Estimating the R-Value of Earth's Atmosphere

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Raymond Pierrehumbert, Director of the Climate Systems Center at the University of Chicago, has pointed out an interesting parallel between heat transfer through planetary atmospheres and heat-transfer from ordinary buildings [1]:

"The temperature of your house is intermediate between the temperature of the flame in your furnace and the temperature of the outdoors, and adding insulation shifts it toward the former by reducing the rate at which the house loses energy to the outdoors. As Fourier already understood, when it comes to relating temperature to the principles of energy balance, it matters little whether the heat-loss mechanism is purely radiative, as in the case of a planet, or a mix of radiation and turbulent convection, as in the case of a house—or a green-house. Carbon dioxide is just planetary insulation."

Motivated by Pierrehumbert's analogy, we can determine the insulative R–value of the Earth's atmosphere with a very straight-forward calculation. This has a personal connection for me: As a young father I almost burned down our house. We lacked shelf space, so I put a shelf in the closet for the kid's winter clothes, ignoring basic physics. The trouble was that the light bulb above the shelf was accidently covered as my kid's winter coats were thrown willy-nilly on the shelf. A covered light bulb raised the temperature to start a smoldering fire. Luckily I smelled smoke in time and saved the day. I had forgotten that the R-value for heat transfer from buildings is defined as

$$dQ/dt = (A/R)\Delta T, \tag{1}$$

where dQ/dt is thermal power passing through the medium in Watts, A is surface area, R is the thermal resistance "R–factor," and ΔT is the temperature drop across the medium. Solving for ΔT, we obtain

$$\Delta T = (R/A)(dQ/dt) \tag{2}$$

If the R–value is doubled, it takes a twice the temperature differential $2\Delta T$ to push the constant heat flux (Watts/m^2) through the medium. This is analogous to doubling the voltage to push a constant current through a doubled electrical resistance.

The MKS unit of the R-value is m^{2}K/W. In customary English units this transforms to (ft^2 hr ^{o}F/BTU); the conversion is $R_{\text{English}} \sim 5.68 R_{\text{MKS}}$. R is related to the thermal conductivity k of elementary textbooks through $R = L/k$, where L is the thickness of the medium. Good insulators have high R-values; polystyrene boards, for example, have $R \sim 0.9$ m^{2}K/W per inch of thickness.

In the case of Earth's atmosphere, it is necessary to radiate the absorbed solar flux at the surface through the atmosphere to space. Earth's average absorbed solar flux $s_{\text{absorbed}} = [(dQ/dt)/A]$ is mostly absorbed at the Earth's surface, giving

$$\Delta T = R\,(dQ/dt)/A = R\,s_{absorbed}. \tag{3}$$

To obtain the R-value, begin with the fact that the solar flux s_o at Earth's orbit is 1367 W/m^2. This is reduced because the albedo (a) reflects about 30% of the incoming flux. Since Earth's perpendicular absorbing area πr^2 is one-fourth the radiating area of $4\pi r^2$ (r = Earth radius), we divide the solar flux by a factor of 4 to get the average flux. This gives a surface-averaged solar absorbed flux of

$$s_{absorbed} = (dQ/dt)/A = (1 - a)(s_o)/4 = (0.7)(1367\ W/m^2)/4 = 239\ W/m^2. \tag{4}$$

The out-going heat flux is transferred from Earth's surface at a mean temperature of 287 K to the middle of the troposphere at 255 K. Combining these facts and using Equation (3) gives the R-value for Earth's atmosphere as

$$R_{atmosphere\text{-}SI} \sim \Delta T/s_{absorbed} = (T_{troposphere} - T_{surface})/s_{absorbed}$$

$$\sim (287\text{ K} - 255\text{ K}) / (239\text{ W/m}^2) \sim 0.13\text{ m}^2\text{K/W}. \tag{5}$$

In English units this corresponds to $R \sim 0.76$ ft^2 hr ^{o}F/BTU. The atmosphere is not a terribly good insulator, equivalent to only about one-seventh of an inch of polystyrene!

Finally, how might global warming affect the R-value? The variation in solar flux over its 11-year cycle is only 0.1% and the average solar constant over the past decades has been very constant. Thus, we will ignore solar corrections. The 2007 Intergovernmental Panel on Climate Change (IPCC) best estimate for Earth's surface temperature rise is 3 K by the year 2100, which is about 10% of the 33 K greenhouse temperature rise before industrialization [2]. Since R is proportional to ΔT, a 10% rise in ΔT will mean the same percentage increase in R.

Calculations along this line appeared in the author's *Physics of Societal Issues* [3]. The second edition will include this work.

[1] R. Pierrehumbert, "Infrared Radiation and Planetary Temperature," Physics Today 64, January 2011, p. 33-38, and Physics of Sustainable Energy, AIP Conference Proceedings 1401, 232-243 (2011), ed. by D. Hafemeister, D. Kammen, B. Levi and P. Schwartz.

[2] Intergovernmental Panel on Climate Change, Climate Change 2007: The Physical Science Basis, Fourth Assessment Review, www.ipcc.ch.

[3] D. Hafemeister, Physics of Societal Issues (Springer, New York, 2007), 282.

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