LASER DOPPLER ANEMOMETRY FOR FLOW MEASUREMENT

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Laser doppler anemometers are non-invasive, linear, and inherently precise. Calibration, in the usual sense, is not necessary; length and frequency measurements suffice to establish velocity at a spatial point. Measurements were made at points in the cross-sections of two square ducts containing water flow. The points were selected in conformance with a numerical integration scheme to be used for volumetric flow rate determination from the velocity measurements. The experiments were performed in a primary calibration facility at flows up to 1.25 m$^3$/sec using ducts with sides 46 and 92 cm. The anemometer, operating in forward scatter differential mode with a 15 mw He Ne laser, was positioned with a special traversing frame. Windows in the ducts allowed transmission of the beams into the flow and reception of scattered light. Two grid patterns, 4 x 11 and 11 x 11, were used so that 44 and 121 velocities were measured for each test. A total of eight tests were conducted covering a Reynolds number range from 1.1 to 3.9 x 10$^8$. After accounting for errors due to the discreet integration scheme of 0.61% and 0.13% for the 4 x 11 and 11 x 11 schemes, respectively, comparison with the calibration facility indicated extreme errors of +0.81/-0.16 and +0.84/-0.61. The major limitation of the set-up used was the time required to move the anemometer and obtain a new velocity value. It was pointed out that either better mechanical positioning or optical scanning could be employed to reduce the time required for a flow determination.
1. Introduction

As part of a test program to evaluate the performance of a four path acoustic flow meter, it was necessary to establish the velocity distribution at the metering section. The measurements were made using a laser doppler anemometer and a pattern which allowed numerical integration for volumetric flow rate. The experiments were set up in a primary flow calibration facility using gravimetric determination of water flow rate. Square ducts were used with windows provided for the anemometer. A traversing mechanism was used to position the anemometer measuring volume at spatial locations where velocity measurements were made. The facilities, procedures, and results of the entire program are reported in [1, 2]*, while this paper discusses only those aspects pertaining to determination of flow rate from laser doppler measurements.

2. Test Facility

Square ducts (46 and 92 cm) were installed in the large primary calibration facility at the Alden Research Laboratories, Figure 1. This facility consisted of a sump with a capacity of 760 m³ of water, which was maintained at temperatures from 30°C to 40°C, a pumping capacity of 1.27 m³/s at a head of 37 m, a straight pipe run of 25 m, and a weigh tank with capacity of 45,000 kg. A diverter was provided to allow static weighing for all tests and a timing activator for mid-stroke operation. At the maximum flow rate, the filling time for a weigh tank run was approximately 30 seconds.

The weighing system was a Toledo beam scale arranged with a direct reading dial and nine individual drop weights. Timing was accomplished by a photoelectric system mounted on the diverter which activated a 100 KH counter synchronized with a precision time base. Dead weight calibration of the scale and load cell was carried out on a routine basis every 6 months, while the time base was checked on a continuing basis each week. The cumulative error associated with this facility was known to be approximately 0.1% so that it formed a good primary standard for the LDA flow measurements.

3. Laser Doppler Anemometer

A laser doppler anemometer (LDA) operating in the forward scatter differential mode was used. The light beam from a 15 mw He Ne laser was split into two beams which passed into the test section through a plane window, Figure 2. The beams were brought to a focus at the crossing point where the velocity measurement was made. A photomultiplier on the opposite side of the duct detected light scattered from particles in the measuring volume modulated at the doppler frequency. Figure 3 shows the 46 cm test section, traversing frame, laser, and transmission optics. The receiving optics were also mounted on the frame, but out of view. Figure 4 shows the measuring volume just inside the window on the 92 cm duct.

*Figures in brackets indicate the literature references at the end of this paper.
The instantaneous particle velocity, $u$, was given by the equation

$$u = \frac{\lambda f_D}{2 \sin \theta/2}$$

where $\lambda$ was the wavelength of the laser radiation, $f_D$ was the doppler frequency, and $\theta$ was the beam angle. The radiation wavelength was known accurately, and the beam angle was determined by projecting the two beams onto a distant screen and using careful geometrical measurements. The accuracy of a velocity measurement thus hinged on measurement of the doppler frequency. This measurement was made with a frequency tracker which utilized a voltage controlled oscillator and an error detector to track the doppler signal. Either the control voltage or the VCO output could be monitored as an indication of velocity. The measurements of mean velocity reported here were made using a digital voltmeter with variable time constants, although it was later found that a frequency meter monitoring the VCO gave better results. In either case, the system was conveniently and accurately calibrated using a crystal frequency source.

Thus, all quantities associated with a velocity measurement were accurately and easily traceable. Furthermore, the velocity and signal output were linearly related, which simplified subsequent processing. An error analysis indicated that the accuracy of the velocity measurements was approximately $\pm 0.18\%$.

4. Integration for Flowrate

The traversing mechanism provided for the LDA facilitated positioning the measuring volume at specific locations in the cross-section. The positions were chosen as $4 \times 11$ and $11 \times 11$ matrices such that the flow rate could be obtained from the mean velocities using Gaussian quadrature [3].

The volumetric flowrate was determined as

$$Q = \sum_{i=1}^{11} \sum_{j=1}^{4} A \mu_i \mu_j u_{ij}$$

where

- $A$ = duct area
- $x_i, x_j$ = fractional location of point from centerline
- $\mu_i, \mu_j$ = weights
- $u_{ij}$ = measured velocity at $i, j$
The Gaussian weights and fractional locations are given in Table I.

<table>
<thead>
<tr>
<th>i, j</th>
<th>$x_i$</th>
<th>$\mu_i$</th>
<th>$x_j$</th>
<th>$\mu_j$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.978</td>
<td>0.056</td>
<td>0.861</td>
<td>0.348</td>
</tr>
<tr>
<td>2</td>
<td>0.887</td>
<td>0.126</td>
<td>0.340</td>
<td>0.652</td>
</tr>
<tr>
<td>3</td>
<td>0.730</td>
<td>0.186</td>
<td>0.340</td>
<td>0.652</td>
</tr>
<tr>
<td>4</td>
<td>0.519</td>
<td>0.233</td>
<td>0.861</td>
<td>0.348</td>
</tr>
<tr>
<td>5</td>
<td>0.269</td>
<td>0.263</td>
<td>0.000</td>
<td>0.273</td>
</tr>
<tr>
<td>6</td>
<td>0.000</td>
<td>0.273</td>
<td>0.269</td>
<td>0.263</td>
</tr>
<tr>
<td>7</td>
<td>0.269</td>
<td>0.263</td>
<td>0.519</td>
<td>0.233</td>
</tr>
<tr>
<td>8</td>
<td>0.519</td>
<td>0.233</td>
<td>0.730</td>
<td>0.186</td>
</tr>
<tr>
<td>9</td>
<td>0.730</td>
<td>0.186</td>
<td>0.978</td>
<td>0.056</td>
</tr>
<tr>
<td>10</td>
<td>0.887</td>
<td>0.126</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>0.978</td>
<td>0.056</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

To the extent that velocity profiles could be represented by Legendre polynomials, the integration would be exact. For more realistic profiles, the accuracy was checked by integrating over a theoretical velocity distribution derived from a law of the wall. An exact integration was made and compared to the 4 x 11 and 11 x 11 schemes corresponding to the measurements. Based on this calculation, the 4 x 11 and 11 x 11 schemes should have overestimated the flow rate by 0.61% and 0.13%, respectively.

5. Results

Mean and rms velocity measurements were made at each of the 165 grid points. Each measurement required approximately two minutes to insure proper time averaging. A flow rate determination was made after each 11 measurements and slight temporal variations accounted for in subsequent data reduction. Three useful runs were made using the smaller duct, while five were made for the larger. The results are summarized in Table II.

<table>
<thead>
<tr>
<th>Size cm</th>
<th>Test No.</th>
<th>Test Date</th>
<th>Flow m$^3$/s</th>
<th>% 11 x 11</th>
<th>% 4 x 11</th>
<th>$\omega$ x 10$^{-6}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>46 x 46</td>
<td>4</td>
<td>1/29/75</td>
<td>1.2013</td>
<td>+0.37</td>
<td>+0.70</td>
<td>3.5</td>
</tr>
<tr>
<td>46 x 46</td>
<td>5</td>
<td>2/05/75</td>
<td>1.2098</td>
<td>-0.01</td>
<td>+0.45</td>
<td>3.9</td>
</tr>
<tr>
<td>46 x 46</td>
<td>6</td>
<td>2/07/75</td>
<td>0.5955</td>
<td>+0.30</td>
<td>+0.73</td>
<td>1.9</td>
</tr>
<tr>
<td>92 x 92</td>
<td>7</td>
<td>5/06/75</td>
<td>1.2470</td>
<td>-0.24</td>
<td>+0.83</td>
<td>1.8</td>
</tr>
<tr>
<td>92 x 92</td>
<td>8</td>
<td>5/12/75</td>
<td>0.6370</td>
<td>-0.12</td>
<td>+0.68</td>
<td>1.0</td>
</tr>
<tr>
<td>92 x 92</td>
<td>9</td>
<td>5/19/75</td>
<td>1.2418</td>
<td>+0.97</td>
<td>+1.42</td>
<td>2.0</td>
</tr>
<tr>
<td>92 x 92</td>
<td>10</td>
<td>5/20/75</td>
<td>0.6222</td>
<td>+0.65</td>
<td>+1.16</td>
<td>1.1</td>
</tr>
<tr>
<td>92 x 92</td>
<td>11</td>
<td>6/17/75</td>
<td>1.2498</td>
<td>-0.48</td>
<td>+0.59</td>
<td>1.8</td>
</tr>
</tbody>
</table>
Keeping in mind that errors of 0.61% and 0.13% should be expected for the 4 x 11 and 11 x 11 schemes, respectively, because of the integration method, it can be seen that flow rate determinations using the LDA are quite good. In fact, after accounting for the integration error, the maximum percentage flow rate error for the 4 x 11 scheme was +0.81/-0.16 while the 11 x 11 scheme was +0.84/-0.61. Similarly, the mean errors were +0.21% and -0.35%, respectively.

6. Conclusions

The precision of the system, including the LDA and the integration, was good. The system required no calibration other than a tape measure to determine beam angle and a frequency standard to calibrate the tracker. The possibility of using standards other than volume or mass in connection with flow metering is thus open. The major limitation is seen to be the length of time required to make a flow rate determination. It is speculated, however, that suitable hardware could be arranged to provide fast scanning of the flow cross-section.

7. Acknowledgements

Much of the work described herein was originally supported by the Oceanic Division of Westinghouse Electric Corporation in connection with the development of their LEFM flow meter. Matching funds from an internal development fund enabled acquisition of the laser Doppler system.

8. References


BREAKDOWN VALVE

FIGURE 1 TEST FACILITY

45,000 kg WEIGHING TANK

FIGURE 2 TEST SECTION

46 x 46 CM DUCT

PHOTOMULTIPLIER

FLOW

15 mw He Ne

DISA OPTIC HEAD

FL = 60 CM

4 x 11 VELOCITY MEASUREMENT PATTERN
FIGURE 3  46 cm DUCT, LDA, AND TRAVERSING FRAME

FIGURE 4  BEAM CROSSING IN 92 cm DUCT