Interactions of a co-rotating vortex pair at multiple offsets

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Two NACA0012 vanes at various lateral offsets were investigated by wind tunnel testing to observe the interactions between the streamwise vortices. The vanes were separated by nine chord lengths in the streamwise direction to allow the upstream vortex to impact on the downstream geometry. These vanes were evaluated at an angle of incidence of 8° and a Reynolds number of 7×10^4 using particle image velocimetry. A helical motion of the vortices was observed, with rotational rate increasing as the offset was reduced to the point of vortex merging. Downstream meandering of the weaker vortex was found to increase in magnitude near the point of vortex merging. The merging process occurred more rapidly when the upstream vortex was passed on the pressure side of the vane, with the downstream vortex being produced with less circulation and consequently merging into the upstream vortex. The merging distance was found to be statistical rather than deterministic quantity, indicating that the meandering of the vortices affected their separations and energies. This resulted in a fluctuation of the merging location. A loss of circulation associated with the merging process was identified, with the process of achieving vortex circularity causing vorticity diffusion, however all merged cases maintained higher circulation than a single vortex condition. The presence of the upstream vortex was found to reduce the strength of the downstream vortex in all offsets evaluated. *Published by AIP Publishing.* [<http://dx.doi.org/10.1063/1.4982217>]

NOMENCLATURE

$R_{0.1}$	Average radius of vortex at 0.1 vorticity threshold
$R_{0.3}$	Average radius of vortex at 0.3 vorticity threshold
$A_{0.1}$	Area of vortex at 0.1 vorticity threshold
$A_{0.3}$	Area of vortex at 0.3 vorticity threshold
Γ	Circulation
X_c	X core location
Y_c	Y core location
C	Chord length
Re	Reynolds number, based off chord length
B_v	Vortex separation.

I. INTRODUCTION

Turbomachinery blade interactions, aircraft taking off in succession, wind turbines, and vortex generators can all produce vortex interactions with multiple streamwise vortices in close proximity to each other.^{9,12,16,20,26} These vortices may be desirable (flow control, heat transfer) or undesirable (aircraft wake vortices). In previous work, both vortices of a vortex pair have been typically deployed from the same streamwise location,^{5,22} limiting the study of their interactions at extremely close core spacings. These close interactions are important conditions to understand in order to provide a knowledge base for practical vortex applications, where upstream vortices may move in locations on either side of a vortex producing obstacle, such as a wing or vane.

As identified previously,^{6,19,21,25} a pair of co-rotating vortices will merge in any viscous flow. The equilibrium states of

interacting and merging vortices were first studied by Saffman and Szeto²⁵ using energy based equations numerically approximated with Newton's method, finding that the vortices will merge in an equilibrium state at a vortex separation to a radius ratio of 3.16. This was found to be different from that of an unsteady state, which was predicted at a ratio of 3.4 by Zabusky *et al.*³⁰ using contour dynamics, and a ratio of 3.4-3.8 by Rossow²⁴ using point vortex methods. All of these evaluations used equal strength and size vortex cores, with two dimensional flow fields and no velocity deficit through the core, limiting their accuracy and resulting in the discrepancies between the methods. It is currently accepted that merging is due to the viscous diffusion causing vorticity to expand from the inner recirculation region to the outer recirculation region.¹⁴ The ghost vortex of the outer recirculation region then stretches the vorticity between the two cores, resulting in the production of a singular vortex core.

Merging of equal strength co-rotating vortices can be broken up into four distinct stages, the first diffusive stage, the convective stage, the second diffusive stage, and the merged diffusive stage.^{3,17,18} The first diffusive stage consists of the two vortex cores increasing in size through viscous diffusion and has no change in core separation distance. The convective stage occurs once the two vortices reach a critical size, and the vortices begin to move towards each other at a rapid rate. During this stage, the advection of vorticity away from the cores forces the cores together due to the conservation of angular momentum, causing their merging. The second diffusive stage then involves the diffusion of the two vortex azimuthal velocity

peaks to form a singular vortex. In the merged diffusive stage, the combined vortices become more axisymmetric; however, now they have the same core location.

Devenport⁵ found by wind tunnel testing of co-rotating vortices deployed from the same upstream location that the unmerged cores of a co-rotating pair were far more turbulent before merging than a single vortex core by itself. Once the two cores have merged, the final structure was found to be larger and more axisymmetric than a single vane vortex. The hot wire measurements showed that post merging, the turbulence of the core was found to decrease; however, the induction of a probe into the core would have increased the sensitivity of the vortices to instabilities. As the spacing between vortices increases, the merging distance is shifted further downstream.^{5,22} Increasing vortex swirl decreases merging distance and also increases the amplitudes of vortex motion (meandering).

In the case of vortices of unequal strength, the mechanism of merging is notably different if the circulation differential is large. In these cases, the weaker vortex has insufficient circulation to support the strain field induced by the stronger vortex, and as such is strained into a spiral tail structure.¹⁴ Using inviscid contour method calculations, Dritschel and Waugh⁷ found that the interaction between two vortices with a large difference in size results in the smaller vortex being torn away, with little increase in the size of the larger vortex. This was identified as a regime of either partial or complete straining out. This is in contrast with more closely sized vortices, which often result in total core growth, under a regime they identified as complete merger or partial merger. In addition to this, equal or similar strength vortex interactions typically produce single vortices, while unequal strength interactions may produce two vortex systems. A critical ratio of core radius and vorticity was also used by Yasuda and Flierl²⁸ in their transient contour dynamics calculations to characterise empirically the likely merging state. Numerical studies of such scenarios have also been performed,¹ finding similar structures and regimes. The mechanism behind these straining actions is a combination of two causes. First, the weaker vortex is stretched and drawn into the stronger vortex by a process of elongation.²⁷ Second, a continuous erosion of vorticity into the primary vortex is caused by the strong strain field and high shear, in a mechanism analytically observed by Legras and Dritschel.¹³

If the total circulation of any vortex pair is non-zero, there will be a net rotation of the vortex system.¹⁴ In the case of a co-rotating vortex pair, both circulations are of the same sign, hence they must add to a non-zero amount, causing an orbital motion of the vortex system. If the circulations are equal, this will cause the two cores to orbit at an equal radius around a central point, while if they are unequal, the vortices will orbit on different radii. These migrations have been seen in the water tunnel testing of Rokhsaz,²² where dye marker injected into the cores of a pair of co-rotating vortices showed negligible change in the location of the orbital centre. While the dye marker can show the location of the core streamline, it cannot predict vorticity strength or the centre of vorticity, making it difficult to ascertain the mechanisms behind merging.

Vortices act as pressure gradient amplifiers, increasing an induced pressure gradient in the freestream at the vortex

core.¹⁰ As such, a probe placed near a vortex causes substantial upstream migration of the breakdown location.² Consequently either Laser Doppler Anemometry (LDA) or Particle Image Velocimetry (PIV) must be used for accurate experimental results. Due to vortex meandering, averaging point measurements can result in errors of up to 35% in tangential velocity, emphasising the importance of a global measurement technique for vortex analysis.²⁹

The work described in this paper investigates the near field interactions of a vortex produced by an upstream vortex with a downstream vane. PIV analyses have been performed for a wide variety of vane offsets at multiple downstream locations, allowing inspection of both the paths of the vortices and the meandering of the vortex pairs. Vortex interactions at very close core spacings have not been previously experimentally observed, as the vortices have been typically 2D or deployed at the same streamwise location. The studies that have deployed vortices from an upstream location have either focussed on the flow characteristics on the downstream wing itself, and/or have been limited in the number of vortex positions run, making trend analysis difficult. The aim of this work is to achieve a better characterisation of near-field co-rotating vortex interactions than has been previously available and to determine the effects of generating a vortex in a flow field with a pre-existing vortex structure. This will facilitate a better understanding of the vortex fields produced by multiple arrays of vortex generators or aircraft in following flight.

II. EXPERIMENTAL SETUP

The present study considers the interaction of two stream-wise vortices produced by two NACA 0012 vanes. One vane was located 10 chord lengths (C) downstream of the other, as can be seen in Figure 1. This configuration was chosen as it allows interactions between vortices to occur at close proximities that cannot be observed if the vortices are deployed at the same location. This is also the representative of the effects of a pre-existing vortex in a flow interacting with a vortex producing device. An angle of attack of 8° on each vane has been used for all cases, with a square-edged tip. Higher angles of attack decreased the vortex stability, with unsteady breakdown becoming observable for a single vortex case at 12° . Multiple offsets were tested from $-0.7C$ to $0.6C$ in an increments of $0.1C$, with a finer spacing of $0.05C$ between $-0.3C$ and $0.05C$.

The X axis is in the direction of the flow, with positive downstream, the Y axis is across the tunnel, and the Z axis is in the vertical direction. As such, the rear vane quarter chord was located at $X = 10C$, with the vane root at $Z = -1.5C$.

FIG. 1. Vane layout diagram, origin is at the quarter chord tip of front vane.

Planar slices of the flowfield were captured using PIV at 0.5C intervals from 1.5C back from the quarter chord of the trailing vane to 7C back. These correspond to 11.5C and 17C from the leading vane, respectively. The laser sheet was not moved closer than 11.5C as the reflections from the vanes began to distort the results. The experiment was performed at a Reynolds number of approximately 7×10^4 based on the chord length. At 7×10^4 the vortex shedding from a NACA0012 airfoil at 8° angle of attack is within the supercritical region¹¹ and therefore any Reynolds number lower than 6×10^4 at this angle of attack will result in a shedding regime that is not indicative of higher Reynolds number scenarios. Running the tunnel as slow as possible within the acceptable Reynolds number range minimised vibration of the diffuser expansion, camera mounting, and test section caused by the operation of the fan, thus minimising imaging errors. It is expected that at higher Reynolds numbers, the merging distance and number of rotations to merger will increase, as identified by Cerretelli and Williamson,³ however the mechanism studied here will be representative of a broader range of flow conditions.

A. Wind tunnel

Experiments were performed in the Macquarie University open return, closed section wind tunnel. This tunnel has a 610×610 mm (24×24 in.) octagonal test section with a 1900 mm (6'3") length. Optical access is through a glass window on the top of the test section and removable windows on the side. The test section was characterised using a Turbulent Flow Instrumentation 100 Series Cobra probe, giving a peak turbulence intensity of 0.35% and an average of 0.25%. Velocity uniformity was measured as better than 1% variance, and flow angularity was found to vary by 1° across the test section inlet. The wind tunnel speed was electronically controlled through a National Instruments MyRIO, with the pressure sensors calibrated against a temperature controlled Baratron 120AD Differential Capacitance Manometer. Streamwise velocity variance was held to within 0.38%.

A separate elevated ground is mounted to the floor of the tunnel with a rounded front splitter to minimise the effects of the pre-existing boundary layer in the test section. This ground is mounted 100 mm above the tunnel floor on two steel rails. The vanes have a chord of 80 mm and a span of 120 mm and are painted matte black to minimise reflections. A schematic of this setup can be seen in Figure 2. The boundary layer at the location of the rear vane was experimentally measured to be 5 mm thick at 80% of the freestream velocity and 20 mm thick at 95% of the freestream velocity.

B. PIV setup

A planar two component PIV system was used to capture the vortex dynamics. Due to the large expansion length of the Macquarie University wind tunnel, the camera was placed inside the expansion itself rather than using a mirror system. This allowed the camera to be positioned 2.1 m downstream of the test section, giving a maximum perspective bias of 6.25° (0.21 mm at furthest edge or 0.0027C) with a 120 mm lens. Focus was controlled remotely. By placing the camera this far downstream of the test section, there was no observable difference to the flow in characterisation measurements obtained through the tunnel section. The expansion section of the tunnel was on isolated mounts from the tunnel fan, minimising vibration. Over 200 image pairs, the tip of the rear vane was found to have a maximum displacement change of 1 pixel during operation, with no observable change between images of an image pair.

Laser access to the tunnel was through a glass window in the top of the test section. The laser beam was sent to this location via a periscope connected to a Dantec 3-axis computer controlled traverse. This traverse was restricted to only allow laser sheet movement along the axis of the tunnel. The laser used was a dual-cavity Nd:YAG laser (Quantel EverGreen) with an output of 200 mJ per pulse at 532 nm wavelength and a repetition rate of 15 Hz. Synchronisation between the laser and camera was performed with an ILA synchroniser. Laser pulses were delivered at 55 μ s apart as any higher resulted in significant out of plane migration of particles. The laser sheet thickness varies throughout the observation window as a result of the focus, with an average thickness of approximately 4 mm through the region of interest. Seeding was performed with a PIVtech generator using Di-Ethyl-Hexyl-Sebacat (DEHS) air soluble particles of 0.2-0.3 μ m typical diameter. This gives a Stokes number of approximately 2×10^{-5} , indicating that the particle size is sufficiently low to follow all flow streamlines accurately.⁴

Scattered laser light was captured by a monochrome cooled CCD pco.1600 camera with 1 GB of RAM. Images were digitised at 14 bits, with a resolution of 1600×1200 pixels. The camera was fitted with a 120 mm lens. The CCD size on the camera was 12.5 mm wide $\times$ 9.38 mm high, giving a field of view at the most downstream plane of approximately 100×133 mm. Image analysis was performed with PIVView software. Multi grid interpolation was used, starting at a coarse grid size of 128 pixels $\times$ 128 pixels windows and finishing with refinement to 32 pixels $\times$ 32 pixels over 3 passes. Standard FFT correlation was used, with two repeated

FIG. 2. Cutaway diagram of tunnel test section (left) and image of in-tunnel setup (right).

correlations on 16 pixels offset grids being performed. Sub-pixel shifting was enabled on all passes with b-spline interpolation and peak detection by a Gaussian least squares fit from 3 points. The final grid size was 99×74 nodes.

Calibration of the camera was performed using a grid that was photographed at all analysis plane locations, compensating for the increase in the plane size due to perspective. The plane was located using the laser sheet and then photographed to give an accurate scale.

C. Sources of error

The sampling error for averaged results was determined to be 3.7% in circulation and 0.0035C in location for the 400 total shots taken against a multiple representative sample of 2000 image pairs. Due to the nature of the manual focussing system, there were induced errors, with differences in focus able to produce up to 0.04C error in core location. By implementing a particle pixel size threshold of no more than 2 pixels at a brightness level of 4.5% of the total dynamic range, this error was reduced to 0.0015C in core location. Total error due to the calibration plane procedure was found to be a maximum of 0.18% in location and 0.22% in scale, due to minute differences in lateral calibration plane location. Seeding levels in the room were convergence tested such that the error from the seeding was not discernible from the randomness induced by the other errors. Camera vibration was not observed at an appreciable level, with a maximum image migration of 0.06% measured over the course of an imaging run. The particle size was measured at an average of 1.5 pixels, giving an uncertainty in position of 0.03 pixels.¹⁵ Quantization errors were negligible due to 14 bit quantization. Any biases inherent in each run were minimised by having the each set of 400 images taken with one forward run of 200 images (plane moving from X17 to X11.5) and one backward run in the opposite direction; this way any errors in seeding or focus would be minimised. The total error in core location was found to be $\pm 0.006C$. The error in lateral vane offset adjustment is $\pm 0.005C$ (10% of the smallest offset change).

D. Vortex analysis methodology

Vortex radii can vary by up to 35% if time averaged results are used due to vortex meandering and local fluctuations in velocity.²⁹ In addition to this, the velocity field will be smoothed, resulting in significant deviations in circulation and core size if time averaged results are used. However, it is still desired to have average values for core location, size, and strength, and as such the results were analysed by a script based evaluation of each individual pair of images. These images were sequentially analysed in Matlab, with peak noise filtered by vorticity gradient as previously mentioned. To eliminate the influence of weak secondary vortex structures, vortex shedding, and low level noise on the calculation of tip vortex properties, all vorticity constructs except the tip vortex were filtered out. This was performed by computing contours at 10% of the peak vorticity and calculating the area enclosed by each individual structure. These data points were then exported to Matlab, where they were then combined and analysed for average values and variances. This allowed for an accurate calculation of real world core size,

as well as time-averaged values that could be used to represent the core characteristics and allow comparison between cases.

The vortex centre within a plane is defined as the integral of the vorticity (ω) multiplied by the displacement (X or Y value, depending on the axis being calculated) divided by the circulation (Γ).¹⁴ This can be seen in Eqs. (1) and (2),

$$X_c = \frac{1}{\Gamma} \int X \omega dS, \quad (1)$$

$$Y_c = \frac{1}{\Gamma} \int Y \omega dS. \quad (2)$$

While this does not always align with the location of zero in-plane velocity, it allows for consistent prediction of the centre of circulation intensity even when the vortex pair is migrating with an in-plane motion, which would otherwise skew the core location significantly. It is also more robust than simply using the value of peak vorticity, as it is not significantly skewed by asymmetrical vortices or vorticity peaks in the result.

As the vortices are co-rotating, they both have the same signed vorticity. This means that identifying the centre of vorticity within a plane will be ineffective as it will only find the centre point between the two vortices. An automated script was used to identify the two separated vorticity peaks and construct a contour line at 0.1 of the peak vorticity and 0.3 of the peak vorticity on a given plane, giving enclosed areas of $A_{0.1}$ and $A_{0.3}$, respectively. In the case that the smaller $A_{0.3}$ was less than a quarter of the larger $A_{0.3}$, the vortices were considered merged. This 1:4 ratio was selected based on the graphical results, which correlated with the observable vortex cores while minimising the influence of signal noise on the results. The area represented by $A_{0.3}$ can be used to track the vortices through the initial stages of the merging process, as it allows for better detection of the secondary peak in a merging and partially strained vortex structure. The single $A_{0.1}$ and two $A_{0.3}$ areas are considered as the vortex core regions for the merging vortex system and individual vortices, respectively. Consequently, for path tracking the weighted centroid of Eqs. (3) and (4) was used,

$$X_c = \frac{1}{\Gamma_{A_{0.3}}} \int X_{A_{0.3}} \omega dS, \quad (3)$$

$$Y_c = \frac{1}{\Gamma_{A_{0.3}}} \int Y_{A_{0.3}} \omega dS. \quad (4)$$

While the vortices remain near a uniform Lamb-Oseen distribution at the far offsets, at nearer offsets significant partial straining occurs from the influence of the vortex interaction. This causes a skew in the shape of the vortex core that changes its primary axis as the vortex pair rotates downstream. This prevents the fitting of a Lamb-Oseen distribution of vorticity to the results. Consequently, the radius of the vortices was calculated using the vortex areas and assuming vortex circularity to give an effective radius. These were $R_{0.1}$ and $R_{0.3}$ for $A_{0.1}$ and $A_{0.3}$, respectively. The vortex circulation was calculated by the integral of the vorticity within the identified core region. For when there are individual vortices identified, this is taken at an $A_{0.3}$ cutoff, as this allows the continued identification of vortex peaks through the merging case. When the vortex is merged,

this is evaluated at $A_{0,1}$ to capture the entire vortex. If $A_{0,3}$ is used to characterise the merged vortex, it excludes the merging tail region of the vortex, causing a significant drop in effective vortex circulation. This is not an issue for the unmerged vortex cases, as the vortices are still approximately circular in shape so there is no vorticity lost to the tail region. This will however cause an effective circulation reduction for the unmerged cases, so should be noted for the results of this section. This reduction was found to be 10.5% as calculated from the single vortex case.

By comparing this method to a Lamb-Oseen approximation on a uniform, circular vortex, it was found that the sampling resolution could result in a 15% maximum error in peak vorticity. This translated to a 1.5% maximum error in the 10% peak vorticity, giving a maximum core radius error of 5% per image pair, which was considered acceptable for this analysis.

III. RESULTS AND DISCUSSION

A. Vortex migration

In all un-merged cases, the vortices followed a helical path as can be seen in Figure 3. Downstream vortex positioning at the start of the domain varied linearly with offset; however, between $0.2C$ and $-0.25C$ the vortices were merged. This merging can be seen in the $0.1C$ offset case, where the downstream vortex disappears after $X12.5$ due to it merging into the upstream vortex. As the offset approached the point of vortex merging, the path length of both the upstream and downstream vortices increased, with the downstream vortex experiencing the most migration. Total path length at

$0.6C$ offset was $0.308C$ and $0.186C$ for the upstream and downstream vortices, respectively. At $0.2C$ offset, this increased to $0.511C$ (66% increase) and $0.330C$ (77%).

While the paths retained their helical migration pattern with a linear orbital rate independently of which side of the vane the vortex passed on, the total circularity of the path varied. When comparing the $-0.3C$ case to the positive $0.3C$ case, the non-circularities of the $-0.3C$ case can clearly be seen, with a near horizontal movement of the downstream vortex for the first 4 data points. There is a translation of $0.1926C$ in the lateral direction for a total movement of only $0.0542C$ in the vertical direction for the upstream vortex across these data points. This is due to the non-linearities associated with the vortices being drawn closer from the initial stages of the merging process, as well as the influence from the wake of the rear vane. The $-0.3C$ offset case is the only case presented in this figure where the vortex paths pass both above and below where the merged vortex is located in the $-0.1C$ offset case. This means that until Z/C drops below -0.025 , the vortex is not being affected by the rear vane downwash, and once it is below this value it will be, thus causing the path non-linearity. This can only occur when the upstream vortex passes on the suction side of the vane, as this will cause orbiting motion induced by the downstream vane to draw it through this region. This effect will dissipate as the downstream vane wake dissipates further downstream.

As opposed to the laterally spaced test configuration of Rokhsaz²³ where negligible centre of rotation migration was observed, the migration of the centre of rotation of the vortices was found to be significant. Total vertical migrations of up to $0.06C$ and lateral migrations of $0.07C$ were observed in the centre of rotation. This was as high as 35% of the total vortex

FIG. 3. Paths of upstream (solid) and downstream (dotted) vortices for various lateral vane offsets. Error in core location is $\pm 0.006C$.

FIG. 4. Schematic of rotation angle calculation for vortex pairs. In-plane vortex trajectory is shown via the red (upstream vortex) and green (downstream vortex) arrows. The viewing plane is normal to the freestream velocity, with the view seen from downstream of the vanes.

migration at an offset of -0.3 . The absolute magnitude of the centroid migration remained roughly constant across the offset range measured; however, it was a significantly higher percentage of the total migration at the nearer offsets of the vortices. The analytical, inviscid results presented by Lewke¹⁴ also show a static core rotation centre. The differences observed can be attributed to the downwash produced by the vane in the creation of the second vortex. This downwash causes a change in the migration of the pair, something not previously observed due to the vortices being created at the same upstream location (in the case of Rokhsaz) or not having any vane influence (Lewke).

The spiralling rate of the vortices was calculated through a linear approximation of the change in the angle of the line drawn between the two vortex cores, as can be seen in Figure 4. Decreasing the offset increased the spiralling rate until the point of merging, as can be seen in the rotational rate in Figure 5. This rotation had a non-linear trend as the point of merging was reached, peaking at approximately 44° per chord length. This is distinctly less than the 1200° per chord length effective rotational rate of the peak azimuthal velocity region of a single vortex, attained at a radius of $0.075C$ and velocity of 37.5 C/s (3 m/s). While an inverse relationship cannot be explicitly confirmed from the offset range investigated, the rotational rate will trend to zero as the vane separation goes to infinity, indicating an extension of the non-linearity observed in the rotation trends. The rotation rate remained constant throughout the domain. The separation linearly varied at the same rate as the offset changed until the point of vortex merging.

By combining the separation distance curves from each unmerged case, the trends of separation distance for the vortex pair can be extrapolated to cover a much longer effective distance. This allows us to simulate how a vortex pair deployed at an initial separation width of $B_v/R_{0.3} \approx 7$ would behave further downstream, as can be seen in Figure 6. The separation data show that there are two different separation rate trends depending on which side of the vane the vortex is passed on. If the vortex passes on the pressure side of the vane, for every chord length travelled downstream, the vortices move together approximately 0.154 of the core radius. However, if the vortex passes on the suction side of the vane, this is decreased to 0.110 core radii, giving a 28% differential in separation rate. This suggests that the wake region of the vane significantly affects the speed of the merger, causing the vortices to be forced together faster. This happened independently of the circulation within the vortex core, which showed similar trends regardless of which side of the vane the vortex approached from.

As the vortices approach merger, the trend deviates from linear. The $-0.25C$ offset case exhibits all the merging regimes discussed in the merging section up to single vortex, combining the second diffusive and convective merging states. However, it does not show the clear levelling off or core separations as observed by Cerretelli and Williamson,³ instead demonstrating a reduced but still significant gradient. As the separation between the cores reaches two core radii apart, the separatrices of the two vortices connect and rapid merging occurs, resulting in the transformation to a singular vortex. The asymmetric mechanism behind these separation trends will be discussed further in the merging section.

FIG. 5. Vortex pair rotational rate (left) and vortex pair separation (right).

FIG. 6. Vortex pair separations for all unmerged cases. Each offset case is indicated by the annotations on the line segments.

In the merged condition, the single vortex path only was tracked, as can be seen in Figure 7. The path of the merged vortex was laterally shifted by approximately half the offset change of the rear vane, demonstrating the influence of the rear vane on vortex trajectory. This indicated that the downstream vortex contributes to approximately half of the vortex total location, despite the fact that the vortices were merged prior to the window of observation. As the downstream vane is angled to direct the flow towards $-Y$, it was anticipated that the merged vortex would be located towards $-Y$ due to the vane downwash, but as can be seen from the $-0.15C$ offset case, the vortex initially starts at a greater Y/C , peaking at $-0.11C$. This is of note as the quarter chord of the vane is located to the negative side of the initial vortex core. When the downstream vane was located at $-0.1C$, the resultant merged vortex starts at $-0.09C$, peaking at $-0.08C$ before dropping to $-0.12C$ by the end of the domain. This is significantly more positive than the single vortex case for the entire observation domain. The curvilinear path is due to the tail of the merged vortex produced by the drawing in of the downstream vortex, as will be discussed in Sec. III B. A component of the curvature is also due to the vortex passing slightly inboard and offset of the wingtip. There is a considerable downwards shift imposed by the presence of the rear vane, as can be seen compared to the path of the single vortex. In all cases, the downwards travel was approximately $0.075C$, with all paths being within error bars of each other.

Vortex path meandering was evaluated through the vortex tracking and analysis of each individual set of image pairs. Uniform circular meandering was observed at the far range of the offsets investigated. A maximum radius of displacement of $0.020C$ was measured at $0.6C$ offset. As the offset was decreased, there was no observable shift in meandering until $0.2C$ offset, where partial merging was present towards the end of the domain. The secondary vortex was drawn around the primary at this point, creating a bias in the meandering. This bias predominantly affected the weaker vortex, with a maximum amplitude of $0.066C$ measured on the axis of bias. This instability was at an average angle of 25° to the line between the two vortex cores. The stronger, upstream vortex was also marginally affected by this instability, with a maximum meandering amplitude along the axis of bias of $0.029C$ at $0.2C$ offset. This gives meandering bias ratios of 3.22 and 1.38 for the downstream and upstream vortices, respectively, indicating an instability with stronger effects on the downstream vortex. The same meandering trends were seen on the negative offsets. The magnitude of the instabilities was increased as the vortices travelled downstream and the vortex proximity was reduced through either offset change or drawing in of the vortex paths.

B. Vortex merging

Time averaged results were inspected to identify the merging pattern. The stronger and weaker vortices were selected from their circulation, with the upstream vortex (red) being the stronger and downstream vortex (green) being the weaker. The evolution of a typical merging pattern can be seen in the planar slices of the $-0.25C$ offset case in Figure 8. Individual vortex identification was performed using the contour lines at 30% of the peak vorticity on the plane ($A_{0.3}$). The stronger and weaker vortices were selected from their circulation, with the upstream vortex (red) being the stronger and downstream vortex (green) being the weaker. The yellow band shown in the figure is the $A_{0.1}$ contour line, with the other contours showing lower levels of vorticity. The scale has been selected to maintain a proportional X and Y axis for visualisation of circularity.

At the start of the domain, the vortices have similar circularity; however, as they travel downstream they are drawn closer together and partial straining of the weaker vortex occurs. This process starts at $X14$, with the secondary peak

FIG. 7. Paths of merged vortex for various lateral vane offsets. Error in core location is $\pm 0.006C$.

FIG. 8. Vortex merging pattern for $-0.25C$ offset, with upstream vortex in red and downstream vortex in green. Note the non-uniform spacing of the planes.

being completely dissipated by X16.5. Throughout the process, the upstream vortex $A_{0.3}$ does not significantly increase in area; however, the $A_{0.1}$ surrounding it does significantly increase. This is from the vorticity of the weaker vortex being diffused and spread around the stronger vortex. Of note is the fact that the upstream vortex is the stronger, while the downstream vortex is weaker. This indicates that the presence of the upstream vortex has caused the strength of the downstream vortex to be weakened. This results in the merger of the downstream vortex into the upstream vortex as the pair progresses downstream, as the upstream vortex is the stronger of the two at the location just behind the rear vane (X11.5). As a consequence, the downstream vane is effectively re-energising the existing upstream vortex after the vortex pair has merged.

The transition of the vortex from a shape with a spiral tail to a circular structure can be better investigated at the $-0.2C$ offset in Figure 9. Moving the vane offset $-0.05C$ closer causes a significant upstream shift in the merging location, with no existence of secondary peaks from the X11.5 plane onwards. As the merged vortices travel downstream, the vorticity is

transferred from the tail to the circular vortex core. Eventually the tail is completely dissipated, with the final core achieving circularity and a larger size than one individual vortex, as can be seen at the X16.5 plane.

The initial stages of the merging can be visualised through the inspection of the $-0.3C$ offset as seen in Figure 10. While this case did not merge within the observation window, the initial drawing in and vorticity transfer was clearly occurring. The lower rotational rate of the vortex cores observed at this further offset significantly slows the rate of merging when compared to the $-0.25C$ case. Initially the two vortices are separate, both at the $A_{0.3}$ and $A_{0.1}$ levels. As they travel downstream, their separations move closer by approximately $0.007C$ per chord length downstream. This equates to approximately 6% of the $R_{0.3}$ per chord length travelled downstream. From the X15 to X16 planes, there is a distinct change in the circularity of the weaker vortex, with the X16 plane showing partial straining and an oval shape occurring at a vortex separation of $0.021C$. Between X16 and X16.5, there is also an observable reduction in the size of the weaker $A_{0.3}$; however,

FIG. 9. Vortex merging pattern for $-0.2C$ offset, with upstream vortex in red and downstream vortex in green. Note the non-uniform spacing of the planes.

FIG. 10. Vortex merging pattern for $-0.3C$ offset, with upstream vortex in red and downstream vortex in green. Note the non-uniform spacing of the planes.

$A_{0,1}$ has largely remained unchanged. This indicates that the vorticity transfer between the two vortices is caused by the diffusion of high level vorticity from the second vortex into the lower energy level $A_{0,1}$. From here it is drawn around the stronger vortex, as was demonstrated in the previous cases. This case also demonstrates the need for tracking the vortex core $A_{0,3}$, as $A_{0,1}$ indicates that the vortices are merged from X12.5, while $A_{0,3}$ can clearly track distinct vortices until the final plane.

These observations of asymmetric merger show similarities to the two dimensional numerical simulations of Brandt and Nomura.¹ Partial straining of the weaker vortex followed by the diffusion of vorticity and absorption into the stronger

vortex were observed at similar circulation ratios. However, the very high vortex eccentricities and aspect ratios observed in the weaker vortex by Brandt and Nomura were not observed before complete merging. This is likely reflecting the increased vorticity transfer in the turbulent, three-dimensional experimental flow, resulting in faster merging.

Inspecting the pathlines in the co-rotating reference frame as seen in Figure 11 allows for further understanding of the uneven merging mechanism. To calculate the rate of rotation of the co-rotating reference frame, the average rotation rate across the entire domain sweep as previously calculated was used. At large separations, the vorticity fields of the two primary vortices are significantly separated ($B_v/R_{0,3} > 2.3$), with

FIG. 11. Pathlines in the co-rotating reference frame and vorticity for different stages of vortex merger.

the streamlines of the two vortices being clearly separated by an inner recirculation region. This inner recirculation region appears to be the origin of the two “ghost vortices” of the outer recirculation region. While not observed in the offset range investigated, it is anticipated that the two ghost vortices will merge at larger offsets, forming a singular recirculation region. As the vortices are drawn closer together, they divide this recirculation region into the two ghost vortices of the outer recirculation region. At this point ($B_v/R_{0.3} \approx 2.3$), the two vortex streamlines connect, as well as their vorticity field. Unlike the stages of Cerretelli and Williamson,³ the unequal three dimensional merger does not appear to enter the well defined diffusive and convective stages, as from this point onwards the vortex separations do not significantly change; however, there is a significant transfer of vorticity from the weaker to stronger vortices. Once the streamlines of the two vortices have joined and the ghost vortices are fully separated ($B_v/R_{0.3} < 2.3$), the flow begins to become significantly asymmetric in the horizontal axis, as opposed to the relative symmetry present in the further separated condition. Once this asymmetry occurs, the transfer of vorticity and modification of the pathline patterns occurs rapidly. As the merger progresses that the rotating pathlines of the weaker vortex are strained out, leaving the previously discussed vorticity tail. After the remnants of the secondary vortex have been strained out, the ghost vortices rapidly migrate to the other side of the vortex configuration and merge into a singular recirculation region. This recirculation region expands and reduces in strength as the vortex slowly normalises itself towards circularity in the merged diffusive state.

The merging lengths identified from the analysis of the time averaged cases can be seen in Figure 12. These are only given for cases where merging was observed within the domain. It can be seen that the offset for merging at the start of the domain is skewed to the positive side of the vortex (passing inherently at $-0.12C$). This shows that passing the vortex on the pressure side of the downstream vane facilitates more rapid merging than passing it on the suction side. The vortex merging length showed a highly non-linear trend with respect to offset, with the merge length rapidly exceeding the $5C$ domain length over just $0.15C$ offset change. This trend and the observed results of the merging pattern indicate that there may be a link between the merging length and rotational rate.

While the analysis of the merging patterns was taken from time averaged data, each individual image pair was analysed

to detect the vortices. It was found that the vortex merging location in the transition regions was probabilistic rather than deterministic, as seen on the right side of Figure 12. The probability of the vortex being merged is simply the percentage of image pairs without a secondary vortex. These probabilities were also tested with a random sample of 200 image pairs and found to be within 5% of the values from the full 400 image pairs, indicating an error in probability of less than $\pm 5\%$. In the $-0.2C$ case there was a 66% occurrence of merging in the first plane, with 100% of image pairs being merged with no secondary peaks by $X15.5$. The time averaged point of merge at $X13$ lies approximately halfway between these points. Similarly, in the $-0.25C$ case, the probability of merging linearly decreases throughout the domain, with a 44% probability of merging at the time averaged merge location. This indicates the presence of a fluctuation side to side of the vortices, similar to that identified in a previous computational study by the authors⁸ producing a sinusoidal fluctuation in the merging point. This meandering of the singular vortices causes them to move towards and away from each other, with a resultant fluctuation in vortex separation. As previously identified, the merging location is very sensitive to offset, and consequently any variance in vortex separation will cause a significant difference in the presence of secondary vortex peaks.

Two interesting findings are apparent from these results. The first is the near linear rate of the probability decay with distance. This rate appears to have minimal skew from the samples taken, and minimal non-linearity. However, when considering the probability distribution for a regular sine wave, there is a quasi-constant region that shows similarity. From -50% to $+50\%$ of a sine wave amplitude, all sample bins of a frequency histogram are within 2%, and at $\pm 75\%$ of the waves amplitude, the samples all fall within a maximum variance of 10%. This means that a sine wave displacement change will appear linear up to 75% of its maximum amplitude. Consequently, the merge is following the sinusoidal oscillation previously discussed, likely caused by a sinusoidal instability in one or both of the vortices. This causes a sinusoidal change in vortex spacings, resulting in the observed merging statistics. The second finding is that the time averaged merge location does not necessarily coincide with the point of 50% merging probability. This is clear in the -0.2 case, where the time averaged case merges at $X13$, while the probability of merging at this point is 89%. However, in the -0.15 case, the time averaged merge

FIG. 12. Distance to vortex merging for time averaged cases (left) and probability distribution for instantaneous measurements at various lateral vane offsets (right).

FIG. 13. Initial vortex radii (left) and final vortex radii (right).

at X12 is reflected in the 100% merging probability from X12 onwards. This indicates the variances in vortex meandering, as well as the change in energy distributions and vortex shapes accounts for significant changes in the transient fluctuations of the vortex merger.

C. Circulations and core radii

The radius results of Figure 13 show the initial $R_{0.3}$ as remaining relatively constant for the unmerged cases, with the downstream vortex radius approximately 9% smaller than the upstream at the start of the domain. The radius of the upstream vortex does not significantly drop throughout the domain, with drops in radius of approximately 3%. The downstream vortex has a similar trend for its size in far offset cases; however, as the offset is reduced, its interaction with the upstream vortex causes a reduction in size of up to 13% over the domain. For the merged case it can be seen that the initial $R_{0.3}$ is significantly higher than the single vortex case; however, by the end of the domain, it has reduced to within the error of the single vortex case. This is due to the dispersion of vorticity from the weaker vortex core to the $A_{0.1}$, as identified in the merging section of this paper.

When inspecting the $R_{0.1}$ this can be seen through the significantly higher radii for both the initial and final cores. The core radius in this merged region is also affected by how merged the vortices are. $R_{0.3}$ in the $-0.2C$ offset case is the largest of the merged cases at the start of the domain, coinciding with the irregular, non-circular shape seen in Figure 9. As the vortex travels downstream, it forms circular and uniform $A_{0.3}$, and this coincides with the final radius observed in the single vane condition. The nearer offset cases have more

significant vortex core relaxation by the initial plane, resulting in their comparatively smaller radii. Applying the same principles to $R_{0.1}$, it would be expected that over the course of a longer domain, merged $R_{0.1}$ would trend towards the single vortex as the vorticity is drawn in from $A_{0.1}$.

The circulation figures seen in Figure 14 show similar trends to the radius; however, there is a greater discrepancy between the upstream and downstream vortices. The loss in circulation from the downstream vortex is very apparent, with drops of 28% along the length of the domain observed for the cases nearest to merging. This was a non-linear trend, showing far more significant decreases than core radius changes. This is indicative of the dissipation of the secondary vorticity peak into the $A_{0.1}$ as part of the energy transfer mechanism. Of note is that the energy transfer out from the secondary vortex is occurring at a far greater offset than the merged cases, with it being clearly observable at the $-0.4C$ and $0.4C$ offsets. The drop in downstream vortex circulation is 4.7% at the $0.4C$ offset and 7.3% at the $-0.4C$ offset. This drop is also skewed to the positive offset, similarly to the merging distance. It is hypothesised that this is due to the low pressure core of the upstream vortex passing on the pressure side of the downstream vane, reducing the magnitude of the high pressure here. This reduces the pressure differential across the downstream vane's tip, thus reducing the strength of the resultant tip vortex. It is also a cause of the skew in vortex merging to positive offset, as the lower strength downstream vortex is more rapidly merged.

While the radius of the upstream vortex remained constant as the vanes approached merging offset, the upstream vortex circulation can be seen to reduce at nearer offsets. At the $0.2C$

FIG. 14. Initial vortex circulation (left) and final vortex circulation (right).

offset, for example, the upstream circulation drops by 9%, as opposed to the 0.5C offset where it drops by only 3.7%. As such, the diffusion of vorticity from both vortex peaks becomes more significant as their proximities are reduced. This circulation has diffused into the $A_{0,1}$ region as part of the secondary diffusive stage of vortex merging.

Inspecting the initial circulation for the merged case, it can be seen that the outer regions of the merged offsets trend towards the sum of the two individual vortex circulations. At $-0.3C$ offset, the initial sum of the upstream and downstream vortex circulations is $0.222 \text{ m}^2/\text{s}$, and at $0.2C$ offset it is $0.227 \text{ m}^2/\text{s}$, which compares similarly to $0.220 \text{ m}^2/\text{s}$ and $0.236 \text{ m}^2/\text{s}$ measured at $-0.2C$ and $0.1C$ offset, respectively. However, at the end of the domain, the merging process has levelled the circulation to closer to that of the $0C$ and $-0.1C$ offsets. This indicates that the shift towards circularity involves a penalty in circulation, although the final circulation of the merged vortex is still significantly higher than a single vortex case. It is important that this is not necessarily considered as a loss of flow energy, as the circulation is proportional to vorticity, which is not a direct measure of flow energy.

IV. CONCLUSION

Wind tunnel experimentation was performed to investigate the behaviour of the interactions between a co-rotating vortex pair produced by two offset vanes. NACA0012 wings of 1.5 aspect ratio, at 8° angle of attack and a Reynolds number of 70 000 were used for this study, spaced $10C$ apart in the streamwise direction. Lateral offsets from $-0.7C$ to $0.6C$ were studied to examine the effects of vortex proximity on the resulting vortex sizes and paths.

For all unmerged cases, the two vortices migrated in a helical pattern. Vortex merging was observed from $-0.25C$ to $0.2C$ offset, equivalent to $-0.15C$ to $+0.3C$ offset from the unobstructed path of the downstream vortex. This demonstrated a bias to faster vortex merging when the upstream vortex passed on the pressure surface of the downstream vane. As the offset was decreased towards the point of merging, the orbital rate of the vortices increased non-linearly to a maximum of $44^\circ/\text{chord length}$ travelled downstream. Vortex separation varied linearly with offset, with the vortices consistently moving closer together throughout the domain for all offsets investigated. As the vortices moved closer together and further downstream, an instability was identified in the meandering of the vortices. For the merged cases, it was found that the merging process imparted a downwards motion and shifted the vortex path to the positive side. Passing the vortex on the pressure side of the vane resulted in the vortices moving towards each other approximately 28% slower than if it was to be passed on the suction side of the vane.

The vortex merging distance was found to be highly sensitive to offset, with a non-linear trend. An unequal merging process was observed, with the downstream vortex diffusing its vorticity to a lower energy level. This diffuse vorticity was then drawn around the stronger upstream vortex, eventually forming a circular structure. Similar patterns were observed for all offsets where merging occurred. The symmetry of the

vortex structure was found to change rapidly once the vortices came with a core separation 2.3 times the core radius, resulting in rapid merging by the time the vortices were 2 core radii apart. The location of merging could not be determined deterministically but was instead statistical phenomena. This was due to the meandering of the vortex location and energy levels shifting the merging location upstream and downstream in a sinusoidal oscillation.

From the circulations, it was found that the presence of the upstream vortex weakened the downstream vortex. As the vortices approached merging, their vorticity peaks were diffused into a larger, lower energy vorticity level. For the fully merged cases, a circulation loss was found to result from transitioning from an irregular shape to a circular one. Despite this penalty, the merged circulation remained higher than that of a single vortex.

While the merging distance is sensitive to offset, these results indicate that the fundamental effects and mechanisms of the merging process remain the same regardless of vortex separation. As such, the re-energisation of an upstream vortex can be performed with a relative insensitivity to offset.

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