An Analysis of N-Body Trajectory Propagation

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An Analysis of N-body Trajectory Propagation

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Trajectories created with n-body orbit models were propagated in geocentric and interplanetary test cases. The n-body models were created in MATLAB® using numerical integration. In the geocentric test case, the n-body codes were compared to a two-body orbit model and to the default HPOP model used in Satellite Tool Kit®. The interplanetary test case compared the n-body model to the HORIZONS ephemeris data from JPL and an equation for ephemeris propagation. Both cases used the same initial positions and velocities and were propagated for the same duration. The results of the analysis showed that while n-body models are capable of creating complex orbits that two-body models cannot create, common perturbations such as drag and non-uniform gravity are still necessary to produce accurate trajectory models.

Nomenclature

HPOP	=	High Precision Orbit Propagator
ICRF	=	International Celestial Reference Frame
STK	=	Satellite Tool Kit
n	=	Number of bodies
r	=	Position, km
μ	=	Gravitational parameter, km^3/s^2

Introduction

Spacecraft trajectories can be modeled in many ways. The basis of all trajectories is that the gravitational attraction of one or multiple bodies creates an acceleration that results in a specific path through space, called a trajectory. The most simple model is the two-body problem, which consists of a small body such as a spacecraft and a larger body that it orbits about. The mass of the spacecraft is assumed to be negligible compared to the mass of the body it is orbiting, meaning it imparts no acceleration on it. The equations of motion for this model can be directly solved and the result is the four conic section orbits: circle, ellipse, parabola, and hyperbola.

The two body model will provide a rough idea of a spacecraft's trajectory, but in most cases, these trajectories are of low accuracy. For example, a trajectory of a satellite orbiting the Earth is mostly due to the acceleration due to Earth's gravity, but the Sun and Moon also will affect the trajectory of the spacecraft. The resulting trajectory still will have the same shape as the trajectory predicted by the two-body model, but it will not remain fixed and will change with time, due to the gravitational acceleration from the Sun and Moon.

N-body models consist of systems that contain more than one large body. The n-body problem was not specifically developed for orbit propagation nor is it a new mathematical model. The problem can just be applied to model the motion of planets and other orbiting bodies and their affect on the trajectory of smaller bodies. These models can have two large bodies (three-body model) or as many bodies as can be computed, such as models of the universe[1]. N-body models do provide a way to model much more complex and exotic trajectories though. Trajectories such as halo orbits and lunar trajectories could not be modeled with a two-body trajectory model. Figure 1 shows an example of this idea. Both trajectories in this figure have the same initial position and velocity, but the red dashed trajectory was modeled in with a two-body model and the black trajectory used a three-body model, which is an n-body model where n=3. Because the Earth is the only large body in the two-body model, the trajectory is not affected by the moon and resembles an escape trajectory from the Earth. In the three-body model, the moon is included and the resulting trajectory leaves the earth, passes the moon, and returns back to the Earth.

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Gravitational acceleration is not the only property that affects spacecraft trajectories. Perturbations must be accounted for to produce trajectories of greater accuracy. Common perturbations include a non-spherical body, solar radiation pressure, and atmospheric drag. These forces, unlike the gravitational forces, depend on external factors such as the shape of the spacecraft and location of the spacecraft, but they can have a significant effect on the trajectory of the spacecraft. A satellite that orbits the earth in Low Earth Orbit will receive a greater affect on its trajectory from atmospheric drag than from the gravitational acceleration of the moon. Also, the effect of the Earth having a non-uniform mass distribution will also have a much greater effect on the spacecraft's trajectory than the gravitational accelerations of the Sun or Moon.

This objective of this analysis is to cover the extent to which additional bodies alter trajectories and to see if their effect is more significant than the perturbations discussed in the previous paragraph. The multiple-body system is modeled in an n-body model developed in MATLAB® with geocentric and interplanetary versions. To evaluate the accuracy of this model and determine its effect compared to the perturbations, the n-body code will be compared to the simple two-body model and the HPOP (high-precision orbit propagator) that is used in Satellite Tool Kit (STK®) software. Trajectories from the HPOP model will be the basis of comparison between the two-body model and the n-body model.

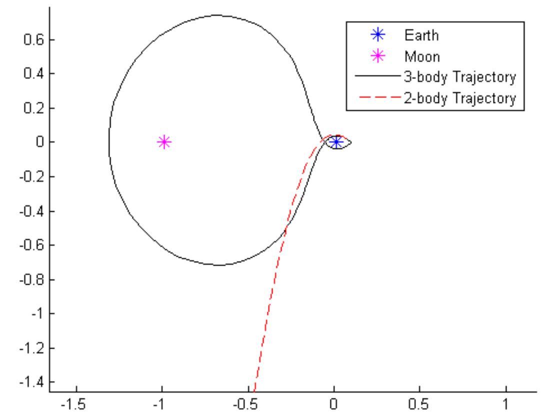

Figure 1. A two-body trajectory and a circular restricted three-body trajectory are modeled. Both trajectories have the same initial conditions, but due to different accelerations in each model, the resulting trajectories differ. Units are canonical.

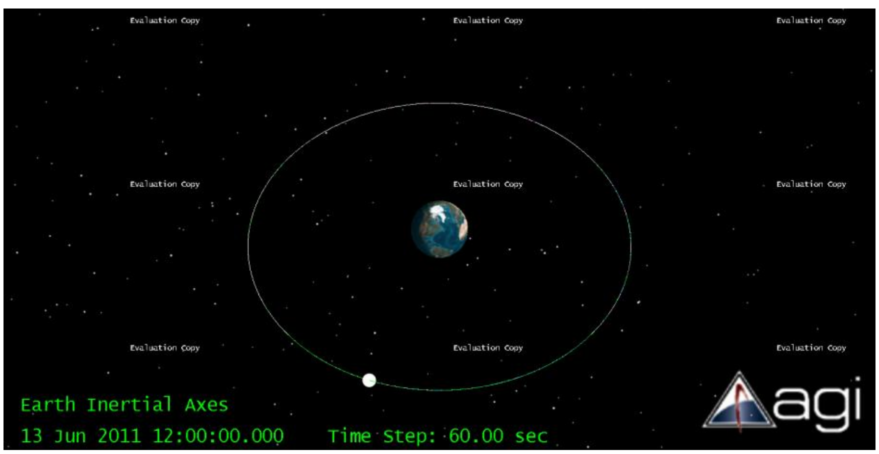

Figure 2. Example results from the HPOP model in STK.

Analysis

Many models were considered and tested for this analysis. The n-body model has no limit to the number of bodies present in a system, but due to computational considerations, the maximum number of bodies simulated was nine, which was a simulation of the eight planets and Pluto. Algorithms to reduce computing time from Aarseth were considered, but because the nine-body system model code ran in less than one minute, the extra coding of the algorithms was deemed unnecessary[1]. The first model was the two-body model, which was numerically integrated as well as calculated with universal variables as a check. For geocentric trajectories, three-body and four-body models were created, with the Sun and Moon acting as the second and third bodies. The interplanetary trajectories used a nine-body model as previously described. A general n-body model was attempted, which would allow for an input of any number of bodies, but was not completed. This would have allowed for one code to run the two, three, four and nine body models, but a problem with the indexing inside of the handle function that ode45 called could not be solved.

The two-body model was the most basic model used, and consisted of the Earth and the spacecraft. The Earth is fixed in the two-body frame and is modeled as a uniform, point mass to simplify the equation of motion. The mass of the spacecraft is assumed to be so small that it is negligible compared to the mass of the main body. Also, the trajectory of the spacecraft is only altered by the acceleration due to gravity of the main body. The equation of motion for this model reduces to Eq. 1.

$$\ddot{\vec{r}} = -\frac{\mu\vec{r}}{\|\vec{r}^3\|} \quad 1)$$

In the n-body model, this main equation still holds true, except that the acceleration, denoted as the second derivative of the position in Eq. 1, is equal to a summation of the gravitational forces of all bodies included in the system. Equation 2 shows the equation of motion of an n-body system. The biggest change compared to the two-

$$\ddot{\vec{r}}(i) = -\sum_{i=1,j\neq i}^{n} \frac{\mu(\vec{r}(i)-\vec{r}(j))}{\|(\vec{r}(i)-\vec{r}(j))\|^3} \quad 2)$$

body system is that the main bodies that alter the trajectory of the spacecraft create gravitational accelerations on each other, meaning that their positions relative to the spacecraft change. This means that the acceleration of the spacecraft is a function of the changing positions of the large bodies.

Each of the models was numerically integrated in MATLAB® using the ode45 function. The two-body problem could also be solved without numerical integration using methods such as universal variables. With exception to the restricted three-body problem, any system with more than two bodies had to be numerically integrated to create a solution to the problem. In the integration, the absolute and relative tolerances used for all models were both 10^{-8}. No perturbations were added to the models. Only one instance of the HPOP propagator in STK® included perturbations for the sake of comparison.

STK® was used to analyze the accuracy of the n-body models in orbit propagation. STK® has multiple propagation options, including a two-body model almost identical to the two-body code created in MATLAB®. HPOP is a unique propagator to STK® and has many propagation schemes available within it. A summary of the propagation options in HPOP is included in Table 1. For the analysis, three versions of HPOP were used. The first

Table 1. Details of the HPOP model used in STK®.

Property/Perturbation	**Details**
Integration Method	Runge-Kutta-Fehlberg 7th order
Oblate Earth	Earth Model with max degree and order of 21
Solar Radiation Pressure	Spherical Model
Drag	Jacchia-Roberts Atmosphere, C_d of 2.2
3rd Body	Sun and Moon
Other Perturbations	Solar Flux, Geomagnetic, Solid Tides, Shadowing

two used were an attempt to compare the three-body and four-body models created in MATLAB®. All perturbations such as drag, solar radiation, geomagnetic, and tides were turned off and the maximum degree and order of the harmonics of the oblateness effects was set to zero. The third-body effects of the Sun and the Moon were included though. The third version used was the default HPOP used in STK®. All available perturbations were used and the Sun and Moon were included for third body effects.

Discussion

The accuracy of the MATLAB® models was compared with a test case replicating the orbit of a GEO satellite. Table 2 shows the details of the test case. Each propagator in STK® and MATLAB® used the same initial position

Table 2. Summary of the GEO test case.

Item	Details
Initial Position	[42164 0 0] km, ICRF coordinates
Initial Velocity	[0 3.07467 0] km/s, ICRF coordinates
Start Date	June 13, 2011 12:00
Durations	12 h, 1 d, 3 d, 5 d, 7 d, 2 w, 3 w, 1 m
Propagators	2-body, 3-body, 4-body, HPOP (all perturbations)

and velocity conditions as well as the same duration to propagate. The resulting position and velocity at the end of the propagation duration was recorded and compared to the value calculated by the HPOP model with all perturbations. Figure 3 shows the results from the test case. The three-body and four-body models, both in STK® and MATLAB® deviated the most from the perturbed HPOP model. After thirty days of propagation the deviation of both the unperturbed STK® model with Sun and Moon effects and the three-body MATLAB® model from the perturbed HPOP model was over 200 km in the Y component. The two-body models never exceeded 100 km in deviation though.

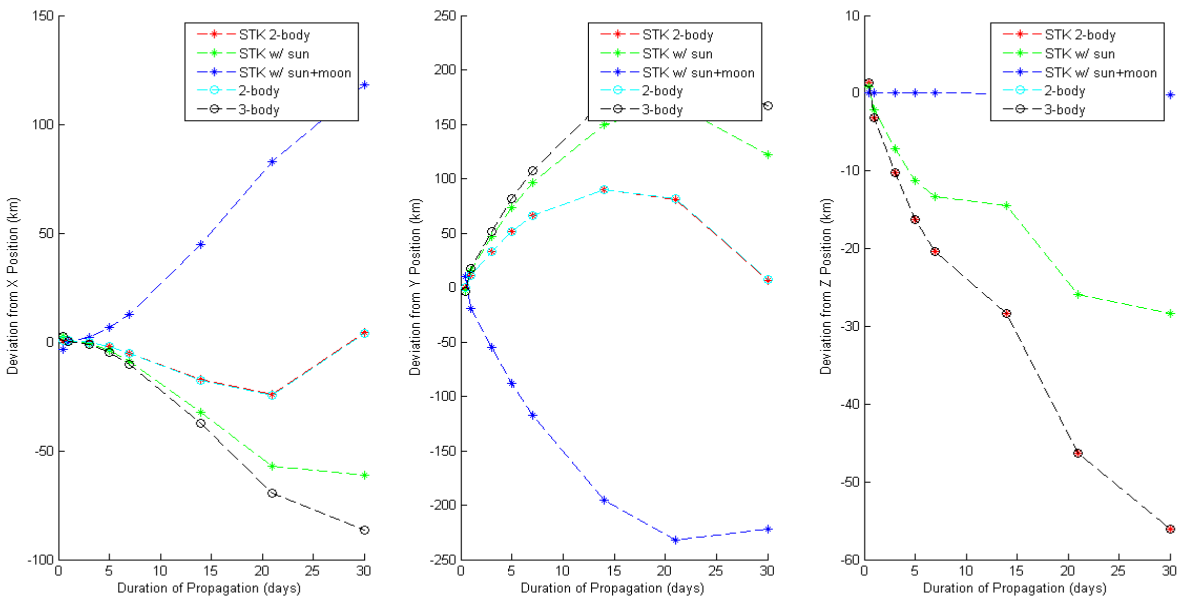

Figure 3. Results from the GEO test case. The values are the deviations in the X, Y, and Z ICRF components from the HPOP results with all perturbations and Sun and Moon third body effects.

The large error can be attributed to the fact that the propagation schemes do not have any perturbations and the HPOP model they are compared to does. Even though the n-body schemes differ from the perturbed HPOP model, they have the same trends in error and only differ from each other slightly. The three-body model from MATLAB and the HPOP model with Sun effects in particular followed similar trends and had similar error values. It is possible that the accelerations due to the perturbations and accelerations due to the gravity of the Sun and Moon differ in direction, causing the error to increase if only n-body effects are considered over the two-body model. In addition, though the maximum deviation reaches over 200 km after thirty days, that deviation is still less than 1 % of the radius of the orbit and may be expected if no perturbations are considered over the course of a month. The results still show that the addition of extra bodies actually increased the deviation from the perturbed HPOP values in the case of a GEO satellite.

The interplanetary model used a nine-body model to simulate the motion of the planets. Each planet imparted and received a gravitational acceleration from every other planet. The model used relative and absolute accuracies of 10^{-10} in integration and still could propagate the positions of the planets forward 10 years in less than one minute of computing time. This model was compared a two-body model, HORIZONS ephemeris data from JPL, and an ephemeris equation found in Curtis, which came from Standish et al. (1992)[2]. HORIZONS was chosen to be the

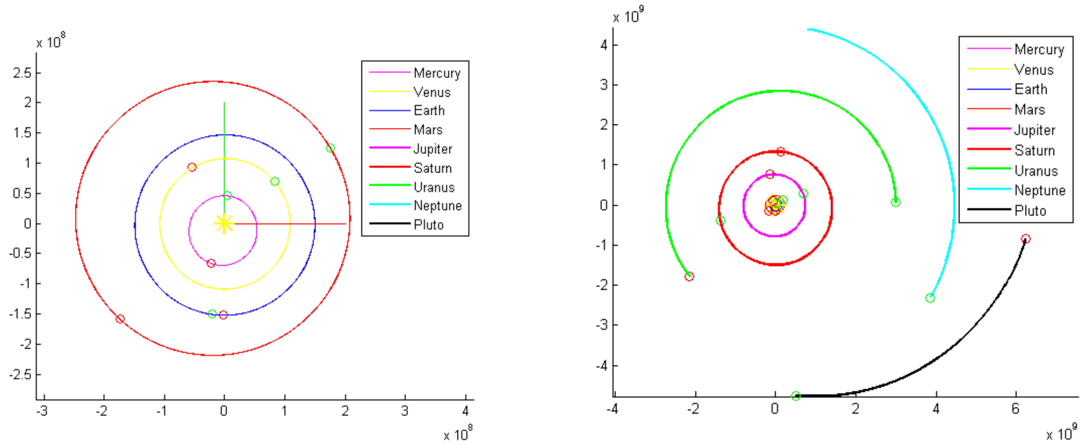

Figure 5. Results from the nine-body propagation. The two graphs show the trajectories of the planets propagated 50 years from June 13, 2011.

basis of comparison of the propagation models. The same trends in deviation as the GEO test case were observed in the results of the comparison. The trajectory results from the nine-body equation still differed from the other trajectories tested. For durations of a year and less, the nine-body results deviates little from the HORIZONS positions and deviates less than the ephemeris code, but for durations longer than a year, the other methods are closer to HORIZONS. Figures 5 and 6 show the inertial X, Y, and Z components of position for Mars for a propagation of 90 days forward and a propagation of 10 years forward.

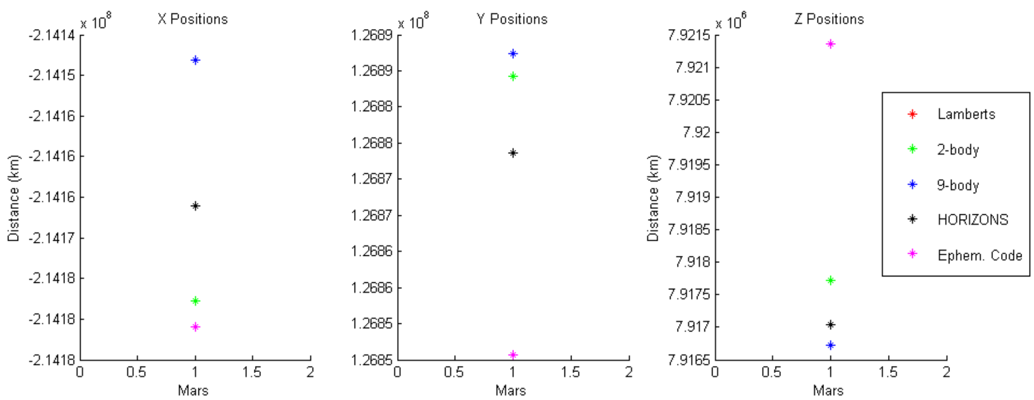

Figure 4. Deviation from HORIZONS positions after 90 days.

The results of both of the cases show that more than an n-body model is necessary for accurate propagation of trajectories. The n-body models allow for the creation of complex orbits that two-body schemes cannot model, but the analysis shows that perturbations need to be included to have an accurate trajectory model.

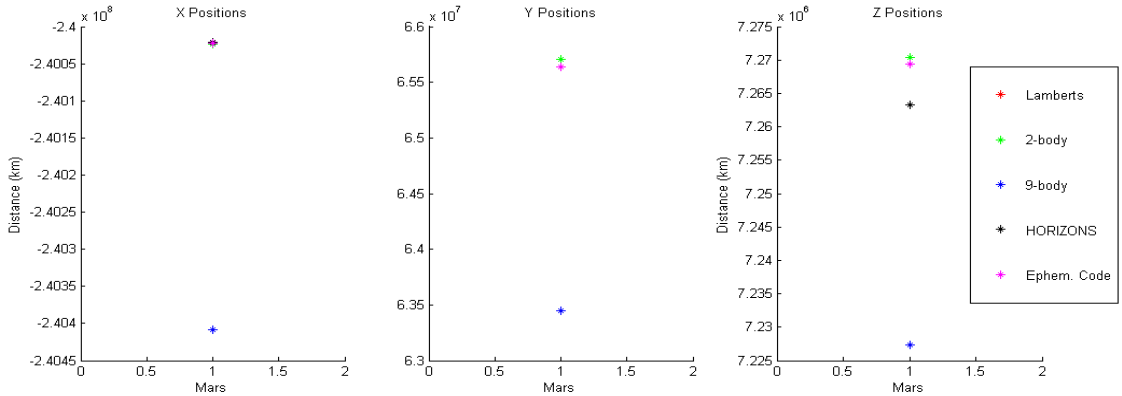

Figure 6. Deviation from HORIZONS positions after 10 years.

Conclusion

N-body trajectories were calculated for a geocentric case and an interplanetary case. The trajectories were compared to trajectories created by a two-body model in deviation from either the perturbed HPOP results from STK or the ephemeris results from JPL's HORIZONS, which served as a baseline for comparison. The results from the analysis showed that though n-body trajectories follow the same trends as the HPOP or HORIZONS data, they still deviated significantly. Though some error could possibly be in the MATLAB® code, the n-body trajectory results differed from the baseline results, sometimes in greater magnitude than the two-body model. The results show that to produce accurate trajectories, perturbations, as well as the gravitational effects of other bodies, are required.

References

[1]Aarseth, Sverre J., *Gravitational N-body Simulations*. Cambridge University Press, Cambridge, 2003.

[2]Vallado, David A., *Fundamentals of Astrodynamics and Applications*, 3rd ed., Microcosm Press, Hawthorne, 2007.

[3]Curtis, Howard, *Orbital Mechanics for Engineering Students*, Elsevier, New York, 2005.

[4]Sellers, Jerry J., "Analytical Graphics Inc. Astrodynamics Primer," Analytical Graphics, Inc.

[5]"HPOP Technical Notes," Analytical Graphics Inc. http://www.stk.com/resources/help/online/stk/source/hpop/hpop-10.htm. 1 June 2011.