The effect of the fastener of different configuration composite panels on failure analysis.

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Abstract

The effect of the fastener of different configuration composite panels on failure analysis.

By

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This study presents the effect of the stacking sequence and fiber orientation on a composite sandwich panel subjected to static in-plane bolt loading. Six plates were constructed with laminates of unidirectional carbon fiber and cross ply weaves of fiberglass. The orientations that were examined included 0, +/- 45, and 90 degrees. Half of the plates had fiberglass lamina on the outside of the laminate while the other three plates had the carbon fiber on the outside. Experimental and analytical tests were performed to determine the best orientations and stacking sequence.

For the numerical analysis, plates with fibers oriented at +/- 45 degrees showed the highest strength. The experimental data also showed high strengths for the +/- 45 degree plates. However the experimental data also showed high strengths for the 90 degree laminate but with very high displacements. These high displacements would not allow the joint to maintain its relative position to the adjacent part. The discrepancy between the strength of the FEA models and the experimental data is attributed to inaccurate strength properties. The effect of *in situ* strength and compression strength was found to have a significant effect on the accuracy of the FEA solution. Good correlation was found between the FEA and experimental data in predicting the trend of the stiffness of the plates.

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List of Symbols

A	Constant
B	Constant
C	Constant
D	Constant
E	Young's Modulus
FEA	Finite Element Analysis
FG	Fiberglass
FV	Field Variable
G	Shear Modulus
N	Number of plies
S	Shear Strength
U	Displacement
X	Longitudinal Strength
Y	Transverse Strength
in	Inch
lbf	Pound Force
psi	Pound per square inch
r	Radius
s	Symmetric
t	Thickness of layer
V_f	Volume fraction of the fibers
W_{FG}	Weight fraction of the fiberglass
W_c	Weight fraction of the carbon fiber material
W_e	Weight fraction of the epoxy
e_B	Failure criterion for fiber breakage
e_f	Failure criterion for fiber/matrix shear
e_m	Failure criterion for matrix tension/compression

Greek Symbols

θ	Fiber orientation
ρ_e	Density of the epoxy
ρ_f	Density of the fibers
ρ_m	Density of the matrix
σ_{ij}	Stress
ν_{ij}	Poisson's Ratio

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1. Introduction

A composite material is one that is composed of one or more different materials. The purpose of the combination is to improve the characteristics of the composite over the performance of either of its constituents alone. In aerospace structures the constituent materials are fibers and matrix. The fibers are typically stronger and stiffer than the matrix and carry most of the load. The matrix supports the fibers, protects the fibers, and transfers the load to the fibers. There are four basic types of composite materials: fibrous composite materials that are fibers in a matrix, laminated composites that are layers of different materials, particulate composites which are particles in a matrix, and any combination of these. In this paper the type of composites that will be focused on are laminated composites.

There are many different types of materials that are used for fibers. A few examples of these include graphite or carbon, fiberglass, boron, and Kevlar. The two types that will be examined here are carbon fiber and fiberglass (Jones, 1999).

Composite materials offer some of the highest specific strength and specific stiffness among aerospace materials. The importance of specific strength and stiffness is because of the strong ties between structural weight and aerospace vehicle payload and performance. Furthermore composite materials can be tailored to provide the required performance in different orientations. This tailoring of strength and stiffness allows greater structural optimization. In the past some composite materials high initial cost has led to use of more classical materials such as aluminum alloys. This has led to increasing use of composite materials in the aerospace industry and other industries such as automobiles and ships (Jones, 1999).

One type of composite structure frequently being used in the aerospace industry is sandwich panels. One such example of this structure is seen on the Lockheed S-3A spoiler. The graphite-epoxy skin layed-up over the honeycomb core gave a 41% weight savings over the traditional metal structure (Jones, 1999). This good strength to weight ratio along with their good thermal and acoustic properties make composite sandwich structures ideal for both load bearing and non-load bearing cases. Panels typically are composed of laminated face sheets separated by some medium. In this case the face sheets are made from carbon/epoxy and fiberglass/epoxy and are separated by a nomex honeycomb core. This arrangement allows the panel to act similar to an I-beam. The face sheets resist the bending loads while the honeycomb core transfers the shear loads between the laminates (Demelio, 2001).

While composite do give excellent structural efficiency, they do not handle concentrated loads as well as other aerospace materials. One such concentrated load that is frequently seen in aerospace structures is from fasteners. Structures can be joined together either by bonding or by using a fastener. Adhesive bonding has fewer pieces and is lighter weight than mechanically fastened joints. However, due to the need for structural parts to be inspected and/or repaired, the use of fasteners is common in aerospace structures (Demelio, 2001).

Typically when designing a structure for maximum efficiency, one must know the load at which the structure will fail. There are three basic modes of failure for a mechanical joint. They are tension, shear-out, and bearing. Crews and Naik examined composite joints to determine factors that affect which failure mode will be observed. They primarily looked at the edge distances and overall width of the specimens and their effect on the failure mode. They found that the initial damage was usually in the bearing mode. The exception was when there was very small edge distances or specimen width, and then the dominant failure mode was tension.

Beyond the stress level of the initial onset, damage in the tension mode appeared. The location of the tension damage showed good correlation with the 2-D finite element computed max tensile stress. For narrow specimens the initial damage mode was tension which led to the section failure in tension. However for moderate and large edge distances the initial damage mode and the ultimate damage mode were different. For wide specimens the initial damage onset was local bearing followed by local tension and finally remote bearing outside the washer. For moderately small edge distances the initial failure mode was local bearing, followed by local tension, and finally remote shear-out from outside the washer (Crews, 1986).

Typically aerospace structures are designed to have joints fail in bearing. This bearing failure will cause the joint to no longer function but the larger area around the hole will be more functional than a net tension failure or a shear-out failure. If the structure is properly designed the load can then be transferred through other joints in the composite. This local bearing failure can easily be identified and repaired as soon as possible.

To properly design the mechanically joined structures great care has to be taken in the analysis of the composite structure. Due to the complexity of the problem and the nonlinearity, it is common for the structures to be analyzed using a finite element model. While most research has been done with a 2-D model, Liu, Shao, and Yue used a 3-D finite element model to examine clamping effects on the joint. As can be seen in Figure 1, the elements are clustered around the hole where the stress is most critical (Liu et al., n.d.).

Figure 1: Finite Element Model Around Bolt Loaded
Hole

To determine the deformations and stress fields after damage occurred, the elastic constants in the elemental stiffness matrices of the elements in appropriate elements were modified as can be seen in Figure 2.

Initial (no damage)	Matrix cracking	Fiber/matrix shear	Fiber breakage
E_{11}	E_{11}	E_{11}	$E_{11} \rightarrow 0$
E_{22}	$E_{22} \rightarrow 0$	E_{22}	$E_{22} \rightarrow 0$
μ_{12}	$\mu_{12} \rightarrow 0$	$\mu_{12} \rightarrow 0$	$\mu_{12} \rightarrow 0$
G_{12}	G_{12}	$G_{12} \rightarrow 0$	$G_{12} \rightarrow 0$
G_{13}	G_{13}	$G_{13} \rightarrow 0$	$G_{13} \rightarrow 0$
G_{23}	G_{23}	G_{23}	$G_{23} \rightarrow 0$
FV1=0	FV1=1	FV1=0	FV1=0
FV2=0	FV2=0	FV2=1	FV2=0
FV3=0	FV3=0	FV3=0	FV3=1

Figure 2: Elastic Constants Damage Progression

For the case presented in this paper, it is necessary to use a 3-D analysis. This is because the effect of stacking sequence is to be examined. For this reason a finite element model similar to that of Liu, Shao, and Yue will be used (Liu et al., n.d.).

Similar to the finite element analysis done by Liu, Shao, and Yue; Aktas and Husnu looked at the experimental effect of stacking sequence in a carbon composite laminate. Analysis done by Liu et al. (n.d.), Aktas and Husnu (2003) tested two laminates of opposite stacking sequence. The effect of stacking sequence and fiber orientation will be examined in this paper.

Laminates can be stacked for various reasons ranging from mechanical, thermal, or electrical. One such example of an electrical reason for having a particular stacking sequence is the different electrical potentials of metals and composites. Carbon composites and aluminum have a large difference in electric potentials therefore carbon composites are susceptible to galvanic corrosion. Fiberglass has a closer electric potential to that of aluminum so fiberglass can be used as a barrier between the carbon composite material and the aluminum material. This is one reason why stacking sequence is being examined in this experiment.

The goal of the investigation is to experimentally and analytically examine the different composite orientations and stacking sequence on failure of a composite joint. Composite sandwich panels were constructed with laminates made from combining fiberglass and carbon-epoxy in different orientations and stacking sequences. The experimental analysis will include loading mechanically fastened joints in shear until failure occurs. The analytical analysis will be performed by modeling the plate with a finite element method. The finite element model will be used to determine failure loads and failure modes for each plate. The experimental and analytical methods will then be compared. From this examination of various panel structures for single lap joints a determination of the most effective structure can be made.

2. Manufacturing

2.1 Design

The two types of fiber reinforcement that were chosen for this experiment were fiberglass and carbon fiber. These are two materials that are in common usage in many aerospace applications. Most reinforcement fibers come from periods 2 and 3 of the periodic table. These fibers typically have good strength but are brittle. Fiber are combined together to form strands that are usually approximately 200 fibers. The strands can then be joined to form a cloth of fabric.

The cloth can be oriented in one direction or cross woven into a weave. The unidirectional carbon fiber cloth can be seen below in Figure . Unidirectional composites have high strength and stiffness in the fiber direction and relatively low strength and stiffness perpendicular to the fiber direction. The cloth is 0.0074 oz/in^2 .

Figure 3: Carbon Fiber Cloth

The fiberglass cloth that was used was a weave that can be seen in Figure 4 below. Cross ply weaves have the same stiffness in the transverse direction as in the longitudinal direction.

Figure 4: Fiberglass Cloth Weave

To protect the fibers and transfer load between them epoxy is used as the matrix in this composite. The epoxy consisted of Aeroepoxy P2032 resin and Aeroepoxy 3660 hardener. These can be seen in the following two figures.

Figure 5: Hardener

Figure 6: Epoxy Resin

The epoxy was mixed at a ratio of 3:1 of resin to hardener as specified by the manufacturer. This was done by weighing the resin and the hardener separately before mixing. Once the epoxy is sufficiently mixed together it is applied to the fibers. Care was taken to ensure that all of the fibers were covered in epoxy so as not to leave any dry spots or gaps in the coverage. These gaps or dry spots could lead to poor performance in the composite. The wet laminates were then laid on the nomex honeycomb core and placed onto the heating table. The vacuum bag was laid over the panel so that heat and pressure could be applied to the specimen.

The composites had heat applied for approximately 2.5 hours. The temperature was set to 130 °F. 80 pounds of weights were evenly dispersed over the top of the composite to increasing bonding. A schematic of the composite can be seen in Figure 7 below.

The heating table is located in the Cal Poly San Luis Obispo Aerospace Engineering Structures lab. The vacuum bag was first laid onto the table and held to the table by sealant tape. On top of the vacuum bag a nonporous material is laid that is larger than the size of the composite plate. This nonporous material is used to prevent the vacuum bag from sticking to the

composite. The composite is then laid in the center of the table and a piece of porous material is placed over it. The porous material is to allow the excess epoxy to pass through to be absorbed by the cloth but keep the cloth from sticking to the finished composite material. Another nonporous material layer is then laid on top of the cloth followed by the top of the vacuum bag. The vacuum bag is then sealed with gum tape. A vacuum is then used to decrease the air pressure around the composite. This helps the composite laminates bond to the honeycomb. Pressure is also applied to the composite in the form of 80 lbf of dead weight evenly distributed over the top of the composite.

The material coupons that were tested in the tensile tests required attaching aluminum tabs using the press seen below in Figure 7. These tabs are clamped by the tensile machine and are used to provide a more uniform load distribution to the composite. The tabs are attached to the composite using the same epoxy that is used in the composites.

Figure 7: Composite Press

2.2 Materials:

The construction of the composite panels was by a hand lay-up method. The manufacturing of the panels took place at the Cal Poly San Luis Obispo Aerospace Structures Lab. The appropriate size of the fiberglass and carbon cloth was cut. Cloth was cut in separate pieces for the top and bottom laminates. One cut of the laminate was slightly larger than the honeycomb planar area so as to be large enough to cover the thickness of the honeycomb. This was done to help protect the composite from moisture effects. The stacking sequence and the fiber orientation of each panel is seen below in Table 1.

Table 1: Plate Stacking Sequence and Orientation

	Material Stacking Sequence (outside, in)	Carbon Orientation	Fiberglass Weave Orientation
Panel 1	[Fiberglass, Carbon]s	[0,0]s	[0/90]s
Panel 2	[Fiberglass, Carbon]s	[-45,45]s	[45/-45]s
Panel 3	[Carbon, Fiberglass]s	[-45,45]s	[45/-45]s
Panel 4	[Carbon, Fiberglass]s	[0,0]s	[0/90]s
Panel 5	[Fiberglass, Carbon]s	[90,90]s	[0/90]s
Panel 6	[Carbon, Fiberglass]s	[90,90]s	[0/90]s

The fastener chosen was 3/8 in diameter plain steel shoulder bolt. This is a standard bolt size common in many industries. A shoulder screw was selected to apply a uniform load to the hole. The shoulder length was selected to be 1 in. which allowed it to slightly protrude through the hole. This ensured that the threaded section of the screw did not interface with composite laminate. Another way to avoid this is to use a threaded insert that is bonded into the laminate and transfers the load to the surrounding laminate. This was not done in this experiment so as to simplify the model as much as possible. The fastener was obtained from McMaster Carr. The dimensions of the bolt can be seen below in Figure 8.

Figure 8: Bolt Dimensions

The fastener that was chosen was made of steel with a shear strength of 84,000 lbs. This ensured that the composite would fail before the fastener did. A steel bolt was chosen because it is common on aerospace structures. Other higher performance bolts can be manufactured out of materials such as titanium although the cost is much higher for these fasteners. Zinc plated washers and nuts were obtained from a local hardware store. The bolt has hand tightened so as not to induce out of plane loading on the laminate.

The joint that was chosen is a single lap joint. A steel plate is attached to the composite through the bolt to allow the Instron machine a surface to clamp to. Care has to be taken when drilling the hole in the composite material. If the speed is too fast then the heat can cause the hole to have frayed edges which will reduce the bearing strength of the laminate. Another consideration is the clearance in the hole between the bolt and the laminate. If the clearance is too tight then the bolt might prestress the laminate due to insertion. This can induce out of plane

loading on the laminate causing a reduction in strength. An example of this type of joint can be seen in Figure 9 below.

Figure 9: Single Lap Joint

3. Experimental Set-up

3.1 Material Properties:

To create an analytical model to compare to the experimental results it is necessary to determine mechanical characteristics of the materials used. This includes finding the volume fiber fraction of all the laminae, the stiffness of all the laminae, and the strength of all the laminae.

3.1.1 Volume fiber fraction

The volume fiber fraction is the ratio of the volume of the fibers to the total volume of the composite material. This is an important parameter since one composite may have more fibers packed together than another composite which would cause the higher volume fiber fraction material to have increased stiffness and strength.

To determine the fiber volume fraction for each composite a burn test was conducted. A small piece of laminate is cut out after the mechanical test is performed. Typically the laminate is a 1 inch by 1 inch square. The laminate is weighed before the test. An example can be seen in Figure 10 below.

Figure 10: Composite Laminate Used in Burn Test

The composite is placed in a burner that can be seen in the following figure.

Figure 11: Burner

The burner is heated to approximately 800 degrees F for 30 minutes. This allows all the epoxy to burn off. The remaining fibers are then weighed. The fibers however are both carbon fiber and fiberglass. To determine the weight of each material, the total fiber was multiplied by the ratio of the thickness of the carbon fiber or fiberglass laminate respectively. Knowing the density of the carbon fiber, the fiberglass, and the epoxy, then the volume fraction can be determined by the following equations.

$$V_{f,c} = \frac{\frac{W_c}{\rho_c}}{\frac{W_c}{\rho_c} + \frac{W_e}{\rho_e}}$$

Equation 1

$$V_{f,FG} = \frac{\frac{W_{FG}}{\rho_{FG}}}{\frac{W_{FG}}{\rho_{FG}} + \frac{W_e}{\rho_e}}$$

Equation 2

The subscripts c, e, and FG denote carbon fiber, epoxy, and fiberglass respectively. The results of the burn tests can be seen in table X below.

Table 2: Fiber Volume Fraction

Plate number	Carbon Volume Fraction	Fiberglass Volume Fraction
1	0.67	0.75
2	0.58	0.67
3	0.66	0.62
4	0.58	0.67
5	0.52	0.62
6	0.75	0.82

3.1.2 Stiffness and Strength

To determine the stiffness and strength characteristics of the materials additional testing had to be conducted. Representative coupons were constructed out of the same materials and in the same construction process as the plates. For each material three coupons were constructed. The results of the three tests were then averaged together. These coupons were typically one inch wide by 8-10 inches long strips. These strips then had aluminum plates bonded to each end to allow the Instron machine area to grip the specimen. Each specimen was tested to failure. The highest load before failure was then taken to be the ultimate stress of that material.

The strength measured in these specimens is used as the strength of each individual ply in the FEA model. However in actuality the correct strength that should be used is the *in situ* strength, which is dependent on the neighboring ply orientations and the thickness of the plies (Chang, 1991). If a laminate does not have all plies in the same direction then the *in situ* strength of a laminate is generally greater than the strength of a laminate determined by a tensile test. These other plies of different orientations can act as lateral constraints to the adjacent plies. However, if plies are clustered in the same orientation then a crack in lamina can propagate further before being arrested by an adjacent ply therefore reducing the *in situ* strength. However,

to determine the *in situ* strength of a laminate requires extensive material testing of various orientations, and therefore will not be done in this study.

The stiffness and strength characteristics for each material can be seen in Table 3 and Table 4 below respectively. The stiffness and strength of the Fiberglass material was assumed to be the same in both the longitudinal and transverse directions since it is cross ply weave.

Table 3: Stiffness Characteristics

	E1 (psi)	E2 (psi)	E3 (psi)	v12	v13	v23	G12 (psi)	G13 (psi)	G23 (psi)
Carbon Fiber	14,840,000	1,592,750	1,592,750	0.45	0.45	0.45	9,471,849	9,471,849	5,846,800
Fiberglass	1,214,200	1,214,200	1,214,200	0.05	0.05	0.05	122,920	122,920	193,400

Table 4: Strength Characteristics

	X (psi)	Y (psi)	S (psi)
Carbon Fiber	211,000	942	1,022
Fiberglass	31,000	31,000	7,309

4. Mechanical Testing

4.1 Experimental Results

Plate 1 consisted of an outside lamina of Fiberglass weave oriented at 0/90 degrees and two inside laminae of carbon fiber oriented at 0 degrees. The load displacement graph is seen below in Figure 12.

Figure 12: Plate 1 Force-Displacement Response

Figure 12 shows a maximum load value of 491.3 lbf at a displacement of 0.0445 in. The non-linear region near the origin is due to the clamp up friction between the washer and the laminate. The graph suggests that the plate failed predominately in matrix tension/compression and fiber/matrix shearing because of the lack sudden drops in load associated with fiber breakage failures. The linear region of plate 1 is taken from 50 lbf to 450 lbf. This region showed a stiffness of 13,287 lbf/in.

Plate 2 consisted of an outside lamina of +/- 45 degrees fiberglass weave and two inside laminae of +/- 45 degrees carbon fiber. The load displacement chart for plate 2 can be seen below.

Figure 13: Plate 2 Force-Displacement Response

Figure 13 shows a maximum load for plate 2 to be 701 lbf. The maximum load corresponds to a displacement of 0.217 in. The 1st load drop value is 544 lbf with a displacement of 0.063 in. The stiffness for the linear region was found to be 12,470 lbf/in. This region was taken to be between loads 200 lbf and 400 lbf. This region showed a coefficient of determinant of 0.99 for a linear regression.

Plate 3 consisted of two outside laminae of ± 45 degree carbon fiber and one inside laminae of ± 45 degree fiberglass weave. The load displacement chart can be seen below in Figure 14.

Figure 14: Plate 3 Force-Displacement Response

Figure 14 shows a maximum load 993 lbf at a displacement of 0.57 in. The 1st drop load is 542 lbf at a displacement of 0.048 in. The linear stiffness was found to be 17,640 lbf/in for the region between 200 lbf and 400 lbf. This region showed a coefficient of determinant of 0.99 for a linear regression.

Plate 4 consisted of a two outside laminae of 0 degree carbon fiber and an inside lamina of 0/90 fiberglass weave. The load displacement chart for plate 4 can be seen in Figure 15 below.

Figure 15: Plate 4 Force-Displacement Response

Figure 15 shows a maximum load of 592 lbs at a displacement of 0.121 in. The 1st drop load was found to be 531 lbf at a displacement of 0.067 in. The stiffness of the linear region was found to be 10,765 lbf/in. This region was considered to be from 150 lbf to 400 lbf.

Plate 5 consisted of an outside lamina of 0/90 fiberglass weave and two inside laminae of 90 degree carbon fiber. The force displacement chart for plate 5 can be seen below in Figure 16.

Figure 16: Plate 5 Force-Displacement Response

Figure 16 shows a maximum load of 994 lbf at a displacement of 0.257 in. The 1st drop load was found to be 593 lbf at a displacement of 0.101 in. The stiffness for the undamaged region was 7,008 lbf/in with a coefficient of determinant of 0.99. The linear region was considered between 200 lbf and 500 lbf.

Plate 6 consisted of two outside laminae of 90 degree carbon fiber and an inside lamina of 0/90 degree fiberglass weave. The force displacement chart for plate 6 can be seen below in Figure 17.

Figure 17: Plate 6 Force-Displacement Response

Figure 17 above shows a maximum load of 895 lbf at a displacement of 0.250 in. The 1st drop load was found to be 714 lbf at a displacement of 0.139 in. The stiffness for the linear region was found to be 7531 lbf/in with a coefficient of determinant of 0.99. The linear region was taken to be from 100 lbf to 400 lbf.

4.2 Comparison of Experimental Results

A comparison of all the failure loads of all six plates can be seen in Table 5 below. The displacement value seen in Table 5 corresponds to the displacement at which the failure load occurred.

Table 5: Comparison of Failure Loads

Plate number	Outside Laminae	Inside Laminae	1st Failure Load (lbf)	Displacement (in)	Maximum Load (lbf)	Displacement (in)	Linear Stiffness (lbf/in)
1	Fiberglass [0/90]	Carbon [0,0]	491	0.045	522	0.090	13,287
2	Fiberglass [-45/45]	Carbon [-45/45]	544	0.064	702	0.217	12,470
3	Carbon [-45/45]	Fiberglass [-45/45]	542	0.048	993	0.570	17,640
4	Carbon [0,0]	Fiberglass [0/90]	531	0.068	592	0.121	10,765
5	Fiberglass [0/90]	Carbon [90,90]	593	0.102	994	0.248	7,008
6	Carbon [90,90]	Fiberglass [0/90]	714	0.139	895	0.250	7,531

As can be seen in Table 5 the highest load for 1st failure occurred in plate 6. However the plate that exhibited the highest maximum load was plate 5 followed closely by plate 3. The difference in maximum load between plates 5 and 3 was one pound, however the displacement was different between plates 5 and 3.

Plates 2 and 3 had the same fiber orientation of +/- 45 degrees but different stacking sequence. The stacking sequence for these plates is shown to have negligible effect on the 1st failure load. The plate with the carbon fiber on top was shown to have a higher maximum load than the plate with fiberglass on top.

Plates 1 and 4 both had carbon fiber oriented at 0 degrees. These configurations were shown to have the worst performance for both 1st failure and maximum load failure. In both cases plate 4, with carbon fiber on top, showed better performance than plate 1. The reason both plates 1 and 4 had less strength than the other plates was because the load had to be transferred purely by shear from the load bearing fibers that were within the diameter of the fastener to the neighboring fibers. This caused the loaded fibers to buckle and fail.

For the plates 1,2, and 5 the carbon fiber is the inside lamina. These plates show a trend that as the carbon fiber orientation angle increases from 0 degrees to 90 degrees then the stiffness decreases. This is expected since the fibers are no longer in the direction of the load. However, for both the first failure load and the maximum failure load the opposite trend is seen. That is as the orientation angle increases the strength also increases.

5. Analytical Method

5.1 Finite Element Model

The finite element model was constructed using Abaqus/CAE version 6.7-3. The laminates were constructed as three dimensional solids while the honeycomb core was represented as a two dimensional shell. This model therefore assumes the thickness effects of the honeycomb are negligible.

For the solid regions C3D8R 8-node linear brick with reduced integration elements and hourglass control were used. Full integration and hybrid elements were also compared but were found to have negligible effects on the results. To model the two dimensional regions of the honeycomb and the epoxy, S4R 4-node doubly curved thin shell elements with reduced integration, hourglass control, and finite membrane strains were used.

To model the honeycomb region the hexagonal shape was approximated with a similar sized square region. This was done to simplify the model and reduce computation time. This change will have minimal effect on the model since only a small amount of the load is transferred through the honeycomb.

To model the interface between the laminate and the honeycomb a thin region of epoxy was modeled. This region makes up the last 0.1 in of the honeycomb region on both laminate-honeycomb interfaces.

Only one quadrant of the plate is analyzed to take advantage of symmetry which reduces computational time. The X-Y plane of symmetry is cut through the middle of the honeycomb laminate and the boundary condition is applied to all of the nodes at this mid plane. In this boundary condition the displacement in the Z direction is set to zero. The displacement in the Y direction was set to zero for the laminate along the X-Z plane. This constrains the model in the Y

direction to prevent rigid body movement of the model. The displacement in the X direction was set to zero along the Y-Z plane. The displacement was also constrained on the top laminate to be zero in the Z direction to prevent any out of plane motion. This can be seen in Figure 18 below.

Figure 18: Finite Element Model

The model was divided into four regions. As seen in the figure below, the red region is the honeycomb, the green region is the epoxy, the white region is the carbon fiber, and the blue region is the fiberglass region. The epoxy region was created to simulate the non-perfect bonding between the honeycomb and the laminate. This region is a shell that is an extension of the honeycomb region. The nodes along the epoxy and the laminate interface are tied together. For plates 3, 4, and 6 the fiberglass and carbon fiber regions are reversed. A diagram of the regions of the model can be seen below in Figure 19.

Figure 19: Model Regions

The honeycomb and the epoxy shells were tied together at the points indicated in Figure 20 below. This means that the displacements at those points will be the same for the honeycomb nodes and the epoxy nodes.

Figure 20: Tie Constraints

The load was applied through a surface traction to only the laminate in the Y direction. The load being carried by the honeycomb was neglected because the stiffness of the honeycomb

is considerably less than the stiffness of the laminate. The load applied to the model is only half the total load since there is symmetry about the Y-Z plane. The load is scaled by a cosine function to decrease as it increases in the X direction as shown in Figure 21 below. The X direction is perpendicular to the direction of the load and in the plane of the laminate. The load distribution is constant through the thickness of the laminate.

Figure 21: Applied Load

The following load distribution along the X direction is used.

$$f(x) = A * \cos\left(\frac{\pi x}{2 * r}\right)$$

Equation 3

Where r is the radius of the circle which is 0.1875 in. and A is the magnitude of the load at x=0. The load goes to zero as x=r. To calculate the magnitude of the load it is necessary to perform a surface integral over the region the load is applied. This is done using the following equation:

$$A = \int_0^t \int_0^r \cos\left(\frac{\pi x}{2 * r}\right) \sqrt{\left(\frac{\partial y}{\partial x}\right)^2 + \left(\frac{\partial y}{\partial z}\right)^2} + 1 dx dz$$

Equation 4

This requires the surface on which the load is applied to be parametrized as

$$y = \sqrt{r^2 - x^2}$$

Equation 5

The coefficient A is a function of geometry and is the same for all the plates at a value of 0.00556.

The mesh was created by using 8 node brick elements for the majority of the laminate. Wedge elements were used in some areas near the hole boundary due to the curvature and the interface of the honeycomb with the laminate. Wedge elements were also used to transition from the region of a fine mesh near the hole to the region of the more coarse mesh farther away from the hole. The mesh transition point was chosen to be just over one radius distance from the edge of the hole. More elements were clustered in the direction of the load due to the higher deflections in this region. The mesh was determined to be sufficient after a convergence study. The mesh can be seen below in Figure 22.

Figure 22: Planar Distribution of Mesh

The mesh is clustered in the laminates more than the honeycomb and epoxy regions because the majority of the load is carried in this region. The distribution of the mesh through the thickness of the plate can be seen in Figure 23 below.

Figure 23: Mesh of Thickness

To simulate the damage onset and the damage progression the user subroutine USDFLD is used in conjunction with model Abaqus. The user routine is called for all material points of the fiberglass and carbon fiber laminates at each increment. This routine calculates when damage begins at each material point and then lowers the respective stiffness of the material to further simulate damage progression. The three failure criteria that are used in this subroutine are matrix tension/compression failure, fiber-matrix shearing failure, and fiber breakage. Since only in plane loading is applied, no out of plane failure criteria will be used. There is a possibility that delamination could have occurred in the test however it is beyond the scope of this model. Thus the FEA model is likely to be too stiff if significant delamination is occurring.

The matrix tension/compression failure criterion is met when the value of e_m equals one in the equation below.

$$e_m = \sqrt{\left(\frac{S_{22}}{Y}\right)^2 + \left(\frac{S_{12}}{S}\right)^2}$$

Equation 6

S_{22} refers to the normal stress component in the transverse direction with respect to the direction of the fibers. The S_{12} refers to the shear stress in the plane of the laminate. The Y and S variables are the transverse and shear strengths respectively.

Once the matrix tension/compression criterion is met the transverse modulus E_2 and the poisson's ratio ν_{12} and ν_{13} are reduced. The value of the poisson's ratios are reduced to zero while the value of the transverse modulus is reduced to a number that is approximately 1 % of its original value. This is done to keep the elements from distorting.

The fiber-matrix shearing failure criterion is met when the value of e_f equals one in the equation below.

$$e_f = \sqrt{\left(\frac{S_{11}}{X}\right)^2 + \left(\frac{S_{12}}{S}\right)^2}$$

Equation 7

S_{11} is the normal stress component in the direction of the fibers.

Once the fiber-matrix shearing failure criterion is met the values of the G_{12} , G_{13} , ν_{12} , and ν_{13} are reduced in the same manner as the case of the matrix tension/compression failure.

The fiber breakage failure criterion is met when the value of e_b equals one in the equation below.

Once the fiber breakage failure criterion is met the values of all the stiffness properties are reduced in the same manner as the case of the matrix tension/compression failure. The fiber breakage criterion can be seen in the equation below.

$$e_b = \frac{S_{11}}{X}$$

Equation 8

To get proper results when using the user subroutine USDFLD it is necessary to use fixed time incrementation. However a small time increment must be used to ensure that the solution converges. A time increment of 0.01 was found to be sufficient for this analysis.

5.2 Finite Element Results

To compare the FEA results with the experimental data an average displacement of the indicated nodes in Figure 24 below were used. In some plates the top node is used instead of the average and will be noted when this is the case. The displacement is only measured in the direction of the load. These nodes correspond to elements that have the highest load applied to them.

Figure 24: Node Region Used in Global Response

The finite element results for all six plates are shown in Figure 26. Plates 1 and 4, and plates 2 and 3 showed similar results for both stiffness and strength. This indicates that the stacking sequence had a negligible effect on these orientations. However, plate 5 showed higher stiffness and strength than plate 6. The strength difference may be because in plate 5 the honeycomb is adding to the stiffness of the failed carbon fiber laminae. However, in plate 6 the honeycomb is bonded to the fiberglass which allows the failed carbon fiber laminae to deform further. This phenomena is most noticeable in plates 5 and 6 because these plates have the carbon fiber oriented at 90 degrees and show the matrix tension/compression failure sooner than the other plates.

The +/- 45 degree orientations showed much higher strength than the other orientations. This is likely because plates 2 and 3 allow for better transfer of the load around damaged regions.

Figure 25: FEA Results

6. FEA-Experimental Comparison

6.1 Plate 1

For plate 1 the global response can be seen below in Figure 26. The FEA shows increased stiffness at lower loads indicating that the material properties are likely too high. However the FEA also shows that failure begins at a much lower level than the experimental data. This likely means that the FEA strength values are too low. These strength values were varied and the carbon fiber shear strength was shown to have significant effect on the overall strength of the plate. The shear strength was increased from a value of 1,022 psi to 15,500 psi. The 15,500 is a published value of shear strength for carbon fiber. The failure load of the plate is highly dependent on the shear strength since both the matrix tension/compression failure and the fiber/matrix shear failure have this parameter in the criterion.

Figure 26: Plate 1 Global Response

The discrepancy in the two different shear strengths can be due to multiple effects. First, there is likely a difference in the method of obtaining the shear strength value. Shear strength cannot be measured directly from a coupon test like longitudinal and transverse strength can. The original value of 1,022 psi was obtained by using the Tsai-Hill equation seen below.

$$\frac{\cos^4(\theta)}{X^2} + \left[\frac{1}{S^2} - \frac{1}{X^2} \right] \cos^2(\theta) \sin^2(\theta) + \frac{\sin^4(\theta)}{Y^2} = \frac{1}{\sigma_x^2}$$

Equation 9

The θ value for this case is 45 degrees so as induce shear on the fibers on the test specimen. The X and Y values were determined in other specimens at orientations of 0 degrees and 90 degrees respectively. The σ_x value is the normal stress at failed cross section of the

specimen. The Tsai-Hill criterion is based off of experimental data for E-glass-epoxy composites. The criterion therefore may not be as suitable for carbon fiber composites and may contribute to some of the discrepancy between the two shear strength values. Determining the shear strength this way also makes the value of the shear strength dependent on the accuracy of the measured longitudinal and transverse strengths.

Secondly, the discrepancy is likely also due to the *in situ* strength which is dependent on the number of adjacent plies in the same orientation. This dependency can be seen in the equations below.

$$Y_t = Y_t^0 \left(1 + A \frac{\sin(\Delta\theta)}{N^B} \right)$$

Equation 10

$$S_c = S_c^0 \left(1 + C \frac{\sin(\Delta\theta)}{N^D} \right)$$

Equation 11

In the equations above the terms S_c and Y_t correspond to the *in situ* shear and transverse strengths of a laminate respectively. The terms S_c^0 and Y_t^0 correspond to the shear strength and transverse strength respectively as measure by typical test specimens. The terms A, B, C, and D must be determined experimentally.

For plate 1 there is considerable failures at the load of 200 lbf. Figure 27 below shows the elements that have failed in matrix tension/compression. The failures are concentrated around an angle of 45 degrees in the laminate plane where positive is counterclockwise. The majority if the FV1 failure is in the carbon fiber laminas. This is most likely due to the fact that the transverse strength of the unidirectional laminate is much lower than the transverse strength of the

fiberglass lamina. This is due to the fiberglass lamina being a cross weave so the strength in the 1 direction is the same as the strength in the 2 direction. This failure will decrease the stiffness of the plate but laminate is still capable of carrying increased load since this failure primarily affects the stiffness perpendicular to the direction of the load.

Figure 27: Plate 1 FV1 at Load 200 lbf

The fiber-matrix shear failure can be seen in Figure 28 below. Like the matrix tension/compression case, the failure is concentrated near 45 degrees and extends outward in the direction of the load. However there are not any failures in the hole at an angle of 90 degrees like there is for the matrix tension/compression failure.

Figure 28: Plate 1 FV2 at Load 200 lbf

There was no fiber breakage in the model at the load of 200 lbf. The normal stress S11 which is in the direction of the load is seen in Figure 29 below. The S11 levels are relatively low at this load. The S11 values are lower than the longitudinal failures of both carbon fiber laminas and the fiberglass lamina. Near an angle of 0 degrees the load is compressive which is expected since this is where most of the bearing damage occurs.

Figure 29: Plate 1 S11 at Load 200 lbf

The normal stress perpendicular to the load can be seen in Figure 30 below. The S22 are typically less than the fiberglass transverse failure load (31,000 psi) but are at some points higher than the carbon fiber transverse failure load (942 psi).

Figure 30: Plate 1 S22 at Load 200 lbf

The shear stress S12 is seen in Figure 31 below. The shear stress is concentrated between 45 and 90 degrees in the hole for the carbon fiber. This shear build up can contribute to the FV1 failure that is seen in the FV1 plot. However it is not enough to cause the FV2 in this region because the S11 is too low at this load.

Figure 31: Plate 1 S12 at Load 200 lbf

The displacement in the direction of the load can be seen in Figure 32 below. The displacement in the carbon fiber laminae is more than the fiberglass lamina near 0 degrees because of the FV1 and FV2 failures. The FV2 failure probably contributes more to the decreased stiffness since this keeps the load from transferring to the adjacent elements through shear.

Figure 32: Plate 1 U2 at Load 200 lbf

The displacement perpendicular to the direction of the load can be seen in Figure 33 below. As expected large U1 displacements can be seen near the regions that showed FV1 failures.

Figure 33: Plate 1 U_1 at Load 200 lbf

The next load that is examined for plate 1 is at 300 lbf. The FV1 failure can be seen below in Figure 34. The FV1 propagated further in the direction of the load from the 200 lbf point to the 300 lbf point.

Figure 34: Plate 1 FV1 at Load 300 lbf

The FV2 failure plot can be seen in Figure 35 below. The FV2 shear failure follows a similar trend between 200 lbf and 300 lbf as to the FV1 failure. The FV2 propagated further in the direction of the load.

Figure 35: Plate 1 FV2 at Load 300 lbf

Figure 36 below shows the fiber breakage failure, FV3. There are only a few elements that show failure in the region that also includes FV1 and FV2 failures.

Figure 36: Plate 1 FV3 at Load 300 lbf

The stress in the direction of the load can be seen in Figure 37 below. The progression between 200 lbf and 300 lbf showed increase in the size of the tension region that is located in the carbon fiber laminas near an angle of 90 degrees. The magnitude of the compressive region near an angle of 0 degrees has increased from the 200 lbf to the 300 lbf points.

Figure 37: Plate 1 S_{11} at Load 300 lbf

The normal stress perpendicular to the direction of the load are seen below in Figure 38. The stresses do not appear to change in magnitude much from the 200 lbf to the 300 lbf cases.

Figure 38: Plate 1 S22 at Load 300 lbf

The in-plane shear stress can be seen in Figure 39 below. The size of the regions containing significant shear stress does not appear to have increased a noticeable amount between the 200 lbf and 300 lbf case.

Figure 39: Plate 1 S12 at Load 300 lbf

The displacement in the direction of the load is seen in Figure 40 below. The distribution of the displacements has not significantly changed from the 200 lbf case to the 300 lbf case. However the magnitude of the displacement has changed. In the regions near 0 degrees the magnitude appears to double in size from the 200 lbf case.

Figure 40: Plate 1 U2 at Load 300 lbf

The displacement perpendicular to the load can be seen in Figure 41 below. The magnitude of the displacement has increased from the 200 lbf case to the 300 lbf case. However the distribution of the displacement does not appear to have changed significantly.

Figure 41: Plate 1 U1 at Load 300 lbf:

The load at 400 lbf showed that the stiffness was greatly decreased and the load was approaching the maximum strength of the plate. The FV1 failure can be seen in Figure 42 below. The FV1 region appears to have increased in the X direction in the carbon fiber laminas.

Figure 42: Plate 1 FV1 at Load 400 lbf

The FV2 failure can be seen in Figure 43 below for the 400 load case. The number of failed elements in the fiberglass laminate near 0 degrees have increased significantly from between the 300 lbf and the 400 lbf cases. This indicates that the plate is near its failure load since it can no longer transfer the load near the 0 degree region to the other regions through shear.

Figure 43: Plate 1 FV2 at Load 400 lbf

The FV3 failures can be seen in Figure 44 below for the 400 lbf case. The regions of FV3 failure appear to have increased in size slightly from the 300 lbf case to the 400 lbf case.

Figure 44: Plate 1 FV3 at Load 400 lbf

The normal stress in the direction of the load is shown in Figure 45 below. The stress region near 90 degrees in the carbon fiber laminas appears to have increased in the X direction between the 300 lbf case and the 400 lbf case.

Figure 45: Plate 1 S11 at Load 400 lbf

The normal stress S22 in the direction perpendicular to the direction of the load can be seen in Figure 46 below. The distribution of the stress does not appear to have significantly changed from the 300 lbf case to the 400 lbf case. However the magnitude of the S22 stress increased in the regions that showed stress concentrations.

Figure 46: Plate 1 S22 at Load 400 lbf

The in-plane shear stress can be seen in Figure 47 below. The magnitude of the S12 stress appears to have increased from the 300 lbf case however the distribution has not significantly changed.

Figure 47: Plate 1 S12 at Load 400 lbf

The displacement in the direction of the load can be seen in Figure 48 below for the 400 lbf case. Similar to the 300 lbf case, the majority of the displacement near 0 degrees is in the carbon fiber laminas.

Figure 48: Plate 1 U2 at Load 400 lbf

The displacement perpendicular to the direction of the load can be seen in Figure 49 below for the 400 lbf case. The distribution of the displacements appears to be similar between the 300 lbf case and the 400 lbf cases, however the magnitudes have increased.

Figure 49: Plate 1 U1 at Load 400 lbf

6.2 Plate 2

The global response of plate 2 can be seen below in Figure 50. The finite element solution shows a failure at a low stress like the other plates, however, plate 2 does decrease its stiffness as dramatically as the other plates. This is most likely due to plate 2 having the carbon fiber laminas in opposite directions. This allows the failed laminas to transfer the load to the adjacent plies of different orientation. The finite element model was also run with an increased shear strength value. This case shows that the linear stiffness parameters are again higher than the experimental data. The stiffness of the linearized experimental data is 12,470 lbf/in while the increased shear strength FEA case had a stiffness of 212,585 lbf/in.

Figure 50: Plate 2 Global Response

The matrix tension/compression failure can be seen below in Figure 51 for plate 2. The failure is shows that almost the entire carbon fiber lamina has failed.

Figure 51: Plate 2 FV1 at Load 240 lbf

The fiber/matrix shear failure can be seen below for 240 lbf. The failure is mostly clustered near the hole region. Most of the failures are in the carbon fiber laminas. There are also failures showing in the fiberglass lamina around 60 degrees in the hole.

Figure 52: Plate 2 FV2 at Load 240 lbf

There is no fiber tension/compression failure at 240 lbf. The displacement in the direction of the load can be seen below in Figure 53 for 240 lbf.

Figure 53: Plate 2 FV3 at Load 240 lbf

The displacement perpendicular to the direction of the load can be seen below in Figure 54 for 240 lbf.

Figure 54: Plate 2 U_2 at Load 240 lbf

The matrix tension/compression failure at 270 lbf can be seen in Figure 55 below. The failures have propagated further away from the hole in the carbon fiber laminate. The failures do not appear to have propagated into the fiberglass lamina.

Figure 55: Plate 2 FV1 at Load 270 lbf

The fiber/matrix shear failure for 270 lbf can be seen below in Figure 56. The failures seem to have propagated along the hole boundary in the fiber direction which is 45 degrees.

Figure 56: Plate 2 FV2 at Load 270 lbf

The fiber tension/compression failure can be seen below in Figure 57. The failure is near the region that has already failed in matrix tension and shear.

Figure 57: Plate 2 FV3 at Load 270 lbf

The displacement in the direction of the load can be seen below in Figure 58. The distribution has not changed significantly but the magnitude has changed.

Figure 58: Plate 2 U2 at Load 270 lbf

The displacement perpendicular to the direction of the load can be seen below in Figure 59. The displacement changes from positive to negative at approximately 45 degrees in the hole.

Figure 59: Plate 2 U1 at Load 270 lbf

The matrix tension/compression failure can be seen below in Figure 60. The failure has propagated away from the hole mostly in the carbon fiber laminate.

Figure 60: Plate 2 FV1 at Load 300 lbf

The fiber/matrix shear failure for 300 lbf can be seen below in Figure 61. The failure has propagated further in the fiber direction for the fiberglass lamina.

Figure 61: Plate 2 FV2 at Load 300 lbf

The fiber tension/compression failure can be seen below in Figure 62. The failure appears to have increased near the 45 degree region.

Figure 62: Plate 2 FV3 at Load 300 lbf

The displacement in the direction of the load for 300 lbf can be seen in Figure 63 below. The distribution appears to have changed so that more of the displacement is around the 0 degree direction.

Figure 63: Plate 2 U2 at Load 300 lbf

The displacement perpendicular to the direction of the load can be seen below in Figure 64. There is a large displacement in the same region as the U2 distribution.

Figure 64: Plate 2 U_1 at Load 300 lbf

6.3 Plate 3

The global response for plate 3 can be seen in Figure 65 below. Plate 3 shows a 1st failure at a low load value however the plate retains a large amount of stiffness. The stiffness after the failure is shown to be almost linear. The catastrophic failure for this plate is most likely at a much higher load.

Figure 65: Plate 3 Global Response

The matrix tension/compression failure for plate 3 at 400 lbf can be seen below in Figure 66. The failure is mostly concentrated in the carbon fiber laminas. There are failures in the fiberglass lamina in the hole at approximately 45 degrees and at 90 degrees further from the hole area.

Figure 66: Plate 3FV1 at Load 400 lbf

The fiber/matrix failure for plate 3 at 400 lbf can be seen below in *Figure 67*. Almost all of the carbon fiber lamina and most of the fiberglass lamina near the hole have failed.

Figure 67: Plate 3 FV2 at Load 400 lbf

The fiber tension/compression failure at 400 lbf can be seen below in *Figure 68*. The failures are mostly in the fiberglass lamina and seem to correspond to similar regions to that of the matrix tension/compression failure.

Figure 68: Plate 3 FV3 at Load 400 lbf

The displacement in the direction of the load can be seen in *Figure 69* below. Most of the displacement is concentrated around 0 degrees in the hole.

Figure 69: Plate 3 U2 at Load 400 lbf

The displacement perpendicular to the direction of the load can be seen below in *Figure 70*. The majority of the displacement is positive, indicating that the hole is deforming away from the Y-Z plane.

Figure 70: Plate 3 U1 at Load 400 lbf

The matrix tension/compression failure for 480 lbf can be seen below in *Figure 71*. The clusters of failures in the fiberglass lamina seem to have increased from the 400 lbf case.

Figure 71: Plate 3 FV1 at Load 480 lbf

The fiber/matrix shear failure at 480 lbf can be seen below in *Figure 72*. The failures seem to have propagated in the direction of the load.

Figure 72: Plate 3 FV2 at Load 480 lbf

The fiber tension compression failure can be seen below in *Figure 73*. The failures remain mostly in the fiberglass lamina which has a smaller tension strength than the carbon fiber laminas.

Figure 73: Plate 3 FV3 at Load 480 lbf

The displacement in the direction of the load can be seen below in *Figure 74*. The distribution of the displacement is similar to that of the 400 lbf case, however, the magnitude has increased.

Figure 74: Plate 3 U2 at 480 lbf

The displacement perpendicular to the load at 480 lbf can be seen in *Figure 75* below.

The distribution of the displacement is similar to that of the 400 lbf case.

Figure 75: Plate 3 U1 at 480 lbf

The matrix tension/compression failure can be seen in *Figure 76* below. The size of the clustered failures seems to have increased from the 480 lbf case.

Figure 76: Plate 3 FV1 at 560 lbf

The fiber/matrix shear failure can be seen in *Figure 77* below. Almost the whole region near the hole has failed under shear.

Figure 77: Plate 3 FV2 at 560 lbf

The fiber tension/compression failure can be seen in *Figure 78*. The failure appears to have propagated into the carbon fiber laminas near 45 degrees in the hole.

Figure 78: Plate 3 FV3 at 560 lbf

The displacement in the direction of the load can be seen in *Figure 79* below. The distribution appears to be similar to the 480 lbf case.

Figure 79: Plate 3 U2 at Load 560

The displacement perpendicular to the load at 560 lbf can be seen in *Figure 80* below. The distribution of the displacement is similar to the previous cases.

Figure 80: Plate 3 U1 at Load 560

6.4 Plate 4

The global response of plate 4 can be seen in *Figure 81* below. The FEA is shown to have failed at a relatively low value as compared to the experimental. This is attributed to the shear strength again being too low in the finite element model. The FEA with the shear strength of 15,500 psi is shown to have comparable strength to that of the experimental data. The stiffness however is too high in the linear region for the FEA indicating that the elastic properties are likely too high.

Figure 81: Plate 4 Global Response

The matrix tension/compression failure for plate 4 at 120 lbf can be seen below in *Figure 82*. All of the failures are confined to the carbon fiber laminas. This most likely due to the fact that the fiberglass has a much higher transverse strength as compared to the carbon fiber.

Figure 82: Plate 4 FV1 at Load 120 lbf

The fiber/matrix shearing failure at 120 lbf can be seen below in *Figure 83*. The failure seems to be clustered around 30 degrees and extends in the direction of the load.

Figure 83: Plate 4 FV2 at Load 120 lbf

There is no fiber tension failure at a load of 120 lbf. The displacement in the direction of the load for 120 lbf can be seen below in *Figure 84*. The plate shows signs of shearout failure as seen by the large displacement in the load direction at an angle of 30 to 45 degrees.

Figure 84: Plate 4 U2 at Load 120 lbf

The displacement perpendicular to the load direction can be seen in *Figure 85* below for a load of 120 lbf. The displacement is negative near 90 degrees in the hole is negative. This means that the displacement is inwards towards where the hole contacts the pin. This is incorrect in a physical sense because the pin acts as a boundary condition limiting the displacement. However, the displacements in this direction are small relative to the loaded direction.

Figure 85: Plate 4 U1 at Load 120 lbf

The matrix tension/compression failure at 180 lbf can be seen in *Figure 86* below. The failure is shown to have propagated into the fiberglass lamina around 45 degrees in the hole. The failures in the carbon fiber lamina have propagated further in the direction perpendicular to the load.

Figure 86: Plate 4 FV1 at Load 180 lbf

The fiber/matrix shear failure can be seen below in *Figure 87*. The failures in the carbon fiber have propagated closer to 0 degrees in the hole and extending outwards in the direction of the load. Shear failure is now shown in the fiberglass lamina at approximately 45 degrees.

Figure 87 : Plate 4 FV2 at Load 180 lbf

There is no fiber tension failure at a load of 180 lbf. The displacement in the direction of the load for plate 4 at 180 lbf can be seen below in *Figure 88*. The distribution of the displacements has not significantly changed however the magnitude of the displacements has.

Figure 88: Plate 4 U2 at Load 180 lbf

The displacement in the direction perpendicular to the load can be seen in Figure 89 below for 180 lbf. The magnitude of the displacements has increased but the distribution has not significantly increased.

Figure 89: Plate 4 U1 at Load 180 lbf

The matrix tension/compression failure can be seen in *Figure 90* below for 240 lbf. The failure in the 45 degree region has increased in the fiberglass lamina. Most significant however is the clustered failures far from the hole. These failures appear to have propagated from the free edge of the plate.

Figure 90: Plate 4 FV1 at Load 240 lbf

The fiber/matrix shear failure at 240 lbf can be seen below in *Figure 91*. Similar to the matrix tension/compression failure for 240 lbf, the shear failure has clustered far from the hole region. Since the matrix tension/compression cluster is larger it is likely that the matrix failure initiated in this region first.

Figure 91: Plate 4 FV2 at Load 240 lbf

There is no fiber tension failure at a load of 240 lbf. The displacement in the direction of the load can be seen in *Figure 92*. The magnitude of the displacement has greatly increased particularly in the region near zero degrees.

Figure 92: Plate 4 U2 at Load 240 lbf

The displacement perpendicular to the direction of the load can be seen below in *Figure 93*. The displacement around 90 degrees in the hole is mostly negative while it transitions to approximately zero near zero degrees in the hole.

Figure 93: Plate 4 U1 at Load 240 lbf

6.5 Plate 5

For plate 5 the points that will be examined are at 300, 400, and 500 lbf. These points correspond to when the plate is shown to have approached its maximum load value. The global

response of plate 5 can be seen below in *Figure 94*. As with the previously mentioned plates the first failure happens far sooner in the finite element model than it does in the experimental. This is likely due to the effect of in situ strength in the carbon fiber laminas. However, because the carbon fiber laminates are oriented at 90 degrees it is likely that the transverse strength has a much greater effect on the overall strength of the laminate as opposed to the shear strength like in plates 1 and 4 which were oriented at zero degrees.

Figure 94: Plate 5 Global Response

Figure 95 below shows the matrix tension/compression failure at a load of 300 lbf for plate 5. The plate already shows considerable failure in the carbon fiber lamina and so clustered around 45 degrees for the fiberglass lamina.

Figure 95: Plate 5 FV1 at Load 300 lbf

The fiber/matrix shear failure can be seen for 300 lbf below in *Figure 96*. Most of the damage is clustered in the direction of the load indicating bearing damage. There is more damage in the carbon fiber lamina than in the fiberglass lamina.

Figure 96: Plate 5 FV2 at Load 300 lbf

Figure 97 below shows the fiber tension/compression failure of plate 5 under a 300 lbf load. Failures are clustered around 90 degrees and 0 degrees. The 90 degree failure is likely due to a tension failure while the 0 degree failure is likely due to a compression failure.

Figure 97: Plate 5 FV3 at Load 300 lbf

The displacement in the direction of the load for 300 lbf can be seen below in *Figure 98*. There is a large displacement in carbon fiber laminae around the 0 degree region. At 300 lbf this region shows to have both matrix tension/compression failure and fiber/matrix shear failures.

Figure 98: Plate 5 U2 at Load 300 lbf

The displacement perpendicular to the direction of the load for 300 lbf can be seen below in *Figure 99*. The highest displacement for the laminate is near the 0 degree region in the hole.

Figure 99: Plate 5 U1 at Load 300 lbf

The matrix tension/compression failure at a load of 400 lbf can be seen in *Figure 100* below. The failure has propagated in the direction of the load in the fiberglass lamina from the state of the 300 lbf case to the 400 lbf case.

Figure 100: Plate 5 FV1 at Load 400 lbf

The fiber/matrix shear failure can be seen in *Figure 101* below. Like the matrix tension/compression case the fiber/matrix shear failure has propagated in the direction of the load in both the carbon fiber and the fiberglass laminae.

Figure 101: Plate 5 FV2 at Load 400 lbf

The fiber tension/compression failure can be seen in Figure X below. The size of the clustered failures at both 90 degrees and approximately 30 degrees has increased from the 300 lbf case to the 400 lbf case.

Figure 102: Plate 5 FV3 at Load 400 lbf

The displacement in the direction of the load can be seen below in *Figure 103*. The majority of the displacement is in the carbon fiber laminate in the region that has both matrix tension/compression failure and fiber/matrix shear failure.

Figure 103: Plate 5 U2 at Load 400 lbf

The displacement perpendicular to the direction of the load for 400 lbf can be seen below in *Figure 104*. Most of the displacement around the hole appears to be negative which indicated that it is displacing towards the hole.

Figure 104: Plate 5 U_1 at Load 400 lbf

The matrix tension/compression failure at 500 lbf can be seen below in *Figure 105*. The failure appears to have propagated in the directions perpendicular to the load in the fiberglass lamina. The carbon fiber lamina appears to be almost completely failed for the matrix tension/compression criterion.

Figure 105: Plate 5 FV1 at Load 500 lbf

The fiber/matrix shear failure can be seen below in Figure 106 for a load of 500 lbf. The failure propagation from 400 lbf to 500 lbf does not appear to have significantly increased for the fiber/matrix shear failure criterion.

Figure 106: Plate 5 FV2 at Load 500 lbf

The fiber tension/compression failure for 500 lbf can be seen below in Figure 107. From the 400 lbf case to the 500 lbf case failures have appeared in the direction of the load near the free edge.

Figure 107: Plate 5 FV3 at Load 500 lbf

The displacement in the direction of the load for 500 lbf can be seen below in Figure 108. As with the 400 lbf case, the majority of the displacement is in the carbon fiber lamina in the region of zero degrees.

Figure 108: Plate 5 U2 at Load 500 lbf

The displacement perpendicular to the direction of the load can be seen in Figure 109 below. The distribution has not significantly changed from the 400 lbf case to the 500 lbf case.

Figure 109: Plate 5 U1 at Load 500 lbf

6.6 Plate 6

For plate 6 the points that will be examined are at 210, 280, and 350 lbf. These points are in the damaged region of the analysis. The global response for plate 6 can be seen below in Figure 110.

Figure 110: Plate 6 Global Response

The matrix tension/compression failure at 210 lbf can be seen below in Figure 111. A large portion of the carbon fiber laminate is shown to have failed at this load.

Figure 111: Plate 6 FV1 at Load 210 lbf

The fiber/matrix shear failure is seen below in Figure 112. The majority of the failures are concentrated near the zero degree region in the hole.

Figure 112: Plate 6 FV2 at Load 210 lbf

There was no fiber tension/compression failure at the load of 210 lbf. The displacement in the direction of the load for 210 lbf can be seen in Figure 113 below. The majority of the displacement is in the zero degree region and in the carbon fiber lamina.

Figure 113: Plate 6 U2 at Load 210 lbf

The displacement perpendicular to the direction of the load can be seen below in Figure 114. Majority of the displacement is in the direction towards the hole. However there is a region at approximately 15 to 30 degrees that shows a displacement away from the hole.

Figure 114: Plate 6 U1 at Load 210 lbf

The matrix tension/compression failure for 280 lbf can be seen below in Figure 115. The failure is mostly concentrated in the carbon fiber laminas but appears to be propagating into the fiberglass lamina.

Figure 115: Plate 6 FV1 at Load 280 lbf

The fiber/matrix shear failure for 280 lbf can be seen below in Figure 116. The failures appear to be propagating perpendicular to the direction of the load in the carbon fiber laminas. In the fiberglass lamina the failure appears to be propagating in the direction of the load.

Figure 116: Plate 6 FV2 at Load 280 lbf

The fiber tension/compression failure can be seen in Figure 117 below. Most of the failure is concentrated around the region of zero degrees.

Figure 117: Plate 6 FV3 at Load 280 lbf

The displacement in the direction of the load for 280 lbf can be seen in Figure 118 below. Like the 210 lbf case, most of the displacement is concentrated around the zero degree region in the carbon fiber laminas.

Figure 118: Plate 6 U_2 at Load 280 lbf

The displacement perpendicular to the direction of the load for 280 lbf can be seen below in Figure 119. The displacement around the circumference of the hole is all in the direction towards the hole.

Figure 119: Plate 6 U1 at Load 280 lbf

The matrix tension/compression failure for 350 lbf can be seen in Figure 120 below. Almost all of the carbon fiber laminas are shown to have failed in matrix tension/compression. The region of failures in the fiberglass lamina near the hole is shown to have increased slightly.

Figure 120: Plate 6 FV1 at Load 350 lbf

The fiber/matrix shear failure can be seen in Figure 121 for 350 lbf. The failures in the fiberglass lamina have propagated in the direction of the load. This may indicate that the failed regions of the carbon fiber laminas have caused the stresses to redistribute into the fiberglass lamina and thus caused them to fail.

Figure 121: Plate 6 FV2 at Load 350 lbf

The fiber tension/compression failure for 350 lbf can be seen below in Figure 122. The failure appears to have propagated in the direction of the load.

Figure 122: Plate 6 FV3 at Load 350 lbf

The displacement in the direction of the load can be seen in Figure 123 below for 350 lbf. The displacement is high in the carbon fiber laminas, particularly as you approach the top of the laminate.

Figure 123: Plate 6 U2 at Load 350 lbf

The displacement perpendicular to the direction of the load can be seen in Figure 124 below. The distribution has significantly changed from the 280 lbf case where all of the displacement was in the direction towards the hole. In the 350 lbf case there is now a region around 30 degrees where the displacement is away from the hole.

Figure 124: Plate 6 U_1 at Load 350 lbf

6.7 Analysis

There is considerable error shown in all of the plates between the finite element analysis and experimental data in both determining the failure load of the plate and of the stiffness of each plate. The error in determining the failure load is largely due to two main factors which are both related to the strength parameters used in the model. The first is the effect of *in situ* ply strength which has been discussed previously. For all the plates the FEA showed a first failure that is lower than that of the experimental. Since the *in situ* strength of a ply is typically larger than the measured tensile strength, this may account for some of the discrepancy. For example, using the experimental data provided by Chang et al. (1991) the *in situ* shear strength is twice the strength as measured by a tensile test. The transverse *in situ* strength showed to be approximately 1.8 times higher than the measured transverse strength. Chang et al. (1991) reported that the fiber buckling strength near the stress concentrations of a hole was 1.78 times higher than that of the

measured buckling strength. The second effect is likely due to the difference between the strength parameters in tension and compression. All values used in this analysis were from tensile testing due to the difficulties involved in determining the compressive strengths of a material. For carbon fiber T300/976, the compressive transverse strength is approximately 2.5 times higher than the tensile transverse strength.

The prediction of failure strength in plates 1 and 4 is shown to be mostly dependent on the shear strength. This trend was also reported by Chang et al. (1984) for laminates with mostly fibers in the 0 degree direction. However, the finite element model for plate 4 is shown to be more dependent on shear strength than plate 1. This is observed from the cases of increasing the shear strength of both plates. Plate 4 showed a large increase in the plate strength due to the increased shear strength.

For all of the plates the stiffness of the finite element model is higher than the experimental data stiffness in the linear region. This region is before any failure takes place and therefore is independent of any discrepancies between the strength of the FEA and the experimental data. One reason for this is that the experimental plates could have been damaged by the hole drilling procedure. This could have caused the fibers on the hole boundary to become frayed and lose strength. However, since the strength is shown to be dependent on how the load is transfer around damaged regions, the effect of the frayed fibers is likely less. However, a similar trend in stiffness is shown between the FEA and the experimental. The highest stiffness is shown for plates 2 and 3 which are oriented at +/- 45 degrees followed by plates 1 and 4 which are oriented at 0 degrees and finally plates 5 and 6 which are oriented at 90 degrees.

One explanation for the stiffness constants being too high is that edge effects are influencing the measured values. This would in particular influence the measured shear modulus. To avoid this in the future, the tested coupons need to have a higher length to width ratio.

Another source of error that would affect both the stiffness and the strength of the laminate is the thickness of each ply in the FEA model. The thickness of each ply was estimated from the thickness of the tensile coupon specimens. However, these specimens were laminates of all the same material whereas the laminates in the experimental tests were made up of both carbon fiber and fiberglass laminae.

7. Conclusion

This study presents the effect of the stacking sequence and fiber orientation on a composite sandwich panel subjected to in-plane bolt loading. Six plates were constructed with laminates of unidirectional carbon fiber and cross ply weaves of fiberglass. The orientations that were examined included 0, +/- 45, and 90 degrees. Half of the plates had fiberglass lamina on the outside of the laminate while the other three plates had the carbon fiber on the outside. Experimental and analytical tests were performed to determine the best orientations and stacking sequence.

For the FEA, plates with fibers oriented at +/- 45 degrees showed the highest strength. This is likely because the +/- 45 degree orientation allows the load to be transferred better from the damaged regions to the undamaged regions. The experimental data also showed higher strengths of the +/- 45 degree plates. However the experimental data also showed high strengths for the 90 degree laminate but with very high displacements. These high displacements would not allow the joint to maintain its relative position to the adjacent part. The discrepancy between the strength of the FEA models and the experimental data is attributed to incorrect strength properties. The effect of *in situ* strength and compression strength was found to have a significant effect on the accuracy of the FEA solution.

Further work recommended includes examining the *in situ* strengths of the appropriate laminates and determination of the compressive strengths of the materials. Greater examination of the effect of the honeycomb on the strength of the plates needs also to be addressed. More orientations such as 30 and 60 degrees and the effect of the laminate thickness should be examined.

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