EFFECTS OF NONLINEARITY IN PRIMARY SYSTEMS ON ACCELERATIONS IN SECONDARY SYSTEMS: PIERS, WHARVES, AND MARINE OIL TERMINALS

By

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ABSTRACT

This investigation examines the effects of nonlinearity in the primary system of the coupled primary-secondary systems on accelerations in the secondary systems during seismic loading. The coupled primary-secondary systems considered in this investigation are those typically found in piers, wharves, and marine oil terminals. This investigation first examines the effects of nonlinearity in the primary system on acceleration at the point of attachment of the secondary system to the primary system and found that:

- The acceleration at the point of attachment of the secondary system to the primary system decreases with increasing level of nonlinearity in the primary system. This occurs because yielding in the primary system limits accelerations that can transmit through it.

- The recommendation by Goel (2017a) provides very good estimate of the acceleration at the point of attachment of the secondary system to the primary system in coupled primary-secondary systems when the primary systems remains linear elastic. However, it provides increasingly conservative estimate of the acceleration with increasing nonlinearity in the primary system.

- The recommendations in ASCE 7-10 significantly over-predict accelerations at the point of attachment of the secondary system. The level of over-prediction increases with increasing level of nonlinearity in the primary system and period of the primary system.

This investigation next examined the effects of nonlinearity in the primary system on amplification of the acceleration in the secondary system due to its flexibility and found that:

- The trends in amplification of acceleration in the secondary system due to its flexibility of linear-elastic system no longer apply when the primary system is deformed beyond the linear elastic range. In particular, amplification of acceleration tends to be much larger when period of the secondary system is longer than period of the primary system and this difference increases with increasing level of nonlinearity in the primary system. This occurs because the effective period of the primary system elongates due to its nonlinearity and thereby reduces the effective period ratio, which has the effect of increasing amplification of acceleration in the secondary system.

- The nonlinearity in the primary system has minimal effect on amplification of acceleration in the secondary system when period ratio is less than 0.6. For such systems, recommendation by Goel (2017a) may be used to accurately estimate amplification of acceleration in the secondary system when the primary system is expected to be deformed beyond the linear elastic range.

Finally, this investigation studied the effects on nonlinearity in the primary system on acceleration in the secondary system and found that:

- The recommendation in the ASCE 7-10 document for flexible secondary system generally lead to significant over-prediction of acceleration in the secondary system. The level of over-prediction increases with increasing level of nonlinearity in the primary system and increasing period of the primary system.

- The recommendation in the commentary of the ASCE 7-10 document also leads to over-prediction, although not as large as that from the recommendation in the main body of the ASCE 7-10 document, of acceleration in the secondary system. The level of over-
prediction increases with increasing level of nonlinearity in the primary system and increasing period of the primary system.

- The recommendation by Goel (2017a) provide a reasonably good estimate of acceleration in the secondary system over the entire range of vibration period of the primary system when the primary system remains in the linear elastic. However, this recommendation provides slight over-prediction when the primary system deforms beyond the linear elastic range.

- The recommendation in ASCE 7-10 document and by Goel (2017a) were developed based on studies of linear-elastic systems. The current investigation, which considers nonlinearity in the primary system, indicates that recommendations based on linear-elastic systems lead to conservative estimates of accelerations in secondary system even when the primary system in the coupled primary-secondary system is deformed beyond the linear-elastic range.

Based on findings in this investigation, it is recommended not to design coupled primary-secondary systems with period ratio between 0.6 and 1.4 and secondary systems weighing less than 20% of the primary system. For such cases, secondary systems may experience excessive accelerations that may equal to or exceed eight times the peak ground accelerations due to strong coupling between primary and secondary systems.
ACKNOWLEDGMENT

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INTRODUCTION AND BACKGROUND

Engineering practice often uses provisions in ASCE 7-10 standard (ASCE, 2010) to compute horizontal seismic forces in nonstructural components or nonbuilding structures, referred to as secondary systems, supported on other structures, referred to as primary systems. For secondary systems that weigh less than 25% of the combined effective weights of the secondary and primary systems, ASCE 7-10 specifies the following formula to compute seismic force in the secondary system:

\[
F_p = \frac{0.4a_p S_{DS} I_p W_p}{R_p} \left(1 + 2 \frac{z}{h}\right)
\]

\[
0.3S_{DS} I_p W_p \leq F_p \leq 1.6S_{DS} I_p W_p
\]

where \( S_{DS} \) = short period spectral acceleration, \( a_p \) = component amplification factor, \( I_p \) = component importance factor, \( R_p \) = component response modification factor, \( W_p \) = component operating weight, \( z \) = height in structure of point of attachment of component with respect to the base, and \( h \) = average roof height of structure with respect to the base. The values of \( a_p \) and \( R_p \) for different types of nonstructural components are available in ASCE 7-10. The coefficient \( a_p \) is typically set equal to 1 for rigid components and 2.5 for flexible components. ASCE 7-10 permits lower value of \( a_p \) for flexible components if justified by detailed dynamic analysis.

The engineering practice also refers to FEMA-356 and FEMA-440 documents for computation of seismic forces in secondary systems. Generally, the FEMA-356 and FEMA-440 provisions are similar to those in the ASCE 7-10 document. However, the types of nonstructural components and, in some cases, values of \( a_p \) and \( R_p \) may differs between these documents.

The term \( 0.4S_{DS} \) in Equation (1) represents the acceleration at the ground level and \( (1 + 2z/h) \) captures amplification of the acceleration from ground to the point of attachment (or base) of the nonstructural component in the building. The term \( a_p \) represents further amplification of the acceleration within the component itself.

Both FEMA-450 and ASCE 7-10 also permit an alternative method to compute \( F_p \):

\[
F_p = \frac{a_i a_x I_x A_w W_p}{R_p}
\]

(2)

in which \( a_i \) = acceleration at the point of attachment of the component from modal (or response spectrum) method, and \( A_x \) = torsional amplification factor computed from

\[
A_x = \left(\frac{\delta_{\text{max}}}{1.2\delta_{\text{avg}}}\right)^2
\]

\[1 \leq A_x \leq 3\]
where $\delta_{\text{max}}$ is the maximum displacement, and $\delta_{\text{avg}}$ is the average of the displacements at the extreme points of the structure (see Figure 1).

Equation (2) essentially replaces $0.4S_{D_{50}}(1 + 2z/h)$ with $a_i$ and considers further amplification because of torsion. Finally, if fundamental period of the structure, $T_n$, and period of the flexible nonstructural component, $T_p$, is known, commentary in the latest printing of the ASCE 7-10 standard provides guidelines for estimating $a_p$ as shown in Figure 2.

![Figure 1](image1.png)

**Figure 1.** Definition of displacements used in computation of torsional amplification factor, $A_x$ (Adopted from ASCE 7-10, 2010).

![Figure 2](image2.png)

**Figure 2.** Formulation of $a_p$ as a function of structural and component periods (Adopted from ASCE 7-10, 2010).

Recent investigations by the author (Goel, 2017a, 2017b) examined seismic forces in secondary systems supported on piers, wharves, and marine oil terminals. It was recognized that piers, wharves, and marine oil terminals are generally one-level structures and thus can be idealized as SDOF systems. These investigations led to several important observations. In particular, it was found that the amplification factor, $a_p$, in the ASCE 7-10 provision for flexible secondary systems tends to be overly conservative for systems when ratio of the vibration period of the secondary and primary system is less than 0.6 or more than 1.4, and is significantly unconservative for lighter secondary systems with period ratios between 0.6 and 1.4. Based on the findings in these investigations, a simple but more appropriate formula was specified to estimate seismic forces in
ancillary components and nonbuilding structures supported on piers, wharves, and marine oil terminals:

\[ F_p = \frac{a_p A_{p} A_{W} W_p}{R_p} \]  

in which \( A \) is the spectral acceleration computed from the design earthquake spectrum at period equal to fundamental vibration period of the primary system, i.e., pier, wharf, or marine oil terminal; and \( a_p \) is the acceleration amplification factor due to flexibility of the secondary system, i.e., ancillary component or nonbuilding structure, given by:

\[
a_p = \begin{cases} 
1.0 & T_p / T_n \leq 0.1 \\
1.0 + 3 \left( T_p / T_n - 0.1 \right) & 0.1 < T_p / T_n < 0.6 \\
2.5 \left( T_p / T_n - 1.4 \right) & 1.4 < T_p / T_n < 2.0 \\
1.0 & T_p / T_n \geq 2 \\
\end{cases}
\]

where \( \mu \) is the ratio of weights of the secondary and primary systems, \( T_p \) is the vibration period of the secondary system, and \( T_n \) is the vibration period of the primary system. Use of equations (4) and (5) to compute seismic forces is not permissible for systems with \( \mu < 0.2 \) and \( 0.6 \leq T_p / T_n \leq 1.4 \) because such systems may exhibit excessive amplification of acceleration in the secondary system.

The ASCE 7-10, FEMA-356, and FEMA-440 provisions or the recommendations from Goel (2017a, 2017b) account for the potential for nonlinearity in the secondary system through component response modification factor, \( R_p \). However, these provisions and recommendations do not appear to account for effects of nonlinearity in the primary system on the input acceleration at the point of attachment of the secondary system, as apparent by use of \( 0.4 S_{DS}(1+2z/h) \) in equation (1), \( a_i \) in equation (2), or use of \( A \) in equation (4), which are all based on linear behavior of the primary system. Furthermore, the component amplification factor, \( a_p \) in ASCE 7-10, FEMA-356, and FEMA-440 documents and recommendations from Goel (2017a, 2017b) is also based on linear behavior of the primary system. These observations appears to be validated by previous work on this topic (e.g., Drake and Bachman, 1996; Miranda and Taghavi, 2005a, 2005b; Singh et al., 2006a, 2006b) which did not consider nonlinearity in the primary system. However, it is well known that most primary systems will exhibit nonlinear behavior during design-level earthquake.

Therefore, this investigation focuses on understanding effects of nonlinearity in the primary system on seismic forces in the secondary system. In particular, it examines the effects on acceleration at the point of attachment of the secondary system to the primary system, amplification of the acceleration in the secondary system due to its flexibility, and the total acceleration of the secondary system. The primary systems considered in this investigation are the piers, wharves, and marine oil terminals.
SYSTEM CONSIDERED

This investigation examines the coupled primary-secondary system model shown in Figure 3 that is appropriate for ancillary and nonbuilding systems supported on piers, wharves, and marine oil terminals. As demonstrated by Goel (2017a, 2017b), the parameters that characterize the response of linear elastic system are: (1) ratio of the mass of the secondary and primary system, \( \mu = m_2 / m_1 \); (2) ratio of the vibration periods, \( T_p / T_n \), where \( T_p = 2\pi \sqrt{m_2 / k_2} \) is the vibration period of the secondary system alone and \( T_n = 2\pi \sqrt{m_1 / k_1} \) is the vibration period of the primary system alone; (3) vibration period of the primary system alone, \( T_n = 2\pi \sqrt{m_1 / k_1} \); (4) damping in each of the two modes of vibration of the system, which is assumed to be 5% in this investigation.

![Figure 3. Coupled primary-secondary system model.](image)

The nonlinearity in the primary system is represented by \( R_y \), which is defined as the ratio of the strength, \( F_u \), required by the primary system to remain linear elastic and its yield strength, \( F_y \).
The force-deformation behavior of the primary system (Figure 4) is represented by Takeda hysteresis model (Takeda et al., 1970) found to be appropriate for piers, wharves, and marine oil terminals in MOTEMS (CSLC, 2016; Kowalsky et al., 1994). The nonlinear response history analysis of the coupled primary secondary system is implemented in the computer program OpenSees (McKenna and Fenves, 2011).

**GROUND MOTIONS CONSIDERED**

This investigation uses the SAC ground motion set consisting of 20 ground motions from 10 sites (Table 1) for 10% probability of exceedance in 50 years for a site in Los Angeles, California (Somerville et al., 1997). The previous study by the author (Goel, 2017a) also considered a second much larger set consisting of 80 ground motions from 40 sites selected from the NGA2-West database (PEER, 2013). This ground motions set was developed to be compatible with the site-specific spectrum in the MOTEMS for Level 2 (or 10% probability of exceedance in 50 years) for the Port of Long Beach, California. It was found that selection of the ground motion set does not affect the overall conclusions on seismic response behavior of coupled primary-secondary system. Therefore, this study considers only the smaller SAC ground motion set. Figure 5 shows the elastic response spectrum for individual ground motions and median for the ensemble of the selected set.

![Figure 5. Response spectrum for SAC ground motions.](image)

### Table 1. Ground motions in the SAC database.

<table>
<thead>
<tr>
<th>Eq. No.</th>
<th>Event</th>
<th>Date</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Imperial Valley</td>
<td>May 19, 1940</td>
<td>El Centro Valley Irrigation District</td>
</tr>
<tr>
<td>2</td>
<td>Imperial Valley</td>
<td>October 15, 1979</td>
<td>El Centro Array 5</td>
</tr>
<tr>
<td>3</td>
<td>Imperial Valley</td>
<td>October 15, 1979</td>
<td>El Centro Array 6</td>
</tr>
<tr>
<td>4</td>
<td>Landers</td>
<td>June 28, 1998</td>
<td>Barstow-Vineyard</td>
</tr>
<tr>
<td>5</td>
<td>Landers</td>
<td>June 28, 1998</td>
<td>Yermo Fire Station</td>
</tr>
<tr>
<td>6</td>
<td>Loma Prieta</td>
<td>October 17, 1989</td>
<td>Gilroy Sewage Plant</td>
</tr>
<tr>
<td>7</td>
<td>Northridge</td>
<td>January 17, 1994</td>
<td>Newhall, LA County Fire Station</td>
</tr>
<tr>
<td>8</td>
<td>Northridge</td>
<td>January 17, 1994</td>
<td>Rinaldi Receiving Station FF</td>
</tr>
<tr>
<td>9</td>
<td>Northridge</td>
<td>January 17, 1994</td>
<td>Sylmar, Olive View FF</td>
</tr>
<tr>
<td>10</td>
<td>North Palm Springs</td>
<td>July 8, 1986</td>
<td>DHSP</td>
</tr>
</tbody>
</table>
EFFECTS ON ACCELERATION AT THE POINT OF ATTACHMENT OF THE SECONDARY SYSTEM

This section examines the effects of nonlinearity in the primary system on the peak acceleration at the point of attachment of the secondary system to the primary system, $\ddot{u}_{io}$, normalized by the peak ground acceleration, $\ddot{u}_{go}$, in coupled primary-secondary systems. This investigation considers the following system parameters: $T_n = 0.25, 0.5, 0.75, 1, 1.25, 1.5, 1.72, \text{ and } 2 \text{ sec}$; $\mu = 0.01, 0.05, 0.1, 0.15, 0.2 \text{ and } 0.25$; $T_p/T_n$ between 0.01 and 3; and $R_y = 1, 2, 4,$ and $8$. The results for ratio, $\ddot{u}_{io}/\ddot{u}_{go}$, are available in Appendix A for various combinations of system parameters and for individual ground motion in the selected set as well as median for the entire ground motion set. As mentioned in previous investigations by the author (Goel, 2017a, 2017b), the trends may be gleaned based on the median values of $\ddot{u}_{io}/\ddot{u}_{go}$ and thus only the median results are included in this section.

This section first examines sensitivity of median values of $\ddot{u}_{io}/\ddot{u}_{go}$ with various system parameters; comprehensive set of results are available in Appendix A. For this purpose, Figures 6 to 8 examine dependence of trends in median values of $\ddot{u}_{io}/\ddot{u}_{go}$ on $\mu$. The results in these Figures are for three selected values of $T_p/T_n = 0.5, 1,$ and $2$. The values of $T_p/T_n = 0.5$ and $2$ are appropriate because $\ddot{u}_{io}/\ddot{u}_{go}$ does not vary with $T_p/T_n$ for $T_p/T_n < 0.5$ or $T_p/T_n > 1.5$; and $T_p/T_n = 1$ is captures the region where coupling between primary and secondary systems strongly influences $\ddot{u}_{io}/\ddot{u}_{go}$ (see detailed results in Appendix A).

The results in Figures 6 to 8 show that $\mu$ has very minor effect on median values of $\ddot{u}_{io}/\ddot{u}_{go}$ for linear-elastic systems, i.e., systems with $R_y = 1$, with the most prominent effects being on systems with $T_p/T_n = 1$ (Figure 7). This effect decreases with increasing value of $R_y$ with the effects of $\mu$ essentially disappearing for $R_y > 2$. These observation suggest that we can essentially consider median values of $\ddot{u}_{io}/\ddot{u}_{go}$ to be independent of $\mu$ and glean trends in median values of $\ddot{u}_{io}/\ddot{u}_{go}$ based on just a single value of $\mu$. Therefore, this investigation considers only $\mu = 0.1$ in rest of this section.
Figure 6. Variation of normalized acceleration at the point of attachment of the secondary system to the primary system. Results are for $T_p / T_n = 0.5$.

Figure 7. Variation of normalized acceleration at the point of attachment of the secondary system to the primary system. Results are for $T_p / T_n = 1$. 
Figure 8. Variation of normalized acceleration at the point of attachment of the secondary system to the primary system. Results are for $\frac{T_p}{T_n} = 2$.

Figure 9. Variation of normalized acceleration at the point of attachment of the secondary system to the primary system. Results are for $\mu = 0.1$. 
This section next examines sensitivity of median values of $\frac{\ddot{u}_{to}}{\ddot{u}_{go}}$ to $T_p/T_n$. For this purpose, Figure 9 presents results for three selected values of $T_p/T_n = 0.5$, 1, and 2. As mentioned in previously, these results are for $\mu = 0.1$ because trends in median values of $\frac{\ddot{u}_{to}}{\ddot{u}_{go}}$ are essentially independent of $\mu$. These results show that the median values of $\frac{\ddot{u}_{to}}{\ddot{u}_{go}}$ are essentially identical for $T_p/T_n = 0.5$ and 2 for all values of $R_y$. The curve for $T_p/T_n = 1$ tends to be lower than those for $T_p/T_n = 0.5$ or 2 when $R_y = 1$ but the differences decrease with increasing values of $R_y$. These observations also indicate that we can glean trends in median values of $\frac{\ddot{u}_{to}}{\ddot{u}_{go}}$ based on just a single value of $T_p/T_n = 0.5$ or 2. Therefore, this investigation considers only $T_p/T_n = 2$ in rest of this section.

Finally, this section examines the effects of primary system period, $T_n$, and level of nonlinearity in the primary system represented by the strength ratio, $R_y$, on median values of $\frac{\ddot{u}_{to}}{\ddot{u}_{go}}$. The results in Figure 10 show that the median values of $\frac{\ddot{u}_{to}}{\ddot{u}_{go}}$ decrease with increasing period of the primary system, $T_n$. This trend is consistent with the spectral shape of the selected set of ground motions; Figure 5 shows that spectral accelerations decrease with increasing period for system with periods in the range of those considered in Figure 10. Figure 10 also shows that median values of $\frac{\ddot{u}_{to}}{\ddot{u}_{go}}$ decrease with increasing value of $R_y$. In other words, increasing nonlinearity in the primary system leads to lower accelerations at the top of the primary system. This is expected as yield strength of the primary system limits accelerations that can transmit through it. Since the acceleration at the top of the primary system is the acceleration at the bottom of the primary system (see Figure 3), we can conclude that secondary system will experience decreasing accelerations at its point of attachment with increasing nonlinearity in the primary system.

It is useful to recall that the secondary systems in the types of primary systems considered in this investigation (i.e., piers, wharves, and marine oil terminals) are attached at the top of the primary system. For such systems, ASCE 7-10 Equation (1) leads to peak acceleration at the point of attachment of the secondary system to be equal to three times the peak ground acceleration. Comparison of the $\frac{\ddot{u}_{to}}{\ddot{u}_{go}}$ obtained from ASCE 7-10 formula of Equation (1) with the values from response history analysis in this investigation (Figure 10) shows that ASCE 7-10 formula is overly conservative for linear elastic system, i.e., $R_y = 1$. This level of conservatism increases with increasing level of nonlinearity in the primary system, as apparent from much lower curves in Figure 10 for $R_y > 1$ and with increasing period of the primary system.

The improved formula of Equation (4) recommended by the author in previous studies (Goel, 2017a, 2017b), which recommends use of spectral acceleration, $A$, instead of $0.4S_{PS}(1+2z/h)$, provides very good estimate of the acceleration at the point of attachment of the secondary system in piers, wharves, and marine oil terminals which remain elastic. This is apparent from essentially identical curves for $\frac{\ddot{u}_{to}}{\ddot{u}_{go}}$ and $A/\ddot{u}_{go}$ when $R_y = 1$ in Figure 10. However, $A/\ddot{u}_{go}$ also provides increasingly conservative estimate of $\frac{\ddot{u}_{to}}{\ddot{u}_{go}}$ with increasing nonlinearity in the primary system as apparent from curves for $\frac{\ddot{u}_{to}}{\ddot{u}_{go}}$ being increasingly lower with increasing $R_y$ value.
Recall that neither ASCE 7-10 formula (Equation 1) nor previous recommendation in Equation (4) by the author (Goel, 2017a, 2017b) consider the effects of nonlinearity in the primary system on the acceleration at the point of attachment of the secondary system. This investigation shows that both these approaches leads to increasingly conservative estimate of the peak acceleration at the point of attachment of the secondary system in piers, wharves, and marine oil terminals with decreasing strength of the primary system.

**Figure 10.** Comparison of 84th-percentile value of peak acceleration at the point of attachment of the secondary system to the primary system from response history analysis of coupled primary-secondary system with recommendation in ASCE 7-10 Equation (1) and \( A \) in Equation (4).

**EFFECTS ON AMPLIFICATION OF ACCELERATION IN SECONDARY SYSTEM**

This section examines the effects of nonlinearity in the primary system on amplification of acceleration in the secondary system. For this purpose, this investigation examines the ratio of peak acceleration in the secondary and primary systems, \( \frac{\ddot{u}_2}{\ddot{u}_1} \), for four different levels of system nonlinearity: \( R_y = 1, 2, 4, \) and 8. The other system parameters considered are: \( T_n = 0.25, 0.5, 0.75, 1, 1.25, 1.5, 1.72, \) and 2 sec; \( \mu = 0.01, 0.05, 0.1, 0.15, 0.2 \) and 0.25; and \( T_p/T_n \) between 0.01 and 3. The comprehensive set of results for ratio, \( \frac{\ddot{u}_2}{\ddot{u}_1} \), is available in Appendix B for various combinations of system parameters and for individual ground motion in the selected set as well as median for the entire ground motion set. As mentioned in previous investigations by the author (Goel, 2017a, 2017b), the trends in this section are also studied based on the median values of \( \frac{\ddot{u}_2}{\ddot{u}_1} \).

This section first examines sensitivity of median values of \( \frac{\ddot{u}_2}{\ddot{u}_1} \) with various system parameters; comprehensive set of results is available in Appendix B. For this purpose, results in Figures 11 to 16 first examine dependence of trends in median values of \( \frac{\ddot{u}_2}{\ddot{u}_1} \) on \( T_n \). These results show that
$\ddot{u}_{2o}/\ddot{u}_{1o}$ is mildly dependent on the vibration period of the primary system, $T_n$. However, $\ddot{u}_{2o}/\ddot{u}_{1o}$ may be considered to be independent of $T_n$ for the purpose of investigating overall trends. Therefore, this investigation uses the cases with $T_n = 1$ sec in rest of this section. This observation is consistent with that by the author in previous studies (Goel, 2017a, 2017b).

**Figure 11.** Variation of amplification of acceleration in the secondary system. Results are for $\mu = 0.01$. 
Figure 12. Variation of amplification of acceleration in the secondary system. Results are for $\mu = 0.05$.

Figure 13. Variation of amplification of acceleration in the secondary system. Results are for $\mu = 0.1$. 
Figure 14. Variation of amplification of acceleration in the secondary system. Results are for \( \mu = 0.15 \).

Figure 15. Variation of amplification of acceleration in the secondary system. Results are for \( \mu = 0.2 \).
This section next examines that sensitivity of $\ddot{u}_{2o}/\ddot{u}_{1o}$ to ratio of the mass of the secondary and primary systems, $\mu$. The results in Figure 17 show that $\ddot{u}_{2o}/\ddot{u}_{1o}$ exhibits strong dependence on $\mu$. For systems with $R_y = 1$, the dependence of $\ddot{u}_{2o}/\ddot{u}_{1o}$ on $\mu$ is strong when ratio the vibration periods of the secondary and primary systems, $T_p/T_n$, is close to one. However, this dependence spreads over a much wider range of $T_p/T_n$ values with increasing nonlinearity in the primary system. Therefore, dependence of $\ddot{u}_{2o}/\ddot{u}_{1o}$ on $\mu$ cannot be ignored.

It is useful to understand how the trends observed in this investigation compare with the previous recommendations on amplification of acceleration in the secondary system due to its flexibility. For this purpose, Figure 18 compares 84th-percentile values of the amplification of acceleration (or $\ddot{u}_{2o}/\ddot{u}_{1o}$) due to flexibility of the secondary system computed from response history analysis with the recommendation in ASCE 7-10, commentary in ASCE 7-10, and the revised proposal in Goel (2017a).

Recall that ASCE 7-10 specifies an amplification factor of 2.5 for flexible secondary systems irrespective of the period ratio, $T_p/T_n$, or the mass ratio, $\mu$. The ASCE 7-10 commentary provides a relationship based on $T_p/T_n$ (see Figure 2). The author in a previous study (Goel, 2017a) proposes an improved relationship described by Equation (5) and imposed restriction on its use for systems with $\mu < 0.2$ and $0.6 \leq T_p/T_n \leq 1.4$ because such systems may exhibit excessive amplification of acceleration in the secondary system.
Figure 17. Variation of amplification of acceleration in the secondary system. Results are for $T_n$ = 1 sec.

Previous study on coupled primary-secondary systems in which the primary system remains linear-elastic (Goel, 2017a) found that amplification factor, $a_p$, in the ASCE 7-10 document for flexible secondary systems tends to be overly conservative when $T_p/T_n < 0.6$ or $T_p/T_n > 1.4$ and significantly unconservative for lighter secondary systems, i.e., $\mu < 0.2$, when $0.6 < T_p/T_n < 1.4$. It further found that ASCE-7 commentary provides improved recommendation for $a_p$ but it tends to be slightly unconservative when $T_p/T_n < 0.6$ and significantly unconservative for lighter secondary systems, i.e., $\mu < 0.2$, when $0.6 < T_p/T_n < 1.4$. Finally, it provided Equation (5) to improve upon the estimate from ASCE 7-10. Since the results in this investigation are identical to those in Goel (2017a) for linear-elastic primary systems, these trends also visible in Figure 18 for cases with $R_y = 1$. However, as level of nonlinearity in the primary system increases, i.e., $R_y$ becomes more than 1, the trends begin to diverge from those for cases with $R_y = 1$. In particular, $a_p$ tends to be much larger for cases with $R_y > 1$ compared to cases with $R_y = 1$ when $T_p/T_n > 1$ and this difference increases with increasing value of $R_y$. Furthermore, recommendations in ASCE 7-10 commentary or Goel (2017a) may no longer be sufficient for $T_p/T_n > 1.4$ as was the case for $R_y = 1$. This occurs because the effective period of the primary system elongates due to its nonlinearity and thereby reduces the effective period ratio, $T_p/T_n$. This reduction in effective period ratio, $T_p/T_n$, has the effect of increasing amplification $a_p$, especially in the region where $T_p/T_n > 1$.

The nonlinearity in the primary system has minimal effect on $a_p$ for cases with $T_p/T_n < 0.6$. Therefore, Equation (5) proposed by Goel (2017a) is still applicable and may be used to accurately
estimate amplification of acceleration in the secondary system when the primary system is expected to be deformed beyond the linear elastic range.

As noted previously, $a_p$ tends to be much larger for cases with $R_y > 1$ compared to cases with $R_y = 1$ when $T_p/T_n > 1$ with the difference increasing with increasing value of $R_y$. However, the effect of $R_y$ is minimal when $T_p/T_n < 0.6$. Therefore, it may the best practice to design secondary system so that $T_p/T_n < 0.6$ as it will permit Equation (5) to accurately predict $a_p$ regardless of level of nonlinearity in the primary system.

**EFFECTS ON TOTAL ACCELERATION IN SECONDARY SYSTEM**

This section examines the effects of nonlinearity in the primary system on the total acceleration in the secondary system. The results considered in this section include the combination of effects on acceleration at the point of attachment of the secondary system to the primary system and the amplification of acceleration within the secondary system studied in the previous two section. This section presents results for a selected set of system parameters; comprehensive set of results for a wide-range of system parameters is available in Appendix C of this report.

**Figure 18.** Comparison of amplification factor, $a_p$, from response history analysis (84th-percentile), ASCE 7-10 recommendations and revised recommendation from Goel (2017a).
Figure 19. Comparison of normalized secondary system acceleration from response history analysis (84<sup>th</sup>-percentile), ASCE 7-10 recommendations and revised recommendation from Goel (2017a). Results are for $T_p/T_n = 0.1$.

Figures 19 to 23 present 84<sup>th</sup>-percentile values of normalized acceleration, $\bar{u}_{2o}/\bar{u}_{go}$, in the secondary system obtained from response history analysis of the coupled primary-secondary system for four different levels of nonlinearity in the primary system. Also included are the results that will be obtained by implementing recommendations in ASCE 7-10 and from that proposed by Goel (2017a). As noted previously (Goel 2017a, 2017b), 84<sup>th</sup>-percentile results from response history analysis are more appropriate when comparing to code-type recommendations. It is also useful to emphasize that the results from the ASCE 7-10 recommendations are obtained by multiplying the amplification factor, equal to 2.5 for recommendation in the ASCE 7-10 or equal to that in Figure 2 for recommendation in the ASCE 7-10 commentary, by 3.0 which represents that ratio of the acceleration at the point of attachment of the secondary system to the primary system and the base acceleration. The results from the recommendation by Goel (2017a) are obtained by multiplying the spectral acceleration, normalized by the peak ground acceleration, by Equation (5). It is also useful to recall that ASCE 7-10 recommendations are independent of the
parameters $\mu$ or $T_n$. Therefore, curves for ASCE 7-10 recommendations are flat to represent independence with $T_n$ and identical for various $\mu$ values to represent independence with $\mu$ in Figures 19 to 23.

**Figure 20.** Comparison of normalized secondary system acceleration from response history analysis (84th-percentile), ASCE 7-10 recommendations and revised recommendation from Goel (2017a). Results are for $T_p/T_n = 0.5$. 
Figure 21. Comparison of normalized secondary system acceleration from response history analysis (84th-percentage), ASCE 7-10 recommendations and revised recommendation from Goel (2017a). Results are for $T_p/T_n = 1$.

The results in Figure 19 for a case with $T_p/T_n = 0.1$ show that recommendation by Goel (2017) accurately predict the total acceleration in the secondary system when the primary system remains elastic, i.e., $R_y = 1$. However, it over-predicts the total acceleration for cases when the primary system experiences nonlinearity, i.e., $R_y > 1$, and the level of over-prediction increases with increasing nonlinearity in the primary system. This observation holds for the range of $T_n$ and $\mu$ parameters considered in this investigation. This trend is consistent with the earlier finding in Figure 10 where it was found that acceleration at the point of attachment of the secondary system to the primary system decreases with increasing level of nonlinearity in the primary system.

The results in Figure 19 also show that ASCE 7-10 recommendation significantly over-predict $\ddot{u}_{2o}/\ddot{u}_g$. Although ASCE 7-10 commentary recommendation lead to lower $\ddot{u}_{2o}/\ddot{u}_g$ values by a factor of 2.5, it still tends to significantly over-predict $\ddot{u}_{2o}/\ddot{u}_g$, especially for longer values of $T_n$. 


Figure 22. Comparison of normalized secondary system acceleration from response history analysis (84th-percentile), ASCE 7-10 recommendations and revised recommendation from Goel (2017a). Results are for $T_p/T_n = 2$.

The results in Figure 20 for a case with $T_p/T_n = 0.5$ show that recommendation by Goel (2017) over-predicts the total acceleration in the secondary system for all levels of nonlinearity in the primary system. The level of over-prediction increases with increasing nonlinearity in the primary system and period of the primary system, $T_n$. The ASCE 7-10 commentary recommendation significantly over-predicts $\ddot{u}_{2o}/\ddot{u}_{go}$ for most cases but the over-prediction is not as large as for the ASCE 7-10 recommendation.

The results in Figure 21 for a case with $T_p/T_n = 1$ show that recommendations in the ASCE 7-10 and ASCE 7-10 commentary, both of which lead to same results, generally over-predict $\ddot{u}_{2o}/\ddot{u}_{go}$ with the level of over-prediction increasing with increasing nonlinearity in the primary system and period of the primary system, $T_n$. The exception occurs for the case with lighter secondary systems, i.e., $\mu < 0.1$, shorter periods of primary system, and when the primary system remains
elastic. In such cases, the ASCE 7-10 recommendations may under-predict $\ddot{u}_{so}/\ddot{u}_{go}$. The recommendation by Goel (2017) follow similar trends but the level of over-prediction tends to reduce with increasing period of the primary system, $T_n$.

**Figure 23.** Comparison of normalized secondary system acceleration from response history analysis (84th-percentile), ASCE 7-10 recommendations and revised recommendation from Goel (2017a). Results are for $T_p/T_n = 3$.

The results in Figure 21 also show that accelerations in secondary systems may be excessive especially in systems with $T_n < 0.5$ sec, $R_y < 4$ and $\mu < 0.2$. The values of $\ddot{u}_{so}/\ddot{u}_{go}$ may approach or exceed 8 implying that the acceleration in the secondary system may be eight times or more than the peak ground acceleration. This occurs because of strong coupling between primary and secondary systems when periods of the two systems are close and the secondary system weighs less than 20% of the primary system. Therefore, it is recommended to avoid designing coupled primary-secondary systems with $0.6 < T_p/T_n < 1.4$ and $\mu < 0.2$. This recommendation is consistent with Equation (5) which is not applicable for these ranges of system parameters.
The results in Figures 22 and 23 for a cases with $T_p/T_n = 2$ and 3 show that recommendation by Goel (2017) reasonably predicts $\bar{u}_{2o}/\bar{u}_{go}$ for systems $R_y = 1$ and slightly over-predicts $\bar{u}_{2o}/\bar{u}_{go}$ for systems $R_y > 1$. The recommendation in ASCE 7-10 commentary over-predicts $\bar{u}_{2o}/\bar{u}_{go}$ with the level of over-prediction increasing with increasing nonlinearity in the primary system and period of the primary system, $T_n$. The ASCE 7-10 recommendation significantly over-predicts $\bar{u}_{2o}/\bar{u}_{go}$.

The results presented so far in this section lead to the following observations for acceleration in the secondary system of the coupled primary-secondary systems in piers, wharves, and marine oil terminals:

- The recommendation in the ASCE 7-10 document for flexible secondary system generally lead to significant over-prediction of acceleration in the secondary system. The level of over-prediction increases with increasing level of nonlinearity in the primary system and increasing period of the primary system.
- The recommendation in the commentary of the ASCE 7-10 document also leads to over-prediction, although not as large as that from the recommendation in the main body of the ASCE 7-10 document, of acceleration in the secondary system. The level of over-prediction increases with increasing level of nonlinearity in the primary system and increasing period of the primary system.
- The recommendation by Goel (2017a) provide a reasonably good estimate of acceleration in the secondary system over the entire range of vibration period of the primary system when the primary system remains in the linear elastic. However, this recommendation provides slight over-prediction when the primary system deforms beyond the linear elastic range.
- The recommendation in ASCE 7-10 document and by Goel (2017a) were developed bases on studies of linear-elastic systems. The current investigation, which considers nonlinearity in the primary system, indicate that recommendations based on linear-elastic systems lead to conservative estimates of accelerations in secondary system even when the primary system in the coupled primary-secondary system is deformed beyond the linear-elastic range.
- The secondary system in coupled primary-secondary systems with $0.6 < T_p/T_n < 1.4$ and $\mu < 0.2$ may experience excessive accelerations that may equal to exceed eight times the peak ground accelerations due to strong coupling between primary and secondary systems. Therefore, it is best to avoid designing secondary systems in these ranges of parameters.

**CONCLUSIONS AND RECOMMENDATIONS**

This investigation on the effects of nonlinearity in the primary system of the coupled primary-secondary systems, found in piers, wharves, and marine oil terminals, on (1) acceleration at the point of attachment of the secondary system to the primary system, (2) amplification of the acceleration in the secondary system due to its flexibility, and (3) the total acceleration of the secondary system has led to the following conclusions:

- The acceleration at the point of attachment of the secondary system to the primary system decreases with increasing level of nonlinearity in the primary system.
- The recommendation by Goel (2017a) provides very good estimate of the acceleration at the point of attachment of the secondary system to the primary system in coupled primary-secondary systems where the primary systems remains linear elastic. However, it provides
increasingly conservative estimate of this acceleration with increasing nonlinearity in the primary system.

- The recommendations in ASCE 7-10 significantly over-predict accelerations at the point of attachment of the secondary system to the primary system in coupled primary-secondary systems. The level of over-prediction increases with increasing level of nonlinearity in the primary system and period of the primary system.

- The trends in amplification of acceleration in the secondary system due to its flexibility based on studies of linear-elastic system no longer apply when the primary system is deformed beyond the linear elastic range. In particular, amplification of acceleration tends to be much larger for the latter when $T_p/T_n > 1$ and this difference increases with increasing level of nonlinearity in the primary system.

- The nonlinearity in the primary system has minimal effect on amplification of acceleration in the secondary system when $T_p/T_n < 0.6$. Therefore, Equation (5) proposed by Goel (2017a) may be used to accurately estimate amplification of acceleration in the secondary system when the primary system is expected to be deformed beyond the linear elastic range.

- The recommendation in the ASCE 7-10 document for flexible secondary system generally lead to significant over-prediction of acceleration in the secondary system. The level of over-prediction increases with increasing level of nonlinearity in the primary system and increasing period of the primary system.

- The recommendation in the commentary of the ASCE 7-10 document also leads to over-prediction, although not as large as that from the recommendation in the main body of the ASCE 7-10 document, of acceleration in the secondary system. The level of over-prediction increases with increasing level of nonlinearity in the primary system and increasing period of the primary system.

- The recommendation by Goel (2017a) provide a reasonably good estimate of acceleration in the secondary system over the entire range of vibration period of the primary system when the primary system remains in the linear elastic. However, this recommendation provides slight over-prediction when the primary system deforms beyond the linear elastic range.

- The recommendation in the ASCE 7-10 document and by Goel (2017a) were developed bases on studies of linear-elastic systems. The current investigation, which considers nonlinearity in the primary system, indicate that recommendations based on linear-elastic systems lead to conservative estimates of accelerations in secondary system even when the primary system in the coupled primary-secondary system is deformed beyond the linear-elastic range.

Based on findings in this investigation, it is recommended not to design coupled primary-secondary systems with $0.6 < T_p/T_n < 1.4$ and $\mu < 0.2$ because secondary systems in such cases may experience excessive accelerations that may equal to or exceed eight times the peak ground accelerations due to strong coupling between primary and secondary systems.

REFERENCES

American Society of Civil Engineers (ASCE), 2014. Seismic Design of Piers and Wharves, ASCE Standard ASCE/COPRI 61-14, Reston, VA.


Port of Long Beach (POLB), 2012. Port of Long Beach Wharf Design Criteria, Version 3.0, Long Beach CA.

Port of Los Angeles (POLA), 2010. The Port of Los Angeles Code for Seismic Design, Upgrade and Repair of Container Wharves, City of Los Angeles Harbor Department, Los Angeles, CA.


Figure A1: Peak accelerations in the primary system normalized by the peak ground acceleration. Results are for $\mu = 0.01$ and $R_y = 1$. 
Figure A2: Peak accelerations in the primary system normalized by the peak ground acceleration. Results are for $\mu = 0.05$ and $R_y = 1$. 
Figure A3: Peak accelerations in the primary system normalized by the peak ground acceleration. Results are for $\mu = 0.1$ and $R_y = 1$. 
Figure A4: Peak accelerations in the primary system normalized by the peak ground acceleration. Results are for $\mu = 0.15$ and $R_y = 1$. 

\[ \frac{A_p}{A_{go}} \]
Figure A5: Peak accelerations in the primary system normalized by the peak ground acceleration. Results are for $\mu = 0.2$ and $R_y = 1$. 
Figure A6: Peak accelerations in the primary system normalized by the peak ground acceleration. Results are for $\mu = 0.25$ and $R_y = 1$. 
Figure A7: Peak accelerations in the primary system normalized by the peak ground acceleration. Results are for $\mu = 0.01$ and $R_y = 2$. 
Figure A8: Peak accelerations in the primary system normalized by the peak ground acceleration. Results are for $\mu = 0.05$ and $R_y = 2$. 
Figure A9: Peak accelerations in the primary system normalized by the peak ground acceleration. Results are for $\mu = 0.1$ and $R_y = 2$. 
Figure A10: Peak accelerations in the primary system normalized by the peak ground acceleration. Results are for $\mu = 0.15$ and $R_y = 2$. 
Figure A11: Peak accelerations in the primary system normalized by the peak ground acceleration. Results are for $\mu = 0.2$ and $R_y = 2$. 
Figure A12: Peak accelerations in the primary system normalized by the peak ground acceleration. Results are for $\mu = 0.25$ and $R = 2$. 
Figure A13: Peak accelerations in the primary system normalized by the peak ground acceleration. Results are for $\mu = 0.01$ and $R_y = 4$. 
Figure A14: Peak accelerations in the primary system normalized by the peak ground acceleration. Results are for $\mu = 0.05$ and $R_y = 4$. 
Figure A15: Peak accelerations in the primary system normalized by the peak ground acceleration. Results are for $\mu = 0.1$ and $R_y = 4$. 
Figure A16: Peak accelerations in the primary system normalized by the peak ground acceleration. Results are for $\mu = 0.15$ and $R_y = 4$. 
Figure A17: Peak accelerations in the primary system normalized by the peak ground acceleration. Results are for \( \mu = 0.2 \) and \( R_y = 4 \).
Figure A18: Peak accelerations in the primary system normalized by the peak ground acceleration. Results are for $\mu = 0.25$ and $R_y = 4$. 
Figure A19: Peak accelerations in the primary system normalized by the peak ground acceleration. Results are for $\mu = 0.01$ and $R_y = 8$. 
Figure A20: Peak accelerations in the primary system normalized by the peak ground acceleration. Results are for $\mu = 0.05$ and $R_y = 8$. 
Figure A21: Peak accelerations in the primary system normalized by the peak ground acceleration. Results are for $\mu = 0.1$ and $R_y = 8$. 
Figure A22: Peak accelerations in the primary system normalized by the peak ground acceleration. Results are for $\mu = 0.15$ and $R_y = 8$. 

\[ \frac{a_{\text{ro}}}{a_{\text{go}}} \]
Figure A23: Peak accelerations in the primary system normalized by the peak ground acceleration. Results are for $\mu = 0.2$ and $R_y = 8$. 
Figure A24: Peak accelerations in the primary system normalized by the peak ground acceleration. Results are for $\mu = 0.25$ and $R_y = 8$. 
Figure B1: Amplification of acceleration due to flexibility of the secondary system. Results are for $\mu = 0.01$ and $R_y = 1$. 
Figure B2: Amplification of acceleration due to flexibility of the secondary system. Results are for \( \mu = 0.05 \) and \( R_y = 1 \).
Figure B3: Amplification of acceleration due to flexibility of the secondary system. Results are for $\mu = 0.1$ and $R_y = 1$. 
Figure B4: Amplification of acceleration due to flexibility of the secondary system. Results are for $\mu = 0.15$ and $R_y = 1$. 
Figure B5: Amplification of acceleration due to flexibility of the secondary system. Results are for $\mu = 0.2$ and $R_y = 1$. 
Figure B6: Amplification of acceleration due to flexibility of the secondary system. Results are for $\mu = 0.25$ and $R_y = 1$. 
Figure B7: Amplification of acceleration due to flexibility of the secondary system. Results are for $\mu = 0.01$ and $R_y = 2$. 
Figure B8: Amplification of acceleration due to flexibility of the secondary system. Results are for $\mu = 0.05$ and $R_y = 2$. 
Figure B9: Amplification of acceleration due to flexibility of the secondary system. Results are for $\mu = 0.1$ and $R_y = 2$. 
Figure B10: Amplification of acceleration due to flexibility of the secondary system. Results are for $\mu = 0.15$ and $R_y = 2$. 
Figure B11: Amplification of acceleration due to flexibility of the secondary system. Results are for $\mu = 0.2$ and $R_y = 2$. 
Figure B12: Amplification of acceleration due to flexibility of the secondary system. Results are for \( \mu = 0.25 \) and \( R_y = 2 \).
Figure B13: Amplification of acceleration due to flexibility of the secondary system. Results are for $\mu = 0.01$ and $R_y = 4$. 
Figure B14: Amplification of acceleration due to flexibility of the secondary system. Results are for $\mu = 0.05$ and $R_y = 4$. 
Figure B15: Amplification of acceleration due to flexibility of the secondary system. Results are for $\mu = 0.1$ and $R_y = 4$. 
Figure B16: Amplification of acceleration due to flexibility of the secondary system. Results are for $\mu = 0.15$ and $R_y = 4$. 
Figure B17: Amplification of acceleration due to flexibility of the secondary system. Results are for $\mu = 0.2$ and $R_y = 4$. 
Figure B18: Amplification of acceleration due to flexibility of the secondary system. Results are for $\mu = 0.25$ and $R_y = 4$. 
Figure B19: Amplification of acceleration due to flexibility of the secondary system. Results are for $\mu = 0.01$ and $R_y = 8$. 
Figure B20: Amplification of acceleration due to flexibility of the secondary system. Results are for $\mu = 0.05$ and $R_y = 8$. 
Figure B21: Amplification of acceleration due to flexibility of the secondary system. Results are for $\mu = 0.1$ and $R_\gamma = 8$. 
Figure B22: Amplification of acceleration due to flexibility of the secondary system. Results are for $\mu = 0.15$ and $R_y = 8$. 
Figure B23: Amplification of acceleration due to flexibility of the secondary system. Results are for $\mu = 0.2$ and $R_y = 8$. 
Figure B24: Amplification of acceleration due to flexibility of the secondary system. Results are for $\mu = 0.25$ and $R_y = 8$. 
Figure B25: Comparison of median values of amplification of acceleration due to flexibility of the secondary system for four values of $R_y = 1, 2, 4, \text{ and } 8$. Results are for $\mu = 0.01$. 
Figure B26: Comparison of median values of amplification of acceleration due to flexibility of the secondary system for four values of $R_y = 1, 2, 4, \text{ and } 8$. Results are for $\mu = 0.05$. 
Figure B27: Comparison of median values of amplification of acceleration due to flexibility of the secondary system for four values of $R_y = 1, 2, 4,$ and $8$. Results are for $\mu = 0.1$. 
Figure B28: Comparison of median values of amplification of acceleration due to flexibility of the secondary system for four values of $R_y = 1$, 2, 4, and 8. Results are for $\mu = 0.15$. 
Figure B29: Comparison of median values of amplification of acceleration due to flexibility of the secondary system for four values of $R_y = 1, 2, 4,$ and $8$. Results are for $\mu = 0.2$. 
Figure B30: Comparison of median values of amplification of acceleration due to flexibility of the secondary system for four values of $R_y = 1, 2, 4, \text{ and } 8$. Results are for $\mu = 0.25$. 
Figure C1. Comparison of normalized secondary system acceleration from response history analysis, ASCE 7-10 recommendations, and revised recommendation from Goel (2017a). Results are for $T_n = 0.25$ sec.
Figure C2. Comparison of normalized secondary system acceleration from response history analysis, ASCE 7-10 recommendations, and revised recommendation from Goel (2017a). Results are for $T_n = 0.5$ sec.
Figure C3. Comparison of normalized secondary system acceleration from response history analysis, ASCE 7-10 recommendations, and revised recommendation from Goel (2017a). Results are for $T_n = 0.75$ sec.
Figure C4. Comparison of normalized secondary system acceleration from response history analysis, ASCE 7-10 recommendations, and revised recommendation from Goel (2017a). Results are for $T_n = 1$ sec.
Figure C5. Comparison of normalized secondary system acceleration from response history analysis, ASCE 7-10 recommendations, and revised recommendation from Goel (2017a). Results are for $T_n = 1.25$ sec.
Figure C6. Comparison of normalized secondary system acceleration from response history analysis, ASCE 7-10 recommendations, and revised recommendation from Goel (2017a). Results are for $T_n = 1.5$ sec.
Figure C7. Comparison of normalized secondary system acceleration from response history analysis, ASCE 7-10 recommendations, and revised recommendation from Goel (2017a). Results are for $T_n = 1.75$ sec.
Figure C8. Comparison of normalized secondary system acceleration from response history analysis, ASCE 7-10 recommendations, and revised recommendation from Goel (2017a). Results are for $T_n = 2$ sec.
Figure C9. Comparison of normalized secondary system acceleration from response history analysis, ASCE 7-10 recommendations, and revised recommendation from Goel (2017a). Results are for $T_p / T_n = 0.01$. 
Figure C10. Comparison of normalized secondary system acceleration from response history analysis, ASCE 7-10 recommendations, and revised recommendation from Goel (2017a). Results are for $T_p/T_n = 0.05$. 
Figure C11. Comparison of normalized secondary system acceleration from response history analysis, ASCE 7-10 recommendations, and revised recommendation from Goel (2017a). Results are for $T_p/T_n = 0.1$. 
Figure C12. Comparison of normalized secondary system acceleration from response history analysis, ASCE 7-10 recommendations, and revised recommendation from Goel (2017a). Results are for $T_p/T_n = 0.2$. 
Figure C13. Comparison of normalized secondary system acceleration from response history analysis, ASCE 7-10 recommendations, and revised recommendation from Goel (2017a). Results are for $T_p/T_n = 0.3$. 

Figure C14. Comparison of normalized secondary system acceleration from response history analysis, ASCE 7-10 recommendations, and revised recommendation from Goel (2017a). Results are for $T_p/T_n = 0.4$. 
Figure C15. Comparison of normalized secondary system acceleration from response history analysis, ASCE 7-10 recommendations, and revised recommendation from Goel (2017a). Results are for $T_p/T_n = 0.5$. 
Figure C16. Comparison of normalized secondary system acceleration from response history analysis, ASCE 7-10 recommendations, and revised recommendation from Goel (2017a). Results are for $T_p/T_n = 0.6$. 
Figure C17. Comparison of normalized secondary system acceleration from response history analysis, ASCE 7-10 recommendations, and revised recommendation from Goel (2017a). Results are for $T_p/T_n = 0.7$. 
Figure C18. Comparison of normalized secondary system acceleration from response history analysis, ASCE 7-10 recommendations, and revised recommendation from Goel (2017a). Results are for $T_p/T_n = 0.8$. 
Figure C19. Comparison of normalized secondary system acceleration from response history analysis, ASCE 7-10 recommendations, and revised recommendation from Goel (2017a). Results are for $T_p/T_n = 0.9$. 
Figure C20. Comparison of normalized secondary system acceleration from response history analysis, ASCE 7-10 recommendations, and revised recommendation from Goel (2017a). Results are for $\frac{T_p}{T_n} = 1$. 
Figure C21. Comparison of normalized secondary system acceleration from response history analysis, ASCE 7-10 recommendations, and revised recommendation from Goel (2017a). Results are for $T_p/T_n = 1.1$. 
Figure C22. Comparison of normalized secondary system acceleration from response history analysis, ASCE 7-10 recommendations, and revised recommendation from Goel (2017a). Results are for $T_p / T_n = 1.2$. 
Figure C23. Comparison of normalized secondary system acceleration from response history analysis, ASCE 7-10 recommendations, and revised recommendation from Goel (2017a). Results are for $T_p/T_n = 2.3$. 
Figure C24. Comparison of normalized secondary system acceleration from response history analysis, ASCE 7-10 recommendations, and revised recommendation from Goel (2017a). Results are for $T_p / T_n = 1.4$. 

$$\frac{a_{20}}{a_{wp}}$$
Figure C25. Comparison of normalized secondary system acceleration from response history analysis, ASCE 7-10 recommendations, and revised recommendation from Goel (2017a). Results are for $T_p/T_n = 1.5$. 
Figure C26. Comparison of normalized secondary system acceleration from response history analysis, ASCE 7-10 recommendations, and revised recommendation from Goel (2017a). Results are for $T_p/T_n = 1.6$. 
Figure C27. Comparison of normalized secondary system acceleration from response history analysis, ASCE 7-10 recommendations, and revised recommendation from Goel (2017a). Results are for $T_p/T_n = 1.7$. 

\[ \frac{a_{2p}}{a_{gp}} \]
Figure C28. Comparison of normalized secondary system acceleration from response history analysis, ASCE 7-10 recommendations, and revised recommendation from Goel (2017a). Results are for $T_p/T_n = 1.8$. 
Figure C29. Comparison of normalized secondary system acceleration from response history analysis, ASCE 7-10 recommendations, and revised recommendation from Goel (2017a). Results are for $T_p/T_n = 1.9$. 
Figure C30. Comparison of normalized secondary system acceleration from response history analysis, ASCE 7-10 recommendations, and revised recommendation from Goel (2017a). Results are for $T_p/T_n = 2$. 
Figure C31. Comparison of normalized secondary system acceleration from response history analysis, ASCE 7-10 recommendations, and revised recommendation from Goel (2017a). Results are for \( T_p/T_n = 2.25 \).
Figure C32. Comparison of normalized secondary system acceleration from response history analysis, ASCE 7-10 recommendations, and revised recommendation from Goel (2017a). Results are for $T_p/T_n = 2.5$. 
Figure C33. Comparison of normalized secondary system acceleration from response history analysis, ASCE 7-10 recommendations, and revised recommendation from Goel (2017a). Results are for $T_p / T_n = 2.75$. 

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Figure C34. Comparison of normalized secondary system acceleration from response history analysis, ASCE 7-10 recommendations, and revised recommendation from Goel (2017a). Results are for $T_p/T_n = 3$. 