Lasers

A Senior Project

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June 7, 2019
Title: Lasers

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Date Submitted: June 07, 2019

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1 Introduction

Short for “Light Amplification by Stimulated Emission of Radiation”, the theory of a laser has gradually transitioned over the last century from being a theoretical possibility to something used in countless modern, everyday applications and appliances. The idea of a laser is rooted in Einstein’s 1917 paper pertaining to his theory of radiation [1]. In his paper, Einstein theorized that the possibility of thermal equilibrium being achieved between electromagnetic waves and matter could be explained by three fundamental processes. These processes are known as spontaneous emission, absorption, and most importantly, stimulated emission. Einstein also made several other predictions regarding the behavior of photons when interacting with matter that suggested that creating a concentrated, coherent beam of light would be possible. The groundbreaking nature of these predictions wasn’t revealed until several decades later when two scientists, Charles Townes and Arthur Schawlow, formulated a plan to improve upon contemporary molecular spectroscopy technology [2]. At the time only coherent light in the microwave wavelength range was able to be produced. Townes and Schawlow were interested in generating coherent light of much higher frequencies through stimulated emission. To do this, they suggested positioning two parallel-facing mirrors along the optical axis of the cavity of a preliminary laser model. The intended purpose of these two mirrors was to reflect photons with desired frequencies back-and-forth through the medium of the laser. This would allow for the production of identical photons via stimulated emission at a rate that increased with each pass through the medium. Schawlow and Townes published a paper regarding their idea of implementing parallel mirrors into a laser cavity in 1958 [2]. In 1960, Theodore Maiman, a Scientist from Stanford University, brought the ideas of Townes and Schawlow to life. Maiman successfully constructed the first working laser using ruby as the gain medium [3, 4].

2 Background

2.1 Light-Matter Interaction

Another enormously important idea Einstein is credited with postulating is that light comes in discrete, quantized packets known as photons. This theory was first introduced by Einstein in a paper published in
1905 regarding the photoelectric effect [5]. This paper was of particularly great significance because it challenged the widely accepted ideas that light was exclusively wave-like in nature and that a mysterious medium known as “ether” was necessary and fundamental to the propagation of light. Thankfully, light and its elusive nature is understood at a much deeper level today than at the time of the publication of Einstein’s paper. For example, the notion of an invisible, intangible medium through which light propagates has been discarded. The wave-particle duality of light has been reconciled and is accepted as fact. We also know that photons are elementary, bosonic particles with no internal structure or mass. Furthermore, photons require no medium to propagate and do so at the speed of light in all reference frames [6].

Not only has our understanding of photons and their elusive behavior been greatly developed since Einstein’s initial conjecture in 1905, but our ability to mathematically describe the physical properties of light has been as well. Light is characterized by many things such as its polarization, color, wavelength, frequency, etc. The relationship between several of these fundamental properties of laser light and other forms of electromagnetic radiation is illustrated in the equation

\[ c = \lambda v, \]  

(1)

where \( c \) is the speed of light \((299,792,458 \text{ m/s})\), \( \lambda \) is wavelength, and \( v \) is frequency.

As shown in the above equation, the wavelength and frequency of light are directly proportional to the speed of light in a vacuum \((c)\). This relationship implies that as one quantity, either wavelength or frequency, increases the other must decrease to compensate. Light of a higher frequency has shorter wavelengths and carries with it a greater energy, whereas light with a lower frequency has a longer wavelength and contains less energy. The energy of a photon used in the generation of laser light is dependent upon the energy levels that the atom responsible for the photon’s emission transitions between, a principle discussed later in greater detail. The energy of a photon can be found using the equation

\[ E = hv, \]  

(2)
where $E$ is energy in Joules, $h$ is Planck’s constant ($6.626 \times 10^{-34}$ Joule – Seconds), and $v$ is frequency in Hertz.

The frequency of a photon not only indicates what its energy is, but also what type of electromagnetic radiation it is classified as. The electromagnetic spectrum is illustrated in Fig. 1.
Figure 1: Electromagnetic Spectrum [7].

The electromagnetic spectrum is partitioned into various different sections that correspond to light of different frequencies. At one end of the spectrum there are ultra-high frequency gamma rays with fre-
quencies reaching up to $3 \times 10^{24} \text{Hz}$, and on the opposite end of the spectrum there are the low frequency radio waves with wavelengths tens of thousands of meters long. The range from roughly $380 \text{nm}$ to $770 \text{nm}$ composes the visible spectrum, or the light able to be detected by a human eye. The range we are most concerned with, that of light emitted by lasers, extends from roughly $3 \times 10^{12} \text{Hz}$ in the infrared region up through $3 \times 10^{18} \text{Hz}$ in the extreme ultraviolet or x-ray region. Each region is host to electromagnetic radiation of varying energy. This energy can be transferred to and from atoms through the aforementioned means postulated by Einstein. These three vehicles of energy transfer are described in the following sections.

2.2 Energy Transfer Between EM Waves and Atoms

There are three main methods by which energy can be transferred between electromagnetic wave and atoms. The first method we will examine is known as stimulated absorption.

2.2.1 Stimulated Absorption

Stimulated absorption is the first process by which light can interact with atoms. During stimulated absorption a photon of energy $E = hv$ is absorbed by an atom. The energy of the absorbed photon is transferred to this atom and as a result the atom becomes excited and its outermost electron populates a higher energy state.
This process may only occur if the energy of the incident photon, \( E = h\nu \), is equivalent to the energy gap between the lower and higher energy levels occupied by the electron before and after the absorption. The rate at which stimulated absorption occurs is given by the equation

\[
R_{Stim.\,Abs.} = B_{12} g(\nu') \frac{I}{c} N_1,
\]

where \( B_{12} \) is the first Einstein B coefficient, \( g(\nu') \) is the lineshape function evaluated at the frequency of the incident photon, \( I \) is the irradiance of the electromagnetic wave, \( c \) is the speed of light, and \( N_1 \) is the population density of the lower energy level.

The process of stimulated absorption is illustrated in Fig. 2, where the dot for each energy level represents the electron’s current state.

![Figure 2: Stimulated Absorption](image)

2.2.2 Stimulated Emission

The second process through which light can interact with an atom is known as stimulated emission. Stimulated emission occurs when a photon of energy equivalent to the gap between adjacent energy levels is incident upon an atom resident in the higher level. This interaction results in the higher energy atom shifting down into the lower level and the release of another photon exactly identical to the incident one. The released photon has the same phase, polarization, energy, propagation, direction, and frequency of the incident photon. This newly created photon increases the energy of the passing electromagnetic wave. The uniformity between the incident and resulting photon is what allows for the creation of laser light. The rate at which stimulated emission occurs in a given volume is given by the equation
\[ R_{St.Em.} = B_{21} g(v') \frac{I}{c} N_2, \]  

where \( I \) is irradiance of the incident electromagnetic wave, \( c \) is the speed of light, \( N_2 \) is the population density of the upper energy level, \( g(v') \) is the lineshape function at the frequency of the incident photon(s), and \( B_{21} \) is the Einstein coefficient of stimulated emission.

The process of stimulated emission described above governed by Eq. 4 is shown in Fig. 3.

2.2.3 Spontaneous Emission

The third and final method by which an electromagnetic wave and atoms can exchange energy is known as spontaneous emission. Spontaneous emission occurs when an atom in an excited energy state unleashes a quanta of energy in the form of a photon and subsequently drops down to a lower energy state. As the name implies, this occurs spontaneously and requires no external motivation as the excited state is one which is energetically unstable and there exists a lower energy state with an allowed transition. As seen in stimulated emission, the energy of the photon emitted is equivalent to the energy gap between the upper and lower levels. However, in contrast to the uniformity of photons emitted via stimulated emission, each photon emitted in this manner travels in a random direction with a random polarization and phase constant.

The rate at which spontaneous emission occurs in a given volume is given by the equation

\[ R_{SponEm} = A_{21} N_2, \]  

where \( A_{21} \) is the third Einstein coefficient and \( N_2 \) is the population density of the upper energy level.
The process of spontaneous emission is shown in Fig. 4.

![Spontaneous Emission](image)

Figure 4: Spontaneous Emission [7].

### 2.3 Conclusion

The connections that exist between the three aforementioned processes through which light interacts with atoms and their corresponding equations and Einstein coefficients help to illuminate the conditions necessary for laser light to be created. The ratio of the third Einstein coefficient, $A_{21}$, to the Einstein coefficient of stimulated emission, $B_{21}$, is given by the equation

$$R_{Coeff Ratio} = A_{21}N_2,$$

(6)

Out of all three types of light-matter interaction we have discussed, stimulated emission is by far the most critical in the process of creating laser light. Unfortunately, under normal conditions stimulated emission is uncommon because of the tendency of atoms to decay down to lower energy levels via spontaneous emission. At equilibrium, the population distribution of atoms across each individual energy level lessens as energy increases usually following Boltzmann statistics. Because there are significantly more atoms in lower energy levels, absorption of a photon is much more likely to be observed than stimulated emission [8]. An illustration of the distribution of atoms across energy levels at equilibrium is shown in Fig. 5.
As previously stated, stimulated emission is necessary for the creation of laser light. Under the distribution conditions shown in Fig. 5 stimulated emission is not very likely to occur. To rectify this problem one of the main components of a laser system, the pump, introduces more energy to the system and excites the atoms to higher energy levels. If sufficiently pumped, and the natural lifetime of the upper state is significantly longer than the lower state, the number of atoms occupying an upper energy level will be greater than the number of atoms occupying a lower energy level. When this is observed there is said to be a population inversion. Achieving a population inversion is the primary purpose of a laser’s pump and is necessary to produce enough instances of stimulated emission to emit laser light [8]. Fig. 6 shows the population density distribution once a population inversion has been achieved.
3 Laser Elements

3.1 What is a Laser?

A laser is essentially a source of unique and extraordinarily useful light. The light emitted from them has high levels of both spatial and temporal coherence. Lasers can be designed to emit light anywhere along the electromagnetic spectrum from the microwave region to the ultraviolet region. They are used in a variety of practical applications across many different industries such as medicine and defense. All lasers are made up of several different key components that work in harmony to produce a highly coveted beam of light.

3.1.1 Pump

The first essential element of a laser that we will discuss is the pump. The pump acts as a source of energy that excites the atoms found within the laser’s gain medium. This is critical to the operation of laser because the excitation of these atoms is what establishes the population inversion necessary to produce laser light. Once a population inversion has been achieved within the gain medium, the amplification of the light generated by photons released as a result of stimulated emission can take place.
Because a population inversion has been established it is also certain that the rate of stimulated emission will surpass that of stimulated absorption, meaning that the irradiance of coherent light generated inside the laser cavity will increase over time. In the absence of a pump one would observe the intensity of the light within the laser cavity decreasing rather than increasing. There are several different types of pumps used to excite the atoms within a laser’s gain medium. All of these pumps accomplish the same exact thing, but the methods by which they do so vary. Several examples of these various pump types include optical, chemical, thermal, and electrical. Interestingly enough, optical pumps use an external source of light to excite the atoms of the gain medium that in turn produce light of their own. Lasers equipped with a chemical pump harness the energy released during a specific chemical reaction and use it to create the necessary population inversion. Chemical pumps are used in a variety of high-powered continuous wave lasers that have numerous industrial applications. Thermal pumps work by dissipating heat into the gain medium. This increase in thermal energy causes the atoms to become excited, again resulting in the necessary population inversion. Finally, electrical pumping, the kind of pumping used in and many gas lasers, works by passing an electric current through the gas. The electrons collide with the atoms during this process. These electron-atom collisions are what excites the electrons in the atom’s lower level into the upper level [7, 12].

3.1.2 Gain Medium

The next critical component used in the construction of a laser is known as the gain medium. Not only is the gain medium a crucial constituent piece of the laser, but the material from which it is made normally determines the name of the laser itself. For example the HeNe laser uses a gain medium that consists of helium and neon atoms, CO2 lasers use a gain medium of carbon dioxide, and Ti:sapphire lasers use a sapphire crystal doped with titanium ions as a gain medium. The gain medium of the laser is housed inside the laser cavity between the hard reflector and output coupler. It hosts the atoms that are to be excited by the pump and in turn produce photons via stimulated emission. Because the energy gap between the upper and lower laser levels found in various different gain media varies, so does the wavelength of laser light produced by them. There are so many different materials from which laser light can be emitted that it is possible to create lasers whose outputs range all the way from the ultraviolet to
infrared regions of the electromagnetic spectrum. Often times the gain medium of a laser has two distinct parts. These are those host atoms into which the energy generated by the pump is transferred, and the laser atoms from which light is produced. Selecting a suitable gain medium is imperative because if the lifetime of the energy levels used to create laser light are not sufficiently long (or short in the case of the lower level), a population inversion will be unattainable regardless of how intensely it is pumped. As mentioned previously, gain media come in all three states of matter. Gaseous gain media are observed in the aforementioned HeNe and CO2 lasers, solid gain media are present in lasers such as the Ti:sapphire laser, and lasers such as dye lasers with broad wavelength spectra use liquid gain media [7, 12].

3.1.3 Resonator (Hard Reflector and Soft Coupler)

The next integral component that allows amplification of laser light to take place is known as the resonator. This component is typically comprised of two planar or slightly curved mirrors that are oriented meticulously so that the photons created through stimulated emission reflect continuously along the same path through the gain medium. The first of the two mirrors used to create the resonator is called the hard reflector. The hard reflector is engineered to have a perfect reflectivity of 100%. This mirror allows none of the electromagnetic wave generated within the laser cavity to escape and reflects all of it back through the gain medium. The second mirror is known as the output coupler. This mirror is engineered to have a slightly less than perfect reflectivity usually around 98-88%. It is called the output coupler because, due to its imperfect reflectivity, it allows some of the electromagnetic wave to escape the cavity. This emission is the desired laser light. The distance between the hard reflector and output coupler is what determines the allowed wavelengths and frequencies of light that exist within the laser cavity. This relationship is shown in the equation

\[ v_m = \frac{mc}{2d}, \]

where \( d \) is the distance between the output coupler and hard reflector, \( m \) is a positive integer, \( c \) is the speed of light, and \( v_m \) is the frequency of the wave. Because only waves with frequencies at or near the values given by this equation, or higher order harmonics, can exist, the resonator can be described as a
“frequency filter” and can be fine-tuned so that only frequencies within a desired range are emitted from a laser [7].

3.1.4 Cooling System

The efficiency and optimal operation of laser is significantly effected by the temperature at which the laser runs. In reference to laser systems, efficiency can be described as the ratio of the amount of power output by the laser to power necessary to operate a laser. Unfortunately, many high-powered lasers with a variety of uses run at very low (<< 1) efficiencies. The main purpose of a cooling system within a laser is to dissipate the heat generated when energy from the pump is not used in the creation of laser light, which, in most cases, is the vast majority of the pump energy. Insufficient cooling of a laser system can be incredibly detrimental to the components of a laser and should be avoided [7].

As with the gain medium of laser, there are several options when choosing how to best cool a laser. Lasers with Nd:YAG (neodymium-doped yttrium aluminum garnet), Ti:sapphire (Titanium-Sapphire), and other solid gain media are often liquid-cooled. The gain medium is encased in a rigid bladder through which water or another liquid with a high specific-heat flows. This liquid essentially acts as a heat sink for the heat generated by the pump and prevents overheating. Other ways of cooling laser systems can be fairly simple like using fans to cool a laser with air, or more complicated as often seen in high-powered carbon dioxide lasers when the gain medium itself is recycled and cooled before returning to the cavity to be pumped again [7]. A rudimentary illustration of the typical arrangement of the aforementioned components can be seen in Fig. 7. The first mirror, $M_1$, is the hard reflector, the second mirror, $M_2$, is the output coupler, the gain medium is the portion between the hard reflector and soft coupler, and the pump is the solenoidal wire wrapped around the gain medium.
3.2 Tying it all Together

Now that all of the individual essential elements of a laser have been discussed, the way in which they operate in unison to create laser light can be discussed. The following figures provide visual representations of the processes that are covered in the following section.
The general process of creating a population inversion and achieving laser light can be seen in Fig. 8. This process begins with a large number of atoms in the ground state, or $E_0$ as seen in the figure. This is the lowest energy state able to be occupied by the atoms found within a laser’s gain medium. The laser’s pump then excites these atoms into a higher energy level through one of the several aforementioned pumping methods. Fig. 8 shows the atoms being pumped to the third excited state, or $E_3$. At this point there are several routes that the atoms can take back down to the ground state. Some of the atoms decay through a series of transitions from $E_3$ down to $E_0$, but many of the atoms instead almost instantly transition down from $E_3$ to $E_2$. Atoms that follow this path are of great interest to us because they
are responsible for the creation of laser light. $E_2$ is known as the upper laser level. The upper laser level has a lifetime roughly six orders of magnitude larger than the lifetime of the other levels shown in Fig. 8. Because of its lifetime is extraordinarily long compared to the other energy levels, $E_2$ acts as a metaphorical red stoplight at which atoms rapidly accumulate. This causes the population density of the second excited energy level, $N_2$, to become increasingly large. Some atoms stuck at the second excited energy level may spontaneously decay down to the first excited energy level, $E_1$, also known as the lower laser level. Because the lifetime of $E_1$ is short, the population density of the first excited state, $N_1$ cannot achieve a value anywhere near comparable to that of $N_2$. Because of this, population inversion, an absolute necessity when creating laser light, is achieved. Whenever $N_2 > N_1$ there exists a population inversion in the gain medium [7].

Now that a population inversion between the upper and lower laser levels has been achieved, light amplification can finally take place. First, a photon of a specific energy is needed to catalyze the process. As shown in Fig.8, for stimulated emission to take place, the incident photon must have an energy given by the equation

$$E_\gamma = h\nu' = E_2 - E_1,$$

(8)

where $h$ is Planck’s constant, and $\nu'$ is the frequency of the incident photon. Once a photon with an energy given by Eq. 8 stimulates an atom to transition down from $E_2$ to $E_1$ another identical photon via stimulated emission has been created, the process of light amplification has officially begun. Although a photon with an energy given by Eq. 8 may also be absorbed by an atom in the lower laser level and excite it to the upper laser level, there still exists a population inversion where $N_2 > N_1$. Because of this and the fact that the Einstein coefficients of stimulated emission and stimulated absorption, $B_{12}$ and $B_{21}$ respectively, are equivalent, the rate at which stimulated emission occurs, given by Eq. 4, is greater than the rate at which stimulated absorption occurs, given by Eq. 3. This in turn allows for the photon population within the laser cavity to be amplified every time it is reflected between the hard reflector and output coupler. As more and more photons are produced via stimulated emission, the atoms that decayed from $E_2$ to $E_1$ during this process further decay down to the ground state, $E_0$, where
they can be pumped up to $E_3$ yet again. Each atom repeats this process over and over again so that a population inversion between the upper and lower laser levels always exists, and the output laser beam is uninterrupted [7].

Because the output coupler is designed to have an imperfect reflectivity so that laser light may escape the cavity, the irradiance, or power of the light produced by the laser decreases by some amount on each pass between the two mirrors. For the irradiance of the laser beam to continue to increase, the amount that the irradiance is increased through stimulated emission needs to be greater than the amount lost through the output coupler. However, because an increase in irradiance means that more and more atoms are undergoing stimulated emission and transitioning from the upper laser level to the lower laser level, it also means that $N_2$ is decreasing which lessens the severity of the population inversion induced by the pump of the laser. The inverse relationship between the irradiance of the laser and the population inversion is referred to as gain saturation. Over time the irradiance generated through stimulated emission asymptotically approaches the rate at which it is lost through the output coupler and can no longer be increased. When a laser reaches this point in its operation cycle it is said to be in steady-state operating condition [7].

3.3 Laser Light and its Properties

Laser light is so ubiquitous in modern day technology because of its unique properties not found in any other light sources. One such property is the high degree of monochromaticity laser light possesses. In short, monochromaticity is the uniformity of the color of the light emitted by a laser. The linewidth, or possible frequencies of photons emitted by stimulated emission typically spans over a decent range, but because the two mirrors that make up the resonator of a laser form a resonant cavity, only photons with frequencies equivalent to the resonant frequencies of the cavity are amplified. This leads to a significant narrowing of the range of frequencies that are amplified within the laser cavity and thus the uniformity of color is heightened. Unfortunately, instances of spontaneous emission are not able to be completely circumvented. This leads to the emission and amplification of a small, but non-negligible amount of photons that broaden the range of frequencies of light emitted by the laser [7, 8].
Another defining property of laser light is its coherence. If a laser or other light source is said to be coherent it means that the temporal and spatial components of the electromagnetic wave emitted from the source have a high degree of phase correlation. Temporal coherence defines the level of monochromaticity exhibited by a laser beam. In other words, temporal coherence is a measure of how uniform the frequency of light emitted from a source remains. This type of coherence can be measured in terms of coherence length, the distance emitted light may travel while remaining monochromatic. Coherence length can be found using the equation

\[ L = \frac{c}{\Delta v}, \]  

(9)

where \( L \) is the coherence length, \( c \) is the speed of light in a vacuum, and \( \Delta v \) is the frequency bandwidth of the laser. You can also describe the temporal coherence of light by its coherence time, \( \tau_0 \), which can be found by dividing the coherence length by the speed of light, \( c \) [7, 8, 10].

Fig. 10 shows an electromagnetic wave with a coherence time \( \tau_0 \). Notice that at precisely each interval of \( \tau_0 \) the light wave experiences a sudden change in phase and no longer retains temporal coherence.

![Figure 10: Wave With Coherence Time \( \tau_0 \) [10].](image)

The second type of coherence observed in laser light is known as spatial coherence. Spatial coherence “is a measure of the uniformity of phase across the optical wave-front.” [7]. Essentially what this means is that for light to be spatially coherent the phase of the wave describing it must be perfectly synchronized...
with the phase of light equidistant and emitted from the same source traveling in another direction. For example, consider the source of light in Fig. 11 to be ideal and monochromatic. Points $P_1$ and $P_2$ are equidistant from the source meaning they are at the same point on the optical wave-front and are traveling in different directions. If the light form the source is to be considered spatially coherent then the phase of the light at point $P_1$ must be exactly that of the phase of the light at point $P_2$ [7].

Emitted light may have one or both characteristics of being spatially and temporally coherent. However, light that is either spatially or temporally coherent cannot automatically be assumed to be temporally or spatially coherent as well, respectively. If an electromagnetic field that is both spatially and temporally coherent is desired the sources of light that constitute the field must emit photons that are of the same frequency and in phase with each other [7]. The high level of spatial and temporal coherence found in laser light is yet another property that can be attributed to the process of stimulated emission. In stimulated emission, the photon elicited by the interaction between an incident photon and an atom occupying the upper laser level has exactly the same frequency, phase, polarization, energy, and many other properties as the incident photon. This means that the light amplified with each pass through a laser’s cavity via stimulated emission is extraordinarily coherent, both spatially and temporally [7, 8, 10].

4 Comparison of Different Types of Lasers

At this point we have covered a sufficient enough amount of theory and background information to compare and contrast the differences between types of common lasers. The two most notable of which
are known as continuous wave (CW) and pulsed lasers.

4.1 Continuous Wave Lasers

Steady-state, or continuous wave lasers are lasers whose outputs are uninterrupted and consistent; i.e., light is continuously emitted from the laser. Hence the name “continuous wave”. Whether or not a laser can be considered a member of the continuous wave family depends on its pump, gain medium, and resonator. The investigation of continuous wave lasers begins with an analysis of a simple two-mirror laser cavity. Such a cavity is shown in Fig. 12.

![Figure 12: Two Mirror Laser Cavity [7].](image)

In Fig. 12, $M_1$ is the hard reflector, $M_2$ is the output coupler, $L$ is the length of the gain medium, and $I_{out}$ is the irradiance of the output laser beam. $I_{out}$ can be found using the equation

$$I_{out} = \frac{T_2 I_s}{2} \frac{\gamma_0 (2L) - \ln \left( \frac{1}{R_1 R_2} \right)}{(1 - \sqrt{R_1 R_2})(1 + \sqrt{R_1 R_2})},$$

(10)

where $T_2$ is the transmittance of the output coupler, $R_1$ and $R_2$ are the reflectivities of the hard reflector and output coupler respectively, $L$ is the length of the gain medium, $\gamma_0$ is the “small-signal gain coefficient”, and $I_s$ is the saturation irradiance [7]. From Eqn. 10 we can determine the threshold gain coefficient of the two-mirror cavity shown in Fig. 12 to be

$$\gamma_{th} = \frac{1}{2L} \ln \left( \frac{1}{R_1 R_2} \right).$$

(11)

The threshold gain coefficient is the amount of gain needed just to offset losses experienced by the cavity arising from things such as absorption and transmission. For laser light to be created, the small-signal
gain coefficient must be larger than the threshold gain coefficient [7].

4.1.1 Broadening

Broadening is a complicated topic and to understand it a discussion of the lineshape function, \( g(\nu') \), is necessary. The lineshape function is a graphical representation of the probability of an atom’s interaction with a photon resulting in stimulated emission. An example of a lineshape function can be see in Fig. 13.

![Figure 13: Lineshape function [7].](image)

Although it was stated previously that in order for stimulated emission to occur as a result of the interaction of a photon with an atom the incident photon must have an energy exactly equal to the energy gap between the upper and lower laser energy levels of the atom, this is not the complete picture. While photons that propagate with an energy exactly equal to the energy gap are the most likely to elicit stimulated emission, photons with energies very close to that of the energy gap may also elicit stimulated emission, albeit at a lower probability. Broadening is a phenomenon through which the lineshape function of the atoms responsible for the production of laser light are, predictably, broadened. This leads to a breakdown and degradation of the properties of the output laser light such as its coherence [7].

The first type of broadening we will discuss is known as homogeneous broadening. It is called homogeneous because the linewidth of all of the effected atoms are all broadened in the same way. Looking at Fig. 13 one can see that the typical lineshape function is gaussian in nature. Homogenous broadening causes the lineshape function to become Lorentzian, a shape given by the equation
\[ g(v) = \frac{\Delta v_H}{2\pi[(v - v_0)^2 + (\frac{\Delta v_H}{2})^2]}, \] 

where \( v_0 \) denotes the point along the function where \( g(v) \) is a maximum, and \( v_H \) is the full-width at half-maximum (FWHM) of the function and is given by the equation

\[ \Delta v_H = \frac{1}{2\pi}\left(\frac{2}{\tau_1} + \frac{1}{\tau_2} + 2r_{col}\right), \] 

where \( \tau_2 \) and \( \tau_1 \) are the lifetimes of the upper and lower laser energy levels respectively and \( r_{col} \) is a term related to the rate of collisions between atoms.

The two terms involving \( \tau \) in Eq. 13 are a result of a specific type of homogenous broadening known as lifetime broadening. Lifetime broadening arises when atoms spontaneously decay from the upper laser energy level to the lower laser energy level. This disrupts the natural, forced oscillation the atom normally incurs as a byproduct of interaction with electromagnetic waves. This means that the charge oscillation of the gain medium is unable to be represented as a single frequency and is better described by a broadened spectrum of frequencies. This broadening is the origin of the two \( \tau \) terms in Eq. 13 [7].

The term involving \( r_{col} \) in Eq. 13 is a result of another type of homogenous broadening known as pressure broadening. The presence of pressure broadening can be attributed to elastic collisions between atoms in the gain medium. Unlike lifetime broadening which involves inelastic collisions and the changing of the internal state of the atoms involved in the collision, these collisions do not change the internal state of the atoms, but they do disrupt the normal charge oscillation of the atoms in the gain medium. The severity of the effect that pressure broadening has on the lineshape function is directly proportional the pressure and temperature experienced by the atoms in the gain medium. Pressure broadening is of particularly great concern in the operation of gas lasers [7].

The second type of broadening we will discuss is known as inhomogeneous broadening. It is called inhomogeneous because the linewidth of the effected atoms differ among the total population of atoms; i.e., not all of the atoms have the exact same properties. Inhomogeneous broadening shifts the center
frequency, \( v(0) \), for a particular atom. Doppler broadening, a common form of inhomogeneous broadening, is a result of the doppler effect. The underlying principle of the doppler effect essentially states that the perceived frequency or wavelength of a wave depends on the waves velocity relative to the point from which it is being perceived. Waves that are traveling towards the point from which they are being perceived appear or sound to the perceiver to be compressed and therefore of a higher frequency than they are in their own frame of reference. Waves that are traveling away from the point from which they are being perceived appear or sound to the perceiver to be elongated and therefore of a lower frequency. Because the atoms in a gaseous gain medium propagate at different velocities, the light incident upon, and emitted from, each atom will vary slightly in frequency from other atoms because of the doppler effect. Thus variations in the velocities of the atoms result in variations in the lineshape functions within the population. This is the driving mechanism behind doppler broadening [7].

### 4.2 Pulsed Lasers

The second major classification of laser output varieties is pulsed lasers. Pulsed lasers use various different mechanisms to concentrate the output beam of light into a highly compact “pulse”. Pulsed lasers boast a higher peak intensity than their continuous wave counterparts, but are incapable of emitting the same amount of electromagnetic energy for obvious reasons. Pulsed lasers are better suited than continuous wave lasers for numerous applications such as industrial cutting and materials processing.

#### 4.2.1 Q-Switching

Pulsing the output of a laser is generally done using either of two primary methods, the first of which is known as Q-switching. The term Q-switching refers to the process by which the quality factor of the cavity, \( Q \), is made to oscillate between high and low values. \( Q \) is a measure of how well the optical cavity of a laser contains the electromagnetic energy produced inside of it. A high \( Q \) is indicative of an optical cavity that contains energy with very low losses, whereas a low \( Q \) is indicative of the opposite. In general, if both components of a laser’s resonator (hard reflector; output coupler) have high reflectivity values then the cavity will have a high quality factor value. If either of the two components of the resonator have a low reflectivity the cavity will have a low quality factor. The equation used to determine
the quality factor of an optical cavity is

\[ Q = \frac{2L}{\lambda(1 - R)}, \tag{14} \]

where \( L \) is the length of the optical cavity, \( R \) is the reflectance, and \( \lambda \) is the wavelength of light produced by the gain media.

The first step to pulsing a laser’s output via Q-switching is to excite the atoms within the gain medium at a sufficiently low Q. The low Q during this phase is necessary to inhibit the irradiance of the electromagnetic wave within the cavity from increasing. This allows the population density of the upper laser energy level to reach a maximum threshold, establishing a sever population inversion. At this point, no laser light can be created let alone emitted because of the ratio of the small-signal gain coefficient to the threshold gain coefficient. Next, the Q of the cavity is instantaneously switched from a low value to a very high value so that the cavity now experiences minimal loss of irradiance. This causes short, incredibly high irradiance pulses of laser light to be emitted from the soft-coupler. Unfortunately, this causes the population inversion within the gain medium to rapidly decrease. It is at this point in the process that the Q is reset back to a very low value. The process described above then starts over and repeats ad infinitum. Switching the Q of a laser cavity between high and low values can be done through various different methods. The most simple of these methods involves the insertion of a rotating mirror within the cavity that only allows lasing to occur when it is perfectly aligned with the adjacent mirror. It is also possible to Q-switch the output of laser by placing a fan along the optical axis of the resonator which acts as a physical impediment and thus lowers and raises the Q of the cavity rapidly. Other more common methods of Q-switching involve acousto-optic and electro-optic techniques, and sometimes even saturable absorbers that act as irradiance filters [7].

### 4.2.2 Mode-Locking

While Q-switching is a certainly adequate method of pulsing the output of a laser and typically results in optical pulses in the nanosecond regime, another method that allows for even narrower pulses to be created is known as mode-locking. A mode-locked laser differs from a Q-switched laser in several key
ways. First, a Q-switched laser cavity is designed to have a high Q for only a single frequency. The Q of the cavity only needs to be high for a wavelength range much less than 1 nm. A mode-locked laser cavity is designed to have a high Q over a very broad range of frequencies. The Q of the cavity needs to be high for a wavelength range of 10’s to over 100nm. In other words, when a mode-locked laser is not pulsing it can serve as a tunable continuous-wave laser capable of lasing over a vast wavelength range. Second, unlike a Q-switched laser system in which the cavity is made to stay in a low Q state for quite some time so that a population inversion may be established as electromagnetic waves reverberate between the two mirrors of the resonator millions of times, the Q of a mode-locked laser is switched from a high value to a low value once during every single reverberation that the electromagnetic wave makes through the cavity. The switching of the Q happens so quickly that the population inversion of the gain medium remains the same throughout the entire process as it would in a continuous wave laser. So, unlike a Q-switched laser, the purpose of mode-locking is not to saturate the population inversion, but rather to lock the phases of the numerous standing waves. All of the frequencies supported in a mode-locked laser cavity are harmonics of the fundamental longitudinal cavity mode. When many harmonics are simultaneously superimposed it is possible for all of them to constructively interfere at a single point in space and in time. When this happens a light pulse is created where the phases of the waves are locked. Depending upon the bandwidth of allowed and phase-locked longitudinal modes, laser pulses in the range of 100’s of femtoseconds down to single digit femtoseconds can be created. Any of the methods used to Q-switch a laser’s output can be use to mode-lock a laser as well, provided that the oscillation of the Q from high to low and back again occurs rapidly enough [7].

5 Laser Applications

As stated previously, lasers have a very wide range of practical, real-world applications, some of which have had monumental impacts on mankind. These applications have only been mentioned briefly thus far, but are significant enough to warrant delving into them further. The following sections pertain to several different industries and the applications lasers have within them.
5.1 Medical

One field in which the application of lasers to solve or workaround is common is medicine. Laser use has become so prevalent in the field of medicine because of their ability to output an incredibly small point of light in which there is contained a massive amount of energy. This allows for lasers to be used in place of scalpels and other instruments used to cut and cauterize human tissue. There are two main different types of laser applications found within medicine. The first type of applications are known as diagnostic applications. In diagnostic applications the interaction that a specific type of tissue has with the incident laser light is observed and then used to discover or determine its physiological or pathological properties. One example of a diagnostic application of lasers in medicine is the determination of cancer severity and development. Human tissue can fluoresce when excited by laser light. This fluorescence is directly correlated to the chemical properties of the tissue and thus can be used to detect the presence of cancerous masses. The second type of applications are known as therapeutic applications. These applications are used to permanently alter or correct inadequacies in human tissue. The most obvious therapeutic application of lasers in medicine is as a cutting instrument instead of a scalpel. Another example of a therapeutic laser application can be found in the branch of medicine known as ophthalmology, a field concerned with the treatment of eye-related deficiencies. Ophthalmologists will often perform photorefractive laser surgery on patients in an attempt to correct disorders such as hyperopia and astigmatism by altering the shape of their corneas [12].

5.2 Industrial

Outside of medicine, lasers are also used in wide-range of applications in the industrial, manufacturing, and materials processing industries. An example of this is the use of a laser as a welding instrument. The energy from a laser’s output beam is directed onto two pieces of separate, initially unconnected pieces of metal. This results in the melting of the two pieces of metal at the site of the laser beam’s output. The melted portions of these metals combine with each other and eventually cool off, solidifying. At the end of this process the seam along which the laser beam was directed now connects the initially unconnected pieces of metal. This method of welding is preferred over traditional methods because of its precision
in the application of energy as well as where the energy is applied on the weld site. Another advantage laser welding has over other methods is small seam width. Seams created through laser welding are much narrower than those created by electric-arc and plasma welding, meaning the physical integrity of the metals being welded together are better preserved.

Another common application of lasers within industry is drilling. Laser drilling is usually preferred over traditional methods of drilling because of its ability to quickly and consistently drill tiny holes into various different materials. Examples of this include drilling holes into circuit boards, cigarette filters, aerosol can nozzles, and aircraft engine blades [12]. Laser drilling is usually done with a pulsed laser and the length of the pulses emitted are responsible for a number of properties, such as the depth of the hole created. The pulsed output of laser can be used to drill in a few different ways. One such way is known as single-pulse drilling. This style of laser drilling uses one single output pulse to drill through the entirety of the material. Another method known as percussion drilling uses many pulses to accomplish the same thing as single-pulse drilling. This style of drilling is employed instead of single-pulse because less energy is transferred into the material being drilled into, yielding holes of higher quality. As the number of pulses used in percussive drilling increases so does the quality of the holes produced [12].
References


