Study Questions for Actuarial Exam 2/FM

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Background

The purpose of my senior project is to prepare myself, as well as other students who may read my senior project, for the financial mathematics actuarial exam. By gaining sufficient knowledge by studying these questions and preparing oneself by taking the classes and or studying the materials mentioned in this report, an actuarial candidate should be sufficiently prepared to be able to pass the financial mathematics exam.

Table of Contents

Executive Summary

The goal of this project was to create resources for actuarial candidates that are preparing for the actuarial financial mathematics exam. I chose to put the areas of study into seven categories: interest rates, annuities, loan amortization, bonds, rates of return, forwards and futures, and options and swaps. The amount of questions in each category that I prepared reflects the amount of subject matter covered in each category. I referenced the Sam Broverman Study Guide for the general framework of the questions. However, the solutions, possible incorrect answers, reworded questions, and numbers put into said questions are my own.

After finishing this project, I have found myself to be adequately prepared to pass the actuary exam. I have thorough knowledge of every category, and very specific and exact knowledge of the questions I created. I feel that any candidate that works out and explores every question should also be sufficiently prepared.

Methods

To begin my research, I researched the actuary exam 2/FM. I found many useful resources at Cal Poly as well as off the internet. Any candidate that wishes to undertake this exam should utilize these resources.

After finding resources, I studied the material that was on the exam. Doing so, I acquired knowledge of the material which I would use as a tool to create the exam questions.

The most integral and time consuming part of this project was the creation of the exam study questions. I started by separating the material into seven different categories. These include interest rates, annuities, loan amortization, bonds, rates of return, forwards and futures, and options and swaps. Interest Rates include effective rates of interest and discount, nominal rates of interest and discount, and force of interest, and inflation. The questions on these topics will build a foundation of knowledge that will prepare you to answer the more difficult questions that incorporate these principles. The annuities section includes annuity-immediate, annuity due, annuity valuation at any point in time, annuities with differing interest, annuities with geometric payments, and annuities with arithmetic payments. Annuities are a set of reoccurring payments over a given time interval. As you may guess, these pop up a lot in finance. Therefore, you should expect to see many of these types of questions on the exam. The loan amortization section consists of the amortization of a loan and the sinking fund method. Amortization of a loan means how a loan is to be paid back. The other method is the sinking fund method which occurs when a borrower only pays interest on the loan and puts money into a separate account which will be used to pay back the ending balance. The bonds section covers bond valuation, bond amortization, and callable bonds. The rates of return section covers measures of return on a fund, term structure, forward rates, and duration. These questions should be looked at carefully as they make many appearances on the exam. The forwards section covers forward and futures contracts. Although this does not seem to be much material, it should be noted that it is one of the heftier sections to study. The final section is options and swaps. This section covers options, option strategies, and swaps. This section is usually covered the least in the exam, but it is definitely worth learning because even if one question is on there, it is worth studying!

After creating the sections and deciding on how many questions I needed from each, I would find a question that I had marked while studying in the category to model a question on. Most of the inspiration for these questions came from the Sam Broverman Study Guide. I also used my experience of the actuary exam to create a few others. The next step was to solve each problem. I would work out each problem first by myself. After doing so, I would check my answer for accuracy with the solution of a similar problem that was provided in the study guide. From there, I would create the incorrect answers. I created four wrong answers for each question. I tried to create answers that a candidate may actually derive rather than simply providing answers that were close in number to the correct answer. The incorrect answers ranged from simple clerical mistakes to more complicated mistakes such as using the wrong formula.

Results

I discovered many resources to prepare myself for this project. My favorite was the Sam Broverman Study Guide. Other resources I found included Business 343- Quantitative Methods in Finance, Actex which provides a plethora of study materials, and Infinite Actuary which is a website that goes through practice exams and shows candidates step-by-step recordings of how to work out each question.

The questions and answers to those questions that I created are attached at the end of this write-up.

Conclusion

I have prepared a set of study questions that will prepare future actuarial candidates for the actuarial exam 2/FM. By doing this, I have become adequately prepared to take the exam myself. By doing these problems and studying materials that are stated in the results section, candidates should be ready to pass the exam. I wish all readers of this project good luck in their actuarial endeavors!

Practice Questions

Interest Rates

Question 1

John deposits money into an account that has a payment of $25,000 at the end of 5 years. Sally deposits money into 2 accounts. One has a payment of 4,000 at the end of year t and one has a payment of $17, 000 at the end of year 2t. The sum of Sally's present value is equal to John's present value and is equal to a deposit with payment of $7,000 at time 0.

Find the value of the payment $14,000 at the end of year t+4 if all interest rates are equal for all deposits.

a.) $2,704
b.) $3,894
c.) $58,956
d.) $26,737
e.) $3,498,106

S. Broverman Study Guide Section 1 Problem 2

Question 1

Answer A

John's deposit: pv = $25{,}000v^5$

Sally's deposit: pv = $4{,}000v^t$

pv = $17{,}000v^{2t}$

setting them equal to 7.000:

$$7{,}000 = 25{,}000v^5 = 4{,}000v^t + 17{,}000v^{2t}$$

$$v^5 = .28$$

Since we want to find the pv at time equals t + 4 of a payment of 14,000: pv = $14{,}000v^{2t}$

We must then solve the quadratic with $x = v^t$: $17{,}000x^2 + 4{,}000x - 7{,}000 = 0$

$$X = .53474 = v^t$$

Thus,

$$\begin{aligned} pv &= 14{,}000v^{t+4} \\ &=14{,}000v^t v^4 \\ &=14{,}000v^t v^{5(4/5)} \\ &=14{,}000 * .53474 * .28^{4/5} \\ pv &= 2{,}703.94 \end{aligned}$$

Answer B

John's deposit: pv = $25{,}000v^5$

Sally's deposit: pv = $4{,}000v^t$

pv = $17{,}000v^{2t}$

setting them equal to 7,000:

$$7{,}000 = 25{,}000v^5 = 4{,}000v^t + 17{,}000v^{2t}$$

$$v^5 = .28$$

Solve for the quadratic: (*candidate used negative i)

$$17{,}000v^{2t} + 4{,}000v^t - 7{,}000 = 0$$

$$V^t = -.77*$$

Thus,

$pv = 14{,}000v^{t+4}$
$=14{,}000(-.77)(.28)^{4/5}$
$= -3893.54$

*Thinking answer should be positive, candidate makes answer positive so,

$pv = 3893.54$

Answer C

John's deposit: $pv = 25{,}000(1 + i)^{5}$*

Sally's deposit: $pv = 4{,}000(1 + i)^{t}$*
$pv = 17{,}000(1 + i)^{2t}$*

$7{,}000 = 25{,}000(1 + i)^5 = 4{,}000(1 + i)^t + 17{,}000(1 + i)^{2t}$
$.28 = (1 + i)^5$
$I = -.225$
(Knowing interest rates can't be negative candidate makes this positive)

Candidate then solves for t:

$17{,}000(1.225)^{2t} + 4{,}000(1.225)^t – 7{,}000 = 0$
$t = -3.0845$

Once again candidate makes this positive because time can't be negative so,

$pv = 14{,}000(1 + i)^{t+4}$
$=14{,}000(1.225)^{3.0845 + 4}$
$= 58956.02$

Answer D

John's deposit: $pv = 25{,}000v^5$

Sally's deposit: $pv = 4{,}000v^t$
$pv = 17{,}000v^{2t}$

setting them equal to 7,000:

$$7{,}000 = 25{,}000v^5 = 4{,}000v^t + 17{,}000v^{2t}$$

$$v^5 = .28$$

$$17{,}000v^{2t} + 4{,}000v^t - 7{,}000 = 0$$

$$V^t = .53474$$

Thus,

$$pv = 14{,}000v^{t+4}$$

$$=14{,}000v^t v^4$$

$$= 14{,}000\ v^t v^{5-1}$$

$$= 14{,}000(.53474)(.28)^{-1}*$$

$$pv = 26{,}737$$

Answer E

John's deposit: $pv = 25{,}000(1/i)^5*$

Sally's deposit: $pv = 4{,}000(1/i)^t*$

$pv = 17{,}000(1/i)^{2t}*$

setting them equal to 7,000:

$$7{,}000 = 25{,}000(1/i)^5 = 4{,}000(1/i)^t + 17{,}000(1/i)^{2t}$$

$$(1/i)^5 = .28$$

$$i = 1.29$$

Solving for time

$$17{,}000(1/1.29)^{2t} + 4{,}000(1/1.29)^t - 7{,}000 = 0$$

$$t= 2.46$$

Candidate uses .29 for I thinking1.29 is too large*

$$pv = 14{,}000(1/i)^{t+4}$$

$$=14{,}000(1/.29)^{2.46 + 4}$$

$$= 14{,}000\ v^t v^{5-1}$$

$$pv = 3{,}498{,}106$$

Interest Rates

Question 2

Eric deposits $6,000 into an account that gives 6% interest annually. He takes out $2,000 at the end of years 7, 14, and 21 at a penalty of 4%. What is the accumulated value of the deposit at the end of year 23?

a.) $16,678.50
b.) $5,507.16
c.) $11,783.36
d.) $8,695.22
e.) $35,053.63

S. Broverman Study Guide p. 4

Question 2

Answer A

Initial deposit: 6,000

Withdrawals: 2,000 at the end of 7, 14, 21

Interest rate = .06   penalty = .04

Accumulated value = $6{,}000(1.06)^{23} - 2{,}000(1.04)^{*} - 2{,}000(1.04)^{*} - 2{,}000(1.04)$

=$16, 678.50

*candidate fails to recognize that the withdrawals affect how much interest is accumulated

Answer B

Initial deposit: 6,000

Withdrawals: 2,000 at the end of 7, 14, 21

Interest rate = .06   penalty = .04

*candidate mixes up the interest rate and penalty

Accumulated value = $6{,}000(1.04)^{23} - 2{,}000(1.06)(1.04)^{16} - 2{,}000(1.06)(1.04)^{9} - 2{,}000(1.06)(1.04)^{2}$

=$5, 507.16

Answer C

Initial deposit: 6,000

Withdrawals: 2,000 at the end of 7, 14, 21

Interest rate = .06   penalty = .04

*candidate mixes up the interest rate and penalty

Accumulated value = $6{,}000(1.06)^{23} - 2{,}000(1.04)(1.06)^{16} - 2{,}000(1.04)(1.06)^{9} - 2{,}000(1.04)(1.06)^{2}$

=$11,783.36

Answer D

Initial deposit: 6,000

Withdrawals: 2,000 at the end of 7, 14, 21

Interest rate = .06 penalty = .04

*candidate thinks it is present value not accumulated value

Accumulated value = $6{,}000v^0 - 2{,}000v_{.06}{}^7v_{.04} - 2{,}000v_{.06}{}^{14}v_{.04} - 2{,}000v_{.06}{}^{21}v_{.04}$

$=6{,}000 + 2{,}000(1/1.06)^7(1/1.04) + 2{,}000(1/1.06)^{14}(1/1.04) + 2{,}000(1/1.06)^{21}(1/1.04)$

=$8695.22

Answer E

Initial deposit: 6,000

Withdrawals: 2,000 at the end of 7, 14, 21

Interest rate = .06 penalty = .04

*candidate adds up withdrawals when they were supposed to be subtracted

Accumulated value = $6{,}000(1.06)^{23} + 2{,}000(1.04)(1.06)^{16} + 2{,}000(1.04)(1.06)^9 + 2{,}000(1.04)(1.06)^2$

=$34,053.63

Interest Rates

Question 3

Michael deposits $20,000 into his bank account. For the first 4 years the bank credits an interest of i convertible quarterly and 3i convertible monthly after that. If he has $80,000 in his account after 14 years, how much does he have after 3 years?

a.) $26,918.25
b.) $22,084.93
c.) $22,873.49
d.) $22,604.63
e.) $22,603.53

S. Broverman Study Guide Section 2 Problem 1

Question 3

Answer A

Deposit: 20,000
i convertible quarterly
80,000 after 14 years

*candidate fails to notice that i is only the rate of interest for 4 years

$80{,}000 = 20{,}000(1 + i/4)^{56}$
$i = .10026$

after 3 years
$20{,}000(1 + i/4)^{12} = \$26{,}918.25$

Answer B

Deposit: 20,000
3i convertible monthly
80,000 after 14 years

*candidate fails to notice the i for the first 4 years

$80{,}000 = 20{,}000(1 + 3i/12)^{168}$
$i = .0331$

after 3 years
$20{,}000(1 + i/12)^{36} = \$22{,}084.93$

Answer C

Deposit: 20,000
i for 4 years quarterly
3i after the 4 years monthly
80,000 after 14 years

*candidate thinks that you don't compound the interest in the 14th year

$80{,}000 = 20{,}000(1 + i/4)^{16}(1 + 3i/12)^{108}$

$4 = (1 + i/4)^{124}$

$i = .045$

after 3 years

$20{,}000(1 + i/4)^{12} = \$22{,}873.49$

Answer D

Deposit: 20,000
i for 4 years convertible quarterly
3i after the 4 years convertible monthly
80,000 after 14 years

*candidate does the problem thinking interest rate is discount rate

$80{,}000 = 20{,}000(1 + d/4)^{-16}(1 + 3i/12)^{-120}$

$4 = (1 + d/4)^{-136}$

$d = .0406$

after 3 years

$20{,}000(1 + d/4)^{-12} = \$22{,}604.63$

Answer E

Deposit: 20,000
i for 4 years quarterly
3i after the 4 mears monthly
80,000 after 14 years

First find i

$$80{,}000 = 20{,}000(1 + i/4)^{16}(1 + 3i/12)^{108}$$

$$4 = (1 + i/4)^{136}$$

$$i = .045$$

after 3 years

$$20{,}000(1 + i/4)^{12} = \$22{,}603.53$$

Interest Rates

Question 4

Eric puts in a deposit where the bank credits him a rate of discount of 13% convertible every 4 years. Billy wants to find the same rate or better, but can only find banks that will give him rates of interest convertible monthly. What is the lowest rate of interest Billy will accept?

a.) .182
b.) .185
c.) .034
d.) .105
e.) .2014

S. Broverman Study Guide Section 2 Problem 7

Question 4

Answer A

Rate of discount: 13% convertible every 4 years

*candidate uses the present value form of the rate of discount by accident when s/he meant to put a negative sign in frontof the exponent

$(1 - .13/(1/4))^{1/4*} = (1 + i/12)^{12}$

$.8324 = (1 + i/12)^{12}$

$-.182 = i$

Knowing interest rates can't be negative candidate makes this positive, so i = .182

Answer B

Rate of discount: 13% convertible every 4 years

Find the equal rate of interest convertible monthly

$(1 – d/n)^{-n} = (1 + i/n)$

$(1 - .13/(1/4))^{-1/4} = (1 + i/12)^{12}$

$1.2014 = (1 + i/12)^{12}$

$.185 = i$

Answer C

Rate of discount: 13% convertible every 4 years

*knowing d = i/1+i, the candidate uses this to try finding the interest rate

The candidate first tries to find the rate of discount for 1 year, although incorrectly

.13/4 = .0325/yeay

Solving for i,

$.0325 = i/1+I$

$.0325 + .0325i = i$

$.03359 = i$

Answer D

Rate of discount: 13% convertible every 4 years

*candidate uses discount as an interest rate when setting them equal

$(1 - .13/(1/4))^{1/4} = (1 + i/12)^{12}$

$1.1104 = (1 + i/12)^{12}$

$.105 = i$

Answer E

Rate of discount: 13% convertible every 4 years

*candidate uses simple interest

$(1 - .13/(1/4))^{-1/4} = (1 + i/12)(12)$

$1.2014 = (1 + i)$

$.2014 = i$

Interest Rates

Question 5

Max and Tyler both make deposits of \$30,000. Max's bank credits his deposit with a simple interest rate of 10% annually. Tyler's bank credits him with an annual compounded interest rate of 6%. At time t the forces of interest are equal. Determine which person's bank account has more money and by how much.

a.) Max: \$5,532.52
b.) Tyler: \$7.44
c.) Max: \$5,946.14
d.) Tyler: \$16,444.87
e.) Insufficient information to solve

S. Broverman Study Guide Section 3 Problem 9

Answer A

Max deposit = 30,000 simple interest = 10%
Tyler deposit = 30,000 compound interst = 6%

*candidate mixes up the two interest rates in his/her calculations

Max's force of interest: ln(1.1) = .0953
Tyler's force of interest: .06/(1 +.06t)

Set equal:

$.0953 = .06/(1 + .06t)$

$t = -6.1735$

Since time can't be negative, candidate makes it positive, so t = 6.1735

Accumulated value of Max's deposit:

$30{,}000(1 + .1(6.1735))$

$= 48{,}520.50$

Accumulated value of Tyler's deposit:

$30{,}000(1 + .06)^{6.1735}$

$= 42{,}987.98$

Max – Tyler = 5,532.52

Answer B

Max deposit = 30,000 simple interest = 10%
Tyler deposit = 30,000 compound interst = 6%

*candidate tries setting the interest rates equal to determine t

$(1 + .1t) = (1.06)^t$

$(1.06)^t - .1t - 1 = 0$

$T = 17.13$

Accumulated value of Max's deposit:

$30{,}000(1 + .1(17.13))$
$= 81{,}390$

Accumulated value of Tyler's deposit:
$30{,}000(1 + .06)^{17.13}$
$= 81{,}397.44$

Max – Tyler = 7.44

Answer C

Max deposit = 30,000 simple interest = 10%
Tyler deposit = 30,000 compound interst = 6%

Max's force of interest: .1/(1 +.1t)
Tyler's force of interest: ln(1.06) = .0583

Set equal:
$.0583 = .1/(1 + .1t)$
$t = 7.153$

Accumulated value of Max's deposit:
$30{,}000(1 + .1(7.153))$
$= 51{,}459$

Accumulated value of Tyler's deposit:
$30{,}000(1 + .06)^{7.153}$
$= 45{,}512.86$

Max – Tyler = 5,946.14

Answer D

Max deposit = 30,000 simple interest = 10%
Tyler deposit = 30,000 compound interst = 6%

Max's force of interest: .1/(1 +.1t)
Tyler's force of interest: ln(1.06) = .0583

Set equal:

$.0583 = .1/(1 + .1t)$

$t = 7.153$

*candidate mixes up the two interest rates in his/her calculations

Accumulated value of Max's deposit:

$30{,}000(1 + .06(7.153))$

$= 42{,}875.40$

Accumulated value of Tyler's deposit:

$30{,}000(1 + .1)^{7.153}$

$= 59{,}320.27$

Max – Tyler = 16,444.87

Answer E

Max deposit = 30,000 simple interest = 10%

Tyler deposit = 30,000 compound interst = 6%

*candidate thinks force of interest is $\delta = \ln(t + i)$

Max's force of interest: $\ln(t + .1)$

Tyler's force of interest: $\ln(t + .06)$

Set equal:

$\ln(t + .1) = \ln(t + .06)$

$t + .1 \neq t + .06$

no t works for candidate

insufficient information to solve

Annuities

Question 6

Joel just won the lottery. He has two options to take the money. He can take the lump sum of \$3,000,000 or he can take the level payments of \$500,000 over 6 years.

If he takes the lump sum, Joel will deposit the money into an account earning i% annually.

If Joel takes the payment plan, he will deposit the payments at the end of each year at a compounded interest of 14%.

After 16 years, the accounts will be equal. Calculate i.

a.) .1095
b.) .0563
c.) .1065
d.) .371
e.) .022

Question 6

Answer A

Lump sum: 3,000,000 interest i
Payments: 500,000 over 6 years i = .14

First find the accumulated value of the payment plan for the 16 years

$500{,}000s_{.14|6}(1 + .14)^{10}$
$= 500{,}000[((1 + .14)^{6} – 1)/.14](1.14)^{10}$
$= 15{,}821{,}528.50$

Set this equal to the lump sum accumulated value:

$15{,}821{,}528.50 = 3{,}000{,}000(1 + i)^{16}$
$1.1095 = 1 + i$
$i = .1095$

Answer B

Lump sum: 3,000,000 interest i
Payments: 500,000 over 6 years i = .14

*candidate uses the function for present value

$500{,}000s_{.14|6}(1 + .14)^{10}$
$= 500{,}000[(1 – (1/1.14)^{6})/.14](1.14)^{10}$
$= 7{,}208{,}075.55$

Set this equal to the lump sum accumulated value:

$7{,}208{,}075.55 = 3{,}000{,}000(1 + i)^{16}$
$i = .0563$

Answer C

Lump sum: 3,000,000 interest i
Payments: 500,000 over 6 years i = .14

*candidate thinks this is an annuity due rather than an annuity immediate

$500{,}000\ddot{s}_{14|6}(1 + .14)^{10}$
$= 500{,}000[(1 - .123)^{6})/.123](1.14)^{10}$
$= 15{,}156{,}778.75$

Set this equal to the lump sum accumulated value:

$15{,}156{,}778.75 = 3{,}000{,}000(1 + i)^{16}$
$i = .1065$

Answer D

*candidate mixes up interest rates

Lump sum: 3,000,000 i = .14
Payments: 500,000 over 6 years interest i

Solve for accumulated value of lump sum

$3{,}000{,}000(1 + .14)^{16} = 24{,}411{,}747.89$

Set the equations to be equal

$24{,}411{,}747.89 = 500{,}000s_{6|i}(1 + i)^{10}$
$48.823 = 500{,}000[((1 + i)^{6} - 1)/i](1 + i)^{10}$

Solving for i:

$i = .371$

Answer E

Lump sum: 3,000,000 interest i
Payments: 500,000 over 6 years i = .14

*candidate fails to add extra 10 years of interest after the 6 payments

$500{,}000 s_{\overline{6|}i}(1 + i)^{10} = 500{,}000[((1 +.14)^{6} – 1)/.14]$
$= 4{,}267{,}759.37$

Equate equations:
$4{,}267{,}759.37 = 3{,}000{,}000(1 + i)^{16}$
$i = .022$

Annuities

Question 7

Joe plans on going to Cal Tech. He will need to pay 4 payments of $50,000 when he goes there. In order to do this he will deposit x into an account every month that earns 8% interest convertible monthly for 7 years. He will take out the payments at the end of the last 4 years at the end of the year. After the last withdrawal the account will be exhausted. Calculate x.

a.) $2,084.95
b.) $2,016.80
c.) $2,019.78
d.) $3,534.01
e.) $1,999.60

Question 7

Answer A

Deposits: x at 8% convertible monthly

*candidate calculates monthly interest wrong
$(1 + .08/12)^{12} - 1 = .0069$ every month

Withdrawals: 50,000 at months 48, 60, 72, 84

$Xs_{84|.0069} - 50{,}000(1.0069)^{36} - 50{,}000(1.0069)^{24}$
$- 50{,}000(1.0069)^{12} - 50{,}000 = 0$
$Xs_{84|.0069} = 227{,}316.26$

X = 2084.95

Answer B

Deposits: x at 8% convertible monthly
= .0067 every month

Withdrawals: 50,000 at months 48, 60, 72, 84

$Xs_{84|.0067} - 50{,}000(1.0067)^{36} - 50{,}000(1.0067)^{24}$
$- 50{,}000(1.0067)^{12} - 50{,}000 = 0$
$Xs_{84|.0067} = 226{,}450.1209$
$X[((1.0067)^{84} - 1)/.0067] = 226{,}450.1209$

X = 2016.80

Answer C

Deposits: x at 8% convertible monthly
= .0067 every month

Withdrawals: 50,000 at months 48, 60, 72, 84 *candidate adds interest to the final withdrawal

$Xs_{84|.0067} - 50{,}000(1.0067)^{36} - 50{,}000(1.0067)^{24}$
$- 50{,}000(1.0067)^{12} - 50{,}000(1.0067) = 0$
$Xs_{84|.0067} = 226{,}785.1209$

$X = 2019.78$

Answer D

Deposits: x at 8% convertible monthly
= .0067 every month

Withdrawals: 50,000 at months 48, 60, 72, 84

*candidate uses present value equation

$Xa_{84|.0067} - 50{,}000(1.0067)^{36} - 50{,}000(1.0067)^{24}$
$- 50{,}000(1.0067)^{12} - 50{,}000(1.0067) = 0$
$Xa_{84|.0067} = 226{,}450.1209$
$X((1 - v^{84})/.0067) = 226{,}450.1209$

$X = 3534.01$

Answer E

Deposits: x at 8% convertible monthly
= .0067 every month

Withdrawals: 50,000 at end of years 4, 5, 6, 7

*candidate uses an annuity for the withdrawals, but uses 8% as the annual interest rate instead of calculating the correct interest rate

$Xs_{84|.0067} - 50{,}000\ s_{84|.0067} = 0$
$Xs_{84|.0067} = 214576.7619$

$X = 1999.60$

Annuities

Question 8

Mat takes out a loan for a car for $35,000. He must make 16 annual payments of $4,000. For the first 7 years the interest rate is 8%, what is the annual effective interest rate for the last 9 years?

a.) .115
b.) .242
c.) .082
d.) .087
e.) .468

S. Broverman Study Guide Section 5 Problem 4

Question 8

Answer A

*candidate thinks it is an accumulated value problem

$35000 = 4000[s_{7|.08}(1 + x)^9 + s_{9|x}]$
$8.75 = 8.9228(1 + x)^9 + ((1 + x)^9 – 1/x)$
$i = -.1154$

Knowing interest rate can’t be negative candidate makes it positive

Answer B

pv= 35000 i = .08 for first 7 years

candidate doesn’t account for changing interest (v_i)

$35000 = 4000[a_{7|.08} + {}^*a_{9|x}]$
$8.75 = a_{7|.08} + a_{9|x}$
$3.5436 = a_{9|x}$
$i = .242$

Answer C

pv = 35000

*candidate fails to acknowledge the interest rate for the first 7 years

$35000 = 4000a_{16|x}$
$8.75 = a_{16|x}$
$i = .082$

Answer D

pv = 35000 i = .08 for first 7 years

$35000 = 4000[a_{7|.08} + v_{.08}^{7} a_{9|x}]$
$8.75 = a_{7|.08} + v_{.08}^{7} a_{9|x}$
$6.073 = a_{9|x}$
$i = .0868$

Answer E

pv = 35000 i = .08 for first 7 years

*candidate uses $(1 + .08)^7$ instead of $v_{.08}^{7}$

$35000 = 4000[a_{7|.08} + (1 + .08)^7 a_{9|x}]$
$8.75 = a_{7|.08} + (1 + .08)^7 a_{9|x}$
$3.5436 = (1 + .08)^7 a_{9|x}$
$2.0677 = a_{9|x}$
$i = .468$

Annuities

Question 9

Andy Z. opens a sketchy rent to own store where his catch phrase is, "I'll divide your cost by 20 an you can pay that amount for 24 months."

In the fine print it says that the first payment is due at purchase and every subsequent payment is due at monthly intervals after that.

What are Andy's store's customers paying on their loans?

a.) .218
b.) .0012
c.) .01655
d.) .268
e.) .182

S. Broverman Study Guide Section 6 Problem 8

Question 9

Answer A

Present value = x
Payments = x/20 or .05x for 24 months
Interest rate = i per month

To calculate annual interest rate we must first set up the equation for this annuity:

$x = .05x\ddot{a}_{24|i}$

$1 = .05\ddot{a}_{24|i}$
$20 = ((1 - v^{24})/d)$
$20d = 1 - (1/1 + i)^{24}$
$i = .01655$ monthly interest rate

To calculate the yearly interest rate:

$(1 + .01655)^{12} - 1 = .218$

Answer B

*candidate thinks present value is what the payments should be and vice versa

Present value = x
Payments = x/20 or .05x for 24 months
Interest rate = i per month

$.05x = x\ddot{a}_{24|i}$

$.05 = \ddot{a}_{24|i}$
$i = .0001$ monthly interest rate

To calculate the yearly interest rate:

$(1 + .0001)^{12} - 1 = .0012$

Answer C

Present value = x
Payments = x/20 or .05x for 24 months
Interest rate = i per month

$x = .05x\ddot{a}_{24|i}$
$20 = \ddot{a}_{24|i}$
$i = .01655$

*candidate thinks this is the yearly interest rate

Answer D

Present value = x
Payments = x/20 or .05x for 24 months
Interest rate = i per month

*candidate does annuity due instead of annuity immediate

$x = .05xa_{24|i}$
$20 = a_{24|i}$
$i = -1.996$

thinking this is the percentage and it can't be negative:
$(1 + .01996)^{12} - 1 = .268$

Answer E

Accumulated value = x
Payments = x/20 or .05x for 24 months
Interest rate = i per month

*candidate uses accumulated value when he should have used present value calculation

$x = .05x s_{\overline{24}|i}$
$20 = s_{\overline{24}|i}$
$20 = [(1 + i)^{24} - 1]/d$
$i = -.014$

thinking this is the percentage and it can't be negative:
$(1 + .014)^{12} - 1 = .1816$

Annuities

Question 10

At the beginning of each year Apple declares a dividend of 7 to be paid semi-annually. An economist forecasts an increase of 9% per year. At the beginning of the year Bob buys some shares at $X per share and optimistically predicts a 22% yield convertible semi-annually. Calculate X.

a.) $54.68
b.) $87.40
c.) $113.62
d.) $65.77
e.) $103.94

S. Broverman Study Guide Section 7 Problem 3

Question 10

Answer A

*candidate starts by finding k but thinks it only needs the semi-annual value.

$K = 7(1 + .11) = 7.77$

$i = (1 + .22/2)^2 – 1 = .2321$
$r = .09$

$pv = k/(i + r) = 7.77/(.2321 - .09) = 54.68$

Answer B

$K = 7s_{2|.11} = 14.77$

$i = (1 + .22/2)^2 – 1 = .2321$
$r = .05$

*candidate doesn't know the equation for the perpetuity so he guesses a number for n → n = 15

$pv = 14.77[(1 – (1 + .09/1 + .2321)^{15}/(.2321 - .09)] = 87.40$

Answer C

$K = 7s_{2|.11} = 14.77$

*candidate uses the annual interest rate convertible monthly when he needs to use annual interest rate

Thus:
$pv = 14.77/(.22 - .09) = 113.62$

Answer D

*when calculating k, candidate uses .22 to calculate because he doesn't see convertible semi-annually

$i = (1 + .22)^{1/2} = 1.105$
$K = 7(1.105)^2 = 8.55$

Thus:

$pv = 8.55/(.22 - .09) = 65.77$

Answer E

The first thing to notice that isn't entirely apparent is that this is a perpetuity. We nest need to find k, which in this case will be the first yearly payment.

This will be:

$K = 7s_{2|.11} = 14.77$

Next we need to find the yearly interest rate:

$i = (1 + .22/2)^2 - 1 = .2321$
$r = .09$

Since $i > r$

$pv = k/(i + r) = 14.77/(.2321 - .09) = 103.94$

Annuities

Question 11

Brian purchases a 7 year annuity with payments at the end of every quarter for $X. The first payment is $350 and each subsequent payment is $50 more. How much did Brian pay for the annuity if the interest was 14% convertible quarterly?

a.) $75,990.43
b.) $16,155.86
c.) $50,816.33
d.) $16,721.00
e.) $1,982.40

Question 11

Answer A

*candidate doesn't recognize the annuity pattern and just multiplies 350 be the increasing annuity.

$pv = 350(I_a)_{28|.035}$
$= 350[(\ddot{a}_{28|.035} - 28v^{28})/.035]$
$= 350(217.1155)$
$=75{,}990.43$

Answer B

The first thing we must do is recognize the arithmetic pattern which we must separate from the other payments.
Thus the annuity payments are:
300, 300, 300, . . .
And the increasing annuity pay is
50, 100, 150, . . .

Thus the present value would be:
$pv = 300a_{28|.035} + 50(I_a)_{28|.035}$
$= 300(1 - v^{28}/.035) + 50(\ddot{a}_{28|.035} - 28v^{28}/.035)$
$= 300(17.667) + 50(217.1155)$
$=16{,}155.55$

Answer C

*candidate doesn't see the amount of years and thus assumes it is a perpetuity

$pv = 300(1/.035) + 50(1/.035 + 1/.035^2)$
$= 50{,}816.33$

Answer D

*candidate thinks it is an annuity due and increasing due when it is actually an annuity and increasing annuity immediate.

$pv = 300\ddot{a}_{28|.035} + 50(I\ddot{a})_{28|.035}$
$= 300(18.285) + 50(224.71)$
$=16{,}721$

Answer E

*candidate does not see that it is done quarterly and not yearly:

Calculates i: $i = (1 + .035)^4 - 1 = .1475$

$pv = 300a_{7|.1475} + 50(I\ddot{a})_{7|.1475}$
$= 300(4.192) + 50(14.4959)$
$=1{,}982.395$

Annuities

Question 12

Jeffery invests $4,000 at an annual effective rate of 7%. The interest is paid every year and Jeffery reinvests it at annual rate i. At the end of 12 years the accumulated interest is $7,500. If Jane invests $1,000 at the end of each year for 25 years at a rate of interest of 10%, and she reinvests his interest that is paid annually into an account at an effective rate of I, what is Jane's accumulated interest at the end of 25 years?

a.) $108,415.03
b.) $125,777.77
c.) $54,641.48
d.) $77,990.75
e.) $123,276.77

S. Broverman Study Guide Section 8 Problem 3

Question 12

Answer A

We first need to calculate Jeffery's accumulated interest at the end of each 12 years:

His interest each year is: 4000(.07) = 280

So,

$280s_{12|i} = 7500$

$s_{12|i} = 26.786$

$i = .137$

Jane's interest would be:

1000(.1) = 100at the end of 2^{nd} year

2000(.1) = 200 or 2(100) at the end of 3^{rd} year

3000(.1) = 300 or 3(100) at the end of 4^{th} year

. . .

24000(.1) = 2400 or 24(100) at the end of 25^{th} year

Which is a 25 year increasing annuity. She puts this money into an account with interest i

$100(I_s)_{24|.137} = 100(1084.15029)$

$=108{,}415.03$

Answer B

Calculating for i:

$280s_{12|i} = 7500$

$i = .137$

*candidate doesn't realize Jane's interest after year 1 is 0 so he calculates it for all 25 years, so it would be a 25 year increasing annuity

Accumulated value = $100(I_s)_{24|.137} = 100(1257.77)$

$=125{,}777.77$

Answer C

*candidate isn't sure what to do with Jeffery's information, but realizes that Jane's interest forms an increasing annuity, so he uses Jeffrey's received interest percentage as i.
So, $i = .07$

Since Jane reinvests at the end of each year for 25 years, her annuity would be increasing with 24 payments:

$100(I_s)_{24|.07}$
$= 100(546.4148)$
$=54{,}641.48$

Answer D

*candidate thinks the 7500 is the present value of the interest earned.

$280a_{12|i} = 7500$
$i = -.106$

Candidate thinks this must be positive so s/he uses $i = .106$
Candidate continues doing problem correctly from here ending up with:

$100(I_s)_{24|.106}$
$= 100(779.9075)$
$=77{,}990.75$

Answer E

$280s_{12|i} = 7500$
$i = .137$

*candidate uses an increasing annuity due with 24 payments when s/he should have used an increasing annuity immediate

$100(I_{s\square})_{24|.106}$
$= 100(1232.7677)$
$=123{,}276.77$

Loan Amortization

Question 13

Juliana takes out a loan for $200,000 with 25 yearly payments at the end of each year. She makes payments which are twice the interest due for the first 24 months and pays off the remaining balance with the 25^{th} payment. If the interest on the loan is 4%, what is the final payment equal to?

a.) $72,079.34
b.) $117,167.27
c.) $78,085.96
d.) $75,082.65
e.) Insufficient information to solve problem

S. Broverman Study Guide Section 9 Problem 2

Question 13

Answer A

$OB_o = 200,000$
$OB_x = OB_o(1 - i)^t$

*candidate just uses this equation for t = 25, 50:

$OB_{25} = 200,000(.96)^{25}$
$= 72, 079.34$

Answer B

$OB_o = 200,000$
$i = .04$

*candidate tries to set up an annuity but doesn't realize the payments will be decreasing: so,

payments = .04(200,000)(2) = 16,000

Thus for the first 24 payments:

$OB_{24} = 200,000(1 + .04)^{24}$
$= -112,660.83$

Thinking this can't be negative candidate makes it positive

$OB_{24} = 112,660.83$

$OB_{25} = 112,660.83(1.04)$
$= 117,167.27$

Answer C

$OB_o = 200,000$
$i = .04$

$OB_1 = OB_0(1 + i) - 2OB_oi = OB_o(1 - i)$
$OB_2 = OB_1(1 + i) - 2OB_1i = OB_1(1 - i) = OB_o(1 - i)^2$
$OB_3 = OB_2(1 + i) - 2OB_2i = OB_2(1 - i) = OB_o(1 - i)^3$

$OB_t = OB_0(1 - i) = OB_o(1 - i)^t$

After the 24th payment

$OB_{24} = 200{,}000(.96)^{24}$

$= 75{,}082.65$

Thus, she will owe

$OB_{25} = OB_{24}(1 + i) = 75{,}082.65(1.04)$

$= 78{,}085.96$

Answer D

$OB_o = 200{,}000$

$i = .04$

$OB_{24} = 200{,}000(.96)^{24}$

$= 75{,}082.65$

*candidate fails to multiply OB_{24} by the compound interest to get the final balance

Answer E

*candidate cannot find answer

Insufficient information to solve problem

Loan Amortization

Question 14

Jake buys a $140,000 home. He must make monthly mortgage payments for 40 years, with the first payment to be made a month from now. The annual effective rate of interest is 8%. After 20 years Jake doubles his monthly payment to pay the mortgage off more quickly. Calculate the interest paid over the duration of the loan.

a.) $241,753.12
b.) $527,803.12
c.) $356,440.43
d.) $136,398.99
e.) $225,440.43

S. Broverman Study Guide Section 9 Problem 9

Question 14

Answer A

*candidate simply divides yearly interest rate by 12

$j = .08/12 = .00667$

$140{,}000 = xa_{450|.00667}$
$x = 973.86$

After 20 years OB is

$973.86a_{240|.00667} = 115{,}873.33$

doubling the payments:

$973.86(2) = 1947.72$

finding remaining number of payments:

$115{,}873.33 = 1947.72a_{n|.00667}$
$.6032 = v^n$
$n = 76$

total paid

$973.86(240) + 1947.72(76) = 381{,}753.12$

interest:

$381{,}752.12 – 140{,}000 = 241{,}753.12$

Answer B

*candidate finds payments for first 20 years and then doubles it for the next 20 years instead of finding the decreased number of years it would have taken to pay off loan

$j = (1 + .08)^{1/12} = .00643$

$140{,}000 = xa_{480|.00667}$

x = 927.513 for first 20 years

doubling the payments:

927.513 (2) = 1855.03for last 20 years

total paid

927.513 (240) + 1855.03 (240) = 667,803.12

interest:

667,803.12– 140,000 = 527,803.12

Answer C

Monthly rate of interest: $j = (1 + .08)^{1/12} = .00643$

$140{,}000 = xa_{480|.00667}$

x = 927.513 for first 20 years

outstanding balance after 20 years

$927.513 = a_{480|.00667} = 114{,}611.417$

doubling the payments:

927.513 (2) = 1855.03

new amount of years:

$114{,}611.417 = 1855.03\ a_{n|.00667}$

n = 77

total paid

927.513 (240) + 1855.03 (77) = 365,440.43

*candidate sees his answer and doesn't calculate what the interest was

Answer D

Monthly rate of interest: $j = (1 + .08)^{1/12} = .00643$

$140{,}000 = xa_{480|.00667}$
$x = 927.513$

*candidate used the accumulated value annuity function when s/he should have used the present value function for the outstanding balance

outstanding balance after 20 years

$927.513 = s_{420|.00667} = 527{,}438.98$

doubling the payments:

$927.513\ (2) = 1855.03$

new amount of years:

$527{,}438.98 = 1855.03\ a_{n|.00667}$
$-.828 = v^n$
$.828 = v^n$
$n = 29$

total paid

$927.513\ (240) + 1855.03\ (29) = 276{,}398.99$

interest

$276{,}398.99 – 140{,}000 = 136{,}398.99$

Answer E

Monthly rate of interest: $j = (1 + .08)^{1/12} = .00643$

$140{,}000 = xa_{480|.00667}$
$x = 927.513$

outstanding balance after 20 years
$927.513 = a_{480|.00667} = 114{,}611.417$

doubling the payments:
$927.513\ (2) = 1855.03$

new amount of years:
$114{,}611.417 = 1855.03\ a_{n|.00667}$
$n = 76.916 = 77$

total paid
$927.513\ (240) + 1855.03\ (77) = 365{,}440.43$

interest
$365{,}440.43 - 140{,}000 = 225{,}440.43$

Loan Amortization

Question 15

Scott takes out a loan with 29 annual payments of $450 each. With the14th payment, Scott pays an extra $1,400, and then pays the balance in 8 years with revised annual payments. The annual effective interest rate is 11%. Calculate the amount of the revised payment.

a.) $2,359.45
b.) $356.75
c.) $288.09
d.) $154.8
e.) $255.31

S. Broverman Study Guide Section 9 Problem26

Question 15

Answer A

*candidate tries finding outstanding balance by seeing what has been already paid

$450s_{14|.11} = 13{,}542$

After the extra 1,400 the balance is:

$13{,}542 - 1400 = 12{,}142$

Thus the revised payments would be:

$12{,}142 = xa_{8|.11}$

$x = 2359.45$

Answer B

First find the amount of the outstanding balance after the 14th payment:

$450a_{15|.11} = 3235.89$

After the extra 1,400 the balance is:

$3235.89 - 1400 = 1835.89$

Thus the revised payments would be:

$1835.89 = xa_{8|.11}$

$x = 356.75$

Answer C

*candidate first finds present value of the entire loan

$450a_{29|.11} = 3{,}892.55$

After the extra 1,400 the balance is:

$3{,}892.55 - 1400 = 2{,}492$

Thus s/he finds the payments that would be do all 29 years

$2{,}492 = xa_{29|.11}$

$x = 288.09$

Answer D

Finds OB after 14th payment

$450a_{15|.11} = 3235.89$

After the extra 1,400 the balance is:

$3235.89 - 1400 = 1835.89$

*candidate uses accumulated value annuity instead of present value annuity when solving:

$1835.89 = xs_{8|.11}$

$x = 154.80$

Answer E

Finds OB after 14th payment

$450a_{15|.11} = 3235.89$

After the extra 1,400 the balance is:

$3235.89 - 1400 = 1835.89$

*candidate uses the 15 year annuity instead of the correct 8 year annuity calculation

$1835.89 = xa_{15|.11}$

$x = 255.31$

Loan Amortization

Question 16

Lauren takes out a loan of $35,000. She pays this back by establishing a sinking fund and making 16 equal payments at the end of each year. The sinking fund earns 9% each year. Immediately after the 9^{th} payment the sinking fund's yield increases to 11%. At this time Lauren adjusts her sinking fund payment to X so that the fund will accumulate to $35,000 16 years after the original loan date. Find X.

a.) $8,057.17
b.) $1,291.33
c.) $647.09
d.) $1,040.86
e.) $2,166.06

S. Broverman Study Guide Section 10 Problem 3

Question 16

Answer A

OB_0 = 35,000 16 years
i = .09 for 9 years then .11 for 7 years

*candidate uses present value function instead of accumulated value function

$35{,}000 = xa_{16|.09} = 4{,}120.50$

Just after the 9th payment the present value would be:

$4{,}120.50s_{9|.09} = 54{,}825.03$

At .11 for next 7 years x would be:

$54{,}825.03(1.11)^7 + xs_{7|.11} = 35{,}000$

$x = -8057.17$

Knowing this can't be negative candidate makes it positive $8,057.17

Answer B

OB_0 = 35,000 16 years

*candidate gets the interest rate mixed up

i = .11 for 9 years then .09 for 7 years

Initial payments would be:

$35{,}000 = xs_{16|.11} = 893.09$

Just after the 9th payment the present value would be:

$893.09s_{16|.11} = 12{,}649.65$

To calculate final 7 payments:

$12{,}649.65(1.09)^7 + xs_{7|.09} = 35{,}000$

$x = 1{,}291.33$

Answer C

$OB_0 = 35{,}000$ 16 years
$i = .09$ for 9 years then .11 for 7 years

Initial payments would be:

$35{,}000 = xs_{16|.09} = 1{,}060.50$

Just after the 9th payment the balance would be:

$1{,}060.50s_{16|.09} = 13{,}808.81$

At .11 for the next 7 years the accumulated amount will be 35,000 if: :

$13{,}808.81(1.11)^7 + xs_{7|.11} = 35{,}000$

$x = 647.09$

Answer D

$OB_0 = 35{,}000$ 16 years

*candidate gets years mixed up

$i = .09$ for 7 years then .11 for 9 years

Initial payments would be:

$35{,}000 = xs_{16|.09} = 1{,}060.50$

Just after the 7th payment the balance would be:

$1{,}060.50s_{7|.09} = 9{,}757.06$

To calculate final 9 payments:

$9{,}757.06(1.11)^7 + xs_{9|.11} = 35{,}000$

$x = 1{,}040.86$

Answer E

$OB_0 = 35{,}000$ 16 years
$i = .09$ for 9 years then .11 for 7 years

Initial payments would be:

$35{,}000 = xs_{16|.09} = 1{,}060.50$

Just after the 9th payment the balance would be:

$1{,}060.50s_{9|.09} = 13{,}808.81$

*candidate doesn't account for interest on original payments
To calculate final 9 payments:

$13{,}808.81 + xs_{7|.11} = 35{,}000$

$x = 2{,}166.06$

Loan Amortization

Question 17

Aiden takes out a 30 year loan for $24,000 to be repaid with payments at the end of each year consisting of interest on the loan and a sinking fund deposit. Interest is charged at a 16% annual rate. The sinking fund's annual rate is 11%. However, beginning in the 13th year, the annual effective interest rate on the sinking fund drops to 8%. As a result, the payments are increased by X. Calculate X.

a.) $228.01
b.) $348.60
c.) $447.12
d.) $273.41
e.) $337.67

S. Broverman Study Guide Section 10 Problem 6

Question 17

Answer A

$OB_0 = 24{,}000$
$i = .16$
j on sinking fund: .11 first 12 years then .08 for last 18 years

Original payments would be:

$Ks_{30|.11} = 24{,}000$
$K = 120.59$

At the end of 12 years:

$120.50s_{12|.11} = 2{,}738.98$

With the new rate of interest, payment increases to: 120.59 + x

The accumulated value is;

$2{,}738.98(1.08)^{18} + (120.59 + x)s_{18|.08} = 24{,}000$
$(120.59 + x)s_{18|.08} = 13{,}054.98$
$x = 228.01$

Question 17

Answer B

$OB_0 = 24{,}000$
$i = .16$
j on sinking fund: .11 first 12 years then .08 for last 18 years

Original payments would be:

$Ks_{30|.11} = 24{,}000$
$K = 120.59$

At the end of 12 years:

$120.50s_{12|.11} = 2{,}738.98$

*candidate finds new payment not the payment increase

The accumulated value is;

$2,738.98(1.08)^{18} + s_{18|.08} = 24,000$
$x = 348.60$

Answer C

$OB_0 = 24,000$
$i = .16$
j on sinking fund: .11 first 12 years then .08 for last 18 years

Original payments would be:
$Ks_{30|.11} = 24,000$
$K = 120.59$

At the end of 12 years:
$120.50s_{12|.11} = 2,738.98$

With the new rate of interest, payment increases to: 120.59 + x

The accumulated value is:
*candidate doesn't add the interest onto the first payments for the last 18 years
$2,738.98 + (120.59 + x)s_{18|.08} = 24,000$
$(120.59 + x)s_{18|.08} = 21,261.02$
$x = 447.12$

Answer D

$OB_0 = 24,000$
$i = .16$

*candidate mixes up interest rates
j on sinking fund: .08 first 12 years then .11 for last 18 years

Original payments would be:
$Ks_{30|.08} = 24,000$
$K = 211.86$

At the end of 12 years:

$211.86s_{12|.08} = 4{,}020.49$

With the new rate of interest, payment increases to: 211.86+ x

The accumulated value is:

$4{,}020.49\ (1.11)^{18} + (211.86 + x)s_{18|.08} = 24{,}000$

$(211.86 + x)s_{18|.08} = -2305.08$

$x = -273.41$

candidate makes this positive

$x = 273.41$

Answer E

$OB_0 = 24{,}000$

$i = .16$

*candidate mixes up interest rates

j on sinking fund: .11 first 18 years then .08 for last 12 years

Original payments would be:

$Ks_{30|.11} = 24{,}000$

$K = 120.59$

At the end of 12 years:

$120.59s_{18|.11} = 6{,}077.25$

With the new rate of interest, payment increases to: 120.59 + x

The accumulated value is:

$6{,}077.25(1.11)^{18} + (120.59 + x)s_{12|.08} = 24{,}000$

$x = 337.67$

Bonds

Question 18

Kyle can buy a zero-coupon bond that will pay $1,600 at the end of 17 years and it is currently selling for 1,050. Instead he purchases a 8% bond with coupons payable quarterly that will pay $1,600 at the end of 13 years. If he pays x he will earn the same annual effective interest rate as the zero coupon bond. Calculate x.

a.) $2,577.94
b.) $1,418.33
c.) $1,600.00
d.) $2,580.80
e.) $2,593.23

S. Broverman Study Guide Section 11 Problem 12

Question 18

Answer A

Suppose the quarterly yield rate on the zero coupon bond is j.

Thus for the zero coupon bond j would equal:

$1{,}050 = 1600v^{68}$

$v^{68} = .65625$

$j = .00621$

Price of the coupon bond would be:

$1600v^{52} + 1600(.02)a_{52|.00621}$

$= 2{,}577.94$

Answer B

Suppose the quarterly yield rate on the zero coupon bond is j.

Thus for the zero coupon bond j would equal:

$1{,}050 = 1600v^{68}$

$v^{68} = .65625$

*candidate only considers the coupons

$1600(.02)a_{52|.00621}$

$= 1418.33$

Answer C

*candidate uses coupon rate as coupon rate

Price of the coupon bond would be:

$1600v^{52} + 1600(.02)a_{52|.02}$

$= 1600$

Answer D

*candidate thinks the present value is the maturity value

Thus for the zero coupon bond j would equal:

$1600 = 1050v^{68}$

$v^{68} = 1.5238$

$j = -.00618$

candidate makes this positive

Price of the coupon bond would be:

$1600v^{52} + 1600(.02)a_{52|.00618}$

$= 2{,}580.80$

Answer E

*candidate uses the yearly rate instead of quarterly interest rate

$1050 = 1600v^{17}$

$j = .0251$

Price of the coupon bond would be:

$1600v^{12} + 1600(.08)a_{13|.0251}$

$= 2{,}593.23$

Bonds

Question 19

Amin buys a 24 year bond with a par value of \$2,300 and annual coupons. The bond is redeemable at par. He pays \$3,200 for the bond assuming an annual effective yield of i. the coupon rate is 4 times the yield rate. At the end of 9 years Amin sells the bond for S, which produces the same annual effective rate of I for the new buyer. Calculate S.

a.) Insufficient information
b.) \$3,051.19
c.) \$3,721.43
d.) \$1,875.37
e.) \$2,156.91

S. Broverman Study Guide Section 11 Problem 1

Question 19

Answer A

P = 3200   F = C = 2300

To calculate i:
*candidate puts coupon rate at i when it should be 4i:

$3200 = 2300v^{24} + 2300(i)a_{24|i}$
$3200 = 2300v^{24} + 2300(1 - v^{24})$
$1.3913 = v^{24} + 1 - v^{24}$
$1.3913 = 1$

insufficient information to complete the problem

Answer B

P = 3200   F = C = 2300

To calculate i:

$3200 = 2300v^{24} + 2300(4i)a_{24|i}$
$3200 = 2300v^{24} + 9200(1 - v^{24})$
$-6000 = v^{24} - 9200v^{24}$
$i = .0058$

$s = 2300v^{15} + 2300(4(.0058))a_{15|.0058}$
$= 3{,}051.19$

Answer C

*candidate gets face value and face amount mixed up with purchase price
P = 2300   F = C = 3200

To calculate i:

$2300 = 3200v^{24} + 3200(4i)a_{24|i}$

$2300 = 3200v^{24} + 12800(1 - v^{24})$

$1.09375 = v^{24}$

$i = -.00373$

candidate makes this positive

calculating s:

$s = 3200v^{15} + 3200(4(.00373))a_{15|.00373}$

$= 3{,}721.43$

Answer D

P = 3200   F = C = 2300

To calculate i:

*candidate does not include coupon payments

$3200 = 2300v^{24}$

$1.3913 = v^{24}$

$i = -.0137$

candidate makes this positive

solving for s:

$s = 2300v^{15}$

$= 1{,}875.37$

Answer E

P = 3200   F = C = 2300

To calculate i:

*candidate only includes coupon payments

$3200 = 2300(4i)a_{24|i}$

$.3478 = (1 - v^{24})$

$i = .01797$

solving for s:

$s = 2300(4(.01797))a_{15|.01797}$
$= 2,156.91$

Bonds

Question 20

Stacia buys a 5 year bond with coupons at 6% convertible monthly which will be redeemed at \$1,500. She buys the bond to yield 9% convertible monthly. The purchase price is \$1,100. Calculate the par value.

a.) \$2,916.84
b.) \$1,060.67
c.) \$2,114.52
d.) \$376.40
e.) \$23.04

S. Broverman Study Guide Section 11 Problem 4

Question 20

Answer A

$P = 1100$ $i = .0075$ $r = .005$
$C = 1,500$

*candidate only includes coupon payments

$1100 = F(.005)a_{60|.0075}$

$F = 2916.84$

Answer B

$P = 1100$ $i = .0075$ $r = .005$
$C = 1,500$

*candidate fails to account for the present value of the interest on the maturity value of the bond

$1100 = 1500 + F(.005)a_{60|.0075}$

$F = -1060.67$

Candidate makes this positive

Answer C

$P = 1500$ $i = .0075$ $r = .005$
$C = 1100$

*candidate fails to account for the present value of the interest on the maturity value of the bond

$1500 = 1100v^{60} + F(.005)a_{60|.0075}$

$F = 2114.52$

Answer D

$P = 1100$ $i = .0075$ $r = .005$

C = 1500

$1100 = 1500v^{60} + F(.005)a_{60|.0075}$
$141.95 = F(.005)\ a_{60|.0075}$
F =376.40

Answer E

*candidate mixes up bond and interest rate
P = 1100 i = .005 r = .0075
C = 1500

$1100 = 1500v^{60} + F(.0075)a_{60|.005}$
$-12.058 = F(.0075)a_{60|.005}$
F = -23.04
Candidate makes this positive

Rates of Return

Question 21

On February 1, Sawyer's investment is worth \$900. On August 1, the value has incre3ased to \$1600 and Sawyer deposits \$D. On December 1, the value is \$1400 and \$400 is withdrawn. On February 1 of the following year, the investment account is worth \$800. The time-weighted interest is 3%. Calculate the dollar-weighted rate of interest.

f.) Insufficient information given to complete problem
g.) -.685
h.) -.033
i.) -.045
j.) -.142

S. Broverman Study Guide p. 199 Example 58

Question 21

Answer A

Feb 1	Aug 1	Dec 1		Feb 1
900	1600	1400		800
	1600+D		1000	

*Candidate doesn't take into account the amounts before deposits and withdrawals when doing calculations

[(1600+D)/900]*[1000/(1600+D)]*[800/1000]-1=.03

⇨ .888≠.03

Insufficient information to complete problem

Answer B

Feb 1	Aug 1	Dec 1		Feb 1
900	1600	1400		800
	1600+D		1000	

Time Weighted Rate of interest:

[1600/900]*[1400/(1600+D)]*[800/1000]-1=.03

⇨ D=333.12

*Instead of using simple interest rates to find dollar-weighted amount, candidate uses compound interest rates.

Dollar-Weighted:

$900(1+i) + 333.12(1+i)^{1/2} - 400(1+i)^{1/6} = 800$

⇨ i= -.685

Answer C

Feb 1	Aug 1	Dec 1	Feb 1
900	1600	1400	800
	1600+D	1000	

Time Weighted Rate of interest:

[1600/900]*[1400/(1600+D)]*[800/1000]-1=.03

- ⇨ D=333.12
- ⇨

Dollar-Weighted:

900(1+i) + 333.12(1+1/2*i) – 400(1+1/6*i) = 800

- ⇨ 999.89*i=-33.12
- ⇨ i= -.033

Answer D

Feb 1	Aug 1	Dec 1	Feb 1
900	1600	1400	800
	1600+D	1000	

Time Weighted Rate of interest:

[1600/900]*[1400/(1600+D)]*[800/1000]-1=.03

- ⇨ D=333.12

*Candidate calculates time from 1st deposit instead of time until last amount.

Dollar-Weighted:

900(1+i) + 333.12(1+1/2*i) – 400(1+5/6*i) = 800

- ⇨ i= -.04

Answer E

Feb 1	Aug 1	Dec 1	Feb 1
900	1600	1400	800
	1600+D	1000	

*Candidate messes up time-weighted function

Time Weighted Rate of interest:

[900/1600]*[(1600+D)/1400]*[1000/800]-1=.03

- ⇨ D=450.84

Dollar-Weighted:

900(1+i) + 450.84(1+1/2*i) – 400(1+1/6*i) = 800

- ⇨ i= -.142

Rates of Return

Question 22

Alex earned an investment income of \$13,000 during 1999. The beginning and ending balances were \$114,000 and \$136,000. A deposit was made at time k during the year. No other deposits or withdrawals were made. The fund made 11% in 1999 using the dollar-weighted method. Determine k.

a.) August 1
b.) May 1
c.) June 1
d.) July 1
e.) March 1

S. Broverman Study Guide Problem Set 13 Problem 1

Question 22

Answer A

Beginning= 114,000 End=136,000

Investment Income= 13,000 i= .11

Total Increase =22,000

Deposit= Total Income- Investment Income= 9,000

*Candidate uses compound interest rates instead of simple.

Dollar-Weighted:

$114{,}000(1.11) + 9000(1.11)^{(1\text{-}k)} = 136{,}000$

- ⇨ $(1.11)^{(1\text{-}k)} = 1.0511$
- ⇨ k= .5224

*Candidate rounds this to the next month which would be k≈.5833

- ⇨ August 1

Answer B

Beginning= 114,000 End=136,000

Investment Income= 13,000 i= .11

Total Increase =22,000

Deposit= Total Income- Investment Income= 9,000

*Candidate doesn't take the beginning balance into account.

Dollar-Weighted:

$9000(1+.11(1\text{-}k)) = 136{,}000$

- ⇨ k= -127.2828

Candidate tries finding the month determined by this: .2828≈.333

- ⇨ May 1

Answer C

Beginning= 114,000 End=136,000

Investment Income= 13,000 i= .11

*Candidate uses Investment Income as the deposit.

Dollar-Weighted:

114,000(1.11) + 13,000(1+.11(1-k))= 136,000
- ⇨ 13,000(1+.11(1-k))= 9,460
- ⇨ k= 3.476

Candidate tries finding the month determined by this: .476≈.417
- ⇨ June 1

Answer D

Beginning= 114,000 End=136,000

Investment Income= 13,000 i= .11

Total Increase =22,000

Deposit= Total Income- Investment Income= 9,000

Dollar-Weighted:

114,000(1.11) + 9000(1+.11(1-k))= 136,000
- ⇨ -10,000*.11k=-530
- ⇨ k= .4818≈.5
- ⇨ July 1

Answer E

Beginning= 114,000 End=136,000

Investment Income= 13,000 i= .11

Total Increase =22,000

*Candidate uses total increase as the deposit.

Dollar-Weighted:

114,000(1.11) + 22,000(1+.11(1-k))= 136,000

- ⇨ k= 6.1818

Candidate tries finding the month determined by this: .1818≈.1667

- ⇨ March 1

Rates of Return

Question 23

On January 1, 2010, Toni deposits \$140 into an account. On June 1, 2010, when the amount in Toni's account is equal to \$X, a withdrawal W is made. No further deposits or withdrawals are made to Toni's account for the remainder of the year. On December 31, 2010, the amount in Toni's account is \$100. The dollar-weighted return over the period is 15%. The time-weighted return over the 1-year period is 11%. Calculate X.

a.) 123.81
b.) 107.91
c.) 98.15
d.) 126.73
e.) 172.02

S. Broverman Study Guide Problem Set 13 Problem 3

Question 23

Answer A

Initial deposit = 140 Withdrawal on June 1: W
Final Amount= 100
Dollar-Weighted= 15%
Time-Weighted= 11%

Dollar-Weighted:

140(1.15) – W[1+7/12*.15]=100
⇨ W(1+7/12*.15)=61
⇨ W= 56.09

Time-Weighted:

(X/140)(85/(X-56.09))-1=.11
⇨ X= 123.81

Question 23

Answer B

Initial deposit = 140 Withdrawal on June 1: W
Final Amount= 100

*Candidate mixes up the dollar-weighted and the time –weighted interest rates.

Dollar-Weighted= 11%
Time-Weighted= 15%

Dollar-Weighted:

140(1.11) – W[1+7/12*.11]=100
⇨ W= 50.94

Time-Weighted:

(X/140)(85/(X-56.09))-1=.15
⇨ X= 107.91

Answer C

*Candidate mixes up the initial deposit and final amount.

Initial deposit = 100 Withdrawal on June 1: W
Final Amount= 140
Dollar-Weighted= 15%
Time-Weighted= 11%

Dollar-Weighted:

$$100(1.15) - W[1+7/12*.15]=140$$

- ⇨ W=-22.99
- ⇨ Candidate thinks this is positive

Time-Weighted:

$$(X/100)(85/(X-22.99))-1=.11$$

- ⇨ X= 98.15

Answer D

Initial deposit = 140 Withdrawal on June 1: W
Final Amount= 100
Dollar-Weighted= 15%
Time-Weighted= 11%
*Candidate uses time from initial deposit when calculating t.

Dollar-Weighted:

$$140(1.15) - W[1+5/12*.15]=100$$

- ⇨ W=57.41

Time-Weighted:

$$(X/140)(85/(X-57.41))-1=.11$$

- ⇨ X= 126.73

Answer E

Initial deposit = 140 Withdrawal on June 1: W

Final Amount= 100

Dollar-Weighted= 15%

Time-Weighted= 11%

Dollar-Weighted:

140(1.15) – W[1+7/12*.15]=100

⇨ W(1+7/12*.15)=61

⇨ W= 56.09

*Candidate flips rates.

Time-Weighted:

(140/X)((X-56.09)/85)-1=.11

⇨ X= 172.02

Rates of Return

Question 24

Bill is looking at yield maturity rates for zero coupon bonds. They are currently quoted at 14% for one-year maturity, 16.5% for two-year maturity, and 11% for 3-year maturity. Let i be the one-year forward rate for year two implied by current yields of these bonds. Calculate i.

a.) .165
b.) .137
c.) .166
d.) .0077
e.) .191

S. Broverman Study Guide Problem Set 14 Problem 7

Question 24

Answer A

*Candidate thinks the one year forward price is the same as the yield for the two- year maturity.

Thus, $j=.165$

Answer B

*Candidate thinks you must average the interest rates and then find the one year forward.

$(1.14 + 1.165 + 1.11) / 3= 1.138$

He then tries making the 1-year forward:

$$1.14(1+j) = (1.138)^2$$

⇨ $j= .137$

Answer C

*Candidate thinks you must average the first two years' interest rates and then take the one year forward after the first year.

$(1.14 + 1.165) / 2= 1.153$

$$1.14(1+j) = (1.153)^2$$

⇨ $j= .166$

Answer D

*Candidate gets confused and thinks he needs to find the three year forward because he has all three interest rates.

$(1.165)^2 (1+j) = (1.11)^3$

⇨ $j= .0077$

Answer E

The expected value for the bond to yield 2 years from now is 16.5%. Thus, the 2 year forward must equal $(1.165)^2$.

Thus the one year forward rate for year two is j, where:

$$(1.14)(1+j)=(1.165)^2$$

⇨ j= .191

Forwards

Question 25

Jarrett took a long position on a forward contract that pays no dividends and is currently priced at $250=s_0$. The delivery price for a one year forward contract on the stock is $F_{0,1}$=$270. Find the payoff at time 1 if s_1=$260

a.) 10
b.) 20
c.) -10
d.) 15
e.) Insufficient information to complete problem

S. Broverman Study Guide Problem Set 15 Problem 1

Question 25

Answer A

S_0= 250 S_1=260

$F_{0,1}$= 270

*Candidate thinks payoff is $S_1 – S_0$

Payoff:

260-250=10

Answer B

S_0= 250 S_1=260

$F_{0,1}$= 270

*Candidate thinks payoff is $F_{0,1} – S_0$

Payoff:

270-250=20

Answer C

S_0= 250 S_1=260

$F_{0,1}$= 270

Payoff:

S_1- $F_{0,1}$ = 260- 270 = -10

Answer D

S_0= 250 S_1=260

$F_{0,1}$= 270

*Candidate averages the two S values and subtracts it from $F_{0,1}$.

Payoff:

270-255= 15

Answer E

*Candidate does not believe enough information is given to solve problem.

Insufficient information to solve problem

Options and Swaps

Question 26

What combination of puts, calls, and or assets is known as the put-call parity?

a.) Long Asset and Short Call
b.) Long Call and Short Put
c.) Short Call and Long Put
d.) Long Put and Long Asset
e.) Short Asset and Short Call

May 2010 Actuary Exam

Question 26

Answer D

This question has popped up on many actuary exams. I suggest every candidate memorize what the put-call parity is. It is a combination of a long put and long asset!

Options and Swaps

Question 27

Which of these options are correct?

I) A Butterfly Spread is a combination of a written straddle and purchased strangle.
II) A written strangle is sometimes called a zero-cost collar
III) A straddle is a combination of a purchased call and put with the same expiry and strike price
IV) A written straddle is the combination of a written call and purchased put with the same strike price

a.) I only
b.) II, III, and IV
c.) I and III
d.) II only
e.) None of the above

Question 27

Answer

I- Correct
II- Incorrect- A written strangle consists of a written put and written call, with the strike price less than the call price. A zero cost collar is where the price of the collar is very close to 0.
III- Correct
IV- A written straddle is the combination of a written call and written put with the same strike price.

Annuities

Question 28

Mason receives $23,000 from a life insurance policy. He uses the fund to purchase different annuities, each costing $11,500. His first annuities is an 18 year annuity-immediate paying K per year. The second annuity is a 7 year annuity paying 2K per year. Both annuities are based on an annual effective interest rate of i, i>0. Determine i.

a.) .053
b.) 2.08
c.) .052
d.) .5
e.) .99

S. Broverman Study Guide Problem Set 5 Problem 1

Question 28

Answer A

PV= 11,500 interest=i

*Candidate uses the equation for annuity due, not annuity immediate.

$11{,}500 = K\ \ddot{a}_{18|i} = 2K\ \ddot{a}_{7|i}$

- ⇨ $\ddot{a}_{18|i} = 2\ \ddot{a}_{7|i}$
- ⇨ i=.053

Answer B

PV= 11,500 interest=i

*Candidate gets the years for the two annuities mixed up

$11{,}500 = K\ a_{7|I} = 2Ka_{18|i}$

- ⇨ i=2.08

Candidate makes this positive

Answer C

PV= 11,500 interest=i

$11{,}500 = K\ a_{18|I} = 2Ka_{7|i}$

- ⇨ $a_{18|i} = a_{7|i}$
- ⇨ i=.052

Answer D

PV= 23,000 interest=i

*Candidate thinks the payments are 11,500 and the present value is 23,000

$$23{,}000=11{,}500a_{18|i}$$

⇨ $i=.5$

Answer E

PV= 23,000 interest=i

*Candidate thinks the payments are 11,500 and the present value is 23,000

$$23{,}000=2*11{,}500a_{7|i}$$

⇨ $i=.99$

Loan Amortization

Question 29

Michelle takes out a loan. It must be repaid with level annual payments based on an annual coupon rate of 4%. The 6th payment consists of \$960 in interest and \$340 of principal. Calculate the amount of interest paid in the 14th payment.

a.) 465.31
b.) 711.23
c.) 588.77
d.) 13.83
e.) 834.69

S. Broverman Study Guide Exam 1 Problem 18

Question 29

Answer A

Payment= 960 +340=1300

Principal repaid grows by 1.04 with every payment.

Principal in 14^{th} payment is:

$340(1.04)^8= 465.31$

*Candidate Stops Here

Answer B

Payment= 960 +340=1300

Principal repaid grows by 1.04 with every payment.

*Candidate takes compounded increase to the 14^{th} power because that's what year we are trying to find for.

Principal in 14^{th} payment is:

$340(1.04)^{14}= 588.77$

Interest Repaid

1,300-588.77= 711.23

Answer C

Payment= 960 +340=1300

Principal repaid grows by 1.04 with every payment.

*Candidate takes compounded increase to the 14^{th} power because that's what year we are trying to find for.

Principal in 14th payment is:

$340(1.04)^{14} = 588.77$

*Candidate stops here

Answer D

Payment= 960 +340=1300

Principal repaid grows by 1.04 with every payment.

*Candidate mixes up the principal and interest.

Principal in 14th payment is:

$960(1.04)^{8} = 1313.826$

Interest Paid:

1300 – 1313.826= .13.826

Candidate makes this positive.

Answer E

Payment= 960 +340=1300

Principal repaid grows by 1.04 with every payment.

Principal in 14th payment is:

$340(1.04)^{8} = 465.31$

Interest Paid:

1300 – 465.31= 834.69

Annuities

Question 30

Which of these are true about annuities?

I) An annuity due is one that requires payments at the end of every month.
II) A geometric perpetuity present value can be represented by K/(i-r).
III) Because an increasing annuity immediate has an annuity due equation in its equation, it becomes an annuity due.

f.) I only
g.) II only
h.) III only
a.) II and III
b.) I, II, and III are all false

Question 30

Answer

I- False: Annuity Due requires payments at the beginning of every month
II- True
III- False: It is still an annuity immediate