RFID ANTENNA COVERAGE OPTIMIZATION

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By
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ABSTRACT

RFID ANTENNA COVERAGE OPTIMIZATION

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This research focuses on the problem of determining the location of RFID antennas required to read RFID tags from all items in a facility, such that the number of antennas is minimized. We formulate the problem as a Set Covering optimization problem. We develop a heuristic algorithm for this NP-Complete problem. We also develop a computerized system, RFIDMIN, which enables for the automated calculation of the minimum number and location of RFID antennas, given the size of the facility and antenna specifications. RFIDMIN can be used by companies to implement an effective RFID system at lowest hardware costs.

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Chapter 1: Introduction

Global supply chains and complex manufacturing systems require sophisticated control methodologies. Accurate information about inventories and supply chain transactions is the basis for such control, which may significantly increase revenue and decrease costs. Therefore, companies have been investing time and money in developing methods for inventory tracking within the company and throughout its supply chain.

Inventory tracking used to be completely manual, on paper, until the invention of the universal product code (UPC), commonly known as barcode, by George Lauer of IBM (Cole et al., 2003). The UPC helped to semi-automate the retail store checkout process, thus revolutionizing the retail industry. UPC technology allows for items to be held a few inches away from a scanner, and the read UPC is then matched by a computer to the UPC number of the product in the company database. The first item ever scanned in a retail establishment, according to the book "Punched Cards To Bar Codes", was at Marsh's supermarket in Troy, Ohio at 8:01 a.m. on June 26, 1974; and was a 10-pack of Wrigley's Juicy Fruit chewing gum (Nelson, 1997). Currently, a UPC can be found on almost every item (clothes, food, cars, etc.).

With the onset of technology in the late 90's, inventory tracking saw a great improvement possibility with the development of automatic identification methods within the wireless communication arena. One such method, known as radio frequency identification or RFID, saw mainstream possibilities through the development of the electronic product code (EPC) developed at the MIT Auto-ID Center (Brock, 2001). RFID technologies allow for items to be scanned by an RFID reader, and the read EPC is then matched by a computer to the EPC number of the product in the company database. The main advantages of RFID over barcode are that

RFID tags can be read from great distances, which depend on the tag type, and that inventory items can be uniquely identified, since the number of potential EPC codes is extremely large.

There are two main tag varieties; passive and active. Passive tags have no battery power, limited read range, and low cost. Active tags are battery powered, have a significantly larger cost, and can be read from large distances. Companies select the tag type based on application and budget. In this study, we focus on passive RFID technologies. More specifically, we focus on the UHF (ultra high frequency) range of passive RFID and its band of frequencies mandated by retail giants (such as Wal-Mart, Target, Best Buy, Metro Group) and the US Department of Defense. This band is 860MHz – 960MHz.

A typical RFID system consists of a reader, antennas, and tags (see **Figure 1**). The tag is usually attached to an object that is to be identified. Radio transmissions are used by the reader to send a query to the tag, and by the tag to return an answer, generally containing identifying information. The reader sends the identifying information to a network, or sometimes directly to a host computer, where it can be displayed in human-readable form, or incorporated into a database to track objects and guide the activities of people and machines.

Figure 1: Schematic depiction of a simplified RFID system (courtesy PolyGAIT)

Implementations of RFID technology for inventory tracking are still rare. In most of these implementations, facilities tend to install readers and antennas at critical stages of movement or handling, such as by a dock door or before packaging. This is a result of the high cost of the technology. RFID readers can range in price from \$1,000 to \$5,000 and are typically connected to up to four antennas, which range in price from \$100 to \$500. RFID tags range in price from \$0.12 to \$300, depending on the tag type, and RFID software ranges in price from \$5000 to several million dollars.

Placing the RFID antennas in “strategic locations” allows the company to continuously capture all or most of the inventory movements. However, once an item is out of the antenna reading range it can easily be lost or misplaced. If a company were able to fully “cover” its facility (i.e., read all RFID tags located within the facility), there would be no lost inventory and work in process (WIP) would ultimately be reduced. This would save a company bottom-line dollars and increase revenues, which are necessary for a company’s survival in an uncertain economy.

With mandates from giant organizations such as Wal-Mart, Target, Metro Group and even the US Department of Defense, requiring all suppliers to tag their inventory in the foreseeable future, RFID is becoming a popular technology. If a system were developed that would logically place antennas to fully cover a facility, more companies would see RFID as the solution for their inventory tracking.

This research focuses on the problem of determining the optimal locations of RFID antennas in a given space, such that all inventory items are “seen” by these antennas and the number of antennas is minimized. Given certain assumptions regarding the conversion of the

given space to a grid of points representing potential inventory item locations, we show that the resulting optimization problem is a set covering problem.

In addition to the mathematical model, we develop a methodology for the space to grid conversion, and a heuristic solution algorithm for the problem. A computerized system that follows this methodology, RFIDMIN, can be used by stores, warehouses, manufacturing facilities, and service operations, in conjunction with linear programming (LP) software or our greedy algorithm, to calculate the optimal number and location of RFID antennas.

While this research is intended for applications within RFID antenna coverage, there are several other areas where this research is applicable. For instance, the 2D aspects of RFIDMIN may be used for problems such as sprinkler system design, where you are trying to minimizing the number of sprinkler heads used to fully cover a field. Another instance is motion sensors within the security realm, where completely covering the facility in 3D is necessary, while minimizing the cost of the technology involved.

This paper is organized in the following manner: chapter two describes previous research in RFID antenna allocation and similar scenarios in wireless sensor networks; chapter three provides background information and modeling assumptions for the following chapters; chapter four explains the formulation of the antenna minimization problem as a set covering problem, and develops the greedy heuristic solution algorithm for this problem; chapter five discusses the design and development of RFIDMIN in two dimensions; chapter six discusses the design and development of RFIDMIN in three dimensions; and the final chapter concludes the research and gives suggestions for further research.

Chapter 2: Literature Review

Wireless Communication

As RFID is a wireless communication technology, we first review articles describing models of 3D coverage in wireless communication and relate them to RFID antenna optimization. For a comprehensive description of wireless technologies, frequency spectrum allocation and general standards, we refer the reader to Goldsmith (2005).

Huang et al. (2004) propose a polynomial algorithm for the α -coverage wireless networks problem. Alpha, in this paper, is a given integer, and every point in the field is covered by at least α sensors. Adickes et al. (2002) describe a methodology for quickly identifying the optimal number of receivers and their locations in a facility. The optimal position of sensors is acquired by using genetic algorithms to run simulations beginning with an initial placement of stations and then to generate new sets of positions until one solution is accepted. Brown et al. (2007) use grid lattice arrangements for ad-hoc sensor networks. In all these situations, however, the sensor field shape is assumed to be a sphere, since cell phone towers in wireless computers networks use only omni-directional or combinations of spectumized antennas that send out RF signal in all directions (Goldsmith, 2005). The characteristics of various types of antennas are further explained below, as well as in chapter 3.

RFID Antenna Coverage Patterns

There is a basic trade-off between antenna polarization and reading range. More specifically, the greater the spread of signal, the weaker the signal becomes (Dobkin, 2007). The transmitted signal power for cellular phone towers and satellite applications must, therefore, be

very large. RFID antennas, on the other hand, are limited in power, thus requiring the use of sectorized or directional antennas that cover a limited range of signal angles (Goldsmith, 2005, and Dobkin, 2005). RFID antennas intended to cover indoor spaces should typically be mounted on walls, posts, or ceilings. For antennas that are mounted on walls, a spherical sensor shape is not practical or applicable, since the signal is stronger for directional antennas. Therefore, RFID antenna research focuses on directional antennas.

Dobkin (2007) provides a method for 3D antenna beam approximation for directional antennas. Sydänheimo et al. (2006) find that read ranges achieved with linearly polarized antennas are longer due to smaller polarization losses as compared with circular polarized antennas. Keskilammi et al. (2003) showed that patch antennas generate the largest read range, and found that by using high gain antenna, the read range can be increased with a maximum value of 2.76m for a dipole (9.05 ft) and 4.64m (15.22ft) for a patch antenna. Nikitin and Rao (2006) confirm these concepts and add that tag range can be maximized by designing a high-gain antenna well matched to the chip impedance. Our experience with later patch antenna models (2006-2008) results in reading distances of up to 30 feet (Freed et al. 2008).

Since we are analyzing facilities where multiple tags and multiple antennas are deployed, we must analyze collision between signals sent by multiple antennas and multiple tags. Engels (2001) solves the antenna collision problem by allocating frequencies to a set of antennas over time, in a distributed, but centrally controlled manner. Sarma et al. (2003) provide a method for collision free tag communication using algorithms similar to those used in networking.

Wang et al. (2007) provide a methodology for optimizing the number and location of multiple antennas to maximize reading accuracy. This is critical in the application of portal

antennas, insuring that all items are read when progressing through the portal. However, this work is only applicable for portal antennas.

Anusha (2005) presents a thesis involving RFID networks for mobile antennas. The antennas are mounted on forklifts or other warehouse vehicles and capture RFID tag reads while driving through the warehouse. However, this work uses a 2D coverage model and circular antennas. Furthermore, the model does not continuously read every item within the facility, as the mobile antennas cover each item only periodically.

Combinatorial Optimization Formulation and Solution Methods

Facility location problems are similar to our problem in that they seek to cover a space using minimum number of locations in that space. Facility Location Optimization is a classical research area in Operations Research, in which problems dealing with the “best” location of different types of facilities in various settings and under various constraints are studied (Ben-Moshe, 2004). Several papers were written on this problem, including Daskin (1995) and Mirchandani and Francis (1990). In these papers, the given space to be covered is translated from continuous space to discrete space. This idea is used in this paper as well (Chapter 3).

Given a discrete space with a set of locations, Garey and Johnson (1979) provide the basis for the formulation of the minimum number of antennas required to cover a given space. In their book, Garey and Johnson describe the Minimum Cover problem, which can be easily shown to match our optimization problem. Francis et al. (1992) provide a formulation for the minimum number of facilities to cover a set of customers. This is very similar to our case and we show the match in Chapter 4. Meguerdichian and Potkonjak (2003) use the Minimum Cover problem to model situations that arise in wireless sensor networks to reduce overall energy consumption. However, the Minimum Cover model has never been used for RFID problems.

Generally there are four broad approaches reported for solving covering problems. Since the problem is NP-Complete (Karp (1972), Garey and Johnson (1979)), seeking optimal solutions for an industrial problem that requires quick solutions may not be appropriate. Of the four categories of solution approaches, three are optimum seeking and one is heuristic. The first solution approach is an implicit enumeration approach, such as branch and bound, as in Lawler (1966) and Pierce (1968). A second approach is to use cutting-plane methods and solve iteratively a number of linear-programming problems, as in Bellmore and Ratliff (1971). The third approach is to employ reduction techniques, as in Balinski (1965) and Roth (1969). The last approach involves the use of heuristic methods, as in Ignizio (1971). We will use a heuristic rule in the form of a greedy heuristic algorithm.

There are several heuristic solution algorithms for the Minimum Cover problem that lend themselves well to our minimum antenna cover problem. These solution algorithms are not application-specific, and include Slavík (1996) and Bar-Yehuda (2000). Solution algorithms demonstrated specifically in wireless communication, include Calégari et al. (2001) and Ben-Shimol et al. (2007). We will discuss these algorithms in detail in Chapter 4.

For small problems, our paper focuses on developing an optimal location assignment for the minimal number of directional RFID antennas, using the Minimum Cover, binary integer linear programming model. For larger problems we develop a greedy heuristic algorithm to quickly find a solution. We also develop RFIDMIN, an automated system that converts the space to a discrete grid, and can be used in conjunction with LP software or an algorithm to determine the minimum number and location of RFID antennas, such that any given 2D and 3D space is covered by at least one antenna.

Chapter 3: Background and Modeling Assumptions

The problem of determining the optimum number and location of RFID antennas required to completely cover any given two-dimensional and three-dimensional space is studied in this paper. Common applications can be found in warehouse environments and manufacturing facilities that want continuous visibility of the work-in-process or other inventory. In such environments RFID antennas can be mounted on walls, ceilings, and support posts. Since a passive RFID UHF antenna can typically read tags located up to 30 feet away (Freed et al. 2008), a single antenna cannot typically cover an entire warehouse, and multiple antennas must be used. We assume that multiple antennas are required, and that the number and layout of these antennas is a function of the facility dimensions (MaxX, MaxY, MaxZ), as well as the antenna read range (RR) and read/coverage angle (RA).

We convert the given space facility to a grid with assumed spacing between measurements (granularity or resolution). This idea of converting a continuous space to a discrete one had been used for facility location problems, as in Francis et al. (1992) and Daskin (1995). We first analyze the problem for two-dimensional spaces, and then for three-dimensional areas. Using 2-D spaces is justified when the height of the tags is fixed (e.g., at desktop level). Using 3-D areas is appropriate for the general case, where tag height varies.

We assume that directional, patch antennas are used. Since the reading angle (RA) of a directional antenna is smaller than 360 degrees, we assume that an antenna location is defined by a grid point and the direction of the vector perpendicular to the antenna. We further assume that this vector can point either North, South, East, or West.

We further assume that antenna collision and interference is negligible. Minimal interference can be achieved by rotating the power to the antennas, so that only a single antenna

is used at any given moment (Asif and Mandviwalla, 2005). We also assume an ideal environment where reflections from conductive surfaces and background noise (Wi-Fi, cellular, radio), are negligible.

Chapter 4: Combinatorial Optimization Formulation, Example, Validation

Given tagged items located in a continuous area of dimensions MaxX, MaxY, MaxZ, our problem is to find the optimal locations for RFID antennas, such that all tags are read by at least one antenna, and the number of antennas is minimized. We simplify the problem by converting the continuous area to discrete space of prespecified resolution (grid). We then identify a subset of the grid associated with each possible antenna location. That is, if we select to place an antenna on grid point A, then several other grid points (e.g., B, C, J, K) constitute the subset of grid point A, reflecting the fact that these points can be read by the RFID antenna located in grid point A. Our problem thus becomes the problem of choosing the minimal number of points on the discrete grid, such that the union of their corresponding subsets includes all the points on the grid. This problem is similar to the Minimum Cover problem. In Garey and Johnson (1979), the Minimum Cover combinatorial optimization problem is described as follows: For a collection C of subsets of a finite set S , and a positive integer $K \leq |C|$ find a cover for S of size K or less, i.e. a subset $C' \subseteq C$ with $|C'| \leq K$ such that every element of S belongs to at least one member of C' .

In our situation S is the set of grid points. Every possible location of an RFID antenna covers a subset of S ; thus C is the collection of all possible antenna locations and their resulting subsets of S . C' is a potential solution, i.e., a set of grid-point locations of RFID antennas, such that the union of the resulting subsets of S covers all of the grid point of S . K is the size of C' , which is the number of selected RFID antenna locations. The goal is to minimize K .

Formulation

Our Minimum Cover formulation is similar to the formulation by Meguerdichian and Potkonjak (2003), who try “to find the smallest number of sensors that must be active in order to guarantee that every observable point in the sensor field is observed by at least one active sensor.” This formulation can be stated as follows:

- Given:*
1. A set S , where $S = \{s_1, s_2, \dots, s_{|S|}\}$ is a partitioning of S to its elements (grid points).
 2. A set A , where $A = \{a_1, a_2, \dots, a_{|A|}\}$ of subsets of S , such that each element of S is included in at least one element of A ;
 3. An area coverage matrix $C_{|A| \times |S|}$, where an element $C_{ij} = 1$ if antenna $a_i \in A$ and $s_j \in S$, $s_j \in a_i$, and $C_{ij} = 0$ otherwise.

Problem: Find the minimum set of antennas, represented by the vector $X_{1 \times |A|}$ where X_i is 1 if antenna $a_i \in A$ is in the selected set and 0 otherwise.

Minimize: $X_{1 \times |A|} \cdot \mathbf{1}_{|A| \times 1}$

Subject to: $X_{1 \times |A|} \cdot C_{|A| \times |S|} \geq \mathbf{1}_{1 \times |S|}$

Example

A simplified case study with known optimal values was created to validate the model.

The parameters of this case study are:

Given: $S = 4 \text{ ft} \times 4 \text{ ft}$, with 1 ft granularity for a total of 25 grid points

$A = 100$ possible antenna locations (25 possible grid points $\times$ 4 possible antenna directions- n,s,e,w)

$C_{|A| \times |S|} =$ See Appendix A: Validation of ILP

While this case may appear to be complex, it is indeed very simple. A facility size of sixteen square feet, with each antenna covering four square feet, means that four antennas is the minimum number to cover every location (See **Figure 2**). Antenna 1 is located at (1,0), antenna 2 is at (3,0), antenna 3 is at (1,4), antenna 4 is at (3,4).

Y/X	0	1	2	3	4
0	1	1	1+2	2	2
1	1	1	1+2	2	2
2	1+3	1+3	1+2+3+4	2+4	2+4
3	3	3	3+4	4	4
4	3	3	3+4	4	4

Figure 2: 4 ft x 4ft space being covered by 4 antennas

Given the facility size and antenna components, the formulation turned into:

$$\text{Minimize: } X_{1 \times |100|} \cdot \mathbf{1}_{|100| \times 1}$$

$$\text{Subject to: } X_{1 \times |100|} \cdot C_{|100| \times |25|} \geq \mathbf{1}_{1 \times |25|}$$

After defining the coverage area of 100 antenna locations, the solution reached by Excel Solver verified four antennas is the minimum number to adequately cover all points (Refer to Appendix A: Validation of ILP).

Greedy Algorithm for Antenna Cover

Given that the antenna allocation problem is similar to facility location problems and thus NP-complete (Savendy, 2001), we developed a greedy heuristic algorithm that finds a solution relatively quickly. Slavík (1996) states the greedy algorithm for minimum covering as:

The greedy algorithm for approximating a minimum subcover, at each step, simply chooses the covering set with the maximum number of elements left, deletes these elements from the remaining covering sets, and repeats this process until the ground set U is covered. We can assume that in case of a tie, the set with smaller subscript is chosen.

Chvátal (1979) proves that the difference between the upper and lower bounds turns out to be less than 1.1. Therefore, we develop a similar algorithm that results in near optimal solutions and is faster than excel solver optimization. Our greedy algorithm for antenna coverage optimization is as follows:

Step 1: Generate binary tables for given facility size using RFIDMIN.

Table 1: Greedy Algorithm Step 1

	f 0,0	f 0,1	f 0,2	f 0,3	f 0,4	f 1,0	f 1,1	f 1,2
$R_{0,0}$	1	1	1	0	0	1	1	1
$R_{0,1}$	0	0	0	1	0	0	0	0
$R_{0,2}$	0	0	0	0	1	0	0	0
$R_{0,3}$	0	0	0	0	0	0	0	0
$R_{0,4}$	0	0	0	0	0	0	0	0

Step 2: Eliminate suboptimal antennas at locations with multiple antennas (e.g. if two antennas, A & B, at the same location with antenna A covering the same subset of B, eliminate antenna B).

Table 2: Greedy Algorithm Step 2

	f 0,0	f 0,1	f 0,2	f 0,3	f 0,4	f 1,0	f 1,1	f 1,2
$R_{0,0}$	1	1	1	0	0	1	1	1
$R_{0,1}$	0	0	0	1	0	0	0	0
$R_{0,2}$	0	0	0	0	1	0	0	0
$R_{0,3}$	0	0	0	0	0	0	0	0
$R_{0,4}$	0	0	0	0	0	0	0	0

Step 3: Select an antenna with the largest set of unique locations. The first antenna selected will always be one of the corner antennas.

Step 4: Eliminate the appropriate columns associated with the set of unique locations the first antenna covered.

Table 3: Greedy Algorithm Steps 3 and 4

	f 0,0	f 0,1	f 0,2	f 0,3	f 0,4	f 1,0	f 1,1	f 1,2
R 0,0	1	1	1	0	0	1	1	1
R 0,1	0	0	0	1	0	0	0	0
R 0,2	0	0	0	0	1	0	0	0
R 0,3	0	0	0	0	0	0	0	0
R 0,4	0	0	0	0	0	0	0	0

Step 5: Select the next antenna with the largest set of unique locations remaining in the binary data table. In the event of a tie between multiple antennas, randomly select the next antenna.

Table 4: Greedy Algorithm Step 5

	f 0,0	f 0,1	f 0,2	f 0,3	f 0,4	f 1,0	f 1,1	f 1,2
R 0,0	1	1	1	0	0	1	1	1
R 0,1	0	0	0	1	0	0	0	0
R 0,2	0	0	0	0	1	0	0	0
R 0,3	0	0	0	0	0	0	0	0
R 0,4	0	0	0	0	0	0	0	0

Step 6: Repeat Steps 4 and 5 until all locations within the sensor field are covered.

Table 5: Greedy Algorithm Step 6

	f 0,0	f 0,1	f 0,2	f 0,3	f 0,4	f 1,0	f 1,1	f 1,2
R 0,0	1	1	1	0	0	1	1	1
R 0,1	0	0	0	1	0	0	0	0
R 0,2	0	0	0	0	1	0	0	0
R 0,3	0	0	0	0	0	0	0	0
R 0,4	0	0	0	0	0	0	0	0

A separate, larger example of 30 grid points including the optimal solution with the Excel Solver and the greedy algorithm is included in Appendix A: 2D: 4 ft x 5 ft, granularity =1, RR=1, RA=180°, RRC=2.

Optimal versus Greedy Solution

This research proposes two separate methods of finding a solution to RFID antenna coverage, either via optimization or a heuristic algorithm. Optimization is the preferred solution technique in all cases. However, given a large facility with a small granularity, a very expensive, comprehensive LP package would be needed to solve the problem. Therefore, we suggest using the heuristic algorithm to easily find a solution. The heuristic algorithm would be practically in any application where a common LP package, such as Solver, cannot find a solution.

We propose two options for RFID antenna placement, an optimal, which would have the lowest cost and greedy solution, which could have 50% more antennas than the optimal solution. But in implementation of the greedy solution, this cost would be significantly larger due to the additional readers that would be needed, along with wiring and installation costs. Given that a large facility could need several hundred antennas to fully cover the facility; this could mean a difference in tens of thousands of dollars. Specific analysis of which method to use would have to be done on a case by case basis; as the difference in antennas along with the cost of the LP software package needed, varies per application.

Chapter 5: Design and Development of RFIDMIN in 2D

Design and development of RFIDMIN began with analysis of whether to use circular or directional antennas. There has been much research into circular antenna's and its coverage field, however circular antennas have a smaller read range and must be positioned facing into the field. This would work with smaller facilities, but given a typical warehouse's ceiling height of greater than 20 feet; circular antennas would not cover the complete ceiling to floor distance. Therefore, it was determined, that directional antennas would work best since they can be mounted on roof support posts or attached to overhead electrical piping.

Given that we will be only working with directional antennas, we then had to interpret an antenna sensor field in terms of all of the specific locations it covers. We decided that we could determine the sensor field from the antenna read range (RR) and read angle (RA), since only one antenna would be powered on at a given time. This turns the situation into essentially a huge geometry problem. Since RR and RA are fixed, we can determine every point within the read cloud. Furthermore, given that the sensor field is a translation about the location of the antenna and its projection into the field, we only need to find one of each antenna type.

In the two dimensional situation, there are 8 separate antenna orientations; one for each corner and one for each separate wall of the facility. The corners are special cases, since the RA is always 90 degrees, which generates a larger RR (Sydänheimo et al., 2006). Therefore, we denote the RR of all Corner Antennas as RRC. Corner antennas are mounted at orientations of (45, 135, 225, and 315 degrees) with a RA of $\pm 45^\circ$. A wall antenna can be mounted on four separate orientations (90, 180, 270, 360 degrees), which means that each antenna is essentially just rotated about a fixed axis.

RFIDMIN, in the 2D situation, should be used in environments where manufacturing height has minimal variation (e.g. identifying WIP in cellular manufacturing) or conveyor application where the field of interest is with the items location on the conveyor.

Terminology

Before understanding how the conversion of the space is performed, we must first understand the notation that will be described in the following section (see Table 6).

Table 6: RFIDMIN Notation

Notation	Definition
x', y', z'	The location on the grid of the specific antenna
$xmin, xmax, ymax, ymin, zmin, zmax$	Minimum and Maximum values for the whole antenna field
$\underline{min\ y}, \underline{max\ y}, \underline{min\ z}, \underline{max\ z}$	Minimum and Maximum values of the antenna field at a specific location within the field
$RR?, RRC?$	The new read range (RR) at each cross-sectional cut of 3D antenna shapes ($RR1, RR2, \dots, RRn$ – See Figure 16, page 31)

Wall Antennas

The first antenna to be analyzed is the Left Wall Antenna or the antenna with a projection into the facility at 90° (see Figure 3). At this location (x', y') , $xmin$ will always be zero and the y values vary in increments of granularity. We know that at the mounting location on the wall, $xmax$ equals RR . We can also determine that $ymin$ and $ymax$ equals $y' \pm RR \cdot \sin(RA/2)$, respectfully. When we increase one level of granularity $(x' + \text{granularity})$, $\underline{min\ y}$ and $\underline{max\ y}$ have two possible outcomes. If $(x' + \text{granularity}) \leq RR \cdot \cos(RA/2)$, then $\underline{min\ y}$ equals $y' - (x' + \text{granularity}) \cdot \tan(RA/2)$; otherwise $\underline{min\ y}$ equals $y' - (RR^2 - (x' + \text{granularity})^2)^{1/2}$. If $(x' + \text{granularity}) \leq RR \cdot \cos(RA/2)$, then $\underline{max\ y}$ equals $y' + (x' + \text{granularity}) \cdot \tan(RA/2)$;

otherwise max y equals $y' + (RR^2 - (x' + \text{granularity})^2)^{1/2}$. Keep repeating the previous step varying the number of granularities until $(x' + \text{granularity}) > RR$. These calculations give us the range of y values at each and every x value that antenna covers. These steps need to be repeated for each and every antenna at locations $(0, \text{granularity})$ to $(0, \text{MaxY-granularity})$. If the facility is greater than twice the RR, it will be used for locations $(0, \text{granularity})$ to $(\text{MaxX}, \text{MaxY-granularity})$; with x' changed in the previous steps to the x location of the antenna.

Figure 3: Left Wall Antenna Geometry

Next, we will focus on the Bottom Wall Antenna or the antenna with a projection into the facility at 180° (see **Figure 4**). At this location $(x', y' = \text{MaxY})$, $y_{\max}$ will always be MaxY and the x values vary in increments of granularity. We know that at the mounting location on the wall, $y_{\min}$ equals $\text{MaxY} - RR$. We can also determine that $x_{\min}$ and $x_{\max}$ equals $x' \pm RR \sin (RA/2)$, respectively. When we increase one level of granularity $(x' + \text{granularity})$, min y equals $y' - (RR^2 - (x' + \text{granularity})^2)^{1/2}$ and max y equals $y' - (x' + \text{granularity}) \tan (90 - RA/2)$. Keep repeating the previous step varying the number of granularities until $(x' + \text{granularity}) > RR \sin (RR/2)$. The values you found for min y and max y at each $(x' + \text{granularity})$ are the same at $(x' -$

granularity)). These calculations give us the range of y values at each and every x value that antenna covers. These steps need to be repeated for each and every antenna at locations (granularity, MaxY) to (MaxX - granularity, MaxY). If the facility is greater than twice the RR, it will be used for locations (granularity, 0) to (MaxX-granularity, MaxY); with y' in the previous steps changes to the y location of the antenna.

Figure 4: Bottom Wall Antenna Geometry

The third antenna to be analyzed is the Right Wall Antenna or the antenna with a projection into the facility at 270° (see Figure 5). At this location ($x' = \text{MaxX}$, y'), x_{max} will always be MaxX and the y values vary in increments of granularity. We know that at the mounting location on the wall, x_{min} equals $\text{MaxX} - \text{RR}$. We can also determine that y_{min} and y_{max} equals $y' \pm \text{RR} \cdot \cos(\text{RA}/2)$, respectfully. When we increase one level of granularity ($x' + \text{granularity}$), min y and max y have two possible outcomes. If $(x' + \text{granularity}) \leq \text{RR} \cdot \cos(\text{RA}/2)$, then min y equals $y' - (x' + \text{granularity}) \cdot \tan(\text{RA}/2)$; otherwise min y equals $y' - (\text{RR}^2 - (x' + \text{granularity})^2)^{1/2}$. If $(x' + \text{granularity}) \leq \text{RR} \cdot \cos(\text{RA}/2)$, then max y equals $y' + (x' + \text{granularity}) \cdot \tan(\text{RA}/2)$; otherwise max y equals $y' + (\text{RR}^2 - (x' + \text{granularity})^2)^{1/2}$. Keep repeating the previous step varying the number of granularities until $(x' + \text{granularity}) > \text{RR}$.

These calculations give us the range of y values at each and every x value that antenna covers.

These steps need to be repeated for each and every antenna at locations (MaxX, granularity) to (MaxX, MaxY-granularity). If the facility is greater than twice the RR, it will be used for locations (0, granularity) to (MaxX, MaxY-granularity); with x' in the previous steps changed to the x location of the antenna.

Figure 5: Right Wall Antenna Geometry

The last wall antenna to be analyzed is the Top Wall Antenna or the antenna with a projection into the facility at 0° (see Figure 6). At this location (x', y'=0), ymin will always be 0 and the x values vary in increments of granularity. We know that at the mounting location on the wall, ymax equals RR. We can also determine that xmin and xmax equals $x' \pm RR \cdot \cos(RA/2)$, respectfully. When we increase one level of granularity (x'+granularity), min y equals $y' + (x'+granularity) \cdot \tan(90 - RA/2)$ and max y equals $y' + (RR^2 - (x'+granularity)^2)^{1/2}$. Keep repeating the previous step varying the number of granularities until $(x'+granularity) > RR \cdot \sin(RR/2)$. The values you found for min y and max y at each (x'+granularity) are the same at (x'-granularity). These calculations give us the range of y values at each and every x value that antenna covers. These steps need to be repeated for each and every antenna at locations

(granularity, 0) to (MaxX - granularity, 0). If the facility is greater than twice the RR, it will be used for locations (granularity, 0) to (MaxX-granularity, MaxY); with y' changed in the previous steps to the y location of the antenna.

Figure 6: Top Wall Antenna Geometry

Corner Antennas

The first corner antenna is the Left Bottom Corner Antenna or the antenna with projection onto the facility at 45° (see Figure 7). At this location ($x'=0$, $y'=\text{MaxY}$), y_{max} will always be MaxY and the x values vary in increments of granularity. We know that at the mounting location on the wall, y_{min} equals $y' - \text{RRC}$, y_{max} equals y' , x_{min} equals x' , and x_{max} equals $x' + \text{RRC}$. When we increase one level of granularity ($x'+\text{granularity}$), min y equals $y' - (\text{RRC}^2 - (x'+\text{granularity})^2)^{1/2}$ and max y equals y' . Keep repeating the previous step varying the number of granularities until $(x'+\text{granularity}) > \text{RRC}$. These calculations give us the range of y values at each and every x value that antenna covers.

Figure 7: Left Bottom Corner Antenna Geometry

The second corner antenna is the Right Bottom Corner Antenna or the antenna with projection onto the facility at 135° (see Figure 8). At this location ($x' = \text{MaxX}$, $y' = \text{MaxY}$), y_{max} will always be MaxY and the x values vary in increments of granularity. We know that at the mounting location on the wall, y_{min} equals $y' - \text{RRC}$, y_{max} equals y' , x_{min} equals x' , and x_{max} equals $x' - \text{RRC}$. When we decrease one level of granularity ($x' - \text{granularity}$), $\text{min } y$ equals $y' - (\text{RRC}^2 - (x' + \text{granularity})^2)^{1/2}$ and $\text{max } y$ equals y' . Keep repeating the previous step varying the number of granularities until $(x' - \text{granularity}) > \text{RRC}$. These calculations give us the range of y values at each and every x value that antenna covers.

Figure 8: Right Bottom Corner Antenna Geometry

The third corner antenna is the Right Top Corner Antenna or the antenna with projection onto the facility at 225° (see **Figure 9**). At this location ($x'=MaxX$, $y'=0$), $ymin$ will always be 0 and the x values vary in increments of granularity. We know that at the mounting location on the wall, $ymin$ equals y' , $ymax$ equals $y' + RRC$, $xmin$ equals $x' - RRC$, and $xmax$ equals x' . When we decrease one level of granularity ($x' - granularity$), min y equals y' and max y equals $y' + (RRC^2 - (x' + granularity)^2)^{1/2}$. Keep repeating the previous step varying the number of granularities until $(x' - granularity) > RRC$. These calculations give us the range of y values at each and every x value that antenna covers.

Figure 9: Right Top Corner Antenna Geometry

The last corner antenna is the Left Top Corner Antenna or the antenna with projection onto the facility at 315° (see **Figure 10**). At this location ($x'=0$, $y'=0$), $ymin$ will always be 0 and the x values vary in increments of granularity. We know that at the mounting location on the wall, $ymin$ equals y' , $ymax$ equals $y' + RRC$, $xmin$ equals x' , and $xmax$ equals $x' + RRC$. When we increase one level of granularity ($x' + granularity$), min y equals y' and max y equals $y' - (RRC^2 - (x' + granularity)^2)^{1/2}$. Keep repeating the previous step varying the number of granularities until $(x' + granularity) > RRC$. These calculations give us the range of y values at each and every x value that antenna covers.

Figure 10: Left Top Corner Antenna Geometry

Range to Value Conversion

With each of the eight possible antennas, we found out the maximum and minimum values of y given x or the maximum and minimum values of x given y , that the antenna covered. Since we had more than a couple antennas, we decided to undertake this task using Microsoft Excel. This helped to create a table (See Table 7) for each antenna at every location. Table 8 helps explain the missing columns in Table 7. Now we need to determine all of the locations in-between that the antenna covers, given its granularity.

Table 7: Data Table for a Antenna at (x', y')

x''	x'' value	$ x' - x $	In x Range	min y	MIN Y	max y	MAX Y

Table 8: Values for Data Table

Column Value	Description
x''	All the possible x values in the grid (from $x'' + \text{MaxX}$ to $x'' - \text{MaxX}$ in increments of Granularity)
x'' value	From <u>MaxX</u> to <u>-MaxX</u> in increments of Granularity .
$ x'' - x $	Absolute value x'' value.

<u>In x Range</u>	AND(<u>x' value</u> >=xmin, <u>x' value</u> <= xmax), gives TRUE or FALSE outcome
<u>min y</u>	From antenna equations
<u>MIN Y</u>	IF <u>In x Range</u> = TRUE, THEN IF <u>min y</u> < ymin, THEN <u>ymin</u> , ELSE ROUNDUP(<u>min y</u> ,0), ELSE -1
<u>max y</u>	From antenna equations
<u>MAX Y</u>	IF <u>In x Range</u> = TRUE, THEN IF <u>max y</u> > ymax THEN <u>ymax</u> , ELSE ROUNDDOWN(<u>max y</u> ,0), ELSE -1

To determine all of the locations in-between, we must first divide the facility into a grid. For the x axis, the facility will begin at 0 and end at MaxX. The number of total locations on the axis is determined by the ratio of (MaxX/granularity) +1. For instance, in a facility with MaxX = 21 and granularity=3, there are 8 locations (0,3,6,9,12,15,18,21). For the y axis, the facility will begin at 0 and end at MaxY. The number of total locations on the axis is again determined by the ratio of (MaxY/granularity) +1. For instance, in a facility with MaxY = 21 and granularity=3, there are also 8 locations (0, 3, 6, 9, 12, 15, 18, 21). To find the total number of antenna locations, you use the product of the two ratios or [(MaxX/granularity) +1] * [(MaxY/granularity) +1]. In the aforementioned case, *F* is equal to 64 locations (see Figure 11).

Y/X	0	3	6	9	12	15	18	21
0	0	0	0	0	0	0	0	0
3	0	0	0	0	0	0	0	0
6	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0
12	0	0	0	0	0	0	0	0
15	0	0	0	0	0	0	0	0
18	0	0	0	0	0	0	0	0
21	0	0	0	0	0	0	0	0

Figure 11: x=(0,21) y=(0,21) granularity=3.

Now that we have a table, we must reference the minimum and maximum values, given the initial antenna location. In most cases, you can easily analyze the range to determine what values of granularity are within it. However, it is much simpler using the offset function, which, according to the Excel Bible 2003, “returns a reference to a range that is a specified number of rows and columns from a cell or range of cells.” (Walkenback, 2003) In this case, the offset function must be used in conjunction with an IF and an AND function, which ensures that if two conditions are true we will get a 1; meaning that the antenna covers that certain (x, y) location. If one or both conditions were not true, we would get a 0; meaning that the antenna does not cover that certain (x, y) location (see *Table 9*). This converts Figure 11 into Figure 12.

Table 9: Example Offset Functions

Location	Excel Formula
(x=0,y=0)	IF AND (y=0>=OFFSET(MIN Y,(x=0 - x')/Granularity,0), y=0<=OFFSET(MAX Y,(x=0 - x')/Granularity,0)), THEN 1, ELSE 0
(x=21,y=0)	IF AND (y=0>=OFFSET(MIN Y,(x=21 - x')/Granularity,0), y=0<=OFFSET(MAX Y,(x=21 - x')/Granularity,0)), THEN 1, ELSE 0
(x=0,y=21)	IF AND (y=21>=OFFSET(MIN Y,(x=0 - x')/Granularity,0), y=21<=OFFSET(MAX Y,(x=0 - x')/Granularity,0)), THEN 1, ELSE 0
(x=21,y=21)	IF AND (y=21>=OFFSET(MIN Y,(x=21 - x')/Granularity,0), y=21<=OFFSET(MAX Y,(x=21 - x')/Granularity,0)), THEN 1, ELSE 0

Y/X	0	3	6	9	12	15	18	21
0	0	1	1	1	0	0	0	0
3	1	1	1	1	1	0	0	0
6	0	1	1	1	0	0	0	0
9	0	1	1	1	0	0	0	0
12	0	1	1	0	0	0	0	0
15	0	0	0	0	0	0	0	0
18	0	0	0	0	0	0	0	0
21	0	0	0	0	0	0	0	0

Figure 12: Left Wall Antenna Located at (0,3) with $x=(0,21)$ $y=(0,21)$ granularity =3

Now that we have a chart showing what locations an antenna covers, given its location; we can easily create a table (see *Table 10*) of the entire sensor field F , and show the individual antennas partitioning of it. Simply name each location $f_{x,y}$ and correlate that location with the value from the chart. This information will be used in the ILP for each and every antenna.

Table 10: Individual Antenna's Binary Coverage

$f_{0,0}$	$f_{0,3}$	$f_{0,6}$	$f_{0,9}$	$f_{0,12}$	$f_{0,15}$	$f_{0,18}$	$f_{\dots, \dots}$	$f_{\text{MaxX}, \text{MaxY}}$
0	1	0	0	0	0	0	...	...

Formulation Mapping to RFIDMIN

At this point, we have created Table 4 for each and every location; thus giving us every variable for our formulation. Each antenna needs to be named based on the type of antenna and its location (L 0,3 for a left wall antenna at location $x=0$ and $y=0$). All of the individual locations from Table 4 will be multiplied by the binary variable of the antenna and its location. Thus, if that antenna is selected, all of the points in the antenna coverage matrix will become a 1 if it covers that location or a zero otherwise. When then sum all of individual locations and set that sum greater than or equal to one. This ensures that each and every location is covered.

Chapter 6: Design and Development of RFIDMIN in 3D

RFIDMIN, in the 3D situation, should be used in environments where you want complete and total visibility of inventory. Two main applications areas include warehouses, such as distribution facilities, and retail stores, where inventory can be located at multiple heights. In order to transform the two dimensional model into a three dimensional model, some initial analysis was needed to be determined. First, what type of shape does the 3D antenna represent? Second, how does the solution technique learned from the 2D antenna apply to 3D? Third, how many antennas shapes are possible in the 3D case and what values does each shape contain? In order to answer these questions, we must first analyze the true antenna cloud of an RFID antenna.

The true antenna cloud of an RFID antenna (**Figure 13**) is based upon the gain. Furthermore, a true antenna cloud cannot be easily represented due to the side lobes. Dan Dobkin provided the necessary information to translate from a true antenna to an approximation, with:

The higher the gain of a directional antenna the more narrowly focused is the energy radiated from it. We can express the relationship mathematically by making the approximation that all the energy radiated by the antenna is uniformly distributed across a beam with some solid angle Ω_{beam} , and no energy is radiated elsewhere (2007).

We refer to Ω_{beam} as RA in our paper. When analyzing the directional antenna read cloud, we noticed that represented a cone (**Figure 14**). We assumed the field of the antenna to have a constant cone shape, just reducing in size by the change in the RR, at each respective granularity. With typical directional antennas used in RFID, the antenna reads a majority of the points close to the mounting location as you increase or decrease the granularity, due to the gain. This

translates into a large RA in the x-z axis. Accordingly, we assumed that the RA of the antenna in the x-z axis is the same as the RA in the x-y axis.

Figure 13: Pseudo-3D Radiation Pattern for Directional Antenna (courtesy Dobkin)

Figure 14: Beam Approximation of a Directional Antenna (courtesy Dobkin)

After deciding what shape to mimic each antenna, we then needed to determine how to figure out what points in the space that specific antenna covered. Dan Dobkin gave the idea of how to do this in The RF in RFID with “it is traditional to extract slices of the true radiation pattern in planes that pass through symmetry axes of the antenna” (2007). Since we used granularity as the basis for distance between points, we determined that each difference in z

height was essentially just a slice of the cross-section (See Figure 15). At each respective granularity, a new RR will be found. Accordingly, we will use each RR to mimic the antenna field found in the two dimensional case (See Figure 16).

Figure 15: Antenna field from the x,z axis

Figure 16: Antenna Field from x, y axis showing different cross-sectional cuts

When translating into a three dimensional situation, there are now 26 different antenna shapes. If we imagine the facility space as a cube, it would have 6 sides, 12 two-sided corners, and 8 three-sided corners. This translates into 6 wall mounted antennas and 20 corner antennas. It could be argued that you don't need antennas that are mounted on the floor and facing up, but it is possible in certain facilities that may be the only possible location to mount. Therefore, we will model all possible antennas.

Wall Antennas

The first antenna to analyze is the Left Wall Antenna or the antenna with a projection into the facility at 90°. At this location (x', y', z') , x_{min} will always be equal to 0, at the mounting location. However, when you vary the granularity in the z direction, this will not hold true. In the three dimensional case, x_{min} equals $ABS(?granularity / \tan(RA/2))$; changing the k to match the respective iteration. This point becomes the absolute zero when computing the cross-sectional area. Using simply geometry, you can find x_{max} to equal $(RR^2 - granularity^2)^{1/2}$. You can now find $RR?$ at that granularity by subtracting $x_{max} - x_{min}$.

After finding $RR?$, we can also determine that y_{min} and y_{max} equals $y' \pm RR? * \sin(RA/2)$, respectfully. When we increase one level of granularity $(x' + granularity)$, min y and max y have two possible outcomes. If $(x' + granularity - x_{min}) \leq RR? * \cos(RA/2)$, then min y equals $y' - (x' + granularity - x_{min}) * \tan(RA/2)$; otherwise min y equals $y' - (RR?^2 - (x' + granularity - x_{min})^2)^{1/2}$. If $(x' + granularity - x_{min})$ is $\leq RR? * \cos(RA/2)$, then max y equals $y' + (x' + granularity - x_{min}) * \tan(RA/2)$; otherwise max y equals $y' + (RR?^2 - (x' + granularity - x_{min})^2)^{1/2}$. Keep repeating the previous step varying the number of granularities until $(x' + granularity - x_{min}) > RR?$. These calculations give us the range of y values at each and

every x value that antenna covers. These steps need to be repeated for each and every slice until $\text{granularity} \geq RR * \sin \theta$. This process need to be repeated for each and every antenna at locations (0, granularity, granularity) to (0, MaxY-granularity, MaxZ-granularity).

The second antenna to analyze is the Bottom Wall Antenna or the antenna with a projection into the facility at 180° . At this location (x', y', z') , ymin will always be equal to y' , at the mounting location. However, when you vary the granularity in the z direction, this will not hold true. In the three dimensional case, ymax equals $y' + \text{ABS}(\text{?granularity} / \tan(RA/2))$; changing the ? to match the respective iteration. This point becomes the absolute zero when computing the cross-sectional area. Using simply geometry, you can find ymin to equal $y' + (RR^2 - \text{granularity}^2)^{1/2}$. You can now find $RR?$ at that granularity by subtracting $\text{ymax} - \text{ymin}$.

After finding $RR?$, we can also determine that xmin and xmax equals $x' \pm RR? * \sin(RA/2)$, respectfully. When we increase one level of granularity $(x' + \text{granularity})$, max y equals $\text{ymin} + (x' + \text{granularity} - \text{xmin}) * \tan(90 - RA/2)$ and min y equals $\text{ymin} + (RR^2 - (x' + \text{granularity} - \text{xmin})^2)^{1/2}$. Keep repeating the previous step varying the number of granularities until $(x' + \text{granularity}) > RR * \sin(RR/2)$. The values you found for min y and max y at each $(x' + \text{granularity})$ are the same at $(x' - \text{granularity})$. These calculations give us the range of y values at each and every x value that antenna covers. These steps need to be repeated for each and every slice until $\text{granularity} \geq RR * \sin \theta$. This process need to be repeated for each and every antenna at locations (0, granularity, granularity) to (0, MaxY-granularity, MaxZ-granularity).

The third antenna to analyze is the Right Wall Antenna or the antenna with a projection into the facility at 270° . At this location (x', y', z') , xmax will always be equal to 0, at the mounting location. However, when you vary the granularity in the z direction, this will not hold true. In the three dimensional case, xmax equals $-\text{ABS}(\text{?granularity} / \tan(RA/2))$; changing the ?

to match the respective iteration. This point becomes the absolute zero when computing the cross-sectional area. Using simply geometry, you can find x_{min} to equal $-(RR^2 - \text{granularity}^2)^{1/2}$. You can now find $RR?$ at that granularity by subtracting $x_{max} - x_{min}$.

After finding $RR?$, we can also determine that y_{min} and y_{max} equals $y' \pm RR? \cdot \sin(RA/2)$, respectfully. When we increase one level of granularity ($x' + \text{granularity}$), min y and max y have two possible outcomes. If $(x' + \text{granularity} - x_{min}) \leq RR? \cdot \cos(RA/2)$, then min y equals $y' - (x' + \text{granularity} - x_{min}) \cdot \tan(RA/2)$; otherwise min y equals $y' - (RR?^2 - (x' + \text{granularity} - x_{min})^2)^{1/2}$. If $(x' + \text{granularity} - x_{min}) \leq RR? \cdot \cos(RA/2)$, then max y equals $y' + (x' + \text{granularity} - x_{min}) \cdot \tan(RA/2)$; otherwise max y equals $y' + (RR?^2 - (x' + \text{granularity} - x_{min})^2)^{1/2}$. Keep repeating the previous step varying the number of granularities until $(x' + \text{granularity} - x_{min}) > RR?$. These calculations give us the range of y values at each and every x value that antenna covers. These steps need to be repeated for each and every slice until $\text{granularity} \geq RR \cdot \sin \theta$. This process need to be repeated for each and every antenna at locations $(0, \text{granularity}, \text{granularity})$ to $(0, \text{MaxY-granularity}, \text{MaxZ-granularity})$.

The last wall antenna to analyze is the Top Wall Antenna or the antenna with a projection into the facility at 0° . At this location (x', y', z') , y_{min} will always be equal to y' , at the mounting location. However, when you vary the granularity in the z direction, this will not hold true. In the three dimensional case, y_{min} equals $y' + \text{ABS}(\text{?granularity} / \tan(RA/2))$; changing the ? to match the respective iteration. This point becomes the absolute zero when computing the cross-sectional area. Using simply geometry, you can find y_{max} to equal $y' + (RR^2 - \text{granularity}^2)^{1/2}$. You can now find $RR?$ at that granularity by subtracting $y_{max} - y_{min}$.

After finding $RR?$, we can also determine that x_{min} and x_{max} equals $x' \pm RR? \cdot \sin(RA/2)$, respectfully. When we increase one level of granularity ($x' + \text{granularity}$), min y equals

$y_{min} + (x' + \text{granularity}) * \tan(90 - RA/2)$ and max y equals $y_{min} + (RR^2 - (x' + \text{granularity})^2)^{1/2}$. Keep repeating the previous step varying the number of granularities until $(x' + \text{granularity}) > RR * \sin(RR/2)$. The values you found for min y and max y at each $(x' + \text{granularity})$ are the same at $(x' - \text{granularity})$. These calculations give us the range of y values at each and every x value that antenna covers. These steps need to be repeated for each and every slice until $\text{granularity} \geq RR * \sin \theta$. This process need to be repeated for each and every antenna at locations $(0, \text{granularity}, \text{granularity})$ to $(0, \text{MaxY-granularity}, \text{MaxZ-granularity})$.

Two-Sided Corner Antennas

We will analyze the Corner antenna from the position they are on a cube. Since there are three pairs of 4 antennas simply just rotated about the axis, we will analyze each set by which axis is being varied (x or y or z). This makes the analysis much simpler. We will start with the z antennas, locations identified as 1 - 4 (see **Figure 17**). As with the wall antennas, we must recalculate RRC? at each granularity.

Figure 17: 3D Facility space showing Corner Antennas 1 to 4

The first corner antenna is the Z Left Wall Corner Antenna or the antenna with projection onto the facility at position 1 (see **Figure 17**). At the mounting location $(x'=0, y'=0, z'=\text{granularity})$, we know that y_{min} equals 0, y_{max} equals RRC, x_{min} equals 0, and x_{max} equals RRC. When we increase 1 level of granularity at the antenna with mounting location

($x'=0$, $y'=0$, $z'=\text{granularity}$), we know that $y_{\min}$ equals $\text{ABS}(\text{?granularity} / \tan(\text{RA}/2))$, $y_{\max}$ equals $(\text{RRC}^2 - (z'+\text{granularity})^2)^{1/2}$, $x_{\min}$ equals $\text{ABS}(\text{?granularity} / \tan(\text{RA}/2))$, and $x_{\max}$ equals $(\text{RRC}^2 - (z'+\text{granularity})^2)^{1/2}$. When we increase one level of granularity ($x'+\text{granularity}$), $\underline{\min} y$ equals $y_{\min}$ and $\underline{\max} y$ equals $y_{\min} + (\text{RRC}^2 - (x'+\text{granularity} - x_{\min})^2)^{1/2}$. Keep repeating the previous step varying the number of granularities until $(x'+\text{granularity}) > \text{RRC}$. These calculations give us the range of y values at each and every x value that antenna covers. These steps need to be repeated for each and every slice until $\text{granularity} \geq \text{RRC} * \sin \theta$ and $\text{granularity} \leq -\text{RRC} * \sin \theta$. This process need to be repeated for each and every antenna at locations $(0, 0, \text{granularity})$ to $(0, 0, \text{MaxZ-granularity})$.

The second corner antenna is the Z Left Wall Corner Antenna or the antenna with projection onto the facility at position two (see Figure 17). At the mounting location ($x'=0$, $y'=\text{MaxY}$, $z'=\text{granularity}$), we know that $y_{\min}$ equals $\text{MaxY} - \text{RRC}$, $y_{\max}$ equals MaxY , $x_{\min}$ equals 0, and $x_{\max}$ equals RRC . When we increase 1 level of granularity at the antenna with mounting location ($x'=0$, $y'=\text{MaxY}$, $z'=\text{granularity}$), we know that $y_{\max}$ equals $\text{MaxY} - \text{ABS}(\text{?granularity} / \tan(\text{RA}/2))$, $y_{\min}$ equals $\text{MaxY} - (\text{RRC}^2 - (z'+\text{granularity})^2)^{1/2}$, $x_{\min}$ equals $\text{ABS}(\text{?granularity} / \tan(\text{RA}/2))$, and $x_{\max}$ equals $(\text{RRC}^2 - (z'+\text{granularity})^2)^{1/2}$. When we increase one level of granularity ($x'+\text{granularity}$), $\underline{\max} y$ equals $y_{\max}$ and $\underline{\min} y$ equals $y_{\min} - (\text{RRC}^2 - (x'+\text{granularity} - x_{\min})^2)^{1/2}$. Keep repeating the previous step varying the number of granularities until $(x'+\text{granularity}) > \text{RRC}$. These calculations give us the range of y values at each and every x value that antenna covers. These steps need to be repeated for each and every slice until $\text{granularity} \geq \text{RRC} * \sin \theta$ and $\text{granularity} \leq -\text{RRC} * \sin \theta$. This process need to be repeated for each and every antenna at locations $(0, \text{MaxY}, \text{granularity})$ to $(0, \text{MaxY}, \text{MaxZ-granularity})$.

The third corner antenna is the Z Right Wall Corner Antenna or the antenna with projection onto the facility at position three (see **Figure 17**). At the mounting location ($x'=\text{MaxX}$, $y'=\text{MaxY}$, $z'=\text{granularity}$), we know that $y_{\min}$ equals $\text{MaxY} - \text{RRC}$, $y_{\max}$ equals MaxY , $x_{\min}$ equals 0, and $x_{\max}$ equals RRC . When we increase 1 level of granularity at the antenna with mounting location ($x'=0$, $y'=\text{MaxY}$, $z'=\text{granularity}$), we know that $y_{\max}$ equals $\text{MaxY} - \text{ABS}(?granularity / \tan(RA/2))$, $y_{\min}$ equals $\text{MaxY} - (\text{RRC}^2 - (z'+granularity)^2)^{1/2}$, $x_{\max}$ equals $-\text{ABS}(?granularity / \tan(RA/2))$, and $x_{\min}$ equals $-(\text{RRC}^2 - (z'+granularity)^2)^{1/2}$. When we increase one level of granularity ($x'+granularity$), max y equals $y_{\max}$ and min y equals $y_{\min} - (\text{RRC}^2 - (x'+granularity - x_{\min})^2)^{1/2}$. Keep repeating the previous step varying the number of granularities until $(x'+granularity) > \text{RRC}$. These calculations give us the range of y values at each and every x value that antenna covers. These steps need to be repeated for each and every slice until $\text{granularity} \geq \text{RRC} * \sin \theta$ and $\text{granularity} \leq -\text{RRC} * \sin \theta$. This process need to be repeated for each and every antenna at locations (MaxX , MaxY , granularity) to (MaxX , MaxY , MaxZ-granularity).

The fourth corner antenna is the Z Right Wall Corner Antenna or the antenna with projection onto the facility at position four (see **Figure 17**). At the mounting location ($x'=\text{MaxX}$, $y'=0$, $z'=\text{granularity}$), we know that $y_{\min}$ equals 0, $y_{\max}$ equals RRC , $x_{\min}$ equals $\text{MaxX}-\text{RRC}$, and $x_{\max}$ equals MaxX . When we increase 1 level of granularity at the antenna with mounting location ($x'=\text{MaxX}$, $y'=0$, $z'=\text{granularity}$), we know that $y_{\min}$ equals $\text{ABS}(?granularity / \tan(RA/2))$, $y_{\max}$ equals $(\text{RRC}^2 - (z'+granularity)^2)^{1/2}$, $x_{\max}$ equals $-\text{ABS}(?granularity / \tan(RA/2))$, and $x_{\min}$ equals $-(\text{RRC}^2 - (z'+granularity)^2)^{1/2}$. When we increase one level of granularity ($x'+granularity$), min y equals $y_{\min}$ and max y equals $y_{\min} + (\text{RRC}^2 - (x'+granularity - x_{\min})^2)^{1/2}$. Keep repeating the previous step varying the number of granularities

until $(x' + \text{granularity}) > \text{RRC}$. These calculations give us the range of y values at each and every x value that antenna covers. These steps need to be repeated for each and every slice until $\text{granularity} \geq \text{RRC} * \sin \theta$ and $\text{granularity} \leq -\text{RRC} * \sin \theta$. This process need to be repeated for each and every antenna at locations $(\text{MaxX}, 0, \text{granularity})$ to $(\text{MaxX}, 0, \text{MaxZ} - \text{granularity})$.

We will now move to the y antennas, locations identified as 5 - 8 (see Figure 18). As with the wall antennas, we must recalculate RRC? at each granularity.

Figure 18: 3D Facility space showing Corner Antennas 5 to 8

The fifth corner antenna is the Y Bottom Wall Corner Antenna or the antenna with projection onto the facility at position 5 (see Figure 18). At the mounting location $(x'=0, y'=\text{granularity}, z'=0)$, we know that $x_{\min}$ equals 0, $x_{\max}$ equals RRC, $z_{\min}$ equals 0, and $z_{\max}$ equals RRC. When we increase 1 level of granularity at the antenna with mounting location $(x'=0, y'=\text{granularity}, z'=0)$, we know that $x_{\min}$ equals $\text{ABS}(\text{?granularity} / \tan(\text{RA}/2))$, $x_{\max}$ equals $(\text{RRC}^2 - (y' + \text{granularity})^2)^{1/2}$, $z_{\min}$ equals $\text{ABS}(\text{?granularity} / \tan(\text{RA}/2))$, and $z_{\max}$ equals $(\text{RRC}^2 - (y' + \text{granularity})^2)^{1/2}$. When we increase one level of granularity $(x' + \text{granularity})$, min z equals $z_{\min}$ and max z equals $z_{\min} + (\text{RRC}^2 - (x' + \text{granularity} - x_{\min})^2)^{1/2}$. Keep repeating the previous step varying the number of granularities until $(x' + \text{granularity}) > \text{RRC}$. These calculations give us the range of z values at each and every x value that antenna covers. These steps need to be repeated for each and every slice until

granularity $\geq \text{RRC} * \sin \theta$ and granularity $\leq -\text{RRC} * \sin \theta$. This process need to be repeated for each and every antenna at locations (0, granularity, 0) to (0, MaxY-granularity, 0).

The sixth corner antenna is the Y Top Wall Corner Antenna or the antenna with projection onto the facility at position 6 (see **Figure 18**). At the mounting location ($x'=0$, $y'=\text{granularity}$, $z'=\text{MaxZ}$), we know that $z_{\min}$ equals $\text{MaxZ} - \text{RRC}$, $z_{\max}$ equals MaxZ , $x_{\min}$ equals 0, and $x_{\max}$ equals RRC . When we increase 1 level of granularity at the antenna with mounting location ($x'=0$, $y'=\text{granularity}$, $z'=\text{MaxZ}$), we know that $z_{\max}$ equals $\text{MaxZ} - \text{ABS}(?granularity / \tan(\text{RA}/2))$, $z_{\min}$ equals $\text{MaxZ} - (\text{RRC}^2 - (y'+granularity)^2)^{1/2}$, $x_{\min}$ equals $\text{ABS}(?granularity / \tan(\text{RA}/2))$, and $x_{\max}$ equals $(\text{RRC}^2 - (y'+granularity)^2)^{1/2}$. When we increase one level of granularity ($x'+granularity$), max z equals $z_{\max}$ and min z equals $z_{\min} - (\text{RRC}^2 - (x'+granularity - x_{\min})^2)^{1/2}$. Keep repeating the previous step varying the number of granularities until ($x'+granularity$) > RRC . These calculations give us the range of z values at each and every x value that antenna covers. These steps need to be repeated for each and every slice until granularity $\geq \text{RRC} * \sin \theta$ and granularity $\leq -\text{RRC} * \sin \theta$. This process need to be repeated for each and every antenna at locations (0, granularity, MaxZ) to (0, MaxY-granularity, MaxZ).

The seventh corner antenna is the Y Top Wall Corner Antenna or the antenna with projection onto the facility at position 7 (see **Figure 18**). At the mounting location ($x'=\text{MaxX}$, $y'=\text{granularity}$, $z'=\text{MaxZ}$), we know that $z_{\min}$ equals $\text{MaxZ} - \text{RRC}$, $z_{\max}$ equals MaxZ , $x_{\min}$ equals 0, and $x_{\max}$ equals RRC . When we increase 1 level of granularity at the antenna with mounting location ($x'=\text{MaxX}$, $y'=\text{granularity}$, $z'=\text{MaxZ}$), we know that $z_{\max}$ equals $\text{MaxZ} - \text{ABS}(?granularity / \tan(\text{RA}/2))$, $z_{\min}$ equals $\text{MaxZ} - (\text{RRC}^2 - (y'+granularity)^2)^{1/2}$, $x_{\max}$ equals $-\text{ABS}(?granularity / \tan(\text{RA}/2))$, and $x_{\min}$ equals $-(\text{RRC}^2 - (y'+granularity)^2)^{1/2}$. When we

increase one level of granularity ($x' + \text{granularity}$), max z equals z_{max} and min z equals $z_{\text{min}} - (\text{RRC}^2 - (x' + \text{granularity} - x_{\text{min}})^2)^{1/2}$. Keep repeating the previous step varying the number of granularities until $(x' + \text{granularity}) > \text{RRC}$. These calculations give us the range of z values at each and every x value that antenna covers. These steps need to be repeated for each and every slice until $\text{granularity} \geq \text{RRC} * \sin \theta$ and $\text{granularity} \leq -\text{RRC} * \sin \theta$. This process need to be repeated for each and every antenna at locations (MaxX , granularity, MaxZ) to (MaxX , MaxY -granularity, MaxZ).

The eighth corner antenna is the Y Bottom Wall Corner Antenna or the antenna with projection onto the facility at position 8 (see **Figure 18**). At the mounting location ($x' = \text{MaxX}$, $y' = \text{granularity}$, $z' = 0$), we know that z_{min} equals 0, z_{max} equals RRC , x_{min} equals $\text{MaxX} - \text{RRC}$, and x_{max} equals MaxX . When we increase 1 level of granularity at the antenna with mounting location ($x' = \text{MaxX}$, $y' = \text{granularity}$, $z' = 0$), we know that z_{min} equals $\text{ABS}(? \text{granularity} / \tan(\text{RA}/2))$, z_{max} equals $(\text{RRC}^2 - (y' + \text{granularity})^2)^{1/2}$, x_{max} equals $-\text{ABS}(? \text{granularity} / \tan(\text{RA}/2))$, and x_{min} equals $-(\text{RRC}^2 - (y' + \text{granularity})^2)^{1/2}$. When we increase one level of granularity ($x' + \text{granularity}$), min z equals z_{min} and max z equals $z_{\text{min}} + (\text{RRC}^2 - (x' + \text{granularity} - x_{\text{min}})^2)^{1/2}$. Keep repeating the previous step varying the number of granularities until $(x' + \text{granularity}) > \text{RRC}$. These calculations give us the range of z values at each and every x value that antenna covers. These steps need to be repeated for each and every slice until $\text{granularity} \geq \text{RRC} * \sin \theta$ and $\text{granularity} \leq -\text{RRC} * \sin \theta$. This process need to be repeated for each and every antenna at locations (MaxX , granularity, 0) to (MaxX , MaxY -granularity, 0).

We will now move to the x antennas, locations identified as 9 - 12 (see **Figure 19**). As with the wall antennas, we must recalculate RRC at each granularity.

Figure 19: 3D Facility space showing Corner Antennas 9 to 12

The ninth corner antenna is the X Bottom Wall Corner Antenna or the antenna with projection onto the facility at position 9 (see Figure 19). At the mounting location $(x'=\text{granularity}, y'=0, z'=0)$, we know that $y_{\min}$ equals 0, $y_{\max}$ equals RRC, $z_{\min}$ equals 0, and $z_{\max}$ equals RRC. When we increase 1 level of granularity at the antenna with mounting location $(x'=\text{granularity}, y'=0, z'=0)$, we know that $y_{\min}$ equals $\text{ABS}(\text{?granularity} / \tan(\text{RA}/2))$, $y_{\max}$ equals $(\text{RRC}^2 - (x'+\text{granularity})^2)^{1/2}$, $z_{\min}$ equals $\text{ABS}(\text{?granularity} / \tan(\text{RA}/2))$, and $z_{\max}$ equals $(\text{RRC}^2 - (x'+\text{granularity})^2)^{1/2}$. When we increase one level of granularity $(y'+\text{granularity})$, $\underline{\min z}$ equals $z_{\min}$ and $\underline{\max z}$ equals $z_{\min} + (\text{RRC}^2 - (y'+\text{granularity} - y_{\min})^2)^{1/2}$. Keep repeating the previous step varying the number of granularities until $(y'+\text{granularity}) > \text{RRC}$. These calculations give us the range of z values at each and every y value that antenna covers. These steps need to be repeated for each and every slice until $\text{granularity} \geq \text{RRC} * \sin \theta$ and $\text{granularity} \leq -\text{RRC} * \sin \theta$. This process need to be repeated for each and every antenna at locations $(\text{granularity}, 0, 0)$ to $(\text{MaxX}-\text{granularity}, 0, 0)$.

The tenth corner antenna is the X Top Wall Corner Antenna or the antenna with projection onto the facility at position 10 (see Figure 19). At the mounting location $(x'=\text{granularity}, y'=0, z'=\text{MaxZ})$, we know that $y_{\min}$ equals 0, $y_{\max}$ equals RRC, $z_{\min}$ equals $\text{MaxZ}-\text{RRC}$, and $z_{\max}$ equals MaxZ . When we increase 1 level of granularity at the antenna with mounting location $(x'=\text{granularity}, y'=0, z'=\text{MaxZ})$, we know that $y_{\min}$ equals

$\text{ABS}(\text{granularity} / \tan(RA/2))$, y_{max} equals $(RRC^2 - (x' + \text{granularity})^2)^{1/2}$, z_{max} equals $-\text{ABS}(\text{granularity} / \tan(RA/2))$, and z_{min} equals $-(RRC^2 - (x' + \text{granularity})^2)^{1/2}$. When we increase one level of granularity ($y' + \text{granularity}$), max z equals z_{max} and min z equals $z_{\text{max}} - (RRC^2 - (y' + \text{granularity} - y_{\text{min}})^2)^{1/2}$. Keep repeating the previous step varying the number of granularities until $(y' + \text{granularity}) > RRC$. These calculations give us the range of z values at each and every y value that antenna covers. These steps need to be repeated for each and every slice until $\text{granularity} \geq RRC * \sin \theta$ and $\text{granularity} \leq -RRC * \sin \theta$. This process need to be repeated for each and every antenna at locations (granularity, 0, MaxZ) to (MaxX-granularity, 0, MaxZ).

The eleventh corner antenna is the X Top Wall Corner Antenna or the antenna with projection onto the facility at position 11 (see **Figure 19**). At the mounting location ($x' = \text{granularity}$, $y' = \text{MaxY}$, $z' = \text{MaxZ}$), we know that y_{min} equals $\text{MaxY} - RRC$, y_{max} equals MaxY , z_{min} equals 0, and z_{max} equals RRC . When we increase 1 level of granularity at the antenna with mounting location ($x' = 0$, $y' = \text{MaxY}$, $z' = \text{granularity}$), we know that y_{max} equals $\text{MaxY} - \text{ABS}(\text{granularity} / \tan(RA/2))$, y_{min} equals $\text{MaxY} - (RRC^2 - (x' + \text{granularity})^2)^{1/2}$, z_{max} equals $-\text{ABS}(\text{granularity} / \tan(RA/2))$, and z_{min} equals $-(RRC^2 - (x' + \text{granularity})^2)^{1/2}$. When we increase one level of granularity ($y' + \text{granularity}$), max z equals z_{max} and min z equals $z_{\text{min}} - (RRC^2 - (y' + \text{granularity} - y_{\text{min}})^2)^{1/2}$. Keep repeating the previous step varying the number of granularities until $(y' + \text{granularity}) > RRC$. These calculations give us the range of y values at each and every x value that antenna covers. These steps need to be repeated for each and every slice until $\text{granularity} \geq RRC * \sin \theta$ and $\text{granularity} \leq -RRC * \sin \theta$. This process need to be repeated for each and every antenna at locations (granularity, MaxY, MaxZ) to (MaxX-granularity, MaxY, MaxZ).

The twelfth corner antenna is the X Bottom Wall Corner Antenna or the antenna with projection onto the facility at position 12 (see **Figure 19**). At the mounting location $(x'=granularity, y'=0, z'=0)$, we know that $ymin$ equals 0, $ymax$ equals RRC , $zmin$ equals 0, and $zmax$ equals RRC . When we increase 1 level of granularity at the antenna with mounting location $(x'=granularity, y'=0, z'=0)$, we know that $ymin$ equals $MaxY - ABS(?granularity / \tan(RA/2))$, $ymax$ equals $MaxY - (RRC^2 - (x'+granularity)^2)^{1/2}$, $zmin$ equals $ABS(?granularity / \tan(RA/2))$, and $zmax$ equals $(RRC^2 - (x'+granularity)^2)^{1/2}$. When we increase one level of granularity $(y'+granularity)$, $\underline{min} z$ equals $zmin$ and $\underline{max} z$ equals $zmin + (RRC^2 - (y'+granularity - ymin)^2)^{1/2}$. Keep repeating the previous step varying the number of granularities until $(y'+granularity) > RRC$. These calculations give us the range of z values at each and every y value that antenna covers. These steps need to be repeated for each and every slice until $granularity \geq RRC * \sin \theta$ and $granularity \leq -RRC * \sin \theta$. This process need to be repeated for each and every antenna at locations $(granularity, MaxY, 0)$ to $(MaxX-granularity, MaxY, 0)$.

Three-Sided Corner Antennas

The three-sided corners can be modeled very similar to two-sided corners, however have the shape of 1/8 of a circle. Specifically, there are 8 corners with locations of: $(0,0,0)$, $(MaxX,0,0)$, $(0,MaxY,0)$, $(MaxX,MaxY,0)$, $(0,0,MaxZ)$, $(MaxX,0,MaxZ)$, $(0,MaxY,MaxZ)$, $(MaxX,MaxY,MaxZ)$. At each location, the two components being varied will have a max value equal to the RR .

Chapter 7: Conclusion and Further Research

This research focuses on the problem of determining the optimal locations of RFID antennas in a given space, such that all inventory items are “seen” by these antennas and the number of antennas is minimized. Given certain assumptions regarding the conversion of the given space to a grid of points representing potential inventory item locations, we show that the resulting optimization problem is a set covering problem.

In addition to the mathematical model, we develop a methodology for the space to grid conversion, and a heuristic solution algorithm for the problem. A computerized system that follows this methodology, RFIDMIN, can be used by stores, warehouses, manufacturing facilities, and service operations, in conjunction with linear programming (LP) software or our greedy algorithm, to calculate the optimal number and location of RFID antennas.

This research could be enhanced by including true antenna shapes. While our paper uses an approximation of the main lobe of the antenna, true antenna clouds have side lobes as well (see **Figure 13**). Future research should also consider antenna and tag collision. This greatly increases the complexity of the situation.

Future research should consider the effects of the facility environment. Each real world facility has various factors that affect RF reception. These factors may include Wi-Fi and cellular networks, as well as environmental factors, such as water, metal, etc. These factors can greatly change the shape of the antenna clouds and must be accounted for in real world applications.

This research could be further developed to include non-rectangular shapes. Given that facilities are generally either a rectangle or a composition of rectangles, RFIDMIN could be

enhanced by running a step function. This step function would run one area and then reference the shared points as covered when it ran the next area.

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Validation of ILP

50

51

2D: 4 ft x 5 ft, granularity = 1, RR=1, RA=180°, RRC=2.

[illegible]

Greedy Algorithm: Step 1

Greedy Algorithm: Step 2

	X000	X010	X020	X030	X040	X100	X110	X120	X130	X140	X200	X210	X220	X230	X240	X300	X310	X320	X330	X340	X400	X410	X420	X430	X440	X500	X510	X520	X530	X540	Total Points
L0,1,0	1	1	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4
L0,2,0	0	1	1	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4
L0,3,0	0	0	1	1	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4
L1,1,0	0	0	0	0	0	1	1	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4
L1,2,0	0	0	0	0	0	0	1	1	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4
L1,3,0	0	0	0	0	0	0	0	1	1	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4
L2,1,0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	4
L2,2,0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	4
L2,3,0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	4
L3,1,0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	1	0	0	0	0	0	0	0	4
L3,2,0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	1	0	0	0	0	0	0	4
L3,3,0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	1	0	0	0	0	0	4
L4,1,0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	1	0	0	0	4
L4,2,0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	1	0	0	4
L4,3,0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	1	1	0	4
R5,2,0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	1	1	0	4
R5,3,0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	1	1	0	4
R4,1,0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	1	1	0	0	0	0	0	0	4
R4,2,0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	1	1	0	0	0	0	0	4
R4,3,0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	1	1	1	0	0	0	0	0	4
R3,1,0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	4
R3,2,0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	4
R3,3,0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	4
R2,1,0	0	0	0	0	0	0	1	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4
R2,2,0	0	0	0	0	0	0	0	1	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4
R2,3,0	0	0	0	0	0	0	0	0	1	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4
R1,1,0	0	1	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4
R1,2,0	0	0	1	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4
R1,3,0	0	0	0	1	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4
T1,0,0	1	0	0	0	0	1	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4
T1,1,0	0	1	0	0	0	0	1	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4
T1,2,0	0	0	1	0	0	0	0	1	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4
T1,3,0	0	0	0	1	0	0	0	0	1	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4
T2,0,0	0	0	0	0	0	1	0	0	0	1	0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	4
T2,1,0	0	0	0	0	0	0	1	0	0	0	0	1	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	4
T2,2,0	0	0	0	0	0	0	0	1	0	0	0	0	1	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	4
T2,3,0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	4
T3,0,0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	1	0	0	0	1	0	0	0	0	0	0	0	0	0	4
T3,1,0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	1	0	0	0	1	0	0	0	0	0	0	0	0	4
T3,2,0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	1	0	0	0	1	0	0	0	0	0	0	0	4
T3,3,0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	1	0	0	0	1	0	0	0	0	0	0	4
T4,0,0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	1	0	0	0	1	0	0	0	0	4
T4,1,0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	1	0	0	0	1	0	0	0	4
T4,2,0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	1	0	0	1	0	0	0	4
T4,3,0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	1	0	0	1	0	0	0	4
B1,1,0	0	1	0	0	0	1	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4
B1,2,0	0	0	1	0	0	0	1	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4
B1,3,0	0	0	0	1	0	0	0	1	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4
B1,4,0	0	0	0	0	1	0	0	0	1	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4
B2,1,0	0	0	0	0	0	0	1	0	0	0	1	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	4
B2,2,0	0	0	0	0	0	0	0	1	0	0	0	1	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	4
B2,3,0	0	0	0	0	0	0	0	0	1	0	0	0	1	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	4
B2,4,0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	4
B3,1,0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	1	0	0	0	0	1	0	0	0	0	0	0	0	0	4
B3,2,0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	1	0	0	0	1	0	0	0	0	0	0	0	0	4
B3,3,0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	1	0	0	0	1	0	0	0	0	0	0	0	4
B3,4,0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	1	0	0	0	1	0	0	0	0	0	0	4
B4,1,0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	1	0	0	0	0	1	0	0	0	4
B4,2,0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	1	0	0	0	1	0	0	0	4
B4,3,0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	1	0	0	0	0	1	0	0	

Greedy Algorithm: Steps 3 and 4

	X _{0,0}	X _{0,1}	X _{0,2}	X _{0,3}	X _{0,4}	X _{1,0}	X _{1,1}	X _{1,2}	X _{1,3}	X _{1,4}	X _{2,0}	X _{2,1}	X _{2,2}	X _{2,3}	X _{2,4}	X _{3,0}	X _{3,1}	X _{3,2}	X _{3,3}	X _{4,0}	X _{4,1}	X _{4,2}	X _{5,0}	X _{5,1}	Total Points
L 0,1,0	1	1	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4
L 0,2,0	0	1	1	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4
L 0,3,0	0	0	1	1	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4
L 1,1,0	0	0	0	0	0	1	1	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	4
L 1,2,0	0	0	0	0	0	0	1	1	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	4
L 1,3,0	0	0	0	0	0	0	0	1	1	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	4
L 2,1,0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	1	0	0	0	0	0	0	0	4
L 2,2,0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	1	0	0	0	0	0	0	4
L 2,3,0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	0	1	0	0	0	0	0	4
L 3,1,0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	1	0	0	0	4
L 3,2,0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	1	1	0	0	4
L 3,3,0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	2
L 4,1,0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	1	4
L 4,2,0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	2
L 4,3,0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1
R 5,2,0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	2
R 5,3,0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
R 4,1,0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	1	1	0	0	4
R 4,2,0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	1	0	0	3
R 4,3,0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	2
R 3,1,0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	1	1	0	0	0	0	0	0	4
R 3,2,0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	1	0	0	0	0	0	0	4
R 3,3,0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	1	0	0	0	0	0	3
R 2,1,0	0	0	0	0	0	0	1	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	4
R 2,2,0	0	0	0	0	0	0	0	1	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	4
R 2,3,0	0	0	0	0	0	0	0	0	1	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	4
R 1,1,0	0	1	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4
R 1,2,0	0	0	1	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4
R 1,3,0	0	0	0	1	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4
T 1,0,0	1	0	0	0	1	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	4
T 1,1,0	0	1	0	0	0	0	1	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	4
T 1,2,0	0	0	1	0	0	0	0	1	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	4
T 1,3,0	0	0	0	1	0	0	0	0	1	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	4
T 2,0,0	0	0	0	0	0	1	0	0	0	0	1	1	0	0	0	1	0	0	0	0	0	0	0	0	4
T 2,1,0	0	0	0	0	0	0	1	0	0	0	0	1	1	0	0	0	1	0	0	0	0	0	0	0	4
T 2,2,0	0	0	0	0	0	0	0	1	0	0	0	0	1	1	0	0	0	1	0	0	0	0	0	0	4
T 2,3,0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	1	0	0	0	1	0	0	0	0	0	4
T 3,0,0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	1	0	0	1	0	0	0	0	4
T 3,1,0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	1	0	0	1	0	0	0	4
T 3,2,0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	1	0	0	1	0	0	4
T 3,3,0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	2
T 4,0,0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	1	0	1	0	4
T 4,1,0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	1	0	1	4
T 4,2,0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	2
T 4,3,0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1
B 1,1,0	0	1	0	0	0	1	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	4
B 1,2,0	0	0	1	0	0	0	1	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	4
B 1,3,0	0	0	0	1	0	0	0	1	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	4
B 1,4,0	0	0	0	0	1	0	0	0	1	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	4
B 2,1,0	0	0	0	0	0	0	1	0	0	0	1	1	0	0	0	0	1	0	0	0	0	0	0	0	4
B 2,2,0	0	0	0	0	0	0	0	1	0	0	0	1	1	0	0	0	0	1	0	0	0	0	0	0	4
B 2,3,0	0	0	0	0	0	0	0	0	1	0	0	0	1	1	0	0	0	0	1	0	0	0	0	0	4
B 2,4,0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	1	0	0	0	0	0	0	0	0	0	3
B 3,1,0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	1	0	0	1	0	0	0	0	4
B 3,2,0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	1	0	0	1	0	0	0	4
B 3,3,0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	1	0	0	0	0	0	3
B 3,4,0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	2
B 4,1,0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	1	0	0	1	4
B 4,2,0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	1	0	0	3
B 4,3,0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	2
B 4,4,0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
TL 0,0,0	1	1	1	0	0	1	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	5
TR 5,0,0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	1	0	1	1	5
BL 0,4,0	0	0	1	1	1	0	0	0	1	1	0	0	0	0	1	0	0	0	0	0	0	0	0	0	6
BR 5,4,0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Greedy Algorithm: Step 5

	X _{0,3,0}	X _{0,4,0}	X _{1,2,0}	X _{1,3,0}	X _{1,4,0}	X _{2,1,0}	X _{2,2,0}	X _{2,3,0}	X _{2,4,0}	X _{3,0,0}	X _{3,1,0}	X _{3,2,0}	X _{3,3,0}	X _{4,0,0}	X _{4,1,0}	X _{4,2,0}	X _{5,0,0}	X _{5,1,0}	Total Points
L 0,1,0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
L 0,2,0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2
L 0,3,0	1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3
L 1,1,0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	2
L 1,2,0	0	0	1	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	3
L 1,3,0	0	0	1	1	1	0	0	1	0	0	0	0	0	0	0	0	0	0	4
L 2,1,0	0	0	0	0	0	1	1	0	0	0	1	0	0	0	0	0	0	0	3
L 2,2,0	0	0	0	0	0	1	1	1	0	0	0	1	0	0	0	0	0	0	4
L 2,3,0	0	0	0	0	0	0	1	1	1	0	0	0	1	0	0	0	0	0	4
L 3,1,0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	1	0	0	0	4
L 3,2,0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	0	1	0	0	4
L 3,3,0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	2
L 4,1,0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	1	0	1	4
L 4,2,0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	2
L 4,3,0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1
R 5,2,0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	2
R 5,3,0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
R 4,1,0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	1	1	0	0	4
R 4,2,0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	1	0	0	3
R 4,3,0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	2
R 3,1,0	0	0	0	0	0	1	0	0	0	1	1	1	0	0	0	0	0	0	4
R 3,2,0	0	0	0	0	0	0	1	0	0	0	1	1	1	0	0	0	0	0	4
R 3,3,0	0	0	0	0	0	0	0	1	0	0	0	1	1	0	0	0	0	0	3
R 2,1,0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	2
R 2,2,0	0	0	1	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	4
R 2,3,0	0	0	0	1	0	0	1	1	1	0	0	0	0	0	0	0	0	0	4
R 1,1,0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
R 1,2,0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2
R 1,3,0	1	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	4
T 1,0,0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
T 1,1,0	0	0	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	2
T 1,2,0	0	0	1	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	3
T 1,3,0	1	0	0	1	1	0	0	1	0	0	0	0	0	0	0	0	0	0	4
T 2,0,0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	2
T 2,1,0	0	0	0	0	0	1	1	0	0	0	1	0	0	0	0	0	0	0	3
T 2,2,0	0	0	1	0	0	0	1	1	0	0	0	1	0	0	0	0	0	0	4
T 2,3,0	0	0	0	1	0	0	0	1	1	0	0	0	1	0	0	0	0	0	4
T 3,0,0	0	0	0	0	0	0	0	0	0	1	1	0	0	1	0	0	0	0	3
T 3,1,0	0	0	0	0	0	1	0	0	0	0	1	1	0	0	1	0	0	0	4
T 3,2,0	0	0	0	0	0	0	1	0	0	0	0	1	1	0	0	1	0	0	4
T 3,3,0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	2
T 4,0,0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	1	0	1	0	4
T 4,1,0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	1	0	1	4
T 4,2,0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	2
T 4,3,0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	1
B 1,1,0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	1
B 1,2,0	0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	2
B 1,3,0	1	0	1	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	4
B 1,4,0	0	1	0	1	1	0	0	0	1	0	0	0	0	0	0	0	0	0	4
B 2,1,0	0	0	0	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0	2
B 2,2,0	0	0	1	0	0	1	1	0	0	0	0	1	0	0	0	0	0	0	4
B 2,3,0	0	0	0	1	0	0	1	1	0	0	0	0	1	0	0	0	0	0	4
B 2,4,0	0	0	0	0	1	0	0	1	1	0	0	0	0	0	0	0	0	0	3
B 3,1,0	0	0	0	0	0	1	0	0	0	1	1	0	0	0	1	0	0	0	4
B 3,2,0	0	0	0	0	0	0	1	0	0	0	1	1	0	0	0	1	0	0	4
B 3,3,0	0	0	0	0	0	0	0	1	0	0	0	1	1	0	0	0	0	0	3
B 3,4,0	0	0	0	0	0	0	0	0	1	0	0	0	1	0	0	0	0	0	2
B 4,1,0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	1	0	0	1	4
B 4,2,0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	1	0	0	3
B 4,3,0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	0	2
B 4,4,0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
TL 0,0,0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
TR 5,0,0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	1	0	1	5
BL 0,4,0	1	1	0	1	1	0	0	0	1	0	0	0	0	0	0	0	0	0	5
BR 5,4,0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Greedy Algorithm: Step 6

	X _{0,3,0}	X _{0,4,0}	X _{1,2,0}	X _{1,3,0}	X _{1,4,0}	X _{2,1,0}	X _{2,2,0}	X _{2,3,0}	X _{2,4,0}	X _{3,1,0}	X _{3,2,0}	X _{3,3,0}	X _{4,2,0}	Total Points
L0,1,0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
L0,2,0	1	0	1	0	0	0	0	0	0	0	0	0	0	2
L0,3,0	1	1	0	1	0	0	0	0	0	0	0	0	0	3
L1,1,0	0	0	1	0	0	1	0	0	0	0	0	0	0	2
L1,2,0	0	0	1	1	0	0	1	0	0	0	0	0	0	3
L1,3,0	0	0	1	1	1	0	0	1	0	0	0	0	0	4
L2,1,0	0	0	0	0	0	1	1	0	0	1	0	0	0	3
L2,2,0	0	0	0	0	0	1	1	1	0	0	1	0	0	4
L2,3,0	0	0	0	0	0	0	1	1	1	0	0	1	0	4
L3,1,0	0	0	0	0	0	0	0	0	0	1	1	0	0	2
L3,2,0	0	0	0	0	0	0	0	0	0	1	1	1	1	4
L3,3,0	0	0	0	0	0	0	0	0	0	0	1	1	0	2
L4,1,0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
L4,2,0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
L4,3,0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
R5,2,0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
R5,3,0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
R4,1,0	0	0	0	0	0	0	0	0	0	1	0	0	1	2
R4,2,0	0	0	0	0	0	0	0	0	0	0	1	0	1	2
R4,3,0	0	0	0	0	0	0	0	0	0	0	0	1	1	2
R3,1,0	0	0	0	0	0	1	0	0	0	1	1	0	0	3
R3,2,0	0	0	0	0	0	0	1	0	0	1	1	1	0	4
R3,3,0	0	0	0	0	0	0	0	1	0	0	1	1	0	3
R2,1,0	0	0	0	0	0	1	1	0	0	0	0	0	0	2
R2,2,0	0	0	1	0	0	1	1	1	0	0	0	0	0	4
R2,3,0	0	0	0	1	0	0	1	1	1	0	0	0	0	4
R1,1,0	0	0	1	0	0	0	0	0	0	0	0	0	0	1
R1,2,0	0	0	1	1	0	0	0	0	0	0	0	0	0	2
R1,3,0	1	0	1	1	1	0	0	0	0	0	0	0	0	4
T1,0,0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
T1,1,0	0	0	1	0	0	1	0	0	0	0	0	0	0	2
T1,2,0	0	0	1	1	0	0	1	0	0	0	0	0	0	3
T1,3,0	1	0	0	1	1	0	0	1	0	0	0	0	0	4
T2,0,0	0	0	0	0	0	1	0	0	0	0	0	0	0	1
T2,1,0	0	0	0	0	0	1	1	0	0	1	0	0	0	3
T2,2,0	0	0	1	0	0	0	1	1	0	0	1	0	0	4
T2,3,0	0	0	0	1	0	0	0	1	1	0	0	1	0	4
T3,0,0	0	0	0	0	0	0	0	0	0	1	0	0	0	1
T3,1,0	0	0	0	0	0	1	0	0	0	1	1	0	0	3
T3,2,0	0	0	0	0	0	0	1	0	0	0	1	1	1	4
T3,3,0	0	0	0	0	0	0	0	1	0	0	0	1	0	2
T4,0,0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
T4,1,0	0	0	0	0	0	0	0	0	0	1	0	0	1	2
T4,2,0	0	0	0	0	0	0	0	0	0	0	1	0	1	2
T4,3,0	0	0	0	0	0	0	0	0	0	0	0	1	0	1
B1,1,0	0	0	0	0	0	1	0	0	0	0	0	0	0	1
B1,2,0	0	0	1	0	0	0	1	0	0	0	0	0	0	2
B1,3,0	1	0	1	1	0	0	0	1	0	0	0	0	0	4
B1,4,0	0	1	0	1	1	0	0	0	1	0	0	0	0	4
B2,1,0	0	0	0	0	0	1	0	0	0	1	0	0	0	2
B2,2,0	0	0	1	0	0	1	1	0	0	0	1	0	0	4
B2,3,0	0	0	0	1	0	0	1	1	0	0	0	1	0	4
B2,4,0	0	0	0	0	1	0	0	1	1	0	0	0	0	3
B3,1,0	0	0	0	0	0	1	0	0	0	1	0	0	0	2
B3,2,0	0	0	0	0	0	0	1	0	0	1	1	0	1	4
B3,3,0	0	0	0	0	0	0	0	1	0	0	1	1	0	3
B3,4,0	0	0	0	0	0	0	0	0	1	0	0	1	0	2
B4,1,0	0	0	0	0	0	0	0	0	0	1	0	0	0	1
B4,2,0	0	0	0	0	0	0	0	0	0	0	1	0	1	2
B4,3,0	0	0	0	0	0	0	0	0	0	0	0	1	1	2
B4,4,0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
TL0,0,0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
TR5,0,0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
BL0,4,0	1	1	0	1	1	0	0	0	1	0	0	0	0	5
BR5,4,0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Greedy Algorithm: Step 7

	X _{1,2,0}	X _{2,1,0}	X _{2,2,0}	X _{2,3,0}	X _{3,1,0}	X _{3,2,0}	X _{3,3,0}	X _{4,1,0}	Total Points
L 0,1,0	0	0	0	0	0	0	0	0	0
L 0,2,0	1	0	0	0	0	0	0	0	1
L 0,3,0	0	0	0	0	0	0	0	0	0
L 1,1,0	1	1	0	0	0	0	0	0	2
L 1,2,0	1	0	1	0	0	0	0	0	2
L 1,3,0	1	0	0	1	0	0	0	0	2
L 2,1,0	0	1	1	0	1	0	0	0	3
L 2,2,0	0	1	1	1	0	1	0	0	4
L 2,3,0	0	0	1	1	0	0	1	0	3
L 3,1,0	0	0	0	0	1	1	0	0	2
L 3,2,0	0	0	0	0	1	1	1	1	4
L 3,3,0	0	0	0	0	0	1	1	0	2
L 4,1,0	0	0	0	0	0	0	0	1	1
L 4,2,0	0	0	0	0	0	0	0	1	1
L 4,3,0	0	0	0	0	0	0	0	1	1
R 5,2,0	0	0	0	0	0	0	0	1	1
R 5,3,0	0	0	0	0	0	0	0	0	0
R 4,1,0	0	0	0	0	1	0	0	1	2
R 4,2,0	0	0	0	0	0	1	0	1	2
R 4,3,0	0	0	0	0	0	0	1	1	2
R 3,1,0	0	1	0	0	1	1	0	0	3
R 3,2,0	0	0	1	0	1	1	1	0	4
R 3,3,0	0	0	0	1	0	1	1	0	3
R 2,1,0	0	1	1	0	0	0	0	0	2
R 2,2,0	1	1	1	1	0	0	0	0	4
R 2,3,0	0	0	1	1	0	0	0	0	2
R 1,1,0	1	0	0	0	0	0	0	0	1
R 1,2,0	1	0	0	0	0	0	0	0	1
R 1,3,0	1	0	0	0	0	0	0	0	1
T 1,0,0	0	0	0	0	0	0	0	0	0
T 1,1,0	1	1	0	0	0	0	0	0	2
T 1,2,0	1	0	1	0	0	0	0	0	2
T 1,3,0	0	0	0	1	0	0	0	0	1
T 2,0,0	0	1	0	0	0	0	0	0	1
T 2,1,0	0	1	1	0	1	0	0	0	3
T 2,2,0	1	0	1	1	0	1	0	0	4
T 2,3,0	0	0	0	1	0	0	1	0	2
T 3,0,0	0	0	0	0	1	0	0	0	1
T 3,1,0	0	1	0	0	1	1	0	0	3
T 3,2,0	0	0	1	0	0	1	1	1	4
T 3,3,0	0	0	0	1	0	0	1	0	2
T 4,0,0	0	0	0	0	0	0	0	0	0
T 4,1,0	0	0	0	0	1	0	0	1	2
T 4,2,0	0	0	0	0	0	1	0	1	2
T 4,3,0	0	0	0	0	0	0	1	0	1
B 1,1,0	0	1	0	0	0	0	0	0	1
B 1,2,0	1	0	1	0	0	0	0	0	2
B 1,3,0	1	0	0	1	0	0	0	0	2
B 1,4,0	0	0	0	0	0	0	0	0	0
B 2,1,0	0	1	0	0	1	0	0	0	2
B 2,2,0	1	1	1	0	0	1	0	0	4
B 2,3,0	0	0	1	1	0	0	1	0	3
B 2,4,0	0	0	0	1	0	0	0	0	1
B 3,1,0	0	1	0	0	1	0	0	0	2
B 3,2,0	0	0	1	0	1	1	0	1	4
B 3,3,0	0	0	0	1	0	1	1	0	3
B 3,4,0	0	0	0	0	0	0	1	0	1
B 4,1,0	0	0	0	0	1	0	0	0	1
B 4,2,0	0	0	0	0	0	1	0	1	2
B 4,3,0	0	0	0	0	0	0	1	1	2
B 4,4,0	0	0	0	0	0	0	0	0	0
TL 0,0,0	0	0	0	0	0	0	0	0	0
TR 5,0,0	0	0	0	0	0	0	0	0	0
BL 0,4,0	0	0	0	0	0	0	0	0	0
BR 5,4,0	0	0	0	0	0	0	0	0	0

Greedy Algorithm: Step 8 and 9

	X _{1,2,0}	X _{2,1,0}	X _{2,2,0}	X _{2,3,0}	Total Points		Total Points
L 0,1,0	0	0	0	0	0	L 0,1,0	0
L 0,2,0	1	0	0	0	1	L 0,2,0	0
L 0,3,0	0	0	0	0	0	L 0,3,0	0
L 1,1,0	1	1	0	0	2	L 1,1,0	0
L 1,2,0	1	0	1	0	2	L 1,2,0	0
L 1,3,0	1	0	0	1	2	L 1,3,0	0
L 2,1,0	0	1	1	0	2	L 2,1,0	0
L 2,2,0	0	1	1	1	3	L 2,2,0	0
L 2,3,0	0	0	1	1	2	L 2,3,0	0
L 3,1,0	0	0	0	0	0	L 3,1,0	0
L 3,2,0	0	0	0	0	0	L 3,2,0	0
L 3,3,0	0	0	0	0	0	L 3,3,0	0
L 4,1,0	0	0	0	0	0	L 4,1,0	0
L 4,2,0	0	0	0	0	0	L 4,2,0	0
L 4,3,0	0	0	0	0	0	L 4,3,0	0
R 5,2,0	0	0	0	0	0	R 5,2,0	0
R 5,3,0	0	0	0	0	0	R 5,3,0	0
R 4,1,0	0	0	0	0	0	R 4,1,0	0
R 4,2,0	0	0	0	0	0	R 4,2,0	0
R 4,3,0	0	0	0	0	0	R 4,3,0	0
R 3,1,0	0	1	0	0	1	R 3,1,0	0
R 3,2,0	0	0	1	0	1	R 3,2,0	0
R 3,3,0	0	0	0	1	1	R 3,3,0	0
R 2,1,0	0	1	1	0	2	R 2,1,0	0
R 2,2,0	1	1	1	1	4	R 2,2,0	0
R 2,3,0	0	0	1	1	2	R 2,3,0	0
R 1,1,0	1	0	0	0	1	R 1,1,0	0
R 1,2,0	1	0	0	0	1	R 1,2,0	0
R 1,3,0	1	0	0	0	1	R 1,3,0	0
T 1,0,0	0	0	0	0	0	T 1,0,0	0
T 1,1,0	1	1	0	0	2	T 1,1,0	0
T 1,2,0	1	0	1	0	2	T 1,2,0	0
T 1,3,0	0	0	0	1	1	T 1,3,0	0
T 2,0,0	0	1	0	0	1	T 2,0,0	0
T 2,1,0	0	1	1	0	2	T 2,1,0	0
T 2,2,0	1	0	1	1	3	T 2,2,0	0
T 2,3,0	0	0	0	1	1	T 2,3,0	0
T 3,0,0	0	0	0	0	0	T 3,0,0	0
T 3,1,0	0	1	0	0	1	T 3,1,0	0
T 3,2,0	0	0	1	0	1	T 3,2,0	0
T 3,3,0	0	0	0	1	1	T 3,3,0	0
T 4,0,0	0	0	0	0	0	T 4,0,0	0
T 4,1,0	0	0	0	0	0	T 4,1,0	0
T 4,2,0	0	0	0	0	0	T 4,2,0	0
T 4,3,0	0	0	0	0	0	T 4,3,0	0
B 1,1,0	0	1	0	0	1	B 1,1,0	0
B 1,2,0	1	0	1	0	2	B 1,2,0	0
B 1,3,0	1	0	0	1	2	B 1,3,0	0
B 1,4,0	0	0	0	0	0	B 1,4,0	0
B 2,1,0	0	1	0	0	1	B 2,1,0	0
B 2,2,0	1	1	1	0	3	B 2,2,0	0
B 2,3,0	0	0	1	1	2	B 2,3,0	0
B 2,4,0	0	0	0	1	1	B 2,4,0	0
B 3,1,0	0	1	0	0	1	B 3,1,0	0
B 3,2,0	0	0	1	0	1	B 3,2,0	0
B 3,3,0	0	0	0	1	1	B 3,3,0	0
B 3,4,0	0	0	0	0	0	B 3,4,0	0
B 4,1,0	0	0	0	0	0	B 4,1,0	0
B 4,2,0	0	0	0	0	0	B 4,2,0	0
B 4,3,0	0	0	0	0	0	B 4,3,0	0
B 4,4,0	0	0	0	0	0	B 4,4,0	0
TL 0,0,0	0	0	0	0	0	TL 0,0,0	0
TR 5,0,0	0	0	0	0	0	TR 5,0,0	0
BL 0,4,0	0	0	0	0	0	BL 0,4,0	0
BR 5,4,0	0	0	0	0	0	BR 5,4,0	0

Excel Solver:

[illegible]

2D: 21ft x 21ft, granularity=3, RR=12, RA=120°

Reader Location (x',y')		x'	x' value	x'-x	In x Range	min y	MIN Y*	max y	MAX Y*
U	12	x' - 21	-21	21	FALSE	-1.00	-1.00	-1.00	-1.00
LEFT		x' - 18	-18	18	FALSE	-1.00	-1.00	-1.00	-1.00
WALL		x' - 15	-15	15	FALSE	-1.00	-1.00	-1.00	-1.00
		x' - 12	-12	12	FALSE	-1.00	-1.00	-1.00	-1.00
Values:		x' - 9	-9	9	FALSE	-1.00	-1.00	-1.00	-1.00
xmin	0	x' - 6	-6	6	FALSE	-1.00	-1.00	-1.00	-1.00
xmax	12	x' - 3	-3	3	FALSE	-1.00	-1.00	-1.00	-1.00
ymn	1.61	x'	0	0	TRUE	12.00	12.00	12.00	12.00
ymax	21.00	x' + 3	3	3	TRUE	6.80	7.00	17.20	17.00
		x' + 6	6	6	TRUE	1.61	2.00	22.39	21.00
		x' + 9	9	9	TRUE	4.06	5.00	19.94	19.00
		x' + 12	12	12	TRUE	12.00	12.00	12.00	12.00
		x' + 15	15	15	FALSE	-1.00	-1.00	-1.00	-1.00
		x' + 18	18	18	FALSE	-1.00	-1.00	-1.00	-1.00
		x' + 21	21	21	FALSE	-1.00	-1.00	-1.00	-1.00

Reader Location (x',y')					0	12		
Y\X	0	3	6	9	12	15	18	21
0	0	0	0	0	C	0	0	0
3	0	0	1	0	C	0	0	0
6	0	0	1	1	C	0	0	0
9	0	1	1	1	C	0	0	0
12	1	1	1	1	1	0	0	0
15	U	1	1	1	1	U	U	U
18	0	0	1	1	C	0	0	0
21	0	0	1	0	C	0	0	0

x _{0,0}	x _{0,3}	x _{0,6}	x _{0,9}	x _{0,12}	x _{0,15}	x _{0,18}	x _{0,21}	x _{3,0}	x _{3,3}	x _{3,6}	x _{3,9}	x _{3,12}	x _{3,15}	x _{3,18}	x _{3,21}	x _{6,0}	x _{6,3}	x _{6,6}	x _{6,9}	x _{6,12}
0	0	0	0	1	0	0	0	0	0	0	1	1	1	0	0	0	1	1	1	1
x _{6,15}	x _{6,18}	x _{6,21}	x _{9,0}	x _{9,3}	x _{9,6}	x _{9,9}	x _{9,12}	x _{9,15}	x _{9,18}	x _{9,21}	x _{12,0}	x _{12,3}	x _{12,6}	x _{12,9}	x _{12,12}	x _{12,15}	x _{12,18}	x _{12,21}	x _{15,0}	x _{15,3}
1	1	1	0	0	1	1	1	1	1	0	0	0	0	0	1	0	0	0	0	0
x _{15,6}	x _{15,9}	x _{15,12}	x _{15,15}	x _{15,18}	x _{15,21}	x _{18,0}	x _{18,3}	x _{18,6}	x _{18,9}	x _{18,12}	x _{18,15}	x _{18,18}	x _{18,21}	x _{21,0}	x _{21,3}	x _{21,6}	x _{21,9}	x _{21,12}	x _{21,15}	x _{21,18}
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
x _{21,21}																				
0																				

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3D: 21ft x 21ft, granularity=3, RR=12, RA=120°, RRC=20

APPENDIX B: RFIDMIN PROGRAM CODE

```
Public RArad, theta, thetaCOMP, RRSQ, RRCOStheta, RRSINtheta, TANtheta, TANthetaCOMP, RACrad, thetac,
    thetacCOMP, RRCSQ, RRCSINthetac, TANthetac, TANthetacCOMP
Dim IR As Boolean
Dim ylocation As Double
Dim xlocation As Double
Dim zlocation As Double

Private Sub quit_Click()
Unload Shell
End Sub

Private Sub run_Click()

If RR = "" Then
    MsgBox "Please enter a positive read range"
    RR.SetFocus
    GoTo endprog
End If
If RA = "" Then
    MsgBox "Please enter a positive read angle"
    RA.SetFocus
    GoTo endprog
End If
If RA >= 180 Then
    MsgBox "Please select a read angle of less than 180 degrees"
    RA.SetFocus
    GoTo endprog
End If
If RRC = "" Then
    MsgBox "Please enter a positive corner read range"
    RRC.SetFocus
    GoTo endprog
End If
If RAC = "" Then
    MsgBox "Please enter a positive corner read angle"
    RAC.SetFocus
    GoTo endprog
End If
If RAC >= 180 Then
    MsgBox "Please select a corner read angle of less than 180 degrees"
    RAC.SetFocus
    GoTo endprog
End If
If MaxXu = "" Then
    MsgBox "Please enter a positive value for Max Value of X"
    MaxXu.SetFocus
    GoTo endprog
End If
If MaxYu = "" Then
    MsgBox "Please enter a positive value for Max Value of Y"
    MaxYu.SetFocus
    GoTo endprog
```

```

End If
If MaxZu = "" Then
    MsgBox "Please enter a positive value for Max Value of Z"
    MaxZu.SetFocus
    GoTo endprog
End If
If Gran = "" Then
    MsgBox "Please enter a positive value for Granularity"
    Gran.SetFocus
    GoTo endprog
End If
If ((MaxYu / Gran) + 1) * ((MaxXu / Gran) + 1) >= 256 Then
    If MsgBox("Warehouse dimentions are very large, and/or the granularity is very small. This may take a while. Continue?", vbYesNo, "Warning") = vbYes Then
        Else:
            GoTo endprog
    End If
End If
Application.DisplayAlerts = False
Sheets("Program's Formulation").Delete
Application.DisplayAlerts = True
Sheets.Add
ActiveSheet.Name = "Program's Formulation"
Sheets("Program's Formulation").Select
rowvalue = 2
numofcell = 0
Range("b1").Select
Call var_name(MaxXu, MaxYu, MaxZu, Gran)

'for all Left-wall type reader locations
lread:
For zlocation = 0 To MaxZu Step Gran
    For bot = 0 To zlocation Step Gran
        If bot < (zlocation - RRSINtheta) Then
            For grans = 0 To (MaxXu / Gran) * (MaxYu / Gran)
                ActiveCell = 0
                ActiveCell.Offset(0, 1).Select
            Next
        End If
    Next
Next
For xlocation = 0 To MaxXu Step Gran
    For ylocation = 0 To MaxYu Step Gran
        If xlocation = 0 And ylocation = 0 And zlocation = 0 Then GoTo enditrl
        If xlocation = MaxXu And ylocation = 0 And zlocation = 0 Then GoTo enditrl
        If xlocation = 0 And ylocation = MaxYu And zlocation = 0 Then GoTo enditrl
        If xlocation = MaxXu And ylocation = MaxYu And zlocation = 0 Then GoTo enditrl
        If xlocation = 0 And ylocation = 0 And zlocation = MaxZu Then GoTo enditrl
        If xlocation = MaxXu And ylocation = 0 And zlocation = MaxZu Then GoTo enditrl
        If xlocation = 0 And ylocation = MaxYu And zlocation = MaxZu Then GoTo enditrl
        If xlocation = MaxXu And ylocation = MaxYu And zlocation = MaxZu Then GoTo enditrl
        'define constant variables
        qmark = 0
        Pi = Excel.WorksheetFunction.Pi
        RArad = Val(RA) * Pi / 180
        theta = RArad / 2
        thetaCOMP = (Pi / 2) - theta
    End For
End For

```



```

RRSQ = Val(RR) ^ 2
RRCOStheta = Val(RR) * Cos(theta)
RRSINtheta = Val(RR) * Sin(theta)
TANtheta = Tan(theta)
TANthetaCOMP = Tan(thetaCOMP)
RACrad = Val(RAC) * Pi / 180
thetac = RACrad / 2
thetacCOMP = Pi / 2 - thetac
RRCSQ = Val(RRC) ^ 2
RRCSINthetac = Val(RRC) * Sin(thetac)
TANthetac = Tan(thetac)
TANthetacCOMP = Tan(thetacCOMP)
Range("a" & rowvalue).Select
ActiveCell = "L " & xlocation & "," & ylocation & "," & zlocation
Range("b" & rowvalue).Select

Do Until qmark * Gran >= RRSINtheta
    If ActiveCell.Offset(-1, 0).Value = "" Then Exit Do
    czlocation = zlocation + qmark * Gran
    xmin = Abs(czlocation - zlocation) / TANtheta + xlocation
    If RRSINtheta > Abs(czlocation - zlocation) Then
        xmax = RR * Cos(asin(Abs(czlocation - zlocation) / RR)) + xlocation
    Else:
        xmax = -1
    End If
    ymin = -RRSINtheta + ylocation
    RRqmark = xmax - xmin
    ymax = RRqmark * Sin(theta) + ylocation
    RRqmarkCOStheta = RRqmark * Cos(theta)
    RRqmarkSINtheta = RRqmark * Sin(theta)

    For xp = 0 To MaxXu Step Gran
        absxp = Abs(xp)
        If xp >= xmin And xp <= xmax Then
            IR = True
        Else:
            IR = False
        End If
        If IR = True Then
            If (absxp - xmin) <= RRqmarkCOStheta Then
                miny = ylocation - (absxp - xmin) * TANtheta
            Else:
                miny = ylocation - Sqr(RRqmark ^ 2 - (absxp - xmin) ^ 2)
            End If
        Else:
            miny = -1
        End If

        If IR = True Then
            If miny <= ymin Then
                MINya = ymin
            Else:
                MINya = Excel.WorksheetFunction.RoundUp(miny, 0)
            End If
        Else:
            MINya = -1
        End If
    Next xp
Next Gran

```

```

End If

If IR = True Then
    If (absxp - xmin) <= RRqmarkCOStheta Then
        maxy = ylocation + (absxp - xmin) * TANtheta
    Else:
        maxy = ylocation + Sqr(RRqmark ^ 2 - (absxp - xmin) ^ 2)
    End If
Else:
    maxy = -1
End If

If IR = True Then
    If maxy > ymax Then
        MAXya = ymax
    Else:
        MAXya = Int(maxy)
    End If
Else:
    MAXya = -1
End If
For y = 0 To MaxYu Step Gran
    Call alternate(xmin, xmax, MINya, MAXya, xp, y, xlocation)
    numofcell = numofcell + 1
Next
Next
qmark = qmark + 1
Loop
totalcell = ((MaxXu / Gran) + 1) * ((MaxYu / Gran) + 1) * ((MaxZu / Gran) + 1)
If ActiveCell.Offset(-1, 0).Value = "" Then
    startofvalues = ActiveCell.Offset(0, -1).Address(False, False)
Else:
    startofvalues = ActiveCell.Address(False, False)
End If
endofrow = ActiveCell.Offset(0, totalcell - numofcell - 1).Address(False, False)
zeroselection = startofvalues & ":" & endofrow
Range(zeroselection).Value = 0
numofcell = 0
rowvalue = rowvalue + 1
enditrl:
Next
Next
Next
'If MsgBox("End Program?", vbYesNo) = vbYes Then GoTo endprog
'for all right-wall type readers
read:
For zlocation = 0 To MaxZu Step Gran
    For bot = 0 To zlocation Step Gran
        If bot < (zlocation - RRSINtheta) Then
            For grans = 0 To (MaxXu / Gran) * (MaxYu / Gran)
                ActiveCell = 0
                ActiveCell.Offset(0, 1).Select
            Next
        End If
    Next
Next
For xlocation = 0 To MaxXu Step Gran

```

```

For ylocation = 0 To MaxYu Step Gran
  If xlocation = 0 And ylocation = 0 And zlocation = 0 Then GoTo enditr
  If xlocation = MaxXu And ylocation = 0 And zlocation = 0 Then GoTo enditr
  If xlocation = 0 And ylocation = MaxYu And zlocation = 0 Then GoTo enditr
  If xlocation = MaxXu And ylocation = MaxYu And zlocation = 0 Then GoTo enditr
  If xlocation = 0 And ylocation = 0 And zlocation = MaxZu Then GoTo enditr
  If xlocation = MaxXu And ylocation = 0 And zlocation = MaxZu Then GoTo enditr
  If xlocation = 0 And ylocation = MaxYu And zlocation = MaxZu Then GoTo enditr
  If xlocation = MaxXu And ylocation = MaxYu And zlocation = MaxZu Then GoTo enditr
  'define constant variables
  qmark = 0
  Pi = Excel.WorksheetFunction.Pi
  RArad = Val(RA) * Pi / 180
  theta = RArad / 2
  thetaCOMP = (Pi / 2) - theta
  RRSQ = Val(RR) ^ 2
  RRCOStheta = Val(RR) * Cos(theta)
  RRSINtheta = Val(RR) * Sin(theta)
  TANtheta = Tan(theta)
  TANthetaCOMP = Tan(thetaCOMP)
  RACrad = Val(RAC) * Pi / 180
  thetac = RACrad / 2
  thetacCOMP = Pi / 2 - thetac
  RRCSQ = Val(RRC) ^ 2
  RRCSINthetac = Val(RRC) * Sin(thetac)
  TANthetac = Tan(thetac)
  TANthetacCOMP = Tan(thetacCOMP)
  Range("a" & rowvalue).Select
  ActiveCell = "R " & MaxXu - xlocation & "," & ylocation & "," & zlocation
  Range("b" & rowvalue).Select

Do Until qmark * Gran >= RRSINtheta
  If ActiveCell.Offset(-1, 0).Value = "" Then Exit Do
  czlocation = zlocation + qmark * Gran
  xmin = Abs(czlocation - zlocation) / TANtheta + xlocation
  If RRSINtheta > Abs(czlocation - zlocation) Then
    xmax = RR * Cos(asin(Abs(czlocation - zlocation) / RR)) + xlocation
  Else:
    xmax = -1
  End If
  ymin = -RRSINtheta + ylocation
  RRqmark = xmax - xmin
  ymax = RRqmark * Sin(theta) + ylocation
  RRqmarkCOStheta = RRqmark * Cos(theta)
  RRqmarkSINtheta = RRqmark * Sin(theta)

  For xp = MaxXu To 0 Step -Gran
    absxp = Abs(xp)
    If xp >= xmin And xp <= xmax Then
      IR = True
    Else:
      IR = False
    End If
    If IR = True Then
      If (absxp - xmin) <= RRqmarkCOStheta Then
        miny = ylocation - (absxp - xmin) * TANtheta
      End If
    End If
  Next xp
  qmark = qmark + 1
End Do

```

```

Else:
    miny = ylocation - Sqr(RRqmark ^ 2 - (absxp - xmin) ^ 2)
End If
Else:
    miny = -1
End If

If IR = True Then
    If miny <= ymin Then
        MINya = ymin
    Else:
        MINya = Excel.WorksheetFunction.RoundUp(miny, 0)
    End If
Else:
    MINya = -1
End If

If IR = True Then
    If (absxp - xmin) <= RRqmarkCOStheta Then
        maxy = ylocation + (absxp - xmin) * TANtheta
    Else:
        maxy = ylocation + Sqr(RRqmark ^ 2 - (absxp - xmin) ^ 2)
    End If
Else:
    maxy = -1
End If

If IR = True Then
    If maxy > ymax Then
        MAXya = ymax
    Else:
        MAXya = Int(maxy)
    End If
Else:
    MAXya = -1
End If
For y = 0 To MaxYu Step Gran
    Call alternate(xmin, xmax, MINya, MAXya, xp, y, xlocation)
    numofcell = numofcell + 1
Next
Next
qmark = qmark + 1
Loop
totalcell = ((MaxXu / Gran) + 1) * ((MaxYu / Gran) + 1) * ((MaxZu / Gran) + 1)
If ActiveCell.Offset(-1, 0).Value = "" Then
    startofvalues = ActiveCell.Offset(0, -1).Address(False, False)
Else:
    startofvalues = ActiveCell.Address(False, False)
End If
endofrow = ActiveCell.Offset(0, totalcell - numofcell - 1).Address(False, False)
zeroselection = startofvalues & ":" & endofrow
Range(zeroselection).Value = 0
numofcell = 0
rowvalue = rowvalue + 1
enditr:
Next

```

```

Next
Next
'for all top-wall type readers
tread:
For zlocation = 0 To MaxZu Step Gran
  For bot = 0 To zlocation Step Gran
    If bot < (zlocation - RRSINtheta) Then
      For grans = 0 To (MaxXu / Gran) * (MaxYu / Gran)
        ActiveCell = 0
        ActiveCell.Offset(0, 1).Select
      Next
    End If
  Next
Next
For xlocation = 0 To MaxXu Step Gran
  For ylocation = 0 To MaxYu Step Gran
    If xlocation = 0 And ylocation = 0 And zlocation = 0 Then GoTo enditrt
    If xlocation = MaxXu And ylocation = 0 And zlocation = 0 Then GoTo enditrt
    If xlocation = 0 And ylocation = MaxYu And zlocation = 0 Then GoTo enditrt
    If xlocation = MaxXu And ylocation = MaxYu And zlocation = 0 Then GoTo enditrt
    If xlocation = 0 And ylocation = 0 And zlocation = MaxZu Then GoTo enditrt
    If xlocation = MaxXu And ylocation = 0 And zlocation = MaxZu Then GoTo enditrt
    If xlocation = 0 And ylocation = MaxYu And zlocation = MaxZu Then GoTo enditrt
    If xlocation = MaxXu And ylocation = MaxYu And zlocation = MaxZu Then GoTo enditrt
    'define constant variables
    qmark = 0
    Pi = Excel.WorksheetFunction.Pi
    RArad = Val(RA) * Pi / 180
    theta = RArad / 2
    thetaCOMP = (Pi / 2) - theta
    RRSQ = Val(RR) ^ 2
    RRCOStheta = Val(RR) * Cos(theta)
    RRSINtheta = Val(RR) * Sin(theta)
    TANtheta = Tan(theta)
    TANthetaCOMP = Tan(thetaCOMP)
    RACrad = Val(RAC) * Pi / 180
    thetac = RACrad / 2
    thetacCOMP = Pi / 2 - thetac
    RRCSQ = Val(RRC) ^ 2
    RRCSINthetac = Val(RRC) * Sin(thetac)
    TANthetac = Tan(thetac)
    TANthetacCOMP = Tan(thetacCOMP)
    Range("a" & rowvalue).Select
    ActiveCell = "T " & xlocation & "," & ylocation & "," & zlocation
    Range("b" & rowvalue).Select

    Do Until qmark * Gran >= RRSINtheta
      If ActiveCell.Offset(-1, 0).Value = "" Then Exit Do
      czlocation = zlocation + qmark * Gran
      ymin = ylocation + Abs(czlocation - zlocation) / TANtheta
      If RRSINtheta > Abs(czlocation - zlocation) Then
        ymax = ylocation + RR * (Cos(asin(Abs(czlocation - zlocation) / RR)))
      Else:
        ymax = -1
      End If
      RRqmark = ymax - ymin
      RRqmarkCOStheta = RRqmark * Cos(theta)

```

```

RRqmarkSINtheta = RRqmark * Sin(theta)
xmin = -RRqmarkSINtheta
xmax = RRqmarkSINtheta
For xp = 0 - xlocation To MaxXu - xlocation Step Gran
    absxp = Abs(xp)
    If xp >= xmin And xp <= xmax Then
        IR = True
    Else:
        IR = False
    End If
    If IR = True Then
        miny = absxp * TANthetaCOMP + ymin
    Else:
        miny = -1
    End If

    If IR = True Then
        If miny <= ymin Then
            MINya = ymin
        Else:
            MINya = Excel.WorksheetFunction.RoundUp(miny, 0)
        End If
    Else:
        MINya = -1
    End If

    If IR = True And RRqmark >= absxp Then
        maxy = ymin + Sqr(RRqmark ^ 2 - (absxp) ^ 2)
    Else:
        maxy = -1
    End If

    If IR = True Then
        If maxy > ymax Then
            MAXya = ymax
        Else:
            MAXya = Int(maxy)
        End If
    Else:
        MAXya = -1
    End If
    For y = 0 To MaxYu Step Gran
        Call alternate(xmin, xmax, MINya, MAXya, xp, y, xlocation)
        numofcell = numofcell + 1
    Next
Next
qmark = qmark + 1
Loop
totalcell = ((MaxXu / Gran) + 1) * ((MaxYu / Gran) + 1) * ((MaxZu / Gran) + 1)
If ActiveCell.Offset(-1, 0).Value = "" Then
    startofvalues = ActiveCell.Offset(0, -1).Address(False, False)
Else:
    startofvalues = ActiveCell.Address(False, False)
End If
endofrow = ActiveCell.Offset(0, totalcell - numofcell - 1).Address(False, False)
zeroselection = startofvalues & ":" & endofrow

```

```

        Range(zerosselection).Value = 0
        numofcell = 0
        rowvalue = rowvalue + 1
enditrt:
    Next
Next
bread:
'for all bottom-wall type readers
For zlocation = 0 To MaxZu Step Gran
    For bot = 0 To zlocation Step Gran
        If bot < (zlocation - RRSINtheta) Then
            For grans = 0 To (MaxXu / Gran) * (MaxYu / Gran)
                ActiveCell = 0
                ActiveCell.Offset(0, 1).Select
            Next
        End If
    Next
Next
For xlocation = 0 To MaxXu Step Gran
    For ylocation = 0 To MaxYu Step Gran
        If xlocation = 0 And ylocation = 0 And zlocation = 0 Then GoTo enditrb
        If xlocation = MaxXu And ylocation = 0 And zlocation = 0 Then GoTo enditrb
        If xlocation = 0 And ylocation = MaxYu And zlocation = 0 Then GoTo enditrb
        If xlocation = MaxXu And ylocation = MaxYu And zlocation = 0 Then GoTo enditrb
        If xlocation = 0 And ylocation = 0 And zlocation = MaxZu Then GoTo enditrb
        If xlocation = MaxXu And ylocation = 0 And zlocation = MaxZu Then GoTo enditrb
        If xlocation = 0 And ylocation = MaxYu And zlocation = MaxZu Then GoTo enditrb
        If xlocation = MaxXu And ylocation = MaxYu And zlocation = MaxZu Then GoTo enditrb
'define constant variables
qmark = 0
Pi = Excel.WorksheetFunction.Pi
RArad = Val(RA) * Pi / 180
theta = RArad / 2
thetaCOMP = (Pi / 2) - theta
RRSQ = Val(RR) ^ 2
RRCOStheta = Val(RR) * Cos(theta)
RRSINtheta = Val(RR) * Sin(theta)
TANtheta = Tan(theta)
TANthetaCOMP = Tan(thetaCOMP)
RACrad = Val(RAC) * Pi / 180
thetac = RACrad / 2
thetacCOMP = Pi / 2 - thetac
RRCSQ = Val(RRC) ^ 2
RRCSINthetac = Val(RRC) * Sin(thetac)
TANthetac = Tan(thetac)
TANthetacCOMP = Tan(thetacCOMP)
Range("a" & rowvalue).Select
ActiveCell = "B " & xlocation & ", " & ylocation & ", " & zlocation
Range("b" & rowvalue).Select

Do Until qmark * Gran >= RRSINtheta
    If ActiveCell.Offset(-1, 0).Value = "" Then Exit Do
    czlocation = zlocation + qmark * Gran
    ymax = ylocation + Abs(czlocation - zlocation) / TANtheta
    If RRSINtheta > Abs(czlocation - zlocation) Then
        ymin = ylocation - RR * (Cos(asin(Abs(czlocation - zlocation) / RR)))
    End If
    qmark = qmark + 1
End Do

```

```

Else:
    ymin = -1
End If
RRqmark = ymax - ymin
RRqmarkCOStheta = RRqmark * Cos(theta)
RRqmarkSINtheta = RRqmark * Sin(theta)
xmin = -RRqmarkSINtheta
xmax = RRqmarkSINtheta
For xp = 0 - xlocation To MaxXu - xlocation Step Gran
    absxp = Abs(xp)
    If xp >= xmin And xp <= xmax Then
        IR = True
    Else:
        IR = False
    End If
    If IR = True Then
        maxy = ymax - absxp * TANthetaCOMP
    Else:
        maxy = -1
    End If

    If IR = True And RRqmark >= absxp Then
        miny = ymax - Sqr(RRqmark ^ 2 - (absxp) ^ 2)
    Else:
        miny = -1
    End If

    If IR = True Then
        If miny <= ymin Then
            MINya = ymin
        Else:
            MINya = Excel.WorksheetFunction.RoundUp(miny, 0)
        End If
    Else:
        MINya = -1
    End If

    If IR = True Then
        If maxy > ymax Then
            MAXya = ymax
        Else:
            MAXya = Int(maxy)
        End If
    Else:
        MAXya = -1
    End If
    For y = 0 To MaxYu Step Gran
        Call alternate(xmin, xmax, MINya, MAXya, xp, y, xlocation)
        numofcell = numofcell + 1
    Next
Next
qmark = qmark + 1
Loop
totalcell = ((MaxXu / Gran) + 1) * ((MaxYu / Gran) + 1) * ((MaxZu / Gran) + 1)
If ActiveCell.Offset(-1, 0).Value = "" Then
    startofvalues = ActiveCell.Offset(0, -1).Address(False, False)

```



```

Else:
    startofvalues = ActiveCell.Address(False, False)
End If
endofrow = ActiveCell.Offset(0, totalcell - numofcell - 1).Address(False, False)
zeroselection = startofvalues & ":" & endofrow
Range(zeroselection).Value = 0
numofcell = 0
rowvalue = rowvalue + 1
enditrb:
Next
Next
Next
tlcread:
'for all top-left corner readers
'define constant variables
qmark = 0
Pi = Excel.WorksheetFunction.Pi
RARad = Val(RA) * Pi / 180
theta = RARad / 2
thetaCOMP = (Pi / 2) - theta
RRSQ = Val(RR) ^ 2
RRCOStheta = Val(RR) * Cos(theta)
RRSINtheta = Val(RR) * Sin(theta)
TANtheta = Tan(theta)
TANthetaCOMP = Tan(thetaCOMP)
RACrad = Val(RAC) * Pi / 180
thetac = RACrad / 2
thetacCOMP = Pi / 2 - thetac
RRCSQ = Val(RRC) ^ 2
RRCSINthetac = Val(RRC) * Sin(thetac)
TANthetac = Tan(thetac)
TANthetacCOMP = Tan(thetacCOMP)
If MaxZu = 0 Then
    MaxZustep = 1
Else:
    MaxZustep = MaxZu
End If
For tlclloop = 0 To MaxZu Step MaxZustep
    xlocation = 0
    ylocation = 0
    zlocation = 0
    Range("a" & rowvalue).Select
    ActiveCell = "TL " & xlocation & "," & ylocation & "," & tlclloop
    Range("b" & rowvalue).Select
    qmark = 0
    Do Until qmark * Gran >= RRCSINthetac
        Range("a" & rowvalue).Select
        ActiveCell = "TL " & xlocation & "," & ylocation & "," & tlclloop
        Range("b" & rowvalue).Select
        If ActiveCell.Offset(-1, 0).Value = "" Then Exit Do
        czlocation = zlocation + qmark * Gran
        xmin = Abs(czlocation - zlocation) / TANthetac
        If RRCSINthetac > Abs(czlocation - zlocation) Then
            xmax = RR * (Cos(asin(Abs(czlocation - zlocation) / RR)))
        Else:
            xmax = -1

```

```

End If
ymin = ylocation + Abs(czlocation - zlocation) / TANtheta
If RRCsintheta > Abs(czlocation - zlocation) Then
    ymax = ylocation + RR * Cos(asin((Abs(czlocation - zlocation) / RR)))
Else:
    ymax = -1
End If
RRCqmark = ymax - ymin
RRCqmarkCOStheta = RRCqmark * Cos(theta)
RRCqmarkSINtheta = RRCqmark * Sin(theta)
For xp = 0 - xlocation To MaxXu Step Gran
    absxp = Abs(xp)

    If xp >= xmin And xp <= xmax Then
        IR = True
    Else:
        IR = False
    End If

    If IR = True Then
        miny = ymin
    Else:
        miny = -1
    End If

    If IR = True Then
        If miny <= ymin Then
            MINya = ymin
        Else:
            MINya = Excel.WorksheetFunction.RoundUp(miny, 0)
        End If
    Else:
        MINya = -1
    End If

    If IR = True Then
        maxy = ymin + Sqr(RRCqmark ^ 2 - (absxp - ymin) ^ 2)
    Else:
        maxy = -1
    End If

    If IR = True Then
        If maxy > ymax Then
            MAXya = ymax
        Else:
            MAXya = Int(maxy)
        End If
    Else:
        MAXya = -1
    End If
    For y = 0 To MaxYu Step Gran
        Call alternate(xmin, xmax, MINya, MAXya, xp, y, xlocation)
        numofcell = numofcell + 1
    Next
Next
qmark = qmark + 1

```

```

    rowvalue = rowvalue + 2
Loop
totalcell = ((MaxXu / Gran) + 1) * ((MaxYu / Gran) + 1) * ((MaxZu / Gran) + 1)
If ActiveCell.Offset(-1, 0).Value = "" Then
    startofvalues = ActiveCell.Offset(0, -1).Address(False, False)
Else:
    startofvalues = ActiveCell.Address(False, False)
End If
endofrow = ActiveCell.Offset(0, totalcell - numofcell - 1).Address(False, False)
zeroselection = startofvalues & ":" & endofrow
Range(zeroselection).Value = 0
numofcell = 0
rowvalue = rowvalue + 1
Next
'If MsgBox("Continue?", vbYesNo, "Message") = vbNo Then GoTo endprog
trcread:
'for all top-right corner readers
'define constant variables
qmark = 0
Pi = Excel.WorksheetFunction.Pi
RARad = Val(RA) * Pi / 180
theta = RARad / 2
thetaCOMP = (Pi / 2) - theta
RRSQ = Val(RR) ^ 2
RRCOStheta = Val(RR) * Cos(theta)
RRSINtheta = Val(RR) * Sin(theta)
TANtheta = Tan(theta)
TANthetaCOMP = Tan(thetaCOMP)
RACrad = Val(RAC) * Pi / 180
thetac = RACrad / 2
thetacCOMP = Pi / 2 - thetac
RRCSQ = Val(RRC) ^ 2
RRCSINthetac = Val(RRC) * Sin(thetac)
TANthetac = Tan(thetac)
TANthetacCOMP = Tan(thetacCOMP)
If MaxZu = 0 Then
    MaxZustep = 1
Else:
    MaxZustep = MaxZu
End If
For trcloop = 0 To MaxZu Step MaxZustep
    xlocation = 0
    ylocation = 0
    zlocation = 0
    Range("a" & rowvalue).Select
    ActiveCell = "TR " & MaxXu & "," & ylocation & "," & trcloop
    Range("b" & rowvalue).Select
    qmark = 0
    Do Until qmark * Gran >= RRCSINthetac
'    Range("a" & rowvalue).Select
'    ActiveCell = "TL " & xlocation & "," & ylocation & "," & tlcloop
'    Range("b" & rowvalue).Select
        If ActiveCell.Offset(-1, 0).Value = "" Then Exit Do
        czlocation = zlocation + qmark * Gran
        xmin = Abs(czlocation - zlocation) / TANthetac
        If RRCSINthetac > Abs(czlocation - zlocation) Then

```

```

    xmax = (RR * (Cos(asin(Abs(czlocation - zlocation) / RR))))
Else:
    xmax = -1
End If
ymin = ylocation + Abs(czlocation - zlocation) / TANthetac
If RRCSINthetac > Abs(czlocation - zlocation) Then
    ymax = ylocation + RR * Cos(asin((Abs(czlocation - zlocation) / RR)))
Else:
    ymax = -1
End If
RRCqmark = ymax - ymin
RRCqmarkCOSthetac = RRCqmark * Cos(thetac)
RRCqmarkSINthetac = RRCqmark * Sin(thetac)
For xp = MaxXu To 0 - xlocation Step -Gran
    absxp = Abs(xp)

    If xp >= xmin And xp <= xmax Then
        IR = True
    Else:
        IR = False
    End If

    If IR = True Then
        miny = ymin
    Else:
        miny = -1
    End If

    If IR = True Then
        If miny <= ymin Then
            MINya = ymin
        Else:
            MINya = Excel.WorksheetFunction.RoundUp(miny, 0)
        End If
    Else:
        MINya = -1
    End If

    If IR = True Then
        maxy = ymin + Sqr(RRCqmark ^ 2 - (absxp - ymin) ^ 2)
    Else:
        maxy = -1
    End If

    If IR = True Then
        If maxy > ymax Then
            MAXya = ymax
        Else:
            MAXya = Int(maxy)
        End If
    Else:
        MAXya = -1
    End If
    For y = 0 To MaxYu Step Gran
        Call alternate(xmin, xmax, MINya, MAXya, xp, y, xlocation)
        numofcell = numofcell + 1
    
```

```

        Next
    Next
    qmark = qmark + 1
    rowvalue = rowvalue + 2
Loop
totalcell = ((MaxXu / Gran) + 1) * ((MaxYu / Gran) + 1) * ((MaxZu / Gran) + 1)
If ActiveCell.Offset(-1, 0).Value = "" Then
    startofvalues = ActiveCell.Offset(0, -1).Address(False, False)
Else:
    startofvalues = ActiveCell.Address(False, False)
End If
endofrow = ActiveCell.Offset(0, totalcell - numofcell - 1).Address(False, False)
zeroselection = startofvalues & ":" & endofrow
Range(zeroselection).Value = 0
numofcell = 0
rowvalue = rowvalue + 1
Next
blcread:
'for all bottom-left corner readers
'define constant variables
qmark = 0
Pi = Excel.WorksheetFunction.Pi
RArad = Val(RA) * Pi / 180
theta = RArad / 2
thetaCOMP = (Pi / 2) - theta
RRSQ = Val(RR) ^ 2
RRCOStheta = Val(RR) * Cos(theta)
RRSINtheta = Val(RR) * Sin(theta)
TANtheta = Tan(theta)
TANthetaCOMP = Tan(thetaCOMP)
RACrad = Val(RAC) * Pi / 180
thetac = RACrad / 2
thetacCOMP = Pi / 2 - thetac
RRCSQ = Val(RRC) ^ 2
RRCSINthetac = Val(RRC) * Sin(thetac)
TANthetac = Tan(thetac)
TANthetacCOMP = Tan(thetacCOMP)
If MaxZu = 0 Then
    MaxZustep = 1
Else:
    MaxZustep = MaxZu
End If
For blcloop = 0 To MaxZu Step MaxZustep
    xlocation = 0
    ylocation = MaxYu
    zlocation = 0
    Range("a" & rowvalue).Select
    ActiveCell = "BL " & xlocation & "," & ylocation & "," & blcloop
    Range("b" & rowvalue).Select
    qmark = 0
    Do Until qmark * Gran >= RRCSINthetac
        If ActiveCell.Offset(-1, 0).Value = "" Then Exit Do
        czlocation = zlocation + qmark * Gran
        xmin = Abs(czlocation - zlocation) / TANthetac
        If RRCSINthetac > Abs(czlocation - zlocation) Then
            xmax = (RR * (Cos(asin(Abs(czlocation - zlocation) / RR))))

```

```

Else:
    xmax = -1
End If
ymax = ylocation - Abs(czlocation - zlocation) / TANthetac
If RRC SINthetac > Abs(czlocation - zlocation) Then
    ymin = ylocation - RR * Cos(asin((Abs(czlocation - zlocation) / RR)))
Else:
    ymin = -1
End If
RRCqmark = ymax - ymin
RRCqmarkCOSthetac = RRCqmark * Cos(thetac)
RRCqmarkSINthetac = RRCqmark * Sin(thetac)
For xp = 0 - xlocation To MaxXu Step Gran
    absxp = Abs(xp)

    If xp >= xmin And xp <= xmax Then
        IR = True
    Else:
        IR = False
    End If

    If IR = True Then
        maxy = ymax
    Else:
        maxy = -1
    End If

    If IR = True Then
        miny = ymax - Sqr(RRCqmark ^ 2 - (absxp - xmin) ^ 2)
    Else:
        miny = -1
    End If

    If IR = True Then
        If miny <= ymin Then
            MINya = ymin
        Else:
            MINya = Excel.WorksheetFunction.RoundUp(miny, 0)
        End If
    Else:
        MINya = -1
    End If

    If IR = True Then
        If maxy > ymax Then
            MAXya = ymax
        Else:
            MAXya = Int(maxy)
        End If
    Else:
        MAXya = -1
    End If
    For y = 0 To MaxYu Step Gran
        Call alternate(xmin, xmax, MINya, MAXya, xp, y, xlocation)
        numofcell = numofcell + 1
    
```

```

        Next
    Next
    qmark = qmark + 1
    rowvalue = rowvalue + 2
Loop
totalcell = ((MaxXu / Gran) + 1) * ((MaxYu / Gran) + 1) * ((MaxZu / Gran) + 1)
If ActiveCell.Offset(-1, 0).Value = "" Then
    startofvalues = ActiveCell.Offset(0, -1).Address(False, False)
Else:
    startofvalues = ActiveCell.Address(False, False)
End If
endofrow = ActiveCell.Offset(0, totalcell - numofcell - 1).Address(False, False)
zeroselection = startofvalues & ":" & endofrow
Range(zeroselection).Value = 0
numofcell = 0
rowvalue = rowvalue + 1
Next
brcread:
'for all bottom-right corner readers
'define constant variables
qmark = 0
Pi = Excel.WorksheetFunction.Pi
RArad = Val(RA) * Pi / 180
theta = RArad / 2
thetaCOMP = (Pi / 2) - theta
RRSQ = Val(RR) ^ 2
RRCOStheta = Val(RR) * Cos(theta)
RRSINtheta = Val(RR) * Sin(theta)
TANtheta = Tan(theta)
TANthetaCOMP = Tan(thetaCOMP)
RACrad = Val(RAC) * Pi / 180
thetac = RACrad / 2
thetacCOMP = Pi / 2 - thetac
RRCSQ = Val(RRC) ^ 2
RRCSINthetac = Val(RRC) * Sin(thetac)
TANthetac = Tan(thetac)
TANthetacCOMP = Tan(thetacCOMP)
If MaxZu = 0 Then
    MaxZustep = 1
Else:
    MaxZustep = MaxZu
End If
For brcloop = 0 To MaxZu Step MaxZustep
    xlocation = 0
    ylocation = MaxYu
    zlocation = 0
    Range("a" & rowvalue).Select
    ActiveCell = "BR " & MaxXu & "," & ylocation & "," & brcloop
    Range("b" & rowvalue).Select
    qmark = 0
    Do Until qmark * Gran >= RRCSINthetac
        If ActiveCell.Offset(-1, 0).Value = "" Then Exit Do
        czlocation = zlocation + qmark * Gran
        xmin = Abs(czlocation - zlocation) / TANthetac
        If RRCSINthetac > Abs(czlocation - zlocation) Then
            xmax = (RR * (Cos(asin(Abs(czlocation - zlocation) / RR))))

```

```

Else:
    xmax = -1
End If
ymax = ylocation - Abs(czlocation - zlocation) / TANthetac
If RRC SINthetac > Abs(czlocation - zlocation) Then
    ymin = ylocation - RR * Cos(asin((Abs(czlocation - zlocation) / RR)))
Else:
    ymin = -1
End If
RRCqmark = ymax - ymin
RRCqmarkCOSthetac = RRCqmark * Cos(thetac)
RRCqmarkSINthetac = RRCqmark * Sin(thetac)
For xp = MaxXu To 0 - xlocation Step -Gran
    absxp = Abs(xp)

    If xp >= xmin And xp <= xmax Then
        IR = True
    Else:
        IR = False
    End If

    If IR = True Then
        maxy = ymax
    Else:
        maxy = -1
    End If

    If IR = True Then
        miny = ymax - Sqr(RRCqmark ^ 2 - (absxp - xmin) ^ 2)
    Else:
        miny = -1
    End If

    If IR = True Then
        If miny <= ymin Then
            MINya = ymin
        Else:
            MINya = Excel.WorksheetFunction.RoundUp(miny, 0)
        End If
    Else:
        MINya = -1
    End If

    If IR = True Then
        If maxy > ymax Then
            MAXya = ymax
        Else:
            MAXya = Int(maxy)
        End If
    Else:
        MAXya = -1
    End If

    For y = 0 To MaxYu Step Gran
        Call alternate(xmin, xmax, MINya, MAXya, xp, y, xlocation)
        numofcell = numofcell + 1
    
```



```

        Next
    Next
    qmark = qmark + 1
    rowvalue = rowvalue + 2
Loop
totalcell = ((MaxXu / Gran) + 1) * ((MaxYu / Gran) + 1) * ((MaxZu / Gran) + 1)
If ActiveCell.Offset(-1, 0).Value = "" Then
    startofvalues = ActiveCell.Offset(0, -1).Address(False, False)
Else:
    startofvalues = ActiveCell.Address(False, False)
End If
endofrow = ActiveCell.Offset(0, totalcell - numofcell - 1).Address(False, False)
zeroselection = startofvalues & ":" & endofrow
Range(zeroselection).Value = 0
numofcell = 0
rowvalue = rowvalue + 1
Next
endprog:
End Sub

Private Sub test_Click()
MaxXu = 21
MaxYu = 21
MaxZu = 21
Gran = 3
RR = 12
RA = 150
RRC = 12
RAC = 150
End Sub

```