
Rakesh K. Goel\textsuperscript{1}, A.M. ASCE, and Anil K. Chopra\textsuperscript{11}, M. ASCE

Abstract

The effects of plan asymmetry on the earthquake response of code-designed, one-story systems are identified with the objective of evaluating how well these effects are represented by torsional provisions in US building codes. The earthquake-induced deformations and ductility demands on resisting elements of asymmetric-plan systems, are compared with their values if the system plan were symmetric. The presented results demonstrate that the design eccentricity in US building codes should be modified in order to achieve the desirable goal of similar ductility demands on asymmetric-plan and symmetric-plan systems. The design eccentricity should be defined differently depending on the design value of the reduction factor $R$.

Introduction

The evaluation of torsional provisions in building codes based on computed responses of elastic as well as inelastic, asymmetric-plan systems has been the subject of numerous studies in the past. However, the conclusions of these studies may not be generally applicable to code-designed buildings because the assumed plan-wise distribution of stiffness and strength is not representative of code-designed buildings and the strength distribution can significantly influence the inelastic structural response (Chopra and Goel 1991). Thus, the main objective of this work is to investigate the effects of plan-asymmetry on the earthquake response of code-designed, one-story systems and to determine how well these effects are represented by torsional provisions in building codes. For this purpose, the deformation and ductility demands on resisting elements of asymmetric-plan systems are compared with their values if the system plan were symmetric. Based on these results, deficiencies in code provisions are identified and improvements suggested. This paper presents a summary of the work that is available in more details.
Torsional Provisions in Seismic Codes

Most building codes require that the lateral earthquake force at each floor level of an asymmetric-plan building be applied eccentrically relative to the center of stiffness (CS). The design eccentricity $e_d$ specified in most seismic codes is of the form (International 1988)

$$e_d = \alpha e_s + \beta b$$

$$e_d = \delta e_s - \beta b$$

where $e_s$ is the eccentricity between the center of mass (CM) and the CS, $b$ is the plan dimension of the building perpendicular to the direction of ground motion; and $\alpha$, $\beta$, and $\delta$ are specified coefficients; US building codes, e.g., Uniform Building Code (UBC-91) and Applied Technology Council (ATC-3) provisions specify $\alpha = \delta = 1$ and $\beta = 0.05$. For each element the $e_d$ value leading to the larger design force is to be used. Consequently, Eq. 1a is the design eccentricity for elements located within the flexible-side of the building and Eq. 1b for the stiff-side elements (Fig. 1).

Figure 1 Idealized one-story system.

Inelastic Response

The deformations $u_i$ and ductility demands $\mu_i$ of resisting elements in the asymmetric-plan system, normalized by $u_o$ and $\mu_o$, the respective response quantities of the corresponding symmetric-plan system (Goel and Chopra 1990) are presented in the form of response spectra for the first 6.3 secs. of the S00E component of the 1940 El Centro ground motion applied in the Y-direction. The yield force for the system is defined as the base shear induced in the elastic symmetric-plan system due to the selected ground motion and reduced by the reduction factor $R$, and the element yield forces are determined in accordance with the torsional provisions of UBC-91. Two types of asymmetric-plan systems are considered: in the first system, the code design force for the stiff-side element can be smaller than the design force of the same element in the corresponding symmetric-plan system; and in the second type, such a reduction is precluded. Several parameters of the system are fixed at: stiffness eccentricity normalized by the radius of gyration, $e_s/r = 0.5$, ratio of the uncoupled torsional and lateral frequencies, $\omega_\theta = 1$, and damping ratio, $\xi = 0.05$.

The deformations of resisting elements in the system designed according to UBC-91 may be significantly affected by plan-asymmetry, as indicated by the deviation of $u_i/u_o$ or $\mu_i/\mu_o$ from unity (Fig. 2). Plan-asymmetry generally tends to reduce the deformation of the stiff-side element and increase the deformation of the flexible-side element compared to their respective deformations in the corresponding symmetric-plan system. Effects of the increased strength of the system resulting from the restriction that the stiff-side element design force must not fall below its symmetric-plan value are negligible.

The ratio $\mu_i/\mu_o$ of the element ductility demands in an asymmetric-plan system and the corresponding symmetric-plan system are also presented in Fig. 2. If the reduction in the design force for the stiff-side element is permitted the element ductility demand tends to be significantly larger due to plan-asymmetry. However, if reduction in the element design force is precluded, $\mu_i/\mu_o = u_i/u_o$ and the above observations on how deformations are affected by plan-asymmetry also apply to ductility demand. The ductility demand on the flexible-side element is generally reduced significantly because of plan-asymmetry. These trends are unaffected by whether the design force reduction for the stiff-side elements is permitted or not (Fig. 2).

![Image of Figure 2](image-url)

Figure 2. Ratio of element deformations, $u_i/u_o$, and ductility demands, $\mu_i/\mu_o$, for asymmetric-plan and corresponding symmetric-plan systems; $R = 4$.

![Image of Figure 3](image-url)

Figure 3. Ratio of element deformations, $u_i/u_o$, for asymmetric-plan and corresponding symmetric-plan systems, and element ductility demands, $\mu_i$, for asymmetric-plan systems; $R = 1$. 

elsewhere (Chopra and Goel 1991).
Torsional Provisions in Seismic Codes

Most building codes require that the lateral earthquake force at each floor level of an asymmetric-plan building be applied eccentrically relative to the center of stiffness (CS). The design eccentricity $e_d$ specified in most seismic codes is of the form (International 1988)

$$e_d = \alpha e_s + \beta b$$  \hspace{1cm} (1a)  

$$e_d = \delta e_s - \beta b$$  \hspace{1cm} (1b)

where $e_s$ is the eccentricity between the center of mass (CM) and the CS, $b$ is the plan dimension of the building perpendicular to the direction of ground motion; and $\alpha$, $\beta$, and $\delta$ are specified coefficients; US building codes, e.g., Uniform Building Code (UBC-91) and Applied Technology Council (ATC-3) provisions specify $\alpha = \delta = 1$ and $\beta = 0.05$. For each element the $e_d$ value leading to the larger design force is to be used. Consequently, Eq. 1a is the design eccentricity for elements located within the flexible-side of the building and Eq. 1b for the stiff-side elements (Fig. 1).

![Figure 1 Idealized one-story system.](image)

Inelastic Response

The deformations $u$ and ductility demands $\mu$ of resisting elements in the asymmetric-plan system, normalized by $u_o$ and $\mu_o$, the respective response quantities of the corresponding symmetric-plan system (Goel and Chopra 1990) are presented in the form of response spectra for the first 6.3 secs. of the SDOE component of the 1940 El Centro ground motion applied in the Y-direction. The yield force for the system is defined as the base shear induced in the elastic symmetric-plan system due to the selected ground motion and reduced by the reduction factor $R$, and the element yield forces are determined in accordance with the torsional provisions of UBC-91. Two types of asymmetric-plan systems are considered: in the first system, the code design force for the stiff-side element can be smaller than the design force of the same element in the corresponding symmetric-plan system; in the second type, such a reduction is precluded. Several parameters of the system are fixed at: stiffness eccentricity normalized by the radius of gyration, $e_s/r = 0.5$, ratio of the uncoupled torsional and lateral frequencies, $\Omega_a = 1$, and damping ratio, $\xi = 0.05$.

The deformations of resisting elements in the system designed according to UBC-91 may be significantly affected by plan-asymmetry, as indicated by the deviation of $u_i/u_o$ or $\mu_i/\mu_o$ from unity (Fig. 2). Plan-asymmetry generally tends to reduce the deformation of the stiff-side element and increase the deformation of the flexible-side element compared to their respective deformations in the corresponding symmetric-plan system. Effects of the increased strength of the system resulting from the restriction that the stiff-side element design force must not fall below its symmetric-plan value are negligible.

The ratio $\mu_i/\mu_o$ of the element ductility demands in an asymmetric-plan system and the corresponding symmetric-plan system are also presented in Fig. 2. If the reduction in the design force for the stiff-side element is permitted the element ductility demand tends to be significantly larger due to plan-asymmetry. However, if reduction in the element design force is precluded, $\mu_i/\mu_o=\mu_i/\mu_o$ and the above observations on how deformations are affected by plan-asymmetry also apply to ductility demand. The ductility demand on the flexible-side element is generally reduced significantly because of plan-asymmetry. These trends are unaffected by whether the design force reduction for the stiff-side elements is permitted or not (Fig. 2).

![Figure 2. Ratio of element deformations, $u_i/u_o$, and ductility demands, $\mu_i/\mu_o$, for asymmetric-plan and corresponding symmetric-plan systems; and element ductility demands, $\mu_i$, for asymmetric-plan systems ; $R = 4$.](image)

![Figure 3. Ratio of element deformations, $u_i/u_o$, for asymmetric-plan and corresponding symmetric-plan systems, and element ductility demands, $\mu_i$, for asymmetric-plan systems ; $R = 1$.](image)
The preceding results have demonstrated that the ductility demand on the stiff-side element may increase significantly because of plan-asymmetry when reduction in the stiff-side element design force is permitted. Since it is desirable that the element ductility demands be similar whether the plan is symmetric or not, the presented results suggest that seismic codes should preclude reduction in the design forces of the stiff-side elements below their values for symmetric-plan systems.

The stiff-side elements are generally believed to be beneficially affected by torsion and are therefore not considered the most critical elements for design purposes. However, the preceding results that the largest ductility demand among all the resisting elements may occur in the stiff-side element. Thus, additional care is required in the design of stiff-side elements for ductility demand.

"Elastic" Response

It is the intent of US seismic codes that buildings suffer no damage during some, usually unspecified, level of moderate ground shaking. Thus, the response of asymmetric-plan systems designed with $R = 1$ is examined next. $R = 1$ implies that the design strength $V$ of the corresponding symmetric-plan system is just sufficient for it to remain elastic during the selected excitation. However, as will be shown in subsequent sections, asymmetric-plan systems designed for the same base shear may not remain elastic.

The deformation of resisting elements may be significantly affected by plan-asymmetry. The deformation of the stiff-side element is generally reduced because of plan-asymmetry whereas deformation of the flexible-side element in such systems is considerably increased (Fig. 3). The ductility demand for stiff-side and flexible-side elements in the asymmetric-plan system exceeds one for some period values (Fig. 3) indicating yielding in these elements, which were designed to remain elastic if the building plan were symmetric. The stiff-side element yields more if its design force is permitted to fall below its symmetric-plan value because this results in smaller yield deformation (Chopra and Goel 1991).

The preceding results indicate that asymmetric-plan systems designed with $R = 1$ may deform into the inelastic range and the element deformation may significantly exceed the deformation of the corresponding symmetric-plan system. Thus, asymmetric-plan systems designed with $R = 1$ may experience structural damage due to yielding and nonstructural damage resulting from increased deformations.

Modifications in Design Eccentricity

The results of preceding sections indicate that deformations and ductility demands on resisting elements in a code-designed asymmetric-plan system differ from those for the corresponding symmetric-plan system. However, it would be desirable that the responses of the two systems be similar so that the earthquake performance of the asymmetric-plan system would be similar to, and specifically no worse than, that of the symmetric-plan system. In order to investigate this issue further, the responses of asymmetric-plan systems with their element yield forces computed with three different values of $\delta = 1, 0.5, \text{ and } 0$ in Eq. 1 are compared in Fig. 4. In all cases, $\alpha = 1$ and four different values of $R = 1, 2, 4 \text{ and } 8$ were considered. The ductility demand on the stiff-side element is the only response quantity presented because other responses are affected very little by $\delta$. It is apparent that the ductility demand $\mu_1$ on the stiff-side element in the asymmetric-plan systems designed with $\delta = 0$ is generally below the element ductility demand, $\mu_0$, if the system plan were symmetric. However, for some period values, precluding reduction of stiff-side element design force ($\delta = 0$) is not sufficient to keep $\mu_1$ below $\mu_0$. In order to achieve this objective, perhaps this design force should be increased relative to its symmetric-plan value, which implies a negative value of $\delta$ in Eq. 1b.

![Figure 4. Ratio of stiff-side element ductility demands, $\mu_1/\mu_0$, for asymmetric-plan and corresponding symmetric-plan systems; $\alpha = 1$ and $\beta = 0$.](image1)

Even if such a reduction in the stiff-side element design force is precluded, earlier inelastic response results for systems designed with $R = 1$ have demonstrated that the ductility demand on the flexible-side element in an asymmetric-plan system may exceed one indicating yielding of the element because of torsional motions (Fig. 3). In order to further investigate this issue, the responses of asymmetric-plan systems with their element yield forces computed with three different values of $\alpha$ are compared in Fig. 5. In addition to $\alpha = 1$, two larger values are considered for systems designed with $R = 1$ or 2; two smaller values are considered when $R = 8$; and one smaller and another larger value is selected when $R = 4$. The ductility demand on the flexible-side element is the only response quantity presented because other response quantities are affected very little by $\alpha$. These results demonstrate that, in order to keep the ductility demand on the flexible-side element in the asymmetric-plan system below its symmetric-plan value, $\alpha$ should be.
The preceding results have demonstrated that the ductility demand on the stiff-side element may increase significantly because of plan-asymmetry when reduction in the stiff-side element design force is permitted. Since it is desirable that the element ductility demands be similar whether the plan is symmetric or not, the presented results suggest that seismic codes should preclude reduction in the design forces of the stiff-side elements below their values for symmetric-plan systems.

The stiff-side elements are generally believed to be beneficially affected by torsion and are therefore not considered the most critical elements for design purposes. However, the preceding results that the largest ductility demand among all the resisting elements may occur in the stiff-side element. Thus, additional care is required in the design of stiff-side elements for ductility demand.

'ELASTIC' Response

It is the intent of US seismic codes that buildings suffer no damage during some, usually unspecified, level of moderate ground shaking. Thus, the response of asymmetric-plan systems designed with \( R = 1 \) is examined next. \( R = 1 \) implies that the design strength \( V \) of the corresponding symmetric-plan system is just sufficient for it to remain elastic during the selected excitation. However, as will be shown in subsequent sections, asymmetric-plan systems designed for the same base shear may not remain elastic.

The deformation of resisting elements may be significantly affected by plan-asymmetry. The deformation of the stiff-side element is generally reduced because of plan-asymmetry whereas deformation of the flexible-side element in such systems is considerably increased (Fig. 3). The ductility demand for stiff-side and flexible-side elements in the asymmetric-plan system exceeds one for some period values (Fig. 3) indicating yielding in these elements, which were designed to remain elastic if the building plan were symmetric. The stiff-side element yields more if its design force is permitted to fall below its symmetric-plan value because this results in smaller yield deformation (Chopra and Goel 1991).

The preceding results indicate that asymmetric-plan systems designed with \( R = 1 \) may deform into the inelastic range and the element deformation may significantly exceed the deformation of the corresponding symmetric-plan system. Thus, asymmetric-plan systems designed with \( R = 1 \) may experience structural damage due to yielding and nonstructural damage resulting from increased deformations.

Modifications in Design Eccentricity

The results of preceding sections indicate that deformations and ductility demands on resisting elements in a code-designed asymmetric-plan system differ from those for the corresponding symmetric-plan system. However, it would be desirable that the responses of the two systems be similar so that the earthquake performance of the asymmetric-plan system would be similar to, and specifically no worse than, that of the symmetric-plan system. In order to investigate this issue further, the responses of asymmetric-plan systems with their element yield forces computed with three different values of \( \delta = 1, 0.5, \) and 0 in Eq. 1 are compared in Fig. 4. In all cases, \( \alpha = 1 \) and four different values of \( R = 1, 2, 4, \) and 8 were considered. The ductility demand on the stiff-side element is the only response quantity presented because other responses are affected very little by \( \delta \). It is apparent that the ductility demand \( \mu_s \) on the stiff-side element in the asymmetric-plan systems designed with \( \delta = 0 \) is generally below the element ductility demand, \( \mu_s^0 \), if the system plan were symmetric. However, for some period values, precluding reduction of stiff-side element design force \( \delta = 0 \) is not sufficient to keep \( \mu_s \) below \( \mu_s^0 \). In order to achieve this objective, perhaps this design force should be increased relative to its symmetric-plan value, which implies a negative value of \( \delta \) in Eq. 1b.

![Figure 4](image_url) **Figure 4.** Ratio of stiff-side element ductility demands, \( \mu_s/\mu_s^0 \), for asymmetric-plan and corresponding symmetric-plan systems; \( \alpha = 1 \) and \( \beta = 0 \).

![Figure 5](image_url) **Figure 5.** Ratio of flexible-side element ductility demands, \( \mu_f/\mu_f^0 \), for asymmetric-plan and corresponding symmetric-plan systems; \( \delta = 0 \) and \( \beta = 0 \).

Even if such a reduction in the stiff-side element design force is precluded, earlier inelastic response results for systems designed with \( R = 1 \) have demonstrated that the ductility demand on the flexible-side element in an asymmetric-plan system may exceed one indicating yielding of the element because of torsional motions (Fig. 3). In order to further investigate this issue, the responses of asymmetric-plan systems with their element yield forces computed with three different values of \( \alpha \) are compared in Fig. 5. In addition to \( \alpha = 1 \), two larger values are considered for systems designed with \( R = 1 \) or 2; two smaller values are considered when \( R = 8 \); and one smaller and another larger value is selected when \( R = 4 \). The ductility demand on the flexible-side element is the only response quantity presented because other response quantities are affected very little by \( \alpha \). These results demonstrate that, in order to keep the ductility demand on the flexible-side element in the asymmetric-plan system below its symmetric-plan value, \( \alpha \) should be...
selected as follows: α=1 if $R=8$; α=1.5 if $R=2$ and 4; and α=2 if $R=1$. However, the optimal α values may differ with the ground motion. Thus, response results should be generated for several ground motions to determine for code use the coefficient α which should depend on the design value of the reduction factor $R$.

Even if the asymmetric-plan system can be designed for significant yielding in such a way that the ductility demand on the flexible-side element does not exceed the symmetric-plan value, the element deformation may still be larger because of plan-asymmetry. It may not be possible to reduce this deformation by increasing the strength of the system because the deformation of a medium-period, velocity-sensitive system is not strongly affected by its strength and it is for such systems that the additional deformation due to plan-asymmetry is most significant (Figs. 2 and 3). Thus, these larger deformations should be provided for in the design of asymmetric-plan structures.

Acknowledgments

This research investigation is supported by the National Science Foundation under Grant BCS-8921932. The authors are grateful for this support.

Appendix. References


