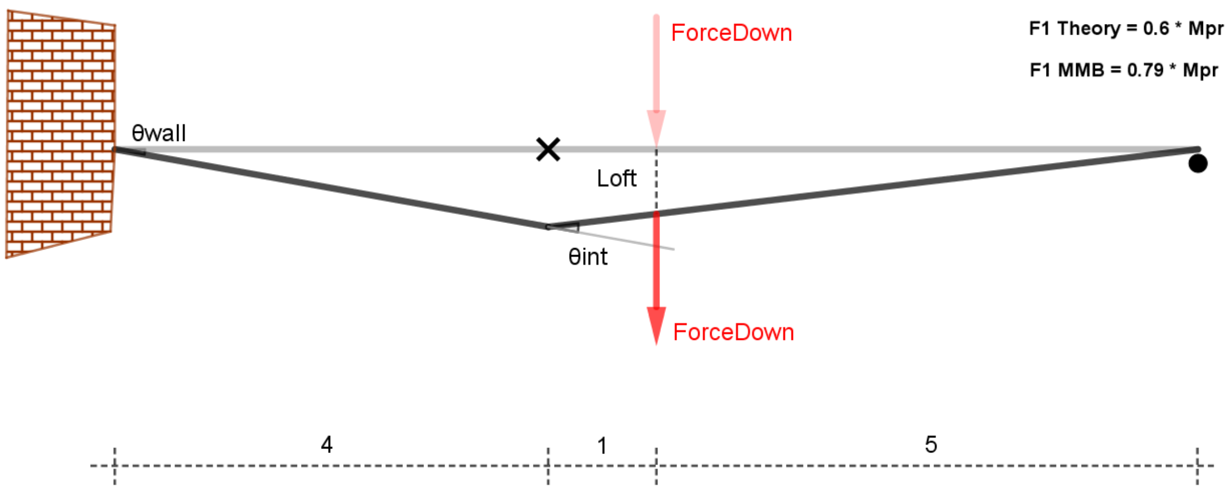


Figure 50. Propped Cantilever Plastic Analysis

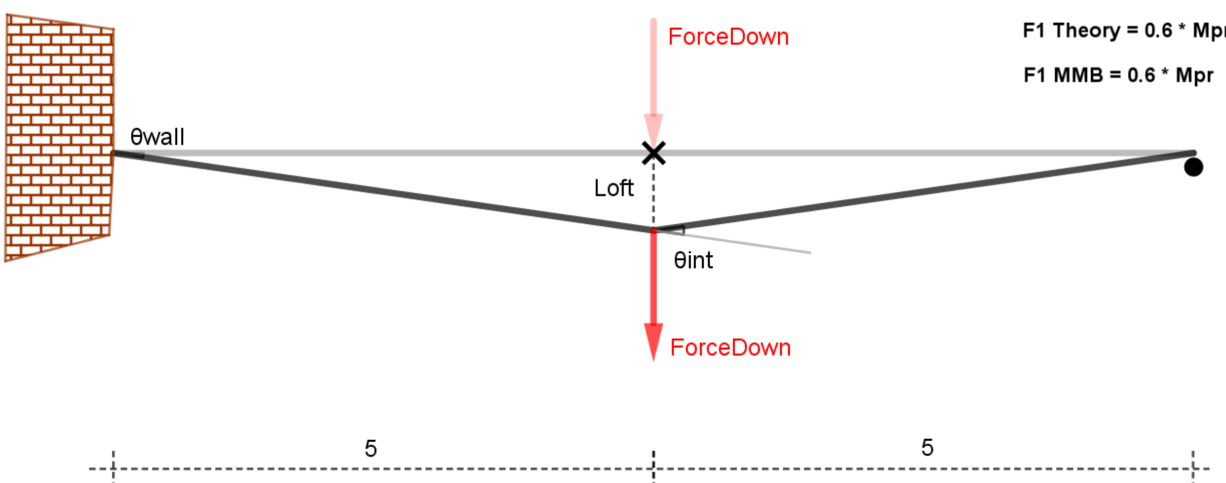


Figure 51. Propped Cantilever Plastic Analysis

Figure 52 and Figure 53 show is a propped cantilever with two point loads of different magnitudes applied. The load on the right is one and a half times as large as the point load on the left. Using Equation 1 once again, FMin can be found by sliding the location of the cut along the length of the beam. As the cut is moving along the beam, one would note where the value of FMin is the smallest and that is the location that the plastic hinge will form.

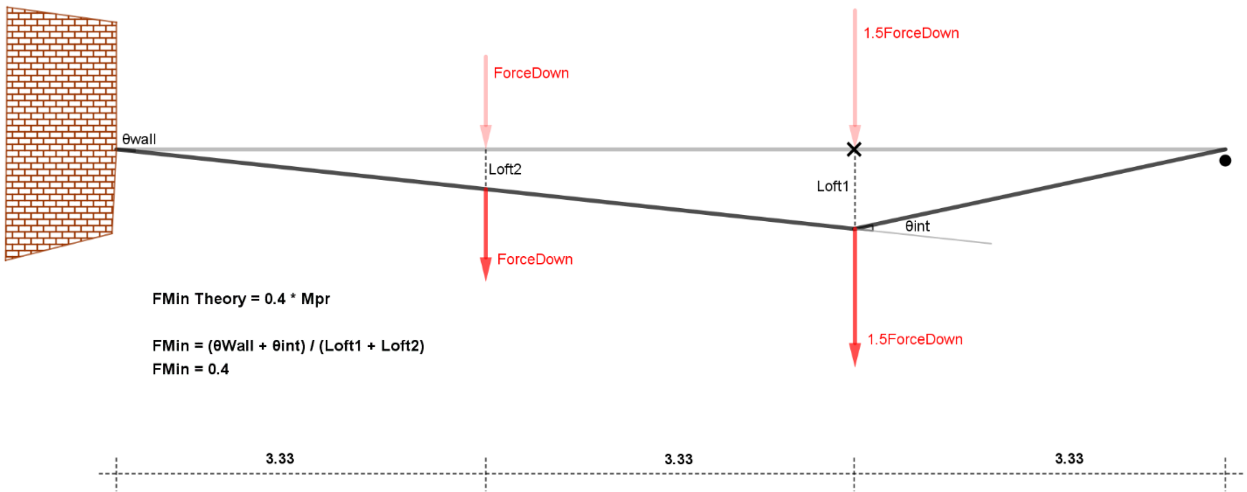


Figure 52. Propped Cantilever Plastic Analysis

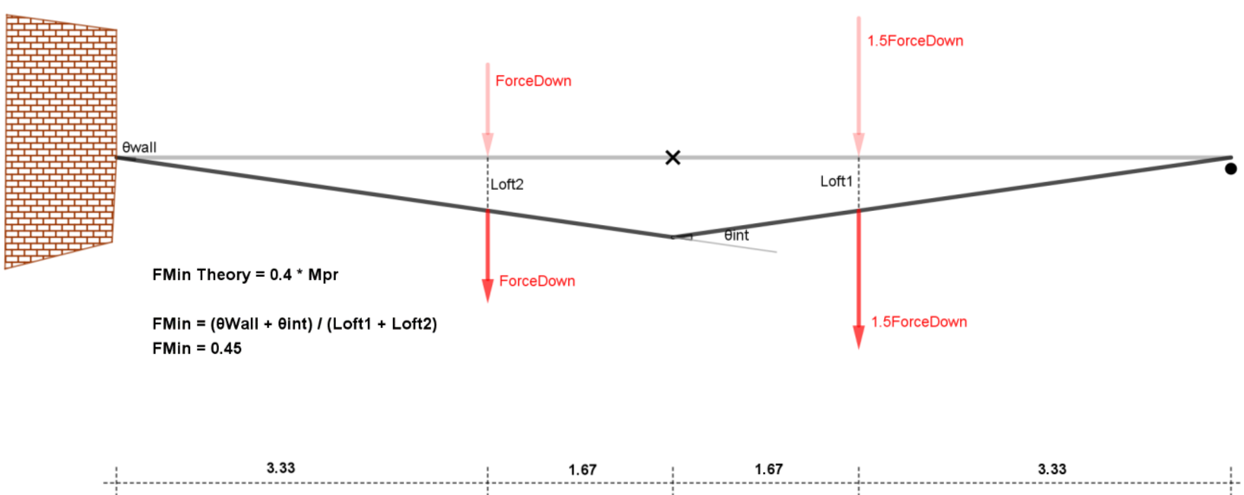


Figure 53. Propped Cantilever Plastic Analysis

Next, this theory was applied to an example where it may not be immediately clear which bay will first form a plastic hinge. The beam shown in Figure 54 and Figure 55 is determinant to the first degree and therefore will need two hinges to fail. For this reason, it is assumed that the two modes for a plastic hinge would be in the midspan of either bay in this beam. The force required to form this plastic hinge is again found by using Equation 1B.

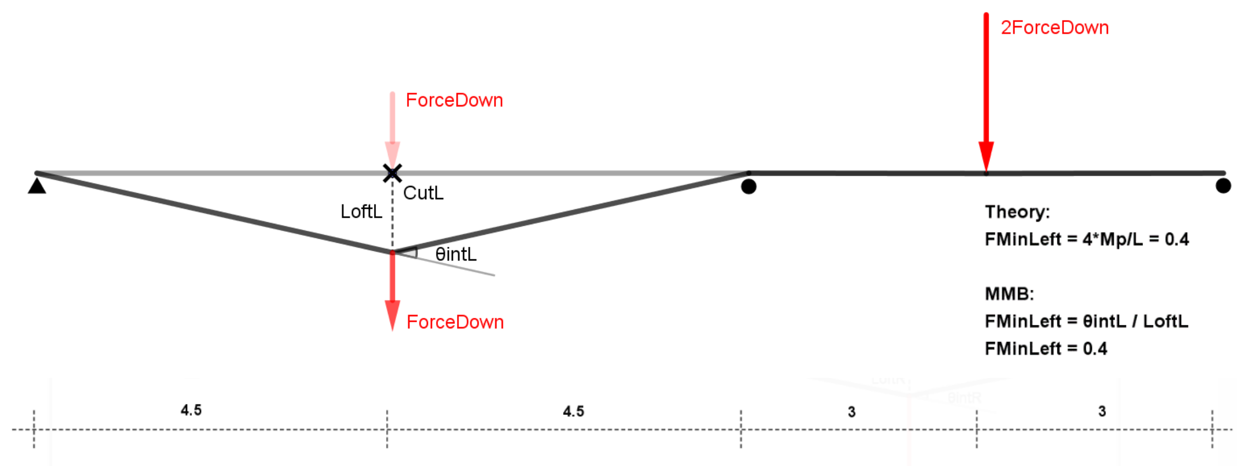


Figure 54. Plastic Analysis of Two-Bay Indeterminate Beam

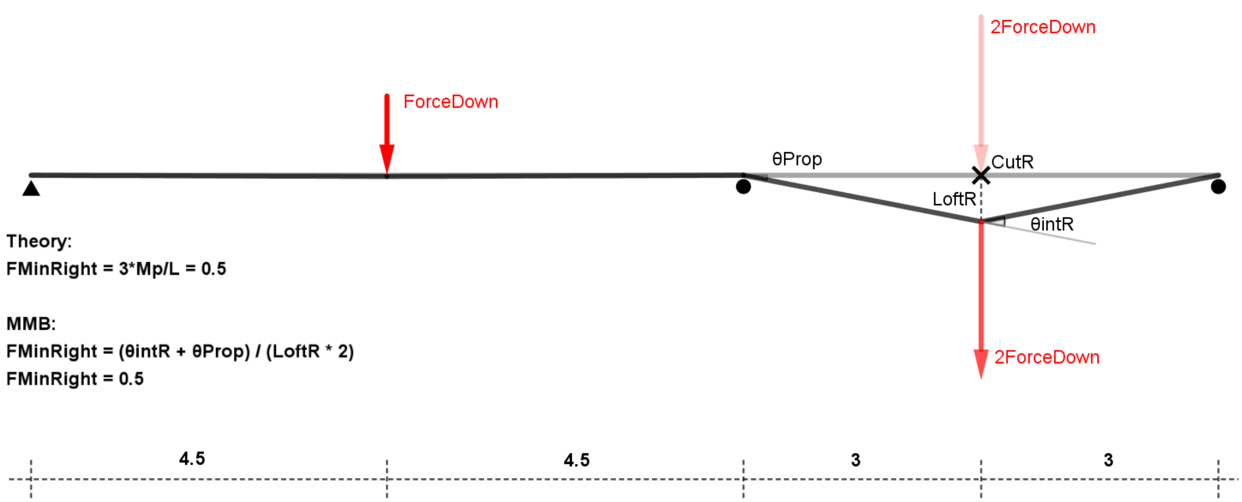


Figure 55. Plastic Analysis of Two-Bay Indeterminate Beam

Figure 56 is an example of a fixed-fixed beam condition. This example is very similar to Figure 50 and Figure 51 in finding the location of the plastic hinge. In this case, the plastic hinge will form at the applied point load. If this location was not already apparent, this can be found by sliding the cut line along the length of the beam to find the minimum force.

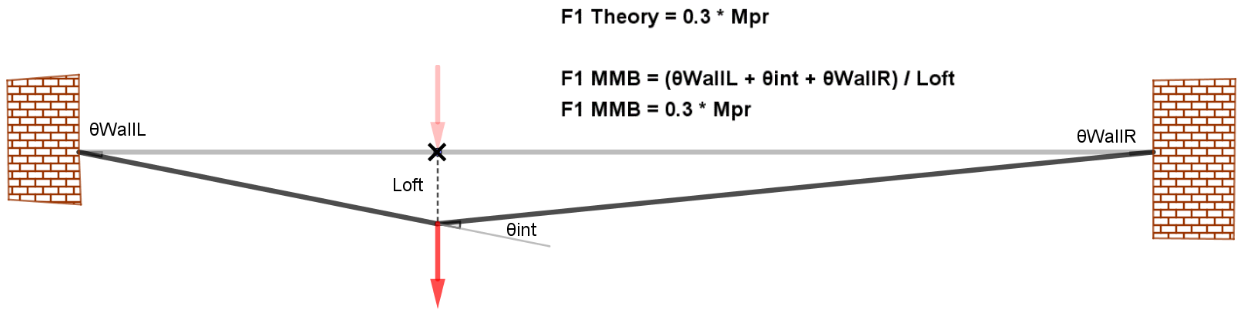


Figure 56. Plastic Analysis of Fixed-Fixed Beam

Another two bay example is shown below, however in this case, the beam is two degrees indeterminate. This means that three hinges will be required and, similar to Figure 54 and Figure 55, the analysis will show one bay at a time. The capacity of the beam also differs between the two segments, with the left being twice as large. This is reflected in solving for FMin. In addition to the differing beam properties, there is a distributed load which adds some complexity. When applying distributed loads in GeoGebra in regards to plastic analysis, it is important to split the load onto either side of the crack. In this case, Figure 58 shows ForceA and ForceB as the resultants of the distributed load on either side of the crack. Since the minimum force in Figure 58 is less than that of Figure 57, that shows that the failure method shown in Figure 58 will be the first mode of failure of this beam.

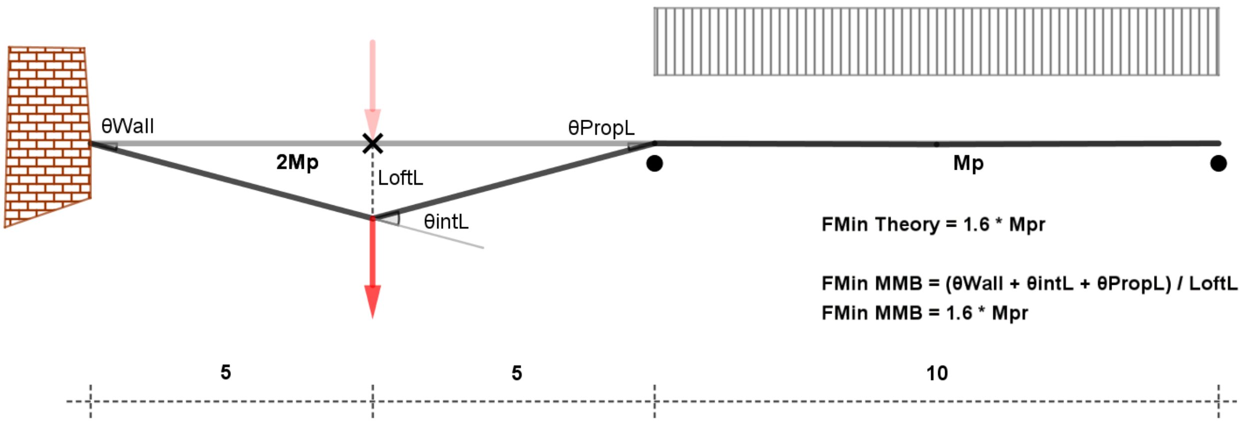


Figure 57. Plastic Analysis of Two-Bay Indeterminate Beam

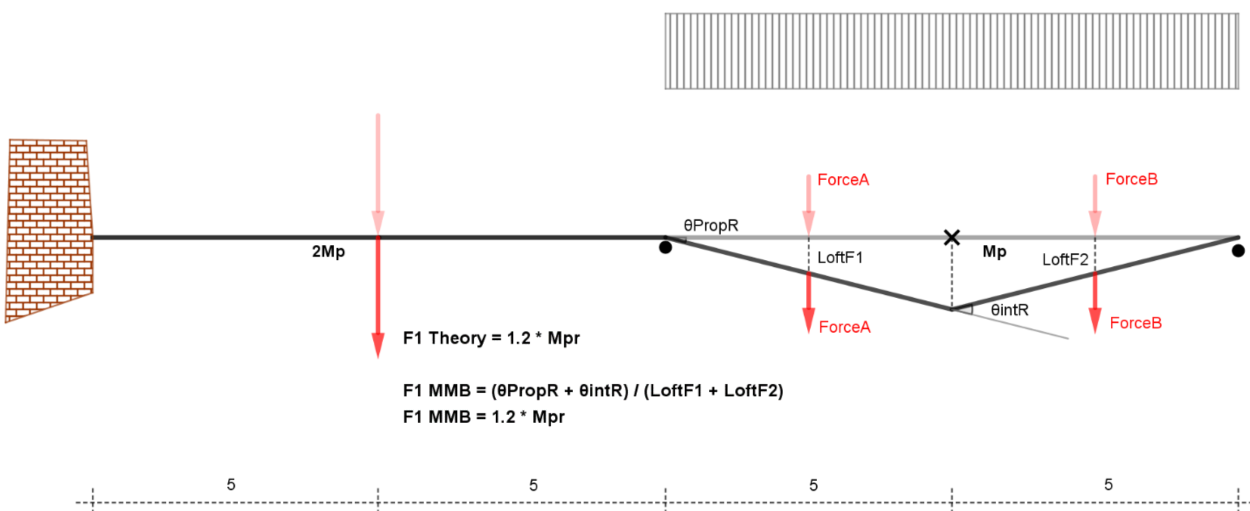


Figure 58. Plastic Analysis of Two-Bay Indeterminate Beam

Conclusion

This report has shown many of the advantages and has also shed some light on a few challenges of the Modern Muller Breslau Method. There are some challenges in this method which were mentioned in the report. Some of the construction in GeoGebra can become a bit cumbersome as the geometry of the problems get more advanced. It can take a bit of planning to ensure the boundary conditions are satisfied and to ensure that the members are simply rotating and are stretching only in the correct circumstances. With that being said, this method is very powerful and the principles can be used on physical models which removes any complexity of GeoGebra. This was shown in the section on Trusses, but with the right members, this could apply to any of the problems.

This method allows a graduating Architectural Engineering student to see a completely different method of solving both basic and complex problems without the standard statics taught in school. With a substantial background in structural analysis using statics, some things came more easily than without. The ability to quickly grasp the way the perturbed shapes should be drawn as well as understanding boundary conditions helped tremendously in the time it took to learn the method. While this method is still developing, it is already very powerful and will only continue grow. The MMB Method would be very beneficial to Architectural Engineering students, or other students interested in structures. Learning this method alongside statics would allow students to grasp a more wholistic view of structural analysis. Using statics, it is easy to get caught up in the numbers, but the Muller Breslau Method allows students to focus on visualizing the problem and use only one equation to solve. In addition, learning the two methods side by side would allow students to make the decision of which method to use when faced with a problem.

My experience in learning the Muller Breslau Method was very positive. After completing the structural analysis classes in the Cal Poly Architectural Engineering department, I was much more confident in solving problems with statics than I was with the Modern Muller Breslau Method. As my understanding and confidence with this method grew, I saw the benefits and the simplicity it creates in seemingly very complex problems.