Two Alternative System Reliability Approaches to the Serviceability Condition Assessment of Spillway Gate Systems on Dams

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INTRODUCTION

The U.S. Army Corps of Engineers is one of the Nation's largest managers of infrastructure. It is responsible for maintaining and operating the Nation's navigable waterways and is the primary agency for maintaining federal flood control dams. This includes a vast amount of infrastructure that includes roughly 270 navigation dams, 350 reservoir dams, and 238 lock chambers (Bullock and Foltz 1995). The navigable inland waterways carry roughly 17% of the Nation's intercity cargo – an important economic role (USACE 2004). Over half of the locks and dams are over 50 years old. The entire inventory is deteriorating over time and requires billions of dollars to upgrade, maintain, and repair. The Corps of Engineers requires that a strength-based reliability analysis be completed to justify major rehabilitation projects. Reliability methods are preferred for cost-benefits analyses and for quantifying risk. More often, however, it is the general serviceability of a structure that dictates the requirements for maintenance and repair.

The Corps of Engineers has developed a Condition Index (CI) rating system for a variety of structures that assesses the general serviceability condition of a structure based on periodic visual inspections. A Condition Index (CI) is a rating between 0 and 100 that describes the condition of a structure at a point in time. The CI is based on a series of observations by an inspector. At the component level, the inspector classifies what he or she sees into the predefined descriptive category that best matches the observation. Some CIs also include measurements. More often, the condition scores are based on descriptive word pictures that are often vague and difficult to quantify. Chouinard *et.al.* (2003) developed a deterministic CI rating system for spillway gate systems on dams. The gate system contained 122 separate inspectable components, each with its own condition rating table. The structure was decomposed into a seven-level structural hierarchy of systems, sub-systems, and components. The highest levels are shown in Figure 1.

Estes *et.al.* (2005) developed a reliability-based approach that assigns probabilistic CI ratings for groups of components, systems, and projects. The approach accounts for the considerable uncertainty associated with the CI process which includes:

- Uncertainty in the ability of different inspectors to reliably choose the correct condition state and to a greater degree, the appropriate score within a condition state

- Uncertainty associated with the condition state tables where a single numerical score is obtained from matching an inspector observation to a word description of the distress.
- Uncertainty in defining at which condition state a component will actually fail and needs to be replaced.
- Uncertainty with how a component will deteriorate over time, although this uncertainty is gradually eliminated as inspections occur and the maintenance plan is updated.

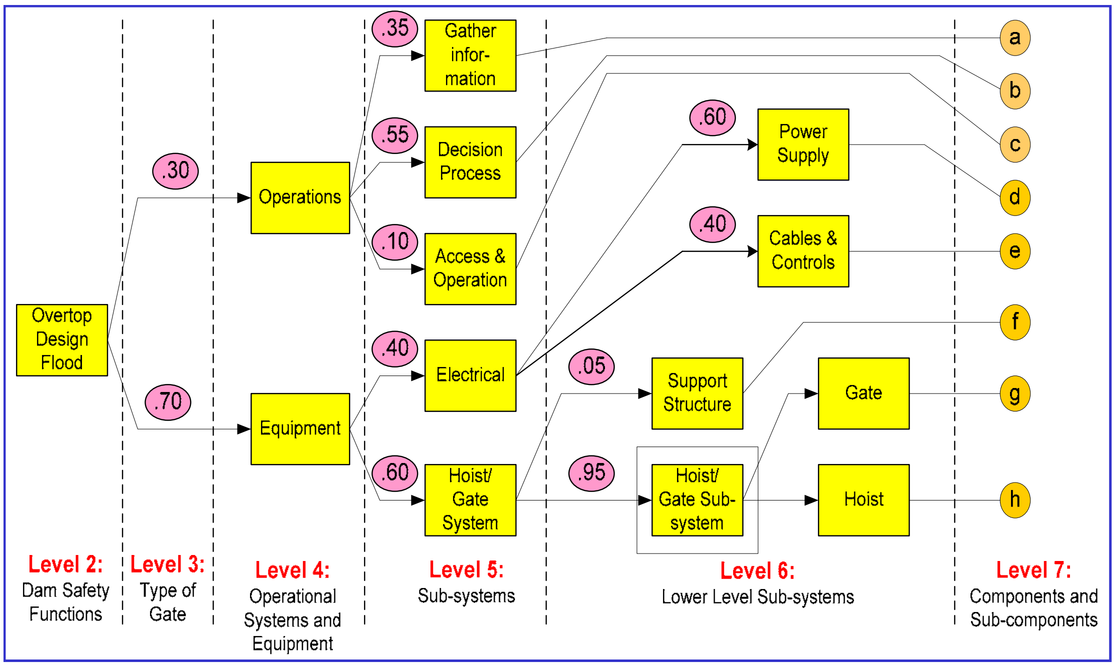

FIGURE 1
STRUCTURAL HIERARCHY FOR THE SPILLWAY GATE SYSTEM FOR A DAM

Estes *et.al.* (2005) makes some reasonable assumptions and addresses these uncertainties. Using the condition index value as the random variable, the reliability index and probability of failure for a component at a point in time can be computed. With some further assumptions about deterioration, a time-dependent reliability analysis can be conducted using hazard functions to facilitate a probabilistic cost-benefit analysis. Estes *et.al.* (2005) illustrates these concepts using a both a simple hypothetical structure and the spillway gate system for the Great Falls dam.

In the deterministic methodology, the higher level condition indices for sub-systems and systems ($CI_{\,system}$) were computed as:

$$CI_{System} = \sum_{j=1}^{n} I_j CI_j = I_A CI_A + I_B CI_B + I_C CI_C \qquad (1)$$

where CI_j is the condition index of the subordinate component and I_j is the importance factor for that component relative to the rest of the system. The composite rating was a weighted average of condition based on importance of the component. The equation was used regardless of whether the components of the system were in series or parallel.

When applying reliability-based methods to this system, there were two available approaches which produced dramatically different results. The first was to use equation (1) for the mean value of the system CI and develop the corresponding standard deviation. The second was a traditional reliability approach where the probability of failure of a series system with statistically indepent components can be computed as

$$p_{f\,series} = 1 - \prod_{a=1}^{z} (1 - p_{f_a}) \quad (2)$$

where $p_{f,\,series}$ is the probability of failure of the series system and p_{fa} is the probability of failure of one of the components that comprise the system. A series system is one where the system fails if any component in the system fails. Conversely, a parallel system is one where every component must fail before the system fails. The probability of failure of a parallel system with independent components ($p_{f,\,parallel}$) is similarly

$$p_{f,parallel} = \prod_{a=1}^{z} p_{f_a} \quad (3)$$

This paper compares and contrasts these two approaches and recommends when each might be appropriate. Examples from a dam spillway gate system are used to illustrate the differences in the two approaches.

PARALLEL SYSTEMS

Figure 2 shows a parallel power system consisting of medium voltage overhead lines and two back-up emergency generators. The importance of the overhead wires is determined to be I=0.70 and each emergency generator has an importance of I = 0.15. Note that within any system or subsystem the sum of the importance factors equals 1.0. The importance factor is typically obtained using expert opinion and the weighting is usually a factor of cost, criticality, and function.

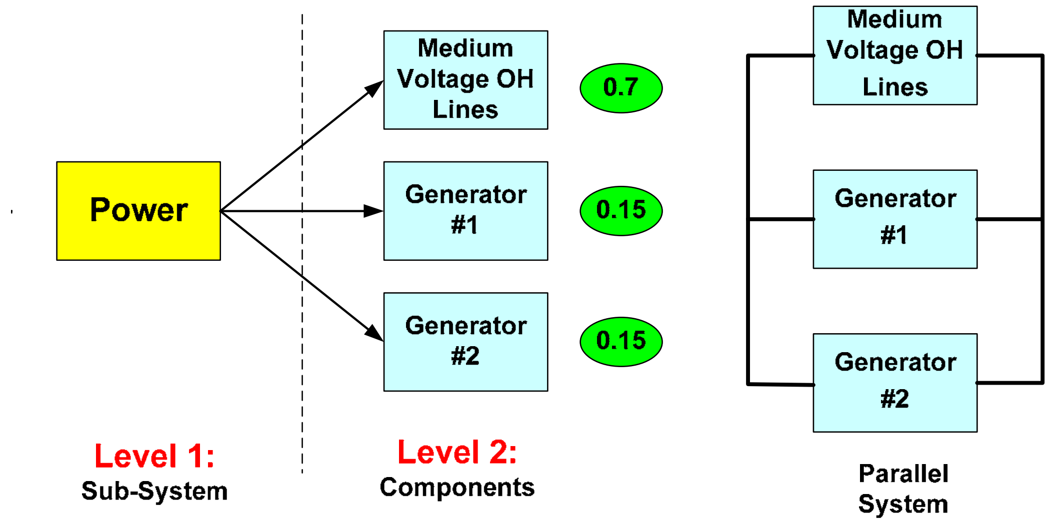

FIGURE 2
A POWER SUB-SYSTEM FOR A DAM SPILLWAY CONSISTING OF OVERHEAD WIRES AND TWO BACK-UP GENERATORS. THE SYSTEM COMPONENTS ARE IN PARALLEL AND HAVE ASSIGNED IMPORTANCE FACTORS

Tables 1 and 2 show the component condition tables for the overhead wires and emergency generators, respectively. Based on inspection data, the overhead wires are considered in condition state 2 and for the two generators: one is in condition state 1 and the other is in condition state 2.

Medium Voltage Overhead Wires									
Condition State	Condition Index Score							Mean Value	Standard Deviation
	0-9	10-24	25-39	40-54	55-69	70-84	85 - 100		
1						x	x	85	7.65
2			x	x	x			47	11.22
3		x						17	3.57
4	x							4.5	2.29

TABLE 1
COMPONENT CONDITION TABLE FOR MEDIUM VOLTAGE OVERHEAD WIRES COMPONENT OF THE POWER SUB-SYSTEM FOR THE SPILLWAY GATE SYSTEM ON A DAM

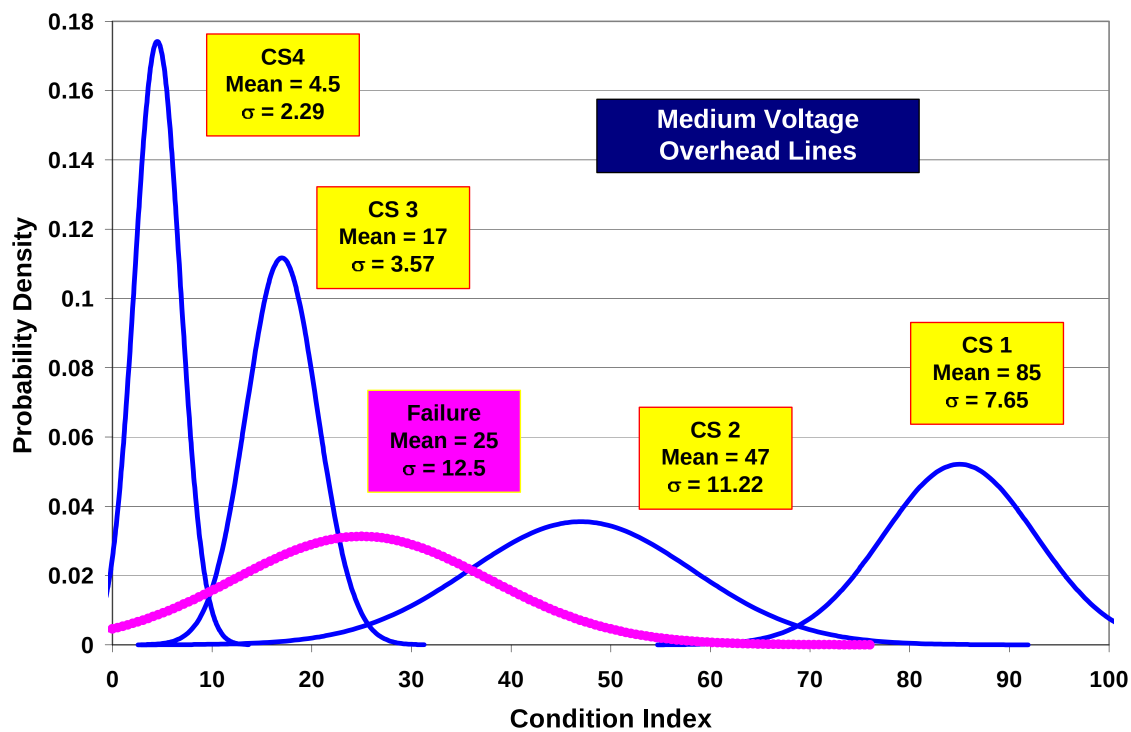

FIGURE 3
PROBABILISTIC CONDITION STATES AND FAILURE DEFINITION FOR THE MEDIUM VOLTAGE OVERHEAD LINES

Emergency Generator									
Condition State	**Condition Index Score**							**Mean Value**	**Standard Deviation**
	0-9	**10-24**	**25-39**	**40-54**	**55-69**	**70-84**	**85 - 100**		
1						x	x	85	7.65
2		x	x	x				32	11.22
3	x							4.5	2.29

TABLE 2
COMPONENT CONDITION TABLE FOR EMERGENCY GENERATOR COMPONENT OF THE POWER SUB-SYSTEM FOR THE SPILLWAY GATE SYSTEM ON A DAM

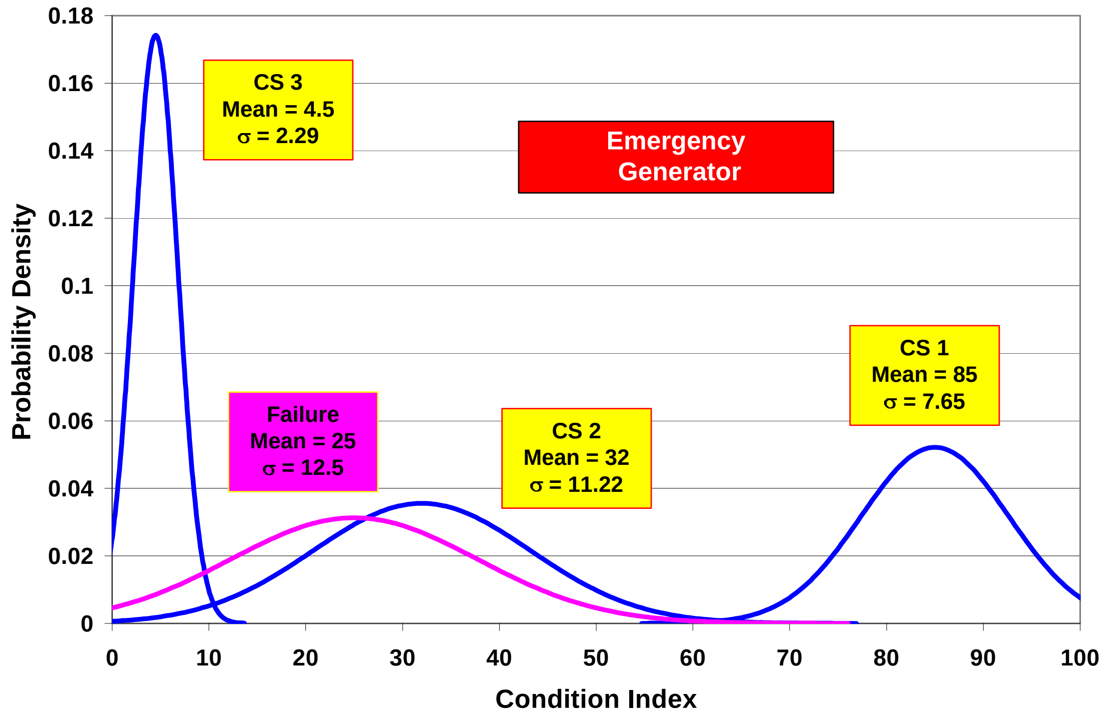

FIGURE 4
PROBABILISTIC CONDITION STATES AND FAILURE DEFINITION FOR THE EMERGENCY GENERATORS

Regardless of the system reliability approach used, the reliabilities of the components are computed as specified in detail in Estes *et.al.* (2005). Assuming that these are the first inspections for this condition state, the initial mean value for the condition index is the middle of the condition state. As a component remains in a condition state, the mean value CI is assumed to deteriorate linearly over time. The range for the medium voltage overhead line condition state is 25-69 (Table 1), so the mean value is CI=47. Assuming a 5% inspector error, the probability of obtaining a value of CI<69 when the structure is actually in this condition state is 97.5%, or 0.975. The standard deviation σ can be computed as:

$$
\begin{aligned}
&P(CI_{OHLines} \leq 69) = 0.975 = \Phi(\frac{CI-\mu}{\sigma}) = \Phi(\frac{69-47}{\sigma_{OHLines}}) \\
&\sigma_{OHLines} = \frac{(69-47)}{\Phi^{-1}(0.975)} = \frac{(69-47)}{1.96} = 11.22
\end{aligned}
\tag{4}
$$

where Φ is the standard normal variate whose value can be found in the standard normal distribution tables, and μ is the mean value of the condition state (Ang and Tang 1975).

The distribution for $CI_{OHLines}$ is assumed to be normally distributed with parameters: N[47, 11.22]. Similarly, the distributions associated with the two generators are N[85, 7.65] and N[32,11.22]. Based on the condition index definitions, failure is a normally distributed variable with the parameters N[25, 12.5]. The failure distribution would ideally be derived from a data base that reflects the historical condition index at time of replacement for similar components. Expert opinion is used in the absence of data, until a sufficient database can be established. Figure 3 shows the probabilistic distributions of the condition states for the Medium Voltage Overhead Wires as well as the probabilistic distribution for failure. Figure 4 shows these distributions for the Generators.

The reliability index (β) for these components with respect to failure can be computed as:

$$
\begin{aligned}
&\beta_{OHLines} = \frac{CI_{Actual} - CI_{Failure}}{\sqrt{\sigma^2_{Actual} + \sigma^2_{Failure}}} = \frac{47-25}{\sqrt{(11.22)^2 + (12.5)^2}} = 1.31 \\
&\beta_{Gen\#1} = \frac{85-25}{\sqrt{(7.65)^2 + (12.5)^2}} = 4.09 \\
&\beta_{Gen\#2} = \frac{32-25}{\sqrt{(11.22)^2 + (12.5)^2}} = 0.42
\end{aligned}
\tag{5}
$$

The probability of failure (the probability that the component will be replaced) for each component is

$$
\begin{aligned}
&p_{f,OHLines} = \Phi(-\beta) = \Phi(-1.31) = 1 - \Phi(1.31) = 1 - 0.9049 = 9.51(10)^{-2} \\
&p_{f,Gen\#1} = \Phi(-\beta) = \Phi(-4.09) = 1 - \Phi(4.09) = 1 - 0.9999788 = 2.12(10)^{-5} \\
&p_{f,Gen\#2} = \Phi(-\beta) = \Phi(-0.42) = 1 - \Phi(0.42) = 1 - 0.662 = 0.338
\end{aligned}
\tag{6}
$$

Weighted Average Reliability Approach

Using the weighted average approach, the mean condition index for this system would be computed using Eq (1)

$$
\begin{aligned}
&CI_{System} = I_{OHLine} CI_{OHLines} + I_{Gen\#1} CI_{Gen\#1} + I_{Gen\#2} CI_{Gen\#2} = \\
&(0.7)(47) + (0.15)(85) + (0.15)(32) = 50.5
\end{aligned}
\tag{7}
$$

Because the equation is linear and the variables $CI_{OH\ Lines}$, $CI_{Gen\#1}$, and $CI_{Gen\#2}$ are independent and normal variates, the standard deviation of the system CI, $\sigma_{CI\ system}$ is (Ang and Tang 1975)

$$\sigma_{System} = \sqrt{\sum_{j=1}^{n} I_j^2 \sigma_j^2} = \sqrt{(0.7)^2(11.22)^2 + (0.15)^2(7.65)^2 + (0.15)^2(11.22)^2} = 8.11 \quad (8)$$

The value of the system condition index will always fall somewhere between the condition of the best and worst component. The reliability index and probability of failure (replacement) of this system would be computed as:

$$\begin{aligned} \beta_A &= \frac{CI_{Actual} - CI_{Failure}}{\sqrt{\sigma_{Actual}^2 + \sigma_{Failure}^2}} = \frac{50.5 - 25}{\sqrt{(8.11)^2 + (12.5)^2}} = 1.71 \\ p_f &= \Phi(-\beta) = \Phi(-1.71) = 0.044 \end{aligned} \quad (9)$$

In this case, the system index was 50.5 which is affected largely by the condition of the overhead lines. Suppose several similar systems were competing for the same scarce maintenance dollars. Table 3 shows the results of three such structures where the condition ratings of the components have been reversed. In case 2, the system index is raised to 71.4 because the condition of the overhead lines is rated so high. Similarly, in case 3, the system index is only 42.2 despite both emergency generators being in the best condition of all three cases. In this example, if these three systems were competing for a complete rehabilitation of the power system, the priority of funding would go towards case 3 followed by cases 1 and 2 in that order. The large difference in importance factors makes the choice clearer. Conversely, if the funding decision was only for replacement of generators, the priorities would have been different and case 2 would receive priority.

Traditional Reliability Approach

In the traditional reliability approach, the reliability of the system is at least as large as the reliability of its strongest member.

$$\begin{aligned} p_{f,power} &= p_{OHLines}(p_{f,Gen\#1})p_{f,Gen\#2} = 9.51(10)^{-2} * 2.12(10)^{-5} * 0.338 = 6.83(10)^{-7} \\ \beta &= \Phi^{-1}(p_{f,power}) = 4.83 \\ \sigma_{Power} &= \sqrt{\sigma_{OHLines}{}^2 + \sigma_{Gen\#1}{}^2 + \sigma_{Gen\#2}^2} = \sqrt{(11.22)^2 + (8.16)^2 + (11.22)^2} = 17.6 \end{aligned} \quad (10)$$

The system condition index can then be back calculated as:

$$CI_{power} = \beta\sqrt{\sigma_{Actual}^2 + \sigma_{Failure}^2} + CI_{Failure} = 4.83\sqrt{(17.6)^2 + (12.5)^2} + 25 = 129.3 \quad (11)$$

The condition index is higher than the condition index of any individual component. Because all three of these components would have to fail for power to be denied to the system, there is little likelihood of occurrence. Table 3 illustrates that this condition index would apply to any of the cases described because traditional reliability does not account for the relative importance of a component to the overall system. In a funding decision

for system replacement, all three cases would receive the same priority and no distinction can be made between them.

	Med. Voltage Overhead Lines		Generator #1		Generator #2		Power System Weighted Average		Power System Traditional Method	
I	0.70		0.15		0.15		← Importance Factors			
Case	Mean CI	Std Dev	Mean CI	Std. Dev	Mean CI	Std. Dev.	Mean CI	Std. Dev.	Mean CI	Std. Dev.
1	47	11.22	85	7.65	32	11.22	50. 5	8.11	129.3	17.6
2	85	7.65	47	11.22	32	11.22	71.4	5.86	129.3	17.6
3	32	11.22	85	7.65	47	11.22	42.2	8.11	129.3	17.6

TABLE 3
COMPARISON OF RESULTS USING A WEIGHTED AVERAGE AND TRADITIONAL SYSTEM RELIABILITY APPROACH FOR A PARALLEL POWER SYSTEM ON A SPILLWAY GATE

SERIES SYSTEMS

Figure 1 shows the hierarchical structure of a typical spillway gate system on a dam. The gate system is divided into the Operations and Equipment portions of the dam. The Equipment which consists of the electrical system and the gate structure is given more importance that the Operations system which involves gathering information, making decisions and the actual access to the site and operation of the equipment. Each of the sub-systems is further decomposed down to the component level. Figure 5, for example, shows the Information Gathering sub-system that includes 19 inspectable components, each with its own component condition table.

The condition index of each of these sub-systems is determined from the importance and condition of the components that comprise the sub-system. In this case, the distributions of the sub-systems were determined to be: Information Gathering (IG), N[84,5]; Decision Making (DM), N[32,10]; and Access and Operations (A&O), N[47,15]. The reliability index (β) for these components with respect to failure can be computed as:

$$\begin{aligned}
\beta_{IG} &= \frac{CI_{Actual} - CI_{Failure}}{\sqrt{\sigma^2_{Actual} + \sigma^2_{Failure}}} = \frac{84-25}{\sqrt{(5.0)^2 + (12.5)^2}} = 4.38 \\
\beta_{DM} &= \frac{32-25}{\sqrt{(10.0)^2 + (12.5)^2}} = 0.44 \\
\beta_{A\&O} &= \frac{47-25}{\sqrt{(15.0)^2 + (12.5)^2}} = 1.13
\end{aligned} \tag{12}$$

The probability of failure (the probability that the component will be replaced) for each component is

$$p_{f,IG} = \Phi(-\beta) = \Phi(-4.38) = 5.87(10)^{-6}$$
$$p_{f,DM} = \Phi(-\beta) = \Phi(-0.44) = 0.331 \quad (13)$$
$$p_{f,A\&O} = \Phi(-\beta) = \Phi(-1.13) = 0.130$$

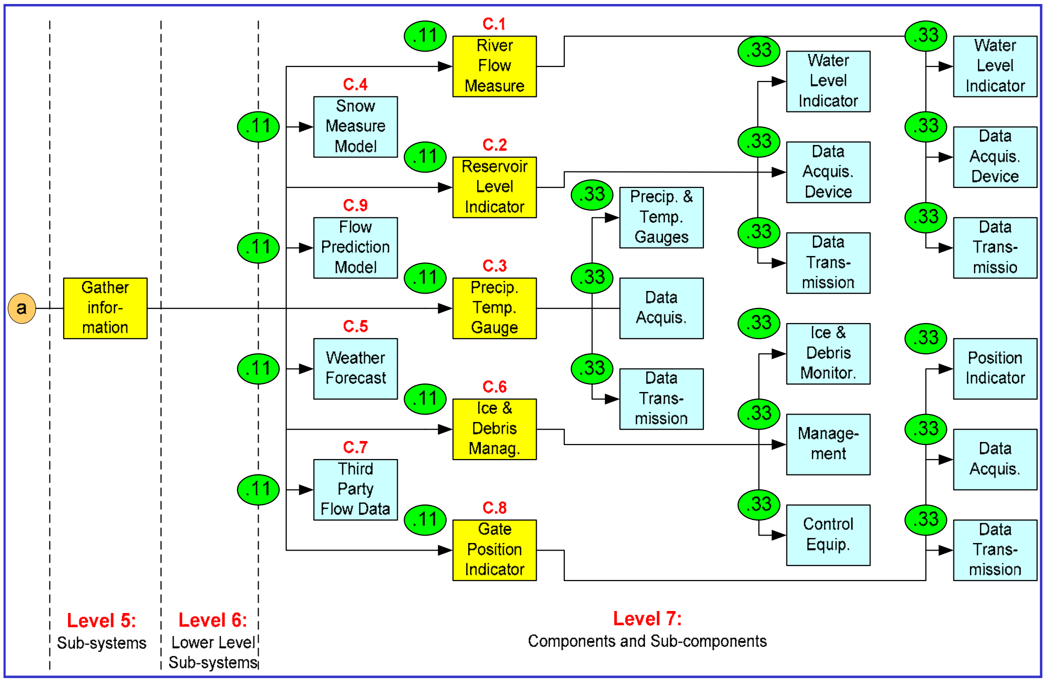

FIGURE 5

THE GATHER INFORMATION SUB-SYSTEM FOR THE OPERATIONS SYSTEM (FIGURE 1) FOR THE SPILLWAY GATE SYSTEM ON A DAM

Weighted Average Reliability Approach

The Gathering Information, Decision Making, and Access/Operation sub-systems are a series system with respect to spillway gate Operations. If any of those sub-systems fail, then the system will fail. The decision making process is considered most important (I=0.55) and the access and operation is the least important (I=0.10). Table 4 address three cases where the subsystems have various condition indices based on the inspection of subordinate components. Looking at Case 1, the condition index of the Operations System is computed using Eq. (1)

$$CI_{Operations} = \sum_{j=1}^{n} I_j CI_j = I_{IG} CI_{IG} + I_{DM} CI_{DM} + I_{A\&O} CI_{A\&O} =$$
$$(0.35)(84) + (0.55)(32) + (0.10)(47) = 51.7 \quad (14)$$

And the standard deviation is computed as

$$\sigma_{Operations} = \sqrt{\sum_{j=1}^{n} I_j^2 \sigma_j^2} = \sqrt{(0.35)^2 (5)^2 + (0.55)^2 (10)^2 + (0.10)^2 (15)^2} = 5.96 \quad (15)$$

Looking at the three cases, the priority of replacement funding would go to case 3 where the system condition index value is 45.5 based on the combined poor conditions of the Information Gathering and Decision Making sub-systems. Even though the Decision Making sub-system is most important (I=0.55) and has the lowest individual score in case 1, the relatively good condition of the Information Gathering portion (I=0.35) keeps it from receiving top priority. The priority of replacement funding for these three systems would be case 3, case 1 and case 2 in that order.

	Information Gathering (IG)		Decision Making (DM)		Access and Operations (A&O)		Spillway Operations Weighted Average		Spillway Operations Traditional Method	
I	0.35		0.55		0.10		← Importance Factors			
Case	Mean CI	Std Dev	Mean CI	Std. Dev	Mean CI	Std. Dev.	Mean CI	Std. Dev.	Mean CI	Std. Dev.
1	84	5.0	32	10.0	47	15.0	51.7	5.96	27.7	18.7
2	47	15.0	84	5.0	32	10.0	65.8	6.01	27.7	18.7
3	32	10.0	47	15.0	84	5.0	45.5	8.97	27.7	18.7

TABLE 4
COMPARISON OF RESULTS USING A WEIGHTED AVERAGE AND TRADITIONAL SYSTEM RELIABILITY APPROACH FOR A SERIES OPERATIONS SYSTEM ON A SPILLWAY GATE

Traditional Reliability Approach

In the traditional reliability approach, the reliability of the system is less than the reliability of its weakest member. Using Eq. (2) the system failure probability is computed as:

$$\begin{aligned} p_{f,Operations} &= 1-(1-p_{f,IG})(1-p_{f,DM})(1-p_{f,A\&O}) = \\ &= 1-(1-5.87(10)^{-6})(1-0.331)(1-0.130) = 0.418 \\ \beta_{Operations} &= \Phi^{-1}(p_{f,Power}) = 0.207 \\ \sigma_{Operations} &= \sqrt{(\sigma_{IG})^2+(\sigma_{DM})^2+(\sigma_{A\&O})^2} = \sqrt{5^2+10^2+15^2} = 18.71 \end{aligned} \tag{16}$$

The system condition index is computed as:

$$CI_{Operations} = \beta\sqrt{\sigma^2_{Actual}+\sigma^2_{Failure}} + CI_{Failure} = 0.207\sqrt{(18.71)^2+(12.5)^2}+25 = 27.7 \tag{17}$$

The condition index is lower than the condition index of any individual component. Because the Operations system would fail if any of the three sub-systems fail, the probability of failure is high and the condition index is extremely low. The results will

be the same for all three cases using this approach as no distinction is possible for component importance. The funding priority would be high for all three cases.

CONCLUSIONS

Two competing methods were introduced for analyzing the condition of a structural system. The results produced by both were very different. In the weighted average approach, the system condition index will always be somewhere between the condition of the best and worst component in the system. In the traditional reliability approach, the condition index will always be lower than the condition of the worst member in a series system and higher than the condition of the best member in a parallel system.

A traditional reliability approach works extremely well for a strength-based system where the importance factors of the components are relatively equal and the consequences of failure are typically catastrophic. In a serviceability context where some failures are more serious than others and some components are clearly more important than others, the information provided is less useful. The extreme values obtained in the traditional approach exaggerate the condition of the structure. A condition index of over 100 for the parallel structure would probably ensure it does not get replaced until every portion of the system is significantly deteriorated. In a multi-tiered series system, such as the spillway gate system presented here, the condition index would be so low that it would appear that every structure was in dire need of replacement. The condition of the worst element in the system, no matter how minor, controls the maintenance decision.

If the goal is to use the overall condition of a structure to prioritize and optimize maintenance funding, the weighted average approach seems to provide better decision making information. By combining the condition of components with their importance to the overall system, it is easier to make a distinction between competing priorities. While series and parallel systems are treated in the same manner, a distinction could be made in the assignment of importance factors where a redundant system might receive a lower importance factor.

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