MEAN REVERSION AND THE ASSET ALLOCATION DECISION

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ABSTRACT

Until fairly recently the conventional wisdom in the finance academic community was that security prices follow a random walk. Some influential papers have uncovered evidence of mean reversion particularly over longer horizons. Siegel (1998) has suggested that given this evidence: "the holding period becomes a crucial issue when the data reveal the mean reversion of the stock returns." In this paper, we explore the pattern of mean reversion in post-World War II U.S. stock returns and find that it peaks in a 4-year cycle. Given this empirical regularity we show that a buy-and-hold investment strategy, which is appropriate under a random walk, is no longer optimal.

Keywords: Predictability, bootstrapping, random walk, portfolio choice.

I. INTRODUCTION

In the last decade we have witnessed an intense interest in the issue of mean reversion in stock returns. When Fama and French (1988) and Poterba and Summers (1988) documented mean reversion in long-horizon (greater than one year) stock returns, the initial debate revolved around what these findings implied for the market efficiency hypothesis. If stock prices have a significantly predictable component, then this could be consistent with models of an irrational market in which prices exhibit long but ultimately temporary swings away from fundamental values.[1] Alternatively, this price behavior could also result from time-varying equilibrium expected returns in an efficient market. If there is some sort of average to which stock prices tend to return, then apart from the market efficiency question, the mean reversion phenomenon could be a significant factor in making optimal asset allocation decisions.

A number of recent papers have explored the issue of predictability in stock returns and the implications for investment decisions. For instance, Barberis (2000) considers both buy-and-hold and dynamic rebalancing strategies for long horizon investors in the context of predictable returns. He shows that the risk-averse investor will allocate a larger proportion to equities, the longer her horizon, even in the presence of parameter uncertainty about the predictor variable.[2] Campbell and Viceira (1999) propose a solution to the multi-period portfolio choice problem under plausible assumptions about the nature of time-varying expected returns and investors' utility functions.

In this paper, we explore mean reversion patterns in an intuitive setting and show that these patterns have direct implications for the investor's asset allocation decision. Mean reversion in stock returns suggests that bad returns over several time periods are likely to be followed by several periods of good returns. With a random walk, the future is a flip of the coin, regardless of the outcomes in earlier periods. If stock returns exhibit mean reversion in long horizons, the volatility of returns is lower than that implied by a random walk model. We use the post-World War II period to study the pattern of mean reversion. Using monthly Large Company U.S. Stock returns, we implement various tests, including those suggested by Fama and French (1988) and Lo and Mackinlay (1988), to report that the evidence of mean reversion is strongest in a 4-year cycle. Apparently, a couple of bad years tend to be followed by a couple of good years and vice versa. This reinforces the result in McQueen and Thorley (1991), who use a Markov chain approach to report that a low annual stock return is three times more likely to be preceded by a sequence of two years of high returns as compared to two years of low returns.

Samuelson (1969, 1994) has shown that if an investor's relative risk aversion is greater than unity, then her asset allocation choice is independent of her investment horizon provided that the risky asset returns follow a random walk. This is a refutation of the popular time diversification argument. Samuelson (1991) qualifies the above result by showing theoretically that in the presence of mean reversion the optimal proportion allocated to equities increases as the investment horizon lengthens. This has been interpreted particularly in the practitioner literature (see Kritzman, 1994 and Reichenstein & Dorsett,1995) as redemption for the time diversification position. However, our tests detect a pattern of mean reversion that does not validate the time diversification position; the optimal allocation to equities reaches a peak at four years, beyond which it declines. Siegel (1998) comments: "it might seem puzzling why the holding period has almost never been considered in portfolio theory. This is because modern portfolio theory was established when the academic profession believed in the random walk theory of security prices...The holding period becomes a crucial issue when the data reveal the mean reversion of the stock returns."

We suggest that that it is important to construct an investment strategy that exploits the dampened volatility in risky asset returns that is brought about by a 4-year mean reversion cycle. As an example, we postulate a 6-year investment horizon with an asset allocation change permitted at year four. In a standard utility maximization framework, we show that an investor with constant relative risk aversion maximizes utility by changing her asset allocation proportions in the fourth year. Evidently, it is preferable to invest 79% and 43% in risky assets for four and two years respectively, instead of allocating a fixed 58% for the entire 6-year period. There would be no particular advantage to changing the initial proportions if stock prices followed a random walk.

The contributions of this study are twofold. First, we provide an intuitive exposition of mean reversion on the basis of two data generating procedures. The first series preserves the serial dependence present in the actual data. The second series is designed to yield serially independent returns (random walk), which serves as a benchmark for

evaluating the actual series. We confirm that there is some evidence of mean reversion in stock returns and that it peaks in a 4-year cycle. Although no formal statistical tests are used, a number of illustrative methods verify this phenomenon and show that it persists with real, nominal as well as excess return data. Second, we show that the mean reversion pattern affects the optimality of the asset allocation decision. We suggest that in addition to the macroeconomic and other technical variables ordinarily employed in tactical asset allocation models, the mean-reverting behavior of security returns should also enter the equation. In the next section, we provide the details of our methodology and the results. The final section contains concluding comments.

II. METHODOLOGY AND RESULTS

As indicated above, the raw data are monthly returns for Large Company U.S. Stocks and U.S. Treasury Bills for the 1947-97 period.[3] We choose post-World War II return data in order to obtain results that can be generalized in a contemporary setting.[4] The monthly Consumer Price Index (inflation rate) numbers are used to calculate real monthly returns on stocks and t-bills. Two different data picking procedures (bootstrapping) are employed. In the first procedure (designated as Data 1), one month is picked randomly and followed for six years to calculate annual returns over a 6-year period. This is repeated 5000 times and the observations generated mirror the actual data because the serial pattern is maintained for each 6-year period. Under this procedure, any mean reversion in the data will be picked up. In the second procedure (designated as Data 2), six different months are picked randomly and each is followed for one year resulting in six non-serial annual returns and this is also done 5000 times. This method creates annual returns that are independent while capturing any monthly patterns that might exist in stock and t-bill returns. This procedure is designed to yield a random walk in annualized returns.

In Table 1, we report the ratio of variances (Data 1 w.r.t Data 2) of annual and cumulative returns for Year 1 through Year 6. Note that the ratio of annual returns is tightly clustered around 1.0 for stock as well as t-bill returns. This result is not particularly surprising because even though Data 1 yields serial returns while Data 2 yields independent returns, the variance measurement has been reported with each year in a 6-year period treated as an isolated unit. However, the cumulative

TABLE 1. Ratio of Variances of Annual and Cumulative Returns: Var(Data 1) / Var(Data 2)

	Annual			*Cumulative*		
Year	*Stocks*	*T-Bills*	*Excess*	*Stocks*	*T-Bills*	*Excess*
1	1.031	0.987	1.025	1.031	0.987	1.025
	(1.031)	(1.018)		(1.031)	(1.018)	
2	0.969	0.950	0.961	0.915	1.498	0.907
	(0.960)	(1.020)		(0.824)	(1.951)	
3	0.944	1.049	0.954	0.814	1.964	0.824
	(0.948)	(1.005)		(0.650)	(2.858)	
4	1.010	1.020	1.003	0.788	2.440	0.805
	(1.018)	(0.965)		(0.591)	(3.569)	
5	0.988	1.020	0.990	0.874	2.834	0.936
	(0.985)	(0.977)		(0.662)	(4.321)	
6	0.965	0.979	0.965	0.957	3.064	1.087
	(0.963)	(0.973)		(0.720)	(4.882)	

These ratios are calculated from Data 1 and Data 2 where these data mirror the actual and the independent returns respectively. Results are reported in real terms; the corresponding results in nominal terms are reported in parentheses. Excess returns are Stock minus T-Bill returns.

return ratio for stocks exhibits a U-shaped pattern with the lowest value of 0.788 in Year 4. This suggests that mean reversion in the actual returns (Data 1) peaks at the fourth year implying volatility that is well below that of a random walk model (Data 2). This pattern is even more pronounced for measurements done in nominal rather than real terms (see numbers in parentheses in Table 1). By contrast, the cumulative return ratio for t-bills shows a continuous upward trend implying that t-bill returns display mean *aversion* with the effect becoming stronger as the horizon lengthens. We also report the ratio of variances for an excess return (stocks minus t-bills) series in Table 1. Once again, the results are striking and consistent. The ratios for annual excess returns

cluster around 1.0 while cumulative excess returns exhibit a U-shaped pattern with the lowest value reached in Year 4. Clearly, the excess return series also reflects a mean reversion pattern similar to that found in real and nominal stock returns.

In order to verify the mean reversion pattern more formally, we turn to the Lo and Mackinlay (1988) and Fama and French (1988) tests. These tests are based on continuously compounded returns, $r_t = \ln(1+R_t)$ where R_t are monthly returns. Real returns are obtained by deflating nominal returns by the corresponding Consumer Price Index. The variance ratio, popularized by Lo and Mackinlay, for a q-year return is:

$$VR(q) = \frac{Var(r_t(q))}{qVar(r_t)} \tag{1}$$

where $r_t(q) = \sum_{j=1}^{q} r_{t-j+1}$.

If annual returns are i.i.d. (random walk), then $Var(r_t(q)) = qVar(r_t)$ and, therefore, $VR(q) = 1$. If security returns are mean reverting (averting), then VR(q) is less than (more than) one. We implement these variance ratios for stock and t-bill returns for q = 2,3...6.

Another intuitive test of mean reversion is suggested by Fama and French (1988). Consider running an OLS on the following:

$$r_{t+p}(p) = \beta_0 + \beta_1 r_t(p) + \varepsilon_{t+p} \tag{2}$$

Notice that the slope coefficient, β_1, of the above regression denotes the first-order autocorrelation of p-period returns. If the slope coefficient is negative (positive) it implies mean reversion (aversion). Further, it should be noted that if annual returns are used and β_1 is negative for p = 2, it suggests that two years of good (bad) returns are likely to be followed by two years of bad (good) returns, implying mean reversion in a 4-year cycle. In order to make this test comparable to the variance ratio test, we use half-yearly returns for implementing the Fama-French test with p = 2,3...6.

The variance ratios as well as the slope coefficients for 2-year through 6-year cycles are reported in Table 2. Once again, a U-shaped pattern in mean reversion shows up for stock returns based on Data 1. The variance ratio starts off at 0.92 for the 2-year return, reaches a minimum of 0.84 for the 4-year return and turns up again reaching 1.11 for the 6-year return. Again, the effect is much more pronounced for nominal returns. Although, the slope coefficient for the Fama-French test is the lowest for a 3-year cycle, it is still negative for Year 4, becoming positive thereafter. Also, both test procedures applied to excess returns yield results confirming a mean reversion effect that peaks in a 4-year cycle.[5] It should be noted that there is no obvious economic explanation for this apparent 4-year cycle. Even though our analysis has been executed in aggregate series, there is mounting evidence in the cross-section that stock returns exhibit short-term momentum (up to twelve months) and reversals over a 3-5 year period (see Jegadeesh & Titman, 2001). A number of theorists have suggested that investors are subject to behavioral biases that induce departures from market efficiency (see Daniel et al., 2001 for a review).

TABLE 2. Tests of Mean Reversion

	L-M Ratio[a]			*F-F Coefficient*[b]		
Year	*Stocks*	*T-Bills*	*Excess*	*Stocks*	*T-Bills*	*Excess*
2	0.922	1.507	0.886	-0.082	0.458	-0.117
	(0.825)	(1.893)		(-0.175)	(0.874)	
3	0.850	1.891	0.801	-0.158	0.388	-0.194
	(0.679)	(2.689)		(-0.281)	(0.821)	
4	0.836	2.355	0.778	-0.095	0.473	-0.130
	(0.628)	(3.397)		(-0.248)	(0.777)	
5	0.971	2.698	0.907	0.151	0.430	0.139
	(0.730)	(4.049)		(0.035)	(0.739)	
6	1.107	2.935	1.023	0.326	0.389	0.328
	(0.828)	(4.652)		(0.237)	(0.712)	

Notes: [a] **L-M Ratio is the variance ratio of the Lo-Mackinlay test; a value less (more) than 1 suggests mean reversion (aversion).**
[b] **F-F Coefficient is the regression coefficient of the Fama-French test; a negative (positive) value suggests mean reversion (aversion). Results are reported in real terms; the corresponding results in nominal terms are reported in parentheses. Excess returns are Stock minus T-Bill returns.**

In this paper, our focus is to explore the implications of mean reversion patterns for the investor's asset allocation decision. We adopt a standard wealth utility maximization framework to identify the optimal allocation between the risk-free (t-bills) and risky assets (stocks) over investment horizons ranging from one to six years. Consider the following terminal wealth at period t:

$$W_t = \alpha W_0(1+R_r)^t + (1-\alpha)W_0(1+R_f)^t \qquad (3)$$

where W_0 represents the value of the initial wealth and annualized returns of stocks and t-bills are given by R_r and R_f respectively. Note that α is the proportion of the initial investment that is allocated to equities. The investor maximizes her expected utility to make an optimal asset allocation decision at time t_0. Let the utility function be:

$$U(W) = \frac{\delta}{\delta - 1}(W)^{1-1/\delta} \qquad (4)$$

with a constant relative risk aversion parameter = $1/\delta$; $\delta > 0$ and $W>0$.

The optimal α is determined by numerically maximizing the sample average utility with $1/\delta = 5$, $W_0 = \$1$, and $t = 1,2...6$. The results are presented in Table 3. The assumption at this point is that the investor pursues a passive buy-and-hold strategy and maintains the initial asset allocation proportions over the entire investment horizon. The optimal α, under the Data 2 scenario is around 62% regardless of the length of the investment horizon. Recall that the Data 2 procedure generates serially independent returns and hence this result is nothing but a confirmation of the Samuelson (1969) proof of the time diversification fallacy. However, the more interesting finding flows from Data 1 or the sample that is designed to reflect the true nature of dependence in annual returns. Once again, an inverted U-shaped pattern emerges.[6] The optimal α rises from 62% for a 1-year horizon to a peak of 88% for the 4-year horizon and declines to 58% for the 6-year horizon. This result contradicts the popular belief that mean reversion in stock returns could perhaps be used to justify the time diversification argument.

The higher allocation to risky assets for a 4-year period stems from the fact that the volatility in a 4-year cycle is reduced due to mean reversion in stock returns. In cross-sectional analyses, one attempts to create a hedge portfolio by including a risky asset that is negatively correlated with the other risky assets in the portfolio. A negative correlation makes the volatility of this hedge asset a risk-reducing property. Similarly, negatively correlated returns in a mean reversion cycle could serve a similar purpose.

TABLE 3. Optimal Allocation to Risky Assets for different (Years 1-6) Investment Horizons

Year	*Data 1*	*Data 2*
1	0.62	0.64
2	0.66	0.62
3	0.76	0.63
4	0.88	0.62
5	0.71	0.61
6	0.58	0.60

Data 1 and Data 2 are simulated to mirror the actual and the independent real returns respectively. The utility, , is maximized to obtain the optimal allocation to risky assets, α, over various investment horizons. Initial wealth, $W_0 = \$1$ and the relative risk aversion parameter, $1/\delta = 5$.

As an example, we postulate a 6-year investment horizon to compare the buy-and- hold strategy for the entire 6-year period (6,0) with one in which asset allocations are changed after a 4-year run. This is achieved by making an asset allocation change either at the end of the second year (2,4) or the fourth year (4,2). The terminal wealth at t = 6 is:

$$W_6 = \alpha_2 W_j(1+R_{r2})^{6-j} + (1-\alpha_2)W_j(1+R_{f2})^{6-j} \qquad (5)$$

$$W_j = \alpha_1 W_0(1+R_{r1})^j + (1-\alpha_1)W_0(1+R_{f1})^j \qquad (6)$$

The R_{r1}, R_{f1} and R_{r2}, R_{f2} represent annualized returns for the first j years and the last 6-j years respectively where j = 2 or 4. The utility function defined earlier in (4) is maximized to yield the optimal asset allocation proportions α_1 and α_2. If no switch is allowed, then $\alpha_1 = \alpha_2$. The

results are reported in Table 4. Note that in the case of Data 2 (serially independent returns) the optimal proportion in risky assets is a near constant 60% regardless of the strategy employed. In stark contrast, for Data 1 (designed to reflect the serial dependence observed in the actual data), the optimal allocations to risky assets vary dramatically with an asset allocation change.

An investment approach designed to exploit the dampened volatility in risky asset returns due to mean reversion involves holding these assets for four years. In a 6-year investment horizon, the (4,2) as well as the (2,4) strategies result in a higher average utility than the (6,0) strategy. We report certainty equivalent wealth levels to risky prospects, evaluated at optimal α levels, to highlight the utility gain. We also report the mean and standard deviations of terminal wealth for the above three strategies. Note that the (2,4) strategy dominates the (6,0) strategy in terms of a higher sample mean and a lower sample standard deviation of terminal wealth. However, the certainty equivalent wealth level is the highest with the (4,2) strategy with an improvement of about four% in total real return over the passive (6,0) strategy.[7] The optimal

TABLE 4. Optimal Allocation to Risky Assets for a 6-year Investment Horizon

	Data 1			*Data 2*		
Strategy	*(6,0)*	*(2,4)*	*(4,2)*	*(6,0)*	*(2,4)*	*(4,2)*
α_1	0.58	0.43	0.79	0.60	0.62	0.61
α_2	0.58	0.74	0.43	0.60	0.60	0.60
$CE(W_6)$	1.179	1.184	1.186	1.214	1.214	1.214
$Mean(W_6)$	1.437	1.446	1.471	1.441	1.430	1.430
$Stddev(W_6)$	0.384	0.383	0.403	0.397	0.372	0.372

Data 1 and Data 2 are simulated to mirror the actual and the independent real returns respectively. The utility, , is maximized to obtain optimal asset allocation to risky assets (α_1 and α_2) for a six year investment period. $CE(W_6)$ represents the certainty equivalent of the risky six year prospect. Sample mean and standard deviation evaluated at the optimal α's are given by $Mean(W_6)$ and $Stddev(W_6)$.

asset allocation is 79% allocated to equities for the first 4-year period followed by a sharply reduced 43% assigned to equities for the remaining 2-year period. Clearly, this is the mean reversion effect at work and it is strongest for the 4-year return.

We emphasize that the 6-year horizon was chosen purely as an example. We also reran the experiment with a 10-year horizon with a switch either at year 6 or year 8. These switch points were chosen to exploit the 4-year mean reversion pattern.[8] The certainty equivalent wealth levels were slightly higher for the (6,4) and (8,2) strategies compared to the (10,0) buy-and-hold strategy. The results based on the 6-year horizon are more reliable due to the data constraints of longer horizon returns. Since we are working with monthly data from 1947-97, we can construct very few 10-year periods even when we consider overlapping periods[9]

It is important to point out that in the above (simplified) example, the investor is following an unconditional buy-and-hold strategy with one asset allocation change at time 4 that is predetermined at time 0. In other words, the investor learns nothing from the actual returns realized in the initial 4-year period. Practically, when an investor contemplates an asset allocation change, quite apart from the general historical pattern of mean reversion in stock returns, the returns immediately preceding the change point are clearly salient. If a 4-year mean reversion cycle is to be interpreted as two years of bad (good) returns following two years of good (bad) returns, one could exploit this pattern in making dynamic asset allocation decisions. We conduct the following simple experiment to demonstrate this fact.

Consider an investor who has a 4-year investment horizon and is generally content to follow a buy-and-hold strategy. According to this specification, she will allocate 88% to equities (see Table 3, Data 1 column). However, this investor reserves the right to make an asset allocation change after two years if her 2-year realized return falls either in the top or bottom quartile of historical 2-year returns. Using a grid search, we find that it is optimal for the investor in the bottom (top) quartile to increase (decrease) the allocation to equities to 100% (58%). The investor who makes this change ends up with a certainty equivalent wealth level of \$1.1665 whereas the investor who does nothing ends up with \$1.1569. Apparently, the gains associated with the 4-year mean reversion cycle under a buy-and-hold scenario can be further enhanced

by a dynamic strategy. However, a detailed analysis of dynamic rebalancing is beyond the scope of this paper.

We feel that investment practitioners ought to factor in mean reversion effects in their asset allocation models for buy-and-hold as well as dynamic rebalancing strategies. This is particularly true for the type of tactical asset allocation described by Sharpe (1992). As he points out: "Tactical changes in asset mix are driven by changes in predictions concerning asset returns." We believe that the prediction models could be improved if they incorporate the mean reversion (or aversion) properties of different asset classes in addition to the macroeconomic and fundamental variables usually employed.

III. CONCLUSION

In this paper, we study the mean reversion phenomenon in security returns and the implications for the investor's asset allocation decision. We use a bootstrapping procedure that yields two different series of annual returns over a 6-year horizon. The first series preserves the serial dependence present in the actual data. The second series is designed to yield serially independent returns (random walk), which serves as a benchmark for evaluating the actual series.

Mean reversion implies that stock return volatility is lower than what is predicted by a random walk model. We find that there is a well defined U-shaped pattern in the mean reversion behavior in stock returns. Using a variety of approaches, we show that mean reversion increases as you lengthen the horizon, peaks at the 4-year return and diminishes in intensity thereafter. This pattern implies that the optimal allocation to equities reaches a peak at four years, beyond which it declines. Therefore, mean reversion cannot be used to justify the popular time diversification argument that the allocation to equities increases continuously with investment horizon.

We suggest that that it is important to construct an investment strategy that exploits the dampened volatility in risky asset returns that is brought about by a 4-year mean reversion cycle. We demonstrate this by postulating a 6-year investment horizon and compare a buy-and-hold strategy with one in which the asset allocation is changed at the fourth year. We show that it is preferable to invest a high proportion in equities for four years and lower this exposure significantly in the remaining two years relative to allocating a fixed proportion for the entire 6-year period. There is no advantage to changing the initial proportions when the random walk series is used.

The utility gains documented in this paper are fairly modest but this is not surprising. If mean reversion and its pattern were very obvious then the predictability in stock returns would be immediately exploited and the pattern broken. The departures from the random walk are small in nature and any exploitation thereof cannot be expected to be dramatic. However, even a weak mean reversion pattern may enable investors to squeeze out a few extra basis points. We do not think that transaction costs are likely to outweigh the utility gains because we are implementing only one index level change over a six-year period. Also, if the rebalancing takes place in a retirement account (401K or IRA), capital gains taxes are not a problem.[10]

It is important to note that we have only considered the mean reversion pattern for the S&P 500 return series in a six-year framework. This has been done purely for illustrative purposes. In a more general setting, the properties of other security return series, both U.S. and international, over investor-specific time horizons, can similarly be used to implement optimal asset allocation strategies.

ACKNOWLEDGEMENTS

We thank Timothy Mech, Steven Thorley and an anonymous reviewer for useful comments. We retain responsibility for any errors.

NOTES

1. See DeLong et al (1990).

2. Also see Kandel and Stambaugh (1996).

3. These returns are from Ibbotson Associates and represent the S&P 500 Composite with dividends reinvested from 1957-97 and the S&P 90 from 1947-56.

4. Kim, Nelson and Startz (1991) point out that the very long-horizon (8 years or longer) mean reversion is driven largely by the unreliable pre-war period. See McQueen (1992) for a similar observation.

5. When Data 2 are used, the L-M ratios and the F-F coefficients are consistently close to 1 and 0 respectively in all cases.

6. See Thorley (1995) for a similar result.

[7] When nominal wealth is used, the improvement of the (4,2) strategy is 4.4% over the (6,0) strategy. This result is consistent with our earlier tests that showed that mean reversion is more evident with nominal returns. We also implemented other switch point strategies, (i.e., (1,5), (3,3), (5,1)) and by all measures the (4,2) strategy was dominant.

[8] Ideally there should be two switches over ten years occurring at either 2,6 or 4,8 but we implemented only one switch point since this exercise is primarily a sensitivity check on our previous result.

[9] In the interest of brevity, we do not report results for the 10-year horizon experiment but they are available from the authors.

[10] Most retirement funds like TIAA-CREF permit several asset-allocation changes without fees.

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