

# LIFE-CYCLE COST DESIGN OF DETERIORATING STRUCTURES

By Dan M. Frangopol,<sup>1</sup> Fellow, ASCE, Kai-Yung Lin,<sup>2</sup> and Allen C. Estes,<sup>3</sup> Member, ASCE

**ABSTRACT:** A lifetime optimization methodology for planning the inspection and repair of structures that deteriorate over time is introduced and illustrated through numerical examples. The optimization is based on minimizing the expected total life-cycle cost while maintaining an allowable lifetime reliability for the structure. This method incorporates: (a) the quality of inspection techniques with different detection capabilities; (b) all repair possibilities based on an event tree; (c) the effects of aging, deterioration, and subsequent repair on structural reliability; and (d) the time value of money. The overall cost to be minimized includes the initial cost and the costs of preventive maintenance, inspection, repair, and failure. The methodology is illustrated using the reinforced concrete T-girders from a highway bridge. An optimum inspection/repair strategy is developed for these girders that are deteriorating due to corrosion in an aggressive environment. The effect of critical parameters such as rate of corrosion, quality of the inspection technique, and the expected cost of structural failure are all investigated, along with the effects of both uniform and nonuniform inspection time intervals. Ultimately, the reliability-based lifetime approach to developing an optimum inspection/repair strategy demonstrates the potential for cost savings and improved efficiency.

## INTRODUCTION

The management of the nation's infrastructure is a vitally important function of government. The inspection and repair of the transportation network is needed for uninterrupted commerce and a functioning economy. With about 600,000 highway bridges in the national inventory, the maintenance of these structures alone represents a commitment of billions of dollars annually. In fact, the nation spends at least \$5,000,000,000 per year for highway bridge design, construction, replacement, and rehabilitation (Status 1993). Given this huge investment along with an increasing scarcity of resources, it is essential that the funds be used as efficiently as possible.

Highway bridges deteriorate over time and need maintenance/inspection programs that detect damage, deterioration, loss of effective strength in members, missing fasteners, fractures, and cracks. Bridge serviceability is highly dependent on the frequency and quality of these maintenance programs. Because the welfare of many people depends on the health of the highway system, it is important that these bridges be maintained and inspected routinely. An efficient bridge maintenance program requires careful planning based on potential modes of failure of the structural elements, the history of major structural repairs done to the bridge, and, of course, the frequency and intensity of the applied loads. Effective maintenance/inspection can extend the life expectancy of a system while reducing the possibility of costly failures in the future.

In any bridge, there are many defects that may appear during a projected service period, such as potholes in the deck, scour on the piers, or the deterioration of joints or bearings. Corrosion of steel reinforcement, initiated by high chloride concentrations in the concrete, is a serious cause of degradation in concrete structures (Ting 1989). The corrosion damage is revealed by the initiation and propagation of cracks, which can be detected and repaired by scheduled maintenance and inspection procedures. As a result, the reliability of corrosive

critical structures depends not only on the structural design, but also on the inspection and repair procedures.

This paper proposes a method to optimize the lifetime inspection/repair strategy of corrosion-critical concrete structures based on the reliability of the structure and cost-effectiveness. The method is applicable for any type of damage whose evolution can be modeled over time. The reliability-based analysis of structures, with or without maintenance/inspection procedures, is attracting the increased attention of researchers (Thoft-Christensen and Sørensen 1987; Mori and Ellingwood 1994a). The optimal lifetime inspection/repair strategy is obtained by minimizing the expected total life-cycle cost while satisfying the constraints on the allowable level of structural lifetime reliability in service. The expected total life-cycle cost includes the initial cost and the costs of preventive maintenance, inspection, repair, and failure.

## MAINTENANCE/INSPECTION

For many bridges, both preventive and repair maintenance are typically performed. Preventive or routine maintenance includes replacing small parts, patching concrete, repairing cracks, changing lubricants, and cleaning and painting exposed parts. The structure is kept in working condition by delaying and mitigating the aging effects of wear, fatigue, and related phenomena. In contrast, repair maintenance might include replacing a bearing, resurfacing a deck, or modifying a girder. Repair maintenance tends to be less frequent, requires more effort, is usually more costly, and results in a measurable increase in reliability. A sample maintenance strategy is shown in Fig. 1, where  $T_1$ ,  $T_2$ ,  $T_3$ , and  $T_4$  represent the times of repair maintenance, and effort is a generic quantity that reflects cost, amount of work performed, and benefit derived from the maintenance.

While guidance for routine maintenance exists, many repair maintenance strategies are based on experience and local practice rather than on sound theoretical investigations. Maintenance

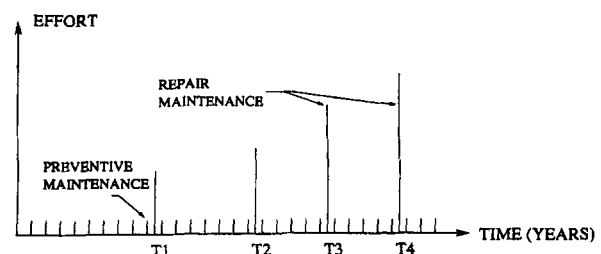


FIG. 1. Maintenance Strategy

<sup>1</sup>Prof., Dept. of Civ., Envir., and Arch. Engrg., Univ. of Colorado, Boulder, CO 80309-0428.

<sup>2</sup>Proj. Mgr., Chung-Shen Inst. of Sci. and Technol., Taiwan, Republic of China; formerly, Grad. Student, Dept. of Civ., Envir., and Arch. Engrg., Univ. of Colorado, Boulder, CO.

<sup>3</sup>Grad. Student, Dept. of Civ., Envir., and Arch. Engrg., Univ. of Colorado, Boulder, CO.

nance/inspections based solely on experience may be more expensive and less safe than those based on a more rational approach. The optimal policy has to be chosen based on minimum expected total life-cycle cost criterion including its effect on structural reliability and the expected costs associated with failure.

### Preventive Maintenance

The cost of routine maintenance is difficult to predict. Traditionally, an engineering cost associated with the routine maintenance expenditure is used for estimating budgets and planning. Such estimates are obtained by summing the products of input quantities and their unit rates (McNeil and Hendrickson 1982). For example, an organization might use the average cost per mile for bridge repair multiplied by the number of miles of bridges as part of an estimate of repair costs. These average cost rates are derived from observed costs and quantities from a large number of bridges. They rarely account for factors such as weather, bridge age, and bridge condition.

The routine maintenance work is proportional to the size and the age of the bridge. It may become more attractive at some point to replace a bridge rather than spend a large sum of money to maintain it. Because the maintenance cost increases with time, an estimate of the routine cost must consider the effect of time. For a given bridge, the cost of routine maintenance at any time  $t$ ,  $C_{\text{main},t}$ , may be assumed a linear function defined as (McNeil and Hendrickson 1982)

$$C_{\text{main},t} = C_{\text{main}}t \quad (1)$$

where  $C_{\text{main}}$  = cost of preventive maintenance at year one; and  $t$  = age of the bridge in years.

Assuming a service life of 75 years and routine maintenance scheduled once every two years, the preventive work starts at  $t = 2$  years and continues until  $t = 74$  years. Consequently, preventive maintenance work will be performed 37 times during the life of the structure. Therefore, the lifetime routine maintenance cost is

$$C_{PM} = C_{\text{main},2} + C_{\text{main},4} + C_{\text{main},6} + \dots + C_{\text{main},74} \quad (2)$$

If the future maintenance costs are converted to their present values, then the lifetime preventive maintenance cost becomes

$$C_{PM} = C_{\text{main},2} \frac{1}{(1+r)^2} + C_{\text{main},4} \frac{1}{(1+r)^4} + \dots + C_{\text{main},74} \frac{1}{(1+r)^{74}} \quad (3)$$

where  $r$  = net discount rate of money.

Numerous factors such as type of bridge, average daily truck traffic, and bridge environment influence the level of bridge maintenance expenditure. Nonlinear cost functions may be necessary to forecast routine maintenance expenditures based on these factors. Additional research is needed to develop a more accurate cost model for preventive maintenance.

### Inspection

While most bridge inspections are visual, it is assumed, in this study, that all inspection and repair work is for the corrosion of steel reinforcement in concrete and thus requires a nondestructive evaluation (NDE). When performing this special inspection, the ability to detect damage is dependent on the quality of the inspection technique being used. A higher quality inspection method will provide a more dependable assessment of damage. No repair will be made unless the damage is detected.

To define the quality of an NDE inspection method, a damage detectability function is needed. In this paper, the damage

intensity  $\eta$ , which defines the degree of existing damage due to corrosion at time  $t$ , is defined as the ratio

$$\eta(t) = \frac{D_{b0} - D_b(t)}{D_{b0}} \quad (4)$$

where  $D_{b0}$  = initial diameter of a bending reinforcement bar in a concrete section;  $D_b(t)$  = diameter of a bending reinforcement bar at time  $t$ ; and  $t$  = time in years.

The impact of corrosion on the bending capacity of a concrete bridge girder is generally greater than on its shear capacity (Lin 1995). The damage intensity can range from a value of zero, which indicates no damage, to a value of one, which indicates no residual strength. If the time required for the chlorides to penetrate the concrete prior to reaching the reinforcement is considered, the damage intensity function becomes

$$\eta(t) = \begin{cases} 0 & \text{for } 0 \leq t \leq T_i \\ \frac{D_{b0} - D_b(t)}{D_{b0}} & \text{for } T_i < t \end{cases} \quad (5)$$

where  $T_i$  = time of corrosion initiation in years.

The corrosion initiation time will be considered prior to the first repair. Under uniform corrosion, the reinforcing bar diameter  $D_b(t)$  is calculated as

$$D_b(t) = D_{b0} - 2v(t - T_i) \quad (6)$$

where  $v$  is the corrosion rate and the factor 2 takes into account the uniform corrosion propagation process from all sides at the level of the rebar. After the first repair,  $D_b(t)$  is no longer a function of  $T_i$  because the chlorides have already penetrated the concrete. Therefore, after the repair, the reinforcing bar diameter is calculated as

$$D_b(t) = D_{br} - 2vt \quad (7)$$

where  $D_{br}$  is the diameter of the repaired reinforcing bar.

Several NDE techniques for monitoring corrosion of reinforcement are available, such as the electrical resistance method, the half-cell potential method, and the polarization resistance method (Tamura and Yoshida 1984; *Manual* 1994). In some structures, a visual examination may suffice, whereas for other structures, a more complex method may be needed to detect small corrosion defects in reinforcement buried beneath the surface of the concrete structure (Tamura and Yoshida 1984). The effectiveness of available inspection methods can vary widely and must be considered in the development of an inspection program.

Due to a shortage of experimental data, detectability functions are not always available for the techniques that detect corrosion in concrete reinforcing bars. The detectability function  $d(\eta)$  can be defined as the probability of detecting damage given the damage intensity  $\eta$  as follows:

$$d(\eta) = P(\text{damage detection}|\eta) \quad (8)$$

The detectability function  $d(\eta)$  is modeled in this paper as a cumulative normal distribution function for each NDE method. The damage intensity at which the NDE method has a 50% probability of detection is defined as  $\eta_{0.5}$  and the coefficient of variation is assumed to be 0.1 (i.e.,  $\sigma = 0.1\eta_{0.5}$ , where  $\sigma$  = standard deviation). The minimum detectable damage intensity is defined as

$$\eta_{\min} = \eta_{0.5} - 3\sigma = 0.7\eta_{0.5} \quad (9)$$

and the value of damage above which the probability of detection is 1 is denoted as

$$\eta_{\max} = \eta_{0.5} + 3\sigma = 1.3\eta_{0.5} \quad (10)$$

Consequently, the normal distribution is only considered in the interval  $(\eta_{\min}, \eta_{\max})$ . In this manner, the imperfect nature of an NDE method is described in probabilistic terms.

As an example, consider three NDE methods *A*, *B*, and *C*, such that  $\eta_{0.5}^A = 0.05$ ,  $\eta_{0.5}^B = 0.1$ , and  $\eta_{0.5}^C = 0.15$ , respectively. Therefore, minimum detectable damage intensities associated with these methods are 0.035, 0.070, and 0.105, respectively. Clearly, methods *A* and *C* have the highest and lowest detectability, respectively. For a structure under a damage intensity  $\eta$  at time  $T_i$  inspected by an NDE method, the probability of damage detection is approximated as

$$d(\eta) = \begin{cases} 0 & \text{for } 0 \leq \eta \leq \eta_{\min} \\ \Phi\left(\frac{\eta - \eta_{0.5}}{\sigma}\right) & \text{for } \eta_{\min} < \eta \leq \eta_{\max} \\ 1 & \text{for } \eta > \eta_{\max} \end{cases} \quad (11)$$

where  $\Phi(\cdot)$  is the standard normal cumulative distribution function.

In general, the cost of inspection is dependent on the quality of the NDE method. A higher quality inspection is usually more expensive. Assuming that the cost for the ideal inspection [i.e.,  $d(\eta) = 1$  for  $\eta > 0$ ] is  $\alpha_{\text{ins}}$ , the cost associated with a real inspection method,  $C_{\text{ins}}$ , can be estimated based on the quality of detectability as follows (Mori and Ellingwood 1994b):

$$C_{\text{ins}} = \alpha_{\text{ins}}(1 - \eta_{\min})^{20} \quad (12)$$

where  $\eta_{\min} > 0$  is the minimum detectable damage intensity. In this study,  $\alpha_{\text{ins}}$  is assumed to be a fraction (i.e., 0.07) of the initial cost  $C_T$ .

## Repair

Inspections in themselves do not affect the probability of failure of a structure. Following an inspection, a decision must be made regarding repair if damage is found. The repair decision will depend on the inspection quality. With advanced inspection methods, the repair work can be effective, since even a small defect can be detected and repaired. The higher quality of inspection may lead to a higher quality of repair, which brings the reliability of the structure closer to its original condition (Mori and Ellingwood 1994a).

In reality, however, the inspection methods are not perfect. Some items that require repair may be overlooked. When the damage intensity is less than  $\eta_{\min}$  for the inspection method being used, the probability of detection is zero and the structure will not be repaired. Consider a repair following an inspection method with median detectability  $\eta_{0.5}$  at time  $T_i$ . The structure has a damage intensity  $\eta_i$  ( $\eta_{\min} < \eta_i < \eta_{\max}$ ). Due to the uncertainties associated with detectability, some of the damage will not be detected. After repair, the damage intensity will be reduced from  $\eta_i$  to  $\eta_{\text{rep}}$  (i.e.,  $\eta_{\text{rep}} < \eta_i$ ). It is assumed that the damage intensity after repair,  $\eta_{\text{rep}}$ , is expressed as

$$\eta_{\text{rep}} = (\eta_{\min} + \eta_i)/2 \quad (13)$$

When the damage has an intensity at least equal to  $\eta_{\max}$ ,  $\eta_i$  is replaced by  $\eta_{\max}$  in (13). Therefore, when  $\eta_i \geq \eta_{\max}$ , the damage of the repaired structure will be reduced to its median value

$$\eta_{\text{rep}} = (\eta_{\min} + \eta_{\max})/2 = \eta_{0.5} \quad (14)$$

In summary, the damage intensity after repair,  $\eta_{\text{rep}}$ , is approximated as

$$\eta_{\text{rep}} = \begin{cases} \eta_i & \text{for } 0 \leq \eta_i \leq \eta_{\min} \\ (\eta_{\min} + \eta_i)/2 & \text{for } \eta_{\min} < \eta_i < \eta_{\max} \\ \eta_{0.5} & \text{for } \eta_i \geq \eta_{\max} \end{cases} \quad (15)$$

Therefore, the repaired structure, a hybrid of new and old materials, is not expected to be as reliable as the new structure (LeDoux et al. 1983). Also, there are other factors that will affect the reliability (such as its internal degradation, accidental collisions, and aging). In this study, these factors are all grouped into the "aging factor." The aging is considered a linear function over time  $t$ . A 0.4% yearly decrease of the original mean bending moment capacity will be used as the aging rate. As such, the mean residual moment capacity due to aging  $\bar{M}_{r,\text{age}}(t)$  is

$$\bar{M}_{r,\text{age}}(t) = (1 - 0.004t)\bar{M}_{r,0} \quad (16)$$

where  $\bar{M}_{r,0}$  = original mean moment capacity; and  $t$  = age of the structure in years.

The effect of both corrosion and aging deterioration must be considered in determining the resistance capacity of the structure after a repair. Assume that a repair action is undertaken at time  $T_i$ . At this time, the remaining mean capacity of a reinforced concrete beam under age deterioration alone is  $\bar{M}_{r,\text{age}}(T_i)$  and the remaining mean capacity under corrosion alone is  $\bar{M}_{r,\text{corr}}$ . Assuming the damage is detected and a repair is made, the corresponding mean moment capacity is denoted as  $\bar{M}_{r,\text{rep}}$ .

There are two situations to consider. If the deterioration due to aging is not serious and  $\bar{M}_{r,\text{rep}} < \bar{M}_{r,\text{age}}(T_i)$ , then the effect of aging is neglected. The capacity of the beam after repair is assumed as  $\bar{M}_{r,\text{rep}}$ . On the other hand, if the mean moment associated with the proposed repair exceeds the remaining mean moment capacity under age deterioration alone,  $\bar{M}_{r,\text{rep}} > \bar{M}_{r,\text{age}}(T_i)$ , then a compromise between  $\bar{M}_{r,\text{rep}}$  and  $\bar{M}_{r,\text{age}}(T_i)$  is reached. The capacity of the beam after repair is set to be  $[\bar{M}_{r,\text{rep}} + \bar{M}_{r,\text{age}}(T_i)]/2$ .

In summary, when considering the effect of both aging and corrosion, the mean moment capacity after repair becomes

$$\bar{M}_{r,a} = \begin{cases} \bar{M}_{r,\text{rep}} & \text{for } \bar{M}_{r,\text{age}}(T_i) \geq \bar{M}_{r,\text{rep}} \\ [\bar{M}_{r,\text{rep}} + \bar{M}_{r,\text{age}}(T_i)]/2 & \text{for } \bar{M}_{r,\text{age}}(T_i) < \bar{M}_{r,\text{rep}} \end{cases} \quad (17)$$

In most inspection and repair works, it is assumed that the repair cost is constant during the life of the structure. This is not generally true. Since the repair is part of the life-cycle cost, it is reasonable to assume that its cost will be a fraction of the replacement construction cost. In this study, the repair cost is considered to be a function of the replacement cost and the effect of the repair activity.

Assuming that the after repair damage reduces from  $\eta_{br}$  to  $\eta_{ar}$  (i.e.,  $\eta_{ar} < \eta_{br}$ ), where  $\eta_{br}$  and  $\eta_{ar}$  are the damage intensities before and after repair, respectively, the corresponding mean moment capacity increases from  $\bar{M}_{r,b}$  to  $\bar{M}_{r,a}$  (i.e.,  $\bar{M}_{r,a} > \bar{M}_{r,b}$ ), where  $\bar{M}_{r,b}$  and  $\bar{M}_{r,a}$  are the mean moment capacities before and after repair, respectively. The effect of a repair activity,  $e_{\text{rep}}$ , is defined as the amount by which this activity improves the condition of a structural component. Because most structural components are evaluated based on their moment resistances, the effect of a repair activity can be quantified as

$$e_{\text{rep}} = \frac{\bar{M}_{r,a} - \bar{M}_{r,b}}{\bar{M}_{r,0}} \quad (18)$$

where  $\bar{M}_{r,0}$  is the original mean moment capacity of the beam, and  $0 < e_{\text{rep}} < 1$ . The repair cost can be expressed in terms of the repair effect as follows (Mori and Ellingwood 1994b):

$$C_{\text{rep}} = \alpha_{\text{rep}} \left( \frac{\bar{M}_{r,a} - \bar{M}_{r,b}}{\bar{M}_{r,0}} \right)^\gamma = \alpha_{\text{rep}} e_{\text{rep}}^\gamma \quad (19)$$

where  $\gamma$  = a model parameter; and  $\alpha_{\text{rep}}$  = replacement cost. In this study it is assumed that  $\alpha_{\text{rep}}$  is equal to the initial cost,  $\alpha_{\text{rep}} = C_T$ , and  $\gamma = 0.5$ .

## LIFETIME COST

The lifetime (also called life-cycle) cost, target lifetime reliability, inspection interval, and quality of repair must all be considered when optimizing the inspection/repair strategy of structural systems. There is a trade-off between a higher reliability and minimum expected total cost. The goal of an optimal inspection and repair strategy is to minimize the lifetime cost of a given structure while ensuring that the structure maintains an acceptable reliability level throughout its expected service life.

An event tree is used to investigate all possible repair events associated with the inspections. For each case, the structural cross sectional dimensions, corrosion rate  $\nu$ , number of inspections, loads, allowable reliability level, and median detectability of the inspection method  $\eta_{0.5}$  are given. The expected total costs associated with different inspection/repair strategies are obtained.

The assumptions used to compute the optimal lifetime solution are as follows:

1. The initial design is given and the associated reliability index  $\beta_{i=0}$  under nondeteriorating condition is computed.
2. The reliability index  $\beta$  is assumed to be a nonincreasing function with time  $t$  if no repair is performed (Thoft-Christensen and Sørensen 1987).
3. The deterioration mechanism considered is associated with general corrosion.
4. The loading, material properties, and time-dependent limit state function that describes the moment capacity of a reinforced concrete T-girder subjected to corrosion are those described in Lin (1995) and Frangopol et al. (1997).
5. The time value of money is considered using a constant interest rate over time. The net discount,  $r$ , is used to convert the future cost to present cost.
6. If damage is found then a repair action will follow. If the damage is not found then the repair action will be postponed until the next inspection.

## Event Tree Analysis

The event tree model provides a systematic means of structuring and evaluating the repair possibilities related to an uncertain inspection/repair environment. It clearly and precisely defines the total environment. In this study, the event tree is used as a model to represent all possible events associated with repair or no repair actions.

To construct an event tree, it is recognized that a decision to either repair or not repair needs to be made after every inspection. Repair decisions made after every new inspection are influenced by decisions made in the past. For example, the decision whether or not to repair after the second inspection will be influenced by whether or not the structure was repaired after the first inspection. As the number of inspections,  $m$ , increases, the number of branches,  $2^m$ , in the event tree increases much faster.

Fig. 2 shows the inspection and repair event tree when there are five inspections,  $m = 5$  (and therefore  $2^5 = 32$  branches  $B_j$ ), during the lifetime of the structure. The values 0 and 1 represent no repair and repair action, respectively, and  $T_i$  is the time of inspection.

Consider a bridge with three inspections during its entire lifetime using an inspection method with  $\eta_{0.5}$ . Let  $b_j^i$  represent the event corresponding to the occurrence of branch  $j$  at time  $T_i$ . The event tree is shown in Fig. 3, where the inspection interval is  $t_i$  and the  $i$ th inspection occurs at time

$$T_i = \sum_{j=1}^i t_j \quad (20)$$

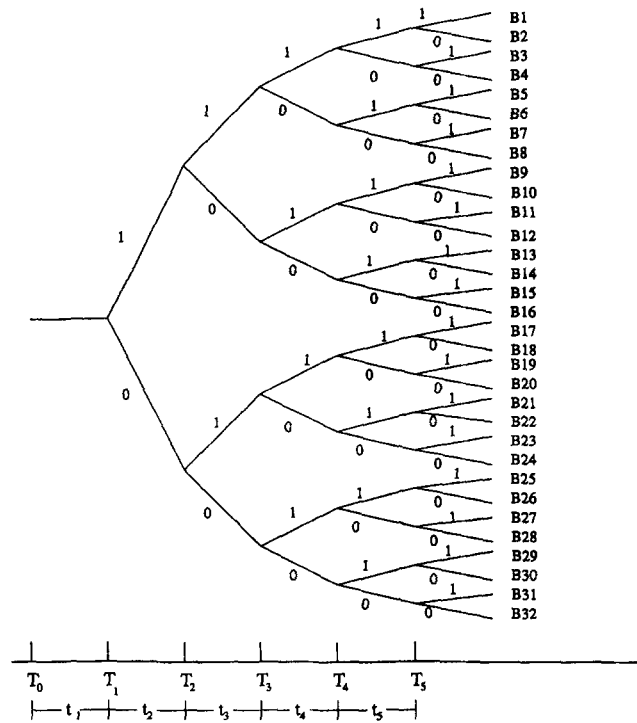


FIG. 2. Event Tree

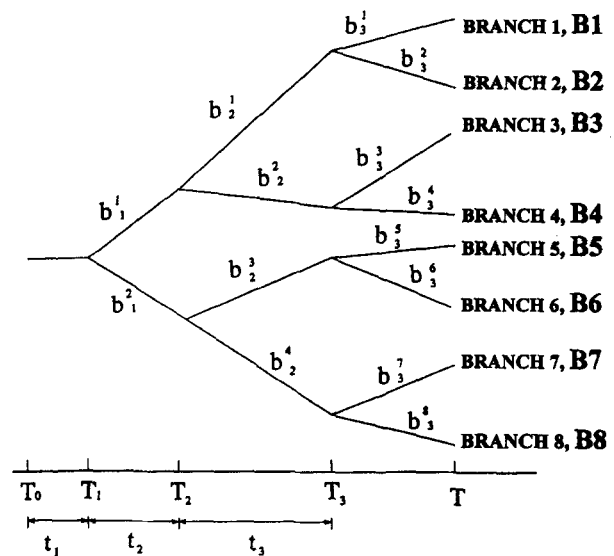


FIG. 3. Event Tree for Bridge with Three Inspections

Assume that the bridge is placed in service at  $t = T_0$ . At time  $t = T_1 - \epsilon_b = T_{1-}$ , where  $\epsilon_b$  represents a small time interval (e.g., 1 day) before the first inspection occurs, the probability of failure of this structure is

$$P_{f,T_{1-}} = P[g_M(T_{1-}) \leq 0] \quad (21)$$

where  $g_M(T_{1-}) \leq 0$  defines the failure of the concrete bridge girder due to moment for the reference time interval  $(T_0, T_{1-})$ . The damage intensity  $\eta$  at  $t = T_{1-}$  is calculated using (5).

The probabilities associated with the events in Fig. 3, the probabilities of failure before each inspection and at the end of lifetime, the lifetime failure probability, and the expected total failure and repair costs are computed as follows:

1. At  $t = T_1 + \epsilon_a = T_{1+}$  where  $\epsilon_a$  represents a small time interval (e.g., 1 day) after the first inspection has been

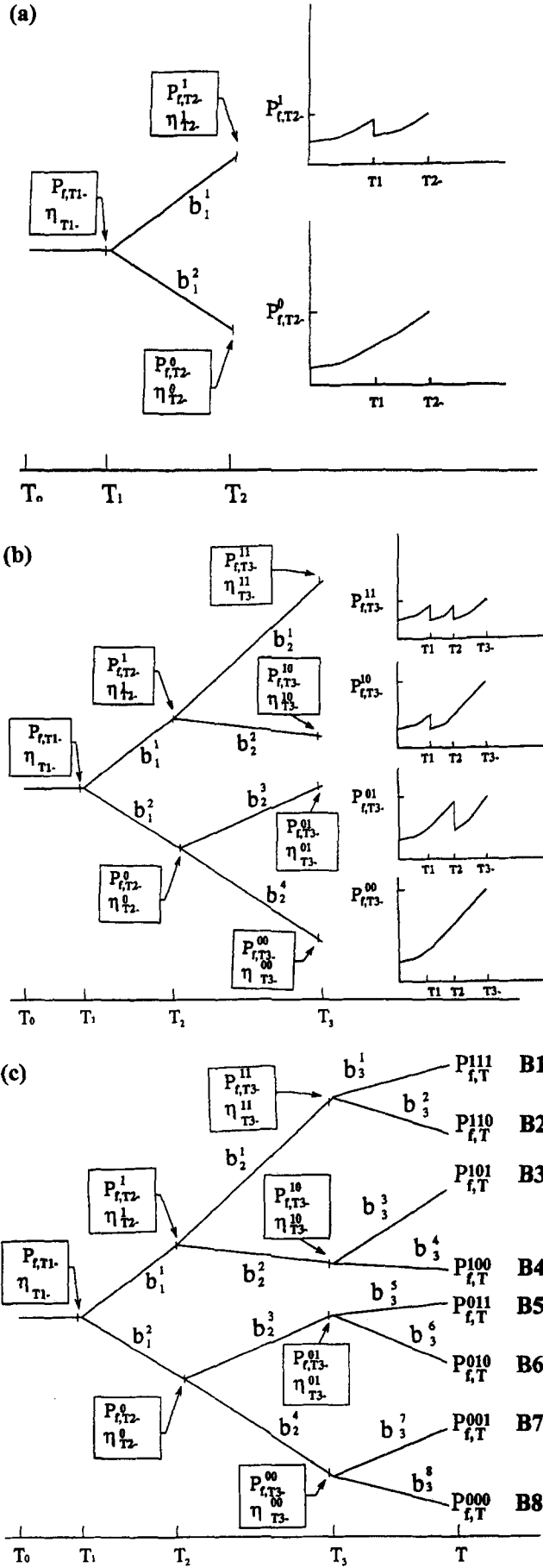


FIG. 4. Event Tree Analysis: (a) Before Second Inspection; (b) Before Third Inspection; (c) At Lifetime

performed, there are two possible events depending on the result of inspection. In Fig. 4(a), the event  $b_1^1$  indicates that the structure is repaired and event  $b_1^2$  indicates no repair.  $P(b_1^1)$  and  $P(b_1^2)$  are, respectively, the probabilities that events  $b_1^1$  and  $b_1^2$  occur at time  $t = T_{1+}$ . According to the repair policy adopted by Lin (1995), the probability of event  $b_1^1$  is

$$P(b_1^1) = \Phi\left(\frac{\eta_{T1-} - \eta_{0.5}}{\sigma}\right) \quad (22)$$

The repair effort  $e_{\text{rep},1,1}$  associated with the event  $b_1^1$  can be estimated from (18). Clearly, the probability of event  $b_1^2$  is

$$P(b_1^2) = 1 - P(b_1^1) \quad (23)$$

The repair effort  $e_{\text{rep},1,2}$  associated with the event  $b_1^2$  is nil since no repair effort is required.

Prior to the second inspection at time  $t = T_2 - \epsilon_b = T_{2-}$  [see Fig. 4(a)], the probability of failure is calculated for both branches. Given  $b_1^1$ , the failure probability is

$$P_{f,T2-}^1 = P[g_M^1(T_{2-}) \leq 0] \quad (24)$$

where  $g_M^1(t)$  defines failure given repair at time  $T_1$ . The damage intensity is  $\eta_{T2-}^1$ . The superscripts 1 and 0 indicate whether a repair has or has not been performed after an earlier inspection, respectively. In the same manner, given  $b_1^2$  [Fig. 4(a)], the structure was not repaired at  $T_1$  so that the probability of failure is

$$P_{f,T2-}^0 = P[g_M^0(T_{2-}) \leq 0] \quad (25)$$

and the damage intensity is  $\eta_{T2-}^0$ . Fig. 4(a) clearly shows that  $P_{f,T2-}^1 < P_{f,T2-}^0$ .

- After the second inspection given the occurrence of event  $b_1^1$ , there are two possibilities represented by the complementary events,  $b_2^1$  and  $b_2^2$  [see Fig. 4(b)] that indicate repair and no repair, respectively. The probabilities of these events are

$$P(b_2^1) = 1 - P(b_2^2) = \Phi\left(\frac{\eta_{T2-}^1 - \eta_{0.5}}{\sigma}\right) \quad (26)$$

The associated repair effects are  $e_{\text{rep},2,1}$  and  $e_{\text{rep},2,2} = 0$ .

In the same manner, there are also two complementary events,  $b_2^3$  and  $b_2^4$ , that follow the occurrence of event  $b_1^2$ . The probabilities that events  $b_2^3$  and  $b_2^4$  occur are

$$P(b_2^3) = 1 - P(b_2^4) = \Phi\left(\frac{\eta_{T2-}^0 - \eta_{0.5}}{\sigma}\right) \quad (27)$$

The repair effects associated with the events  $b_2^3$  and  $b_2^4$  are  $e_{\text{rep},2,3}$  and  $e_{\text{rep},2,4} = 0$ , respectively. The four probabilities of failure before the third inspection at  $t = T_{3-}$  [see Fig. 4(b)] associated with damage intensities  $\eta_{T3-}^{11}$ ,  $\eta_{T3-}^{10}$ ,  $\eta_{T3-}^{01}$ , and  $\eta_{T3-}^{00}$  are

$$P_{f,T3-}^{11} = P[g_M^{11}(T_{3-}) \leq 0] \quad (28a)$$

$$P_{f,T3-}^{10} = P[g_M^{10}(T_{3-}) \leq 0] \quad (28b)$$

$$P_{f,T3-}^{01} = P[g_M^{01}(T_{3-}) \leq 0] \quad (28c)$$

$$P_{f,T3-}^{00} = P[g_M^{00}(T_{3-}) \leq 0] \quad (28d)$$

- There are eight possible events after the third inspection at  $t = T_3$ , namely  $b_3^1, b_3^2, \dots, b_3^8$  [see Fig. 4(c)]. The probabilities of these events are

$$P(b_3^1) = 1 - P(b_3^2) = \Phi\left(\frac{\eta_{T3-}^{11} - \eta_{0.5}}{\sigma}\right) \quad (29a)$$

$$P(b_3^3) = 1 - P(b_3^4) = \Phi\left(\frac{\eta_{T_3-}^{10} - \eta_{0.5}}{\sigma}\right) \quad (29b)$$

$$P(b_3^5) = 1 - P(b_3^6) = \Phi\left(\frac{\eta_{T_3-}^{01} - \eta_{0.5}}{\sigma}\right) \quad (29c)$$

$$P(b_3^7) = 1 - P(b_3^8) = \Phi\left(\frac{\eta_{T_3-}^{00} - \eta_{0.5}}{\sigma}\right) \quad (29d)$$

with repair effects  $e_{\text{rep},3,1}$ ,  $e_{\text{rep},3,2}$ ,  $\dots$ ,  $e_{\text{rep},3,8}$ , respectively.

Each branch in the event tree represents a specific sequence of events  $b_i^j$ . Assuming that these events are mutually statistically independent, then the probabilities of occurrence of paths  $B_1$ ,  $B_2$ ,  $\dots$ ,  $B_8$ , respectively, are

$$\begin{aligned} P(B_1) &= P(b_3^1)P(b_2^1)P(b_1^1) \\ P(B_2) &= P(b_3^2)P(b_2^2)P(b_1^1) \\ &\dots \\ P(B_8) &= P(b_3^8)P(b_2^4)P(b_1^2) \end{aligned} \quad (30)$$

4. The probabilities of failure at the end of lifetime  $t = T$  associated with the eight paths shown in Fig. 4(c) are as follows:

$$\begin{aligned} P_{f,T}^{11} &= P[g_M^{111}(T) \leq 0] \\ P_{f,T}^{10} &= P[g_M^{110}(T) \leq 0] \\ &\dots \\ P_{f,T}^{000} &= P[g_M^{000}(T) \leq 0] \end{aligned} \quad (31)$$

For each branch, there are four probabilities of failure as follows: before first inspection, before second inspection, before third inspection, and at the end of lifetime. The lifetime probabilities of failure for branches 1–8 are defined as

$$\begin{aligned} P_{f,\text{life},1} &= \max(P_{f,T_1-}, P_{f,T_2-}^1, P_{f,T_3-}^{11}, P_{f,T}^{11}) \\ P_{f,\text{life},2} &= \max(P_{f,T_1-}, P_{f,T_2-}^1, P_{f,T_3-}^{11}, P_{f,T}^{10}) \\ &\dots \\ P_{f,\text{life},8} &= \max(P_{f,T_1-}, P_{f,T_2-}^0, P_{f,T_3-}^{00}, P_{f,T}^{000}) \end{aligned} \quad (32)$$

Finally, the lifetime probability of failure of the structure is

$$P_{f,\text{life}} = \sum_{i=1}^8 P_{f,\text{life},i} P(B_i) \quad (33)$$

5. If the failure cost is  $C_f$ , then the expected failure cost is

$$C_F = C_f P_{f,\text{life}} \quad (34)$$

The costs of repair associated with branches 1–8 are [see Eqs. (3) and (19)]

$$\begin{aligned} C_{\text{rep},1} &= \frac{\alpha_{\text{rep}} e_{\text{rep},1,1}^{0.5}}{(1+r)^{T_1}} + \frac{\alpha_{\text{rep}} e_{\text{rep},2,1}^{0.5}}{(1+r)^{T_2}} + \frac{\alpha_{\text{rep}} e_{\text{rep},3,1}^{0.5}}{(1+r)^{T_3}} \\ C_{\text{rep},2} &= \frac{\alpha_{\text{rep}} e_{\text{rep},1,1}^{0.5}}{(1+r)^{T_1}} + \frac{\alpha_{\text{rep}} e_{\text{rep},2,1}^{0.5}}{(1+r)^{T_2}} + \frac{\alpha_{\text{rep}} e_{\text{rep},3,2}^{0.5}}{(1+r)^{T_3}} \\ &\dots \\ C_{\text{rep},8} &= \frac{\alpha_{\text{rep}} e_{\text{rep},1,2}^{0.5}}{(1+r)^{T_1}} + \frac{\alpha_{\text{rep}} e_{\text{rep},2,4}^{0.5}}{(1+r)^{T_2}} + \frac{\alpha_{\text{rep}} e_{\text{rep},3,8}^{0.5}}{(1+r)^{T_3}} \end{aligned} \quad (35)$$

Then the expected total repair cost  $C_{\text{REP}}$  is

$$C_{\text{REP}} = \sum_{i=1}^8 C_{\text{rep},i} P(B_i) \quad (36)$$

where  $P(B_i)$ ,  $i = 1, \dots, 8$ , are defined in (30).

### Expected Total Cost

During the life of the structure (e.g., 75 years), preventive maintenance occurs 37 times (every two years). The expected cost of preventive maintenance  $C_{PM}$  is defined in (3). For a strategy involving  $m$  lifetime inspections, the total expected inspection cost is

$$C_{\text{INS}} = \sum_{i=1}^m C_{\text{ins}} \frac{1}{(1+r)^{T_i}} \quad (37)$$

where  $C_{\text{ins}}$  = inspection cost based on the inspection method used [see Eq. (12)]; and  $r$  = net discount rate.

Finally, the expected total cost  $C_{ET}$  is the sum of its components including the initial cost of the structure  $C_T$  [see Lin and Frangopol (1996)], the expected cost of routine maintenance  $C_{PM}$  [see Eq. (3)], the expected cost of inspection and repair maintenance, which includes the cost of performing the inspection  $C_{\text{INS}}$  [see Eq. (37)] and the cost of repair  $C_{\text{REP}}$  [see Eq. (36)], and the expected cost of failure  $C_F$  [see Eq. (34)]. Accordingly,  $C_{ET}$  can be expressed as

$$C_{ET} = C_T + C_{PM} + C_{\text{INS}} + C_{\text{REP}} + C_F \quad (38)$$

The objective remains to develop a strategy that minimizes  $C_{ET}$  while keeping the lifetime reliability of the structure above a minimum allowable value.

### OPTIMUM STRATEGY

To implement an optimum lifetime strategy, the following problem must be solved:

$$\text{minimize } C_{ET} \quad \text{subject to } P_{f,\text{life}} \leq P_{f,\text{life}}^* \quad (39)$$

where  $P_{f,\text{life}}^*$  = maximum acceptable lifetime failure probability (also called lifetime target failure probability). Alternatively, considering the reliability index

$$\beta = \Phi^{-1}(1 - P) \quad (40)$$

where  $\Phi$  is the standard normal distribution function, the optimum lifetime strategy is defined as the solution of the following mathematical problem

$$\text{minimize } C_{ET} \quad \text{subject to } \beta_{\text{life}} \geq \beta_{\text{life}}^* \quad (41)$$

where  $\beta_{\text{life}}$  and  $\beta_{\text{life}}^*$  are the lifetime reliability index and the lifetime target reliability index, respectively.

Studies by Thoft-Christensen and Sørensen (1987) and Mori and Ellingwood (1994b) proposed solutions to formulations (39) and (41) for metallic structures subjected to fatigue and for simple reinforced concrete beams under flexure, respectively. However, several aspects, such as the effects of inspection methods and intervals, degradation rates, and costs of failure, were not fully covered in these studies. These aspects should be considered in designing a robust and reliable optimum lifetime strategy.

In this study, the best lifetime strategy is found by solving the optimization problem (39). The solution takes into account the quality of various inspection techniques, all repair possibilities based on an event tree, the effects of aging and corrosion deterioration, the damage intensity, the effects of repair on structural reliability, and the time value of money. The cost analysis includes all components of the overall life-cycle cost according to (38).

As previously indicated, the number of branches in the

event tree is  $2^m$ , where  $m$  is the number of inspections. The probabilities of failure of the structure before each inspection  $i$  ( $P_{f,T_i}$ ) [see Eqs. (21), (24), (25), and (28)] and at the end of lifetime ( $P_{f,T}$ ) [see Eq. (31)] are calculated using the Monte Carlo simulation software MCREL (Lin 1995). Automated Design Synthesis (ADS) by Vanderplaats (1986) is a general purpose optimization software capable of solving linear, nonlinear, constrained, and unconstrained optimization problems. ADS was linked to MCREL for solving the reliability-based optimization formulation (39). Both uniform and nonuniform time interval inspection strategies are considered in the solution of the lifetime optimization problem. The sensitivity of the expected total cost to changes in such factors as inspection quality, corrosion rate, failure cost, and number of lifetime inspections is examined.

### UNIFORM INTERVAL INSPECTION STRATEGY

Initially, the inspection strategy is restricted to uniform time intervals. Assuming a lifetime of 75 years, 10 inspection strategies are considered in the analysis. The number of inspections  $m$  during the service of a structure is considered to be given. The optimization problem is solved for all values of  $m$ . The value of  $m$  that produces the smallest expected total cost  $C_{ET}$  is designated as  $m_{opt}$ . This indicates the optimum number of inspections to minimize the life-cycle cost of the structure. A prefabricated reinforced concrete T-girder bridge (Fig. 5) is considered for the inspection/repair maintenance analysis. Two lanes of HS-20 trucks provide the loading. Girder spacing  $S$  is 2.44 m, total width of the bridge is 7.32 m, and the span  $L$  is 18.30 m.

The interior girder in Fig. 5 was designed in Lin and Frangopol (1996). The design shown in Fig. 6 was based on reliability and optimization according to the constraints specified by the American Association of State Highway and Transportation Officials (AASHTO) (Standard 1992).

This design is characterized by a mean bending capacity of 282.51 kNm and a reliability index  $\beta = 3.76$  ( $P_f = 0.000085$ ) and produces the minimum initial cost  $C_T = 692.7$ . The initial cost is associated with a steel to concrete cost ratio  $C_s/C_c = 50$  and a unit cost of concrete  $C_c = 1$ . For illustrative purposes, let us assume that: (a) during its projected service life cycle of 75 years, the bridge is placed in an aggressive environment characterized by a uniform corrosion rate of  $\nu = 0.0089$  cm/year; (b) the net discount rate is  $r = 0.02$ ; (c) the cost of failure  $C_f = 50,000C_c$ ; (d) the cost of preventive maintenance during the first year is  $C_{main} = 0.001C_T$ ; (e) the corrosion initiation time is three years; (f) the damage intensity at which the NDE method has a 50% probability of detection is  $\eta_{0.5} = 0.1$  with the coefficient of variation of 0.1 (i.e., inspection method B); and (g) the lifetime target reliability index is  $\beta_{life}^* = 2.0$  ( $P_{f,life}^* = 0.02275$ ).

Fig. 7(a) shows the results of the cost analysis for different numbers of inspections. The optimum inspection/repair strategy for this beam is identified by the minimum expected total cost,  $C_{ET}$ . Note that as the number of inspections  $m$  increases, the cost of repair  $C_{REP}$  increases, whereas the expected cost of failure  $C_F$  decreases. Accordingly, there is a trade-off point at which the expected total cost has a minimum. Fig. 7(a) indicates that the optimum number of lifetime inspections is six, where the expected total cost is a minimum. For six inspections, the branch in the event tree with the highest probability of occurrence is shown in Fig. 7(b). The optimum inspection/repair strategy ( $m = 6$ ,  $n = 4$ , where  $n =$  number of repairs) in Fig. 7(b) is associated with no repair after the first two inspections and repair after each of the remaining four inspections.

To demonstrate the effect of corrosion rate, two other cor-

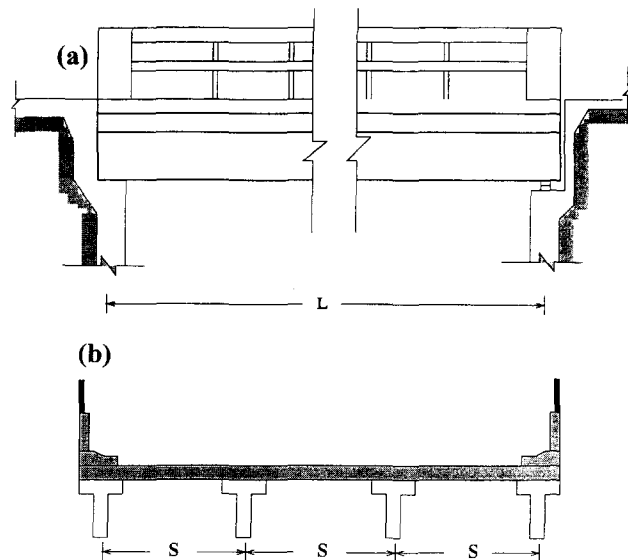
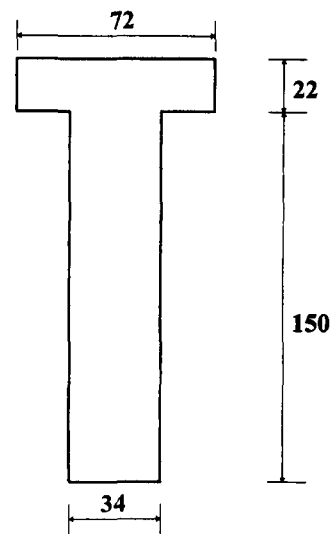


FIG. 5. Reinforced Concrete T-Girder Bridge: (a) Elevation,  $L = 18.3$  m; (b) Cross Section,  $S = 2.44$  m



$$\beta = 3.76$$

$$A_s = 90.71 \text{ cm}^2 \text{ 9\#11}$$

$$\text{INITIAL COST} = 692.7$$

FIG. 6. Optimum Design Solution; Dimensions Are in cm

rosion rates ( $\nu = 0.0064$  and  $0.0114$  cm/year) are considered. The three corrosion rates used in this study indicate a mean of  $0.0089$  cm/year and a standard deviation of  $0.0025$  cm/year. Figs. 8(a) and 9(a) show the projected costs as a function of the number of inspections for  $\nu = 0.0064$  and  $0.0114$  cm/year, respectively. The optimum inspection/repair strategies associated with the corrosion rates  $\nu = 0.0064$  and  $0.0144$  cm/year are shown in Figs. 8(b) and 9(b), respectively. As expected, the optimum total cost and the number of optimum lifetime inspections both increase as the rate of corrosion increases. For corrosion rates of  $\nu = 0.0064$ ,  $0.0089$ , and  $0.0114$  cm/year, the optimum number of lifetime inspections  $m$  is four, six, and seven, respectively, and the minimum total expected costs are 1,473.2, 1,682.5, and 1,890.6, respectively. As the corrosion rate increases, the optimum number of repairs  $n$  is also increasing.

The quality of the inspection technique has an effect on the optimum inspection/repair strategy. This effect is illustrated using three different inspection methods A, B, and C, whose

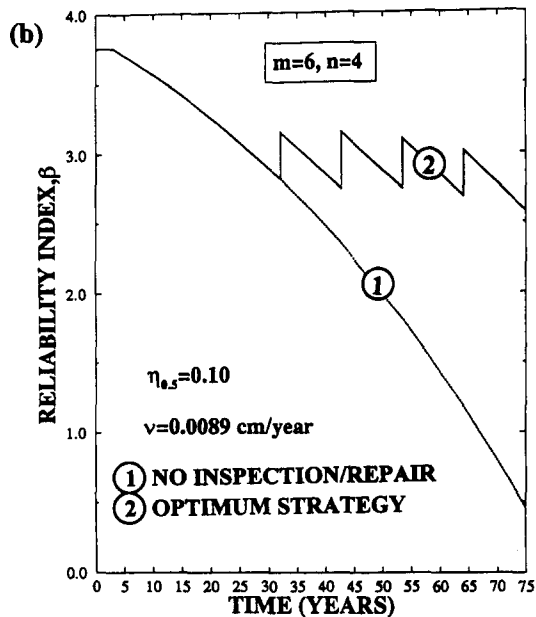
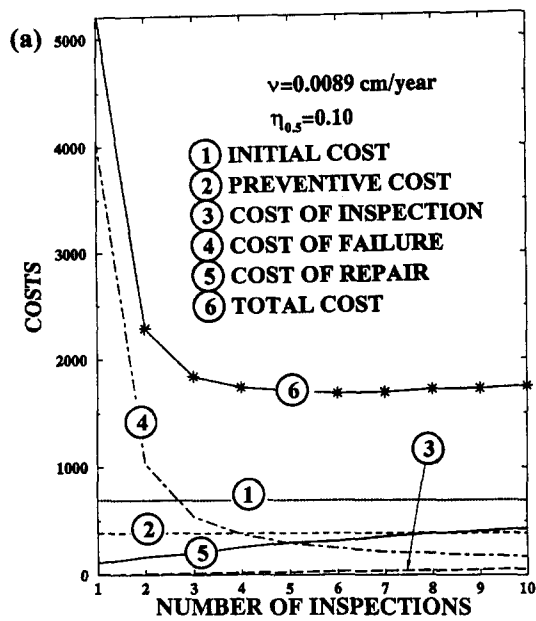


FIG. 7. (a) Costs as Function of Number of Uniform Interval Inspections; (b) Optimum Inspection/Repair Strategy for  $\nu = 0.0089$  cm/Year,  $C_t = 50,000C_e$ , and Inspection Method B ( $\eta_{0.5} = 0.10$ )

defect detectability parameters ( $\eta_{0.5}$ ;  $\sigma$ ) are (0.05; 0.005), (0.1; 0.01), and (0.15; 0.015), respectively. It is clear that inspection method A has the highest quality and is the most expensive ( $C_{ins} = 23.78$ ) and method C has the lowest quality and is the least expensive ( $C_{ins} = 5.27$ ).

For the corrosion rate of  $\nu = 0.0089$  cm/year, Figs. 10(a) and 11(a) show the resulting costs associated with inspection methods A and C, respectively. The costs associated with inspection method B were shown in Fig. 7(a). Figs. 10(b) and 11(b) show the optimum inspection/repair strategies associated with inspection techniques A and C, respectively. As the quality of the inspection technique increased (even though the technique itself was more expensive), the total cost and the optimum number of lifetime inspections decreased. For inspection techniques A, B, and C, the optimum number of lifetime inspections is five, six, or eight, and the minimum total expected costs are 1,673.3, 1,682.5, and 1,792.8, respectively. The minimum expected total cost appears to be much more

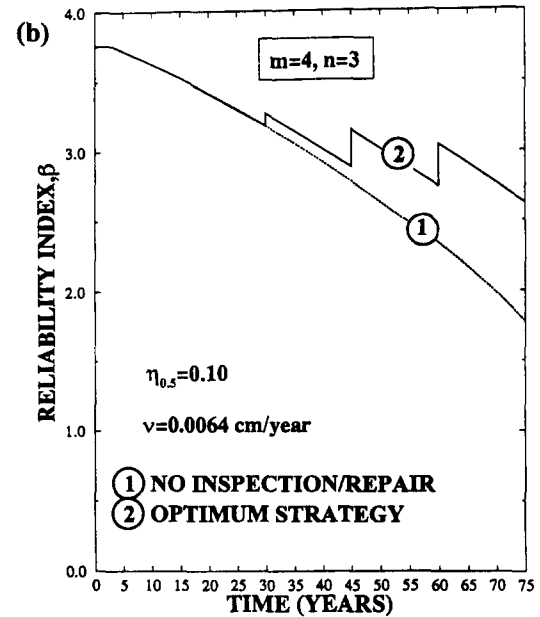
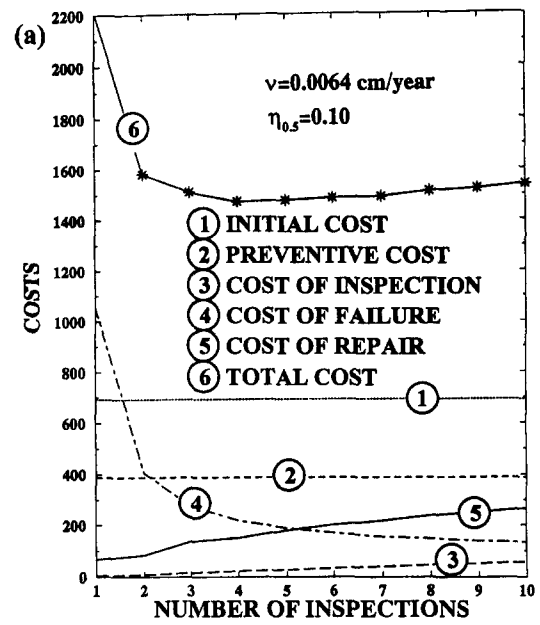


FIG. 8. (a) Costs as Function of Number of Uniform Interval Inspections; (b) Optimum Inspection/Repair Strategy for  $\nu = 0.0064$  cm/Year,  $C_t = 50,000C_e$ , and Inspection Method B ( $\eta_{0.5} = 0.10$ )

sensitive to the rate of corrosion than to the quality of the inspection technique.

### NONUNIFORM INTERVAL INSPECTION STRATEGY

While uniform inspection intervals are more convenient and easier to manage, there is a potential for less cost and greater efficiency by considering nonuniform time intervals. For nonuniform inspection intervals, the lifetime optimization process consists of finding the optimum number of inspections  $m$  and the optimum times at which inspections/repairs are carried out  $T_1, T_2, \dots, T_m$  such that

$$\min C_{ET} = C_T + C_{PM} + C_{DNS} + C_{REP} + C_F \quad (42)$$

subject to

$$\sum_{i=1}^m t_i \leq T; \quad T_j = \sum_{i=1}^j t_i \quad (43, 44)$$



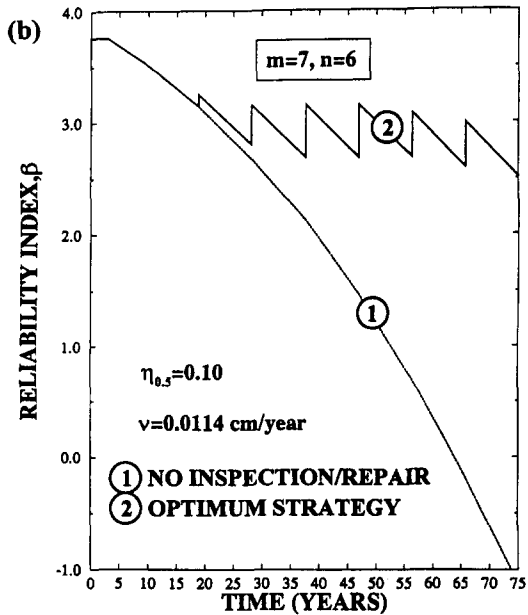
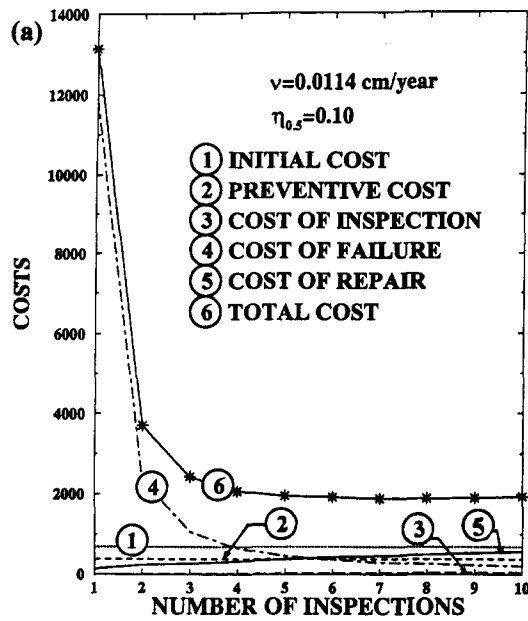


FIG. 9. (a) Costs as Function of Number of Uniform Interval Inspections; (b) Optimum Inspection/Repair Strategy for  $\nu = 0.0114$  cm/Year,  $C_f = 50,000C_c$ , and Inspection Method B ( $\eta_{0.5} = 0.10$ )

$$t_{\min} \leq t_i \leq t_{\max}, \quad i = 1, 2, \dots, m; \quad \beta(t) \geq \beta_{\text{life}}^* \quad (45, 46)$$

where  $T$  = life-cycle of the structure (e.g.,  $T = 75$  years);  $t_j = T_j - T_{j-1}$  = time interval between the inspections  $j - 1$  and  $j$ ;  $t_{\min}$  and  $t_{\max}$  are minimum and maximum inspection time intervals (e.g.,  $t_{\min} = 2$  years and  $t_{\max} = T$ );  $\beta(t)$  = reliability index at time  $t$ ; and  $\beta_{\text{life}}^*$  = lifetime target reliability index (e.g.,  $\beta_{\text{life}}^* = 2.00$ ).

As in the case of uniform inspection intervals, the optimum number of inspections  $m_{\text{opt}}$  is found by solving the optimization formulation for a number of different values of  $m$ . The value that minimizes the total expected cost  $C_{ET}$  is  $m_{\text{opt}}$ . Again, MCREL (Lin 1995) and ADS (Vanderplaats 1986) are used to optimize the nonuniform inspection time intervals.

For the same reinforced concrete T-girder used earlier (see Fig. 6) and for the same parameters considered in the uniform inspection interval example (i.e., projected life-cycle of 75 years, aggressive environment characterized by a uniform corrosion rate of  $\nu = 0.0089$  cm/year, cost of failure  $C_f =$

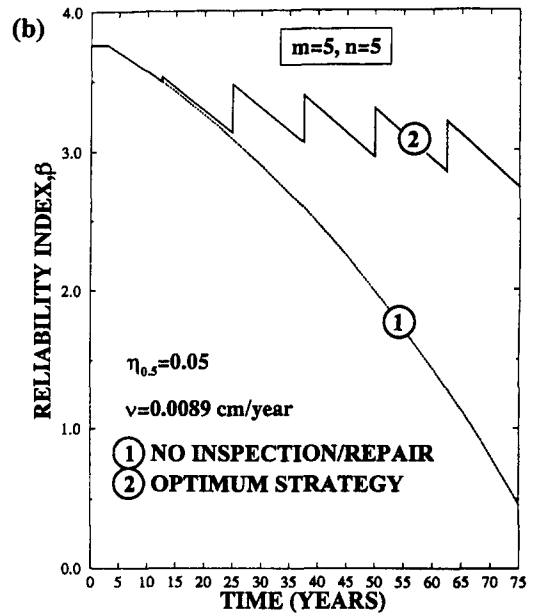
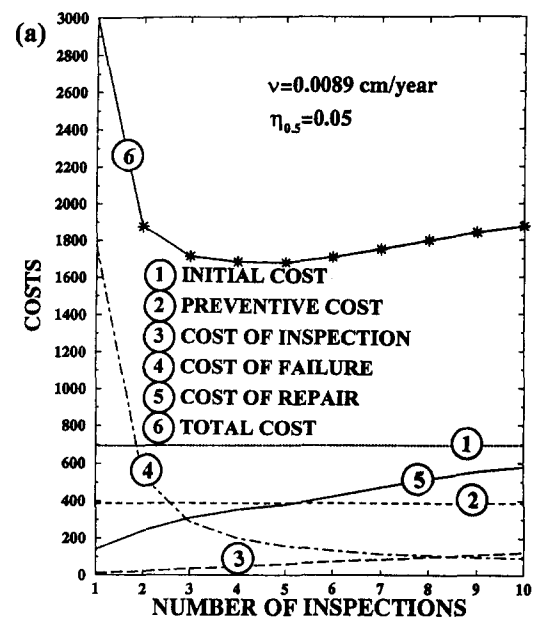


FIG. 10. (a) Costs as Function of Number of Uniform Interval Inspections; (b) Optimum Inspection/Repair Strategy for  $\nu = 0.0089$  cm/Year,  $C_f = 50,000C_c$ , and Inspection Method A ( $\eta_{0.5} = 0.05$ )

$50,000C_c$ , cost of preventive maintenance during the first year  $C_{\text{main}} = 0.001C_T$ , corrosion initiation time of three years,  $\eta_{0.5} = 0.1$ ,  $\sigma = 0.01$ , and  $\beta_{\text{life}}^* = 2.0$ ), Table 1 and Fig. 12(a) show the optimum inspection times (Table 1) and the expected total lifetime costs for different numbers of lifetime inspections associated with nonuniform time intervals. The optimal inspection/repair strategy is shown in Fig. 12(b). It corresponds to four inspections ( $m = 4$ ) and four repairs ( $n = 4$ ) at 35, 46.9, 58.5, and 67.9 years of service life and a minimum expected total cost of 1,573.4. Note that the expected total cost is rather insensitive to the number of lifetime inspections  $m$  after the optimum number of inspections  $m_{\text{opt}}$  has been reached. Comparing Figs. 12 and 7, it is observed that the optimum nonuniform inspection interval strategy produces a cheaper solution (1,573.4 instead of 1,682.5) with fewer inspections (four instead of six) conducted later in the life of the structure (at 35, 46.9, 58.5, and 67.9 years, instead of 10.7, 21.4, 32.1, 42.9, 53.6, and 64.3 years). It is also interesting to

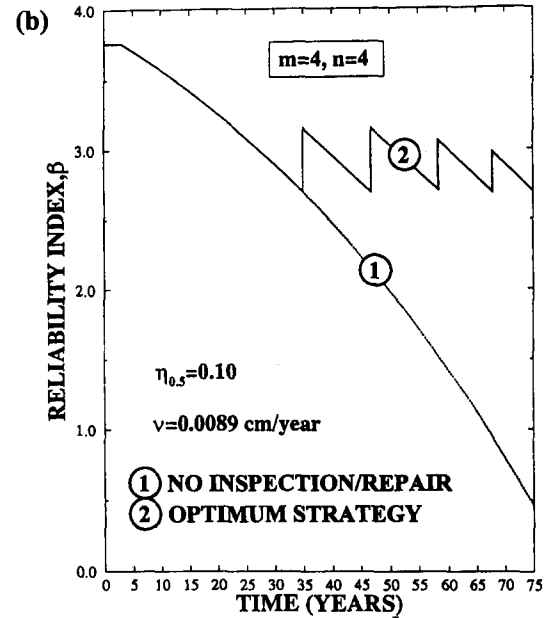
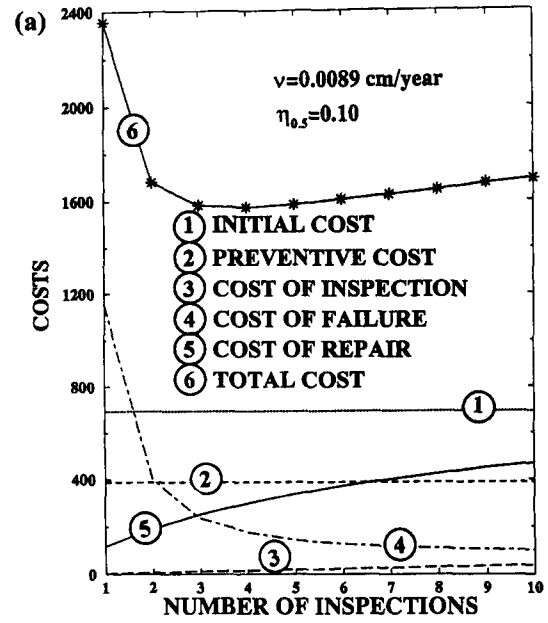
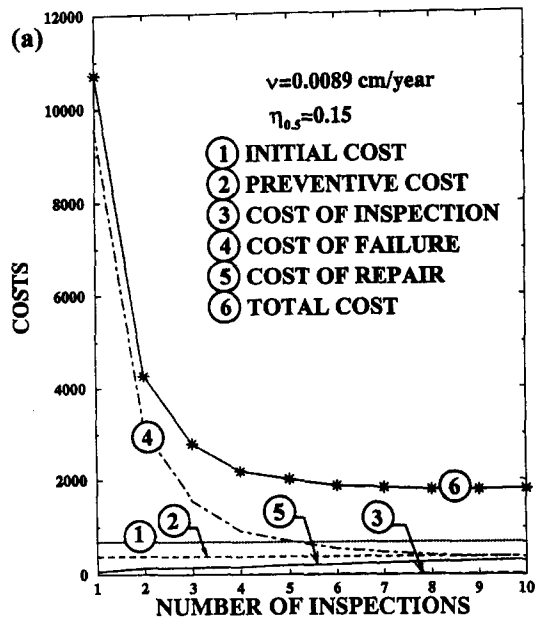


FIG. 11. (a) Costs as Function of Number of Uniform Interval Inspections; (b) Optimum Inspection/Repair Strategy for  $v = 0.0089$  cm/Year,  $C_r = 50,000C_e$ , and Inspection Method C ( $\eta_{0.5} = 0.15$ )

FIG. 12. (a) Costs as Function of Number of Nonuniform Interval Inspections; (b) Optimum Inspection/Repair Strategy for  $v = 0.0089$  cm/Year,  $C_r = 50,000C_e$ , and Inspection Method B ( $\eta_{0.5} = 0.10$ )

TABLE 1. Optimum Solution for Nonuniform Inspection Intervals:  $v = 0.0089$  cm/Year,  $C_r = 50,000$ , and Inspection Method B ( $\eta_{0.5} = 0.10$ )

Number of inspections (m) (1)	Optimum Inspection Times (years)										Total cost ( $C_{ET}$ ) (12)
	$T_1$ (2)	$T_2$ (3)	$T_3$ (4)	$T_4$ (5)	$T_5$ (6)	$T_6$ (7)	$T_7$ (8)	$T_8$ (9)	$T_9$ (10)	$T_{10}$ (11)	
1	48.5	—	—	—	—	—	—	—	—	—	2,356.9
2	41.5	68.2	—	—	—	—	—	—	—	—	1,681.2
3	37.4	51.6	64.8	—	—	—	—	—	—	—	1,581.5
4	35.0	46.9	58.5	67.9	—	—	—	—	—	—	1,573.4
5	33.6	44.0	54.4	63.1	69.9	—	—	—	—	—	1,584.8
6	32.5	41.8	51.1	59.6	66.1	71.2	—	—	—	—	1,605.4
7	31.7	40.4	49.1	57.3	63.6	68.5	72.2	—	—	—	1,625.6
8	31.3	39.5	47.7	55.7	61.9	66.6	70.2	72.9	—	—	1,645.5
9	30.8	38.1	45.4	52.7	59.1	64.1	68.1	71.1	73.2	—	1,672.3
10	30.3	37.5	44.7	51.9	58.2	63.1	66.9	69.7	71.9	73.5	1,689.9

TABLE 2. Effect of Corrosion Rate on Optimum Solutions

Corrosion rate (cm/year) (1)	Optimum Number of Inspections		Minimum Expected Total Cost	
	Uniform intervals (2)	Nonuniform intervals (3)	Uniform intervals (4)	Nonuniform intervals (5)
0.0064	4 <sup>a</sup>	3 <sup>b</sup>	1,473.2	1,395.5
0.0089	6 <sup>c</sup>	4 <sup>d</sup>	1,682.5	1,573.4
0.0114	7 <sup>e</sup>	5 <sup>f</sup>	1,890.6	1,785.0

<sup>a</sup>At 15.0, 30.0, 45.0, and 60 years.

<sup>b</sup>At 44.6, 58.2, and 68.2 years.

<sup>c</sup>At 10.7, 21.4, 32.1, 42.9, 53.6, and 64.3 years.

<sup>d</sup>At 35.0, 46.9, 58.5, and 67.9 years.

<sup>e</sup>At 9.4, 18.8, 28.1, 37.5, 46.9, 56.3, and 65.6 years.

<sup>f</sup>At 29.4, 40.1, 50.2, 59.6, and 68.2 years.

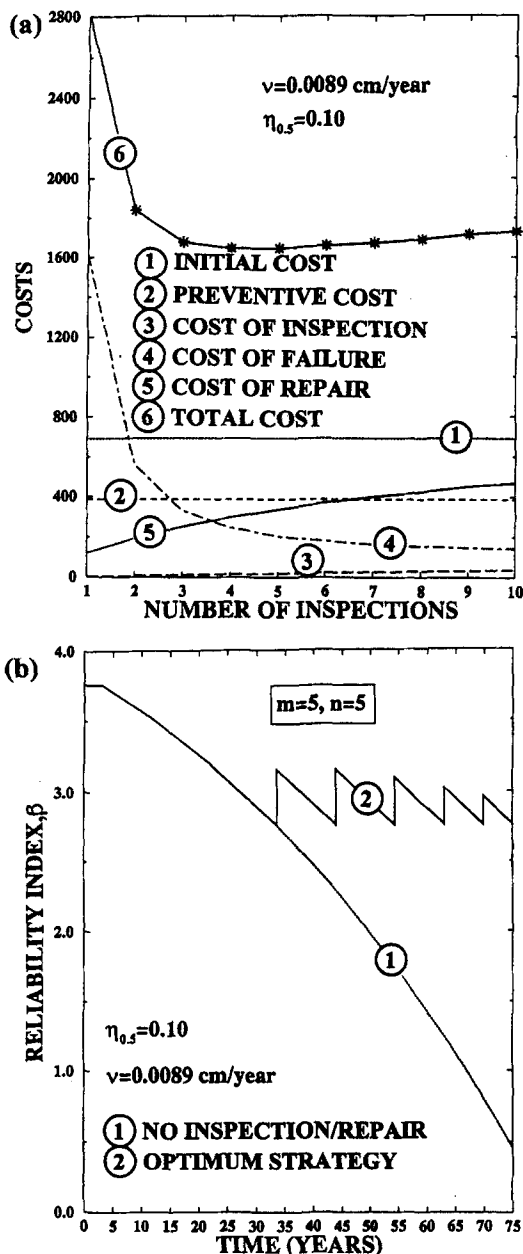


FIG. 13. (a) Costs as Function of Number of Nonuniform Interval Inspections; (b) Optimum Inspection/Repair Strategy for  $v = 0.0089$  cm/Year,  $C_f = 70,000C_c$ , and Inspection Method B ( $\eta_{0.5} = 0.10$ )

note (Figs. 12 and 7) that the number of repairs ( $n = 4$ ) is the same for both optimum nonuniform and uniform interval strategies but, again, conducted at later times (at 35, 46.9, 58.5, and 67.9 years, instead of 32.1, 42.9, 53.6, and 64.3 years) in the service life of the structure.

Table 2 compares the optimum number of inspections and the minimum expected total cost for different corrosion rates for both uniform and nonuniform time intervals. In all cases, the nonuniform time intervals were optimized with fewer lifetime inspections and less total cost. Also, as the corrosion rate increases, the first inspection appears sooner in the life of the structure and the time intervals between successive inspections are smaller.

Finally, to illustrate the effect of the cost of failure, let us assume  $C_f = 70,000C_c$  instead of  $50,000C_c$ . Fig. 13(a) shows the expected total lifetime costs for different numbers of lifetime inspections at nonuniform time intervals. The optimum inspection/repair strategy is shown in Fig. 13(b). The optimum

number of lifetime inspections is five, and the minimum expected total cost is 1,643.4. Under the same conditions when  $C_f = 50,000C_c$ , the optimum number of inspections was four at a minimum expected total cost of 1,573.4 (Fig. 12). The increased cost of failure causes both the optimum number of lifetime inspections and the minimum expected total cost to increase.

## CONCLUSIONS

A conceptual framework for reliability-based life-cycle cost design of deteriorating concrete structures has been presented. Reinforced concrete T-girders subject to corrosion were used to illustrate the approach. The optimization is based on minimizing the expected total life-cycle cost that includes the initial cost and the costs of preventive maintenance, inspection, repair, and failure. The analysis incorporates the quality of inspection methods, all possible repair options, the effects of aging, corrosion damage, and repair on structural reliability, and the time value of money.

In the T-girder analysis, results were obtained for both uniform inspection time intervals (where only the number of inspections was optimized) and nonuniform inspection time intervals (where both the number of inspections and the time intervals themselves were optimized). The effects of varying corrosion rates, different inspection techniques, and alternative costs of failure on the optimum solution were all examined. Regarding these effects, the following conclusions can be made:

1. The optimal nonuniform time interval inspection/repair strategy is more economic and requires fewer lifetime inspections/repairs than that based on uniform time interval inspections.
2. Numerical results indicate that the optimum number of inspections and the optimum expected total cost both increase as the corrosion rate increases. Also, as the quality of the inspection method increases, the optimum number of inspections decreases.
3. The cost of failure significantly affects the optimum inspection and repair strategy. A higher failure cost leads to an optimum solution requiring more inspections and repairs at a higher total cost.
4. The expected total cost  $C_{ET}$  was most sensitive to the corrosion rate and the cost of failure. Also,  $C_{ET}$  was relatively insensitive to the quality of inspection and the number of lifetime inspections above the optimum number.

The conceptual framework of the proposed reliability-based approach for life-cycle cost design of degrading concrete structures could be easily modified to accommodate degrading steel, masonry, or timber structures. The challenge in using the proposed approach is quantifying the uncertainties in the input variables. The rate of corrosion, quality of inspection method, and cost of failure are often subjective and difficult to obtain, but, as demonstrated, their values may have a great effect on the final result. With reliable input data, the methodology described here offers the real potential for integrating economic and safety issues in structural design.

Concluding, it should be mentioned that this study serves as an initial base on which to develop improved life-cycle cost design models. These models have to address additional issues such as serviceability limit states, use of spatially distributed random fields for describing the corrosion process, use of Bayesian theory for estimating the probability of damage detection, use of improved time-variant bridge reliability models, reliability updating in a Bayesian light, selection of repair policy, selection of target reliability level, and development of user costs, among others.

## ACKNOWLEDGMENTS

The support from the Chung-Shen Institute of Science and Technology, Taiwan, Republic of China, and from the U.S. Army is gratefully acknowledged. It made it possible for the second and third writers, respectively, to pursue PhD work at the Department of Civil, Environmental, and Architectural Engineering, University of Colorado at Boulder. The partial financial support of the National Science Foundation through grants CMS-9319204, CMS-9506435, and CMS-9522166 is also gratefully acknowledged.

## APPENDIX. REFERENCES

- Frangopol, D. M., Lin, K.-Y., and Estes, A. C. (1997). "Reliability of reinforced concrete girders under corrosion attack." *J. Struct. Engrg.*, ASCE, 123(3), 286–297.
- LeDoux, F. S., Levine, A. S., and Kamp, R. N. (1983). "New York State Department of Transportation bridge inspection and rehabilitation design program." *Transp. Res. Rec.* 899, 35–38.
- Lin, K.-Y. (1995). "Reliability-based minimum life-cycle cost design of reinforced concrete girder bridges." PhD thesis, Department of Civil Engineering, University of Colorado, Boulder, Colo.
- Lin, K.-Y., and Frangopol, D. M. (1996). "Reliability-based optimum design of reinforced concrete girders." *Struct. Safety*, Amsterdam, The Netherlands, 18(2/3), 239–258.
- Manual for condition evaluation of bridges.* (1994). American Association of State Highway and Transportation Officials, Washington, D.C.
- McNeil, S., and Hendrickson, C. (1982). "Prediction of pavement maintenance expenditure by using a statistical cost function." *Transp. Res. Rec.* 846, 71–76.
- Mori, Y., and Ellingwood, B. R. (1994a). "Maintaining reliability of concrete structures. I: Role of inspection/repair." *J. Struct. Engrg.*, ASCE, 120(3), 824–845.
- Mori, Y., and Ellingwood, B. R. (1994b). "Maintaining reliability of concrete structures. II: Optimum inspection/repair." *J. Struct. Engrg.*, ASCE, 120(3), 846–862.
- Standard specifications for highway bridges.* (1992). 15th Ed., American Association of State Highway and Transportation Officials, Washington, D.C.
- The status of the nation's highways, bridges and transit: Condition and performance.* (1993). Federal Highway Administration, Washington, D.C.
- Tamura, H., and Yoshida, M. (1984). "Nondestructive method of detecting steel corrosion in concrete." *In Situ/Nondestructive Testing of Concrete, SP-82*, American Concrete Institute, Detroit, Mich., 689–702.
- Thoft-Christensen, P., and Sørensen, J. D. (1987). "Optimal strategies for inspection and repair of structural systems." *Civ. Engrg. Sys.*, 4, 94–100.
- Ting, S.-C. (1989). "The effects of corrosion on the reliability of concrete bridge girders," PhD thesis, Department of Civil Engineering, University of Michigan, Ann Arbor, Mich.
- Vanderplaats, G. N. (1986). *ADS—A FORTRAN program for automated design synthesis. Version 1.10.* Engineering Design Optimization, Inc., Santa Barbara, Calif.