Extension of Modal Pushover Analysis to Compute Member Forces

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This paper extends the modal pushover analysis (MPA) procedure for estimating seismic deformation demands for buildings to compute member forces. Seismic demands are computed for six buildings, each analyzed for 20 ground motions. A comparison of seismic demands computed by the MPA and nonlinear response history analysis (RHA) demonstrates that the MPA procedure provides good estimates of the member forces. The bias (or error) in forces is generally less than that noted in earlier investigations of story drifts and is comparable to the error in the standard response spectrum analysis (RSA) for elastic buildings. The four *FEMA-356* force distributions, on the other hand, provide estimates of member forces that may be one-half to one-fourth of the value from nonlinear RHA.

INTRODUCTION

The nonlinear static procedure (NSP) (ASCE 2000) has become a standard method in structural engineering practice for performance-based seismic evaluation of structures. In the NSP or pushover analysis, the structure is subjected to monotonically increasing lateral forces with an invariant height-wise distribution until a target displacement is reached. The seismic demands are computed at the target displacement and compared against acceptability criteria. These criteria depend on the material (e.g., concrete, steel, etc.), type of member (e.g., beam, column, panel zones, connections, etc.), importance of the member (e.g., primary or secondary), and the structural performance levels (e.g., immediate occupancy, life safety, or collapse prevention).

The acceptability criteria specified in the *FEMA-356* document (ASCE 2000) are in terms of the deformation demands such as story drift or plastic hinge rotation. Therefore, past work on evaluating the NSP and developing improved procedures focused on deformation demands (Gupta and Krawinkler 2000, Gupta and Kunnath 2000, Chopra and Goel 2002, Goel and Chopra 2004). Recently, the profession has expressed the need to estimate force demands, such as bending moment, shear force, and axial force, in various members of the lateral load-resisting system (Heintz 2002). Furthermore, various inelastic analysis procedures, including the NSP, are being evaluated in the ATC-55 project (ATC 2003) based not only on the deformations but also on the forces. Therefore, it is useful to develop procedures for computing force demands in the NSP.

In the *FEMA-356* NSP, force demands can be computed easily; they are given by member forces in the structure pushed to the target displacement. However, the seismic demands in the recently developed modal pushover analysis (MPA) procedure (Chopra and Goel 2002) are computed by combining contributions of all significant modes. This procedure was further refined (Goel and Chopra 2004) by making several improvements over the original version (Goel and Chopra 2002). It was shown that the MPA procedure provides estimates for deformations—story drifts and beam plastic rotations—demands that are much superior to the *FEMA-356* procedures with similar computational effort (Goel and Chopra 2004). In its present form, the MPA procedure is not applicable to estimating member forces because forces computed by this procedure may exceed the specified member capacity. Therefore, there is a need to extend the MPA procedure to compute member forces that provide estimates consistent with the specified capacity. This paper is aimed at filling this need.

EXTENSION OF MPA TO COMPUTE MEMBER FORCES

While the deformation demands estimated by combining modal contributions according to established modal combination rules compare well with the demands estimated from the nonlinear RHA (Goel and Chopra 2004), the member forces computed in this manner may exceed the member capacity, implying that they are unrealistic. Therefore, the previously presented MPA procedure is extended for calculating member forces. The member forces are first computed by the standard MPA procedure (see Goel and Chopra, 2004) and compared with the specified member capacity. If the computed member force exceeds the member capacity, it is obviously unrealistic. Therefore, the member force is recomputed from the member deformation(s)—determined by the MPA procedure (see Goel and Chopra, 2004)—using the member force-deformation (or moment-rotation) relationship. With this modification, the MPA procedure is able to capture strain-hardening (or strain-softening) effects in forces in members deformed beyond the elastic limit.

Presented next are the procedures to compute (1) bending moment in hinge (rotational connection) element used to model concentrated plasticity at the beam ends; (2) shear forces in beams modeled as elastic elements with hinge elements at the two ends; and (3) shear forces, axial forces, and bending moments in nonlinear beam-column elements with axial-force-bending-moment (P-M) interaction. These elements are available in several widely used computer programs for nonlinear analysis of structural systems, such as DRAIN-2DX (Powell 1993), SAP2000 (CSI 2003), and OpenSees (McKenna and Fenves 2000).

BENDING MOMENT IN HINGE ELEMENT

The hinge element (Figure 1a) is generally modeled with a bilinear moment-rotation relationship (Figure 1b). The moment in the hinge is computed as follows:

1. Compute the moment, M , in the hinge element by combining the peak “modal” moments, M_n and the moments due to gravity load, M_g , according to the combination rule specified in the MPA procedure (see Goel and Chopra, 2004).

Figure 1. (a) Hinge element, and (b) moment-rotation relationship for hinge element.

2. Compare the moment, M , computed in Step 1 with the specified moment capacity (or yield moment), M_y , of the hinge element. If $M \leq M_y$, the moment computed in Step 1 is the moment demand. Otherwise, compute the moment demand according to Step 3 and 4.
3. Compute the total rotation, θ , of the hinge by combining peak “modal” rotations, θ_n , and rotations due to gravity loads, θ_g , according to the combination rule specified in the MPA procedure (see Goel and Chopra, 2004).
4. Compute the moment in the hinge element corresponding to the computed rotation in Step 3 utilizing the hinge moment-rotation relationship (Figure 1b): $M = (1-\alpha)M_y + \alpha k\theta$.

A much simpler, although less accurate, procedure is used to compute the total hinge rotation in Step 3 compared to the procedure presented earlier (Appendix A in Goel and Chopra 2004); in the more accurate procedure, the total hinge rotation is obtained by adding the yield rotation to the plastic hinge rotation that is computed indirectly from the total story drift. Although not appropriate for estimating hinge rotation, this simple procedure is appropriate for computing the hinge moment because it varies slowly with rotation for hinges deformed beyond the elastic limit (see Figure 1b). As a result, even a large error in the hinge rotation (computed from the simple procedure) leads to only small error in the computed moment.

SHEAR FORCE IN BEAMS WITH HINGE ELEMENTS

The nonlinear behavior of the beams is often modeled in structural analysis by an elastic beam element with nonlinear hinge elements at the two ends. In such a nonlinear beam model, the bending moments at the two beam ends are equal to the hinge moments, and the maximum shear in the beam is limited by the moment capacity of the hinge elements. Therefore, accurate estimation of the beam shear requires proper consideration of hinge moments. This procedure is described as follows:

Figure 2. Shear equilibrium of a beam under uniformly distributed span loading, w , and end moments M_I and M_J .

1. Compute the shears, V_I and V_J , at ends I and J of the beam by combining the peak “modal” shears, V_n and the shear due to gravity load, V_g , according to the combination rule specified in the MPA procedure (see Goel and Chopra, 2004).
2. Compute the moment, M_I and M_J , in hinge elements located at ends I and J of the beam using the procedure described in the preceding section. If moments at both end M_I and M_J are $\leq M_y$, the shears computed in Step 1 are the shear demands. Otherwise, compute the shear demands according to Step 3.
3. Compute the shear demands from equilibrium of the beam under external loads and internal moments M_I and M_J computed in Step 2.

The computation required in Step 3 for a beam under gravity loads distributed uniformly over its span and end moments M_I and M_J is demonstrated in Figure 2. For such a beam, the shears are given by

$$V_I = \frac{wL}{2} + \frac{M_I + M_J}{L} \quad \text{and} \quad V_J = \frac{wL}{2} - \frac{M_I + M_J}{L} \quad (1)$$

Note that algebraic signs of the moments M_I and M_J are lost in the combination process. Therefore, Equation (1) assumes bending of the beam in double curvature, which is a reasonable assumption for buildings deformed by horizontal earthquake excitation.

AXIAL FORCE IN NONLINEAR BEAM-COLUMN ELEMENT

The columns are typically designed so that the axial-force demand does not exceed the axial load capacity, and the axial forces are computed assuming elastic behavior. Therefore, axial force, P , in the nonlinear beam-column element is obtained by combining the peak “modal” axial forces, P_n , and the axial force due to gravity load, P_g , according to the combination rule specified in the MPA procedure (see Goel and Chopra, 2004). Note that the axial forces computed in this manner may lead to a conservative estimate, and, because of P-M interaction, lead to an unconservative estimate of other forces such as bending moment or shear force. However, the MPA procedure—an approximate procedure—still provides good estimates of the bending moments and shear forces as demonstrated later in the paper.

Figure 3. (a) Parallel-component model for the nonlinear beam-column element, and (b) moment rotation relationship for the beam-column element.

BENDING MOMENT IN NONLINEAR BEAM-COLUMN ELEMENT

The nonlinear beam-column element in many computer programs (e.g., DRAIN-2DX) is modeled as a system with two components in parallel (Figure 3a): elastic-plastic component, and elastic component. The moment-rotation relationship for the first component is elastic/ perfectly plastic, whereas the second component models the post-yield stiffness (Figure 3b). The axial-force-bending-moment (P-M) interaction relationships typically used for the elastic-plastic component are shown in Figure 4.

The procedure to compute bending moments in the parallel model for the nonlinear beam-column element is as follows:

1. Compute the axial force in the nonlinear beam-column element, P as described in the preceding section.
2. Compute the bending moment at ends I and J , M_I and M_J , of the nonlinear beam-column element by combining the peak “modal” moments, M_n , and the moments due to gravity load, M_g , according to the combination rule specified in the MPA procedure (see Goel and Chopra, 2004).
3. Determine the yield moment, M_{yp} , of the elastic-plastic component corresponding to the axial force, P , computed in Step 1 from the specified P-M interaction diagram (Figure 4) and add the moment in the elastic component to obtain the total yield moment: $M_y = M_{yp}/(1-\alpha)$ (Figure 3b). If the axial forces at two ends of the nonlinear beam-column element are different, the yield moment obtained in this step may also be different for the two ends.
4. Compare the bending moments, M_I and M_J , computed in Step 2 with the yield moments, M_y , computed in Step 3 at the corresponding end. If M_I and M_J are smaller than M_y , they represent the bending-moment demands. Otherwise, compute the bending moments from Steps 5 to 7.
5. Compute the rotations θ_I and θ_J at ends I and J of the nonlinear beam-column

Figure 4. P-M interaction diagrams for nonlinear beam-column elements: (a) steel columns, and (b) reinforced-concrete (R/C) columns.

element of the column by combining the peak “modal” rotations, θ_n , and the rotations due to gravity load, θ_g , according to the combination rule specified in the MPA procedure (see Goel and Chopra, 2004).

6. Compute the bending moment in the elastic component from

$$M_{e,I} = \alpha \frac{EI}{L} (4\theta_I + 2\theta_J) \quad \text{and} \quad M_{e,J} = \alpha \frac{EI}{L} (4\theta_J + 2\theta_I) \quad (2)$$

7. Compute the total bending moment by adding the elastic component (Step 6) and plastic component (Step 3) as

$$M_I = M_{yp} + M_{e,I} \quad \text{and} \quad M_J = M_{yp} + M_{e,J} \quad (3)$$

Implicit in Equation 2 is the assumption that the nonlinear beam-column element deforms in double curvature; this assumption is generally appropriate for columns deformed due to horizontal earthquake excitation.

Note that while the procedure presented here is appropriate for computer programs that use a parallel element model (e.g., DRAIN-2DX), these concepts can be extended to compute bending moments in elements available in other programs, e.g., OpenSees (McKenna and Fenves 2000), where strain hardening is considered at the material level.

SHEAR FORCE IN NONLINEAR BEAM-COLUMN ELEMENT

The shear force in a nonlinear beam-column element is also limited by the moment capacities of the element and is computed by the procedure described previously for computing beam shear, but using the moments M_I and M_J at ends I and J , and the yield moment, M_y , determined from the preceding section.

SELECTED BUILDINGS, GROUND MOTIONS, AND RESPONSE STATISTICS

This study analyzed 9- and 20-story buildings designed for Boston, Seattle, and Los Angeles sites for ground motions with exceedance probability of 2% in 50 years (return period of 2475 years) developed during the SAC project. The selected buildings were modeled in computer program DRAIN-2DX (Prakash et al. 1993), wherein the beams were considered to be elastic with nonlinear hinge elements at the two ends, the columns were modeled with nonlinear beam-column elements with a P-M interaction diagram appropriate for steel columns. Further description of SAC buildings, their modeling, and ground motions is available elsewhere (Gupta and Krawinkler 1999).

The member forces for each building due to each of 20 ground motions are determined by nonlinear response history analysis (RHA), the extended MPA procedure (described above), and NSP using force distributions specified in *FEMA-356* (ASCE 2000), which were summarized by Goel and Chopra (2004). The “exact” peak value of structural response or demand, r , determined by nonlinear RHA is denoted by $r_{\text{NL-RHA}}$, the approximate value from MPA by r_{MPA} , and the approximate value from *FEMA-356* analyses by r_{FEMA} ; the same notation r_{FEMA} is used for all *FEMA-356* force distributions.

The bias in an approximate procedure is quantified by the median of the ratio of structural response values determined by approximate and “exact” procedures: $r_{\text{MPA}}^* = r_{\text{MPA}} / r_{\text{NL-RHA}}$ for the MPA procedure and $r_{\text{FEMA}}^* = r_{\text{FEMA}} / r_{\text{NL-RHA}}$ for *FEMA-356* analyses. The approximate procedure is biased toward underestimating the response if this ratio is less than one and overestimating the response if the ratio exceeds one.

The median value of the approximate-to-exact ratio of demands is defined as the geometric mean of the 20 data values (Goel and Chopra 2004). In the case where one or more excitations caused collapse of the building or its first-“mode” SDF system, the median values were estimated by a counting method. The 20 data values were sorted in ascending order, and the median was estimated as the average of the 10th and 11th values starting from the lowest value.

BIAS IN MPA ESTIMATE OF SEISMIC DEMANDS

Presented next is the height-wise variation of the bias in the MPA estimate of forces: bending moment in hinges, shear force in beams, axial forces in columns, shear force in columns, and bending moment in columns. The results are presented for selected locations: exterior end of the exterior beams for hinge bending moment and beam shear force, and base of the exterior columns for column axial force, column shear force, and column bending moment. Also included for benchmark comparison is the height-wise distribution of bias in the story drifts estimated by MPA (Goel and Chopra 2004).

For convenient reference, we first summarize conclusions of the earlier study (Goel and Chopra 2004) regarding bias in the story drifts estimated by the MPA procedure. It was found that the MPA procedure estimates the values of story-drift demands for five of the six buildings—Boston and Seattle 9- and 20-story buildings, and Los Angeles

9-story building—to a degree of accuracy that is comparable to the standard response spectrum analysis (RSA) procedure; the total bias was found to be no more than 28%. However, the bias was found to be unacceptably large for the Los Angeles 20-story building, a building that was deformed far into the inelastic range, far enough that its lateral capacity is degraded significantly.

The onset of significant degradation in lateral capacity is evident from the pushover curves developed in the MPA procedure. While a detailed discussion on this subject is available in the earlier study (Goel and Chopra 2004), following is a brief summary for the Seattle and Los Angeles buildings; a summary for the Boston buildings is omitted because they remained essentially elastic during the selected ground motions.

The pushover curves for the Seattle and Los Angeles 9-story buildings develop a small plateau after yielding, followed by gradual decay in the lateral capacity and eventually a region of rapid decay in the lateral capacity. The region of rapid decay in the lateral capacity starts at roof drift (roof displacement expressed as a percentage of building height) of about 4.5% for the Seattle building and 6% for the Los Angeles building. The pushover curves for the Seattle and Los Angeles 20-story buildings exhibit a short plateau after yielding, followed by a rapid decay in the lateral capacity. The region of rapid decay in the lateral capacity starts at the roof drift of about 1.4% for the Seattle building and 1.5% for the Los Angeles building. If the building is deformed beyond this limit, the errors in the estimates from the MPA or any other NSP procedure are expected to be large. It is useful to emphasize that the roof drift value associated with onset of significant (or rapid) degradation the lateral capacity depends on the building. However, this value can easily be gleaned from the pushover curves developed in the MPA procedure.

The results presented in Figure 5 show that the MPA procedure provides very good estimates of the hinge bending moments throughout the height of all buildings. This is apparent from the mean value of the ratio M_{MPA}^* being close to unity. The deviation is generally less than 10% for all buildings except for a few stories of the Los Angeles 20-story building where the deviation approaches about 20%. More importantly, the bias in the MPA estimate of hinge bending moments is significantly less compared to the story drifts throughout the building height. The preceding observations also apply to results for beam shear presented in Figure 6.

The MPA procedure generally (except for the Boston 9-story building) overestimates the column axial force with the bias rarely exceeding 25%, which is smaller than the bias in estimating story drifts (Figure 7). The column axial forces in the Boston 9-story building are underestimated slightly. For a few upper or middle stories of the Seattle and Los Angeles 20-story buildings, the bias exceeds the 25% overestimation.

The MPA procedure also provides good estimates of the column shear and column bending moment (Figures 8 and 9) throughout the height of all buildings, except for the Los Angeles 20-story building. The bias in estimating column shear and column bending moment is generally smaller than for story drifts. The larger bias in estimating seismic demands for the Los Angeles 20-story building is expected. The larger bias has been noted earlier even in estimating story drifts (Goel and Chopra 2004) because this build-

Figure 5. Median hinge bending moment ratios M_{MPA}^* and story drift ratios Δ_{MPA}^* for MPA.

Figure 6. Median beam shear force ratios V_{MPA}^* and story drift ratios Δ_{MPA}^* for MPA.

Figure 7. Median column axial force ratios P_{MPA}^* and story drift ratios Δ_{MPA}^* for MPA.

Figure 8. Median column shear force ratios V_{MPA}^* and story drift ratios Δ_{MPA}^* for MPA.

Figure 9. Median column bending moment ratios M_{MPA}^* and story drift ratios Δ_{MPA}^* for MPA.

ing was deformed well into the inelastic range with significant degradation in lateral capacity (see earlier discussion on the subject). For Boston 9- and 20-story buildings, which responded essentially in the elastic range, the bias in estimating column shear and column bending moment is similar to that for story drift.

In summary, the extended MPA procedure provides good estimates of member forces for five of six SAC buildings—Boston and Seattle 9- and 20-story buildings, and Los Angeles 9-story building. The bias (or error) in the forces is generally no more than that in story drifts. Recall that the bias in the story drift estimates of the MPA procedure is comparable to that in the standard RSA procedure for elastic systems (Goel and Chopra 2004). Therefore, it may be concluded that the MPA procedure estimates member forces for these five buildings to a degree of accuracy that is also comparable to the RSA procedure for elastic systems, a standard tool in structural engineering practice.

However, the bias in the MPA estimate of member forces and story drifts is unacceptably large for buildings that are deformed well into the inelastic range with significant degradation in lateral capacity—such an example is the Los Angeles 20-story building subjected to severe ground motions. For such cases, MPA (and most other pushover analysis procedures) cannot be expected to provide satisfactory estimates of seismic demands, and should be abandoned; nonlinear RHA becomes necessary.

Figure 10. Median hinge bending moment ratios M_{MPA}^* for the MPA procedure and M_{FEMA}^* for the four *FEMA-356* distributions: 1st Mode, ELF, SRSS, and Uniform.

BIAS IN DEMAND ESTIMATES FROM *FEMA-356* FORCE DISTRIBUTIONS

The selected SAC buildings exceed the *FEMA-356* criterion for higher mode effects, as shown by Goel and Chopra (2004). Because the *FEMA-356* NSP is permitted for such buildings, even though it cannot be used alone, its results are included for comparison with MPA and nonlinear RHA.

Presented in this section is the bias in member forces computed using the four *FEMA-356* force distributions—1st Mode, ELF, SRSS, and Uniform—which are described in an earlier investigation (Goel and Chopra 2004). To provide a meaningful comparison of MPA and *FEMA-356* estimates of seismic demands, member forces in *FEMA-356* analyses were computed at target displacement equal to the roof displacement determined by MPA. Results are presented for hinge bending moments, column axial forces, and column bending moments. Although the results for beam and column shear forces were generated, they have not been included in this paper for the sake of brevity.

The height-wise distributions of bias in *FEMA-356* and MPA estimates of seismic demands are compared in Figures 10 to 12. The MPA procedure provides estimates of all member forces that are much superior compared to FEMA results. The four *FEMA-356* force distributions significantly underestimate all seismic force demands in upper stories of the Seattle and Los Angeles 9- and 20-story buildings. The underestimation exceeds 50% and in some cases approaches 75%, indicating that *FEMA-356* force distributions provide member forces that are one-half to one-fourth the value from nonlinear RHA.

Figure 11. Median column axial force ratios P_{MPA}^* for the MPA procedure and P_{FEMA}^* for the four *FEMA-356* distributions: 1st Mode, ELF, SRSS, and Uniform.

For Boston 9- and 20-story buildings, which responded essentially in the elastic range, the *FEMA-356* force distributions grossly underestimate most member forces by 50% to 75% in upper as well as lower stories (see Figures 10 and 12). For these two buildings, the bias in the force estimates from the MPA procedure (see Figures 5 and 9) does not exceed 25%. Recall that the MPA procedure for elastic buildings reduces to the standard response spectrum analysis (RSA) procedure (Goel and Chopra 2004). Therefore, the force results for the two Boston buildings from the MPA are consistent with those from the RSA for elastic buildings. The *FEMA-356* distributions, on the other hand, failed to provide accurate estimates for the elastic buildings. The column axial forces, however, are underestimated only in the upper stories by less than 50% (Figure 11). Among the four *FEMA-356* force distributions, the “Uniform” distribution provides the worst results. The results for beam and column shear forces, not presented for the sake of brevity, led to similar observations.

CONCLUSIONS

The MPA procedure, previously shown to be effective in estimating deformation demands (Goel and Chopra 2004), has been extended to estimate member forces—hinge bending moment, beam shear force, column axial force, column shear force, and column bending moment—consistent with the specified member capacity. The member forces are first computed by the standard MPA procedure to estimate deformation—by combining the “modal” contributions using established “modal” combination rule—and compared with the specified member capacity. If the computed member force exceeds

Figure 12. Median column bending moment ratios M_{MPA}^* for the MPA procedure and M_{FEMA}^* for the four *FEMA-356* distributions: 1st Mode, ELF, SRSS, and Uniform.

the member capacity, it is recomputed from the MPA estimate of member deformations using the member force-deformation (or moment-rotation) relationship.

The accuracy of the extended MPA procedure in estimating the member forces is evaluated for six SAC buildings, 9-story and 20-story buildings, designed for Boston, Seattle, and Los Angeles subjected to ensembles of 20 ground motions. This evaluation showed that the extended MPA procedure provides good estimates of member forces for five of six SAC buildings. The bias (or error) in forces is generally less than that in story drifts. Furthermore, the MPA procedure estimates member forces for these five buildings to a degree of accuracy that is comparable to the RSA procedure for elastic systems, a standard tool in structural engineering practice.

However, the bias is unacceptably large for buildings that are deformed far into the inelastic range with significant degradation in lateral capacity; such an example is the Los Angeles 20-story building subjected to severe ground motions. For such cases, MPA (and most other pushover analysis procedures) cannot be expected to provide satisfactory estimates of seismic demands, and should be abandoned; nonlinear RHA becomes necessary. Note that the onset of the region with significant degradation in lateral capacity, which depends on the building, can be easily gleaned from the pushover curves developed in the MPA procedure.

A comparison of member forces computed from the MPA procedure and the *FEMA-356* NSP showed that MPA provides much superior estimates. The four *FEMA-356* force distributions grossly underestimate member forces in upper stories, and also in the lower

stories of some of the buildings considered. The underestimation exceeds 50% and in some cases approaches 75%, indicating that member forces estimated by these force distributions are one-fourth to one-half the value from nonlinear RHA. Among the four *FEMA-356* distributions, the “Uniform” distribution provides the worst results.

ACKNOWLEDGMENTS

This research investigation is funded by the National Science Foundation under Grant CMS-9812531, a part of the U.S.-Japan Cooperative Research in Urban Earthquake Disaster Mitigation. This financial support is gratefully acknowledged. Our research has benefited from discussions with Jon Heintz of Degenkolb Engineers, Inc. Comments from the anonymous reviewers and especially the responsible editor helped in significantly improving the manuscript.

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