Investigation of the Supercritical Bifurcation in a Simple Magneto-Mechanical System

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Abstract

At the point of supercritical bifurcation, a system with one stable state diverges into two separate stable states, with the original state no longer stable. The experiment reported here illustrates the supercritical bifurcation using a spring steel strip with a vertically applied load as the mechanism to be subjected to supercritical bifurcation. Increasing the load causes the strip to buckle to one side or another. We introduce a novel method for laterally perturbing the strip and creating an imperfection in the bifurcation. Our results show that beyond the point of bifurcation the system exhibits hysteresis and lays the groundwork for future studies.

Introduction

The buckling of a beam introduces the concept of bifurcation. When a heavy load is added to the top of a vertical beam, in our case a strip of spring steel, it may become unstable and buckle under the weight. This point of “bending” or “buckling” is known as a bifurcation. In analyzing the bifurcation of a beam, we find that after a certain critical load has been reached, the beam is no longer stable and will bifurcate either to the left or to the right, resulting in the system adopting one of two possible stable states. This “branching” of the stable states of the beam characterizes what is known as a supercritical pitchfork bifurcation, where the vertical state is no longer stable, but states of being bent to the left or to the right are both stable [1]. In graphical form, the supercritical pitchfork bifurcation appears as demonstrated in Figure 1, showing that as you increase the load, the stable state branches into two separate stable states at angles of “deflection” to either side of the previously stable state, which is no longer stable once bifurcation has occurred.

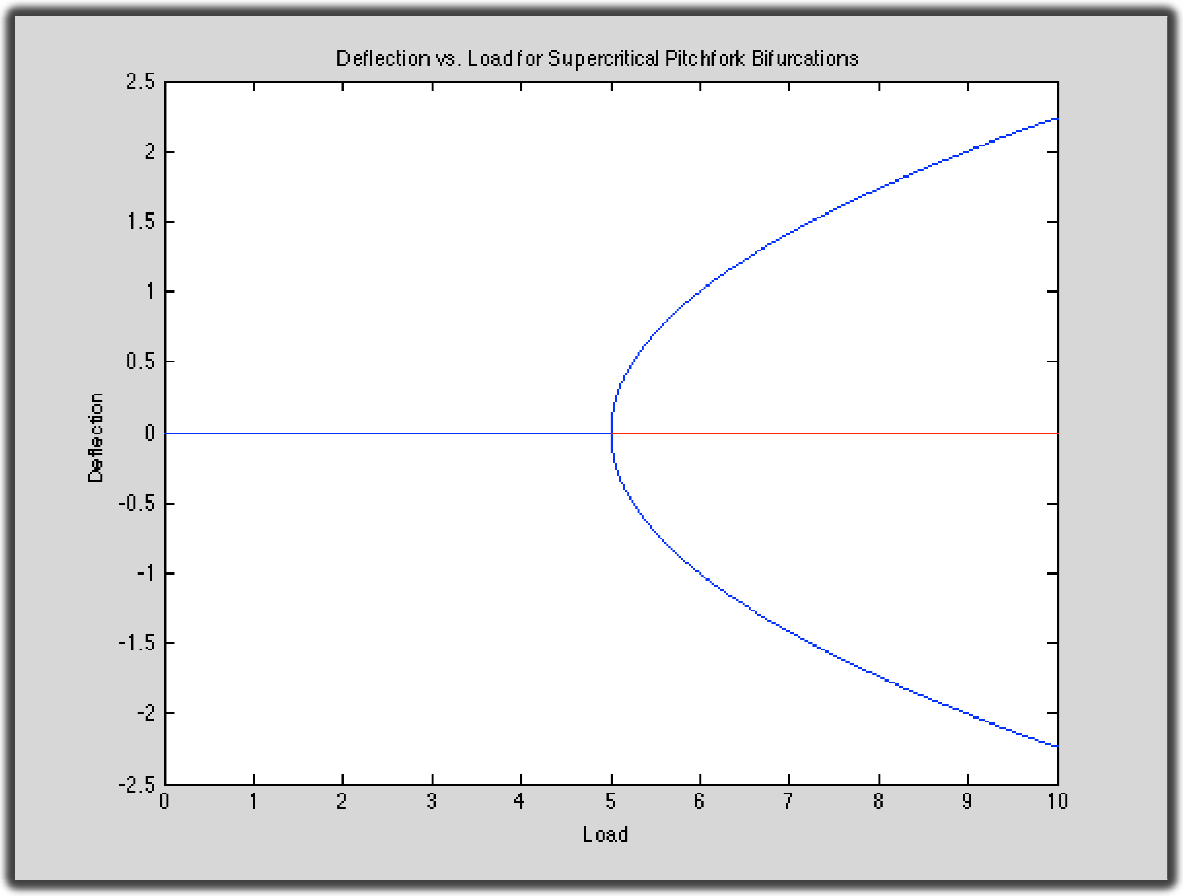

Figure 1. Branching of the stable states in a supercritical pitchfork bifurcation. The blue lines represent that the system is stable; the red line indicates that the system is unstable.

Furthermore, a distinction is made between perfect and imperfect supercritical bifurcations. If a bifurcation is "perfect", the system has an equal probability of bifurcating to either state as the load is increased from below the point of bifurcation. In the case of the metal strip, it has an equal chance of bifurcating to the left or to the right if the bifurcation is perfect. A bifurcation is "imperfect" if something causes it to favor bifurcating in one direction. For the metal strip example, a small lateral displacement of the load can cause the bifurcation to favor one side of the other, creating an imperfect bifurcation [1].

In the past, experiments have been conducted to demonstrate and analyze the nonlinear behavior of such supercritical pitchfork bifurcations. One such experiment employed an inverted pendulum as the mechanism of movement, whose angular motion was measurable by the strip's angular displacement [2]. To cause the strip to bend or bifurcate, weight was added to the top of the strip. Furthermore, to create an imperfect supercritical bifurcation, the load was given a small lateral displacement. By measuring the angular displacement as a function of the load added to the strip at given lateral displacements of the load, the strip's nonlinear behavior could be measured.

In this paper, we describe a conceptually similar experiment in which we use the magnetic field produced by a Helmholtz coil to provide the lateral force on the strip and control the imperfect supercritical pitchfork bifurcation. By providing this imperfection or favoring of one side or the other, we investigate the nonlinear behavior of this magneto-mechanical system. Using a magnetic field to produce the lateral force, and thus torque on the strip, has the advantages of remote control as well as a well-controlled force.

Experimental Design

Our experimental setup, presented in Figures 2 and 3, is designed to measure the buckling of a twelve-inch strip of spring steel. The spring steel strip was 13mm wide and 1mm thick. The spring steel strip was prepared by manipulating and bending a commercially available steel strip to make it as straight as possible prior to usage for experimentation. The ultimate test of the strip's "straightness" was to place it on the surface of an optics table in a dark room and shine a light from behind the strip. We molded the strip until we observed no light emanating from between the strip and the table surface.

The strip was mounted by clamping it to a base, as shown in the overall experimental setup in Figure 2. This clamping base has the purpose of holding the bottom of the strip firmly in place.

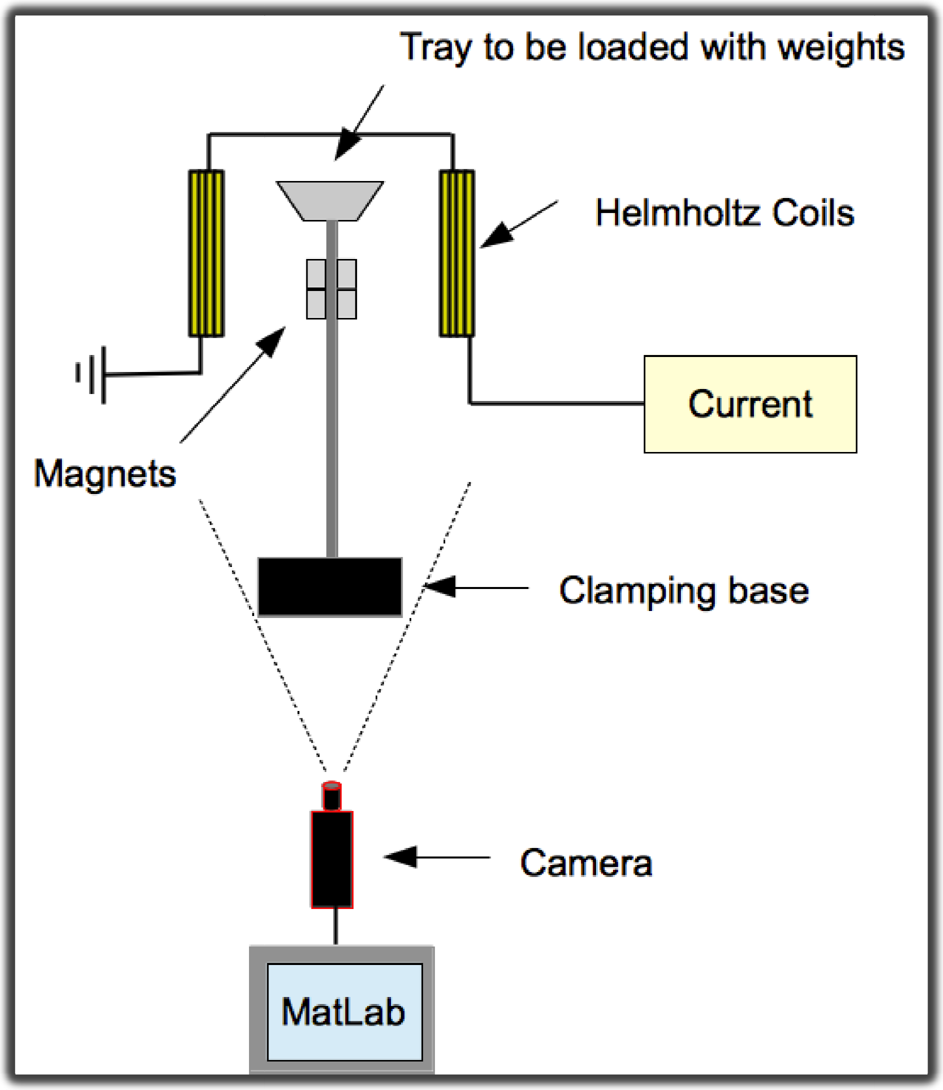

Figure 2. Overall experimental design, demonstrating apparatus used to buckle a metal strip and measure its angle of bifurcation.

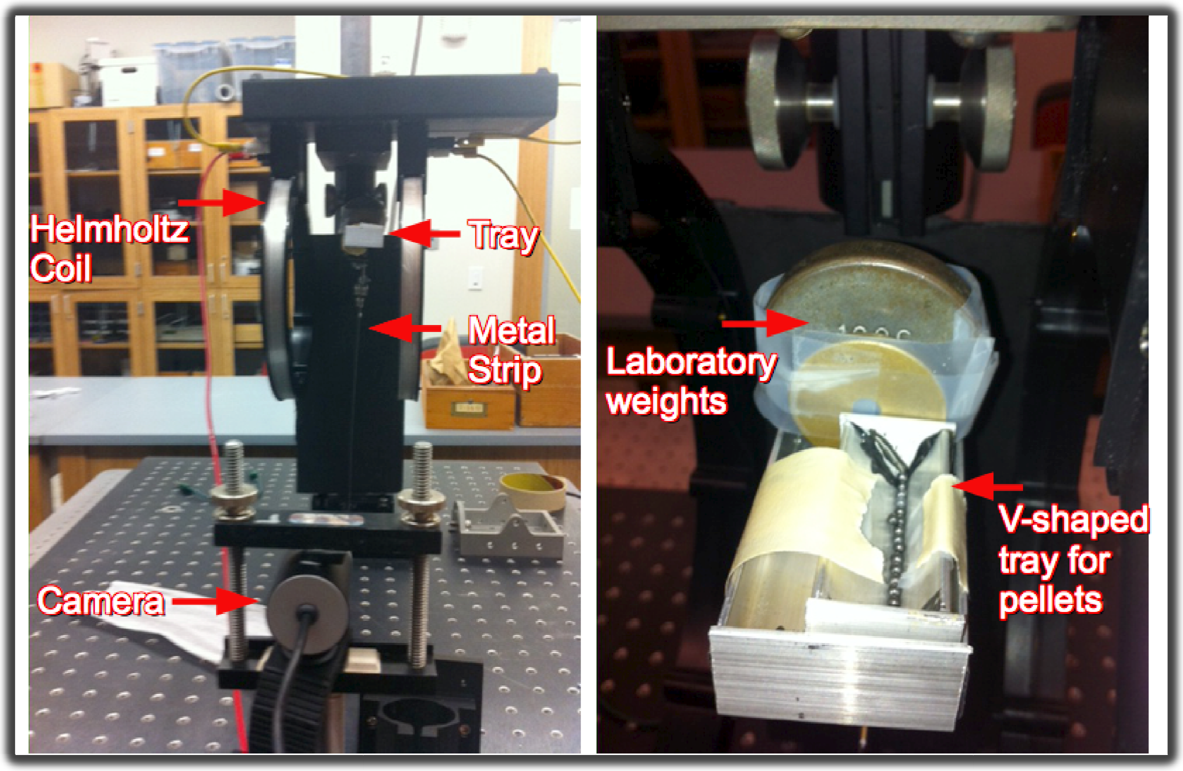

Figure 3. Picture of overall experimental design (left) with important parts labeled. Close-up picture of the tray (right) to see arrangements of weights loaded to the top of the strip.

To be able to load the system, we added a tray to the top of the strip. Inside this tray was another V-shaped tray, as shown in Figures 3 and 4. This tray design allowed us to load the strip until it was close to the point of bifurcation with heavy weight in the main part of the tray and then add smaller weights to adjust it very slightly. For the heavier weights, we used brass laboratory weights of differing but known weights. For the lighter weights, we used spherical lead pellets, which we weighed manually, determining the weight of five lead pellets to be 0.370   0.009g. The range of weights that we used began with 180g in laboratory weights and reached a maximum of 190g in laboratory weights with 100 pellets in the heaviest experimental trial.

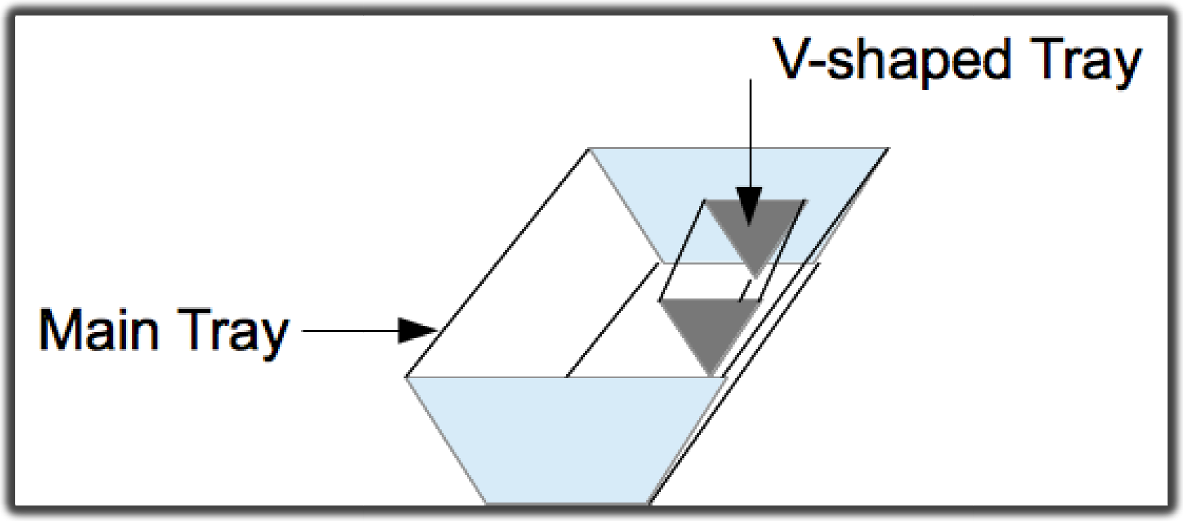

Figure 4. Tray design, showing the main tray to be loaded with steel plates and the V-shaped tray, set inside the main tray, to be loaded with lead pellets.

We attached a small translation stage to the top of the metal strip and epoxied the large tray to the top of the stage, as illustrated in Figure 5, to make fine adjustments in the point where the load was applied to the top of the strip. The translation stage allowed us to move the tray very slightly to center the load on top of the strip, in an attempt to make the bifurcations due solely to the weight of the load and reducing any torque due to off-center load application.

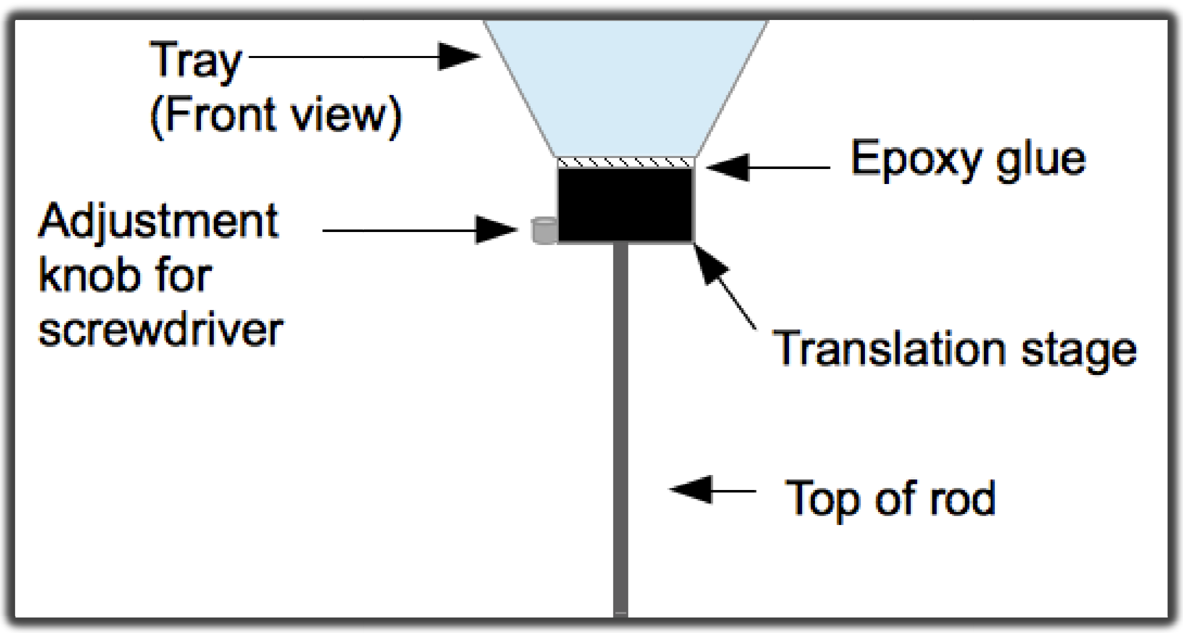

Figure 5. By adjusting the knob shown in the diagram with a screwdriver, the translation stage causes the tray to move left and right, thus adjusting for any uneven weight distributions within the tray that may cause torque on the strip.

To move the strip and thus analyze its dynamical behavior, we used Helmholtz coils to produce a uniform magnetic field around the top of the strip. The Helmholtz coils each had a 500 turn field coil with a radius of 10cm, set to a coil separation of the same length as the radius, the ideal separation to produce a uniform magnetic field between the coils. We placed six magnets near the top of the strip to interact with this magnetic field. This design is shown in Figure 6. The magnets, which stuck directly to the steel strip, were neodymium disk magnets (called Super Magnets), cylindrical in shape with dimensions 0.47" in diameter and 0.11" in height. By changing the current that goes through the Helmholtz coil, we controlled the magnetic field and therefore the force acting on the strip. To generate the current, we used a power supply that produced a current ranging from -0.75 A to 0.75 A. The change in force acting on the strip subsequently caused it to bend or "buckle" one way or another, depending on the direction of the current through the Helmholtz coil.

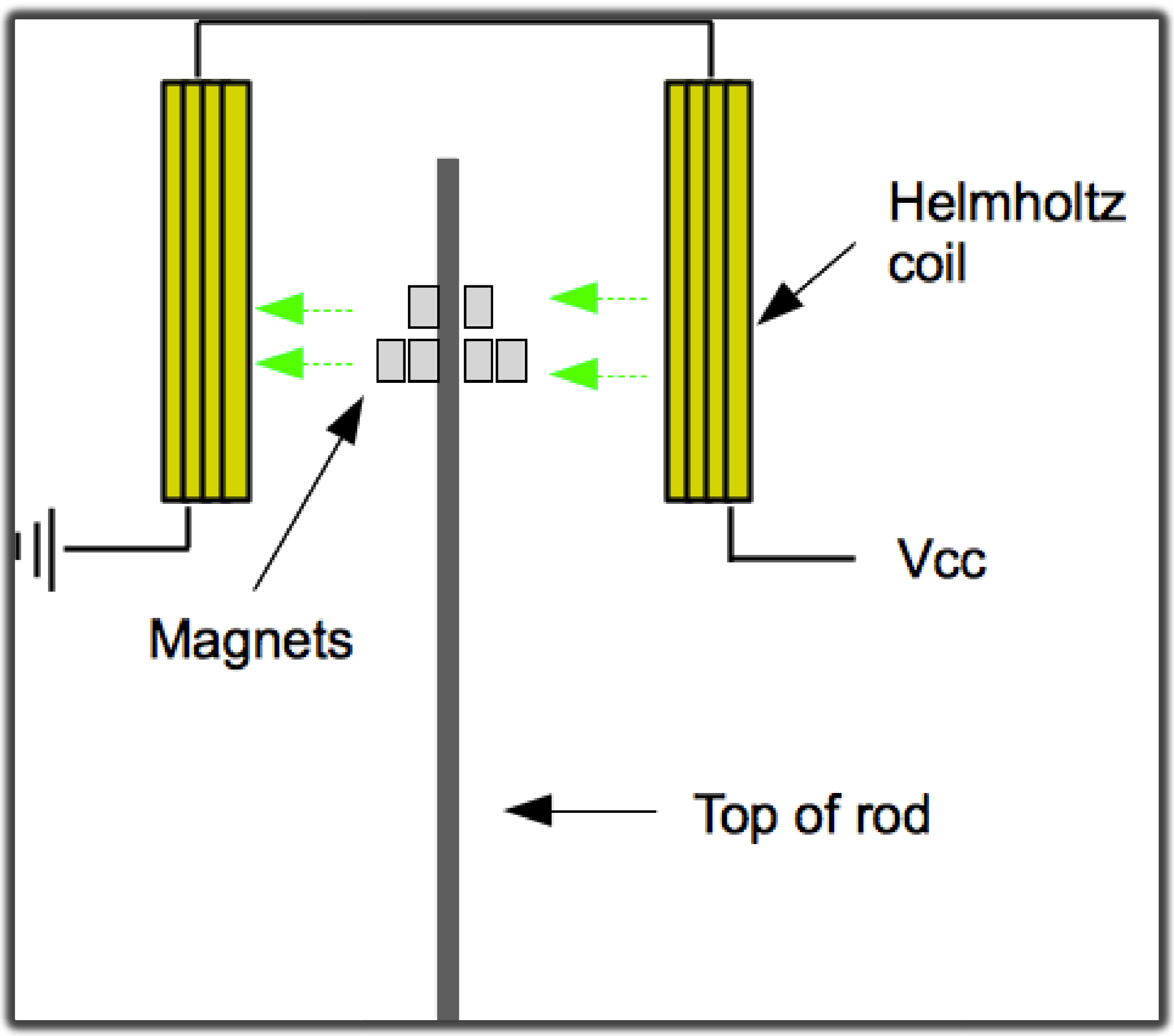

Figure 6. Interaction of the Helmholtz coil with magnet fields, shown by the green arrows, to push the strip to the left. By reversing the current through the coils, the interaction would push the strip to the right.

We took pictures of the buckled strip to accurately measure the angle at which the strip is buckled for a given current (and thus a given magnetic field and force on the strip). We used a simple WebCam (Microsoft Lifecam Cinema) with 640 pixels in the horizontal dimension and 480 pixels in the vertical dimension with images acquired directly into MATLAB. The programs used to acquire and analyze the images are contained in the appendix to this report. In Figure 7, we provide examples of pictures of the strip being buckled to the left (7a) and to the right (7b). Using the camera to capture the buckling of the strip allows a consistent, objective measurement of the angle of bifurcation. Care was taken to ensure that after each adjustment of force, images were only acquired once the system had settled.

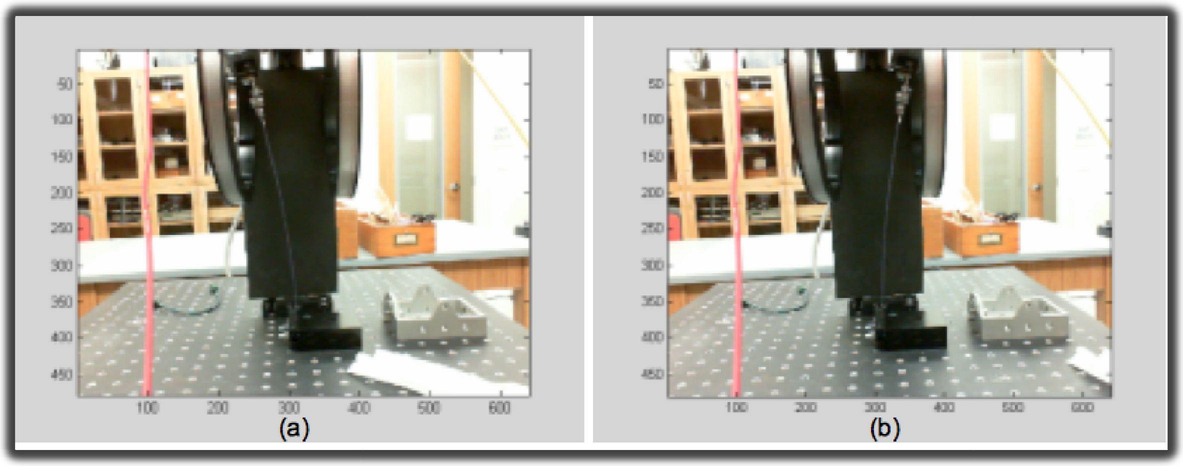

Figure 7. View of the apparatus from the perspective of the camera, showing its capture of a leftward deflection in (a) and a rightward deflection in (b).

In Figure 8, we illustrate our methodology in determining the angle of deflection. Essentially, the MATLAB program first finds the location of the brightest point in the picture taken by the camera. We placed a white dot (using Whiteout) near the top of the strip (just below the magnets) and placed a black cardboard background behind the strip to ensure that the camera would not pick up any bright regions behind the apparatus. The MATLAB code then compares the pixel location of the white dot to the known pixel location of the base of the strip and computes the angle, , as depicted in Figure 8. We call this angle the angle of deflection at which the strip is buckled, with the arbitrary labeling of rightward buckling as a positive angle of deflection (as in Figure 7b) and leftward buckling as a negative angle of deflection (as in Figure 7a).

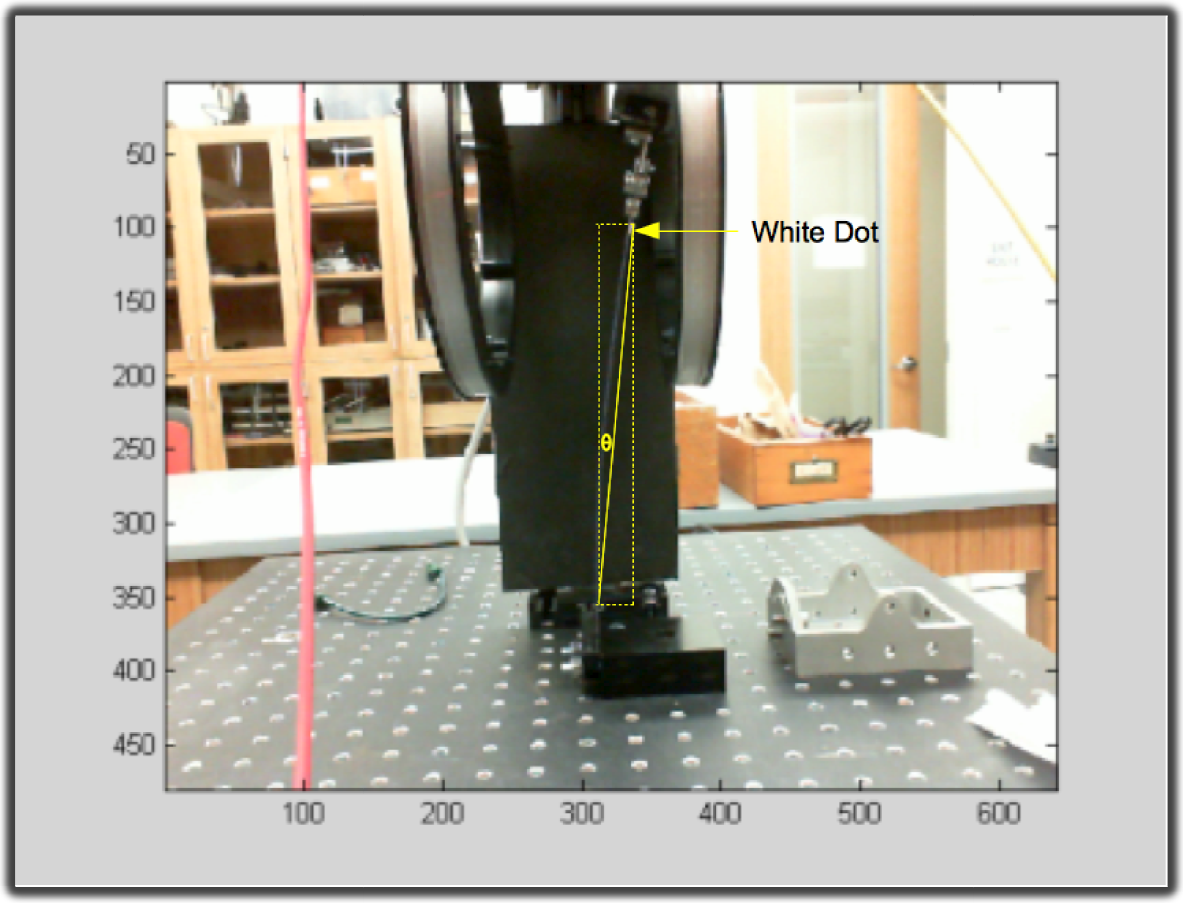

Figure 8. Definition of the angle, , calculated by a comparison of the pixel location of the white dot near the top of the strip and a known pixel location of the base of the strip.

Analysis

Since the key parameter in working with the imperfection of the system is the transverse force acting on the strip, we had to find the relationship between the current in the coils and the force on the magnets. To do this we turned the Helmholtz coil apparatus onto its side, making the axis of the coils vertical. We then constructed a setup from which we supported a laboratory "Scout" weight scale, with a hook on the bottom; any downward or "pulling" force applied to this hook was registered by the scale. From this hook, we suspended the strip using string, positioning the strip horizontally, halfway between the coils. This arrangement in demonstrated with the picture presented in Figure 9.

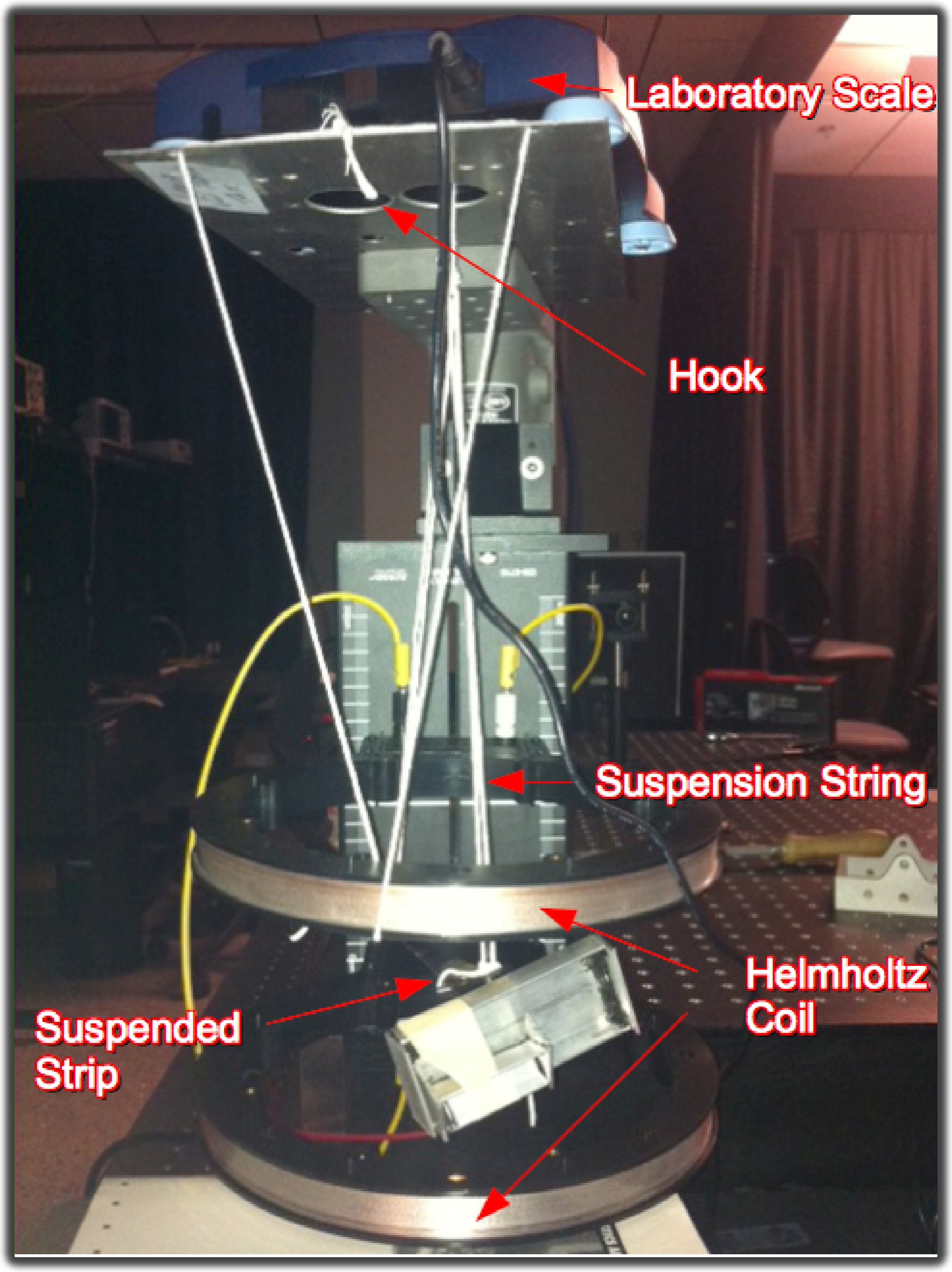

Figure 9. Experimental arrangement used to determine the relationship between the force and current of the Helmholtz coil by measuring the force acting on the suspended strip for varying currents.

With the strip suspended by the suspension strings, we zeroed the scale so that the weight due to gravity of the strip would not be registered. Thus, any additional force on the strip as indicated by the scale, would be the force applied to the strip by the Helmholtz coil. Note that we kept the magnets, translation stage, and anything else that might be at all magnetic on the strip. The point of retaining these aspects of the strip is to recreate the

interaction between the Helmholtz coil and the strip during the experimental trials as closely as possible.

Next, we applied various currents through the Helmholtz coil using the power supply. This generated a series of data points for the force, in Newtons, as registered by the scale at each current in amperes. We provide these results in Table 1 and plot the results in Figure 10. Notably, the force registered by the scale did not change much once the strip had settled under the new magnetic force each time that the current was changed. As a result, there are very small error bars – corresponding to a weight of 0.03g; this error represents the maximum that the registered weight ever changed after the strip had settled, whether due to air current currents in the room, fluctuations in the magnetic force, inaccuracies deriving from the internal mechanisms of the scale, or any other conceivable source of error.

Current (A)	Mass (g)	Force (N)
0.000	0.00 ± 0.04	0.0000 ± 0.0002940
0.100	0.28 ± 0.04	0.0027 ± 0.0002940
0.200	0.57 ± 0.04	0.0056 ± 0.0002940
0.300	0.85 ± 0.04	0.0083 ± 0.0002940
0.400	1.13 ± 0.04	0.0111 ± 0.0002940
0.500	1.42 ± 0.04	0.0139 ± 0.0002940
0.600	1.71 ± 0.04	0.0168 ± 0.0002940
0.700	2.00 ± 0.04	0.0196 ± 0.0002940

Table 1. Force acting on the rod suspended in the Helmholtz coil for various currents.

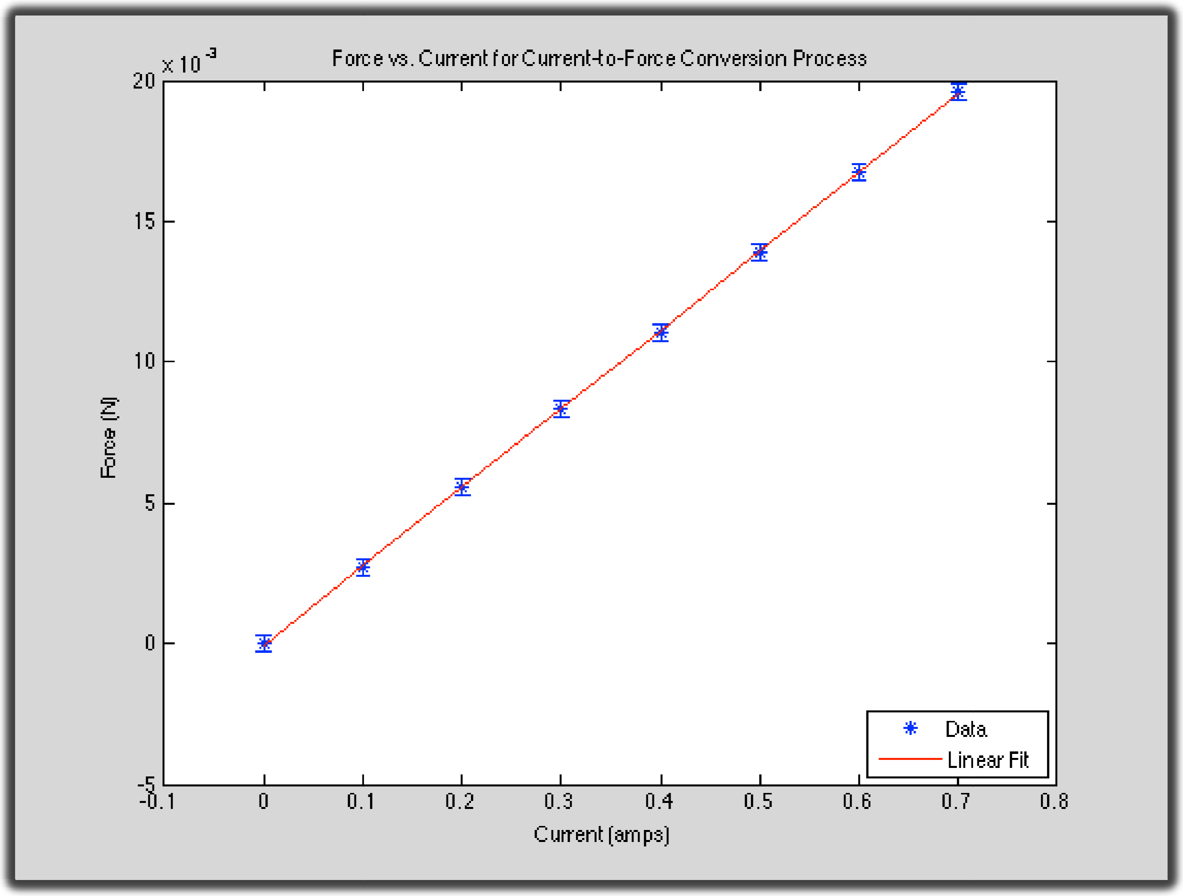

Figure 10. Results from experiment devised to determine the relationship between the force exerted by the Helmholtz coil on the strip and the current through the Helmholtz coil.

As is evident from the plot displayed in Figure 10, the relationship is very linear. The linear fit that we applied generated a conversion formula that allowed us to convert the current through the Helmholtz coil to the force applied on the strip. This equation is given by,

$$\text{Force} = 0.0280 \quad \text{Current}, \tag{1}$$

where current is measured in amperes and force is measured in Newtons. This conversion formula is simply the formula for the best fit line that is plotted in Figure 10. We used this formula to convert the currents that we applied to the Helmholtz coil to the force that it exerted on the strip.

Going back to our original experiment with the loaded strip, we measured the angle of deflection for varying lateral forces for four essential cases. First, we conducted the experiment for a “far below bifurcation” case, in which the tray was loaded with 180g of laboratory weights and 20 pellets. Second, we implemented a “just below bifurcation” case, in which the tray was loaded with 190g of laboratory weights and 20 pellets. Third, we conducted a “just above bifurcation” trial, in which the tray was loaded with 190g of

laboratory weights and 60 pellets. Fourth and finally, we repeated the experiment for a "far above bifurcation" case, with the tray loaded with 190g of laboratory weights and 100 pellets.

In each of our four experimental trials, we began with a strong, positive current, where "positive" current is defined as current that produces a magnetic force acting on the strip to the right and "negative" current generates a force pushing the strip to the left. Analogously, positive angles of deflection are defined here as angles to the right, while negative angles of deflection are angles to the left, and a zero degree angle of deflection represents the state of the strip standing vertically upright. After beginning with a strong positive current and positive angle of deflection, we decreased the current in intervals of 0.02A until the strip was completely deflected to the left. Next, we began increasing the current at the same intervals until the strip was again deflected to the right, as it was in its initial state. For all cases, we present our data in plots with blue stars representing data points taken while we were in the phase of decreasing the current and red circles representing data points taken during the increasing current phase of the experiment.

For our initial experimental trial, we implemented the "far below bifurcation" case, with results plotted in Figure 11. In this case, the strip easily stood vertically upright, demonstrating that it was not near the point of bifurcation. As we conducted the experiment, we found that the angle of deflection changed gradually and came in small, consistent intervals, roughly linear with the force applied to the strip. Furthermore, there seemed to be no "jumps" in the angle of deflection from the right-leaning state to the left-leaning state or vice versa; as we decreased the current, the strip gradually shifted from leaning to the right to leaning to the left. Similarly, as we increased the current, the strip gradually shifted from the negative angle of deflection state back to its original positive angle of deflection state. For comparison to our results for later trials, note the fairly small angles of deflection. Furthermore, note the fact that the paths taken under increasing and decreasing current are not the same. This is indicative of some small "imperfection" in the system, favoring one side or the other.

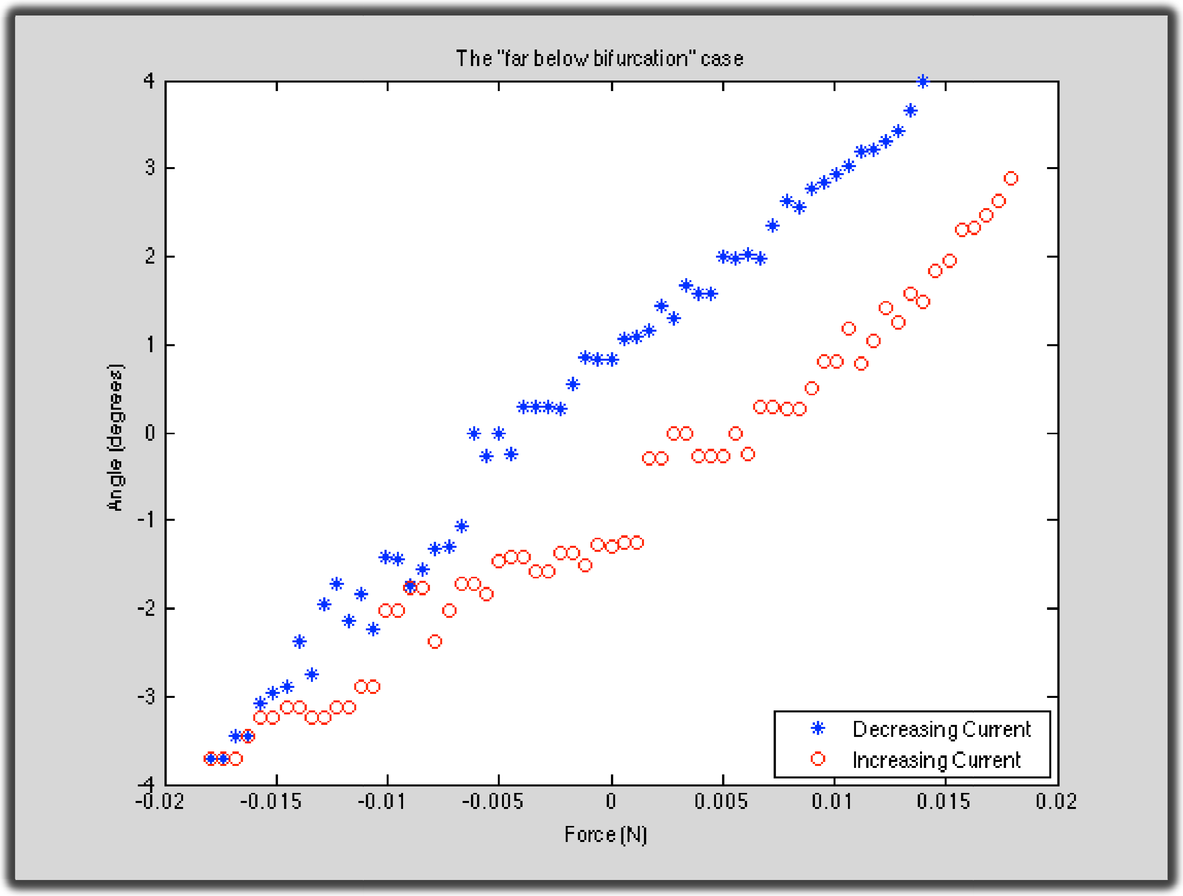

Figure 11. Data plotting the angle of deflection versus current for the "far below bifurcation" case.

Second, we conducted the experimental trial for the "just below bifurcation" state. Prior to introducing current through the Helmholtz coils, we aligned the translation stage so that the strip was standing vertically; although we did manage to make it stand upright, this state was barely stable; only a small shift in the translation stage would cause the beam to come to rest at one side or the other, indicating that it was very near the point of bifurcation. From conducting this experiment, we found that the angle of deflection was not as linear with force as the previous trial, as seen in our results plotted in Figure 12. This trial was characterized by several small jumps in the angle of deflection and an overall nonlinear dependence of the angle on the applied force.

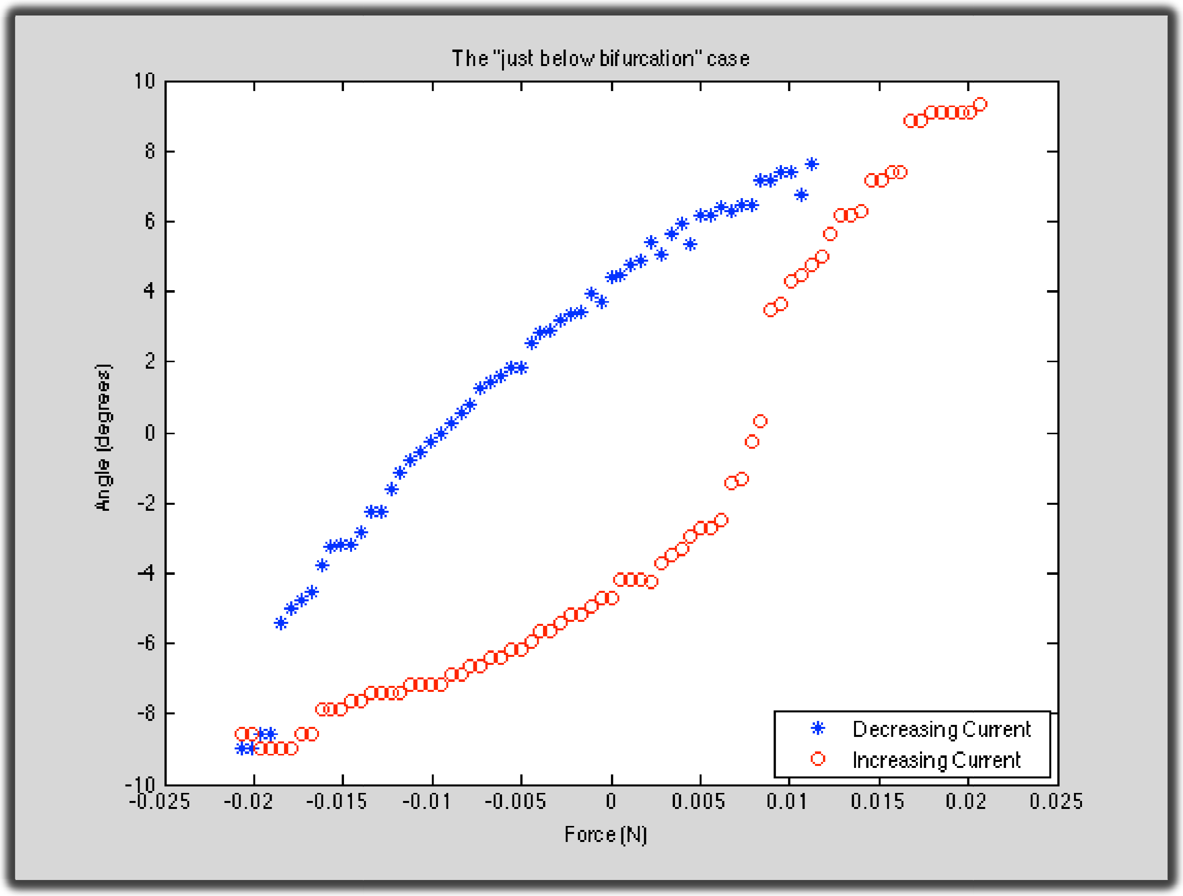

Figure 12. Data plotting the angle of deflection versus current for the "just below bifurcation" case.

For the case of "just above bifurcation", the strip was just past the point of being stable in the center; the translation stage could not be adjusted so as to make the strip stand straight up, suggesting that the strip was past the point of bifurcation. As such, it was necessary to begin this trial with the strip in one of the two stable states, i.e. either stably deflected with a positive or negative angle of deflection – but not an angle of deflection of zero as in the previous experimental trials. Starting with the strip buckled to the right, we applied a high, positive current of 0.36A to the system, then gradually decreased the current as we had in the previously trials until it was deflected to the left. We then repeated the process, increasing the current in the same intervals until it was bifurcated with a positive angle of deflection again. The results, presented in Figure 13, demonstrate the highly nonlinear relationship of the angle of deflection and the applied force. When the strip was deflected to either side, the gradual application of force in the opposite direction did not cause much of a shift in the angle of deflection until it reached a certain point at which two or three more increments of current caused the strip to shift from one stable state to the other stable state of the pitchfork bifurcation.

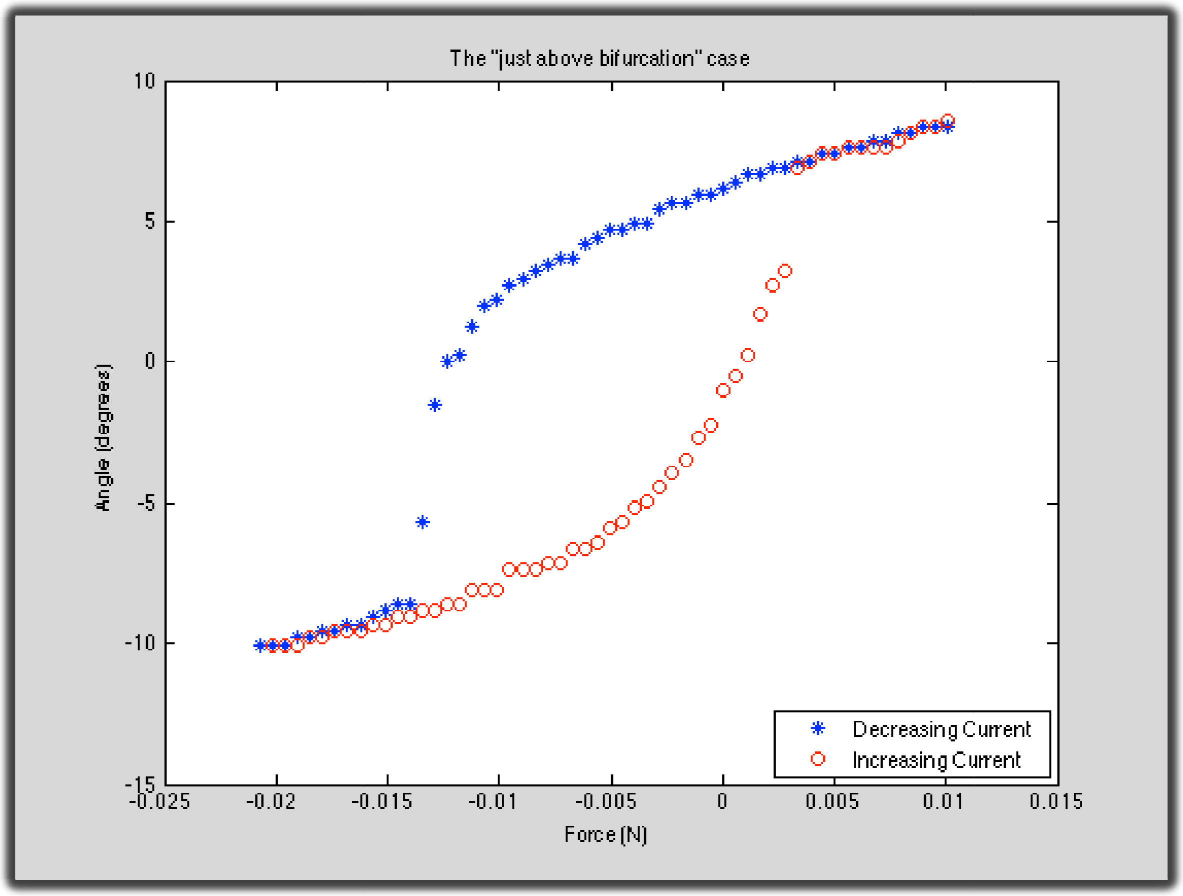

Figure 13. Data plotting the angle of deflection versus current for the "just above bifurcation" case.

Fourth and lastly, we conducted the experimental trial for the "far above bifurcation" case, with data presented in Figure 14 below. This case exemplifies the extreme case of the "jumps" seen in the data from the previous two experimental trials. While fully bifurcated to one side, it takes a fairly high amount of force to cause even small changes in the angle of deflection. However, at a certain point, only one or two more intervals in the force cause the strip to bifurcate completely to the other side. That is, the strip "jumps" from one stable state in the pitchfork bifurcation plot to the other stable state with only a small change in the applied force. This effect is graphically evident in the flat, slowly sloping regions of the plot in which we applied force to no effect, but within one or two intervals of applied force, the angle of deflection exhibited large jumps from positive to negative and vice versa.

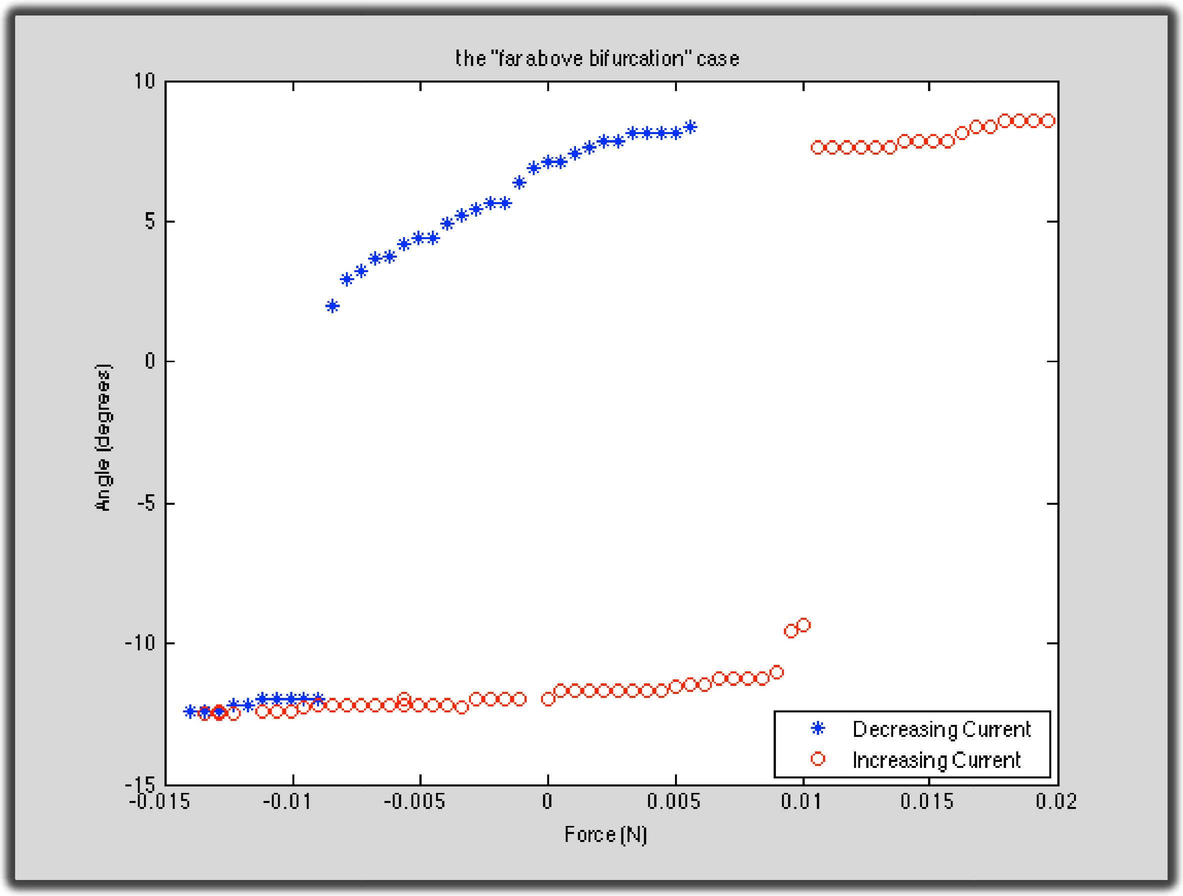

Figure 14. Data plotting the angle of deflection versus the current for the "far above bifurcation" case.

Conclusion

Our experimental apparatus was successful in providing a mechanism with one stable state at loads below bifurcation and two stable states at loads above the point of bifurcation. From the progression of our experimental trials, we found an increasingly nonlinear relationship between the force and the angle of deflection of the strip. This nonlinear relationship ties back to the concept of the supercritical pitchfork bifurcation, in which after a certain load has been added to the system, its previously singular stable state divides into two separate stable states. As we continually increased the vertically applied load to our spring steel strip, we found increasingly drastic jumps as we applied the imperfection to push the system between stable states.

At relatively light loads, i.e. in the "far below bifurcation" and "just below bifurcation" cases, we found that the system exhibited roughly linear behavior. That is, as we increased or decreased the vertically applied load, we found that the angle of deflection changed accordingly. In the latter two cases with heavier loads, i.e. the "just above bifurcation" and "far above bifurcation" cases, we found that the system behaved nonlinearly. This is due to the division of the stable states, a division that increases with load as seen in the model of the pitchfork bifurcation provided in the introduction. As the

point of bifurcation is approached, the system begins to behave nonlinearly. After the system has been loaded to the point of bifurcation and beyond, it behaves completely nonlinearly in accordance with the diverging stable states of the supercritical pitchfork bifurcation model. Furthermore, note the relative increase in the angles of deflection present in the trials with heavier loads as compared to the trials with lighter loads, again suggesting that the divergence of the stable states increases with load.

From our analysis of our results, we found a strong hysteresis in our system. Depending on the initial state, the amount of force required to buckle the strip to the opposite side increased with load. In the case of the heaviest load, the large jump in the angle of deflection is indicative of the significant amount of force required to shift the state from its previously buckled state to the opposite state. Furthermore, even after shifting it from its initial state (buckled to the right) to the opposite stable state (buckled to the left), it required as much in the positive (rightward) direction to push the system back to its original, rightward state. This dependence of the system on its previous position or the system's "past" is characteristic of the phenomenon of hysteresis, in which a system's behavior is governed by its past as well as its present environment.

Another noteworthy aspect in our analysis of our results is the fact that the angle of deflection at zero lateral force is nonzero, even in the "just below bifurcation" case. In this case, we might expect that since the system has not yet reached the point of bifurcation, it should be pushed back to its stable, vertical state with no application of magnetic force. However, the experiment demonstrates that as the point of bifurcation is approached, some residual imperfection from the previous deflection of the strip has an effect on the strip's current behavior. It is a subtle point that the effects of this residual imperfection must still be altering the strip's behavior, despite not having reached the load corresponding to the point of bifurcation.

Lastly, from inspection of the plots for each of the four experimental trials, it is clear that there is an asymmetry in the system, i.e. a favoring of one side of the other. We found this especially evident in our observation of the "just past bifurcation" case, in which the point of zero force is clearly unaligned with the center of our data points. This lack of alignment indicates that the system requires more force to move from one of the two stable states to the other. This asymmetry likely derives from an intrinsic asymmetry in the strip itself, an aspect that is very difficult to eliminate, but ultimately does not prevent our ability to use a controllable imperfection to shift the strip from one stable state to the other and experimentally observe the system's behavior.

References

1. Steven H. Strogatz, *Nonlinear Dynamics and Chaos.* (Addison-Wesley Publishing Company, NY, 1994).

2. Sharpe, J. P. and N. Sungar. "Supercritical bifurcation in a simple mechanical system: An undergraduate experiment". Am. J. Phys. 78, 520 (2010).

Appendix A: Matrix Laboratory Codes

Script 1: Data Measurement

```
clear all; close all; clc
imaqhwinfo

 obj=videoinput('winvideo');
 %preview(obj) %Must close preview before running or you will get an
error

 frame = getsnapshot(obj);
 colormap(gray)
 image(frame);

 for i=1:10
     frame = getsnapshot(obj);
     f=frame(:,:,1)+frame(:,:,2)+frame(:,:,3);
     imagesc(frame); drawnow
 end

  full_hsv = rgb2hsv(frame); %converts picture variable from rgb to hsv
  hsv = full_hsv(103:150,260:350,:); %Takes only the pixels in front of
the black cardboard
  v = hsv(:,:,3); %Takes only the illumination values at each pixel
  [val, loc] = max(v(:)); %Finds the location and value of brightest
pixel
  [small_R,small_C] = ind2sub(size(v),loc); %Finds index within v of
brightest pixel
  R = small_R + 102; %Converts row-value to row within entire picture
  C = small_C + 259; %Converts column-value to column within entire
picture
  d = 48; %distance from camera to apparatus in inches
  xhalfang = 0.569; %half the angular spread of the camera in the x
direction
  totx = 2*d*tan(xhalfang); %total distance in x direction
  xdppix = totx/640; %x distance per pixel
  startx = 312; %starting x pixel of the dot
  xpixdist = abs(startx - C); %number of pixels the dot has moved in x
direction
  xdist = xpixdist*xdppix; %actual x distance the dot has moved
  %Now for y dimension:
  yhalfang = 0.367; %half angular spread in y direction
  toty = 2*d*tan(yhalfang); %total distance in y direction
  ydppix = toty/480; %y distance per pixel
  starty = 392; %starting y pixel of the base
  ypixdist = starty - R; %number of y-direction pixels between base and
dot
  ydist = ypixdist*ydppix; %y distance between base and dot
  radians = atan(xdist/ydist); %angle of bifurcation
  degrees = radians*(180/pi); %angle of bifurcation in degrees
```

Script 2: Data Collection and Display

```
%180g20pellets - the "far below bifurcation" case
current1 = [0.5000 0.4800 0.4600 0.4400 0.4200 0.4000 0.3800 0.3600
0.3400 0.3200 0.3000 0.2800 0.2600 0.2400 0.2200 0.2000 0.1800 0.1600
0.1400 0.1200 0.1000 0.0800 0.0600 0.0400 0.0200 0.0000 -0.020 -0.040 -
0.060 -0.080 -0.100 -0.120 -0.140 -0.1600 -0.1800 -0.2000 -0.2200 -
0.2400 -0.2600 -0.2800 -0.3000 -0.3200 -0.3400 -0.3600 -0.3800 -0.4000 -
0.4200 -0.4400 -0.4600 -0.4800 -0.5000 -0.5200 -0.5400 -0.5600 -0.5800 -
0.6000 -0.6200 -0.6400];
angle1   = [3.9812 3.6541 3.4255 3.3156 3.2126 3.1860 3.0394 2.9275
2.8460 2.7689 2.5516 2.6352 2.3427 1.9851 2.0171 1.9695 1.9930 1.5767
1.5767 1.6751 1.2903 1.4469 1.1671 1.0914 1.0551 0.8217 0.8217 0.8477
0.5563 0.2638 0.2837 0.2942 0.2942 -0.2553 0.00000 -0.2628 0.00000 -
1.0709 -1.3092 -1.3188 -1.5482 -1.7493 -1.4411 -1.4071 -2.2253 -1.8460 -
2.1411 -1.7291 -1.9465 -2.7476 -2.3642 -2.8782 -2.9659 -3.0830 -3.4591 -
3.4591 -3.7055 -3.7055];
current2 = [-0.6400 -0.6200 -0.6000 -0.5800 -0.5600 -0.5400 -0.5200 -
0.5000 -0.4800 -0.4600 -0.4400 -0.4200 -0.4000 -0.3800 -0.3600 -0.3400 -
0.3200 -0.3000 -0.2800 -0.2600 -0.2400 -0.2200 -0.2000 -0.1800 -0.1600 -
0.1400 -0.1200 -0.1000 -0.0800 -0.0600 -0.0400 -0.0200 -0.0000 0.02000
0.04000 0.06000 0.08000 0.1000 0.1200 0.14000 0.16000 0.18000 0.2000
0.22000 0.2400 0.2600 0.2800 0.3000 0.3200 0.3400 0.3600 0.3800 0.4000
0.4200 0.4400 0.4600 0.4800 0.5000 0.5200 0.5400 0.5600 0.5800 0.6000
0.6200 0.6400];
angle2   = [-3.7055 -3.7055 -3.7055 -3.4591 -3.2462 -3.2462 -3.1300 -
3.1300 -3.2462 -3.2462 -3.1300 -3.1300 -2.8888 -2.8888 -2.0171 -2.0171 -
1.7720 -1.7720 -2.3729 -2.0171 -1.7085 -1.7085 -1.8460 -1.4707 -1.4126 -
1.4182 -1.5825 -1.5767 -1.3641 -1.3589 -1.5154 -1.2764 -1.2903 -1.2629 -
1.2497 -0.2930 -0.2837 0.0000 0.0000 -0.2739 -0.2638 -0.2619 0.0000 -
0.2526 0.2837 0.2815 0.2619 0.2628 0.5088 0.8186 0.8186 1.1719 0.7914
1.0512 1.4071 1.2453 1.5767 1.4943 1.8393 1.9618 2.3143 2.3332 2.4638
2.6363 2.8888];
figure(1)
plot(current1,angle1,'b*')
hold on
plot(current2,angle2,'ro')
title('The "far below bifurcation" case')
xlabel('Current (amps)')
ylabel('Angle (degrees)')
legend('Decreasing Current','Increasing Current','Location','SouthEast')

%190g20pellets - the "just below bifucation" case
current3 = [0.4000 0.3800 0.3600 0.3400 0.3200 0.3000 0.2800 0.2600
0.2400 0.2200 0.2000 0.1800 0.1600 0.1400 0.1200 0.1000 0.0800 0.0600
0.0400 0.0200 0.0000 -0.020 -0.040 -0.060 -0.080 -0.100 -0.120 -0.140 -
0.160 -0.180 -0.200 -0.220 -0.240 -0.260 -0.280 -0.300 -0.320 -0.340 -
0.3600 -0.3800 -0.4000 -0.4200 -0.4400 -0.4600 -0.4800 -0.5000 -0.5200 -
0.5400 -0.5600 -0.5800 -0.6000 -0.6200 -0.6400 -0.6600 -0.6800 -0.7000 -
0.7200 -0.7400];
angle3   = [7.6235 6.7389 7.3808 7.3808 7.1614 7.1614 6.4717 6.4717
6.2829 6.4051 6.1819 6.1607 5.3754 5.9160 5.6712 5.0742 5.4261 4.8570
4.7387 4.4593 4.4439 3.7294 3.9655 3.4004 3.3394 3.2126 2.8994 2.8266
2.5336 1.8393 1.8393 1.5825 1.4239 1.2367 0.7715 0.5651 0.2638 0.0000 -
0.2848 -0.5674 -0.7914 -1.1347 -1.5825 -2.2253 -2.2253 -2.8266 -3.1625 -
3.1625 -3.2462 -3.7880 -4.5060 -4.7378 -4.9997 -5.4261 -8.5933 -8.5933 -
8.9533 -8.9533];
current4 = [-0.7400 -0.7200 -0.7000 -0.6800 -0.6600 -0.6400 -0.6200 -
0.6000 -0.5800 -0.5600 -0.5400 -0.5200 -0.5000 -0.4800 -0.4600 -0.4400 -
0.4200 -0.4000 -0.3800 -0.3600 -0.3400 -0.3200 -0.3000 -0.2800 -0.2600 -
0.2400 -0.2200 -0.2000 -0.1800 -0.1600 -0.1400 -0.1200 -0.1000 -0.0800 -
0.0600 -0.0400 -0.0200 0.00000 0.02000 0.04000 0.06000 0.08000 0.10000
0.12000 0.14000 0.16000 0.18000 0.20000 0.22000 0.24000 0.26000 0.28000
```

```
0.3000 0.3200 0.3400 0.3600 0.3800 0.4000 0.4200 0.4400 0.4600 0.4800
0.5000 0.5200 0.5400 0.5600 0.5800 0.6000 0.6200 0.6400 0.6600 0.6800
0.7000 0.7200 0.7400];
angle4   = [-8.5933 -8.5933 -8.9533 -8.9533 -8.9533 -8.9533 -8.5933 -
8.5933 -7.8664 -7.8664 -7.8664 -7.6235 -7.6235 -7.3803 -7.3803 -7.3803 -
7.3803 -7.1369 -7.1369 -7.1369 -7.1369 -6.8932 -6.8932 -6.6493 -6.6493 -
6.4051 -6.4051 -6.1819 -6.1819 -5.9569 -5.6712 -5.6712 -5.4261 -5.1987 -
5.1808 -4.9354 -4.6897 -4.6897 -4.1979 -4.1979 -4.1979 -4.2125 -3.7055 -
3.4591 -3.2805 -2.9659 -2.7191 -2.7191 -2.4723 -1.4587 -1.3188 -0.2826
0.2942 3.4953 3.6830 4.2866 4.4757 4.7929 5.0094 5.6712 6.1607 6.1483
6.3059 7.1369 7.1369 7.3803 7.3803 8.8350 8.8652 9.0764 9.0764 9.1074
9.0764 9.1074 9.3174];
figure(2)
plot(current3,angle3, 'b*')
hold on
plot(current4,angle4,'ro')
title('The "just below bifurcation" case')
xlabel('Current (amps)')
ylabel('Angle (degrees)')
legend('Decreasing Current','Increasing Current','Location','SouthEast')

%190g60pellets - the "just above bifurcation" case
current5 = [0.36 0.34 0.32 0.30 0.28 0.26 0.24 0.22 0.20 0.18 0.16 0.14
0.12 0.10 0.08 0.06 0.04 0.02 0 -0.02 -0.04 -0.06 -0.08 -0.10 -0.12 -
0.14 -0.16 -0.18 -0.20 -0.22 -0.24 -0.26 -0.28 -0.30 -0.32 -0.34 -0.36 -
0.38 -0.40 -0.42 -0.44 -0.46 -0.48 -0.50 -0.52 -0.54 -0.56 -0.58 -0.60 -
0.62 -0.64 -0.66 -0.68 -0.70 -0.72 -0.74];
angle5   = [8.3799 8.3799 8.3799 8.1090 8.1090 7.8664 7.8664 7.6235
7.6235 7.3803 7.3803 7.1369 7.1369 6.8932 6.8932 6.6493 6.6493 6.4051
6.1607 5.9160 5.9160 5.6712 5.6712 5.4261 4.9354 4.9354 4.6897 4.6897
4.4439 4.1979 3.7055 3.7055 3.4591 3.2126 2.9659 2.7191 2.2253 1.9783
1.2367 0.2474 0 -1.4839 -5.6712 -8.5933 -8.5933 -8.8350 -9.0764 -9.3174
-9.3174 -9.5582 -9.5582 -9.7986 -9.7986 -10.0386 -10.0386 -10.0727];
current6 = [-.72 -.70 -.68 -.66 -.64 -.62 -.60 -.58 -.56 -.54 -.52 -.50
-.48 -.46 -.44 -.42 -.40 -.38 -.36 -.34 -.32 -.30 -.28 -.26 -.24 -.22 -
.20 -.18 -.16 -.14 -.12 -.10 -.08 -.06 -.04 -.02 0 .02 .04 .06 .08 .10
.12 .14 .16 .18 .20 .22 .24 .26 .28 .30 .32 .34 .36];
angle6   = [-10.0386 -10.0386 -10.0386 -9.7986 -9.7986 -9.5582 -9.5582 -
9.5582 -9.3174 -9.3174 -9.0764 -9.0764 -8.8350 -8.8350 -8.5933 -8.5933 -
8.1090 -8.1090 -8.1090 -7.3803 -7.3803 -7.3803 -7.1369 -7.1369 -6.6493 -
6.6493 -6.4051 -5.9160 -5.6712 -5.1808 -4.9354 -4.4439 -3.9518 -3.4711 -
2.7191 -2.2253 -0.9894 -0.4947 0.2474 1.7311 2.7191 3.2126 6.8932 7.1369
7.3803 7.3803 7.6235 7.6235 7.6496 7.6496 7.8664 8.1090 8.3799 8.3799
8.5933];
figure(3)
plot(current5,angle5,'b*')
hold on
plot(current6,angle6,'ro')
title('The "just above bifurcation" case')
xlabel('Current (amps)')
ylabel('Angle (degrees)')
legend('Decreasing Current','Increasing Current','Location','SouthEast')

%190g100pellets - the "far above bifurcation" case
current7 = [0.2 0.18 0.16 0.14 0.12 0.10 0.08 0.06 0.04 0.02 0.0 -0.02 -
0.04 -0.06 -0.08 -0.10 -0.12 -0.14 -0.16 -0.18 -0.20 -0.22 -0.24 -0.26 -
0.28 -0.30 -0.32 -0.34 -0.36 -0.38 -0.40 -0.42 -0.44 -0.46 -0.48 -0.50];
angle7   = [8.3513 8.1090 8.1090 8.1090 8.1090 7.8664 7.8664 7.6235
7.3803 7.1369 7.1369 6.8932 6.4051 5.6712 5.6712 5.4261 5.1808 4.9524
4.4439 4.4439 4.1979 3.7184 3.7055 3.2126 2.9659 1.9783 -11.9455 -
11.9455 -11.9858 -11.9858 -11.9858 -12.1821 -12.1821 -12.4182 -12.4182 -
```

```
12.4182];
current8 = [-0.48 -0.46 -0.44 -0.46 -0.40 -0.38 -0.36 -0.34 -0.32 -0.30
-0.28 -0.26 -0.24 -0.22 -0.20 -0.18 -0.16 -0.14 -0.12 -0.10 -0.08 -0.06
-0.04 -0.2 0.0 0.02 0.04 0.06 0.08 0.10 0.12 0.14 0.16 0.18 0.20 0.22
0.24 0.26 0.28 0.30 0.32 0.34 0.36 0.38 0.40 0.42 0.44 0.46 0.48 0.50
0.52 0.54 0.56 0.58 0.60 0.62 0.64 0.66 0.68 0.70];
angle8   = [-12.4600 -12.4600 -12.4600 -12.4182 -12.4182 -12.4182 -
12.4182 -12.2231 -12.1821 -12.1821 -12.1821 -12.1821 -12.1821 -12.1821 -
12.1821 -12.1821 -12.1821 -12.1821 -12.2231 -11.9858 -11.9858 -11.9858 -
11.9455 -11.9455 -11.9455 -11.7085 -11.7085 -11.7085 -11.7085 -11.7085 -
11.7085 -11.7085 -11.7085 -11.5099 -11.4711 -11.4711 -11.2333 -11.2333 -
11.2333 -11.2333 -10.9951 -9.5582 -9.3174 7.6235 7.6235 7.6235 7.6235
7.6496 7.6496 7.8664 7.8664 7.8664 7.8664 8.1090 8.3799 8.3799 8.5933
8.5933 8.5933 8.5933];
figure(4)
plot(current7,angle7, 'b*')
hold on
plot(current8,angle8,'ro')
title('the "far above bifurcation" case')
xlabel('Current (amps)')
ylabel('Angle (degrees)')
legend('Decreasing Current','Increasing Current','Location','SouthEast')
```