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BODNER-PARTOM VISCOPLASTIC CONSTITUTIVE MODEL AND THE NON LINEAR FINITE ELEMENT ANALYSIS OF A STRESS CONCENTRATION AT HIGH TEMPERATURE

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ABSTRACT

The design and analysis of structural components to operate at elevated temperature and severe stress levels, such as a low-cycle fatigue-limited jet engine disk, require an accurate prediction of the nonlinear stress-strain response encountered during the cyclic loading conditions. Nonlinear analysis of such components is normally carried out by a finite element code making use of constitutive theories in which the material response is separated into the two important groups of phenomena known as rate dependent "creep" and rate independent "plasticity." A number of viscoplastic constitutive theories in which "creep" and "plasticity" effects are combined into a unified plastic strain model have recently been proposed and are still undergoing active development.
In this paper, an elastic-plastic finite element model incorporating the Bodner-Partom model of nonlinear time dependent material behavior is presented. The parameters in the constitutive model are numerically extracted by a least-square fit to experimental data obtained from uniaxial stress-strain and creep tests at 650°C. The finite element model of a double notched specimen is employed to determine the elastic-plastic strain and comparison is made to experimental data.
The constitutive model parameters evaluated in this paper are found to be in good agreement with those obtained by other investigators. However, this numerical technique tends to give better agreement with the response curves than does the graphical methods used by the other investigators. The model calculated elastic-plastic strain agreed well with the experimental.

INTRODUCTION

Inconel 718 is a high temperature superalloy developed for low-cycle fatigue limited components operating at high temperature and severe stress in a hostile environment. To properly use the alloy, it is necessary to know and be able to properly characterize its time dependent elastic-plastic properties. A number of viscoplastic constitutive theories combining creep and plasticity effects into a unified plastic strain model have recently been proposed and are undergoing active development. One of these theories, used in this study, is the constitutive theory of Bodner and Partom [1].

Bodner [1], utilized the constitutive equations of Bodner and Partom to represent the inelastic behavior of Rene 95 at 1200F. Stouffer [2], used the state variable constitutive equations of Bodner and Partom to calculate the mechanical response of IN 100 at 1350F. Hennerichs [3], estimated the materials constants in IN 100 by using Bodner and Partom constitutive equations. Milly [4], represented the experimental data for Inconel 718 at 1200F, from which the material constants were determined by the method given by Stouffer [2]. Milly applied the Bodner constitutive equations and compared the theory and experimental data and concluded the overall behavior was good.

There are a large number of finite elements in use today for both plane stress and plane strain analysis,

for example, the constant strain triangular elements, hybrid elements and higher order isoparametric elements. For linear elasticity, the mathematical development of these elements is simple. Since the constant strain triangular elements have advantages of being simple and economical, they were selected to be employed in this study. The elastic-plastic code was used in the numerical determination of the elastic stress concentration (K_t) for benchmark notched specimen under constant tensile load and for the elastic-plastic response of that specimen for cyclic loading.

Domas, Sharpe, Ward and Yau [7] in the analytical task of their study, used the finite element method and calculated stress and strain at element centroids for good estimation of the elastic stress concentration factor (K_t).

Hennerichs [3], incorporated the Bodner constitutive model into a constant-strain triangular elements model to analyze creep crack growth in a nickel alloy at 1350F.

The Constitutive Theory of Bodner and Partom

The constitutive theory of Bodner a(1) is based on the assumption of small strain and that the total strain rate $\dot{\varepsilon}_t(t)$ is separable into elastic (reversible), $\dot{\varepsilon}^e(t)$, and plastic (irreversible), $\dot{\xi}^P(t)$, components, both non-zero for all loading/unloading conditions.

$$\dot{\varepsilon}_t(t) = \dot{\varepsilon}^e(t) + \dot{\varepsilon}^P(t) \qquad (1)$$

where,

$$\dot{\varepsilon}^P(t) = \dot{\sigma}(t)/E \qquad (2)$$

For the plastic strain rate, .ep(t), the specific representation used by Bodner and Partom is given by

$$\dot{\varepsilon}^P(t) = \frac{2}{\sqrt{3}}\ \frac{\sigma(t)}{\sigma(t)} D_o\ \exp\ [\frac{1}{2}(\frac{Z(t)}{\sigma(t)})^{2n}(\frac{n+1}{n})] \qquad (3)$$

where,

σ (t)=The current value of the stress

D_o =a constant representing the limiting value of the plastic strain rate in shear. Generally it is taken at 104 sec^{-1}, except for conditions of very high rates of strain.

n =a constant related t vi scosity of the dislocation motion. It controls the strain rate sensitivity.

Z(t) =the plastic state variable measure of the overall resistance to plastic flow, that is, hardness.

The evolution equation, i.e. history dependence, of the plastic state variable is generally sought in the form of a differential equation for the hardening rate, .Z, that depends on stress, temperature and hardness, Z. A more specific representation is based on the concept that only the plastic rate of working $.W^P$, and current hardness, Z, control the rate of hardening. The complete expression for, Z, can be written as [1],

$$Z = Z_1 - (Z_1 - Z_0)\exp(-mWp)$$

or

$$(Z - Z_1) = (Z_1 - Z_0)\exp(-mWp) \qquad (4)$$

where,

m =a material constant controlling the rate of work hardening.

Z_1 =the saturation value of Z for large Wp, i.e., it is the maximum value of Z, taken to be constant.

Z_0 =the initial value of Z, corresponding to the reference state from which Wp is measured. Bounds are ($0 \le Z_0 \le Z_1$).

Wp is defined by the differential equation

$$\dot{W}^P = \sigma\dot{\varepsilon}^P + \frac{Z_{recovery}}{m(Z_1 - Z)} \qquad (5)$$

to give

$$\dot{W}^P = \int\sigma\dot{\varepsilon}^P dt + \int\frac{Zrecovery}{m(Z_1 - Z)}dt \qquad (6)$$

where,

$$\dot{Z}_{recovery} = -AZ_1(\frac{Z - Z_i}{Z_1})^r \qquad (7)$$

Therefore, the complete differential expression or Z, is written as

$$\dot{Z} = m(Z_1 - Z)\dot{W}^P - AZ_1(\frac{Z - Z_i}{Z_1})^r \qquad (8)$$

where,

A =the coefficient controlling the rate of hardening recovery

r =the exponent controlling the rate of hardening recovery

Z_1 =the state variable value corresponding to the complete non-work hardened condition, a function of temperature.

In (8), the second term is hardening recovery, negligible during brief load histories, but for long time response, such as creep, the second term is necessary. During a tensile test, fast compared to creep, equation (8) reduces to the first term only. For a tensile test equation (8) becomes

$$\dot{Z} = m(Z_1 - Z)\dot{W}^P \qquad (9)$$

In this case, W^P is determined by only

$$\dot{W}^P = \sigma\dot{\varepsilon}^P \qquad (10)$$

In order to determine the viscoplastic material constants in these constitutive equations, the constants are considered to be in two groups, "creep response" and "short time response." The short time response constants are Do, n, m, Z_o and Z_1 and they are determined based on stress-strain test data. The creep response constants are Z_i, r, A, and they are determined based on data from at least two creep tests at two different stress levels. Step-by-step theoretical evaluation of the material parameters was developed by Bodner [1] for Rene 95, and by Stouffer [2] for In 100. Also, Stouffer and Bodner [5] studied the relationship between theory and experiment for the state variable constitutive equation. Therefore, to complete the study of the constitutive equations, a numerical evaluation of the material parameters was undertaken.

Numerical Evaluation of the Material Parameters

A numerical study of nonlinear time dependent material was considered, where the material variables are numerically calculated using a direct curve fitting technique.

In general, the Bodner material parameters are dependent on temperature, but by performing the material characterization tests (stress-strain and creep) at the same temperature that the Bodner model will be applied, the temperature dependence is suppressed.

To determine the Bodner variables (n, Z_o, Z_1, m, A, r, Z_i and Do) numerically, we consider the measured and the simulation of the plastic strain. For the evaluation of simulated $\dot{\varepsilon}^P(t)$, the total strain rate is considered the sum of elastic and plastic strain rates

$$\dot{\varepsilon}_t(t) = \dot{\varepsilon}^e(t) + \dot{\varepsilon}^P(t) \tag{1}$$

where,

$$\dot{\varepsilon}^e(t) = \dot{\sigma}(t)/E \tag{2}$$

Therefore, we can rewrite equation (1) in the form

$$\dot{\varepsilon}^P(t) = \dot{\varepsilon}^t(t) - \dot{\sigma}(t)/E \tag{11}$$

where, σ (t) is extracted from the data.

For the evaluation of theoretical $\dot{\varepsilon}^P$ we consider the flow law

$$\dot{\varepsilon}^P(t) = \lambda S \tag{12}$$

where, S is the deviatoric stress and λ is a scalar function of the state of the material.

$k = k(X)$

where,

$$< X > = <n, Z_0, Z_1, m, A, r, Z_i, D_0> \tag{13}$$

Equation (12) becomes

$$\dot{\varepsilon}^P(t) = \lambda(X)\, S(t) \tag{14}$$

and

$$\varepsilon^t(t) = \varepsilon^t(0) + \int \dot{\varepsilon}^P(t)dt + \frac{r(t)}{E} \tag{15}$$

where, $\dot{\varepsilon}^t(0)$ is taken from the data.

Now, let

$$Q = \sum_{Data} W(t) \quad (\varepsilon^t_{model} - \varepsilon^t_{data.})^2 \tag{16}$$

where, w(t) is a positive weight function, and $(\varepsilon^t_{model.} - \varepsilon^t_{data.})$ is the time domain strain error. Q was minimized in a computerized numerical scheme by varying values of the material coefficients. In this analysis, a Runge Kutta (fourth order) algorithm was employed for the numerical time integration of the Bodner equations in the following system:

$$\dot{Z} = m(Z_1 - Z)\, \dot{W}^P - AZ_1 \left(\frac{Z - Z_i}{Z_1}\right)^r \tag{8}$$

$$\dot{W}^P = \sigma \dot{\varepsilon}^P \tag{10}$$

$$\dot{\varepsilon}^P(t) = \lambda S \tag{12}$$

$$D_2^P - D_o^2 \exp\left[-\left(\frac{Z^2}{3J_2}\right)^n \left(n + \frac{1}{n}\right)\right] \tag{17}$$

where, J_2 is the second stress invariant and equation (17) is from reference [1].

Using a variant of Powell's [6] search algorithm coupled with the integral square error function, the specific material variables for the Bodner model were determined to best fit the tensile and creep data.

The Experimental Data

In the experimental part of this research, a group of tensile and creep tests were performed for Inconel 718 at 650^{0}C. The Bodner model, coupled with this set of data, was employed to determine the material variables of Inconel 718 to best fit the data.

To help eliminate inconsistencies in data, the same specimen geometry was used for all tests. A drawing of the button-head specimen used in this study, is presented in figure 1.

All experiments were performed in an electrohydraulic testing machine equipped with a

special high-temperature furnace. Special attention was given to the alignment of the specimen to minimize the eccentricity of the load and to obtain a uniform temperature profile in the test section. The machine was run under strain control. The data (stress, strain, time) was obtained by using the Interferometric Strain/Displacement Gage (ISDG) technique developed and conducted by W. N. Sharpe.

The Elastic-Plastic Finite Element Model.

The locally written finite-element program generates constant strain triangular elements with very fine elements in the root of the notch. The 2-D program was formulated for plane stress calculation and uses the plastic force method to enforce plastic strain. Only one quarter of the specimen was modelled due to symmetry, and an automatic mesh generation algorithm permitted easy variation of the total mesh size. It was thus possible to easily select the proper balance between mesh fineness and computation time. A mesh of 176 nodes and 300 elements was eventually chosen, figure 2.

Results and Conclusions

Uniaxial tensile tests were run at strain rates of 1.6 x 10^{-3}, 0.67 x 10^{-4}, 1.0 x 10^{-5}, 1.1 x 10^{-6}, and 3.3 x 10^{-7} sec^{-1}. Two creep tests were run at 758 and 862 MPa. The data for each test was smoothed and approximately 20 data points used for each test in the fitting procedure. The modulus of elasticity was considered as a parameter, increasing the number of constants to be determined to nine. Different starting values were tried, producing slightly different values of constants though no appreciable difference in the agreement with individual curves. A unique set of constants was not already determined to cover the complete data base.

The final parameters determined are listed in Table 1 where the values from Milly [4] are also listed. Figures 3, 4, 5 and 6 show the response curves from the model and the experimental data. Agreement in figures 3 and 4 are reasonable except for the fastest tensile test. Figures 5 and 6 are not good but the agreement is considerably better than other comparisons [2,4]. The model modulus of elasticity, 172 x 10^3 MPa, is high for this material at this temperature. Other determinations place it at 155 x 10^3 MPa[3]. As can be observed from figures 3 and 4, the modulus for the experimental data is higher still at the faster rates. Nevertheless, the Bodner Model describes the results fairly well given the wide range of strain rates and the fact that 650° is near a phase transition.

This numerical scheme for identifying the Bodner parameters is straightforward and tends to give better agreement with the response curves than graphical methods. The Bodner model does reasonably well in describing stress-strain curves covering a wide range of strain-rates (almost 10^4 in these experiments) and creep curves. Actually, it is a stiff requirement to ask a model to cover such a wide variety of material response. The eight parameters of the Bodner model give it the requisite range, but a unique set of parameters was not obtained. In general, one can make the following conclusions:

(1)A numerical evaluation of the material variables can be made by using Bodner constitutive theory through a numerical simulation of the tensile and creep response with reasonable curve fitting to the experimental response.

(2)Bodner Constitutive theory is sensitive to the variability of the experimental data. Since the stress, σ , is the driving force in the constitutive model, special attention should be given to the time data for a smooth (σ -t) curve.

(3)As in Reference [1], Bodner constitutive theory may need further work to decide on improvements which can be made to include effects that would lead to tertiary creep in the representations.

(4)From the fact that the calculated values for some of the variables are different for the same value of fitting error (and they should not be) and from the fact that for some runs "Z_0" and "Z_1" have the same value, (and they should not), it can be recognized that the material variables in Bodner's Constitutive model are not well defined. More specifically, they are not universal, which may address that 650°. is near a transition temperature for this material.

The finite-element model does a good job of evaluating the theoretical elastic stress concentration, K_t, and predicting the notch behavior under low cycle loading at high loads. The response at the notch is very complicated given the finite plastic strains generated. Figure 7 shows the experimental data and finite-element prediction for the comparison of measured notch strain with that predicted by the finite element during the first cycle of continuous cyclic loading at 650°C.

For further application, using the Bodner-Partom state variable material model and model parameters developed from uniaxial test data, creep crack growth in Inconel 718 was analyzed by using the constant strain finite element for compact tension specimen [8].

Crack growth behavior identified solely from crack opening displacement measurements made on pre-cracked specimens was analyzed by examining various parameters around the model's crack tip. The

J-integral was applied and used to predict crack growth initiation. Promising results were obtained.

In reviewing the response, figure 8, one must keep in mind the initial material uniaxial data. The cracked specimens were actually pre-cracked by fatigue loading and the crack propagation material state was not the same as the initial state of the uniaxial test specimen.

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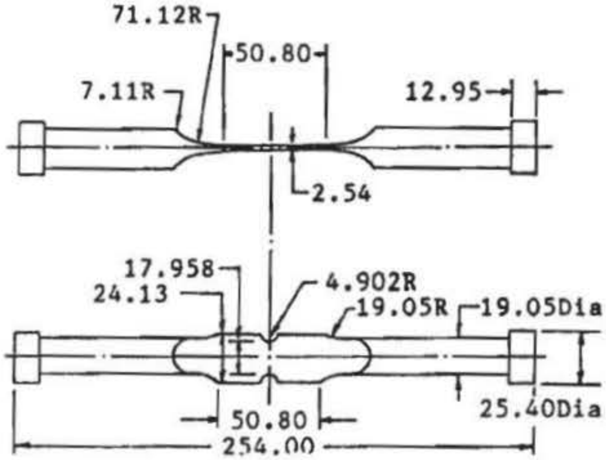

Fig. 1 - Double-notched specimen (dimensions in mm)

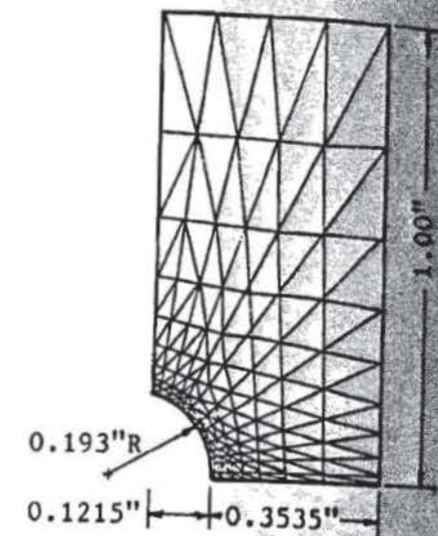

Fig. 2 - Finite element grid

TABLE 1

Constant	Units	Milly (8)	This Work
N	-	1.167	0.7374
Z_0	MPa	3130	6520
Z_1	MPa	4140	7030
m	MPa	2.43×10^{-2}	6.86×10^{-1}
A	sec^{-1}	1.1×10^{-4}	6.82×10^{-4}
R	-	2.857	4.734
Z_I	MPa	2760	3690
D_0	sec^{-1}	10^4	1.03×10^4
E	MPa	165×10^3	172×10^3

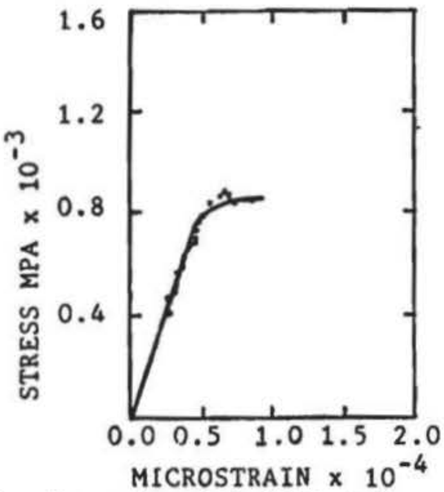

Fig. 3 - Experimental data and numerical tensile response ($\dot{\varepsilon}$ =3.3 x 10^{-7} sec^{-1})

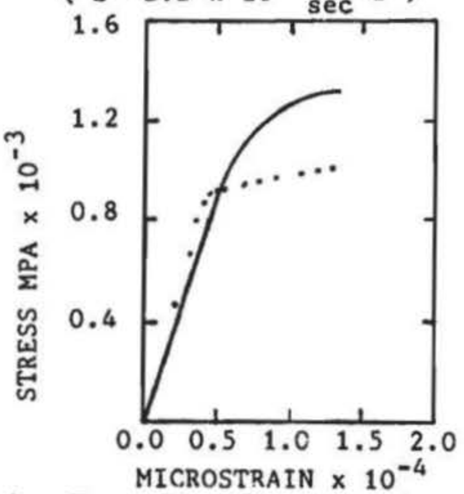

Fig. 4 - Experimental data and numerical tensile response ($\dot{\varepsilon}$ =1.6 x 10^{-3} sec^{-1})

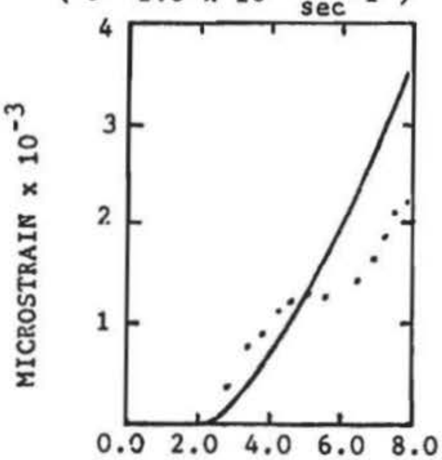

Fig. 5 - Experimental data and creep response (σ =758mpa)

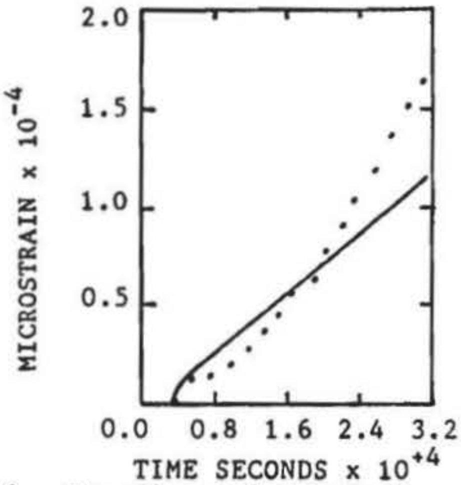

Fig. 6 - Experimental data and creep response (σ =862mpa)

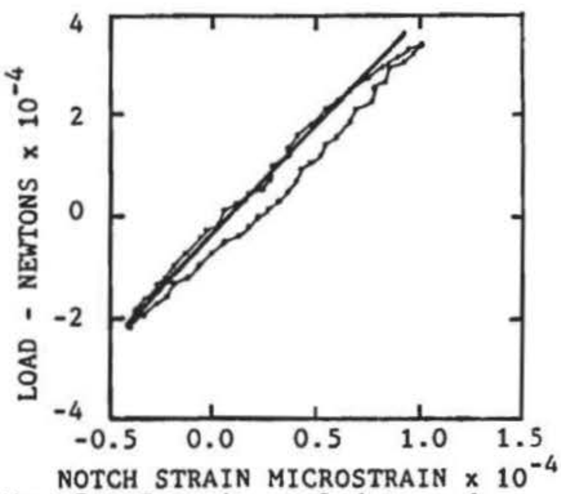

Fig. 7 - Experimental data and finite element prediction

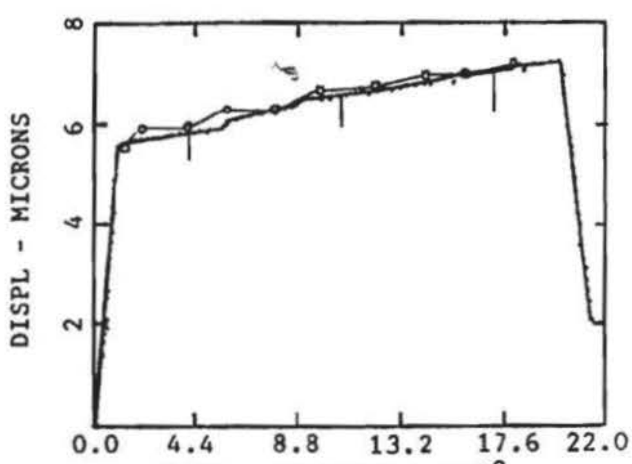

Fig. 8 - Experimental results for creep crack growth