Direct Displacement-Based Design Using Inelastic Design Spectrum

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Objectives

- Demonstrate application of inelastic design spectra to direct displacement-based design (DDBD)
- Demonstrate potential limitations of current DDBD that use elastic design spectra and equivalent linear systems

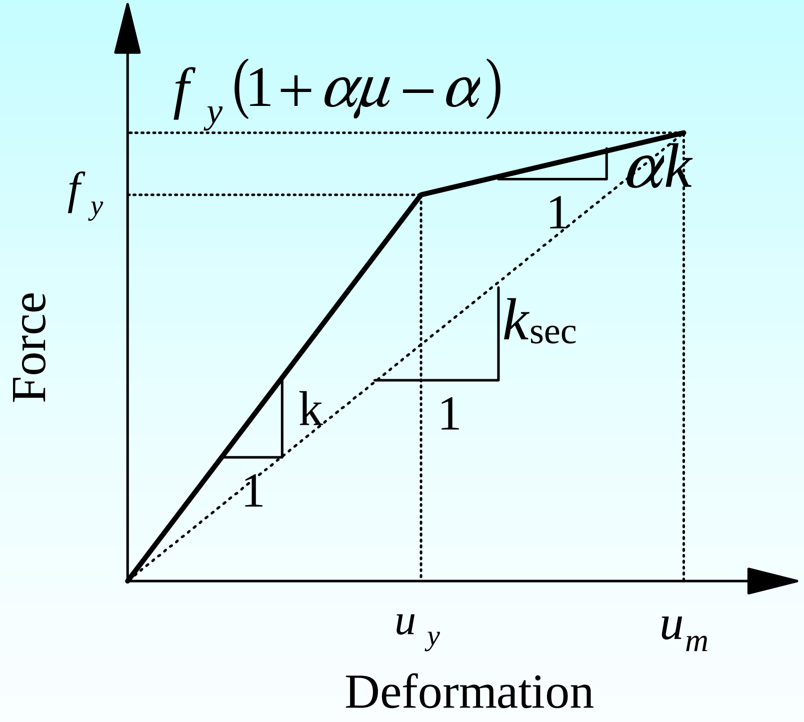
Equivalent Linear System: Period

- For bilinear systems

$$T_{eq} = T_n \sqrt{\frac{\mu}{1 + \alpha\mu - \alpha}}$$

- For elasto-plastic systems

$$T_{eq} = T_n \sqrt{\mu}$$



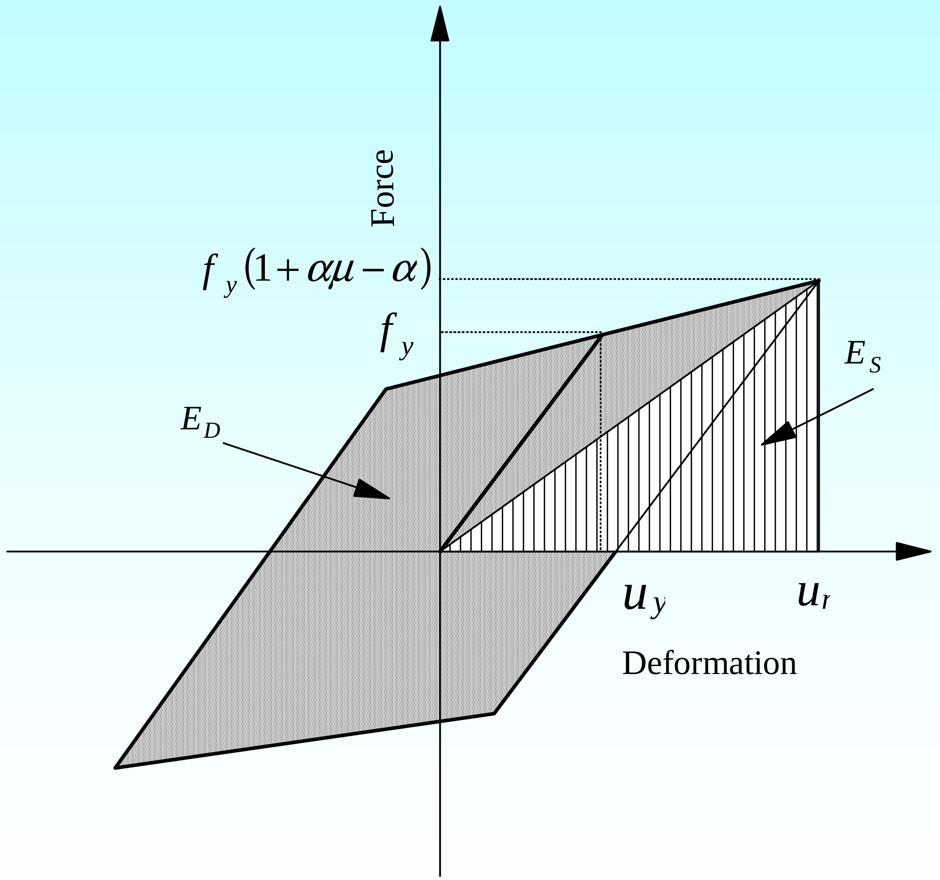
Equivalent Linear System: Damping

- For bilinear systems

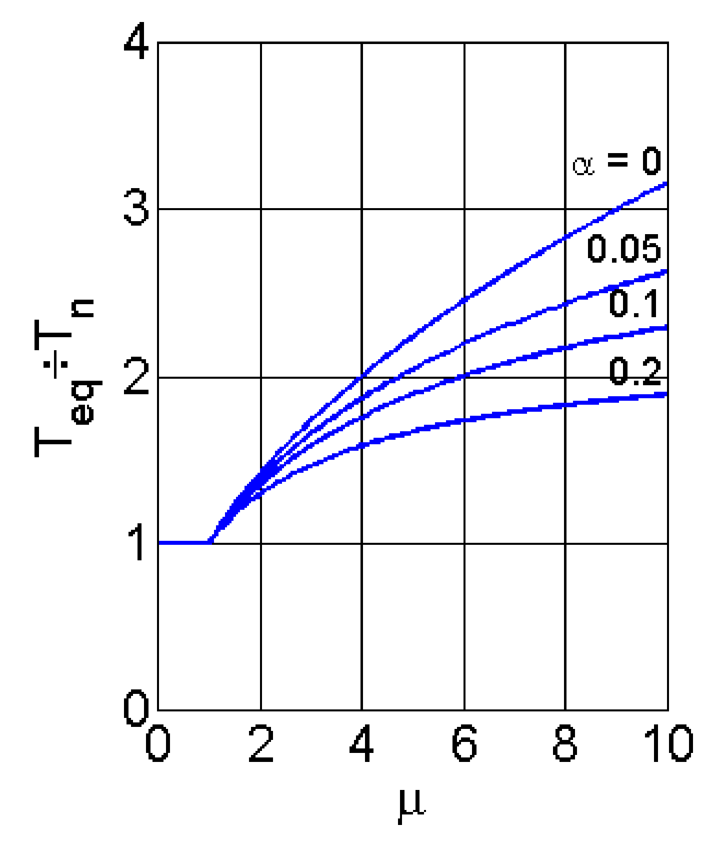
$$\zeta_{eq} = \frac{2}{\pi} \frac{(\mu - 1)(1 - \alpha)}{\mu(1 + \alpha\mu - \alpha)}$$

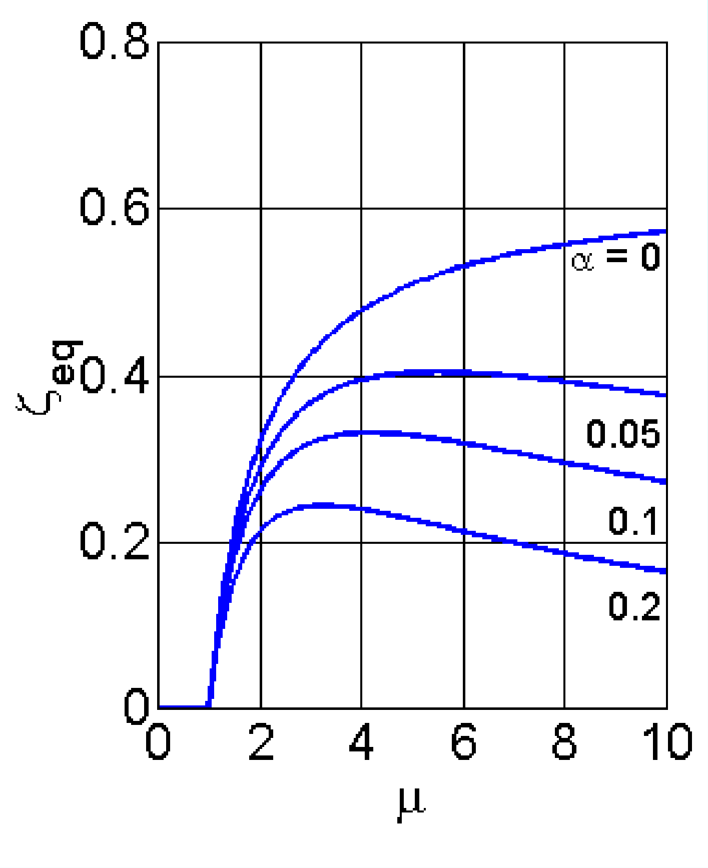
- For elasto-plastic systems

$$\zeta_{eq} = \frac{2}{\pi} \frac{(\mu - 1)}{\mu}$$

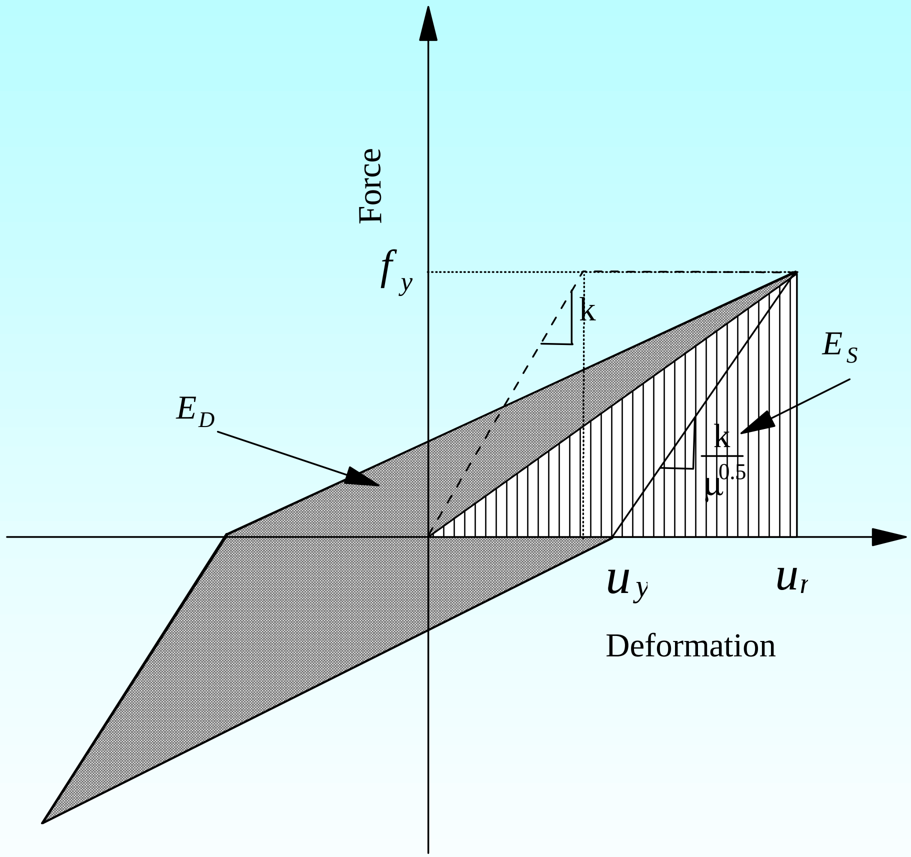


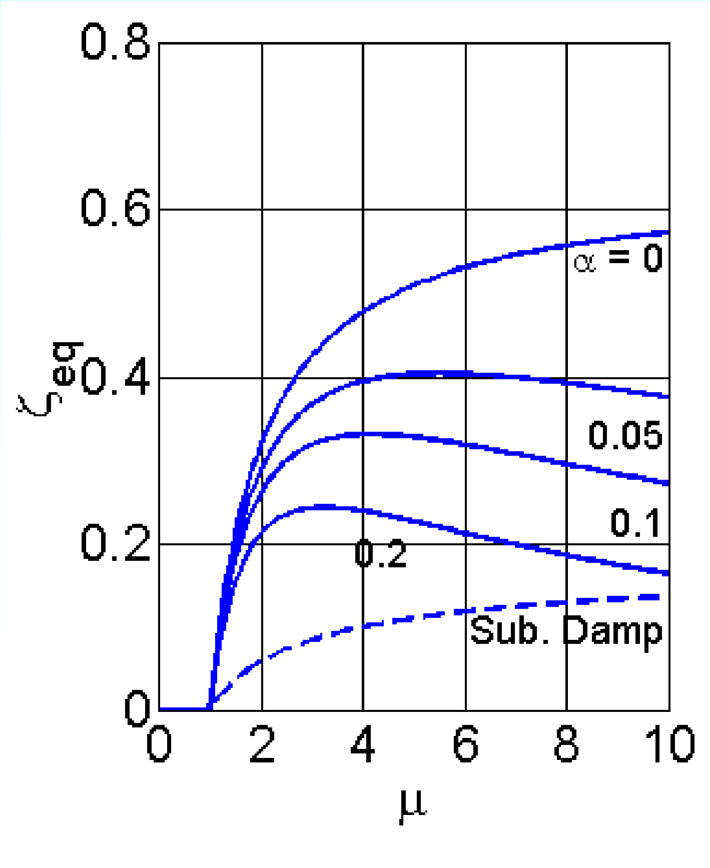
Equivalent Linear System





Substitute Damping

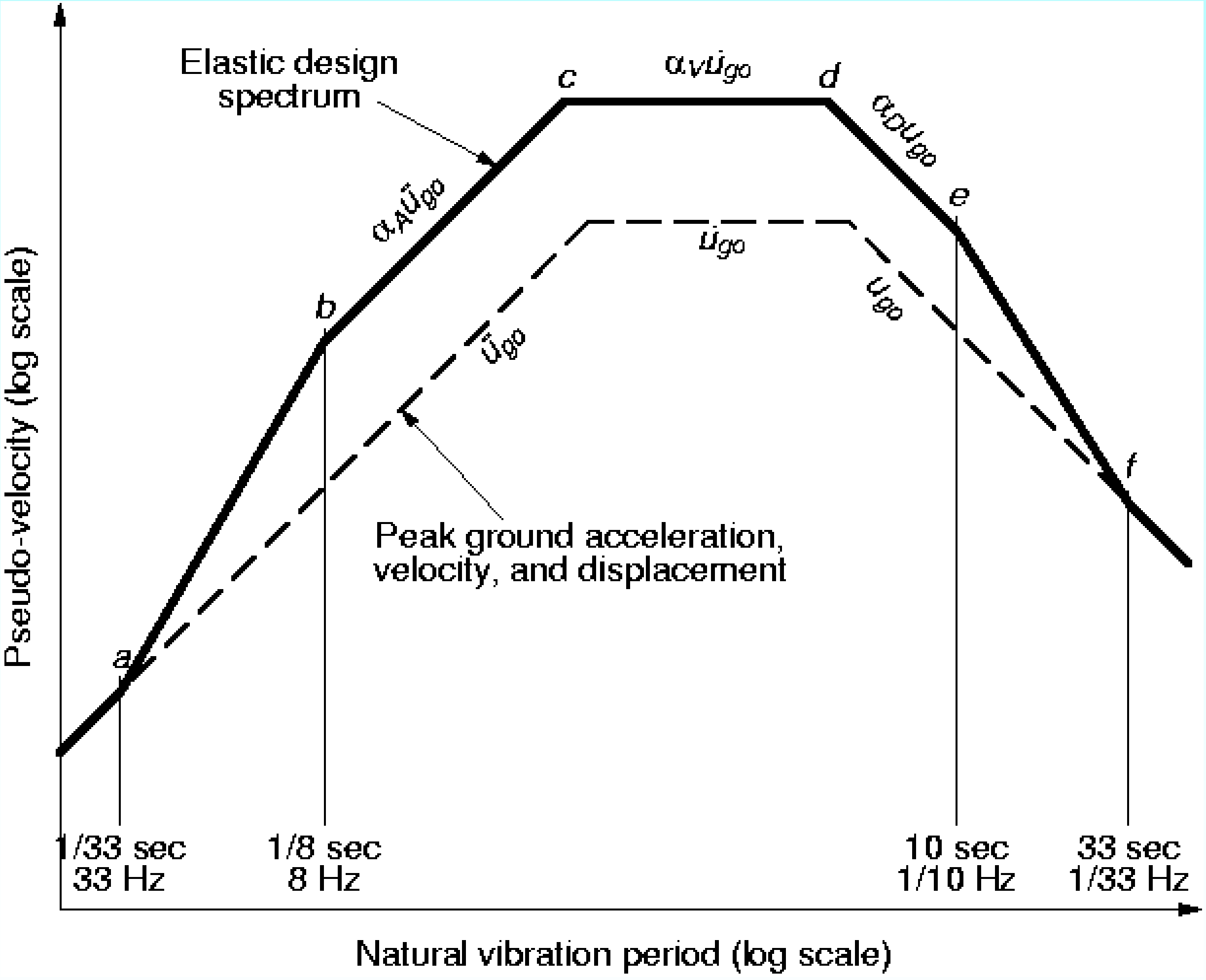




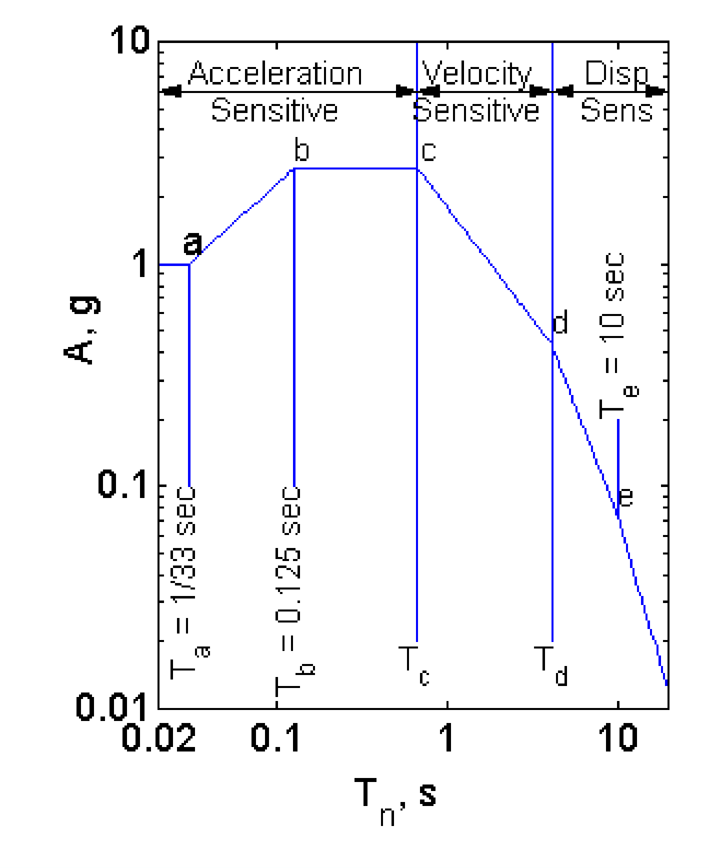
Gulkan & Sozen, Shibata & Sozen

(Takeda model for R/C structures)

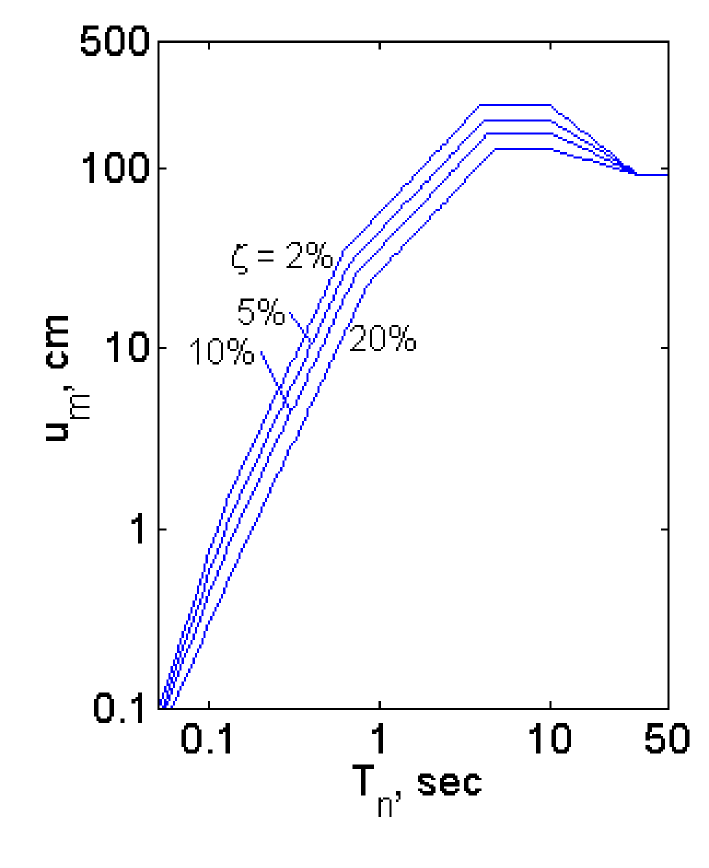
Elastic Design Spectrum



Elastic Design Spectrum



Pseudo-Acceleration



Deformation

DDBD Using Elastic Spectra: Step-by-Step Procedure

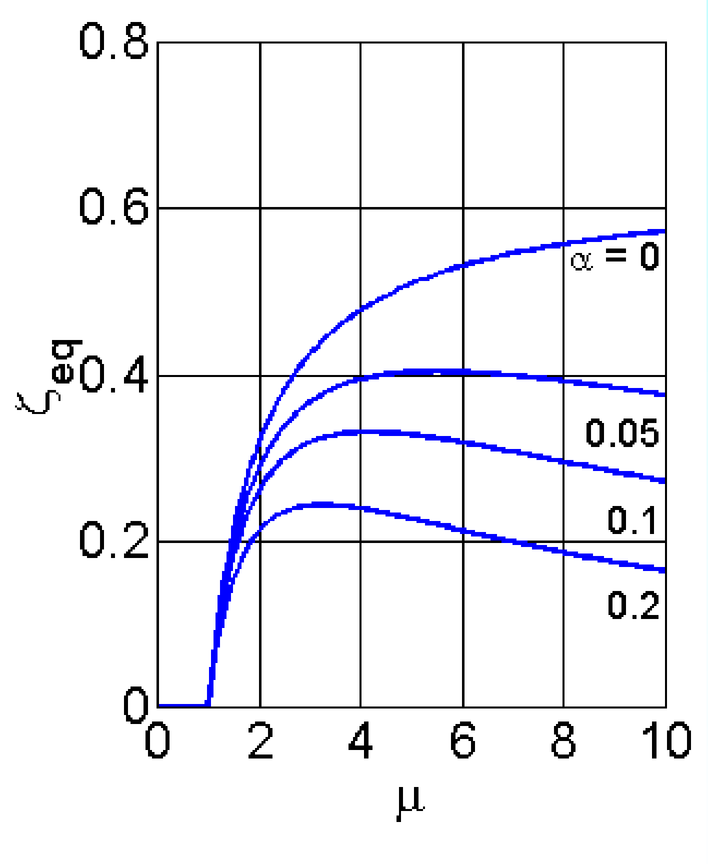
1. Estimate the yield deformation for the system
2. Establish acceptable plastic rotation, θ_p
3. Determine design displacement and ductility factor: $u_m = u_y + h\ \theta_p$ and $\mu = u_m / u_y$

DDBD Using Elastic Spectra: Step-by-Step Procedure

4. Estimate the total equivalent viscous damping:

$$\zeta_{eq} = \frac{2}{\pi} \frac{(\mu-1)(1-\alpha)}{\mu(1+\alpha\mu-\alpha)}$$

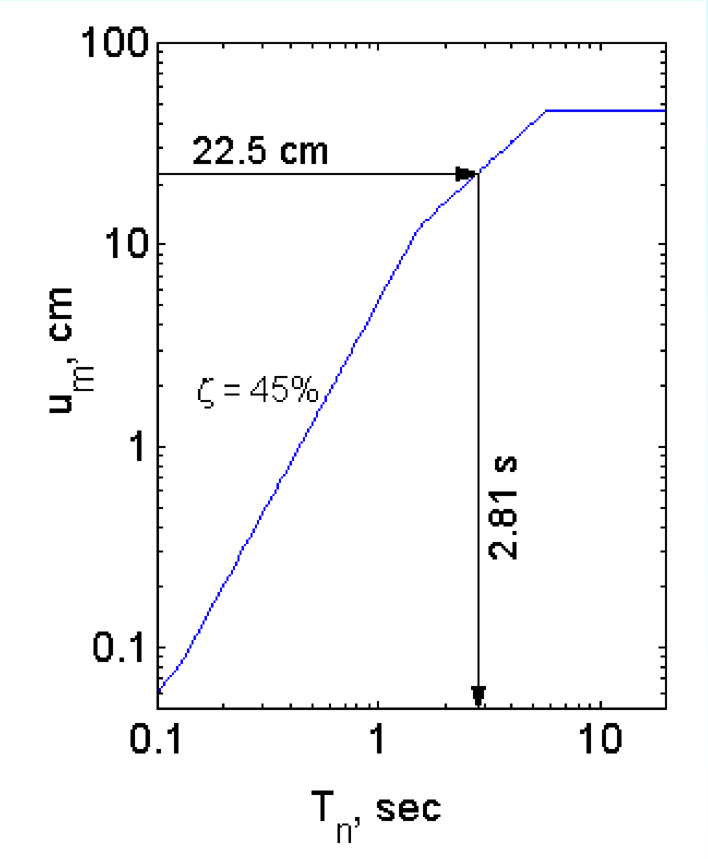
$$\hat{\zeta}_{eq} = \zeta + \zeta_{eq}$$



DDBD Using Elastic Spectra: Step-by-Step Procedure

5. Enter deformation design spectrum and read T_{eq}.
   ➔ Determine the secant stiffness

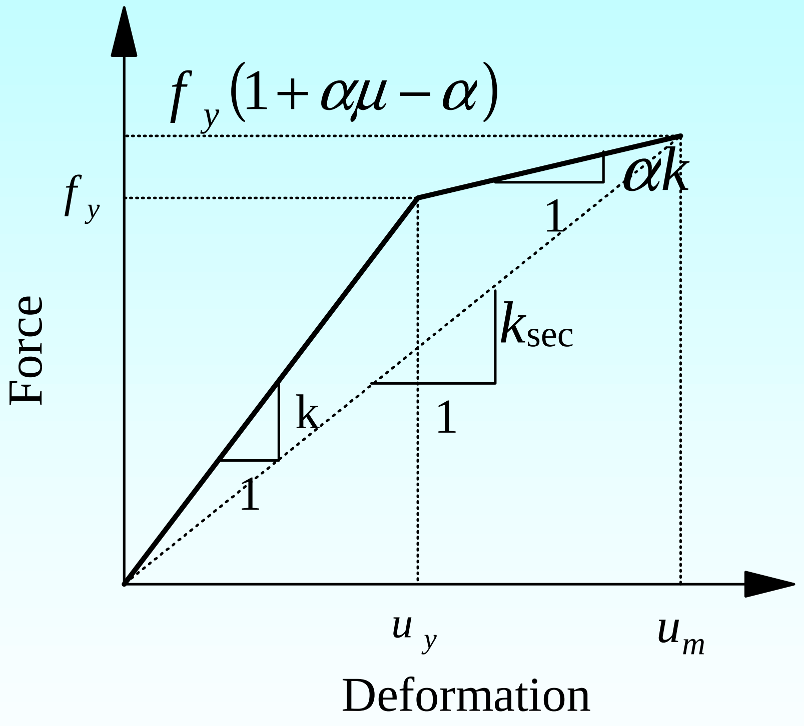
$$k_{\text{sec}} = \frac{2\pi^2}{T_{eq}^2} m$$



DDBD Using Elastic Spectra: Step-by-Step Procedure

6. Determined the required yield strength:

$$f_y = \frac{k_{\mathrm{sec}} u_m}{1 + \alpha\mu - \alpha}$$

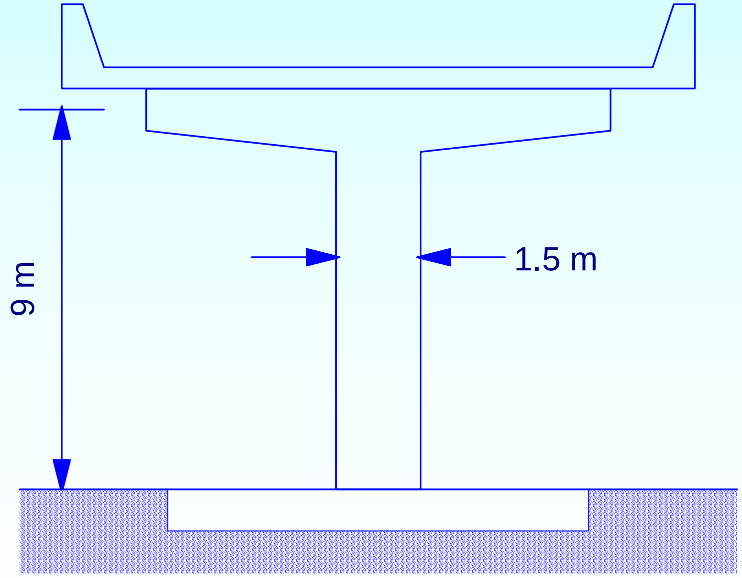


DDBD Using Elastic Spectra: Step-by-Step Procedure

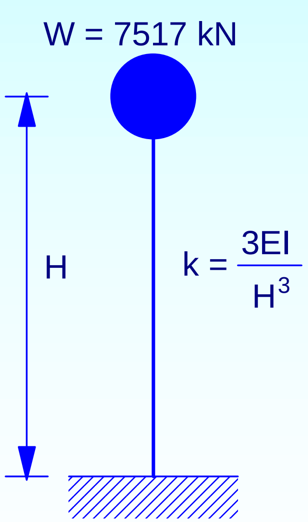
7. Estimate member size and detail (reinforcement in R/C structures, connections in steel structures) to provide f_y.
   - ➔ Calculate initial elastic stiffness k.
   - ➔ Calculate yield deformation: $u_y = f_y / k$
8. Repeat steps 3 to 7 until a satisfactory solution is obtained.

Example

- R/C viaduct
- Superstructure weight = 190 kN/m
- Bent spacing = 39.6 m



(a)

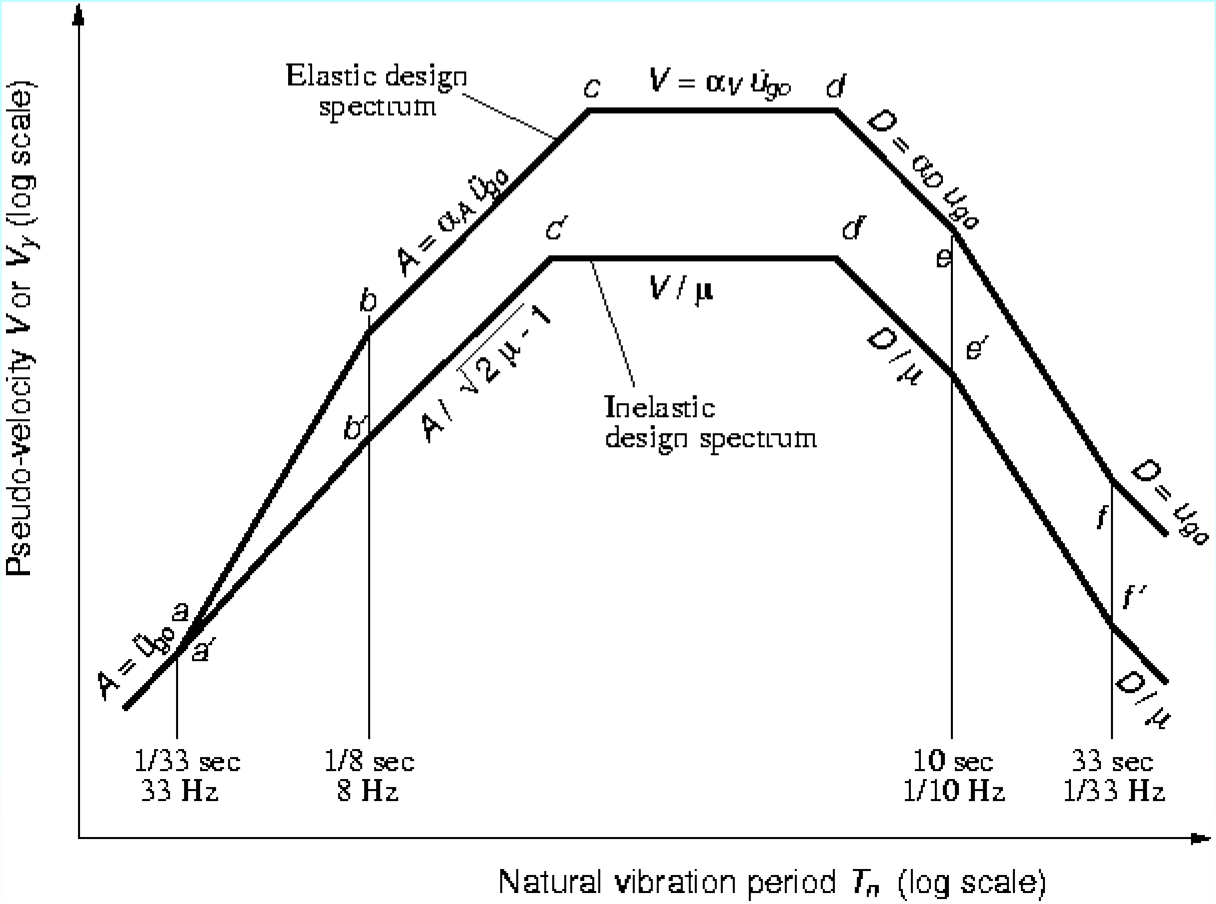


(b)

Design Summary: DDBD Using Elastic Spectra

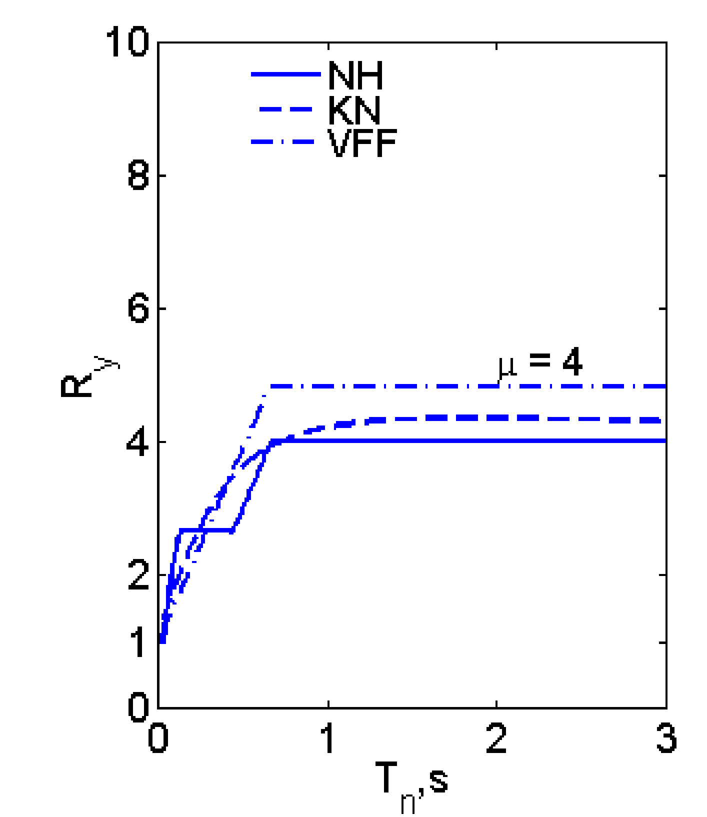
- Starting yield displacement = 4.5 cm.
- Convergence achieved after three iterations
- The final design has:
  - ➔Longitudinal column reinforcement = 1.3%
  - ➔Initial stiffness = 95.17 kN/cm
  - ➔Lateral yield strength = 839.7 kN
  - ➔Yield displacement = 8.82 cm, Design displacement = 26.8 cm
  - ➔Elastic period = 1.78 sec, Secant period = 3.14 sec

Inelastic Design Spectrum

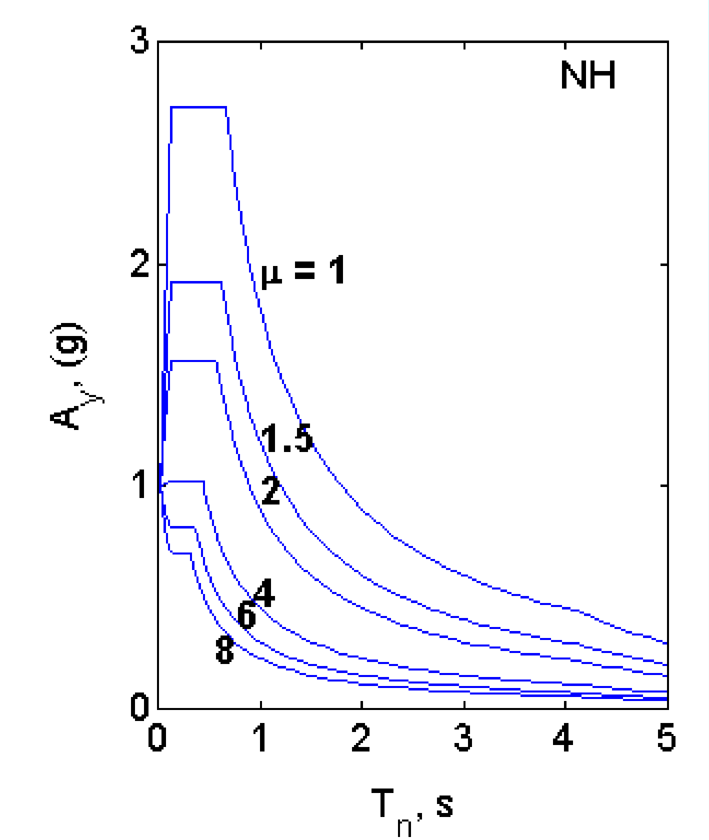


Conversion of Elastic to Inelastic Design Spectrum

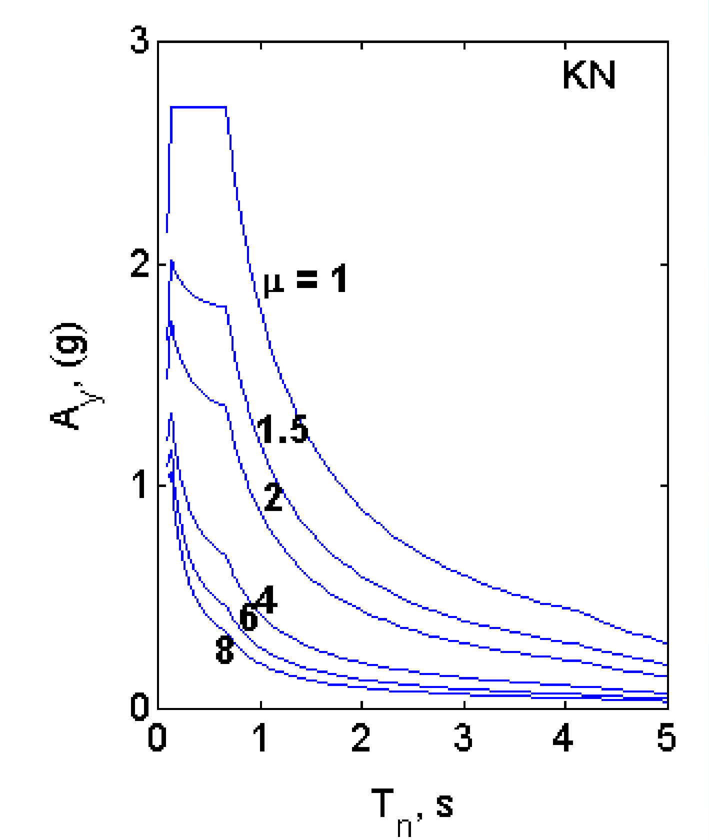
- Use available R_y-μ-T_n relationships
  - ➔ Newmark and hall
  - ➔ Krawinkler et al
  - ➔ Fajfar et al.
  - ➔ Miranda & Bertero
- Relationships based on nonlinear analysis of SDF systems



Constant Ductility Inelastic Design Spectrum

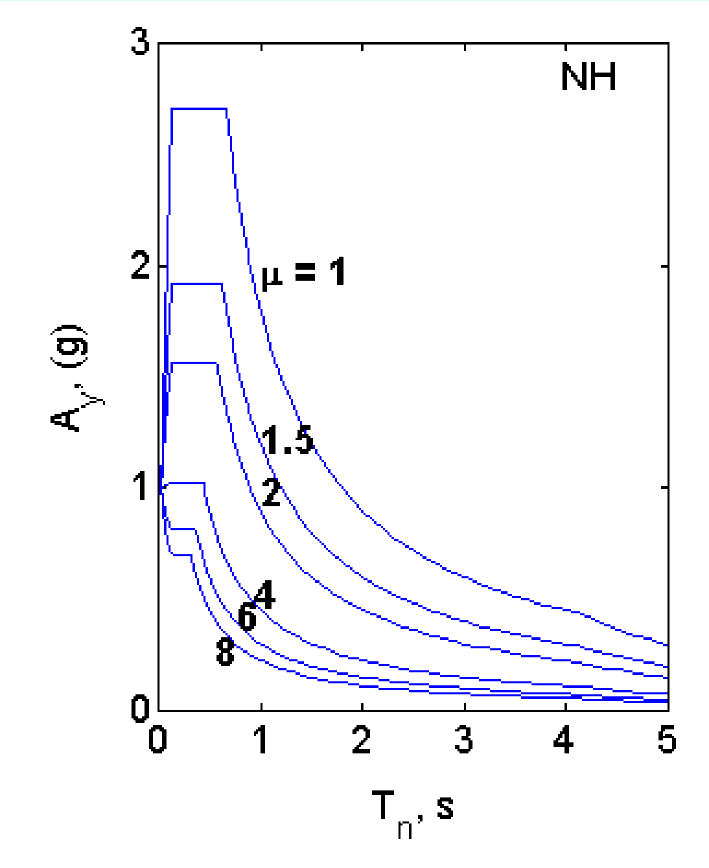


Newmark & Hall

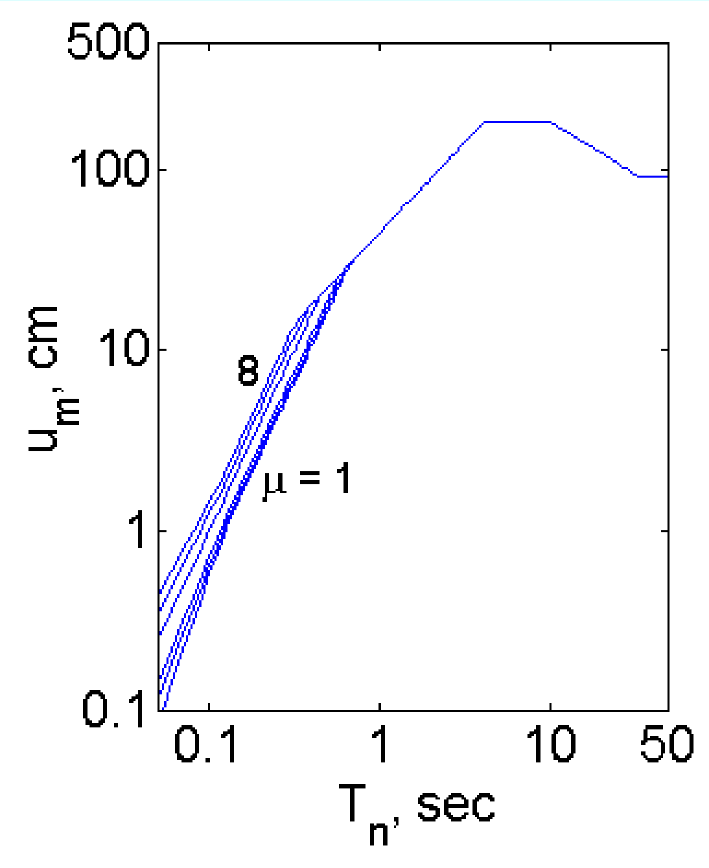


Krawinkler et al.

Inelastic Design Spectrum



Pseudo-Acceleration



Deformation

DDBD Using Inelastic Spectra: Step-by-Step Procedure

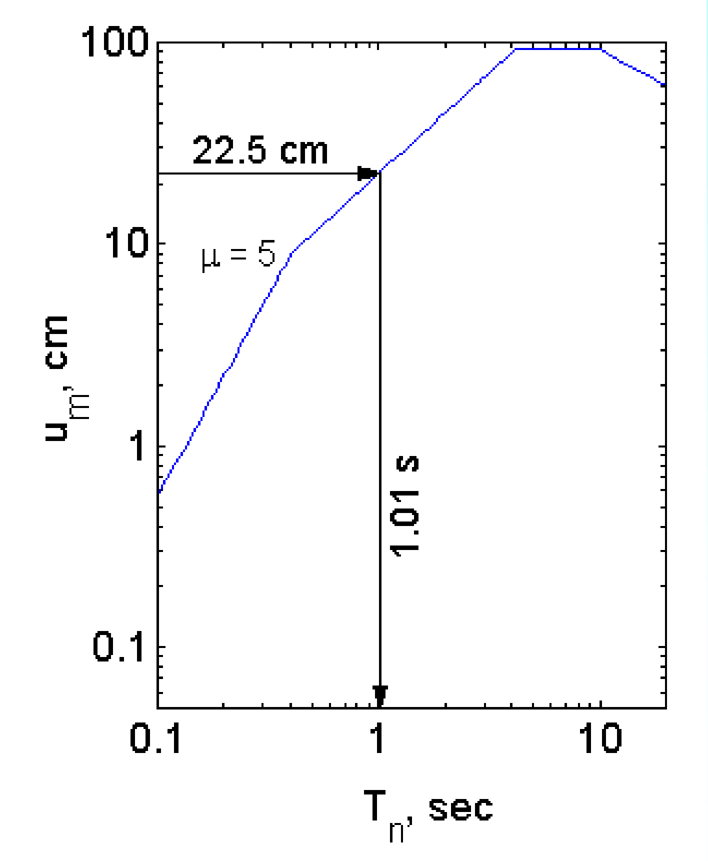
1. Estimate the yield deformation for the system
2. Establish acceptable plastic rotation, θ_p
3. Determine design displacement and ductility factor: $u_m = u_y + h\ \theta_p$ and $\mu = u_m / u_y$

DDBD Using Elastic Spectra: Step-by-Step Procedure

4. Enter deformation design spectrum and read T_n.

   ➔ Determine the elastic stiffness

$$k = \frac{2\pi^2}{T_n^2} m$$



DDBD Using Inelastic Spectra: Step-by-Step Procedure

5. Determine the required yield strength

   ➔ $f_y = ku_y$

6. Estimate member size and detail (reinforcement in R/C structures, connections in steel structures) to provide f_y.

   ➔ Calculate initial elastic stiffness k.

   ➔ Calculate yield deformation: $u_y = f_y / k$

7. Repeat steps 3 to 6 until a satisfactory solution is obtained.

Design Summary: DDBD Using Inelastic Spectra

- Starting yield displacement = 4.5 cm.
- Convergence achieved after five iterations
- The final design has:
  - Longitudinal column reinforcement = 5.5%
  - Initial stiffness = 238.6 kN/cm
  - Lateral yield strength = 1907 kN
  - Yield displacement = 7.99 cm, Design displacement = 26.0 cm
  - Elastic period = 1.16 sec

Evaluation of Design: Inelastic Analysis

1. Calculate initial elastic period from *m* and *k*
2. Determine *A* from elastic design spectrum
   ➔ Elastic Design Force: $f_o = mA$
3. From known f_y, calculate: $R_y = f_o / f_y$
4. Determine ductility demand μ from R_y-μ-T_n relationships
5. Calculate displacement and plastic rotation
   ➔ $u_m = (\mu / R_y)(T_n / 2\pi)^2 A$
   ➔ $\theta_p = (u_m - u_y)/h$

Evaluation of Example Design: DDBD Using Elastic Spectra

- Demands from inelastic analysis of the design structure
  - u_m = 39.7 cm.
  - μ = 4.52
  - θ_p = 0.0343 rad.
- Design using elastic design spectrum
  - u_m = 26.8 cm (**32.6% underestimation)**
  - μ = 3.04 (**32.6% underestimation)**
  - θ_p = 0.02 rad (**Demand exceeds acceptable value by > 72%)**

Evaluation of Example Design: DDBD Using Inelastic Spectra

- Demands from inelastic analysis of the design structure
  - ➔ u_m = 25.9 cm.
  - ➔ μ = 3.25
  - ➔ θ_p = 0.0199 rad.
- Design using elastic design spectrum
  - ➔ u_m = 26.0 cm
  - ➔ μ = 3.06
  - ➔ θ_p = 0.02 rad
  - ➔ **Predictions are nearly the same as the inelastic demands**

Conclusions

- A direct displacement-based design procedure is presented
  - ➔Uses well-known inelastic design spectrum
  - ➔Provides displacement estimates consistent with those from inelastic analysis
  - ➔Produces design that satisfies the design criteria of acceptable plastic rotation
  - ➔The procedure is as simple as the current DDBD procedure using elastic design spectra

Conclusions

- DDBD procedure based on elastic design spectra
  - ➔ Uses equivalent linear systems
    - Secant stiffness and equivalent damping
  - ➔ Provides displacement estimate which can be significantly smaller than that from inelastic analysis
  - ➔ Plastic rotation demand may exceed the acceptable value
    - Leaves an erroneous impression that the allowable plastic rotation constraint has been satisfied