EVALUATION OF TWO MODELS FOR LIGHT TRANSMISSION IN BIOLOGICAL MATERIALS

by

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SUMMARY:
Two light transmission models for biological materials are developed and evaluated. The energy balance model showed promise. It's developed as a 3 coefficient model permitting varied sample absorption and scattering properties. The second model performed unsatisfactorily.
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INTRODUCTION

The use of optics as a non-destructive means to determine the quality of agricultural products has increased over the last thirty years. More precise equipment is available for color sorting a wide variety of products such as tomatoes, lemons, oranges, etc. The color of the products have been related to a quality index such as maturity, ripeness, or the lack of or presence of some specific defect. Most of this work has been developed empirically. Models are being developed to better understand the process involved in light reflection from and transmission through agricultural products. Proper models can help engineers design more efficient equipment for less cost than previously possible. This paper evaluates two models proposed by us.

MODELS

The two models that are proposed approach light scattering from different perspectives. Both models are similar in that they use a discretised sample on which to perform the prediction. Two coefficients, $\alpha$ (scattering) and $\beta$ (absorption) are used to describe the material's light scattering properties. Scattering is the material's ability to receive visible light energy and redistribute it as visible light in all directions. The process of receiving visible light energy and dissipating it as heat is called absorption. This absorbed energy is unavailable to the scattering process.

Several assumptions were made in developing these two coefficient models. Later it will be shown how these restrictions can be relaxed permitting greater flexibility in using the models. The assumptions are:

1. The regular reflectance from the light source at the point of the incidence is ignored;
2. The incident radiation is a non-diffuse monochromatic light source;
3. Sample is homogenous, thus permitting the use of a single absorption and scattering coefficient;
4. Absorption and scattering coefficients for the incident light and the diffused light are the same;
5. The light scattering is isotropic.

The first model studied was described originally in a paper entitled "A Light Transmission Model for Biological Materials" by Cavaletto, et al., 1984. This model is called a multiple-event model because of how it is developed in several stages or events.
The model starts with the assumption that a beam of light strikes the sample in a normal direction. This beam propagates through the sample with diminishing intensity setting up a line source for multiple scattering. The creation of this line source is termed "level 0" in the model.

As the beam of light is transmitted through the sample, setting up the line source, its direction is unchanged. The energy contained in this line source is dissipated as the light travels through the sample due to absorption and scattering. The equation that describes this process is:

\[ I = I_0 e^{-(\alpha + \beta)ax} \]  

Level 0 is the formation of this line source within the sample due to the incident radiation. Each point along the line can be treated as a light scattering source contributing to the light intensity at some point \( P_i \) within the sample. This process is termed "Level 1 Radiation" (see Figure 1).

Every point within the body receives light energy due to the Level 1 radiation. Once again, a part of Level 1 radiation is absorbed by the material, and a part is scattered. The scattered radiation from points such as \( P_i \) sets up an intensity at point \( P_j \). This process is termed Level 2 radiation. Thus point \( P_j \) receives light energy due to Level 1 radiation from the line source, and Level 2 radiation from all other points within the body.

A part of the light energy received due to Level 2 radiation at some point \( P_k \) is absorbed and a part is scattered. The scatter part of the energy behaves similar to the scatter part in the Level 2 radiation. This process is termed Level 3 radiation. Just as Level 2 radiation is a consequence of Level 1 radiation, Level 3 radiation is a consequence of Level 2 radiation. We can think of Level 4 radiation as a result of Level 3 radiation, etc., thus the model treats light propagation through a biological material as a result of several levels (infinite levels), of radiation, each resulting from the previous one. The light intensity at any point \( P \) is the sum of the intensities due to the various levels of radiation. Note that in adopting this additive model, we have assumed that the intensities at a given point, due to two rays of intensity, \( I_1 \) and \( I_2 \) is \( I_1 + I_2 \). The interaction term \( C^2 (E_1 \cdot E_2) \cos \theta \) is assumed to be zero (0) because of the random manner in which the light is scattered through the material. The following equations are the final form used in describing the light propagation for the model:

\[ \text{Level } 0, \quad I = I_0 e^{-(\alpha + \beta)ax} \]
SINGLE EVENT MODEL

This model attempts to describe multiple light scattering as a single event. A power or energy flow variable $\phi_i$ is defined for each element. This variable is defined as the quantity of energy in the element that is scattered and absorbed. Each element gives energy to and receives from its surrounding elements in the process of multiple scattering. It is assumed that the incoming energy is absorbed by the element. A portion of this energy is lost due to absorption $\alpha_i \phi(\text{vol})$, and the rest, $\phi_i \phi(\text{vol})$, is scattered back to the surrounding elements. It is also assumed that no energy is transmitted through the element.

Each internal element of the sample, $E_{i,j,k}$, makes contact with 26 additional elements (Figure 2). Of these elements, 8 have a single point in contact, 12 have a line in contact and the remaining 6 have a face in contact with element $E_{i,j,k}$. It is hypothesised that the elements not having face-to-face contact with the element $E_{i,j,k}$ do not directly influence it as the face elements do, but do so through mutual face elements (see Figure 3).

The energy flowing through element $E_{i,j,k}$ can be accounted for by analyzing what flows through its six faces. The energy leaving and being absorbed by an element is described by the following expression:

$$(\alpha + \beta) \text{vol } \phi_{i,j,k}$$

where $\alpha$ = the scattering coefficient - scatter/volume
$\beta$ = the absorption coefficient - absorption/volume
$\text{vol}$ = element volume
$\phi_{i,j,k}$ = element energy with element coordinates, $i,j,k$

One sixth of the scattered energy leaves through each face of cubic elements. The element may also have an internal energy source, $S$. An energy balance on element $E_{i,j,k}$ is as follows:
\[ \text{vol}(a+\beta)\phi_{i,j,k} = (a/6)\text{vol}(\phi_{i,j,k-1} + \phi_{i,j,k-1} + \phi_{i,j,k+1} + \phi_{i,j,k+1} + \phi_{i,j,k+1} + \phi_{i-1,j,k+1} + \phi_{i+1,j,k}) + S_{i,j,k} \] (6)

Rearranging and simplifying the equation results in:

\[ (a+\beta)\phi_{i,j,k} - (a/6)(\phi_{i,j,k-1} + \phi_{i,j,k-1} + \phi_{i,j,k+1} + \phi_{i,j,k+1} + \phi_{i,j,k+1} + \phi_{i-1,j,k+1} + \phi_{i+1,j,k}) = S_{i,j,k} \] (7)

Notice that for constant volume elements, the volume disappears from the equation. However, the equation development assumes that the element volume size is such that no energy is transmitted through it. An equation of this form can be written for each element of the sample. Assembling the equations into a matrix format the following is obtained:

\[ AX = B \] (8)

where 
- \( A \) = the material property matrix
- \( X \) = a column matrix - \( \phi \), energy in each element
- \( B \) = a column matrix - contains the initial energy source of each element

Matrix \( A \) has dimensions \( N \times N \) where \( N \) is the number of elements. It is a banded matrix. Its band width is dependent on the element numbering scheme. If the elements are numbered in layers, the band width is two times the number of elements in a layer plus one (Figure 4). This matrix will be symmetric when boundary conditions are ignored (Figure 5). Matrices \( X \) and \( B \) have dimensions \( N \times 1 \).

The matrix \( X \) can be solved by using well-known matrix solution routines. If matrix \( B \) is zero, then there is no energy flow. Matrix \( B \) is similar to the forcing function of the system.

Matrix \( B \) represents the effective external sources on the system. A laser is used as an external light source. The size of the element selected is equal to the diameter of the laser beam. The depth of the element is such that the incoming radiation is completely absorbed and scattered by the element. This method of element selection makes it possible to replace the external source by an internal source located within the first element upon which the laser beam strikes. In fact, this criterion imposes a severe restriction on the element size selected. The element size in the direction of light propagation can be selected smaller than necessary to absorb all the light energy. In such a case, more than one element should be treated as a light source. The energy associated with each line source element can be estimated from Equation 1.
BOUNDARY CONDITIONS

The external boundary conditions play a very important role and should be considered. When light passes from one medium to another, its path is modified due to the incident angle and the refractive indices of the materials. The Snell and Fresnel equations quantify this modification.

Parallel incident radiation entering normal to a surface loses a portion of its energy to regular reflection. This reflection occurs due to the difference in the refractive indices of the sample and the medium surrounding the sample. Assuming the incident radiation enters normal to the surface, the correction can be estimated by the Fresnel equation:

\[ R = \frac{(n_2 - n_1)^2}{n_2^2 + n_1^2} \]  

The refractive index for air \( n_1 \) is approximately 1.00 and for potato, \( n_2 \) is between 1.4 and 1.5. This estimates that the regular reflection \( R \) is between 2 and 2.8 percent.

The diffused light leaving the sample has no preferred direction. This type of light has rays impinging the surface at all angles. Snell's law can be used to describe the reflection and refraction that occurs, Figure 6. The governing equation is:

\[ n_1 \sin(\theta_1) = n_2 \sin(\theta) \]  

When \( \theta_1 \) becomes large, a critical \( \theta_c \) angle is reached. This angle is defined by:

\[ \theta_c = \sin^{-1}(n_2/n_1) \]  

where \( n_1 \) = the refractive index of the denser material
\( n_2 \) = the refractive index of the less dense material.

At this critical angle total reflection begins to occur. Angles greater than \( \theta_c \) give complete reflection. The critical angle for potato air is approximately 41.8°.

Looking at an element on the sample surface, Figure 7, light leaving at an angle less than \( \theta_c \) is unattenuated. Light trying to leave at greater angles is internally reflected. The percentage of reflected light can be estimated by comparing the area of the end of a solid cone formed by the 41.8° half angle and the area of an enclosing hemisphere. The area of the hemisphere is \( \pi R^2 \). The area of the cone \( A_2 = 2\pi rh = 2\pi R (R - R \cos \theta) \approx 0.509 \pi R^2 \). The percentage of internally reflected light is 75%. Both models implement this
boundary condition. This is accomplished by determining the number of external faces of each boundary element and letting the reflected energy flow through the non-external faces.

**EXPERIMENTAL PROCEDURE**

An Oriel Corporation Model 7020 photodetection system was used to make the necessary light measurements. The system incorporates a photodiode detector, signal amplifier, output display, and power supply. The photodiode detector was replaced with a photomultiplier tube (R928-Hamamatsu Corporation) and appropriate power supply in order to increase the system sensitivity. The detector measures the incoming energy and the measurement system outputs a relative voltage reading. To prevent burning out the detector when making measurements on thin samples, neutral density filters were used to limit the input to the sample. Neutral density filters are rated by their optical density. A 2.5 optical density filter was necessary for the potato samples examined.

The light source was a helium-neon laser (Melles Griot Corporation). The laser output is 5 mW at a wavelength of 632.8 nm. The beam has a diameter of 1mm at 1/E. The laser is mounted on an optical bench so that its position could be held rigid and still allow flexibility to easily adjust its position.

Precision positioning of the fiber optic was accomplished using a 2 axis translation table. A pair of 32 threads per inch lead screws position the table. The table position could be measured within 0.005 inches (.13mm).

Accurately measuring the light intensity of a given location on the sample proved to be a very difficult task. Because of the difficulties in measuring light intensity accurately, we decided to use fiber optics. The fiber optic had a diameter of 0.25 inches. In order to limit the area viewed by the fiber optic, a 1mm aperture was used. The use of fiber optics has the advantage of flexibility, accurate determination of area and location being measured. The configuration shown in (Figure 8) proved to be the best for minimizing measurement errors. This orientation permitted small samples to be held without being clamped or inserting a holding pin in them. Clamps did often act as a light guide funnelling transmitted light between the sample and the clamp. Over-tightening the clamp just resulted in deforming the sample. Measurement of cubic samples with a dimension less than 13mm was found to be unreliable. For this reason we chose to measure the light transmitted to the far side of larger samples. The sample sizes measured are indicated in Table 1. We measured on a radial line from the edge of the sample through the center to the opposite edge. Measurements were made at .0156 inch (.397mm) increments. This provided two sets of measurements per sample and a way to check the homogeneity of the sample.
RESULTS

The results of the computer simulations are dependent on the selection of the absorption and scattering coefficients. Experimental procedure was unable to provide these coefficients. It was not possible to separate out the effects of scattering from absorption in the sample material. To determine the coefficients, a heuristic approach was undertaken. Different sets of coefficients were tried until a set was obtained that allowed for close approximation to the experimental data for the thicker samples. The experimental data was most accurately predicted for these sizes. The coefficients used for the comparison of the simple model were $a = 1.00$, $b = 0.01$, and line source decay $= 1.00$. The term "Decay" was estimated from the experimental data. Figure 9 shows that for the 2mm thick sample, some of the light from the line source is not absorbed and scattered and is thus transmitted through (change in slope of line). Note that the data from the 5mm sample is represented by a straight line for the whole sample. Decay was chosen to put all the energy from the source in the first 5mm of the sample (forming the line source).

In Figures 10-15 the single event model simulation results are compared to the experimental data. The model over-predicted the experimental data for all samples. Generally, the thicker the sample, the better the comparison. It's interesting to note that for the most part, the simulation results paralleled the experimental data. This result suggests that a second boundary condition is needed. It is unclear what that boundary condition should be.

The input energy $I$ was $2.38 \times 10^{-3}$ after going through a 2.50 OD neutral density filter. The neutral density filter was necessary to limit the amount of energy reaching the photo multiplier tube when measuring the thinner samples.

The multiple event model was unable to come close predicting the light scattering distribution regardless of the coefficients used. The model was modified to include a separate set of scattering and absorption coefficients for the line source and the diffusing medium. Again, the model performed poorly. It did not permit the energy to scatter far enough from the line source. Figure 16 shows a typical model response.

The effect of the internal reflection is illustrated in Figure 17. With no internal reflection (i.e. the energy is free to leave the sample into the air) the distribution of energy was low in the outer elements. The 80 percent internal reflection retained proper energy within the sample.

Varying the element size (the length of the element - X dimension) was also investigated. Making the X dimension larger reduced the accuracy of the solution. A smaller X dimension
(<1mm) resulted in solutions that are not physically correct. There is no constraint forcing the elements to have positive net energy flows, thus some became negative.

**DISCUSSION**

The single event model shows a promise in being able to predict light scattering in biological materials. The model best predicts the experimental data for the 9mm and 13mm thick samples. The model over-predicts everywhere by 140 percent. Using the same coefficients for the thinner samples, results in increased error (Figure 18). Note that the error is generally constant for each individual sample. This indicates that there is probably another factor dependent upon sample thickness which the model doesn't presently include. This factor is most likely the second boundary condition.

The use of line source elements appears to be an acceptable method of representing the effect of the external laser light source on the sample. The cross-sectional area of the line source elements was 1mm². This approximated the cross-sectional area of the laser. For simplicity the simulations divided the whole sample into 1mm cubic elements. Larger elements could be used but with reduced resolution.

The coefficients control the distribution of light energy in the sample. The line source decay coefficient determines which elements in the line source receive energy and what level. A value of one (1) distributed more than 99 percent of the energy in the first 5 elements. This appears to be reasonable. The experimental data from the 2mm samples shows transmission of unscattered light in the center (The sharp downward slope near the center - Figure 9). The experimental data from the 5mm & 7mm samples shows no excess transmission (constant slope). The 9mm and thicker samples show no direct light from the source reaching the far surface (increase in negative slope as you go from the center).

The ratio of $\alpha$ to $\beta$, $\Gamma$ affects the distribution of energy from the source elements. The lower this value, the more energy is absorbed in each interaction and the less energy available to be scattered. A value of $\Gamma$ equal to 100 gives reasonable answers. $\Gamma = 10$ absorbs too much of the energy. A value of 1000 gives nearly the same result as 100 (Figure 19). The values of $\alpha$ and $\beta$ have no noticeable effect; only their ratio seem to have an effect on the energy distribution in the sample.

The multiple event model was unable to even come close in predicting light scattering. Attempts to find coefficients that would allow this model to accurately predict the resultant energy distribution were unsuccessful. The main problem was to get the energy to distribute away from the line source. The value of $\alpha_1 + \beta_1$ was chosen so that the scattered energy from the
last line element was equal to the experimental data. The ratio of the coefficients $a_2$ to $\beta_2$. $\Gamma$ again affects the distribution of energy in the sample. The larger $\Gamma$, the greater the distribution. Unlike the single event model, the value of $a$ also effects the distribution. This is because $a_2$ multiplies directly in the equations describing the scatter (Equations 3-5).

From Equation 5 we tried to estimate what value of $a$ would give maximum scattering. The equation to maximize is the following:

$$ = \frac{A}{R^2} a \exp(-BR^3 a)$$

(12)

where $A$ and $B$ are constants.

The value of $a$ that gives a maximum scattering is dependent upon $R$. Thus the value of $a$ that gives maximum scattering for one sample shape/geometry may not be correct for another.

From Figure 20, note how the energy distribution from the initial line source reaches only three elements away. The subsequent levels of multiple scattering serve to distribute the energy to further elements. Each level of multiple scattering has less energy to redistribute. After three levels of multiple scattering most of the energy has been scattered by the model.

The boundary condition affects the resultant distribution of energy by internally reflecting a portion of it. Theoretically a value of 75 percent reflection from the exterior surface of a boundary element was calculated. The data in Figure indicates that a value of approximately 80 percent gives the correct distribution. If the percent of internal reflection is set equal to zero (0), too much of the energy exits the sample and the resulting energy distribution at the edges is too low. A 90 percent reflection results in too much energy remaining in the sample.

CONCLUSIONS

This study has shown that a simple single event (energy balance) model might be useful in predicting the distribution of light scattering in potato samples. The model permits the use of varying geometric shapes and sizes. Both concentrated and distributed defects can easily be included in the model. The laser light source can be modeled as a line source of energy. Other light sources most likely can be modeled as inputs into the sample.
The present method to select the coefficients is by "trial and error." The coefficients selected gave adequate results when considering the lack of homogeneity of the samples and the differences between samples. A better method to select coefficients would be helpful.

The multiple event model proposed is not able to accurately predict light scattering in the potato samples. It is unable to scatter enough light away from the line source to be distributed in the rest of the sample.

Additional development and testing of the model needs to be done in the future. Some of the details that need to be investigated are:

1. Why is there an error in the simulation results that appears to be dependent on the sample thickness?
2. How well does the model "handle" concentrated defects?
3. Develop a method to determine the material coefficients necessary for the model.
4. Test the model on materials other than potato.

REFERENCES

Table 1. Sizes of potato samples used to obtain experimental data.

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<th>Thickness mm</th>
<th>Width mm</th>
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Figure 1  Light is scattered from the line source to $P_i$. Scattering also occurs from $P_i$ to $P_j$, $P_j$ to $P_k$ etc.
Figure 2  Element $E_{i,j,k}$ and its 26 neighbors.
Figure 3  Paths of energy transfer between elements.

Figure 4  Element numbering scheme for rectangular solids.

Cubic sample divided into 27 elements.
Figure 5  Material property matrix for single event model. (Matrix "A")
Incident ray

Reflected ray

\( \alpha_1 \)

\( \alpha_2 \)

Refracted ray

Interface

\( n_1 \)

\( n_2 \)

Figure 6 Reflection and refraction, \( n_1 < n_2 \).

Air

Figure 7 The difference in the index of refraction of the sample and air cause light leaving the element with an angle greater than \( \alpha_c \) to be internally reflected.
Figure 8  Apparatus used to measure light scatter.
Figure 9: Samples less than 5mm thick permit some light energy to be transmitted directly through the sample.

Figure 10: Comparison of experimental data and single event simulation data for 2mm thick sample.
Figure 11: Comparison of experimental data and single event simulation results for 5mm sample.

Figure 12: Comparison of experimental data and single event simulation results for 7mm sample.
Figure 13 Comparison of experimental data and single event simulation results for 7mm sample.

Figure 14 Comparison of experimental data and single event simulation results for 11mm sample.
Figure 15 Comparison of experimental and simulation data for 13mm thick sample.

Figure 16 Comparison of experimental data, multiple event model and single event model - 2mm sample.
Figure 17 The effect of the boundary condition on energy distribution.

Figure 18 Average error between single event model and the experimental data. The model consistently over predicts the experimental data.
Figure 19 The effect of gama on the distribution of energy.

Figure 20 The energy distribution for each level in the multiple event model.