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Effects of second sound on acoustic transmission at the solid-liquid ^{4}He interface

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We have calculated the reflection and transmission coefficients of acoustic waves propagating across a solid-liquid interface of ^{4}He, using an extension of the treatment of Castaing and Nozières in which we include pressure changes associated with second sound. These calculations account well for the experimental results obtained over the temperature range 0.83 K$\leq T \leq$1.46 K and provide an alternative explanation to the one we offered previously for the lack of agreement between experiment and the theory of Castaing and Nozières.

Crystal growth from the melt has been studied for many years.[1] Recently, particular attention has been directed toward the problem of growth of quantum crystals. Phenomenologically, growth can be described by $J = k\Delta\mu$, where J is the mass freezing per unit time, $\Delta\mu$ is the difference in chemical potential between liquid and solid, and k is a kinetic coefficient. In a classical system where growth is diffusion and/or nucleation dependent, k increases with temperature, typically according to an Arrhenius law. For quantum solids, such as helium, Andreev and Parshin[2] proposed an entirely different growth process in which the solid-liquid interface has a very high mobility, and at $T = 0$ K the interface remains rough, thus the process of growth is continuous and reversible, i.e., without dissipation. So at $T = 0$ K the coefficient k is infinite; however, as the temperature increases, thermal excitations in the liquid and solid interact with the interface and cause dissipation. Therefore, in contrast to classical systems, the coefficient k, for rough interfaces in the case of helium, *decreases* with increasing temperature.

This proposal led to several theoretical and experimental investigations of helium crystal growth.[3-10] Of particular interest to this work was the realization[4,5] that a sound wave propagating from the liquid to solid, in a medium such as helium, could cause rapid freezing and melting for a highly mobile interface. The pressure changes would be taken up by the advancing or receding interface and sound transmission between the two phases would be substantially reduced or even suppressed entirely. The reduction in transmission provides a method for studying the growth kinetics with ultrasonic techniques.

We have recently reported measurements of the temperature dependence of the reflection and transmission coefficients of high-frequency (10 MHz) sound waves at the rough ^{4}He superfluid-solid interface.[10] In our work, as well as previous studies[5] of transmission alone, it was observed that sound incident from liquid into solid was transmitted much less efficiently than would be expected from standard acoustic impedance mismatch theory.[11] Furthermore, our measurements of reflection, along with those of transmission, allowed us to calculate the relative acoustic energy loss at the interface, yielding results not in agreement with existing theoretical predictions.[4] To account for our observations we proposed a phenomenological interpretation based on a relaxation process which was consistent with the energy-loss data. We present here an alternative explanation based on an extension of the theory of Castaing and Nozières[4] (CN). This extension accounts for our reflection data as well as the relative acoustic energy loss.

The theory of CN which treats the interaction of sound with He-4 superfluid-solid interface cannot be applied directly to our measurements. Their treatment considers the melting and freezing of the crystal in response to the sound (pressure) wave. The latent heat produced on freezing is considered to be carried away by a second-sound (temperature) wave in the superfluid phase. Their approach does not include a pressure contribution due to the second sound wave as they take the thermal expansion coefficient equal to zero. As a result they have a relation between R, the reflection coefficient, and τ, the transmission coefficient, given by $1+R=\tau$. Our data are not consistent with this result, in fact discrepancies as large as 50% exist (see Figs. 1 and 2).

We present here an expanded, but similar, approach to that of CN in which we include the effect of pressure associated with second sound. The boundary conditions can be

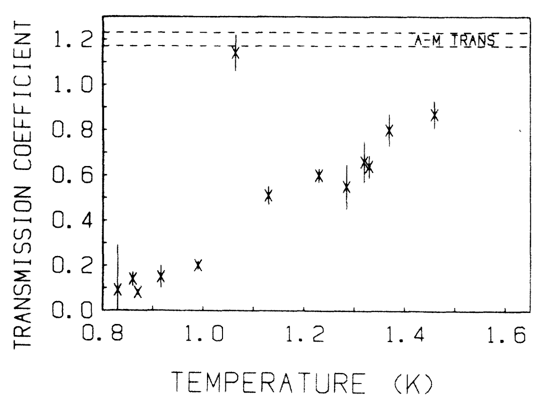

FIG. 1. Transmission coefficient τ vs temperature. $\times$'s are measured values (from Ref. 10). Horizontal dashed lines indicate range of values expected from acoustic impedance mismatch theory (see Ref. 11).

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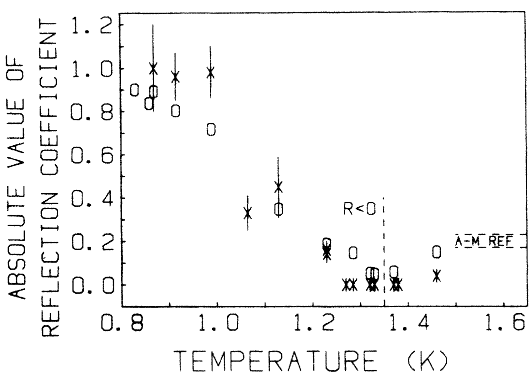

FIG. 2. Absolute value of the reflection coefficient $|R|$ vs temperature. ×'s are measured values (from Ref. 10), O's are values obtained from present theory. Theoretical values to the left of vertical dashed line are negative. See text for details. Horizontal dashed lines indicate the range of values expected from acoustic impedance mismatch theory (see Ref. 11).

written as follows:

$$P_1 = P_2 \ , \tag{1}$$

$$J = k(\mu_1 - \mu_2) = \rho_2(v_2 - v_{\rm int}) \ , \tag{2}$$

$$\rho_n v_n + \rho_s v_s + (\rho_2 - \rho_1) v_{\rm int} = \rho_2 v_2 \ , \tag{3}$$

$$v_n = v_{\rm int} \ . \tag{4}$$

Where P_i, μ_i, and ρ_i are the pressure, chemical potential per unit mass, and density of phase i ($i = 1 =$liquid, $i = 2 =$solid), J is the mass freezing per unit time, ρ_n and ρ_s are the normal and superfluid densities, v_n and v_s are the normal and superfluid particle velocities, v_2 is the particle velocity in the solid, $v_{\rm int}$ the velocity of the interface, and k is the kinetic growth coefficient.

For ordinary (first) sound in a superfluid $v_n = v_s = \delta p/(\rho_1 c_1)$ where δp is the pressure amplitude and c_1 the speed of first sound in the liquid. The particle velocities for second sound in a superfluid are given by $v_n = c_{ii} C_1 \delta T/(s_1 T)$ and $v_s = -\rho_n c_{ii} C_1 \delta T/(\rho_s s_1 T)$, where δT is the temperature amplitude, c_{ii} the speed of second sound in the liquid, C_1 the specific heat per unit mass of the liquid, s_1 the entropy per unit mass in the liquid, and T the temperature.[12,13] We can also make use of the thermodynamic identity: $d\mu = dp/\rho - s dT$. The entropy of the solid, s_2, can be taken as a reference which we take equal to zero, then $L = s_1 T$ is the latent heat (this is equivalent to saying that the temperature of the two phases is the same at the interface). The values of the solid and liquid entropies only appear as their difference so the latent heat is the quantity which appears in the results.

In contrast to the treatment of CN we take the pressure in the liquid to be composed of three rather than two terms. Two of these are due to the incident and reflected waves, as used by CN. In order to account for the "missing" pressure amplitude, however, we also include a contribution to the pressure field from the second sound wave via the thermal expansion coefficient: $\delta p' = \rho_1 c_1^2 \alpha \delta T$, where $\alpha = (1/V)(dV/dT)_p$ is the isobaric thermal expansion coefficient. Assuming an initial pressure (first sound) wave of δp_0 and resulting temperature (second sound) wave δT_0, we obtain the following results:

$$\tau = 2z_2/[z_1 + z_2(1-\eta) + z_1 z_2 \xi] \ , \tag{5}$$

$$R = [z_2(1-\eta) - z_1 - z_1 z_2 \xi]/[z_1 + z_2(1-\eta) + z_1 z_2 \xi] , \tag{6}$$

$$\delta T_0/\delta p_0 = 2 z_2 \eta/\{[z_1 + z_2(1-\eta) + z_1 z_2 \xi]\rho_1 c_1^2 \alpha\} \ . \tag{7}$$

We have used the definition of the acoustic impedance $z_i \equiv \rho_i c_i$ where ρ_i and c_i are the mass density and sound velocity of phase i, and following the notation of CN we have defined

$$\xi \equiv k[(\rho_2 - \rho_1)/\rho_1 \rho_2]^2/[1 + kL^2/(\rho_1 c_{ii} C_1 T)] \ , \tag{8}$$

and we introduce

$$\eta \equiv c_1^2 \alpha L \rho_1 \rho_2 \xi/[c_{ii} C_1(\rho_2 - \rho_1)] \tag{9}$$

with all other variables as above.

With these new expressions for R and τ we can calculate the relative acoustic energy loss:

$$\varepsilon = 1 - |R|^2 - |\tau|^2 z_1/z_2 = \frac{4 z_1 z_2^2 [\xi(1-\eta) - \eta/z_2]}{[z_1 + z_2(1-\eta) + z_1 z_2 \xi]^2} \ . \tag{10}$$

(Note that for all of the above equations taking $\alpha = 0$ implies that $\eta = 0$ and the results all reduce to those of CN.) We can now compare these theoretical predictions with measured quantities. It is clear from Eqs. (5)–(7) that once ξ and η are known the values of R, τ, and $\delta T_0/\delta p_0$ can be determined. Equation (9) contains physical quantities which are all known[14-16] except for ξ. By inserting the definition of η, Eq. (9), into Eqs. (5)–(7) and (10) they can all be expressed in terms of known quantities with the one unknown ξ. By using the measured values of the transmission coefficient, τ, we can obtain ξ for all of the temperatures investigated via Eq. (5). [In principle, the same could have been done with the reflection data via Eq. (6), except that we only measure the absolute value of R and cannot use this relation.] By putting ξ into Eqs. (6), (7) and (10) we are able to predict ε, R, and $\delta T_0/\delta p_0$ for the present treatment.

The expression for $\delta T_0/\delta p_0$, Eq. (7), gives a relation between the amplitude of the incident pressure (first sound) wave δp_0 and the amplitude of the temperature (second sound) wave generated at the interface δT_0. The values of $\delta T_0/\delta p_0$ vary as a function of temperature, as expected from the temperature dependnce of ξ and the other material parameters. (Since the externally generated δp_0 was essentially constant for all temperatures the temperature dependence is mostly in δT_0.) The values ranged from a minimum of 7.4×10^{-8} K cm^2/dyn to a maximum of 2.05×10^{-7} K cm^2/dyn. For our experiment we estimate the amplitude of our incident pressure wave at $\delta p_0 = 2 \times 10^4$ dyn/cm^2. This gives values of δT_0 from a minimum of 1.48 mK to a maximum of 4.1 mK. These values are consistent with those of other workers.[13]

The theoretical values for $|R|$ and ε obtained from the present treatment are shown as circles in Figs. 2 and 3, respectively. As seen in Fig. 3 the measured values of ε have the largest uncertainty at the lower temperatures and

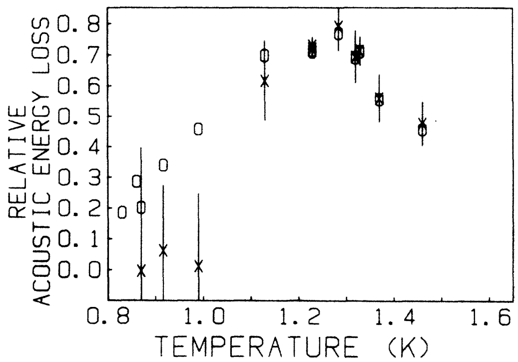

FIG. 3. Relative acoustic energy loss ε vs temperature. $\times$'s are measured values (from Ref. 10), O's are values obtained from present theory. See text for details.

this is where the largest disagreement with the present theory occurs. At the higher temperatures theory and experiment are in close agreement. While the quantitative agreement for $|R|$ (Fig. 2, circles) is not quite as good, the general trend is well reproduced. In fact we gain some new information from this approach. Equation (6) gives the value of R for a given ξ. We have taken the absolute value for comparison with the data; however, for all temperatures to the left of the vertical dashed line in Fig. 2 the reflection coefficient R is less than zero. From this we infer that our data also were less than zero; however, in our experiments we measured only absolute values and this was not apparent. Hence, our data combined with the theory suggest that at low temperatures R is less than zero and at higher temperatures R is greater than zero (with a crossing at approximately 1.3 K).

[This is not surprising as seen from the following argument. As discussed above for $T \rightarrow 0$ the growth coefficient $k \rightarrow \infty$. In this case the interface acts as if it were the boundary between an elastic medium and vacuum so we expect that $\tau = 0$ and $R = -1$. On the other hand, we expect that at higher temperatures the growth coefficient is small and the behavior of R should approach that expected from acoustic impedance mismatch, which predicts $R > 0$ for a wave incident from a medium of low impedance to one with higher impedance. So we expect $R < 0$ for low temperatures and $R > 0$ at high temperatures, with a crossing ($R = 0$) in between; this behavior is not inconsistent with our data.]

As mentioned above when the temperature rises it is predicted[2,9] that the interaction of the interface with the thermal excitations (phonons and rotons) in the bulk phases will cause dissipation and hence lower the value of the growth coefficient as

$$(Km)^{-1} = aT^4 + A\exp(-\Delta/k_B T) \ , \tag{11}$$

where we introduce the notation of Ref. 3, $k = \rho_2 mK$, with m the atomic mass, a and A constants, Δ the roton energy gap, and k_B Boltzmann's constant. The first term in Eq. (11) is that due to phonons, and the second arises from interactions with rotons. Using Eq. (8) we evaluated k for the values of ξ determined from the transmission data. For our temperature region the effects due to phonons are less significant than those due to rotons, so we fit our values of k to just the roton term:

$$(Km)^{-1} = A\exp(-7.8/T) \ ,$$

where $\Delta = 7.8$ K was chosen for comparison with other results.[17] We obtain $A = 5.3(\pm 0.4) \times 10^5$ cm/s which is to be compared with the results of Castaing *et al.*[5,17] of $A = 4.6(\pm 0.9) \times 10^5$ cm/s for their ultrasonic experiment. Finally, we note that Keshishev *et al.*[3,17] made measurements of the growth coefficient in crystallization wave experiments and got a best fit with $\Delta = 7.8$ K and $A = 3.3 \times 10^5$ cm/s. It should be noted that the value of the roton energy gap from neutron scattering experiments is approximately 7.2 K for these temperatures and pressures.[15] [Note that if we take $\Delta = 7.2$ K in order to agree with the neutron data we obtain $A = 3.3(\pm 0.2) \times 10^5$ cm/s.] It is clear that fitting an exponential over such a small range in temperature is difficult at best and these results should be considered with caution.

We have presented an extension of the theory of Castaing and Nozières which includes a contribution to the pressure field due to second sound. The extended theory results in satisfactory agreement with our data for the reflection and transmission coefficients as well as the relative acoustic energy loss. The agreement shows that this formulation is an alternative to our previous interpretation[10] of the experimentally observed failure of the relation $1 + R = \tau$. (We note also that our data, together with the extended theory, imply that the reflection coefficient R is negative at low temperatures and becomes positive as the temperature is increased.) Finally, the values of the growth coefficient k, determined from our data, agree with those of previous studies.

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